The Science of Flexible Retirement Choices: Switching Retirement Savings into an Annuity

by[1]

Andrew Clare*,

James Seaton*,

Peter N. Smith†

and

Stephen Thomas*

*Bayes Business School, City, University of London

†University of York and Centre for Applied Macroeconomic Analysis (CAMA).

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Abstract

In this paper we consider the choice that retirees might make between drawing down from their pension pot and the purchase of an annuity. A key finding of our research that in a world of 'loss aversion', across a very wide range of assumptions, there is almost always a 'crossover point' during retirement at which moving out of drawdown into an annuity can be the optimal strategy. This suggests that the pensions industry should investigate the construction of a hybrid, '*flex-first, fix-later*' pension product. We show that a 'hybrid' approach can produce much higher levels of happiness, especially at older ages, than staying wholly in drawdown or from buying an annuity at the point of retirement.

Key words:
Sequence Risk; Longevity Risk, Withdrawal Risk, Delayed Annuities, Adaptive Withdrawals, Residual Sum

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1. Introduction

Retirement annuity sales have fallen dramatically over the last 2 decades in many countries such as the UK, especially so when prompted by regulatory changes such as the 'pension freedoms' which occurred in that country in 2015. According to figures from the Association of British Insurers (ABI), in 2012 over 90% of DC assets which were accessed were used to buy an annuity. The number of annuities sold by ABU members that year was 412,000 and this had fallen to 49,000 by 2020. This phenomenon has been accompanied by a rich vein of academic research, often based on behavioural finance, exploring the 'annuity puzzle', in which the theoretical 'superiority' of the annuity stands in sharp contrast to the empirical reality of sharply declining sales (see Chen et al, 2022, Inkmann et al, 2011, Horneff et al, 2008, O'Dea and Sturrock, 2023, Michaud and St. Amour, 2023).

But as society struggles to find the 'best' way to fund retirement for long-lived individuals in a low interest rate environment there is a growing sense that annuities do have an important role to play and have not suddenly become a 'bad' product: having the certainty of an income for life, possibly with protection against inflation and a payment for a surviving spouse, is in many ways an attractive way of using pension savings. *What we do have to consider is that annuities may have to be used more flexibly than was considered in an earlier era.* In particular, as a product they may be bought later in life (say, aged 80) to follow income drawdown from a pot of wealth (see Clare et al, 2021) or indeed at the same time as drawdown commences (say, aged 65) in the form of a deferred annuity (see Chen et al, 2022). These options may be termed 'hybrid' or 'flexible' retirement solutions (Boyle and Webb, 2022).

If we are considering the annuity as a useful retirement solution, albeit at different possible stages of retirement for different people, then we have to consider whether the attractiveness of an annuity changes as you go through retirement. This opens up the possibility of an 'optimal' time to switch from a pot of wealth providing drawdown possibilities into an annuity and this is the issue we explore here: is there always such an optimal switching time and if so what determines it?

As described by Webb and Boyle (2021) a retirement product which '*might not look good value to someone coming up to retirement, could look very different to someone ten or twenty years into retirement*'. Why is this? Quite simply mortality risk increases as an investor gets older

and hence the relative uncertainty of how long an individual may live with age. This has the unfortunate, though rarely discussed, implication that managing a drawdown pot to make sure that you don't run out of money (on the one hand) or end up living excessively frugally to avoid the risk of running out (on the other hand) becomes steadily *more* difficult as you get older. Hence purchasing an annuity to remove mortality risk might start to represent better value for money as you get older through retirement.

Boyle and Webb (2022) give the example of a man currently aged 60 who, on average, can expect to live for a further 26 and ask: what are the chances of him living twice the expected period, i.e., to the age of 112? The chances are effectively zero. They then consider a man aged 80 who on average can expect to live for nearly nine more years. What are his chances of living for twice the expected period, i.e. to age 98? Answer: about 6%. So even if you have a good idea how long you might live on average, the chance of your individual outcome being significantly different from that average rises as you get older! Hence, applying this fact to the challenge of managing wealth in drawdown, the lesson is clear: as you get older, the risk increases that your individual outcome will differ significantly from the average and buying an annuity is the best way for you to remove this longevity risk.

In our analysis, the calculation takes place in the form of comparing expected outcomes at each age between those who buy an annuity and those who remain in drawdown. In each case we come up with a measure of the 'economic utility' or 'satisfaction' which a saver would derive from each course of action. So, given that the relative uncertainty about how long you will live rises with age, the attractiveness of an annuity will also rise with age. Whether the attractiveness increases sufficiently to tempt people into buying an annuity later in retirement requires a model and assumptions and we now explore how this works. Perhaps not surprisingly, Boyle and Webb (2022) find that at age sixty this individual can expect to be happier in drawdown than by buying an annuity. However, as they get older, the relative attraction of drawdown decreases as relative life expectancy becomes more uncertain and at some point during retirement there is a crossover point where the individual would expect to be happier if they switched to an annuity. In the Boyle and Webb (2022) parameterisation this turns out to be 67. And if the comparison is with an index-linked annuity then the individual should probably only think about switching to such an annuity beyond the age of 75! Further, introducing a state pension also pushes out the date of crossover.

In this paper we explore the optimal time for a retiree to switch from a pot of wealth invested in equities and bonds into the purchase of an annuity. Unsurprisingly, this will depend on the risk appetite of the retiree, the annuity rate and the expected return and volatility of the risky assets. Addressing this the question requires some way of comparing different outcomes using a metric along the lines of investor utility or loss aversion metrics and we follow Boyle and Webb (2022) in adopting the latter approach. Within the confines of our parameterisation, this allows us to address some very important practical questions:

- Is there a point at which it is advantageous to switch away from a drawdown portfolio and purchase an annuity?
- Does this optimal switching point change dramatically as the macroeconomic context changes; in particular, has the point at which to switch changed substantially in recent times as annuity rates have adjusted to the shift in interest rates and inflation?
- How do individual risk appetites (i.e., loss aversion) play a role in choosing when to move away from drawdown? Are such changes substantial in delaying the desire to switch?
- How is the switching decision affected by the nature of the drawdown portfolio? We show for UK data that the switching decision is fairly insensitive to the asset allocation between equities and bonds but one loses much of the benefit of drawdown if the equity allocation falls below half. We suggest that this would seem to be a good reason to stay invested in stocks late in life. Much of the literature shows little appetite for exploring the impact of alternative investing portfolios on these decisions: e.g., Chen and Munnell (2021) focuses on glidepath portfolios switching between equities and cash driven by Target Date commercial products.
- For reasonable levels of loss aversion, we find that replacing the conventional equity portion of the drawdown portfolio with trend following equity (i.e., 'smoothed' returns) delays the switch to holding an annuity (often by around three years) and adds considerable value for the retiree. This would seem to be a potentially important policy insight to enhance the income of retirees.
- We further suggest that with an increased availability of deferred annuity products, the drawdown phase could be made more attractive for less risk tolerant retirees.

We begin with a description of our research methodology before looking at the impact of annuity rate variation, risk appetite and asset price volatility on the decision to annuitize. We

then investigate the impact of changing asset allocation between bonds and equites on the timing and find two powerful and indeed potentially surprising conclusions:

(i) The timing of buying the annuity is relatively unaffected by asset allocation in drawdown once one has over 50% equities in the portfolio.

(ii) The 'smoothing' of returns (here by a trend following filter) has a powerful impact on delaying the switch to annuitisation, allowing the retiree extra years of enhanced consumption.

2. Methodology

We start by assuming that a retiree aged 65 has the choice between either running a drawdown portfolio with their accumulated defined contribution pension pot, or otherwise accumulated savings, or purchasing an annuity. Each year they will reassess this choice based upon which they expect to offer the higher cash flow subject to their risk tolerance. This will continue until they have either purchased the annuity or reached the age of eighty. It is assumed that if they reach aged 80 and are still in drawdown then they will buy the annuity regardless, in order to hedge longevity risk on the basis that such a product may not be widely available to them on the market after this point.

We envisage that the conversation between a retiree and an adviser regarding the desired level of annual income will come from well-informed discussions based on easily available and widely researched sources such as the UK-based Retirement Living Standards Project, (https://www.retirementlivingstandards.org.uk/).We take these quantities as given in our analysis and proceed to organise drawdown conditional on a choice of required income.

(i) Introducing the Conventional Annuity as part of the Drawdown Decision

Table 1 shows the prevailing annuity rates on 27th October 2022 for a single life, RPI linked, 5-Year guarantee annuity[2]. We assume that this will be the only choice if the retiree wishes to purchase a product. In order to facilitate making decisions within the gaps of the five-year data spacing, and up to an age of 80, we have plotted the values in Figure 1 and interpolated and

[2] Retrieved from www.hl.co.uk/retirement/annuities/best-buy-rates on 28th October 2022

extrapolated using the equation shown. All annuity amounts going forward will be expressed in real terms per £100,000.

(ii) The Drawdown Portfolio(s)

The data used in this paper runs from 1971 to 2021 inclusive, with all observations being monthly. Equity indices throughout are gross values from MSCI UK, Gilts returns are from the FTSE Actuaries All Stocks Index from 1976 onwards and 20-year gilts prior to this date. Where cash rates are referred to, these are 3-month UK Treasury Bills. Throughout the paper all returns quoted are in real terms and are relative to UK Retail Price Index inflation. All values are in British Pounds. For the purposes of the drawdown portfolio, it will be assumed that this will be a combination of equities and bonds *initially set at 75% and 25%* respectively. For this period stocks have a compound annual real return of 4.9% with an annualised real volatility of 18.4% compared to bonds with a return of 2.3% and a volatility of 9.8%. Throughout the paper, wherever Monte Carlo simulations are used these will be monthly return draws with replacement from our data series described earlier. All results reported are for 50,000 simulations.

(iii) The Practical Choice: Comparing the Drawdown and the Annuity

At 65 one could have fifteen years before purchasing an annuity. From Table 1 we can see that the annuity rate if it is bought immediately is £4,290, however, at age 80 (using the equation from Figure 1) the rate has increased to £7,773. Therefore, to achieve the same income at the older age one would only require a pot of (4,290 ÷ 7,773) × £100,000 = £55,191. If we assume that the best expectation of future annuity curves is the current one, then a retiree looking to maximise cash flow would say: "*Can the investment returns from the drawdown portfolio over a fifteen-year period plus depleting the capital to £55,191 (per £100,000 starting capital) give a greater income than purchasing the annuity immediately*?" From eighty years old onwards they would have exactly the same income regardless of the method chosen.

In order to facilitate this comparison, we use Monte Carlo to generate fifteen years of investment returns of the drawdown portfolio. We then calculate the Perfect Withdrawal Rate (PWR) for a pot decumulating from £100,000 to £55,191 over the time period described. The PWR multiplied by the starting pot gives us a constant annual income (Perfect Withdrawal Amount, PWA) which can be directly compared with the annuity rate (for more details on the

formula and examples of this calculation see Suarez et al, 2015 and Clare et al, 2016). Volatility has not suddenly been eradicated from the investment portfolio though, instead it shows up when we run our multitude of simulations. Sometimes there will be an abundance of higher-than-average returns and other times the opposite will be true.

Figure 2 shows the distribution of PWAs from the Monte Carlo analysis. The average of all the PWAs is £6,421 which is considerably higher than the comparable annuity rate of £4,290. On this simple basis it is clearly a 'win' for the drawdown portfolio. There is a fairly large range of outcomes though, from the simulations. Approximately 95% of the results lie between £2,000 and £14,000 with 24.5% returning less than the benchmark annuity rate. If one is risk-indifferent then this is inconsequential, however, that is unlikely to be the case for most retirees. Webb and Boyle (2021) address this by assuming that any excess amounts above the baseline annuity rate are penalised by being divided by a factor of two, i.e., retirees only derive half the utility of relative gains compared to relative losses. We adopt this same approach and thus, any PWA above £4,290 has the excess halved, e.g., an unrisked PWA of £8,290 would have a risk-adjusted value of £6,290. It is assumed that this risk-adjustment also takes into account the utility that a retiree would have from being able to make a bequest should they die before being able to purchase an annuity and also the flexibility of being able to adjust spending whilst remaining in drawdown if such a need arises. These are positive benefits as opposed to the negative of volatility. Figure 2 also shows the distribution of PWAs after this adjustment[3]. The average of this group is £5,160 which is still above the annuity rate and so drawdown remains the more favourable approach. Going forward we will assume this risk aversion factor of two is the standard.

These preferences can be seen as a simplified version of the approach of Barberis and Huang (2009) and Ebner et al (2022). Ebner et al (2022) include an additional stock market loss framing component in an otherwise standard Epstein-Zin specification of preferences in a one-parameter simplification of the original Barberis and Huang (2009) specification. Here, the utility gained from increased income above the annuity baseline can be thought of as being modelled in a constant relative risk aversion form with coefficient of relative risk aversion of

[3] There is an element of rounding and smoothing in order to produce these distributions but the general principle remains intact.

2, whilst the narrow-framing component with embedded loss aversion applies to income below that level. This is also applied to bequests.

Each year one reassesses the relative merits of remaining in drawdown or purchasing the annuity. So, at age 66, from Figure 1, the annuity rate has increased to £4,441. The residual balance now required at eighty in order to purchase the same level of income is thus, (4,441 ÷ 7,773) × £100,000 = £57,134. We run our Monte Carlo simulations again using this new end balance, adjust for risk aversion and then calculate the average PWA which is now £5,265. Again, this is a 'win' for drawdown so one would stay with the investment portfolio and delay the annuity purchase. Figure 3 shows how this plays out with increasing age. We find that the point of indifference between the propositions is around age seventy-three with the annuity being more favourable thereafter. This would be the time to make the purchase.

3. Annuity Rate Variation

Just like interest rates, annuity income can vary quite considerably. Cannon and Tonks (2004, 2005, 2010) provide plenty of background to the history of annuity rates in the UK. At the time of writing the market is offering considerably more income than it was just two years earlier. Table 2 provides comparable rates to Table 1 from February 2020. At age 65 the real income available was just £2,922 or some 32% less than the October 2022 equivalent. For additional context, inflation was fairly benign in the former with 15-year gilt yields under 1% whereas in the latter annual general price levels were rising at double-digit percentages and bond yields had more than tripled. Boyle and Webb (2022) go a step further by also including the receipt of a state pension in their analysis, with this 'triple-locked' inflation-linked payment become increasingly valuable when real annuity rates are very low. The point is that choosing when to switch from drawdown to an annuity is unlikely to be at a fixed age but rather a function of the economic conditions prevalent.

We now run the same Monte Carlo analysis as before but using the values from Table 2[4]. Figure 4 displays the fitted curve with the equation used for interpolation and extrapolation. Starting

[4] There is a small amount of data within our Monte Carlo simulation that wouldn't have been known at February 2020, however, the point is to show how varying annuity rates affect the calculation outcomes rather than to provide a precise point-in-time analysis.

at sixty-five, the residual balance this time to purchase the annuity at age 80 is £45,340, or nearly £10k less than in October 2022. Intuitively, one would expect this to favour drawdown since one can use up more capital before purchasing the annuity and reflects the lower rate in the market. Figure 5 confirms this notion showing that the point of indifference between the risk-adjusted PWA and annuity now occurs just before seventy-eight or nearly five years later than in Figure 3. We also observe that the gap between the two lines is much wider in the early years in February 2020. The initial difference is around £2,000 and is still around £1,000 in favour of drawdown at age 75. Having shown how annuity rate fluctuations affect the point of indifference between drawdown and annuity we will revert back to analysing the October 2022 data for the remainder of the paper.

4. Risk Aversion

We previously discussed how our baseline risk aversion parameter was set at two, making gains relative to the annuity income level half as attractive compared to if one was indifferent about volatility. Attitudes towards risk are likely to vary quite considerably between retirees, and indeed if one had zero tolerance for risk then choosing an annuity becomes the only viable option. Most investors though, will likely fall somewhere between the two extremes. To this extent we now examine how variations in risk aversion might affect the decision about when to make the switch away from drawdown.

Figure 6 shows PWA curves with varying levels of risk aversion (RA). RA (2) is the baseline risk-adjusted PWA that we use in Figure 3 with a risk factor of 2. The uppermost line, RA (1), is if the retiree is indifferent to risk, with the lower three lines showing increasing aversion to relative losses. We observed earlier in our baseline that the point where annuity became more attractive than drawdown was approximately seventy-three years of age. If one was indifferent to risk then this point moves out to about 78.5 years. Going in the other direction, a risk factor of three results in the intersection between PWA and annuity occurring at around an age of seventy, with RA (4) and RA (5) resulting in a switch at sixty-eight and sixty-seven respectively.

It is clear from Figure 6 that one's risk aversion is a critical factor in choosing when to make the move from drawdown to annuity. If risk aversion is above about five then it is probably not even worth contemplating the investment portion and instead going straight to purchasing the fixed income. At the opposite end of the spectrum where one is risk indifferent then drawdown

is the preferred method for virtually the entire fifteen-year period. Most retirees will probably fall somewhere between these two extremes and so determining attitudes to risk, probably in conjunction with a financial advisor, is a very important step in making the transition from drawdown to annuity.

5. Volatility

In the previous section we observed how risk-adjusted PWAs rise initially during the decumulation period but then curl over and decline with time. The greater the level of risk aversion, the more pronounced this became. Figure 7 provides added insight into this by plotting RA (1) and RA (2) curves again but this time also showing the volatility of PWAs for the unrisked case. It is readily observed that over time the volatility increases relative to the level of unrisked PWA with a particular acceleration near the end of the period. As the volatility increases so the gap between the risked and unrisked PWA curve widens.

The increasing volatility that one experiences in decumulating towards a targeted balance is one of the major challenges of a flexible approach. Clare et al (2021a) provide more detail on this in earlier work. To illustrate the point, Figure 8 compares the volatility from Figure 7 with exactly the same methodology but this time decumulating to a zero balance. We have standardised the volatility by dividing through by the average PWA (unrisked) for each year. The contrast is very stark with the zero residual balance experiencing lower initial volatility which declines over time compared to the higher and rapidly increasing volatility of the residual balance.

From a practical standpoint, decumulating to a zero balance provides no hedge against longevity risk. Either one decumulates at a very low rate such that the chance of outliving the investment pot is essentially zero or else one runs the risk of having insufficient funds later in life. The case of the former is probably a viable option for those with a relatively large pot available, particularly if they would derive utility from leaving a meaningful bequest. For others it is a less attractive option and it can make sense to effectively pay the annuity provider to insure the longevity risk rather than do it oneself.

There appears to be an opening in the UK annuity market for more providers to offer *deferred* annuities (this is essentially what the state pension is). By spending a proportion of one's investment pot at, say age 65, to purchase an appropriately sized annuity which only generates

income from say 80 onwards it would allow retirees to decumulate the remaining investment pot to zero over the next fifteen years safe in the knowledge that they have their longevity risk covered. As we have seen from Figure 8, it is much easier to determine the likely PWA when the final balance is zero compared to a residual amount. Clearly one would have less money within the drawdown phase as some would have been spent on the deferred annuity, but it would all be available for consumption rather than having to conserve a portion. The lower volatility with decumulation to zero during the drawdown phase would open this possibility up to less risk tolerant retirees than would otherwise be the case.

6. Asset Allocation

Thus far we have assumed that all investments in the drawdown portfolio have been 75% stocks and 25% bonds. We now allow for this to vary to assess the implications for making the switch into an annuity purchase. Figure 9 displays the PWAs moving from 100% stocks to 100% bonds in 25% increments[5]. As before, the annuity line is shown on the chart as well with all drawdown factors using the risk aversion factor of two.

We observe that there is not a huge difference between being entirely invested in stocks or being equally-weighted across the two asset classes. As we saw previously, the ideal age to make the switch was around aged 73 for the 75-25 portfolio whereas this is approximately 72.5 years for both 100-0 and 50-50. It is only when bond allocations exceed 50% that there becomes a meaningful movement in where the switch occurs. A 25-75 portfolio has a point of indifference at around aged 70. If one was entirely invested in bonds then the PWA is below the annuity curve the entire time and thus one would not even entertain the prospect of commencing drawdown with such a portfolio. We thus conclude that in order to gain the full benefits of a flexible pension approach containing a drawdown phase, one needs to be willing to bear a certain amount of equity risk.

Trend Following

Clare et al (2017, 2021) have shown in both the US and UK that a useful investment technique for enhancing the drawdown experience is trend following. This involves a simple mechanical

[5] Note that we have zoomed in on the y-axis scale compared to other Figures in order to enable better vision of the various crossover points.

rule of owning a long position in the underlying asset when it is in an uptrend and switching to treasury bills when it is in a downtrend. Applying trend following to the UK equity series in this paper results in an annualised real return of 5.5% and annualised real volatility of 13.4%, this compared to the 4.9% and 18.4% respectively for equities without trend following described earlier. Typically, one might expect a fairly similar return with approximately 70% of the volatility. As we have seen earlier in the paper, volatility is very much the enemy of decumulation to a residual balance, particularly where risk aversion is an important factor.

Figure 10 shows the PWA of the standard 75-25 portfolio with RA (2) and the same portfolio but this time with the equity component replaced with trend following equities (75TF-25). We observe that the adoption of trend following leads to the drawdown-annuity crossover now taking place at almost age 76 or close to three years later than in the original portfolio. Furthermore, the trend following line is meaningfully higher than the conventional portfolio resulting in considerably more income being available for the drawdown phase before making the annuity switch. Investors can take this further by trying to build portfolios with even better return-volatility characteristics through adding additional asset classes such as real estate and commodities and applying trend following to some or all of the components (see Clare et al ,2021) .

7. Conclusion

A key finding of this and similar research (e.g. Boyle and Webb, 2022), is that in a world of 'loss aversion', across a very wide range of assumptions, there is almost always a 'crossover point' during retirement at which moving out of drawdown into an annuity can be the optimal strategy. And this should also be considered against the well-recognised (and researched) reality of cognitive decline affecting decision-making as one ages. This suggests that a hybrid, '*flex-first, fix-later*' pension product should be investigated further. Compared with just buying an annuity at retirement, the 'hybrid' approach produces much higher levels of happiness, especially at older ages, than staying wholly in drawdown. Even for those who partially annuitise at retirement, there is still a point during retirement where it may make sense to annuitise the remaining pot; in this case, the cross over point is in the early 70s rather than late 60s.

In this paper we have examined a flexible approach to retirement with an initial drawdown phase followed by an annuity purchase to protect against longevity risk. We found that at current annuity rates, using a conventional portfolio, that the ideal time to switch was around the age of seventy-three. Interest rates have risen quite considerably over 2022/23 and this has affected the decision of when to make the move away from drawdown. For example, using the same analysis, but with annuity rates based on the levels available in February 2020, we found that the age where one would have moved away from drawdown would have been close to seventy-eight, [or nearly five years difference with more recent data]. We suggest that one has to be cognisant of market conditions when using a flexible pension approach rather than relying on a fixed time to make the change. Decisions have to adapt to market conditions.

Attitudes to risk are very important with regards to choosing between drawdown and annuity purchase. We observed that very risk averse retirees should probably buy an annuity almost straight away at retirement whereas risk indifferent participants would likely want to stay with drawdown for almost the entire decumulation period. The vast majority of people probably fall between these two extremes and thus trying to estimate the level of risk that one is willing to bear plays a major role in choosing when to purchase an annuity. This also suggests that a loss aversion parameter could be a useful product of any risk questionnaire which would help inform the conversation with an adviser regarding the time to switch to an annuity.

One of the biggest challenges with decumulating towards a residual balance is dealing with the volatility near the end of the path. We find that this sharp increase in variability of outcomes is what can trigger the move to buy an annuity. The volatility is much lower if one is decumulating to a zero balance, both initially, and importantly, this volatility declines with time in stark contrast to the residual balance method. We suggest that this creates an opportunity for the UK financial services industry to offer more deferred annuities to the marketplace. By purchasing a deferred product at the beginning of decumulation it allows for certainty at the end of the drawdown period and thus the balance of this pot can be taken to zero or as required with much less volatility in annual income than buying a delayed annuity from a residual balance.

Finally, we found that the choice of when to move away from drawdown was relatively insensitive to the allocation of assets between stocks and bonds. It was only when the bond portion of the portfolio rose above half that there was a meaningful change in the outcome. Retirees wishing to make the most of a flexible pension approach therefore probably need to

be willing to accept a reasonable amount of equity risk within their investments. We demonstrated that conventional portfolios can be improved, though, by applying trend-following methods (see Clare et al, 2016, 2017). This approach reduces rate of return volatility in a meaningful way, without impacting the level of returns, such that the annuity purchase *is delayed by almost three years* and a substantial amount of additional income is thereby received over the whole drawdown period.

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Table 1	
Annual Annuity Income from a £100k Pension (Single Life, RPI, 5-Year Guarantee, Oct '22)	
Age	Amount (£)
55	3094
60	3535
65	4290
70	5212
75	6372

Table 2	
Annual Annuity Income from a £100k Pension (Single Life, RPI, 5-Year Guarantee, Feb '20)	
Age	Amount (£)
55	1732
60	2149
65	2922
70	3846
75	5018

Figure 1.

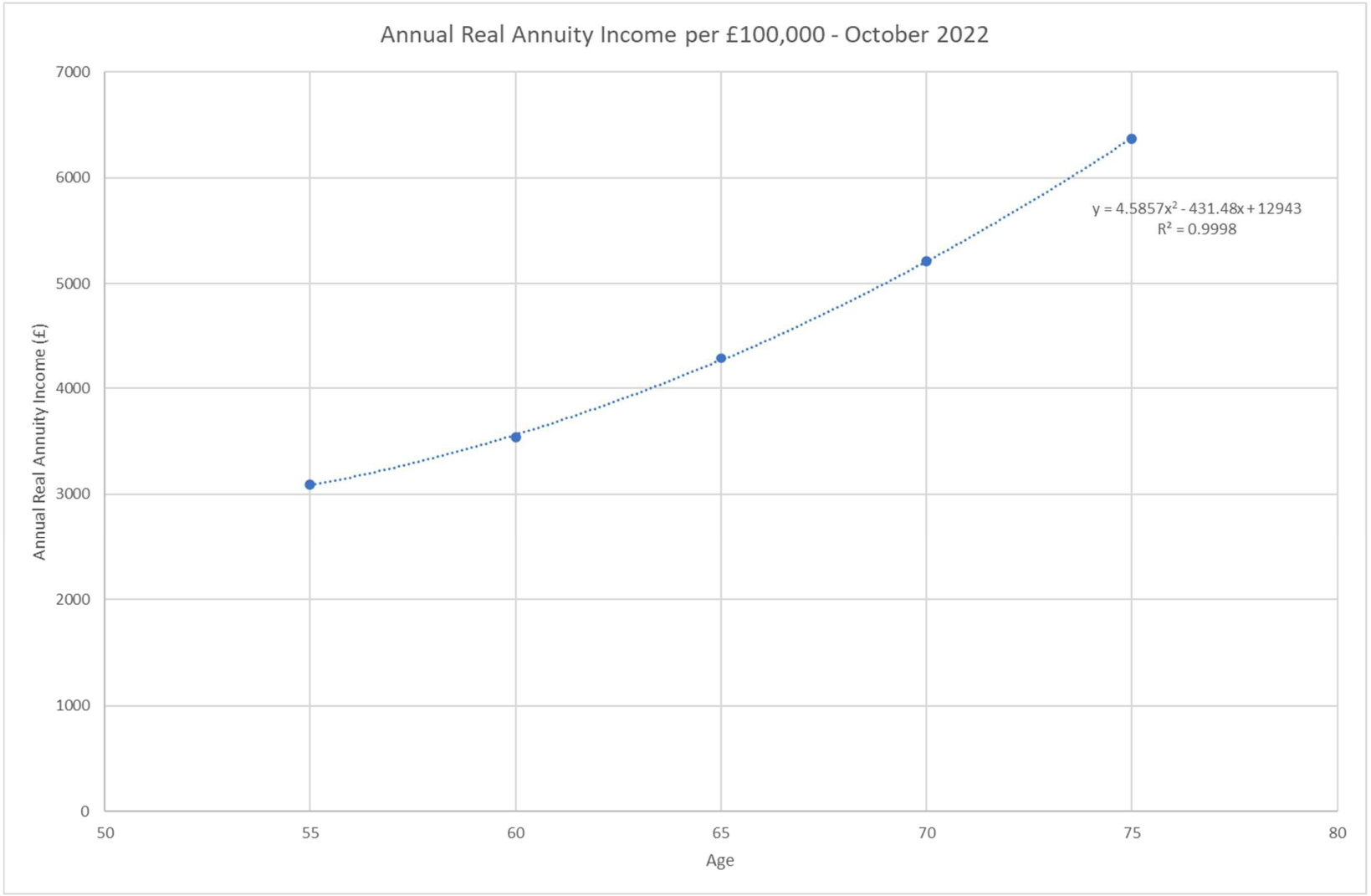

Annual Real Annuity Income per £100,000 - October 2022
Annual Real Annuity Income (£)
7000
6000
5000
4000
3000
2000
1000
0
50
55
60
65
70
75
80
Age
y = 4.5857x² - 431.48x + 12943
R² = 0.9998

Figure 2.

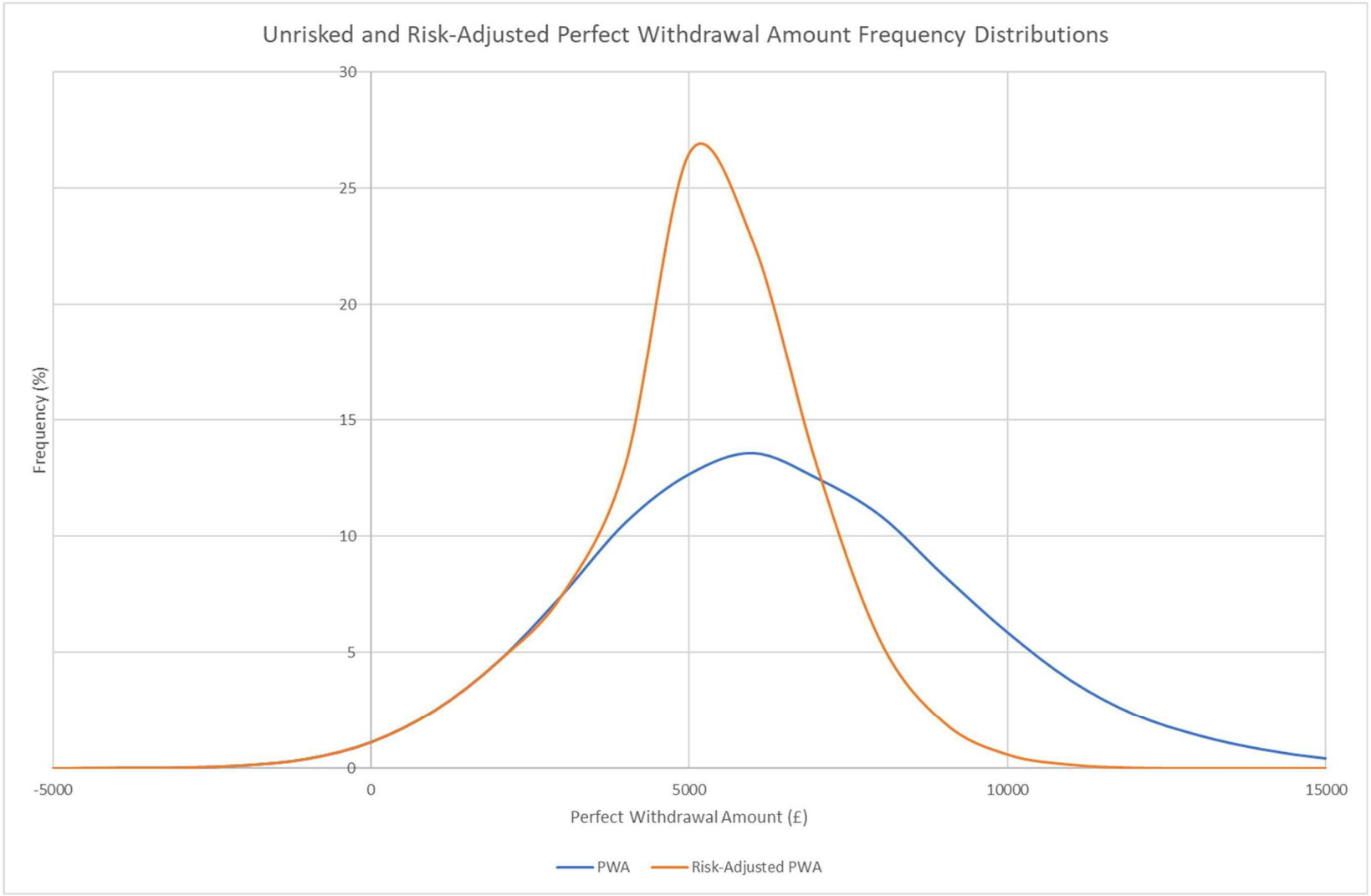

Unrisked and Risk-Adjusted Perfect Withdrawal Amount Frequency Distributions
30
25
20
15
10
5
0
Frequency (%)
-5000
0
5000
10000
15000
Perfect Withdrawal Amount (£)
PWA
Risk-Adjusted PWA

Figure 3.

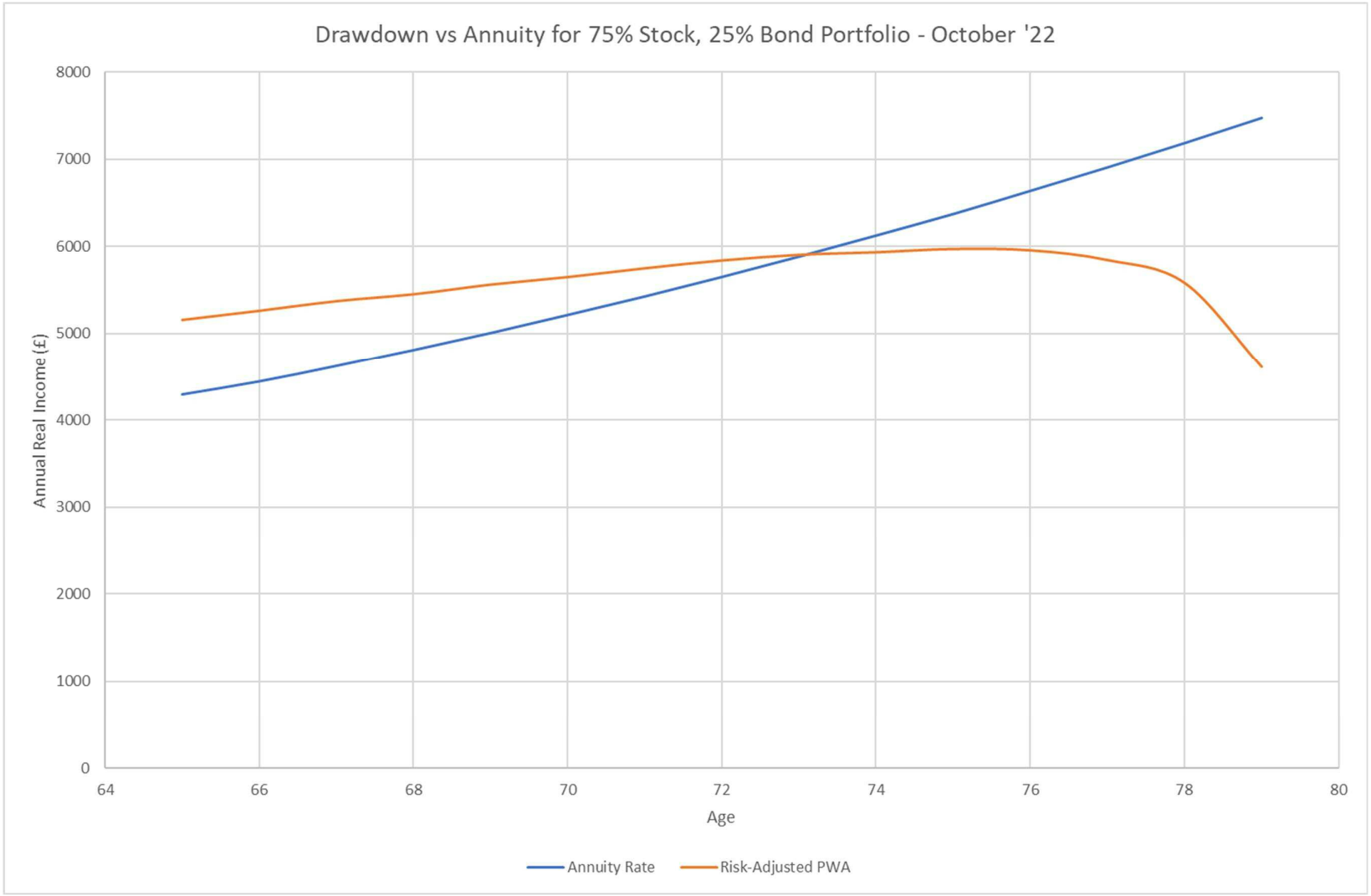

Drawdown vs Annuity for 75% Stock, 25% Bond Portfolio - October '22
Annual Real Income (£)
8000
7000
6000
5000
4000
3000
2000
1000
0
64
66
68
70
72
74
76
78
80
Age
Annuity Rate
Risk-Adjusted PWA

Figure 4.

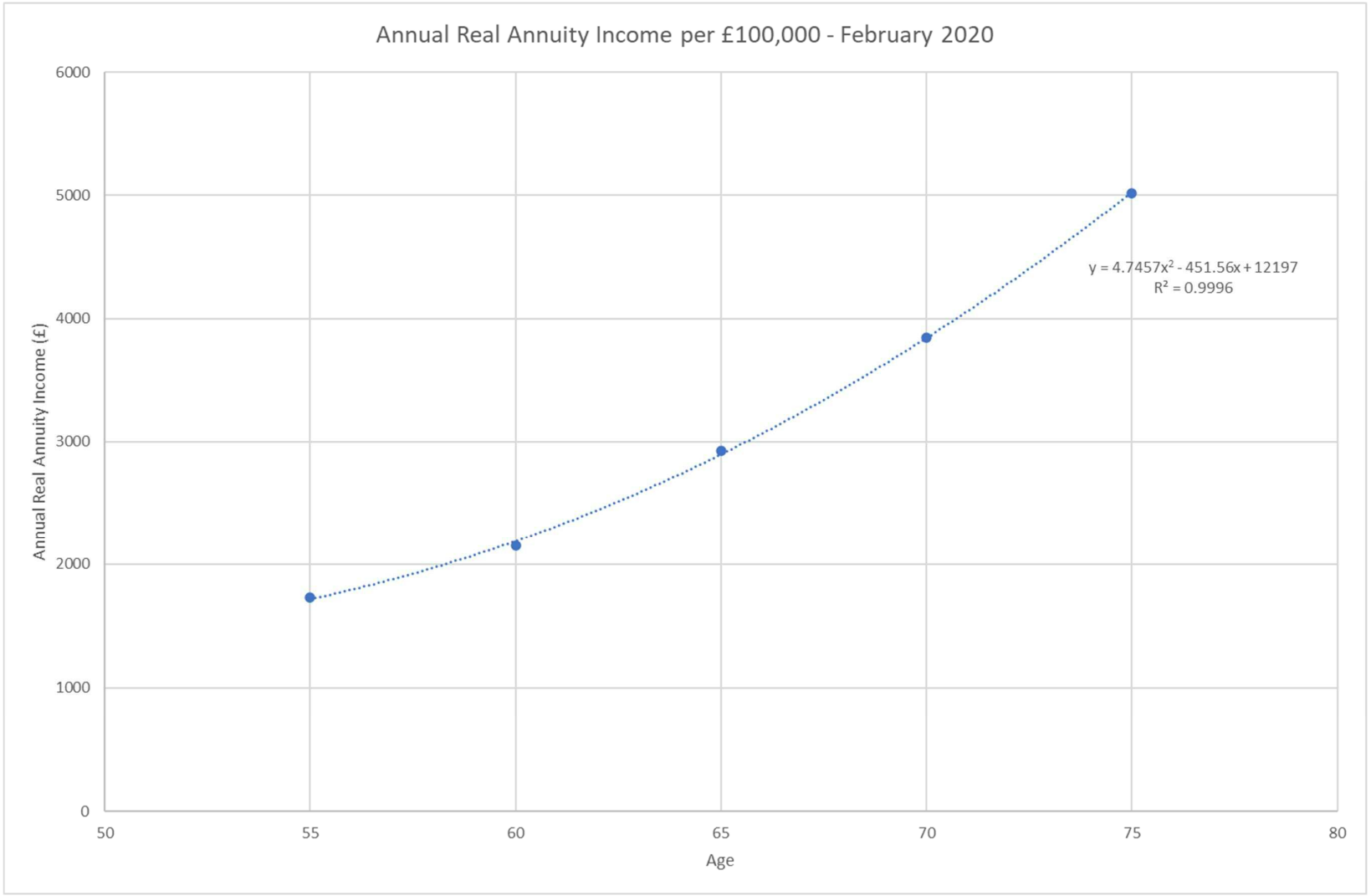

Annual Real Annuity Income per £100,000 - February 2020
6000
5000
4000
3000
2000
1000
0
Annual Real Annuity Income (£)
50
55
60
65
70
75
80
Age
y = 4.7457x² - 451.56x + 12197
R² = 0.9996

Figure 5.

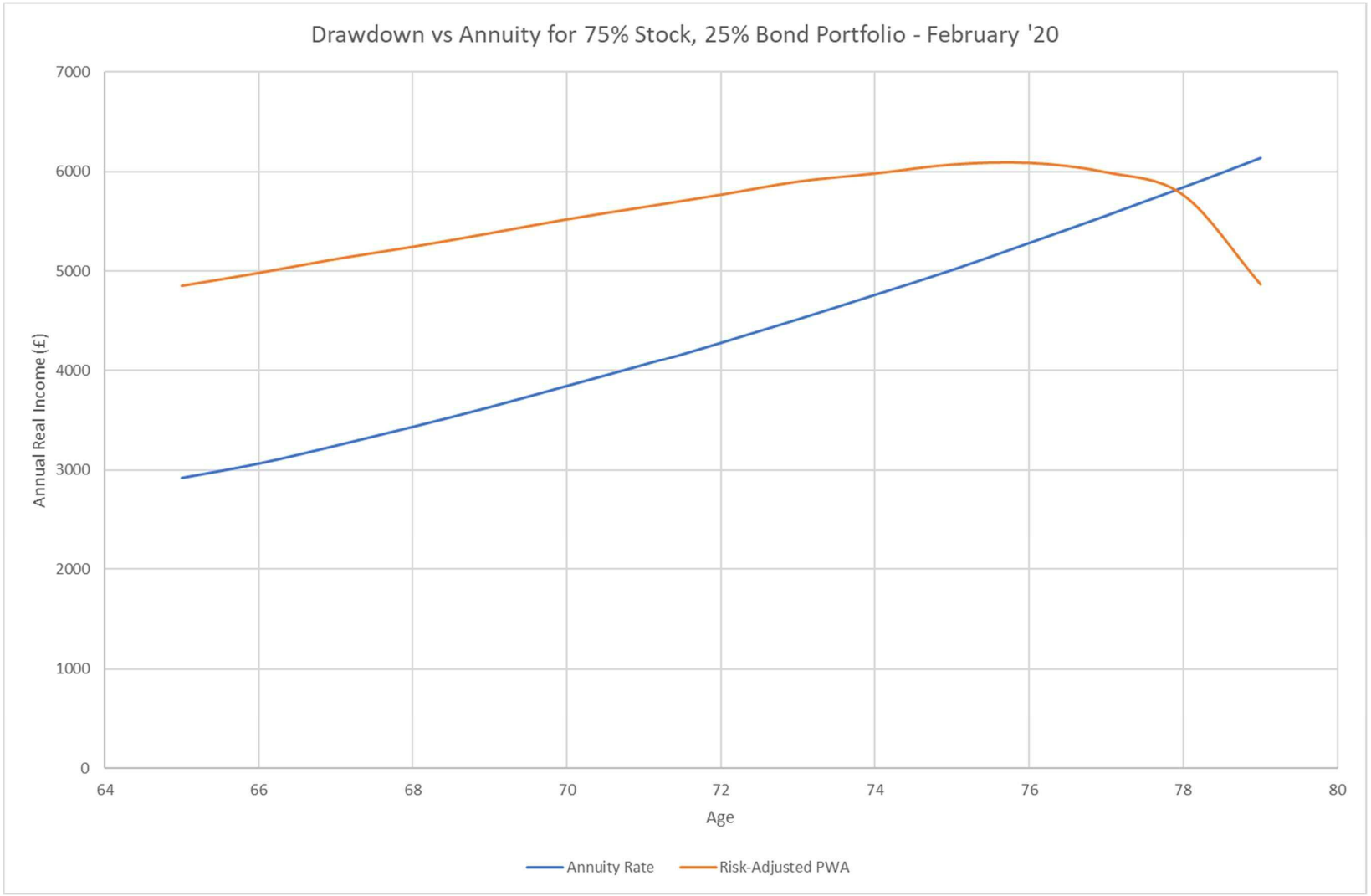

Drawdown vs Annuity for 75% Stock, 25% Bond Portfolio - February '20
Annual Real Income (£)
7000
6000
5000
4000
3000
2000
1000
0
64
66
68
70
72
74
76
78
80
Age
Annuity Rate
Risk-Adjusted PWA

Figure 6.

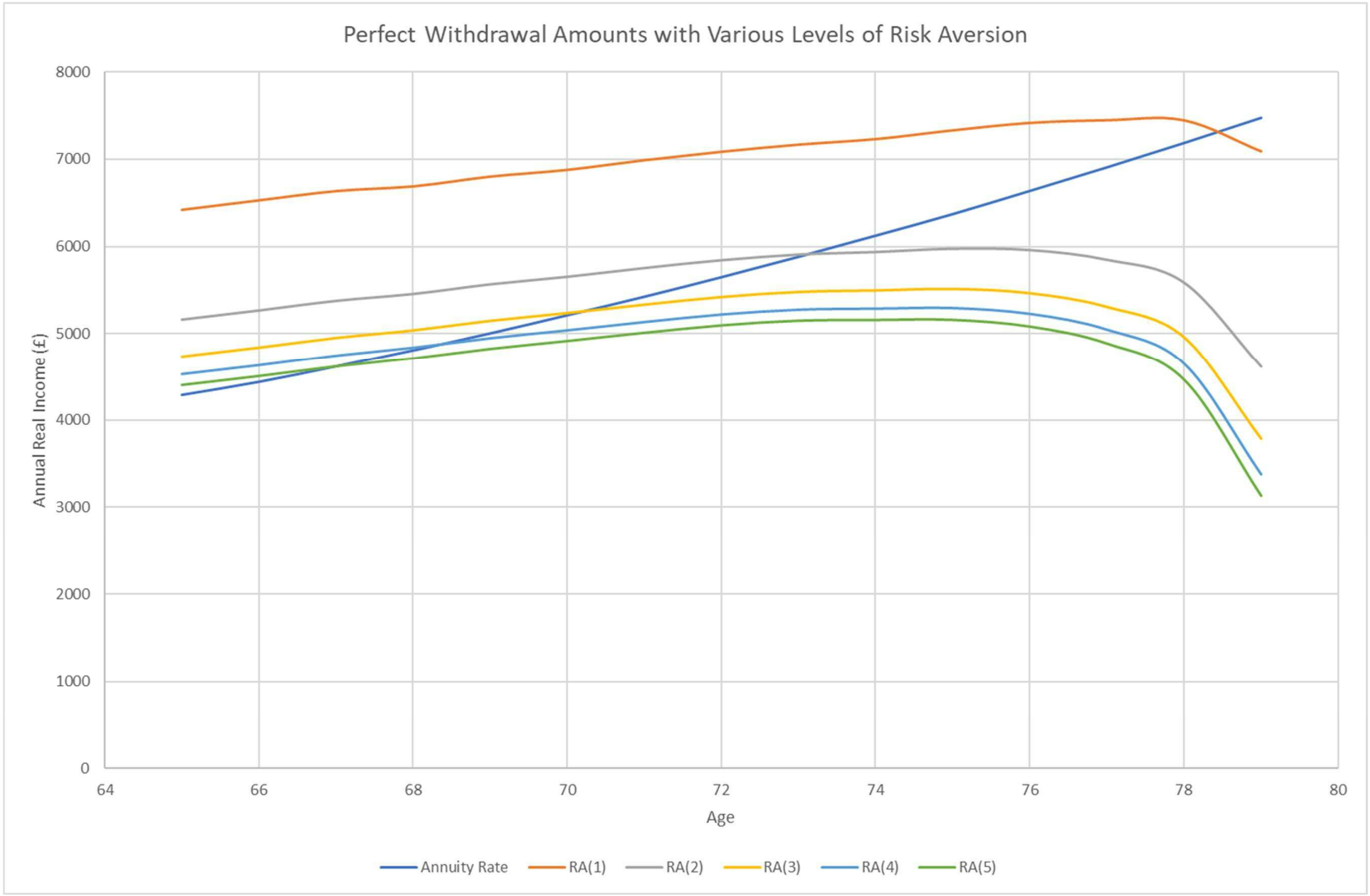

Perfect Withdrawal Amounts with Various Levels of Risk Aversion
Annual Real Income (£)
8000
7000
6000
5000
4000
3000
2000
1000
0
64
66
68
70
72
74
76
78
80
Age
Annuity Rate
RA(1)
RA(2)
RA(3)
RA(4)
RA(5)

Figure 7.


The Effect of Volatility on Risk Adjustments to Perfect Withdrawal Amounts
Annual Real Income (£)
14000
12000
10000
8000
6000
4000
2000
0
64
66
68
70
72
74
76
78
80
Age
PWA
PWA Volatility
Risk-Adjusted PWA

Figure 8.

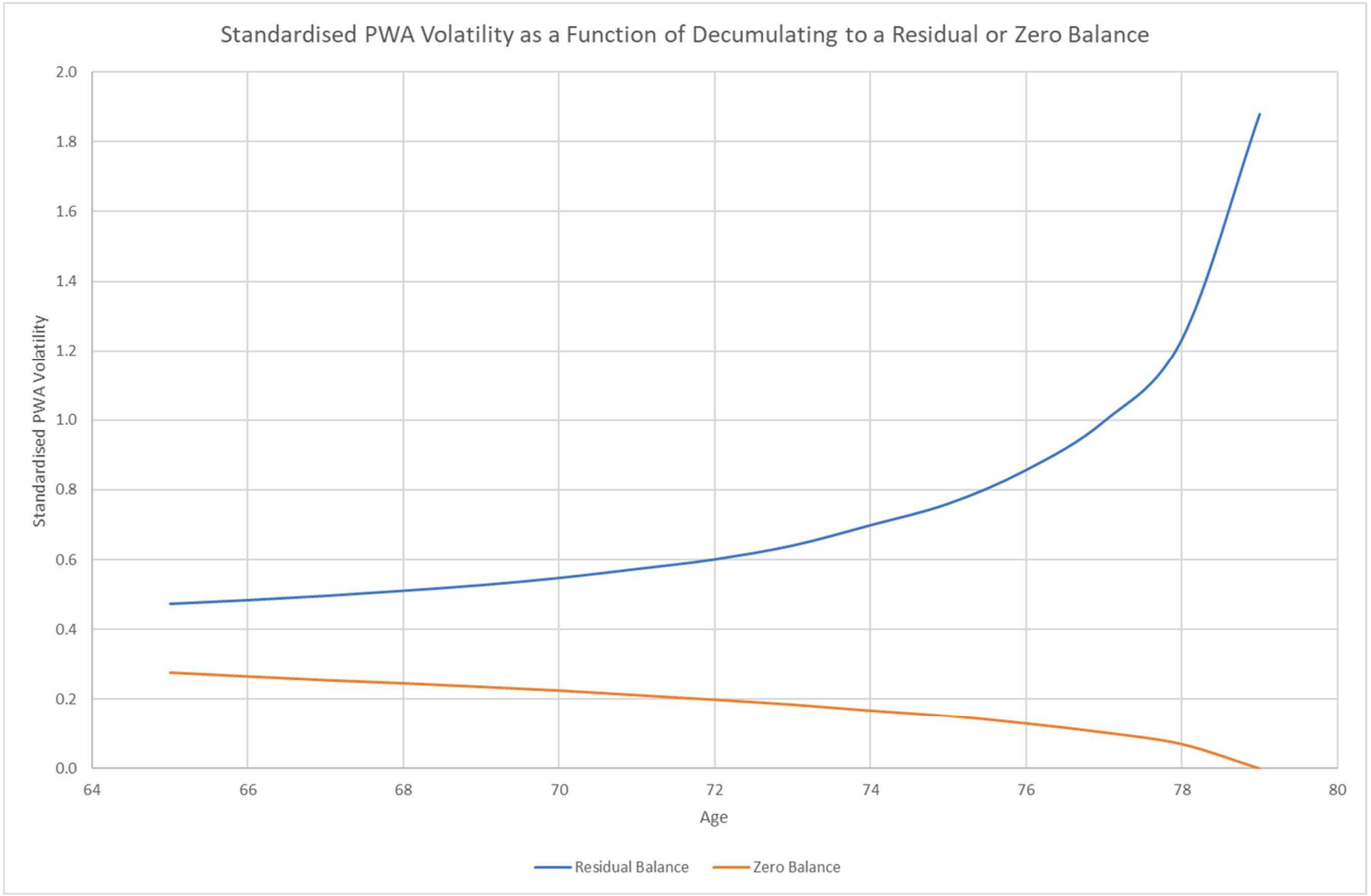

Standardised PWA Volatility as a Function of Decumulating to a Residual or Zero Balance
Standardised PWA Volatility
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
64
66
68
70
72
74
76
78
80
Age
Residual Balance
Zero Balance

Figure 9.

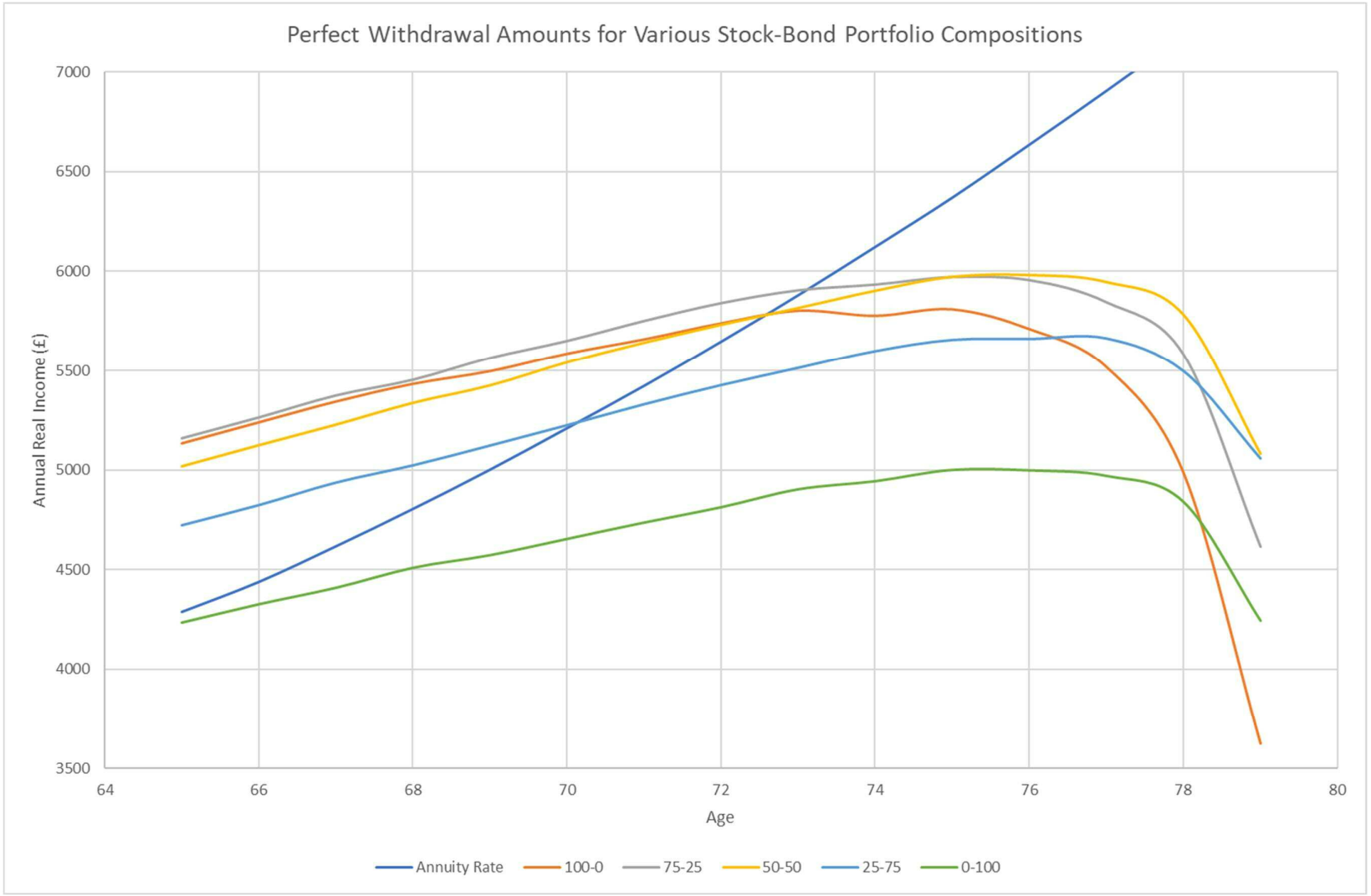
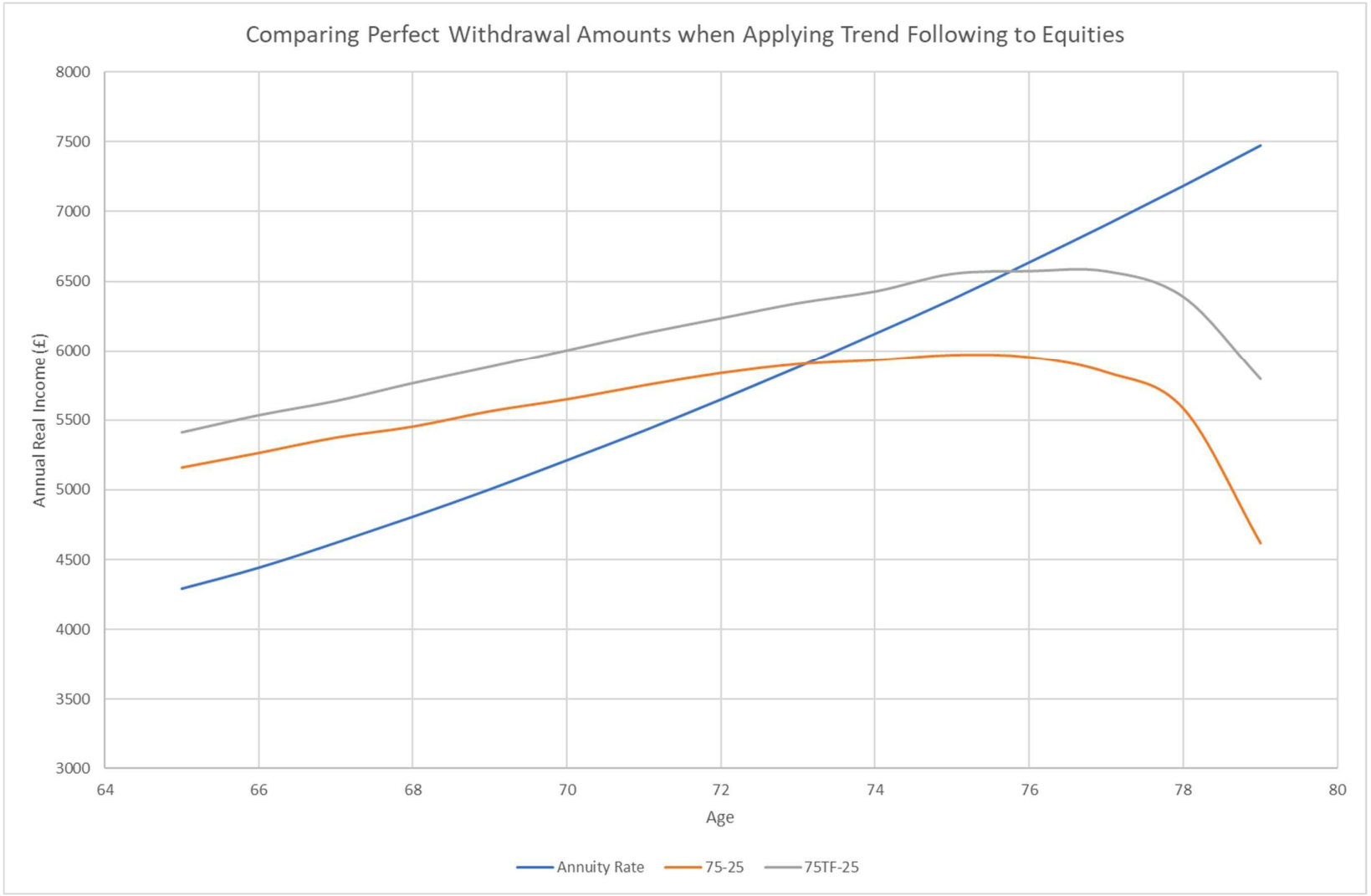

Perfect Withdrawal Amounts for Various Stock-Bond Portfolio Compositions
Annual Real Income (£)
7000
6500
6000
5500
5000
4500
4000
3500
64
66
68
70
72
74
76
78
80
Age
Annuity Rate
100-0
75-25
50-50
25-75
0-100

Figure 10.


Comparing Perfect Withdrawal Amounts when Applying Trend Following to Equities
Annual Real Income (£)
8000
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
64
66
68
70
72
74
76
78
80
Age
Annuity Rate
75-25
75TF-25