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On the Reliability of Estimated Taylor Rules for Monetary Policy Analysis

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JEL Classification

E50, E52, E58

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On the Reliability of Estimated Taylor Rules for Monetary Policy Analysis

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Abstract

Taylor rules and their implications for monetary policy analysis can be misleading if the inflation target is held fixed while being in fact time-varying. We offer a theoretical analysis showing why assuming a fixed inflation target in place of a time-varying target can lead to a downward bias in the estimated policy rate response to the inflation gap and wrong statistical inference about indeterminacy. Our analysis suggests the bias is stronger in periods where inflation target movements are large. This is confirmed by simulation evidence about the magnitude of the bias obtained from a New Keynesian model featuring positive trend inflation. We further estimate medium-scale NK models with positive trend inflation and a time-varying inflation target using a novel population-based MCMC routine known as parallel tempering. The estimation results confirm our theoretical analysis while favouring a determinacy outcome for both pre and post-Volcker periods and shedding new light about the type of rule the Fed likely followed.

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1 Introduction

In March 2021 US inflation began to rise sharply, reaching its highest level in forty years in June 2022. While inflation is expected to remain above an annual rate of 2.5% until the mid-2020s, a growing concern among policy makers and policy observers is whether the Fed should return to the type of rules-based policy many believe helped lowering inflation between the mid-1980s and mid-2000s (see for example, Taylor, 2022)

With this in mind, our paper raises the following questions: Are previously estimated monetary policy rules reliable tools to assess how strongly the Fed adjusted the policy rate in reaction to the inflation gap (i.e., the difference between actual inflation and the inflation target) and to measures of economic activity? Are these rules reliable in determining whether monetary policy prevented or led to self-fulfilling inflationary expectations during the post-WWII period? We offer new theoretical analysis and empirical evidence suggesting the answers to these questions are likely negative depending on the estimation period considered, and this due to a bias that potentially contaminated previous Taylor rule estimates.

Our main contributions are twofold. First, using a standard three-equation New Keynesian (NK) model, we derive theoretical expressions pertaining to the Ordinary Least Squares (OLS) estimator of the central bank's response parameter to the inflation gap and establish a relation between this estimator and the treatment of the inflation target. Our theoretical analysis identifies conditions under which the OLS policy response to the inflation gap is biased downwards if the inflation target is mistakenly held fixed. In turn, this bias may lead to wrong statistical inference as to whether the US economy experienced self-fulfilling inflationary expectations and indeterminacy during the post-WWII era.

Second, we report new empirical evidence supporting our theoretical analysis. For this purpose, we use several NK models with positive trend inflation, imperfectly competitive goods and labor markets, nominal wage and price rigidities and real adjustment frictions. Our estimated models allow distinct probabilities of determinacy and indeterminacy using a novel population-based MCMC sampling technique known as parallel tempering (Brault, 2024).

Historically, the Fed has not disclosed whether it is aiming at a fixed or time-varying inflation target when adjusting the policy rate. Following Taylor (1993)'s analysis of rules versus discretion in monetary policy-making, several monetary models have assumed a fixed target. This approach has been followed notably by Clarida et al. (2000) and Lubik and Schorfheide (2004), their evidence suggesting the Fed accommodated inflation during the 1960s and 1970s, which presumably led to self-fulfilling inflationary expectations and indeterminacy.

Meanwhile, several theories have been put forth supporting the view that the Fed's inflation target has been time-varying. One says the Fed chooses its target as it learns about the structure of the economy, with changing beliefs about the output-inflation trade-off generating a low-frequency, hump-shaped pattern in inflation (Sargent, 1999; Cogley and Sargent, 2005; Primiceri, 2006; Sargent et al., 2006). Cogley et al. (2010) approximate the outcome of this learning process by a near-unit root process which is consistent with the evidence they provide. Another states the Fed accommodated persistent adverse supply shocks by raising its target (Bomfim and Rudebusch, 2000; Orphanides and Wilcox, 2002; Ireland, 2007).[1] Still another holds that due to differences in the preference distributions of the FOMC members, the rotating voting eligibility of members has resulted in a time-varying inflation target (Kozicki and Tinsley, 2009). Finally, in Bianchi et al. (2023), the inflation target is time-varying because the Fed accommodates unfunded fiscal expansions.

Notwithstanding what the preferred explanation is, in several empirical NK models the inflation target has been assumed to be time-varying. For instance, Ireland (2007) estimates a NK price-setting model to infer variations in the inflation target from 1959 to 2004. Aruoba and Schorfheide (2011) generate a time-varying inflation target measure from low-frequency inflation dynamics and inflation expectations, which they use to estimate a DSGE model with centralized and decentralized markets and analyse policy trade-offs. Del Negro and Eusepi (2011) estimate a NK model with zero trend inflation, sticky wages, sticky prices and a time-varying inflation target showing it predicts inflation expectations more in-line with the data than those of models assuming either a fixed inflation target or imperfect information.[2] In these models, the inflation target is inferred ex-post using historical data and econometric techniques. We follow a similar approach.

The first part of the paper shows the Ordinary Least Squares (OLS) estimator of the policy response to the inflation gap can be prone to a bias stemming from two different sources if the inflation target is fixed. A first source identified by Carvalho et al. (2021) stems from an endogeneity problem which arises even in the case where a fixed inflation target would be the correct assumption. That is, if in setting the policy rate the Fed reacts to variables responding to monetary policy shocks, then the regressors of the policy rule and the policy error term are correlated. In turn, this can result in an asymptotically biased policy response to inflation. Nonetheless, their evidence suggests that monetary policy shocks contribute to a small fraction of the variance of typical Taylor rule regressors, so the bias tends to be small and the OLS parameter estimates of Taylor rules can be seen as reliable.

We identify a second source of bias in case the inflation target is mistakenly held fixed in the estimation when the data generating process for the inflation target is time-varying. Our theoretical analysis focuses on the central bank's response to the inflation gap which might be prone to a severe downward bias whose severity quickly increases with the size of variations in the inflation target.

We offer simulation evidence concerning the potential magnitude of biases from a model broadly similar to the canonical three-equation New Keynesian model that served for our theoretical analysis. Using a model with a time-varying inflation target to generate simulated data, we assume inflation target shocks which are either "small", "moderate", or "large". We then estimate the model with this simulated data assuming a fixed inflation target. From small to large shocks, we show the policy response parameter to the inflation gap varies from nearly equal to the true parameter value to just one half of that value. We

[1] Eo and Lie (2020) study time-varying target inflation as a stabilization tool in an NK framework, and find that movements in the target can be welfare improving, particularly in response to supply shocks.

[2] Other studies where the inflation target is time-varying include Erceg and Levin (2003), Cogley and Sbordone (2008), and Del Negro et al. (2015).

find that other policy rule and model parameter estimates are also affected when ignoring time-varying inflation target in estimations.

A simple intuition for the bias in the policy response parameter to inflation, supported by an impulse-response analysis, is the following: A positive shock to the inflation target triggers a positive response of inflation which is larger than the ensuing positive response of the nominal interest rate. This shock is thus followed by a negative response of the real interest rate. With large inflation target movements, one could be misled to conclude from the observed time series for inflation and the nominal interest rate that the Fed was accommodating higher inflation by a passive policy, while in fact it was implementing a policy relatively responsive to the inflation gap within a time-varying inflation target monetary policy regime.

The second part of the paper is devoted to evidence that the theoretical intuition offered in the first part of the paper applies to more general medium-scale NK models. For this purpose, we turn our attention to NK models with positive trend inflation, nominal wage and price stickiness, selected real frictions, and a policy reaction to alternative measures of economic activity.

The models are log-linearized around non-zero steady-state inflation for two reasons. One is essentially factual since the annual rate of inflation has always been positive during the post-WWII era, except in 2009. Furthermore, the average rate of inflation has varied quite significantly over sufficiently long periods. The other is somewhat more fundamental given the scope of our paper. Based on model simulations, Ascari et al. (2018) and Khan et al. (2020) show that positive trend inflation can have significant cyclical and long-run implications, and this even at moderate rates of trend inflation like 2-4%. They show that positive trend inflation generates steady-state distorting effects mainly through its interaction with nominal wage stickiness. Therefore, omitting positive trend inflation, sticky wages, or both could result in biased model and policy rule estimates.

We first report simulation evidence from our medium-scale NK model offering insights about key factors determining the minimum policy response to the inflation gap consistent with determinacy. As in Coibion and Gorodnichenko (2011) and Khan et al. (2020), we show that with a policy rate responding to output gap and output growth rather than to output growth only, a stronger policy response to inflation is needed to achieve determinacy.

We emphasize another factor, overlooked so far in the literature, which increases the prospect of indeterminacy when the policy rate adjusts in response to output gap and output growth. We show that if the inverse Frisch elasticity of labor supply is "high", as some of our estimates seem to suggest, the minimum policy response to inflation consistent with determinacy gets significantly larger even at trend inflation rates like 3% and 4%. Interestingly, the determinacy region is much less affected by the value of this elasticity when the policy rate responds to output growth only.

We next provide empirical evidence using a Bayesian estimation technique allowing for distinct probabilities of determinacy and indeterminacy. Standard algorithms like the Metropolis-Hastings can struggle to accurately characterize the posterior distribution due to discontinuities in the likelihood function around the boundary between determinacy and indeterminacy regions. Hence, we use a novel sampling method known as parallel tempering. The algorithm is a population-based Markov chain Monte Carlo (MCMC) method that is particularly well-suited for problems with ill-behaved posterior distributions. Parallel tempering approximates a target posterior using a family of Markov chains with tempered posteriors that allow exchanging information between chains. The chains with a high amount of likelihood tempering flatten the posterior surface permitting large moves around the parameter space. This makes crossing the boundary between determinacy and indeterminacy regions feasible, while transmitting information to chains in search of a more confined parameter space via an exchange step.[3]

Using this Bayesian method, we estimate several model versions in which the inflation target is time-varying. Our models differ by their assumptions about trend inflation, whether it is positive or zero, and which measure(s) of economic activity the Fed is aiming at when adjusting interest rates, whether it sets the policy rate based on mixed reactions to output gap and output growth (labelled MO-rule model) or output growth only (labelled OG-rule model). We report model estimates for two periods, namely 1960:I-1979:II and 1983:I-2007:IV.

Our estimates suggest that movements in the inflation target have been significantly larger during the pre-Volcker period. Based on estimated marginal data densities, we find that models with positive trend inflation are strictly preferred to their counterparts with zero trend inflation, confirming the relevance of accounting for non-zero trend inflation in model estimation. Also, we find that OG-rule models are marginally preferred to MO-rule models.

Our estimated policy responses to the inflation gap are broadly consistent with the conclusions of our theoretical analysis. While the estimated policy response parameters to inflation previously reported in the literature were generally positive and smaller than 1 prior to 1980 (Clarida et al., 2000; Lubik and Schorfheide, 2004; Coibion and Gorodnichenko, 2011; Hirose et al., 2020), and occasionally after 1982 (Nicoló, 2023), our estimates are larger than 2 both for the pre and post-Volcker periods.[4] We also find that the estimated model parameters display greater stability with OG-rules than MO-rules. We find that both periods were characterized by determinacy with estimated probabilities ranging from .87 to 1. We also discuss how and why our main results differ from others reported in the literature.

The rest of the paper is organized as follows. Section 2 analyzes a basic three-equation New Keynesian model, which allows us to analytically characterize the bias associated with incorrectly assuming a fixed inflation target. Section 3 provides simulation evidence on the quantitative magnitude of the bias. Section 4 outlines a medium-scale DSGE model with positive trend inflation. Section 5 discusses the determinacy regions of the medium scale model. Section 6 describes the data and estimation methodology. Section 7 discusses the estimation results and finally Section 8 concludes.

[3] The interested reader is referred to Brault (2024) for a detailed description of this method.

[4] One exception is the estimate from the MOT-rule model with zero inflation for the period 1983-2007.

2 Omitted Variable Bias in a Basic NK Model

To illustrate theoretically how Taylor rule estimates can be biased when the inflation target is mistakenly assumed fixed, we work with the standard three-equation NK price-setting model (e.g, Galí, 2008, Chapter 3). For expository purposes, the model contains only two sources of exogenous variation: one shock to the central bank's inflation target and one to the policy rule. Since this model is standard apart from the inflation target shock, we only report the log-linearized equations below:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \psi \hat{x}_t, \tag{1}$$

$$\hat{x}_t = \mathbb{E}_t \hat{x}_{t+1} - \frac{1}{\tau}(\hat{i}_t - E_t \hat{\pi}_{t+1}), \tag{2}$$

$$\hat{i}_t = \phi_\pi (\hat{\pi}_t - \hat{\pi}^\star_t) + \hat{v}_t, \tag{3}$$

$$\hat{\pi}^\star_t = \rho_\pi \hat{\pi}^\star_{t-1} + \epsilon^\pi_t, \tag{4}$$

$$\hat{v}_t = \rho_v \hat{v}_{t-1} + \epsilon^v_t. \tag{5}$$

Equations (1) and (2) are respectively the New Keynesian Phillips Curve (NKPC) and IS equations, β denotes the discount factor, ψ is the slope of the NKPC, and τ is the intertemporal elasticity of substitution. Equation (3) is a simplified Taylor rule wherein the policy rate adjusts to deviations of inflation from a time-varying target, and ϕ_π is the parameter that governs the policy response to the inflation gap. Equations (4) and (5) describe the stochastic processes for the inflation target and monetary policy shocks, with $\epsilon^\pi_t \sim i.i.d.\ N(0, \sigma^2_\pi)$ and $\epsilon^v_t \sim i.i.d.\ N(0, \sigma^2_v)$. We assume that $\mathbb{C}ov(\epsilon^\pi_t, \epsilon^v_t) = 0$.[5]

Using the method of undetermined coefficients, equilibrium inflation evolves according to

$$\hat{\pi}_t = \psi \phi_\pi \Lambda_\pi \hat{\pi}^\star_t - \psi \Lambda_v \hat{v}_t, \tag{6}$$

where

$$\Lambda_\pi = \frac{1}{\tau(1-\rho_\pi)(1-\beta\rho_\pi) + \psi(\phi_\pi - \rho_\pi)}, \tag{7}$$

$$\Lambda_v = \frac{1}{\tau(1-\rho_v)(1-\beta\rho_v) + \psi(\phi_\pi - \rho_v)}. \tag{8}$$

Assumptions *The parameters here satisfy:*

[5] In this model, determinacy requires the Fed's policy response to inflation to strictly comply with the Taylor principle ($\phi_\pi > 1$).

(i) $\phi_\pi > 1$,

(ii) $1 - \phi_\pi \psi \Lambda_v > 0$,

(iii) $\phi_\pi \psi \Lambda_\pi - 1 > 0$.

A(i) implies that the policy rate response parameter to the inflation gap satisfies the Taylor principle, such that the equilibrium is unique. A(ii) means that the sign of the policy rate response is always the same as the monetary policy shock, effectively ruling out cases with very persistent monetary policy shocks. A(iii) ensures the response to the inflation gap, and consequently the policy rate response to an inflation target shock is positive.[6] Under Assumptions A(i)-A(iii), Λ_π, Λ_v, and ψ are all positive and equilibrium inflation responds positively to movements in the inflation target and negatively to monetary policy shocks.

2.1 OLS Estimate of ϕ_π

Consider an econometrician estimating ϕ_π with OLS, but mistakenly assuming that the inflation target is fixed. In this case, an estimate of ϕ_π can be obtained by regressing the interest rate on inflation

$$\hat{\phi}_\pi = \frac{\mathbb{C}ov(\hat{i}_t, \hat{\pi}_t)}{\mathbb{V}ar(\hat{\pi}_t)}. \tag{9}$$

Substituting the true monetary policy rule into (9) gives

$$\hat{\phi}_\pi = \frac{\mathbb{C}ov(\phi_\pi(\hat{\pi}_t - \hat{\pi}_t^\star) + \hat{v}_t, \hat{\pi}_t)}{\mathbb{V}ar(\hat{\pi}_t)}. \tag{10}$$

From (10) it is clear there is a bias in the estimator of ϕ_π, as both $\pi_t^\star$ and v_t contribute to movements in inflation. Substituting in (6), the probability limit of ϕ_π is given by

$$\operatorname*{plim}_{T\to\infty} \hat{\phi}_\pi = \phi_\pi - \phi_\pi^2 \psi \Lambda_\pi \frac{\mathbb{V}ar(\hat{\pi}_t^\star)}{\mathbb{V}ar(\hat{\pi}_t)} \hat{\pi}_t^\star - \psi \Lambda_v \frac{\mathbb{V}ar(\hat{v}_t)}{\mathbb{V}ar(\hat{\pi}_t)}, \tag{11}$$

where the first negative term on the right-hand side of equation (11) represents the bias in the OLS estimator of ϕ_π arising from the time-varying inflation target while the second negative term is the bias from monetary policy shocks, with $\mathbb{V}ar(\pi_t)$ given by

$$\mathbb{V}ar(\hat{\pi}_t) = (\psi \phi_\pi \Lambda_\pi)^2 \mathbb{V}ar(\hat{\pi}_t^\star) + (\psi \Lambda_v)^2 \mathbb{V}ar(\hat{v}_t). \tag{12}$$

[6] Assumptions A(ii) and A(iii) can be relaxed, but this would change the interpretation for why the bias occurs. Further, these assumptions are consistent with the empirically relevant cases.

Equation (11) highlights that both inflation target and monetary policy shocks bias the OLS estimator of ϕ_π downwards, as ϕ_π, ψ, Λ_π, and Λ_v are all positive. However, from an intuitive standpoint, the reasons for this downward bias are different for the two shocks. To illustrate this, we generate impulse response functions of inflation, the nominal interest rate, and ex-ante real interest rate to a positive monetary policy shock as well as a positive inflation target shock. Figure 1 plots the impulse response functions for a standard calibration of model parameters.

A positive (contractionary) monetary policy shock raises the nominal interest rate, and due to sticky prices, the real interest rate, reducing both output and inflation, and hence generates a negative comovement between the central bank's policy rate and inflation. If unaccounted for, these shocks make the observed time series between the policy rate and inflation appear less procyclical, and thus bias the estimate of the policy rate's response to inflation downwards.

In contrast, a shock to the inflation target induces a positive comovement between inflation and the policy rate. On impact a positive inflation target shock implies that current inflation is below target, and the central bank responds by lowering the policy rate to stimulate output and inflation. But agents also understand that a positive inflation target shock signals that the central bank will tolerate higher inflation in the future, raising inflation expectations. The second effect dominates the first, and inflation rises above the new target on impact, and in equilibrium, leads the central bank to raise its policy rate. In this case a downward bias occurs because the central bank is raising the policy rate in response to the inflation gap and the increase in policy rate is *less* than the increase in inflation. An econometrician observing interest rates and inflation under the assumption of a fixed inflation target would mistakenly conclude the central bank was raising its policy rate less than one-for-one with increases in inflation and not satisfying the Taylor principle. This is pictured in the bottom row of Figure 1. The left panel shows the responses of both inflation and the inflation target. Comparing the left and middle sub-figures in the bottom row, it is clear that the peak response in inflation is greater than the peak response in the policy rate.

While this is a highly stylized example comprising only two sources of exogenous variation, the intuition extends to larger models (both in structure and number of shocks) where the variance of inflation is simply driven by a larger number of shocks.

Carvalho et al. (2021) identify a bias in OLS estimates of Taylor rules emanating from monetary policy shocks under a fixed inflation target. They argue that since monetary policy shocks contribute to a small fraction of output fluctuations and inflation variability, the endogeneity bias in OLS estimates from this particular source tends to be small. Their conclusion is based on the empirical fact that monetary policy shocks are not very persistent and relatively small in size.

Relative to their work, our findings suggest the bias associated with OLS estimates of Taylor rule parameters can be significantly stronger when the inflation target is time-varying. The reason is that empirically inflation target shocks are highly persistent and subsequently even moderate movements in the inflation target can generate substantial inflation variation, leading to a large bias. For instance, Ireland (2007) finds that inflation target shocks dominate the low frequency movements in inflation during the 1959-2004 period, accounting for roughly 56% and 84% of variation in inflation at horizons of 8 and 40 quarters.

The bias resulting from the omission of a time-varying inflation target can be dealt with by defining the policy relevant inflation gap measure as the difference of inflation from a time-varying inflation target. To illustrate this, suppose the econometrician estimates the policy rate response to the inflation gap (ϕ_π) under the correct assumption that target inflation is time-varying. Further, for expository purposes, assume the time-varying target is observed perfectly.[7] In this case the estimate of ϕ_π is obtained by regressing interest rates on the inflation gap

$$\hat{\phi}_\pi = \frac{\mathbb{C}ov(\hat{i}_t, \hat{\pi}_t - \hat{\pi}_t^\star)}{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)}. \tag{13}$$

As before, it is straightforward to show that the asymptotic OLS estimator of ϕ_π in this case is given by

$$\operatorname*{plim}_{T\to\infty} \hat{\phi}_\pi = \phi_\pi - \psi\Lambda_v \frac{\mathbb{V}ar(\hat{v}_t)}{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)}, \tag{14}$$

where

$$\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star) = (\psi\phi_\pi\Lambda_\pi - 1)^2\mathbb{V}ar(\hat{\pi}_t^\star) + (\psi\Lambda_v)^2\mathbb{V}ar(\hat{v}_t). \tag{15}$$

According to (14), there is no bias in the estimation of ϕ_π from inflation target shocks, and the only remaining source of bias is related to the endogeneity bias introduced by monetary policy shocks.

Proposition 1. *Assume $\sigma_\pi > 0$. Then for plausible shock sizes, the OLS estimate of ϕ_π from (11) is strictly smaller (i.e., more biased) than the estimate of ϕ_π from (14).*

See Appendix B for proof.

Proposition 1 establishes that if the inflation target is time-varying, the OLS estimator of the inflation response parameter ϕ_π will be smaller when the target is taken to be fixed compared to the estimator that accounts for time-variation in the target.

[7] In practice, the target has to be estimated from an econometric model, as we do in the following sections.

3 Simulation Evidence

In this section we offer simulation evidence about the quantitative magnitude of the bias in Taylor rule parameters if the inflation target is mistakenly assumed fixed when the true data-generating process has a time-varying inflation target. While the previous section illustrated the bias in the policy response to inflation using ordinary least squares, the simulation evidence in this section is based on a Bayesian estimated DSGE model.

We favour this approach for the following reasons. First, the model accounts for the simultaneity bias emanating from monetary policy shocks, meaning that any remaining bias in parameter estimates results entirely from the omission of the time-varying target. Second, this approach permits evaluating how the omission of a time-varying inflation target could possibly impact other model parameter estimates, and not only those of the Taylor rule. Third, in practice the target is unobserved and needs to be estimated. Using an estimated DSGE approach, we can simultaneously estimate model parameters and the inflation target series.[8]

To remain consistent with the theoretical analysis presented in the previous section, we assess the magnitude of the bias using a NK model similar to that of Ascari and Sbordone (2014). In this respect, the model has a structure which is nearly identical to the basic NK model used to illustrate the bias analytically, but adds some empirically relevant ingredients like positive trend inflation, stochastic trend output growth and external habit formation.[9] There are four sources of exogenous variation: a neutral technology shock, a preference shock, a monetary policy shock, and an inflation target shock.

Following the evidence in Coibion and Gorodnichenko (2012) and Brault and Phaneuf (2022) showing that policy inertia is better captured by second-order interest rate smoothing, the policy rule used for our model includes two smoothing lags.[10] The log-linearized Taylor rule is

$$\hat{i}_t = \rho_1 \hat{i}_{t-1} + \rho_2 \hat{i}_{t-2} + (1 - \rho_1 - \rho_2)\{\phi_\pi(\hat{\pi}_t - \hat{\pi}_t^\star) + \phi_{gy}(\hat{Y}_t - \hat{Y}_{t-1} + \hat{g}_{A,t})\} + \hat{v}_t, \tag{16}$$

where hatted variables denote log-deviations from the steady state, $\hat{i}_t$ is the nominal interest rate, $\hat{\pi}_t$ is the inflation rate, $\hat{\pi}_t^\star$ is the monetary authority's time-varying inflation target, $\hat{Y}_t$ is output, $\hat{g}_{A,t}$ is the growth rate of neutral technology, and $\hat{v}_t$ is a monetary policy shock. The time-varying inflation target and monetary policy shock follow exogenous processes given by

[8] In contrast, simulation evidence from least squares regressions would require us to either assume the target is known, or estimate it separately. This leads to other challenges, such as dealing with the fact that the target series is a generated regressor.

[9] The log-linearized model can be found in the Appendix.

[10] The evidence in Brault and Phaneuf (2022) also shows NK models with interest rate rules featuring second-order smoothing fit the data better in terms of marginal data densities relative to first-order smoothing.

$$\log \pi_t^\star = (1-\rho_\pi)\log \pi + \rho_\pi \log \pi_{t-1}^\star + \epsilon_t^\pi, \tag{17}$$
$$\log v_t = (1-\rho_v)\log v + \rho_v \log v_{t-1} + \epsilon_t^v. \tag{18}$$

Equation (17) implies that in the absence of shocks, the inflation target equals steady state inflation or trend inflation, π. It is important to note that there is a conceptual difference between the two. Trend inflation is a general level of inflation around which the model is log-linearized (see Ascari and Ropele, 2009), while target inflation is a process allowing the central bank's target to temporarily deviate from this level of trend inflation.

The simulation proceeds as follows. We calibrate the model's parameters to standard values used in the literature. The exact calibration is reported in the column labeled "True value" in Table 1. We consider three different sizes of inflation target shocks: small ($\sigma_\pi = 0.005$), moderate ($\sigma_\pi = 0.02$), and large ($\sigma_\pi = 0.04$). These shock sizes (along with the other calibrated parameters) lead to inflation target shocks accounting for roughly 5%, 50%, and 85% of inflation volatility respectively. For each calibration, we generate 5,000 observations of output growth, inflation, and the nominal interest rate. It is worth noting that we treat the inflation target as unobserved, as it is in practice. We then estimate the model's parameters using Bayesian estimation for two Taylor rules: one assuming the inflation target is positive and fixed (equal to steady-state inflation) and the other that the inflation target is positive and time-varying (though unobserved). The prior distributions are standard and described in Table 2. The posteriors are estimated using the Bayesian estimation technique outlined in Section 6. Throughout the rest of the paper we calibrate the persistence in the inflation target process to $\rho_\pi = 0.995$ following the evidence in Cogley et al. (2010).[11]

Table 1 reports posterior parameter estimates for the six scenarios. Since our simulated samples are very large, we omit reporting posterior density intervals because they are extremely small. Our simulation results are consistent with the intuition provided in Section 2.

The true value of the policy response parameter to the inflation gap is 2.25. With small inflation target shocks ($\sigma_\pi = 0.005$), the difference between the estimated response to inflation (ϕ_π) under a fixed and a time-varying inflation target is small (2.11 vs 2.26), implying the bias is negligible. With moderate inflation target shocks ($\sigma_\pi = 0.02$), the posterior mean for the policy response parameter to inflation is 1.52 with a fixed inflation target and 2.26 with a time-varying inflation target, resulting in a significant bias. With large inflation target shocks ($\sigma_\pi = 0.04$), the corresponding posterior mean is 1.18 with a fixed inflation target and 2.31 with a time-varying target, resulting in a larger downward bias.

As the inflation target shock gets larger, there is also a noticeable change in some other parameter estimates. First, the model with a fixed inflation target overestimates the magnitude of price stickiness (ξ_p). While the true value of this parameter is 0.60, under moderate inflation target shocks ($\sigma_\pi = 0.02$), the mean estimate for the Calvo parameter is 0.70 while under large target shocks ($\sigma_\pi = 0.04$) the estimate is 0.75. So, the larger is the inflation target

[11] We provide a discussion of this chosen calibration in Section 6.3.

shock, the stronger is the upward bias in the price stickiness parameter when assuming a fixed inflation target.

As we have shown, assuming a fixed inflation target when target movements are large leads to a strongly downward biased estimate of the policy response parameter to the inflation gap and a strongly upward biased estimate of price stickiness. The estimated probability of indeterminacy is then likely to be biased upwards as both biases tend to favour the indeterminacy outcome.

Second, with larger inflation target shocks, the mean estimates of the AR(1) parameters of the monetary policy (ρ_v) and preference (ρ_b) shocks rise when incorrectly assuming a fixed inflation target. Under a moderate target shock the degree of persistence in monetary policy shocks is 0.41 while under a large shock it rises to 0.47, and this compared to a true value of 0.20. For preference shocks, the most noticeable change is with a large inflation target shock for which the estimated persistence is 0.96 compared to a true value of 0.90.

Lastly, estimates of first- and second-order interest smoothing (ρ_1 & ρ_2) and the policy rate response to output growth (ϕ_{gy}) are found to be lower than their true values when assuming a fixed inflation target. Overall, these findings suggest that assuming a fixed inflation target when in fact the target is time-varying leads to a number of distortions in the estimated Taylor rule coefficients and other model parameters.

4 A Medium-scale NK Model with Positive Trend Inflation

This Section describes the NK model that will serve for the simulation of determinacy regions (Section 5) and model estimation (Sections 6 and 7). In view of our previous arguments, we allow the inflation target to be potentially time-varying. For the sake of generality and to circumvent the possible criticism that NK price-setting models omit key propagation mechanisms and structural shocks that can lead to misinterpreting the evidence about indeterminacy (Nicoló, 2023), we use a framework commonly referred to as a medium-scale NK model.

Its basic structure is similar to the model in Ascari et al. (2018) used to analyze the welfare costs of long-run or positive trend inflation. Common to other medium-scale NK models (Erceg et al., 2000; Christiano et al., 2005), our model features imperfectly competitive goods and labor markets, Calvo staggered wage and price setting, habit formation in consumption, investment adjustment costs and variable capital utilization. However, the main difference with most existing medium-scale models is that we allow for positive trend inflation to have cyclical and long-run implications.

The model is driven by seven structural shocks, namely shocks to the discount rate, labor hours, government spending, neutral technology, marginal efficiency of investment (MEI), policy rate and inflation target.

Neutral productivity obeys a process embedding both a trending and stationary component. Investment specific technology (IST) follows a deterministic trend with no stochastic component. The deterministic trend captures the downward trend in the relative price of investment goods observed in the data. The shock to the marginal efficiency of investment follows a stationary AR(1) process.

Justiniano et al. (2011) distinguish between IST and MEI shocks, as they show IST shocks map one-to-one into the relative price of investment goods, while MEI shocks do not affect the relative price of investment. They find that MEI shocks are significant drivers of business cycle fluctuations, while IST shocks contribute almost nothing to fluctuations in output, consumption, investment and hours.[12] These considerations form the basis for modeling the MEI component as stochastic and the IST term as deterministic.

4.1 Aggregate Output and Intermediate Goods Producers

Aggregate output, Y_t, is produced by a perfectly competitive firm using a continuum of intermediate goods, Y_{jt}, $j \in (0,1)$, and follows a constant elasticity of substitution (CES) production technology:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{19}$$

where $\theta > 1$ is the elasticity of substitution between differentiated goods.

Profit maximization and a zero-profit condition leads to a downward sloping demand for intermediate good Y_{jt}:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t, \tag{20}$$

where P_{jt} is the price of intermediate good Y_{jt}, and P_t is the aggregate price index with the two being related by:

$$P_t = \left(\int_0^1 P_{jt}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{21}$$

A monopolist produces intermediate good Y_{jt} according to the production function:

$$Y_{jt} = A_t K_{jt}^{\alpha} L_{jt}^{1-\alpha} - \gamma_t F, \tag{22}$$

where F is the fixed cost, α is the capital share of income, γ_t is the growth factor (see below) and production is required to be non-negative. Given γ_t, F is chosen to keep zero profits along a balanced growth path. K_{jt} represents capital services defined as the product of physical capital and the rate of capital utilization. L_{jt} is the labor input.

[12]See Table 3 in Justiniano et al. (2011).

Neutral productivity (A_t) comprises both a trending component (A_t^τ) and a stationary component $(\tilde{A}_t)$. The trending component A_t^τ follows a deterministic process, $A_t^\tau = g_A A_{t-1}^\tau$, with gross growth rate g_A. The stationary component follows an AR(1) process

$$\tilde{A}_t = \rho_a \tilde{A}_{t-1} + \epsilon_t^a \qquad 0 \leq \rho_a < 1 \tag{23}$$

where ϵ_t^a is an *i.i.d* $N\left(0, \sigma_a^2\right)$ technology shock.

The trend growth factor γ_t is given by the composite technological progress

$$\gamma_t = \left(A_t^\tau\right)^{\frac{1}{1-\alpha}} \left(\epsilon_{I,t}^\tau\right)^{\frac{\alpha}{1-\alpha}} \tag{24}$$

where $\epsilon_{I,t}^\tau$ denotes investment-specific technological progress, which follows a deterministic process, $\epsilon_{I,t}^\tau = g_I \epsilon_{I,t-1}^\tau$, where g_I is the gross growth rate of IST.

4.1.1 Cost minimization

An intermediate goods firm chooses its price and quantities of capital services and labor input. In choosing its inputs, a firm minimizes total cost subject to the constraint of meeting demand. The cost minimization problem is then given by:

$$\min_{K_{jt}, L_{jt}} \left(R_t^k K_{jt} + W_t L_{jt}\right)$$

subject to:

$$A_t K_{jt}^\alpha L_{jt}^{1-\alpha} - \gamma_t F \geq \left(\frac{P_{jt}}{P_t}\right)^{-\theta} Y_t \tag{25}$$

where R_t^k denotes the nominal rental price of capital services and W_t the nominal wage index. Solving the cost minimization problem yields the following real marginal cost

$$mc_t = \alpha^{-\alpha} (1-\alpha)^{\alpha-1} A_t^{-1} \left(r_t^k\right)^\alpha w_t^{1-\alpha} \tag{26}$$

and demand functions for factor inputs

$$K_{jt} = \alpha \frac{mc_t}{r_t^k} \left(Y_{jt} + \gamma_t F\right) \tag{27}$$

$$L_{jt} = (1-\alpha) \frac{mc_t}{w_t} \left(Y_{jt} + \gamma_t F\right) \tag{28}$$

where $mc_t = \frac{MC_t}{P_t}$ is the real marginal cost, $r_t^k = \frac{R_t^k}{P_t}$ is the real rental price of capital services, and $w_t = \frac{W_t}{P_t}$ is the real wage, which are common to all firms.

4.1.2 Price Setting

Each period an intermediate good firm is allowed to re-optimize its price with probability $(1 - \xi_p)$. Firms allowed to re-optimize their price choose the same price P^*_{jt}, while non-price-setting firms keep their prices the same as in the previous period. Our price-setting formulation implies that there is no automatic indexation of non-reset prices.[13]

When re-optimizing its price, an intermediate good firm j chooses a price so as to maximize the present discounted value of future profits subject to the demand for its good from the final good firm and to cost minimization

$$\max_{P^*_{jt}} E_t \sum_{s=0}^{\infty} \xi_p^s \mathcal{D}_{t,t+s} \left[\frac{P^*_{jt}}{P_{t+s}} Y_{j,t+s} - mc_{t+s} Y_{j,t+s} \right] \tag{29}$$

where $\mathcal{D}_{t,t+s}$ is the stochastic discount factor, and mc_{t+s} is the real marginal cost in period $t+s$ as defined above. Letting $p^*_{jt} = \frac{P^*_{jt}}{P_t}$ denote the relative price of the optimizing firm at t, the first-order condition of this problem can be written as

$$p^*_{jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \xi_p^s \mathcal{D}_{t,t+s} Y_{j,t+s} \Pi^{\theta}_{t,t+s} mc_{t+s}}{E_t \sum_{s=0}^{\infty} \xi_p^s \mathcal{D}_{t,t+s} Y_{j,t+s} \Pi^{\theta-1}_{t,t+s}} \tag{30}$$

where $\Pi_{t,t+s}$ denotes the cumulative gross inflation rate over s periods:

$$\Pi_{t,t+s} = \begin{cases} 1 & \text{for } s = 0 \\ \left(\frac{P_{t+1}}{P_t}\right) \times \cdots \times \left(\frac{P_{t+s}}{P_{t+s-1}}\right) & \text{for } s = 1, 2, \cdots \end{cases} \tag{31}$$

4.2 Households

Households supply differentiated labor input and face a downward-sloping demand for their labor type. In each period a household i is allowed to re-optimize its nominal wage with probability $(1 - \xi_w)$. As in Erceg, Henderson and Levin (2000), we assume that utility is separable in consumption and labor. State-contingent securities insure households against idiosyncratic wage risk arising from staggered wage-setting. Households are therefore identical along all dimensions other than labor supply and wages.

The problem of a typical household (omitting dependence on i except for labor and wage) is

$$\max_{C_t, L_{it}, K^p_{t+1}, B_{t+1}, I_t, Z_t} E_0 \sum_{t=0}^{\infty} \beta^t b_t \left(ln\left(C_t - hC_{t-1}\right) - \eta_t \frac{L_{it}^{1+\chi}}{1+\chi} \right) \tag{32}$$

[13]In NK models with Calvo wage and price setting, the indexation assumption implies that all wages and prices change once every three months. Ascari, Phaneuf and Sims (2018) offer survey evidence suggesting this is simply at odd with the literature on wage and price changes from micro data.

subject to the following budget constraint

$$P_t \left[C_t + I_t + \left(\gamma_1 (Z_t - 1) + \frac{\gamma_2}{2} (Z_t - 1)^2 \right) \frac{K_t^p}{\varepsilon_{I,t}^\tau} \right] + \frac{B_{t+1}}{i_t} \leq W_{it} L_{it} + R_t^k Z_t K_t^p + \Pi_t + B_t, \quad (33)$$

the physical capital accumulation process

$$K_{t+1}^p = m_t \epsilon_{I,t}^\tau \left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - g_I \right)^2 \right) I_t + (1-\delta) K_t^p, \quad (34)$$

and a downward sloping demand for labor. Here b_t denotes a shock to the discount factor, η_t a shock to the disutility of labor, and m_t a shock to the marginal efficiency of investment (MEI). C_t is consumption, I_t is investment measured in units of consumption, K_t^p is the physical capital stock, Z_t is the level of capital utilization, W_{it} is the nominal wage paid to labor of type i, R_t^k is the common rental price of capital services, B_t is the stock of nominal bonds the household enters with in period t, i_t is the one-period risk-free nominal interest rate, and Π_t represents dividends received from firms. β is the discount factor, χ is the inverse Frisch elasticity of labor supply, h is the degree of (internal) habit formation in consumption, γ_1 and γ_2 are parameters related to the resource cost of capital utilization (measured in units of physical capital), κ captures the cost of investment adjustment relative to trend growth g_I, and δ is the depreciation rate.

Each of the three shocks x_t, $x \in \{b, \eta, m\}$ is assumed to follow a stationary AR(1) process

$$\log x_t = (1 - \rho_x) \log x + \rho_x \log x_{t-1} + \epsilon_t^x \quad (35)$$

where $\rho_x \in [0, 1)$ is the autoregressive parameter and $\epsilon_t^x \sim N(0, \sigma_x^2)$ is the innovation to each shock.

4.3 Employment Agencies and Wage Setting

A large number of competitive employment agencies combine differentiated labor types into one homogeneous labor input which is sold to intermediate goods firms according to

$$L_t = \left(\int_0^1 L_{it}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (36)$$

where $\sigma > 1$ is the elasticity of substitution between differentiated types of labor.

Profit maximization by perfectly competitive employment agencies implies the following labor demand function

$$L_{it} = \left(\frac{W_{it}}{W_t} \right)^{-\sigma} L_t \quad (37)$$

where W_{it} is the wage paid to labor of type i and W_t is the aggregate wage index given by

$$W_t = \left(\int_0^1 W_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \quad (38)$$

Households allowed to re-optimize their wage all choose the same wage W_{it}^*, while non-wage setting households keep their wages the same as that in the previous period. When re-optimizing its wage, households choose a wage for its labor type that maximizes the present discounted value of lifetime utility subject to the demand for its labor. Letting $w_{it}^* = \frac{W_{it}^*}{P_t}$ denote the real wage for labor type i at time t, the first-order condition of this problem is written as

$$(w_{it}^*)^{1+\sigma\chi} = \frac{\sigma}{\sigma-1} \frac{E_t \sum_{s=0}^{\infty} \beta^s \xi_w^s \eta_t \pi_{t,t+s}^{\sigma(1+\chi)} w_{t+s}^{\sigma(1+\chi)} L_{t+s}^{1+\chi}}{E_t \sum_{s=0}^{\infty} \beta^s \xi_w^s \pi_{t,t+s}^{\sigma-1} w_{t+s}^{\sigma} \lambda_{t+s}^r L_{t+s}} \tag{39}$$

where $\Pi_{t,t+s}$ denotes the cumulative gross inflation rate over s periods as before and λ_t^r is the marginal utility of an additional unit of real income received by the household.

4.4 Monetary and Fiscal Policies

Monetary policy is set according to a Taylor rule wherein the monetary authority follows an inertial rule and responds to deviations of inflation from target, to output gap and output growth, or to output growth only. As we mentioned before, the policy rule includes second-order interest rate smoothing.

We consider two different types of rules when simulating determinacy regions and estimating models. One is the MO-rule where monetary policy reacts to both output gap and output growth, and the other is the OG-rule where the policy rate responds only to output growth. The general formulation for the policy rule is

$$\frac{i_t}{i} = \left(\frac{i_{t-1}}{i}\right)^{\rho_1} \left(\frac{i_{t-2}}{i}\right)^{\rho_2} \left[\left(\frac{\pi_t}{\pi_t^*}\right)^{\phi_\pi} \left(\frac{Y_t^{GDP}}{Y_t^{GDP*}}\right)^{\phi_y} \left(\frac{Y_t^{GDP}}{Y_{t-1}^{GDP}} g_Y^{-1}\right)^{\phi_{gy}} \right]^{1-\rho_1-\rho_2} v_t \tag{40}$$

where π_t is the inflation rate, π_t^* is the time-varying inflation target, Y_t^{GDP} is real gross domestic product (GDP), Y_t^{GDP*} is the counterfactual level of real GDP assuming flexible prices and wages, i is the steady state nominal rate, g_Y is steady-state output growth rate, and ρ_1, ρ_2, ϕ_π, ϕ_y and ϕ_{gy} are monetary policy parameters. Here, v_t is a monetary policy shock that follows an AR(1) stochastic process as before

$$\log v_t = (1-\rho_v)\log v + \rho_v \log v_{t-1} + \epsilon_t^v, \tag{41}$$

where $\epsilon_t^v \sim N\left(0, \sigma_v^2\right)$. The inflation target is also assumed to follow the same stochastic process as before

$$\log \pi_t^\star = (1-\rho_\pi)\log \pi + \rho_\pi \log \pi_{t-1}^\star + \epsilon_t^\pi, \tag{42}$$

where π is the steady-state inflation rate and $\epsilon_t^\pi \sim N\left(0, \sigma_\pi^2\right)$ is a shock to the inflation target.

The MO-rule models assume ϕ_y and ϕ_{gy} are possibly different from zero. The OG-rule models impose $\phi_y = 0$ while ϕ_{gy} is different from zero.

Fiscal policy is fully Ricardian and the government finances its budget deficit by issuing short-tem bonds. Public spending is assumed to be a time-varying fraction of aggregate output Y_t:

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t, \tag{43}$$

where g_t is a government spending shock that follows an AR(1) process given by

$$\log g_t = \left(1 - \rho_g\right) \log g + \rho_g \log g_{t-1} + \epsilon_t^g \tag{44}$$

where g is the steady-state level of government spending and $\epsilon_t^g \sim N\left(0, \sigma_g^2\right)$ is a government spending shock.

4.5 Market Clearing and Equilibrium

Market clearing for capital and labor inputs requires $\int_0^1 K_{jt} dj = K_t$ and $\int_0^1 L_{jt} dj = L_t$, respectively.

Aggregate output can be written as:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} - \gamma_t F. \tag{45}$$

The resource constraint in the economy is given by

$$Y_t = C_t + I_t + G_t + \left(\gamma_1 \left(Z_t - 1\right) + \frac{\gamma_2}{2} \left(Z_t - 1\right)^2\right) \frac{K_t^p}{\varepsilon_{I,t}^{\tau}} \tag{46}$$

Finally, real GDP is defined as

$$Y_t^{GDP} = C_t + I_t + G_t. \tag{47}$$

4.6 Detrending and Log-linearization

Most variables in the model inherit trend growth, defined by the growth factor γ_t, from the deterministic trends in neutral and investment-specific technological progress. In particular, output, consumption, investment, intermediate inputs, and the real wage all grow at the rate of this trend factor on a balanced growth path. The capital stock grows faster due to growth in IST with $\tilde{K}_t = \frac{K_t}{\gamma_t \varepsilon_{I,t}^{\tau}}$ being stationary. Solving the model requires detrending the variables and log-linearizing the resulting stationary equilibrium conditions around the non-stochastic steady state. The full set of log-linearized equilibrium conditions can be found in the Appendix.

5 Determinacy Regions

In this Section we offer simulation evidence about the minimum policy response to the inflation gap consistent with determinacy. For this, we simulate determinacy regions using the medium-scale model described in the previous section.

Our simulation analysis focuses on determinacy regions conditioned on three factors: i) a level of trend inflation ranging from an annual rate of 0% to 8%, ii) MO and OG policy rules, and iii) a high and low inverse Frisch elasticity of labor supply.[14] The remaining parameters are calibrated at the prior mean (see discussion on priors in the next Section for more details).

Figure 2 displays determinacy regions for the MO-rule model, with Panel A conveying information about the minimum policy rate response to the inflation gap consistent with determinacy for different levels of trend inflation and a high inverse Frisch elasticity of labor supply ($\chi = 1.75$). With zero trend inflation the minimum policy response parameter to the inflation gap consistent with determinacy is about 1.15. With 2.0% trend inflation, it rises to 1.5, with 3% trend to 1.75, and with 4% to 2.25. An inflation trend of 6.5% would require a minimum policy response to inflation of 5.0.

Panel B of Figure 2 displays determinacy regions with a low inverse Frisch elasticity of labor supply ($\chi = 0.55$). A higher labor supply elasticity lowers the minimum response parameter to inflation consistent with determinacy. With zero trend inflation, a response parameter of 1.1 achieves determinacy. With 4% trend inflation rate, it needs to be 1.35 or more. With 8% trend inflation, determinacy obtains with a parameter of 2 or more.

Panel A of Figure 3 displays determinacy regions for the OG-rule model and a high inverse Frisch elasticity of labor supply. For trend inflation from 0% to 8%, the minimum policy response parameter to inflation consistent with determinacy ranges from 1.0 to 1.23.

Panel B of Figure 3 displays determinacy regions implied by the OG-rule model and a low inverse Frisch elasticity. In this case, determinacy is guaranteed with a minimum policy response parameter of 1.0 for up to 8% trend inflation.

The minimum policy response to inflation consistent with determinacy is thus much higher when assuming a MO-rule than an OG-rule. This is broadly consistent with the analysis of Coibion and Gorodnichenko (2011) based on a NK model with positive trend inflation, flexible nominal wages and sticky prices, and that of Khan et al. (2020) which is based on a NK model with sticky wages and sticky prices, suggesting that targeting the output gap can be destabilizing relative to a policy targeting output growth.

Our evidence also suggests that labor supply elasticity is a critical factor affecting determinacy regions when monetary policy is described by a MO-rule. This is because the inverse Frisch elasticity of labor supply is a key parameter governing the strength of the interaction between nominal wage rigidity and positive trend inflation. A higher elasticity alleviates the effects of positive trend inflation on steady-state monopolistic distortions. The reason is that positive trend inflation distorts the relative allocation of labor across households through an effect on wage dispersion. A stronger curvature in preferences over labor ($\chi = 1.75$) leads to a stronger wage dispersion and labor misallocation. When this curvature is relatively unimportant ($\chi = 0.55$), misallocation of labor arising from wage dispersion has much weaker steady-state effects on the determinacy outcome. A MO-rule magnifies these distortion effects relative to an OG-rule.

[14] The high and low values of χ corresponds to the lowest and highest estimates we later report.

6 Estimation Strategy

The posterior distributions of the structural parameters are estimated using Bayesian methods. This section describes the data used in the estimation, the construction of the linear rational expectations (LRE) solution when the equilibrium is indeterminate, the prior distributions, and the posterior sampling method.

6.1 Data

The model is estimated using seven observables: real output growth, consumption growth, investment growth, wage growth, hours worked, inflation, and the central bank's policy rate. The raw data were obtained from the Federal Reserve Bank of St. Louis (FRED) database and mnemonics are reported in brackets. Nominal output is Gross Domestic Product (GDP) and nominal consumption is the sum of personal consumption expenditures on non-durable goods (PCND) and services (PCESV). Nominal investment is the sum of gross private domestic investment (GPDI) and personal consumption expenditures on durable goods (PCDG). Wages are the hourly compensation for all workers in the nonfarm business sector (COMPNFB) and hours worked is the hours worked for all workers in the nonfarm business sector (HOANBS). Inflation is the growth rate of the GDP deflator (GDPDEF) and the federal funds rate is used as the measure of the central bank's policy rate (FEDFUNDS).

All nominal quantities are converted to real values using the GDP deflator and expressed in per capita terms. Our population measure is a smoothed trend obtained from fitting the Civilian Non-institution Population (CNP16OV) series with a Hodrick-Prescott filter (with a smoothing parameter equal to 10,000). The rationale for this, as noted by Pfeifer (2020), is that population levels are periodically updated due to censuses or benchmarking in the Current Population Survey. These updates cause spikes in population growth rates not related to changes in the actual population.

6.2 LRE Solution

A number of findings reported in the previous literature suggest that prior to 1980 the Fed did not respond strongly enough to inflation to achieve equilibrium determinacy. To permit equilibrium indeterminacy, we use the approach outlined in Bianchi and Nicoló (2021) and append the following autoregressive equation to our model:

$$\omega_t = \frac{1}{\alpha_{BN}}\omega_{t-1} + \epsilon_t^s - (\pi_t - \mathbb{E}_{t-1}\pi_t), \tag{48}$$

where ϵ_t^s is a sunspot shock with $\epsilon_t^s \sim N\left(0, \sigma_s^2\right)$. The added equation works to ensure there exists an appropriate number of explosive roots such that there is a unique Linear Rational Expectations (LRE) solution to the model in both the determinacy and indeterminacy regions of the parameter space. The solution to the model in the determinacy region features structural parameters consistent with equilibrium determinacy and a value for α_{BN}

higher than one. In this case equation (48) is not explosive and the sunspot shock plays no role in the model dynamics. The solution to the model in the indeterminacy region features structural parameters leading to indeterminacy and a value for α_{BN} smaller than one. In this case equation (48) is explosive, providing the missing explosive root, so that endogenous variables are impacted by the sunspot shock which is linked to the expectation errors of inflation. We allow the sunspot shock introduced under indeterminacy to be arbitrarily correlated with other fundamental shocks in the model.[15]

6.3 Priors

Prior to estimation we calibrate a small number of parameters to values commonly used in the literature. The quarterly depreciation rate of capital is set to 0.025, the discount factor at 0.994, the steady state government spending to GDP ratio at 0.22, the trend growth rate of investment-specific technological change at 0.37% per quarter, and the parameters that capture the elasticity of substitution between goods and labour to 6.

The remaining parameters are estimated and their prior distributions are summarized in Table 3. The prior densities, means, and standard deviations are almost identical to those of Justiniano et al. (2011) for the capital share, consumption habit, inverse Frisch labor supply elasticity, Calvo probabilities of wage and price non-reoptimization, capital utilization elasticity, investment adjustment cost and the Taylor rule parameter responses to output gap and output growth. Based on evidence in Coibion and Gorodnichenko (2011) and Brault and Phaneuf (2022), we use a Normal distribution with mean of 0.8 and standard deviation of 0.2 for the AR(1) smoothing parameter, and a mean prior of -0.1 and standard deviation of 0.2 for the AR(2) smoothing parameter.

Since we allow the possibility of equilibrium indeterminacy, we use a Normal distribution centered at 1.1 with a standard deviation of 0.5 for the response parameter to the inflation gap. Given other parameter priors, this implies a prior probability of determinacy of .5 conditioned on the OG-rule model and .25 on the MO-rule model. These different prior probabilities can be explained in light of our simulation analysis of determinacy regions. We saw when trend inflation is positive that determinacy generally requires a much higher response to the inflation gap conditioned on the MO-rule model than on the OG-rule model, explaining these differing prior probabilities.

Targeting a prior probability of determinacy of 0.5 for the MO-rule model and for the OG-rule model would mean assigning different prior values to model parameters. We choose instead to keep the same prior means and standard deviations for both model parameters, except for the policy response parameter to the output gap which is centered at 0.13 with a standard deviation of 0.05 when using the MO-rule model, while it is set to 0 when using the OG-rule model. While the prior probability of determinacy implied by the MO-rule model is significantly lower than 0.5, our evidence will later suggest that the MO-rule model with positive trend inflation and a time-varying inflation target is nonetheless consistent with determinacy with a high estimated probability.

[15] Bianchi and Nicoló (2021) show that when the covariance matrix of the shocks is left unrestricted, changing which expectations errors are included in equation (48) will not alter the fit to the data.

Following Cogley et al. (2010), we calibrate the persistence parameter of the inflation target process to 0.995. This calibration restricts target movements so they capture the low-frequency movements in inflation, but also implies that long-run changes in the level of inflation cannot occur without a corresponding change in the central bank's inflation target (Ireland, 2007). Assuming a highly persistent target process, either through calibration or a prior distribution, is common in the literature. Both Cogley et al. (2010) and Justiniano et al. (2013) calibrate the persistence to 0.995, as does Del Negro et al. (2022) for the New York Fed's DSGE model. In Christiano et al. (2014) the persistence is calibrated to 0.975 for the post-1985 period. In Ireland (2007) and Aruoba and Schorfheide (2011), the target is treated as a random walk process ($\rho_\pi = 1$). Lastly, Haque (2022) estimates target persistence with a prior centered at 0.95.

For the additional parameters under indeterminacy we use the following priors. The parameter α_{BN} has a uniform prior over the interval $[0.5, 1.5]$. The correlations between the sunspot shock and fundamental shocks have uniform priors over the interval $[-1, 1]$.

6.4 Estimation Methodology

We employ Bayesian methods to estimate the models using techniques that allow the joint estimation of determinacy and indeterminacy regions of the parameter space. Standard algorithms like Metropolis-Hastings can struggle to accurately characterize the posterior distribution because the likelihood function features discontinuities around the boundary between indeterminacy and determinacy.

For this reason, we adopt a parallel tempering algorithm (Brault, 2024). This algorithm is a population-based MCMC method which is particularly well-suited for problems with ill-behaved posteriors, such as those with discontinuities in the likelihood or multi-modal distributions. Further, the algorithm does not require finding the posterior mode or Hessian prior to initializing the MCMC routine.

Parallel tempering approximates a target distribution using a family of Markov chains arranged according to a temperature ladder. Each Markov chain has a specified temperature used to temper the likelihood in the posterior, and each posterior is approximated by

$$P_m(\theta|Y) \propto (P(Y|\theta))^{\xi_m} P(\theta), \tag{49}$$

where ξ_m is the (inverse) temperature parameter. Temperatures can range from zero to one. Chains with lower values of ξ_m afford relatively more weight to the prior distribution in the posterior. These chains have flatter posterior surfaces and are therefore able to make relatively larger moves around the parameter space, while chains with higher values of ξ_m search in a more confined space. The target posterior distribution is a single chain in the ladder which is characterized by $\xi_m = 1$.

The algorithm iterates over two types of updates: exchange and mutation. Exchange involves randomly selecting two chains in the family and proposing a swap between their parameter vectors, which is accepted according to a Metropolis criterion. Mutation involves a pre-specified number of random walk Metropolis-Hastings steps for each chain. The auxiliary chains store information on the target distribution, but also allow the target chain to easily cross between the indeterminacy and determinacy regions of the parameter space.

To tune the parallel tempering algorithm, we use a family of 10 Markov chains with a uniformly spaced temperature ladder ranging from zero to one. Each chain is initialized with draws from the prior distribution and a covariance matrix for the proposal density equal to the prior covariance matrix. We use 10 thinning iterations, implying that exchange steps are only proposed every 10 mutation steps. We run the algorithm for 2,000,000 iterations, using the first 500,000 iterations as a warm-up and to tune the scaling parameters for each chain to ensure a reasonable mutation acceptance rate.

7 Estimation Results

This Section presents and discusses the estimation results for the OG-rule and MO-rule models including a time-varying inflation target, and this assuming either positive or zero trend inflation for the periods 1960:I-1979:II and 1983:I-2007:IV.[16]

We first present estimates of the inflation target obtained from our estimated OG-rule model prior to 1980 and after 1982 on the left panel of Figure 4 and compare them to those estimated by Ireland (2007), Aruoba and Schorfheide (2011), and Coibion and Gorodnichenko (2011) on the right panel of the Figure.[17]

Figure 4 shows that the Fed's inflation target was rising throughout the 1960s and 1970s, exhibiting a peak of over 8% around 1974. The rise in the inflation target could be interpreted as a systematic tendency for Fed policy to avoid some of the contractionary impact of adverse supply shocks on real economic activity (Ireland, 2007) or the Fed's changing beliefs about the output-inflation trade-off (Cogley and Sargent, 2005). By the mid-1980s, the Fed's target had fallen to roughly 4% and continued to decline to around 2% by 2000. Thereafter, the target rose in the early-to-mid 2000s, which was an extended period of low interest rates.[18] Overall, one sees that movements in the inflation target implied by our OG-rule model are broadly similar and consistent with those obtained by others despite a variety of approaches, samples and data.

Tables 4 and 5 present the estimated parameters for the OG-rule and MO-rule models with positive trend inflation for the two sub-samples, respectively. Table 6 presents parameter estimates for similar models when assuming zero trend inflation.

[16]The period between the latter half of 1979 and 1982 is commonly excluded since it is generally accepted that the Fed was targeting non-borrowed reserves and not the Federal Funds rate during this period (Bernanke and Mihov, 1998). However, our findings are very similar if we include these years, so we do not explicitly report them for the sake of brevity.

[17]The estimated inflation target from the MO-rule and OG-rule models are very similar and so we show just the latter to conserve space.

[18]Eggertsson et al. (2003) note that keeping interest rates low for an extended period of time is equivalent to a rise in the inflation target, as we find. For alternative interpretations of monetary policy during the 2000s, see Groshenny (2013); Belongia and Ireland (2016); Doko Tchatoka et al. (2017).

Conditioned on estimated marginal data densities, we find that OG-rule and MO-rule models with positive trend inflation are relatively similar in a statistical sense for both periods, although OG-rule models are marginally preferred to MO-rule models. Moreover, estimated model data densities indicate that models with positive trend inflation are quite decisively preferred statistically to models with zero trend inflation. When comparing model data densities for the first period, we get for the OG-rule model -518.0 $>$ -522.1 in favour of positive over zero trend inflation, while for MO-rule models we obtain -518.7 $>$ -524.2 favouring positive trend inflation. For second period models, we report for OG-rule models -428.5 $>$ -437.9 and for MO-rule models -429.6 $>$ -444.8, both favouring models with positive trend inflation.

Of the eight models we have estimated, we find that seven have policy response parameters to the inflation gap of 2.0 or more in terms of the posterior mean estimates. The one exception is the MO-rule model with zero trend inflation estimated for the period 1983-2007 for which the posterior mean of the policy parameter to inflation is 1.35, a point to which we return below.

OG-rule models with positive trend inflation rate predict determinacy with probability 1 for both periods, while MO-rule models imply determinacy with probability .93 for the first period and .90 for the second period. When assuming zero trend inflation, OG-rule models predict determinacy with probability 1 for the first period and .87 for the second period, while MO-rule models imply determinacy with probability 1 and .51, respectively.

Now, previous studies have often reported response parameters to the inflation gap between 0.4 and 1 for the pre-Volcker period, suggesting the Fed did not increase the nominal interest rate strongly enough in response to inflation, which possibly led to indeterminacy. These studies also report evidence showing that this policy response was much stronger after 1982, which possibly prevented self-fulfilling inflationary expectations and indeterminacy from prevailing.

Several of these studies assumed a fixed inflation target. The intuition described previously about the conditions potentially leading to a significant downward bias in the policy response parameter to the inflation gap helps understand the reasons for differing estimates with pre-1980 and post-1982 data. Based on a simpler model, we argued that inflation target shocks generate a positive comovement between inflation and the policy rate, and a positive impulse-response of inflation which is larger than the response of the policy rate (see Figure 1).

Figure 5 shows there are similar effects of a positive inflation target shock at work in our estimated medium-scale NK model. The figure plots the mean impulse-response functions to a positive inflation target shock of inflation, the policy rate, and the ex-ante real interest rate along with their 90% HPD intervals based on our estimated OG-rule model for the two periods. The added nominal wage stickiness and real frictions help generate hump-shaped responses relative to the standard three-equation NK model. A positive inflation target shock generates a procyclical comovement of inflation and the policy rate. The increase in the policy rate is smaller than the increase in inflation, so the real interest rate declines. The impulse-responses to a positive target shock are consistent with our stylized model in Section 2, and similar to those implied by other small scale NK models (see for example Figure 3 in Ireland (2007)).

This being said, there are a few exceptions to the evidence that the US economy moved from a state of indeterminacy prior to 1980 to determinacy after 1982. For instance, Haque (2022) and Brault and Phaneuf (2022) report evidence suggesting the US economy did not experience indeterminacy prior to 1980 and after. Their evidence is based on estimated NK models including positive trend inflation, a time-varying inflation target, sticky prices and flexible nominal wages. However, no formal theoretical explanation is offered for the relation between the treatment of the inflation target, whether fixed or time-varying, and potential biases in Taylor rule estimates. Furthermore, as mentioned earlier, the interaction of positive trend inflation with sticky wages is known to be the one having the most significant impact on the cyclical and long-run properties of medium-scale NK models, and this even at moderate levels of trend inflation (Ascari et al., 2018; Khan et al., 2020). Therefore, omitting sticky wages can significantly alter model parameter estimates.

One exception is Haque et al. (2021) who estimate a sticky-price model with positive trend inflation, commodity price shocks and sticky wages and document that the Fed responded aggressively to inflation but negligibly to the output gap in the pre-Volcker period such that monetary policy-induced indeterminacy and sunspots were not causes of macroeconomic instability during the Great Inflation. However, they assume a fixed inflation target equal to the trend inflation rate in the economy and so their parameter estimates are prone to the bias we identify in this paper.

Another exception is Nicoló (2023), who estimates the Smets and Wouters (2007) (hereafter SW) model with a Bayesian method allowing for the possibility of determinacy and indeterminacy within a single estimation. His evidence suggests the US economy experienced indeterminacy in 1955:IV-1979:II and 1982:IV-2007:IV.[19] Obviously our findings differ. This can be explained by some key modeling assumptions and estimation methods.

A major difference between our models is that the SW model is counterfactually log-linearized around zero steady-state inflation, while our model is realistically log-linearized around non-zero steady-state inflation. As mentioned earlier, modeling developments that took place since SW wrote their paper has established that positive trend inflation significantly affects the cyclical and long-run properties of medium-scale NK models. Our evidence which allows comparing models with positive and zero trend inflation indicates that models with positive trend inflation are preferred in a statistical sense to models with zero trend inflation.

A second difference is that the SW model is estimated solely with a MO-rule. This type of rule has been adopted almost systematically in the NK literature afterwards. Instead, we contrast estimation results from OG-rule and MO-rule models. When estimating a MO-rule model with zero trend inflation for the period 1983:I-2007:IV, we find that the probability of determinacy drops down to .5 and hence closer to Nicolò's estimate. Our simulation evidence based on the MO-rule model with zero trend inflation suggests that with an inverse

[19]Using data for the period 1979:III to 2007:IV, Nicolò also reports evidence of determinacy assuming a fixed inflation target and indeterminacy with time-varying inflation target.

Frisch elasticity of labor supply of 1.75, and Calvo wage and price probabilities of .66, the minimum policy response parameter to the inflation gap required for determinacy is 1.2. Nicolò reports several estimates for the inverse Frisch elasticity which are nearly 2 or higher. Our own estimated elasticity based on the estimated MO-rule model with zero trend inflation is 2.37. In this particular case, the estimated posterior mean policy response to inflation is 1.35, and is thus much lower than the estimated policy responses for our other models.

A third difference is when we estimate the OG-rule model with zero trend inflation over the period 1983:I-2007:IV. Here we obtain two main results. One is that the OG-rule model with zero trend inflation is decisively preferred to the MO-rule model based on estimated model data densities (-437.9 $>$ -444.8). The other is that the OG-rule model with zero trend inflation implies an estimated probability of determinacy of .87, which is significantly higher than .5 estimated with the MO-rule model.

A final difference concerns the determinacy results based on our differing Bayesian methods. Nicolò contrasts the solution method ensuring an appropriate parametrization under indeterminacy proposed by Lubik and Schorfheide (2004) with the method of Bianchi and Nicoló (2021), concluding from simulated data of an indeterminate version of the SW model that estimation of the model with the former method points to evidence of determinacy while that with the later method recovers the indeterminacy result. He goes on estimating the SW model with the later method.

We also use the solution method of Bianchi and Nicoló (2021), which we combine with a parallel tempering, population-based MCMC routine. Our intent is also to allow simultaneous estimation of determinacy and indeterminacy regions of the parameter space. Of the eight model versions we estimate, we find four cases where the estimated probabilities of determinacy and indeterminacy lie between 0 and 1. Paradoxically, of the 26 different model versions estimated by Nicolò, there is not a single case where the estimated probability of indeterminacy is not either 1 or 0.

8 Conclusion

In this paper we have questioned the reliability of estimated Taylor rules for monetary policy analysis. Our paper has uncovered a potential source of bias in Taylor rule estimates when the Fed is assumed to be targeting a constant long-run inflation rate in setting nominal interest rates while its target is actually time-varying.

According to our analysis, the bias in policy rule estimates is stronger in periods when inflation target shocks are "large". This, in turn, could lead to wrong statistical inference about the determinacy outcome. The problem is that the adjustment of inflation and the nominal interest rate following a positive target shock in a time-varying inflation target policy regime can be confounded with those of passive policy in a fixed inflation target policy regime. In periods where inflation target shocks are "small", this bias should be negligible.

We have offered evidence that inflation target shocks have been large during the period 1960-1979 but much smaller in the period 1983-2007. Consistent with our analysis, a substantial strand of the literature has reported evidence of policy response parameters to the inflation gap somewhat smaller than 1 prior to 1980 and significantly higher than 1 after 1982. Unsurprisingly, a number of studies have concluded that a change in the Fed's policy stance against inflation has initiated a transition from a state of indeterminacy to one of determinacy after 1982.

While in this paper the focus of our empirical application has been retrospective, we believe our findings have important implications for contemporary research due to the shift in the stance of monetary policy by many central banks around the world. Specifically, the emphasis on the medium-term inflation outcomes suggests a moving target for central banks depending on a variety of factors. Our findings suggest that if the movements are modest, assuming a fixed target is likely to be harmless for model estimation. However, if target movements are substantial, obtaining unbiased estimates will require modeling and estimating variation in the inflation target.

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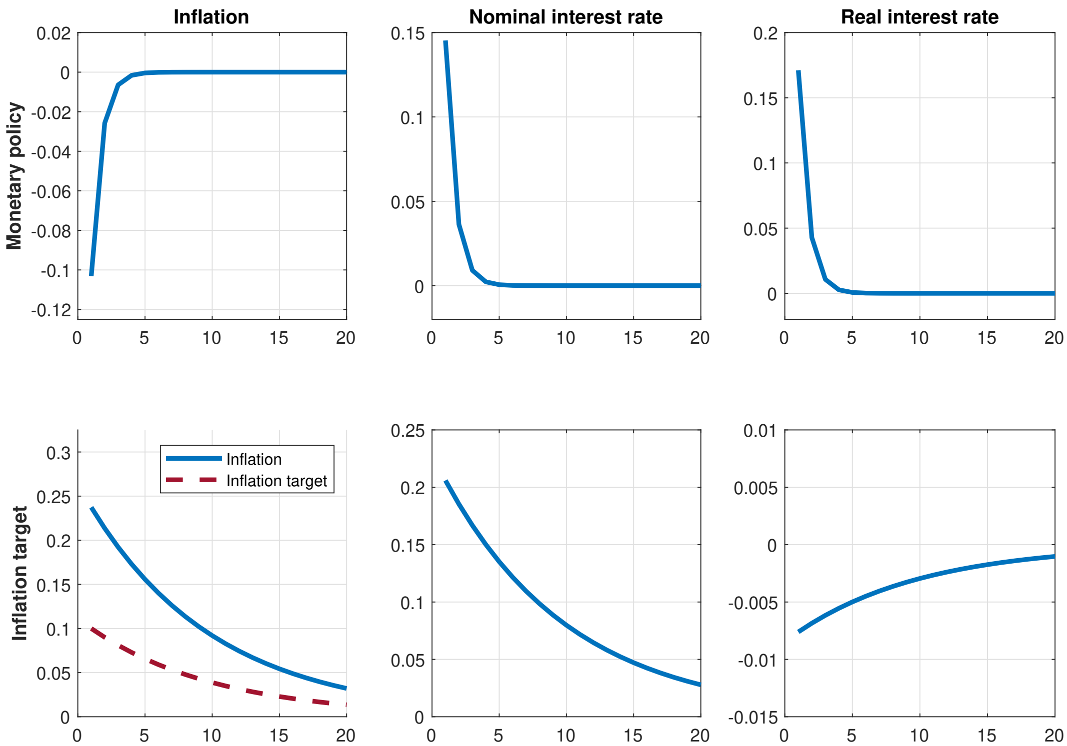
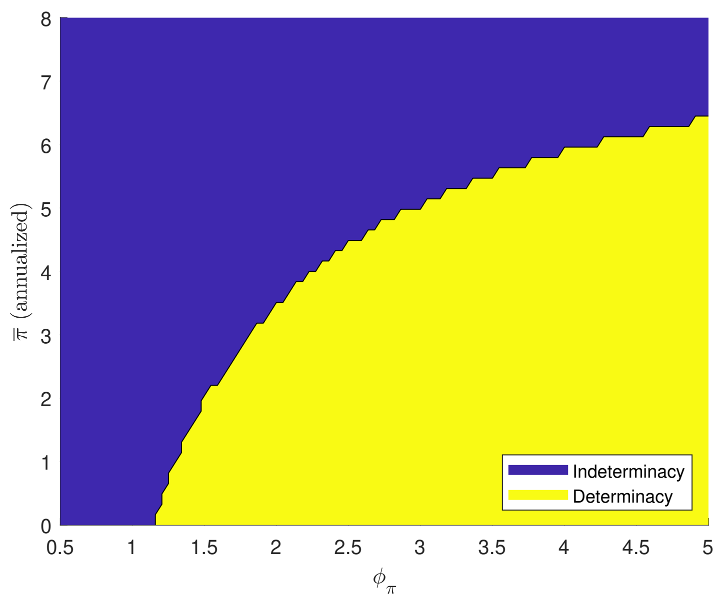
Figure 1: Impulse response functions to a monetary policy and inflation target shock in the basic NK model



Notes: The impulse response functions were generated using a standard calibration given by $\beta = 0.99$, $\tau = 1$, $\phi_\pi = 1.5$, $\rho_v = 0.25$, $\rho_\pi = 0.9$, $\chi = 1$, and $\xi_p = 2/3$. χ is the Frisch elasticity of labour supply and ξ_p is the Calvo price probability. The shock sizes are $\sigma_v = 0.30$ and $\sigma_\pi = 0.10$.

Figure 2: Determinacy region for MO-rule models

(A) $\chi = 1.75$



(B) $\chi = 0.55$

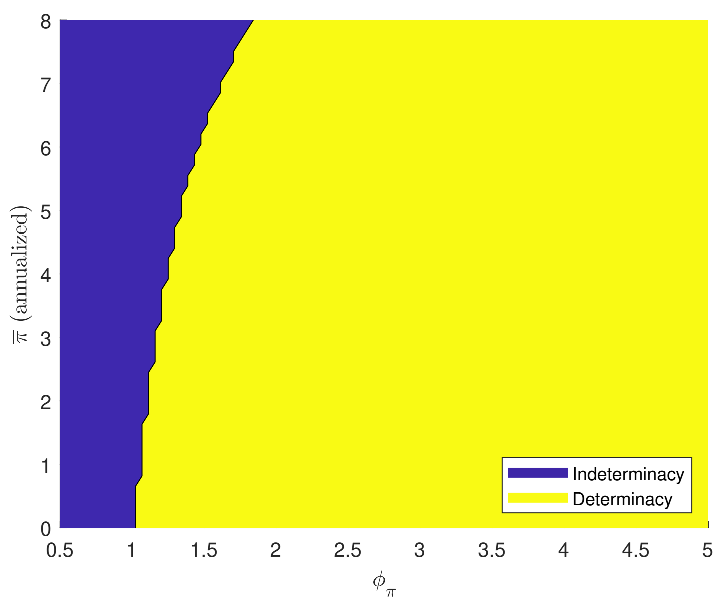
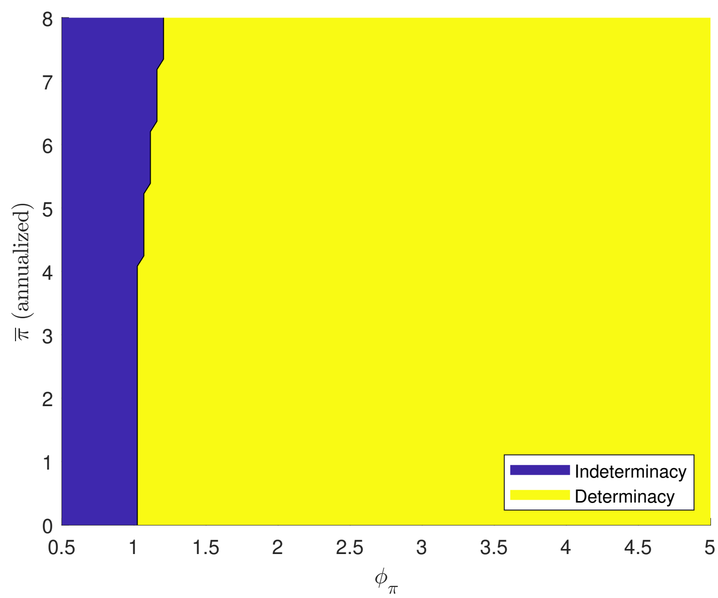


Figure 3: Determinacy region for OG-rule models

(A) $\chi = 1.75$



(B) $\chi = 0.55$

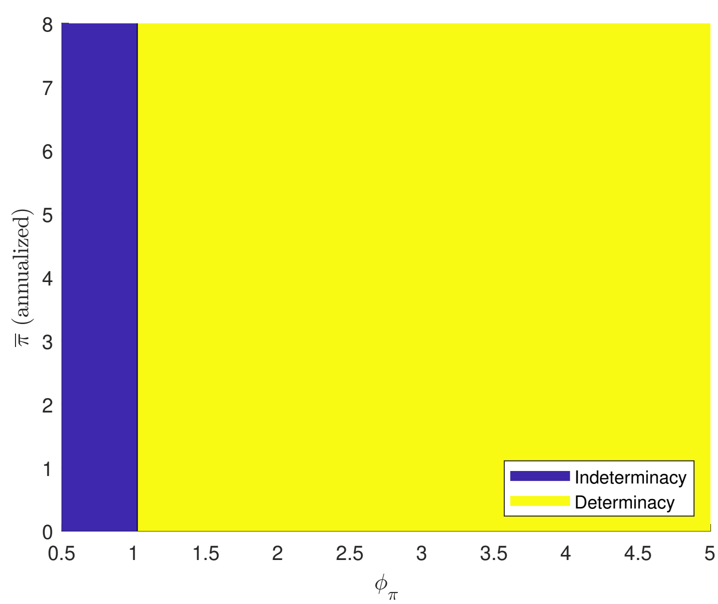
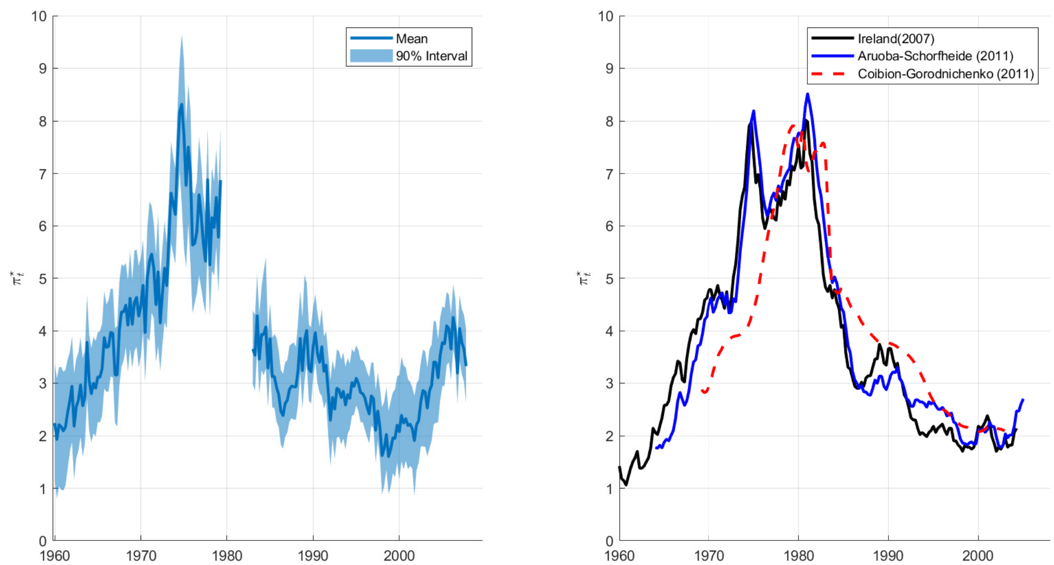


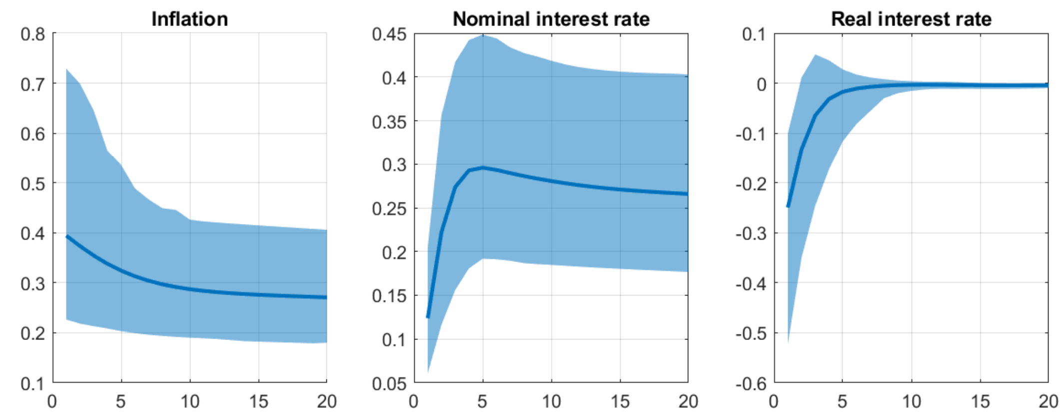
Figure 4: Federal Reserve's inflation target



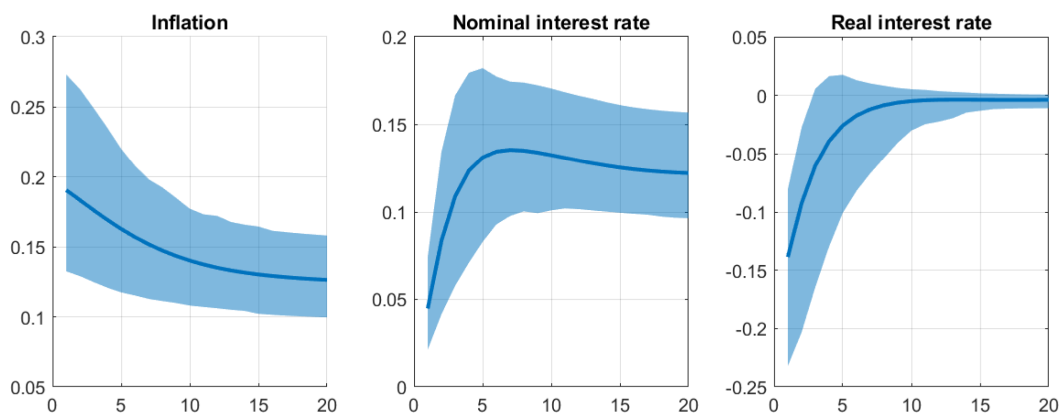
Note: The left panel reports our estimated target process from the two subsamples, 1960I:1979II and 1983I:2007IV. The right panel reports estimated target series from Ireland (2007), Aruoba and Schorfheide (2011), and Coibion and Gorodnichenko (2011). These estimates are obtained using a variety of different approaches. Ireland estimates a fully-specified DSGE model, backing out the target series using the Kalman smoother, as we do. Aruoba and Schorfheide estimate a small state space model containing inflation and inflation expectations (one and ten year), treating the common time-varying intercept as the inflation target. Coibion and Gorodnichenko estimate a generalized Taylor rule, backing out the time-varying target by approximating the equilibrium real rate.

Figure 5: Impulse response functions to an inflation target shock in the medium-scale model

(A) 1960-1979



(B) 1983-2007



Note: The dark blue line and bands are the associated mean response and 90% HPD intervals over 1,000 random draws from the posterior estimates. Posterior estimates are those from the OG-rule models.

Table 1: Simulation results

			$\sigma_\pi = 0.005$		$\sigma_\pi = 0.02$		$\sigma_\pi = 0.04$	
			Fixed	Time-varying	Fixed	Time-varying	Fixed	Time-varying
Name	Description	True value	Mean	Mean	Mean	Mean	Mean	Mean
σ_a	Technology shock std deviation	1.2	1.2	1.19	1.25	1.19	1.27	1.18
σ_b	Preference shock std deviation	1.4	1.32	1.34	1.21	1.32	0.96	1.3
σ_v	Monetary policy shock std deviation	0.2	0.2	0.2	0.21	0.2	0.2	0.21
σ_π	Inflation target shock std deviation	0.005/0.02/0.04	-	0.01	-	0.02	-	0.04
h	Habits	0.5	0.5	0.49	0.52	0.49	0.52	0.49
ξ_p	Calvo price stickiness	0.6	0.62	0.6	0.7	0.6	0.75	0.6
ρ_1	TR interest smoothing, 1st lag	0.8	0.76	0.8	0.58	0.79	0.53	0.78
ρ_2	TR interest smoothing, 2nd lag	-0.2	-0.18	-0.21	-0.12	-0.2	-0.11	-0.2
ϕ_π	TR response to inflation	2.25	2.11	2.26	1.52	2.26	1.18	2.31
ϕ_{gy}	TR response to output growth	0.2	0.2	0.2	0.17	0.2	0.16	0.21
ρ_a	Technology shock persistence	0.25	0.25	0.25	0.28	0.25	0.3	0.25
ρ_b	Preference shock persistence	0.9	0.9	0.9	0.93	0.89	0.96	0.89
ρ_v	Monetary policy shock persistence	0.2	0.23	0.18	0.41	0.19	0.47	0.2
$\bar{\pi}$	Steady state inflation	0.9	0.88	0.88	0.9	0.88	0.84	0.88
$\bar{i}$	Steady state nominal rate	1.4	1.36	1.36	1.37	1.36	1.3	1.35
$\bar{g}_A$	Steady state growth	0.4	0.41	0.41	0.41	0.41	0.42	0.41

Notes: The table reports the posterior mean estimates. We do not report HPD intervals because the sample size is sufficiently large that the upper and lower bounds are very close to the posterior mean.

Table 2: Prior distributions for simulation

Name	Domain	Density	Para(1)	Para(2)
σ_a	$\mathbb{R}^+$	InvGamma	0.5	1
σ_b	$\mathbb{R}^+$	InvGamma	0.5	1
σ_v	$\mathbb{R}^+$	InvGamma	0.5	1
σ_π	$\mathbb{R}^+$	InvGamma	0.1	1
h	[0,1)	Beta	0.7	0.15
ξ_p	[0,1)	Beta	0.66	0.05
ρ_1	[0,2]	Normal	0.80	0.20
ρ_2	[-1,1]	Normal	-0.1	0.2
ϕ_π	$\mathbb{R}^+$	Normal	1.5	0.5
ϕ_{gy}	$\mathbb{R}^+$	Normal	0.125	0.05
ρ_a	[0,1)	Beta	0.5	0.2
ρ_b	[0,1)	Beta	0.5	0.2
ρ_v	[0,1)	Beta	0.5	0.2
$\bar{\pi}$	$\mathbb{R}^+$	Normal	0.75	0.25
$\bar{i}$	$\mathbb{R}^+$	Normal	1.50	0.25
$\bar{g}_A$	$\mathbb{R}^+$	Normal	0.40	0.10

Notes: For the Beta and Normal distributions, Para(1) and Para(2) refer to the means and standard deviations of the prior. For the Uniform distribution, Para(1) and Para(2) refer to the lower and upper bounds. For the Inverse Gamma distribution, Para(1) and Para(2) refer to s and v where $p_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$.

Table 3: Prior distributions for estimation

Parameter	Description	Prior(Mean, Std)
σ_v	Monetary policy shock	IG(0.1,1)
σ_π	Inflation target shock	IG(0.1,1)
σ_a	Technology shock	IG(0.5,1)
σ_g	Government spending shock	IG(0.5,1)
σ_η	Labour supply shock	IG(0.5,1)
σ_b	Discount factor shock	IG(0.5,1)
σ_m	Investment shock	IG(0.5,1)
σ_s	Sunspot shock	IG(0.1,1)
ρ_v	Persistence monetary policy shock	B(0.5,0.2)
ρ_a	Persistence technology shock	B(0.5,0.2)
ρ_g	Persistence government spending shock	B(0.5,0.2)
ρ_η	Persistence labour supply shock	B(0.5,0.2)
ρ_b	Persistence discount factor shock	B(0.5,0.2)
ρ_m	Persistence investment shock	B(0.5,0.2)
$\rho_{v,s}$	Sunspot shock correlation	U(-1,1)
$\rho_{\pi,s}$	Sunspot shock correlation	U(-1,1)
$\rho_{a,s}$	Sunspot shock correlation	U(-1,1)
$\rho_{g,s}$	Sunspot shock correlation	U(-1,1)
$\rho_{\eta,s}$	Sunspot shock correlation	U(-1,1)
$\rho_{b,s}$	Sunspot shock correlation	U(-1,1)
$\rho_{m,s}$	Sunspot shock correlation	U(-1,1)
α	Share of capital	N(0.3,0.05)
$g_{\bar{\gamma}}$	Trend growth	N(0.5,0.1)
h	Consumption habit	B(0.5,0.1)
$\bar{L}$	Steady state hours worked	N(0,0.5)
$\bar{\pi}$	Steady state inflation	N(0.75,0.25)
χ	Frisch elasticity of labour supply	G(2,0.75)
ξ_p	Calvo price probability	B(0.66,0.05)
ξ_w	Calvo wage probability	B(0.66,0.05)
σ_z	Capacity utilization cost	G(5,1)
κ	Investment adjustment cost	G(4,1)
ϕ_y	Taylor rule output gap feedback	N(0.13,0.05)
ϕ_{gy}	Taylor rule output growth feedback	N(0.13,0.05)
ρ_1	Taylor rule interest smoothing, 1st lag	N(0.8,0.2)
ρ_2	Taylor rule interest smoothing, 2nd lag	N(-0.1,0.2)
ϕ_π	Taylor rule inflation feedback	N(1.1,0.5)
α_{BN}	Determinacy parameter	U(0.5,1.5)

Notes: In the column labelled 'Prior', N represents the Normal distribution, B the Beta distribution, G the Gamma distribution, IG the Inverse Gamma distribution, and U the Uniform distribution. For the Uniform distributions, the numbers inside the brackets represent the lower and upper bounds.

Table 4: Posterior estimates for the medium-scale model, 1960-1979

		OG		MOT	
Parameter	Prior	Mean	90% HPD	Mean	90% HPD
σ_v	IG(0.1,1)	0.23	[0.18 , 0.27]	0.23	[0.19 , 0.27]
σ_π	IG(0.1,1)	0.16	[0.11 , 0.21]	0.13	[0.06 , 0.19]
σ_a	IG(0.5,1)	1.01	[0.87 , 1.16]	1.01	[0.85 , 1.15]
σ_g	IG(0.5,1)	0.39	[0.34 , 0.45]	0.39	[0.34 , 0.45]
σ_η	IG(0.5,1)	5.28	[2.83 , 7.67]	5.43	[2.95 , 7.84]
σ_b	IG(0.5,1)	0.14	[0.11 , 0.17]	0.14	[0.11 , 0.17]
σ_m	IG(0.5,1)	8.39	[4.85 , 11.7]	9.36	[5.72 , 13.0]
σ_s	IG(0.1,1)	0.09	[0.02 , 0.18]	0.12	[0.02 , 0.33]
ρ_v	B(0.5,0.2)	0.39	[0.15 , 0.63]	0.43	[0.22 , 0.64]
ρ_a	B(0.5,0.2)	0.97	[0.96 , 0.99]	0.97	[0.95 , 0.98]
ρ_g	B(0.5,0.2)	0.93	[0.90 , 0.97]	0.93	[0.90 , 0.97]
ρ_η	B(0.5,0.2)	0.18	[0.05 , 0.29]	0.25	[0.08 , 0.42]
ρ_b	B(0.5,0.2)	0.18	[0.07 , 0.30]	0.17	[0.06 , 0.28]
ρ_m	B(0.5,0.2)	0.32	[0.18 , 0.46]	0.30	[0.16 , 0.44]
$\rho_{v,s}$	U(-1,1)	0.01	[-0.55 , 0.56]	-0.03	[-0.58 , 0.51]
$\rho_{\pi,s}$	U(-1,1)	0.01	[-0.54 , 0.56]	-0.06	[-0.62 , 0.50]
$\rho_{a,s}$	U(-1,1)	0.00	[-0.56 , 0.55]	-0.02	[-0.56 , 0.55]
$\rho_{g,s}$	U(-1,1)	0.00	[-0.54 , 0.54]	0.01	[-0.53 , 0.52]
$\rho_{\eta,s}$	U(-1,1)	0.00	[-0.55 , 0.55]	0.01	[-0.51 , 0.56]
$\rho_{b,s}$	U(-1,1)	0.01	[-0.54 , 0.55]	0.00	[-0.55 , 0.53]
$\rho_{m,s}$	U(-1,1)	0.00	[-0.55 , 0.55]	0.01	[-0.53 , 0.54]
α	N(0.3,0.05)	0.20	[0.18 , 0.22]	0.20	[0.18 , 0.22]
$g_{\bar{\gamma}}$	N(0.5,0.1)	0.27	[0.20 , 0.34]	0.28	[0.20 , 0.35]
h	B(0.5,0.1)	0.83	[0.77 , 0.88]	0.84	[0.79 , 0.89]
$\bar{L}$	N(0,0.5)	0.05	[-0.73 , 0.83]	-0.07	[-0.84 , 0.70]
$\bar{\pi}$	N(0.75,0.25)	1.02	[0.66 , 1.39]	0.96	[0.59 , 1.33]
χ	G(2,0.75)	0.54	[0.28 , 0.79]	0.66	[0.30 , 1.01]
ξ_p	B(0.66,0.05)	0.73	[0.70 , 0.77]	0.73	[0.69 , 0.77]
ξ_w	B(0.66,0.05)	0.66	[0.60 , 0.74]	0.65	[0.58 , 0.71]
σ_z	G(5,1)	4.99	[3.36 , 6.64]	4.92	[3.21 , 6.50]
κ	G(4,1)	3.09	[1.90 , 4.27]	3.41	[2.13 , 4.63]
ϕ_y	N(0.13,0.05)	—	—	0.09	[0.02 , 0.14]
ϕ_{gy}	N(0.13,0.05)	0.12	[0.05 , 0.19]	0.13	[0.05 , 0.20]
ρ_1	N(0.8,0.2)	0.91	[0.72 , 1.09]	0.93	[0.74 , 1.12]
ρ_2	N(-0.1,0.2)	-0.21	[-0.37 ,-0.06]	-0.20	[-0.36 ,-0.05]
ϕ_π	N(1.1,0.5)	2.24	[1.68 , 2.82]	2.09	[1.43 , 2.89]
α_{BN}	U(0.5,1.5)	1.25	[1.04 , 1.49]	1.22	[1.01 , 1.50]
log p(X^T)		-518.0394		-518.7378	
Prob(det)		1.0000		0.9250	

Table 5: Posterior estimates for the medium-scale model, 1983-2007

		OG		MO	
Parameter	Prior	Mean	90% HPD	Mean	90% HPD
σ_v	IG(0.1,1)	0.12	[0.10 , 0.13]	0.12	[0.10 , 0.14]
σ_π	IG(0.1,1)	0.08	[0.07 , 0.10]	0.05	[0.03 , 0.07]
σ_a	IG(0.5,1)	0.54	[0.48 , 0.61]	0.54	[0.47 , 0.60]
σ_g	IG(0.5,1)	0.29	[0.25 , 0.32]	0.29	[0.25 , 0.32]
σ_η	IG(0.5,1)	17.5	[11.4 , 23.6]	8.86	[3.93 , 13.8]
σ_b	IG(0.5,1)	0.11	[0.09 , 0.13]	0.11	[0.09 , 0.13]
σ_m	IG(0.5,1)	3.88	[2.46 , 5.28]	5.39	[3.49 , 7.21]
σ_s	IG(0.1,1)	0.11	[0.02 , 0.20]	0.10	[0.03 , 0.19]
ρ_v	B(0.5,0.2)	0.54	[0.39 , 0.69]	0.58	[0.44 , 0.71]
ρ_a	B(0.5,0.2)	0.96	[0.93 , 0.99]	0.94	[0.91 , 0.98]
ρ_g	B(0.5,0.2)	0.99	[0.98 , 1.00]	0.99	[0.98 , 1.00]
ρ_η	B(0.5,0.2)	0.27	[0.14 , 0.40]	0.66	[0.42 , 0.88]
ρ_b	B(0.5,0.2)	0.14	[0.05 , 0.23]	0.14	[0.04 , 0.22]
ρ_m	B(0.5,0.2)	0.63	[0.52 , 0.74]	0.57	[0.46 , 0.69]
$\rho_{v,s}$	U(-1,1)	0.00	[-0.53 , 0.58]	0.00	[-0.52 , 0.53]
$\rho_{\pi,s}$	U(-1,1)	-0.01	[-0.55 , 0.55]	-0.05	[-0.61 , 0.51]
$\rho_{a,s}$	U(-1,1)	-0.02	[-0.54 , 0.54]	-0.01	[-0.51 , 0.51]
$\rho_{g,s}$	U(-1,1)	0.01	[-0.54 , 0.58]	0.00	[-0.55 , 0.53]
$\rho_{\eta,s}$	U(-1,1)	0.00	[-0.55 , 0.55]	0.00	[-0.53 , 0.54]
$\rho_{b,s}$	U(-1,1)	0.00	[-0.57 , 0.53]	0.01	[-0.57 , 0.53]
$\rho_{m,s}$	U(-1,1)	0.00	[-0.53 , 0.56]	0.00	[-0.56 , 0.54]
α	N(0.3,0.05)	0.20	[0.18 , 0.22]	0.20	[0.18 , 0.22]
$g_{\bar{\gamma}}$	N(0.5,0.1)	0.37	[0.32 , 0.42]	0.40	[0.36 , 0.44]
h	B(0.5,0.1)	0.92	[0.89 , 0.95]	0.91	[0.88 , 0.95]
$\bar{L}$	N(0,0.5)	-0.02	[-0.82 , 0.81]	-0.12	[-0.90 , 0.70]
$\bar{\pi}$	N(0.75,0.25)	0.86	[0.59 , 1.14]	0.96	[0.65 , 1.27]
χ	G(2,0.75)	1.34	[0.74 , 1.92]	1.73	[0.86 , 2.55]
ξ_p	B(0.66,0.05)	0.84	[0.82 , 0.86]	0.83	[0.80 , 0.86]
ξ_w	B(0.66,0.05)	0.68	[0.62 , 0.74]	0.56	[0.47 , 0.66]
σ_z	G(5,1)	5.20	[3.59 , 6.73]	5.22	[3.54 , 6.79]
κ	G(4,1)	4.19	[2.77 , 5.70]	5.48	[3.81 , 7.05]
ϕ_y	N(0.13,0.05)	—	—	0.07	[0.01 , 0.12]
ϕ_{gy}	N(0.13,0.05)	0.17	[0.10 , 0.24]	0.16	[0.08 , 0.23]
ρ_1	N(0.8,0.2)	0.94	[0.77 , 1.11]	0.94	[0.77 , 1.11]
ρ_2	N(-0.1,0.2)	-0.19	[-0.34 , -0.06]	-0.18	[-0.31 , -0.04]
ϕ_π	N(1.1,0.5)	2.42	[1.94 , 2.92]	2.03	[1.46 , 3.11]
α_{BN}	U(0.5,1.5)	1.25	[1.05 , 1.50]	1.21	[1.00 , 1.50]
log p(X^T)		-428.518		-429.635	
Prob(det)		1.0000		0.9013	

Table 6: Posterior estimates under zero trend inflation

	1960-1979				1983-2007			
	OG		MO		OG		MO	
Parameter	Mean	90% HPD	Mean	90% HPD	Mean	90% HPD	Mean	90% HPD
σ_v	0.22	[0.18 , 0.26]	0.23	[0.19 , 0.27]	0.12	[0.10 , 0.13]	0.12	[0.10 , 0.13]
σ_π	0.18	[0.13 , 0.23]	0.16	[0.10 , 0.22]	0.09	[0.06 , 0.12]	0.06	[0.03 , 0.09]
σ_a	1.01	[0.87 , 1.15]	1.02	[0.87 , 1.15]	0.53	[0.47 , 0.60]	0.53	[0.46 , 0.59]
σ_g	0.39	[0.34 , 0.44]	0.39	[0.33 , 0.44]	0.29	[0.25 , 0.32]	0.29	[0.25 , 0.33]
σ_η	10.41	[4.06 , 16.8]	9.68	[3.31 , 16.1]	35.9	[22.0 , 48.7]	21.29	[11.37 , 31.34]
σ_b	0.14	[0.11 , 0.18]	0.15	[0.11 , 0.18]	0.11	[0.09 , 0.13]	0.11	[0.09 , 0.13]
σ_m	8.49	[5.13 , 11.66]	9.52	[6.02 , 12.88]	4.45	[2.89 , 5.92]	5.71	[3.46 , 7.77]
σ_s	0.11	[0.02 , 0.20]	0.10	[0.02 , 0.18]	0.10	[0.03 , 0.20]	0.14	[0.03 , 0.22]
ρ_v	0.41	[0.18 , 0.64]	0.45	[0.23 , 0.65]	0.56	[0.40 , 0.72]	0.57	[0.40 , 0.74]
ρ_a	0.97	[0.96 , 0.99]	0.97	[0.95 , 0.98]	0.94	[0.90 , 0.98]	0.94	[0.90 , 0.98]
ρ_g	0.94	[0.90 , 0.97]	0.94	[0.90 , 0.97]	0.99	[0.98 , 1.00]	0.99	[0.98 , 1.00]
ρ_η	0.17	[0.05 , 0.29]	0.21	[0.06 , 0.33]	0.28	[0.11 , 0.43]	0.47	[0.26 , 0.69]
ρ_b	0.19	[0.08 , 0.30]	0.18	[0.07 , 0.29]	0.14	[0.05 , 0.24]	0.14	[0.04 , 0.24]
ρ_m	0.30	[0.17 , 0.44]	0.30	[0.16 , 0.43]	0.59	[0.48 , 0.71]	0.55	[0.41 , 0.71]
$\rho_{v,s}$	0.02	[-0.52 , 0.57]	0.02	[-0.49 , 0.58]	-0.03	[-0.61 , 0.48]	-0.15	[-0.58 , 0.32]
$\rho_{\pi,s}$	0.00	[-0.57 , 0.54]	0.00	[-0.53 , 0.55]	-0.02	[-0.59 , 0.51]	-0.16	[-0.70 , 0.40]
$\rho_{a,s}$	-0.01	[-0.58 , 0.51]	-0.01	[-0.57 , 0.54]	0.01	[-0.54 , 0.52]	-0.11	[-0.61 , 0.30]
$\rho_{g,s}$	-0.01	[-0.55 , 0.52]	-0.02	[-0.57 , 0.54]	-0.02	[-0.54 , 0.54]	0.02	[-0.38 , 0.49]
$\rho_{\eta,s}$	0.00	[-0.55 , 0.54]	-0.03	[-0.60 , 0.51]	0.01	[-0.51 , 0.53]	-0.07	[-0.52 , 0.38]
$\rho_{b,s}$	-0.01	[-0.58 , 0.54]	0.00	[-0.56 , 0.53]	-0.01	[-0.51 , 0.54]	0.11	[-0.37 , 0.51]
$\rho_{m,s}$	0.00	[-0.56 , 0.54]	0.02	[-0.53 , 0.56]	-0.03	[-0.52 , 0.52]	0.05	[-0.49 , 0.44]
α	0.20	[0.18 , 0.22]	0.20	[0.19 , 0.23]	0.20	[0.17 , 0.22]	0.20	[0.18 , 0.22]
$g_{\bar{\gamma}}$	0.26	[0.20 , 0.32]	0.27	[0.21 , 0.33]	0.41	[0.38 , 0.45]	0.42	[0.38 , 0.46]
h	0.84	[0.79 , 0.89]	0.85	[0.80 , 0.90]	0.89	[0.85 , 0.95]	0.89	[0.85 , 0.94]
$\bar{L}$	0.08	[-0.67 , 0.82]	-0.03	[-0.77 , 0.72]	-0.08	[-0.92 , 0.74]	-0.21	[-1.07 , 0.65]
$\bar{\pi}$	0.73	[0.32 , 1.16]	0.71	[0.30 , 1.11]	0.67	[0.34 , 1.01]	0.60	[0.25 , 1.01]
χ	1.36	[0.56 , 2.15]	1.36	[0.55 , 2.17]	2.56	[1.50 , 3.53]	2.37	[1.16 , 3.30]
ξ_p	0.75	[0.72 , 0.78]	0.75	[0.72 , 0.78]	0.88	[0.85 , 0.90]	0.86	[0.83 , 0.89]
ξ_w	0.68	[0.60 , 0.75]	0.66	[0.59 , 0.73]	0.74	[0.69 , 0.79]	0.69	[0.61 , 0.75]
σ_z	4.99	[3.30 , 6.59]	5.02	[3.43 , 6.59]	5.13	[3.58 , 6.83]	5.18	[3.78 , 6.63]
κ	3.07	[1.86 , 4.22]	3.41	[2.22 , 4.57]	4.60	[3.28 , 6.09]	5.60	[3.76 , 7.32]
ϕ_y	—	—	0.07	[0.01 , 0.13]	—	—	0.02	[0.01 , 0.04]
ϕ_{gy}	0.13	[0.06 , 0.20]	0.13	[0.06 , 0.20]	0.16	[0.09 , 0.23]	0.17	[0.09 , 0.26]
ρ_1	0.88	[0.70 , 1.07]	0.91	[0.71 , 1.10]	0.95	[0.78 , 1.12]	0.94	[0.74 , 1.18]
ρ_2	-0.18	[-0.33 , -0.03]	-0.18	[-0.33 , -0.03]	-0.21	[-0.36 , -0.07]	-0.19	[-0.35 , -0.05]
ϕ_π	2.19	[1.64 , 2.75]	2.02	[1.44 , 2.59]	2.02	[0.82 , 2.77]	1.35	[0.86 , 1.98]
α_{BN}	1.25	[1.00 , 1.45]	1.25	[1.05 , 1.49]	1.18	[0.87 , 1.50]	1.01	[0.59 , 1.50]
log p(X^T)	-522.078		-524.240		-437.926		-444.766	
Prob(det)	1.0000		1.000		0.8746		0.5061	

Appendix

A Solving for Equilibrium Inflation using the Method of Undetermined Coefficients

To solve for the dynamics of equilibrium inflation for the model in Section 2, start by guessing that the solution is linear in the state variables $\hat{\pi}_t^\star$ and $\hat{v}_t$

$$\begin{aligned}\hat{\pi}_t &= \psi_{\pi\pi}\hat{\pi}_t^\star + \psi_{\pi v}\hat{v}_t,\\ \hat{x}_t &= \psi_{x\pi}\hat{\pi}_t^\star + \psi_{xv}\hat{v}_t,\\ \mathbb{E}_t\hat{\pi}_{t+1} &= \psi_{\pi\pi}\rho_\pi\hat{\pi}_t^\star + \psi_{\pi v}\rho_v\hat{v}_t,\\ \mathbb{E}_t\hat{x}_{t+1} &= \psi_{x\pi}\rho_\pi\hat{\pi}_t^\star + \psi_{xv}\rho_v\hat{v}_t,\end{aligned}$$

where we have used the fact that $\mathbb{E}_t\hat{\pi}_{t+1}^\star = \rho_\pi\hat{\pi}_t^\star$ and $\mathbb{E}_t\hat{v}_{t+1} = \rho_v\hat{v}_t$. Substituting these equations into the NKPC gives

$$\hat{\pi}_t = \left\{\beta\psi_{\pi\pi}\rho_\pi + \psi\psi_{x\pi}\right\}\hat{\pi}_t^\star + \left\{\beta\psi_{\pi v}\rho_v + \psi\psi_{xv}\right\}\hat{v}_t.$$

Next, substituting the monetary policy rule and solution guesses into the consumption Euler equation gives

$$\hat{x}_t = \left\{\psi_{x\pi}\rho_\pi - \frac{1}{\tau}\phi_\pi(\psi_{\pi\pi} - 1) + \frac{1}{\tau}\psi_{\pi\pi}\rho_\pi\right\}\hat{\pi}_t^\star + \left\{\psi_{xv}\rho_v - \frac{1}{\tau}(\phi_\pi\psi_{\pi v} + 1) + \frac{1}{\tau}\psi_{\pi v}\rho_v\right\}\hat{v}_t.$$

Now set coefficients equal to each term in the above equation. This gives

$$\begin{aligned}\psi_{\pi\pi} &= \beta\psi_{\pi\pi}\rho_\pi + \psi\psi_{x\pi},\\ \psi_{\pi v} &= \beta\psi_{\pi v}\rho_v + \psi\psi_{xv},\end{aligned}$$

or more simply

$$\psi_{\pi\pi} = \frac{\psi\psi_{x\pi}}{1-\beta\rho_\pi},$$
$$\psi_{\pi v} = \frac{\psi\psi_{xv}}{1-\beta\rho_v}.$$

Finally substituting these into the consumption Euler equation gives

$$\psi_{x\pi} = \frac{\phi_\pi(1-\beta\rho_\pi)}{\tau(1-\rho_\pi)(1-\beta\rho_\pi)+\psi(\phi_\pi-\rho_\pi)},$$

and

$$\psi_{xv} = \frac{-(1-\beta\rho_v)}{\tau(1-\rho_v)(1-\beta\rho_v)+\psi(\phi_\pi-\rho_v)}.$$

Then the coefficients in the NKPC become

$$\psi_{\pi\pi} = \frac{\psi\phi_\pi}{\tau(1-\rho_\pi)(1-\beta\rho_\pi)+\psi(\phi_\pi-\rho_\pi)},$$
$$\psi_{\pi v} = \frac{-\psi}{\tau(1-\rho_v)(1-\beta\rho_v)+\psi(\phi_\pi-\rho_v)}.$$

Next define Λ_π and Λ_v as

$$\Lambda_\pi = \frac{1}{\tau(1-\rho_\pi)(1-\beta\rho_\pi)+\psi(\phi_\pi-\rho_\pi)},$$
$$\Lambda_v = \frac{1}{\tau(1-\rho_v)(1-\beta\rho_v)+\psi(\phi_\pi-\rho_v)}.$$

Using these defintions, the dynamics of equilibrium inflation can be expressed as

$$\hat{\pi}_t = \psi\phi_\pi\Lambda_\pi\hat{\pi}^\star_t - \psi\Lambda_v\hat{v}_t.$$

B Proof of Proposition 1

This amounts to showing that the downward bias associated with (11) is greater than the downward bias associated with (14)

$$-\phi_\pi^2 \psi \Lambda_\pi \frac{\mathbb{V}ar(\hat{\pi}_t^\star)}{\mathbb{V}ar(\hat{\pi}_t)} - \psi \Lambda_v \frac{\mathbb{V}ar(\hat{v}_t)}{\mathbb{V}ar(\hat{\pi}_t)} < -\psi \Lambda_v \frac{\mathbb{V}ar(\hat{v}_t)}{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)}.$$

Which is equivalent to proving

$$\phi_\pi^2 \frac{\Lambda_\pi}{\Lambda_v} \frac{\mathbb{V}ar(\hat{\pi}_t^\star)}{\mathbb{V}ar(\hat{v}_t)} + 1 > \frac{\mathbb{V}ar(\hat{\pi}_t)}{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)}.$$

Using (12) and (15), it can be shown that

$$\frac{\mathbb{V}ar(\hat{\pi}_t)}{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)} = 1 + \frac{\mathbb{V}ar(\hat{\pi}_t^\star)\left(2\phi_\pi \psi \Lambda_\pi - 1\right)}{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)}.$$

Substituting this in gives

$$\phi_\pi^2 \frac{\Lambda_\pi}{\Lambda_v} \frac{\mathbb{V}ar(\hat{\pi}_t^\star)}{\mathbb{V}ar(\hat{v}_t)} + 1 > 1 + \frac{\mathbb{V}ar(\hat{\pi}_t^\star)\left(2\phi_\pi \psi \Lambda_\pi - 1\right)}{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)}.$$

which simplifies to require that

$$\frac{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)}{\mathbb{V}ar(\hat{v}_t)} > \frac{(2\phi_\pi \psi \Lambda_\pi - 1)}{\phi_\pi^2} \frac{\Lambda_v}{\Lambda_\pi}.$$

This inequality is satisfied for plausible ranges of shock sizes and parameter values. By assumptions A1(i)-A1(iii) $\phi_\pi^2 > 1$, $\frac{\Lambda_v}{\Lambda_\pi} < 1$, and $2\phi_\pi \psi \Lambda_\pi - 1 > 0$. However, the above condition can be violated if the variance of monetary policy shocks is significantly larger than the variance of inflation target shocks. This is because the relative variance of the inflation gap to monetary policy shocks is declining in the variance of monetary policy shocks. Specifically, as the size of monetary policy shocks grows infinitely large we have

$$\lim_{\mathbb{V}ar(\hat{v}_t)\to\infty} \frac{\mathbb{V}ar(\hat{\pi}_t - \hat{\pi}_t^\star)}{\mathbb{V}ar(\hat{v}_t)} = (\psi\Lambda_v)^2 < 1.$$

Using the calibration described in Section 2, for the inequality to be violated would require monetary policy shocks to be over 8 times larger than inflation target shocks. If $\rho_\pi = 0.995$, as it is in our empirical sections of the paper, it would require monetary policy shocks to be about 15 times larger than inflation target shocks, which we view as implausible.

C Log-linearized model for simulation

The following describes the log-linearized equations for the New Keynesian model used in the simulation part of the paper. The model is a small-scale New Keynesian model with positive trend inflation similar to Ascari and Sbordone (2014). The model contains added elements in the form of habit formation and additional shocks. For each variable X_t, we define $\hat{X}_t = log\tilde{X}_t - logX$, where $\tilde{X}_t$ represents the corresponding stationary variable and X its steady state.

$$\begin{aligned}\hat{Y}_t =& \frac{h}{h+g_A}\left(\hat{Y}_{t-1} - \hat{g}_{A,t}\right) + \frac{g_A}{h+g_A}\mathbb{E}_t\left(\hat{Y}_{t+1} + \hat{g}_{A,t+1}\right) \\ &- \frac{g_A - h}{h+g_A}\left(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{b}_t + \mathbb{E}_t\hat{b}_{t+1}\right)\end{aligned} \tag{C.1}$$

$$\begin{aligned}\hat{\pi}_t =& \beta[1+\theta(1-\xi_p\pi^{\theta-1})(\pi-1)]\mathbb{E}_t\hat{\pi}_{t+1} + \beta(1-\xi_p\pi^{\theta-1})(\pi-1)\mathbb{E}_t\hat{X}_{1,t+1} \\ &+ \left(\frac{(1-\xi_p\pi^{\theta-1})(1-\beta\xi_p\pi^{\theta})}{\xi_p\pi^{\theta-1}}\right)((1+\chi)\hat{Y}_t + \chi\hat{s}_t) + \beta(1-\pi)(1-\xi_p\pi^{\theta-1})\hat{b}_t \\ &+ \left(\frac{(1-\beta\xi_p\pi^{\theta-1})(1-\xi_p\pi^{\theta-1})}{\xi_p\pi^{\theta-1}}\right)\left(\frac{h}{g_A - h}\right)(\hat{Y}_t - \hat{Y}_{t-1} + \hat{g}_{A,t})\end{aligned} \tag{C.2}$$

$$\hat{X}_{1,t} = (1-\beta\xi_p\pi^{\theta})(\hat{b}_t + (1+\chi)\hat{Y}_t + \chi\hat{s}_t) + \beta\xi_p\pi^{\theta}\mathbb{E}_t[\hat{X}_{1,t+1} + \theta\hat{\pi}_{t+1}] \tag{C.3}$$

$$\hat{s}_t = \frac{\theta\xi_p\pi^{\theta-1}(\pi-1)}{1-\xi_p\pi^{\theta-1}}\hat{\pi}_t + \pi^{\theta}\xi_p\hat{s}_{t-1} \tag{C.4}$$

$$\hat{i}_t = \rho_1\hat{i}_{t-1} + \rho_2\hat{i}_{t-2} + (1-\rho_1-\rho_2)(\phi_\pi(\hat{\pi}_t - \hat{\pi}_t^\star) + \phi_{gy}(\hat{Y}_t - \hat{Y}_{t-1} + \hat{g}_{A,t})) + v_t \tag{C.5}$$

$$\hat{g}_{A,t} = \rho_a\hat{g}_{A,t-1} + \epsilon_t^a \tag{C.6}$$

$$\hat{b}_t = \rho_b\hat{b}_{t-1} + \epsilon_t^b \tag{C.7}$$

$$\hat{v}_t = \rho_v\hat{v}_{t-1} + \epsilon_t^v \tag{C.8}$$

$$\hat{\pi}_t^\star = \rho_\pi\hat{\pi}_{t-1}^\star + \epsilon_t^\pi \tag{C.9}$$

D Log-linearized model for estimation

This section lists the full set of log-linearized equations for the medium-scale NK model. For each variable X_t, we define $\hat{X}_t = log\tilde{X}_t - logX$, where $\tilde{X}_t$ represents the corresponding stationary variable and X its steady state.

$$\hat{\lambda}_t^r = \frac{\beta h g_\gamma}{(g_\gamma - \beta h)(g_\gamma - h)}\hat{C}_{t+1} - \frac{g_\gamma^2 + \beta h^2}{(g_\gamma - \beta h)(g_\gamma - h)}\hat{C}_t + \frac{g_\gamma h}{(g_\gamma - \beta h)(g_\gamma - h)}\hat{C}_{t-1} + \frac{g_\gamma - \beta h \rho_b}{g_\gamma - \beta h}\hat{b}_t \text{[20]} \tag{D.1}$$

$$\hat{r}_t^k = \sigma_z \hat{z}_t \text{[21]} \tag{D.2}$$

$$\hat{\lambda}_t^r = \hat{\mu}_t + \hat{m}_t - \kappa g_\gamma^2 \left(\hat{I}_t - \hat{I}_{t-1}\right) + \beta\kappa g_\gamma^2 \left(\hat{I}_{t+1} - \hat{I}_t\right) \tag{D.3}$$

$$\hat{\mu}_t = \left[1 - \beta(1-\delta) g_I^{-1} g_\gamma^{-1}\right]\left(\hat{\lambda}_{t+1}^r + \hat{r}_{t+1}^k\right) + \beta(1-\delta) g_I^{-1} g_\gamma^{-1} \hat{\mu}_{t+1} \tag{D.4}$$

$$\hat{\lambda}_t^r = \hat{i}_t - \hat{\pi}_{t+1} + \hat{\lambda}_{t+1}^r \tag{D.5}$$

$$\hat{w}_t^* = \hat{f}_{1t} - \hat{f}_{2t} \tag{D.6}$$

$$\begin{aligned}\hat{f}_{1t} = &\left[1 - \beta\xi_w \pi^{\sigma(1+\chi)} g_\gamma^{\sigma(1+\chi)}\right]\left\{\sigma(1+\chi)(\hat{w}_t - \hat{w}_t^*) + (1+\chi)\hat{L}_t + \hat{\eta}_t\right\} \\ &+ \beta\xi_w \pi^{\sigma(1+\chi)} g_\gamma^{\sigma(1+\chi)}\left\{\sigma(1+\chi)\left(\hat{\pi}_{t+1} + \hat{w}_{t+1}^* - \hat{w}_t^*\right) + \hat{f}_{1t+1}\right\}\end{aligned} \tag{D.7}$$

$$\begin{aligned}\hat{f}_{2t} = &\left[1 - \beta\xi_w \pi^{\sigma-1} g_\gamma^{\sigma-1}\right]\left(\frac{w}{w^*}\right)^{\sigma-1}\left\{\hat{\lambda}_t^r + \sigma(\hat{w}_t - \hat{w}_t^*) + \hat{L}_t\right\} \\ &+ \beta\xi_w \pi^{\sigma-1} g_\gamma^{\sigma-1}\left\{(\sigma-1)\hat{\pi}_{t+1} + \sigma\left(\hat{w}_{t+1}^* - \hat{w}_t^*\right) + \hat{f}_{2t+1}\right\}\end{aligned} \tag{D.8}$$

[20] g_γ is the gross growth rate of the deterministic trend factor in the steady state.
[21] $\sigma_z = \frac{\gamma_2}{\gamma_1}$.

$$\hat{K}_t = \hat{mc}_t - \hat{r}^k_t + \frac{sY}{sY+F}\left(\hat{s}_t + \hat{Y}_t\right) \text{[22]} \tag{D.9}$$

$$\hat{L}_t = \hat{mc}_t - \hat{w}_t + \frac{sY}{sY+F}\left(\hat{s}_t + \hat{Y}_t\right) \tag{D.10}$$

$$\hat{p}^*_t = \hat{g}_{1t} - \hat{g}_{2t} \tag{D.11}$$

$$\hat{g}_{1t} = \left[1 - \xi_p \beta \pi^{\theta}\right]\left\{\hat{\lambda}^r_t + \hat{mc}_t + \hat{Y}_t\right\} + \xi_p \beta \pi^{\theta}\left\{\theta \hat{\pi}_{t+1} + \hat{g}_{1t+1}\right\} \tag{D.12}$$

$$\hat{g}_{2t} = \left[1 - \xi_p \beta \pi^{\theta-1}\right]\left\{\hat{\lambda}^r_t + \hat{Y}_t\right\} + \xi_p \beta \pi^{\theta-1}\left\{(\theta-1)\hat{\pi}_{t+1} + \hat{g}_{2t+1}\right\} \tag{D.13}$$

$$\xi_p \pi^{\theta-1}(\theta-1)\hat{\pi}_t + \left(1-\xi_p\right) p^{*1-\theta}(1-\theta)\hat{p}^*_t = 0 \tag{D.14}$$

$$(1-\sigma)\hat{w}_t = \xi_w g_\gamma^{\sigma-1}\pi^{\sigma-1}\left\{(1-\sigma)\hat{w}_{t-1} + (\sigma-1)\hat{\pi}_t\right\} + (1-\xi_w)\left(\frac{w^*}{w}\right)^{1-\sigma}(1-\sigma)\hat{w}^*_t \tag{D.15}$$

$$\hat{s}_t + \hat{Y}_t = \frac{sY+F}{sY}\left\{\hat{a}_t + \alpha \hat{K}_t + (1-\alpha)\hat{L}_t\right\} \tag{D.16}$$

$$\frac{1}{g}\hat{Y}_t = \frac{1}{g}\hat{g}_t + \frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t + r^k \frac{K}{Y} g_\gamma^{-1} g_I^{-1}\hat{z}_t \tag{D.17}$$

$$\hat{Y}^{GDP}_t = \hat{Y}_t - r^k \frac{K}{Y} g_\gamma^{-1} g_I^{-1}\hat{z}_t \tag{D.18}$$

$$\hat{K}^p_{t+1} = \left[1 - (1-\delta) g_\gamma^{-1} g_I^{-1}\right]\left\{\hat{m}_t + \hat{I}_t\right\} + (1-\delta) g_\gamma^{-1} g_I^{-1}\hat{K}^p_t \tag{D.19}$$

$$\hat{K}_t = \hat{z}_t + \hat{K}^p_t \tag{D.20}$$

$$s\hat{s}_t = -\left(1-\xi_p\right) p^{*-\theta}\theta \hat{p}^*_t + \xi_p \pi^{\theta} s\left\{\theta \hat{\pi}_t + \hat{s}_{t-1}\right\} \tag{D.21}$$

[22] $s_t = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\theta} dj$ denotes the measure of price dispersion in the model.

$$\text{MOT-rule}: \hat{i}_t = \rho_1 \hat{i}_{t-1} + \rho_2 \hat{i}_{t-2} + (1-\rho_1-\rho_2)\Big\{\phi_\pi\left(\hat{\pi}_t - \hat{\pi}_t^*\right) + \phi_y\left(\hat{Y}_t^{GDP} - \hat{Y}_t^{GDP*}\right) + \phi_{gy}\left(\hat{Y}_t^{GDP} - \hat{Y}_{t-1}^{GDP}\right)\Big\} + \hat{v}_t \quad (D.22)$$

$$\text{OGT-rule}: \hat{i}_t = \rho_1 \hat{i}_{t-1} + \rho_2 \hat{i}_{t-2} + (1-\rho_1-\rho_2)\Big\{\phi_\pi\left(\hat{\pi}_t - \hat{\pi}_t^*\right) + \phi_y\left(\hat{Y}_t^{GDP} - \hat{Y}_t^{GDP*}\right)\Big\} + \hat{v}_t \quad (D.23)$$

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_t^a \quad (D.24)$$

$$\hat{b}_t = \rho_b \hat{b}_{t-1} + \epsilon_t^b \quad (D.25)$$

$$\hat{\eta}_t = \rho_\eta \hat{\eta}_{t-1} + \epsilon_t^\eta \quad (D.26)$$

$$\hat{m}_t = \rho_m \hat{m}_{t-1} + \epsilon_t^m \quad (D.27)$$

$$\hat{v}_t = \rho_v \hat{v}_{t-1} + \epsilon_t^v \quad (D.28)$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_t^g \quad (D.29)$$

$$\hat{\pi}_t^* = \rho_\pi \hat{\pi}_{t-1}^* + \epsilon_t^\pi \quad (D.30)$$