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Keywords

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JEL Classification

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Abstract

Idiosyncratic supply shocks often coincide with changes in producers' demand for intermediate inputs. We present a new measure of systemic importance in economies with input-output linkages. *Scale centrality* measures the aggregate impact of simultaneous supply and demand shocks to a producer, which propagate through (in)direct customers and suppliers. Theoretically, scale centrality encompasses *and* extends an existing centrality measure: Domar weights. Empirically, we find: i) Domar weights underestimate sectors' systemic importance, ii) sectors with high scale centralities are typically downstream in the supply chain, and iii) industries' scale centrality can change substantially over time despite the stability of their Domar weights.

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1 Introduction

At least since Leontief (1936), economists have recognized the importance of systematically quantifying interrelationships between the producers of an economy.[1] More recently, studies have shown that these input-output linkages play a crucial role in amplifying microeconomic shocks into aggregate fluctuations (Acemoglu et al., 2012; Atalay, 2017, Baqaee and Farhi, 2019, Huo et al., 2024 and Kinnan et al., 2024). In addition to a producer's share of value added to GDP, its macroeconomic importance depends on how other agents rely on it, directly or indirectly, for intermediate inputs. Moreover, existing measures of systemic importance quantify the impact of an isolated idiosyncratic supply shock to a given producer on aggregate output (see, for example, Hulten, 1978, Liu, 2019, Baqaee and Farhi, 2020b, and Bigio and La'O, 2020).[2] However, in reality, supply shocks rarely occur in isolation and often coincide with reductions in demand for intermediate inputs from upstream suppliers. As a result, the systemic importance of a producer extends beyond the direct and indirect impact of a supply shock to itself and also encompasses the simultaneous effect of changes in its demand for intermediate goods.

This paper presents a new measure of a producer's macroeconomic importance, which we refer to as *scale centrality*, that captures how a shock to a producer impacts real GDP by i) directly affecting the final consumption of its output, ii) indirectly affecting the production of firms that are directly or indirectly connected to it, and iii) affecting direct and indirect suppliers due to changes in its demand for intermediate inputs. To highlight the relevance of our measure, consider the recent Boeing 737 Max groundings that occurred between 2019 and 2020, following the fatal crashes of Lion Air Flight 610 and Ethiopian Airlines Flight 302. In 2019, Boeing temporarily halted the production of the 737 Max aircraft following the Federal Aviation Administration's decision to ground the plane due to a software flaw. The decision had major financial ramifications for Boeing, costing more than USD 18 billion in customer compensation and sales losses (Forbes, 2020). Expectedly, the shock impacted Boeing's customers. Unable to use their 737 Maxes, domestic and international airlines were forced to reduce the number of flights on offer. American Airlines canceled 115 flights per day, affecting more than 20,000 passengers daily. For Southwest Airlines, the grounding

[1] We use the terms producers, sectors, industries, and firms interchangeably in this paper.

[2] While these papers employ different frameworks: efficient vs. inefficient economies (with taxes, markups or financial frictions) and differences in production technologies (Cobb-Douglas vs. CES), they share the common aim of characterizing the change in aggregate output due to an idiosyncratic disturbance to a microeconomic producer.

reduced its second-quarter profit by USD 175 million in 2019 (NPR, 2019).

Perhaps equally importantly, the groundings also placed significant financial pressure on Boeing's suppliers. Spirit AeroSystems, the company that supplied Boeing with the fuselage for its 737 Max jet, announced in early 2020 that it would lay off around 2,800 employees at a facility in Kansas as a result of Boeing's decision to suspend production of the 737 Max (The Washington Post, 2020).[3] Another supplier, German seat maker Recaro Aircraft Seating, which supplied seats for the Max, also scaled its operations as Boeing decreased its monthly production of 737s from 52 to 42. (The Wall Street Journal, 2019). Clearly, the shock to Boeing was felt by its suppliers: in 2020, Spirit AeroSystems made a loss of USD 870 million (Flight Global, 2021), and Recaro's revenues declined by 60% (Recaro, 2020) as a consequence of the disruption to Boeing. The Max groundings highlight that shocks affecting a specific producer can impact not only its customers, who rely directly or indirectly on its output, but also its suppliers by reducing demand for intermediate inputs.

What was the macroeconomic impact of the shock to Boeing? The results of Hulten (1978) imply that the systemic importance of a producer can be approximated by its gross sales as a share of GDP, or Domar weight. While Domar weights sufficiently summarize the change in real GDP in response to idiosyncratic microeconomic productivity shocks, we show that scale centrality is a more appropriate measure of systemic importance in an environment where disruptions to producers entail idiosyncratic demand *and* supply shocks (such as the Boeing groundings). The Domar weight of a producer represents the aggregate impact of a supply shock to itself as it travels through direct and indirect customers. Scale centrality, in addition, encases the general equilibrium impact of changes in the producer's demand for intermediate goods following a shock. Notably, scale centrality encapsulates producers' Domar weight, and all the information contained in it, but also incorporates the systemic importance of a producer due to changes in its demand for intermediate inputs.

While the 737 groundings serve as a motivating example, Figure 1 presents direct empirical evidence that major firm-level shocks simultaneously impact the supply and use of products. As in Barrot and Sauvagnat (2016) and Bahal et al. (2023), we identify exogenous firm-level disruptions with the occurrence of major natural disasters in the US over the period 1978 to 2017. Using Compustat data, we estimate the effect of these shocks on the year-on-year real sales growth of 1) directly affected firms (black

[3] Spirit was heavily reliant on Boeing, with the Max alone accounting for more than 50 percent of the company's annual revenue (The New York Times, 2020).

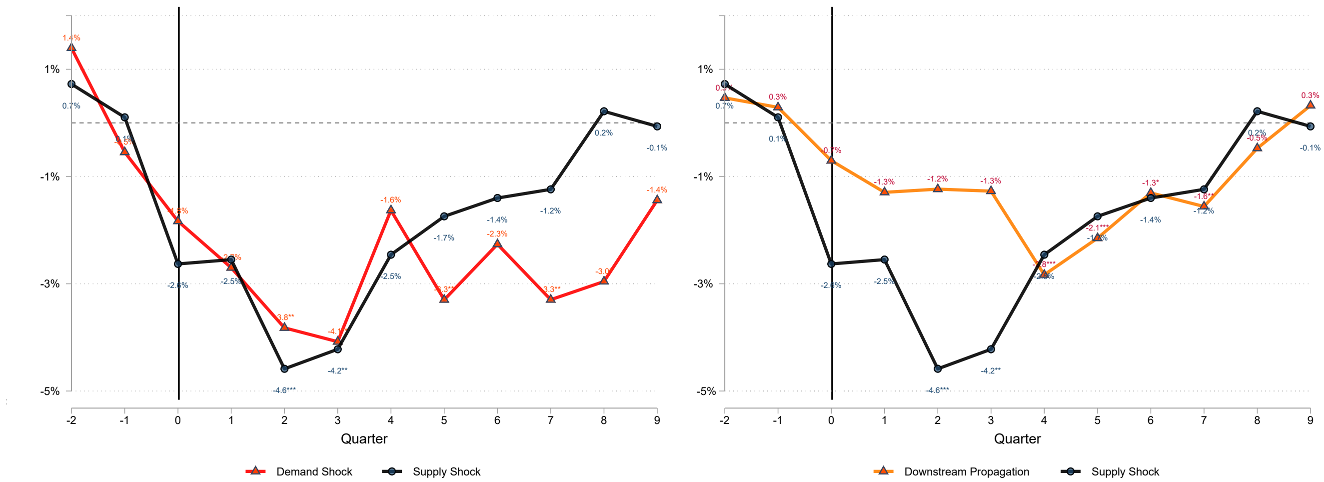


Figure 1: The Effect of Simultaneous Supply and Demand Shocks on Firms' Sales

Note: The figure shows the average impact of natural disasters on the real year-on-year sales growth of i) directly affected firms (black line), ii) affected firms' suppliers (red line, left panel), and iii) affected firms' customers (orange line, right panel). The effect on sales growth is estimated up to nine quarters following a shock. For further details on the data used and our estimation strategy, see Appendix A. Standard errors are clustered at the firm level. *10%; **5%; ***1% significance levels.

line), 2) their immediate suppliers (left panel, red line), and 3) their direct customers (right panel, orange line) up to nine quarters following a shock. See Appendix A for a detailed discussion of the data and identification strategy used in Figure 1.

The figure reveals that directly affected firms and their upstream suppliers experience an immediate and simultaneous decline in output following a major natural disaster.[4] Canonical production network models that employ Cobb-Douglas production technologies (such as Acemoglu et al., 2012) cannot comprehensively explain this finding with negative microeconomic TFP shocks alone. In the benchmark model, TFP shocks propagate downstream, with their effects diminishing as they transmit along the supply chain. Therefore, these shocks can only influence upstream producers through feedback effects, that is, when upstream firms *indirectly* rely on inputs from affected producers.[5] However, the right panel reveals that the shocks propagate to customers with a significant lag. While upstream suppliers are affected immediately, downstream firms experience reductions in sales growth approximately four quarters

[4]To avoid confounding the effects of reduced input demand with the direct impact of shocks on upstream suppliers, we: i) exclude supplier-customer relationships within a 300km radius and ii) control for whether natural disasters strike suppliers themselves.

[5]In a recent paper, Carvalho et al. (2024) show that most of the production economy can be represented as an upstream-to-downstream flow of intermediate goods.

later.[6] Thus, the timing and magnitude of the effect on suppliers is unlikely to be *solely* explained by the downstream propagation of supply shocks.[7]

Our measure of scale centrality is derived within the framework of a macroeconomic production network model in the spirit of Atalay (2017), Bernard and Moxnes (2018), and Baqaee and Farhi (2019). In the model, each sector produces output using a combination of labor and intermediate inputs and is subject to shocks to its use of materials from upstream suppliers, as well as its supply of intermediate and final goods. We capture relationships between industries via constant-elasticity-of-substitution (CES) aggregators of intermediate products. While goods may either be complements or substitutes in our model, a sector's scale centrality does not depend on the value of the elasticity of substitution across intermediate or final goods (to a first-order approximation). In this respect, our formulas align with the nonparametric results of much of the production networks literature, including Hulten (1978), Liu (2019), Baqaee and Farhi (2020b), and Bigio and La'O (2020).

Shocks to producers' supply and demand for goods lead to endogenous changes in the prices and quantities of traded inputs. However, these endogenous changes are macroeconomically irrelevant to a first-order approximation and only matter at higher orders. A key contribution of this paper is to provide a formula that allows us to quantify each sector's scale centrality without having to characterize changes in prices and quantities in response to a shock. Notably, only observable data on intermediate input purchases, nominal gross output, tax payments, and nominal GDP is necessary to estimate an industry's scale centrality. Since this information is available for most countries, our measure is readily computable.

Scale centrality can be decomposed into three distinct components, which we refer to as *direct*, *indirect*, and *supplier* effects. The direct effect captures the impact of a supply shock to the focal industry on real GDP via households' direct consumption of its output. The indirect effect, on the other hand, captures the spillover effect of the supply

[6]The delayed response of customers' sales growth is documented in Barrot and Sauvagnat (2016) and Bahal et al. (2023).

[7]Constant-elasticity-of-substitution (CES) production network models, such as those studied in Baqaee and Farhi (2019) and Baqaee and Rubbo (2023), can generate reductions in output of upstream producers when customers substitute away from the affected suppliers in response to negative TFP shocks (i.e., inputs are gross substitutes for downstream firms). Since customers are impacted two quarters *after* shocked firms' suppliers, the immediate decline in suppliers' sales growth (left panel of Figure 1) is likely a result of a more direct decrease in demand for intermediate inputs from the shocked firm itself. The general equilibrium price effects of Baqaee and Farhi (2019) and Baqaee and Rubbo (2023) may as well be operating and may explain why suppliers' sales growth continues to decline after four quarters.

shock (to the focal industry) on final demand as it spreads to other industries. Lastly, the supplier effect quantifies how changes in the focal producer's demand for intermediate inputs, in response to the shock, impacts real GDP. Supplier effects also spillover to other producers who depend, either directly or indirectly, on products from the focal sector's upstream suppliers. Notably, the sum of a sector's direct and indirect effects corresponds to its Domar weight, a well-established measure of systemic significance (as in Hulten, 1978; Baqaee and Farhi, 2020b).[8] The third component, the supplier effect, represents a sector's ability to shape aggregate fluctuations via changes in demand for inputs and distinguishes a sector's Domar weight from its scale centrality.

In an empirical application, we first estimate scale centrality for approximately 450 sectors in the US between 1982 and 2012 using the detailed input-output accounts provided by the Bureau of Economic Analysis (BEA). We show that Domar weights underestimate the systemic importance of an industry by 40-50%, on average, implying that supplier effects are quantitatively significant for most industries. We further demonstrate that the extent of underestimation increases with the Domar weight of a sector. This result is not immediately obvious as a sector's scale centrality and Domar weight can coincide, even for industries with large Domar weights (say, for an upstream sector that uses no intermediate inputs for production).

We also find the relationship between sectors' Domar weights and scale centrality to be remarkably constant over time. That is, sectors with larger Domar weights have substantially greater supplier effects as well. Thus, as industries grow larger in size (as measured by their Domar weight), they also become increasingly reliant on inputs from other sectors in the economy. This implies that shocks to the input demand of large producers have significant aggregate effects. Our findings reveal that industries such as petroleum refineries, oil & gas extraction, and pharmaceutical manufacturing have a substantial capacity to drive aggregate fluctuations in the US.

Further, changes in a sector's scale centrality need not coincide with changes in its Domar weight. Our analysis shows that key sectors of the US economy, such as motor vehicle production, electronic computer manufacturing, and retail trade, experienced significant changes in their scale centrality over time despite having relatively stable Domar weights. Take, for instance, the retail trade sector. Between 1982 and 1987, re-

[8]In inefficient economies with one factor of production, distortion-adjusted Domar weights are sufficient to characterize the macroeconomic impact of isolated microeconomic TFP shocks (Baqaee and Farhi, 2020b). Since our model includes taxes on sales and input purchases, we use the term "Domar weights" to refer to tax-adjusted sales shares. However, regardless of whether the economy is efficient or inefficient, scale centrality encapsulates the first-order macroeconomic impact of an idiosyncratic microeconomic supply shock to the focal industry.

tail trade experienced a substantial decrease in its reliance on intermediates from other sectors, with its supplier effect declining from around 3.5% of GDP to less than 3% of GDP during this period. Surprisingly, however, the Domar weight of retail trade remained constant. Therefore, when assessing retail trade's systemic importance using our measure of scale centrality, we note a significant decline in its macroeconomic significance, which is not captured by its Domar weight. This finding highlights the relevance of scale centrality in assessing the systemic importance of a sector, in addition to a sector's Domar weight.

Next, using data from the World Input-Output Database between 2000 and 2014, we identify the industries with the greatest scale centrality for six major economies: the United States, Great Britain, Japan, China, Germany, and Australia. For each economy, construction, real estate, public administration, and food & beverages are always among the top five sectors with the largest scale centrality. Notably, the magnitude of direct, indirect, and supplier effects vary significantly across countries and industries. Labor-intensive sectors, such as education and health services, have significant direct effects. This is because they rely less on intermediate inputs and provide services directly to end-consumers, limiting their ability to amplify shocks via indirect and supplier effects. Conversely, sectors like wholesale trade and electricity & gas derive most of their aggregate importance from the indirect effect, reflecting their critical role as input suppliers to other producers. Construction and food & beverages, on the other hand, have relatively large direct and supplier effects, which highlights their central role as both producers of final goods and consumers of intermediate goods.

Related literature. Our article relates to the literature on growth accounting and production networks. Hulten (1978) provided the economic rationale for using Domar aggregation to measure changes in aggregate TFP. Hulten's result was in contrast to Solow (1957), who used an aggregate production function and measured TFP growth as the residual change in output after accounting for the growth of factor inputs. Hulten's theorem has become a benchmark in the macroeconomic literature on production networks, demonstrating that in the presence of intermediate inputs, sales (rather than value-added) shares are the appropriate weights for aggregating microeconomic productivity changes.[9] Specifically, the theorem states that a producer's sales as a share of GDP (also called its Domar weight) is sufficient to capture the first-order macroe-

[9] See Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for a detailed overview of the production networks literature.

conomic impact of a microeconomic productivity shock to that producer. Relatedly, Acemoglu et al. (2012) demonstrate that Domar weights are linked to the economy's input-output network through the Leontief inverse, capturing each industry's direct and indirect dependencies on intermediate inputs from other sectors.[10] Our measure of systemic importance differs from that of Hulten (1978) and Acemoglu et al. (2012) as it accounts for sectors' ability to influence aggregate fluctuations via changes in demand for intermediate inputs.

Since our key measure is derived in the context of an inefficient network model, we contribute to the growing literature on the propagation of shocks through input-output linkages in the presence of market imperfections. Some papers in this literature include Jones (2011, 2013); Bartelme and Gorodnichenko (2015); Caliendo et al. (2018); Liu (2019); Boehm and Oberfield (2020), and Boehm (2022). Bigio and La'O (2020) study the properties of inefficient (Cobb-Douglas) production networks with financial frictions, while Baqaee and Farhi (2020b) study the impact of microeconomic productivity and factor supply shocks on aggregate output in a model with markups. We contribute to this literature by demonstrating analytically that taxes on intermediate input purchases and gross sales increase sectors' scale centrality relative to frictionless economies without such distortions.[11] We also show how to use our framework to quantify the importance of these distortions on sectors' scale centrality. Empirically, however, we find that sectoral taxes had a relatively limited impact on the amplification of microeconomic shocks in the United States between 1982 and 2012.

Our paper also relates to the recent macroeconomics literature that investigates the role of input-output linkages in generating aggregate volatility, including Foerster et al. (2011), Acemoglu et al. (2012), di Giovanni et al. (2014), Acemoglu et al. (2017), Atalay (2017), Grassi (2017), Baqaee (2018), and Altinoglu (2021).[12] A common theme

[10]Baqaee and Farhi (2019) build on the work of Hulten (1978) and Acemoglu et al. (2012), showing that nonlinearities in production have a significant impact on macroeconomic outcomes. While the first-order macroeconomic effect of a microeconomic TFP shock to a sector is given by the sector's Domar weight, the second-order effect requires additional information, such as microeconomic elasticities of substitution and the degree of return to scale.

[11]Taxes on factor payments only affect aggregate volatility when there are factor supply shocks or factor-augmenting productivity shocks.

[12]These papers build on earlier work in macroeconomics that studies aggregate volatility in multi-sector models, such as Long and Plosser (1983); Horvath (1998, 2000); Dupor (1999); Shea (2002). Other papers that model microeconomic behavior to shed light on macroeconomic phenomena include Durlauf (1993) and Jovanovic (1987). Studies such as Gabaix (2011) and Amiti and Weinstein (2018) focus on the role of the firm size distribution in shaping aggregate fluctuations. Finally, Elliott et al. (2022) and Carvalho et al. (2024) examine how supply chain complexity and bottlenecks contribute to macroeconomic fragility.

in this literature is that the interdependence of production through input-output linkages significantly amplifies aggregate volatility: small shocks cascade through supply chains, resulting in larger fluctuations in output.[13] We contribute to this literature in three ways. First, we identify the key sectors that have the greatest impact on aggregate volatility using our measure of scale centrality and demonstrate that these are not necessarily the industries with the largest Domar weights. Second, to gain a deeper understanding of the key determinants of aggregate volatility, we decompose sectors' scale centrality into direct, indirect, and supplier effects. Our analysis underscores the critical role that supplier effects play in shaping aggregate fluctuations. For example, sectors like construction and food & beverages can drive output fluctuations because they are disproportionate consumers of material goods relative to other industries. Lastly, we provide evidence that larger sectors tend to exhibit a higher level of dependence on intermediate inputs from other industries. This finding suggests that as sectors grow in size, they not only contribute more to aggregate fluctuations due to their increased share of GDP but also because they account for a larger proportion of their suppliers' revenues.

The rest of the paper is structured as follows. In Section 2, we present the model and derive our measure of scale centrality. In Section 3, we use our framework to identify the industries that have a significant impact on the propagation of microeconomic shocks. Section 4 concludes. Proofs, a description of the data, and supplementary results appear in the Appendix.

2 Model setup and equilibrium

We consider a static economy with N sectors that each produce one distinct product using some combination of labor and intermediate goods. The output of these sectors can either be consumed directly by households as final goods or used as an intermediate input by other sectors.

Aggregate output. Real GDP is the maximizer of a constant-elasticity-of-substitution (CES) aggregator of final consumption:

[13] The empirical networks literature examines the transmission of microeconomic shocks through input-output linkages using quasi-experiments. For instance, studies such as Barrot and Sauvagnat (2016), Boehm et al. (2019), and Carvalho et al. (2021) estimate the impact of natural disasters on output losses along firm-level supply chains, emphasizing the significance of indirect propagation.

$$Y = \max_{\{c_i\}_{i=1}^N} \left(\sum_{i=1}^N \left(\omega_{\mathcal{D},i} c_i \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{subject to} \quad \sum_{i=1}^N p_i c_i = wL,$$

where Y is aggregate output, c_i is the final consumption of good i, $\omega_{\mathcal{D},i}$ is a sector-specific final demand shifter, σ is the elasticity of substitution, p_i is the price of i's output, w is the wage rate, and L is aggregate labor supply (which is inelastically supplied).

Producers. Sectors produce output using Cobb-Douglas technologies that combine labor and intermediate goods

$$y_i = A_i l_i^{\alpha_i} M_i^{1-\alpha_i},$$

where y_i is sector i's output, A_i is a Hicks-neutral productivity shifter, l_i is labor use, M_i is a bundle of intermediate goods used by i and $\alpha_i \in [0,1]$ is the importance of labor in i's production. Furthermore, M_i is a CES aggregator of intermediate goods from other sectors, defined as

$$M_i \equiv \left(\sum_{j=1}^N \left(\omega_{ij} x_{ij} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$

where x_{ij} is the quantity of good j used by sector i and θ is the elasticity of substitution between inputs. As in Bernard and Moxnes (2018) and Acemoglu and Azar (2020), ω_{ij} is a relationship-specific shock to i's use of inputs from j. We model shocks to sectors' input use/supply via changes in the parameters ω_{ij}. We define an $N+1 \times N$ matrix $\mathbf{X}_i$ that captures all dependencies between sector i and its direct customers, suppliers, and final consumers *at the initial equilibrium*. Specifically, the i^{th} column of $\mathbf{X}_i$ contains the coefficients $(\omega_{1i}, ..., \omega_{Ni}, \omega_{\mathcal{D},i})'$, representing other sectors' reliance on intermediates from i, and households' dependence on final products from i. These parameters capture i's importance as a supplier of intermediates and final goods. The i^{th} row of $\mathbf{X}_i$ contains the coefficients $(\omega_{i1}, ..., \omega_{iN})$, that summarize sector i's demand for intermediate inputs from other sectors. All other elements in $\mathbf{X}_i$ are zeros. There are N matrices of this kind, one for each sector. In what follows, we measure sector i's scale centrality by characterizing the change in aggregate output Y in response to a uniform change in the elements of matrix $\mathbf{X}_i$, $\frac{d \log Y}{d \log \mathbf{X}_i}$.[14] We term this effect *scale centrality*

[14]Both $\mathbf{X}_i$ and $\mathbf{X}_j$ contain a common element, ω_{ij} and shocks to both sectors i and j entail changes in

since, for $d \log \mathbf{X}_i < 0$, it measures the change in GDP resulting from a reduction in the operations of industry i.[15]

Similar to Liu (2019), Baqaee and Farhi (2020b), and Bigio and La'O (2020), we model distortions as taxes on sectors' sales and input use. In our model, distortions leave the economy in the form of deadweight losses. Industry i's profits are therefore given by

$$\pi_i = (1-\tau_{y,i})p_i y_i - (1+\tau_{L,i})wl_i - \sum_{j=1}^{N}(1+\tau_{x,ij})p_j x_{ij},$$

where $\tau_{y,i}$, $\tau_{L,i}$, and $\tau_{x,ij}$ denote taxes imposed on sector i's sales, labor use, and intermediate good purchases, respectively. The market-clearing conditions for goods $1 \le i \le N$ and labor are given by

$$y_i = c_i + h_i + \sum_{j=1}^{N} x_{ji} \quad \text{and} \quad L = \sum_{i=1}^{N}(l_i + b_i) = 1,$$

where $h_i = \tau_{y,i}y_i + \sum_{j=1}^{N} \tau_{x,ji}x_{ji}$ is sector i's output and intermediate inputs that leave the economy as deadweight losses and $b_i = \tau_{L,i}l_i$ is the labor input of sector i that exits the economy. Our treatment of deadweight losses is similar to Liu (2019) and Bigio and La'O (2020).[16]

General equilibrium. Given taxes $\{\tau_{y,i}, \tau_{L,i}, \tau_{x,ij}\}$, productivities A_i, technology parameters ω_{ij}, and final demand shifters $\omega_{\mathcal{D},i}$, a general equilibrium is a set of prices $\{p_i, w\}$, input choices $\{l_i, x_{ij}\}$, outputs y_i and final demands c_i, such that: final demand maximizes the consumption aggregator subject to the budget constraint, producers maximize profits taking prices as given and the markets for labor and goods clear.

the parameter ω_{ij}. In the case of a shock to sector i, the change in ω_{ij} reflects a shock to sector i's use of inputs from j. Conversely, in the case of a shock to j, the change in ω_{ij} captures a change in the supply of inputs from j to i. However, $d \log \mathbf{X}_i$ should not be interpreted as confounding idiosyncratic shocks to i and other sectors. Instead, as we motivate in the introduction, $d \log \mathbf{X}_i$ should be understood as an idiosyncratic production disruption to sector i that entails a supply shock to sector i *and* a demand (for intermediate inputs) shock from sector i. We characterize the scale centrality of each sector *independently* of other sectors, i.e., $d \log \mathbf{X}_i$ is independent of $d \log \mathbf{X}_j$.

[15] In an earlier version of the paper, we referred to our measure as 'removal centrality,' which quantified the macroeconomic impact of removing a specific producer from the economy.

[16] Our key results that (i) scale centrality encompasses direct, indirect, and supplier effects, and (ii) Domar weights are the sum of direct and indirect effects, hold even when payments from distortions are rebated lump sum to the household, as in Jones (2011, 2013) and Baqaee and Farhi (2020b).

Remarks on the model. The above model is a relatively standard multi-sector production network model with tax wedges and CRS production functions. Our key contribution is to derive a new network statistic that accounts for the *empirical evidence* that an idiosyncratic production disruption to a firm often simultaneously impacts its supply and demand for intermediate inputs (Figure 1).[17] Given the empirical support, scale centrality summarizes the macroeconomic effect of such production disruptions to producers. Hulten's theorem and its generalizations continue to hold if the objective is to characterize the aggregate impact of changes in the supply of a producer's output following a microeconomic technology/productivity shock.

Crucially, our key results are invariant to specific modeling choices. Specifically, variants of scale centrality can be derived in models with additional distortions such as nominal rigidities and financing frictions (e.g., Liu, 2019 and Baqaee and Farhi, 2022) or in economies with heterogeneous households (Baqaee and Farhi, 2018) and international trade (Baqaee and Farhi, 2024 and Huo et al., 2024). The result that Domar weights (or variants thereof) are embodied within producers' scale centrality continues to hold across these different models.

Input-output definitions. We now introduce some input-output notation that is central to our analysis. First, we define the $N \times N$ tax-adjusted equilibrium input-output matrix $\mathbf{\Omega} \equiv [\Omega_{ij}]$, where

$$\Omega_{ij} = \frac{(1+\tau_{x,ij})p_j x_{ij}}{(1-\tau_{y,i})p_i y_i}. \tag{1}$$

For brevity, we will refer to the matrix $\mathbf{\Omega}$ as the *input-output matrix* throughout the rest of the paper. Notably, Ω_{ij} captures the direct exposure of sector i to sector j, after accounting for the taxes $\tau_{y,i}$ and $\tau_{x,ij}$ that separate prices from marginal costs.[18] The first-order condition with respect to intermediate inputs from sector j (x_{ij}), implies

$$\Omega_{ij} = (1-\alpha_i)(1+\tau_{x,ij})^{1-\theta} p_j^{1-\theta} P_{M,i}^{\theta-1} \omega_{ij}^{\theta-1}, \tag{2}$$

[17] The existing literature primarily focuses on the propagation and aggregation of productivity shocks in multi-sector models with input-output linkages. Baqaee and Farhi (2020a) and Baqaee and Farhi (2022) are exceptions that study supply and demand shocks simultaneously. However, in their context, demand shocks correspond to changes in household preferences. To the best of our knowledge, we are the first to study *production disruptions* that simultaneously change a producer's supply of and demand for intermediate inputs.

[18] The input-output matrix $\mathbf{\Omega}$ is similar to the cost-based input-output matrix of Baqaee and Farhi (2020b).

where $P_{M,i}$ is the price index associated with the intermediate goods bundle M_i.[19] The input-output parameter Ω_{ij} is therefore a function of the price of j's output relative to i's intermediate goods price index $p_j/P_{M,i}$, the tax wedge $(1+\tau_{x,ij})$, the elasticity of substitution θ, the importance of labor in i's production α_i, and the shock to i's use of inputs from j ω_{ij}.

As discussed above, to measure a sector's scale centrality, we consider negative shocks to i's *use* ($d\log\omega_{ij}<0$) and *supply* ($d\log\omega_{ji}$ & $d\log\omega_{\mathcal{D},i}<0$) of goods. For instance, a shock $d\log\omega_{ij}<0$ reflects a decrease in i's use of intermediate inputs from sector j. Equation (2) implies that in response to the shock $d\log\omega_{ij}<0$, the corresponding entry in the input-output matrix, Ω_{ij}, will also decrease. However, it is important to note that the input-specific shock will have a ripple effect on other elements in the input-output matrix $\mathbf{\Omega}$, as it will alter the prices of all sectors' outputs. Since intermediate inputs can be either complements or substitutes, changes in the parameters ω_{ij} will prompt producers to adjust the mix of inputs they use, leading to price changes across all sectors. As a result, these relative price adjustments affect final demand.[20]

Associated with the input-output matrix is the economy's Leontief inverse $\mathbf{\Psi}\equiv\left[\psi_{ij}\right]$, where

$$\mathbf{\Psi}\equiv(I-\mathbf{\Omega})^{-1}=I+\mathbf{\Omega}+\mathbf{\Omega}^2+\ldots$$

Intuitively, a typical element of the Leontief inverse ψ_{ij} captures all direct and indirect ways that sector i uses sector j's output.[21] In this respect, the Leontief inverse summarizes all production chains of any length. To see this, note that $(\mathbf{\Omega}^n)_{ij}$ measures the weighted sum of all paths of length n linking sector j to sector i through the production network.

Related to the matrix $\mathbf{\Omega}$ is the economy's *off-diagonal input output matrix* $\tilde{\mathbf{\Omega}}$, defined

$$\tilde{\mathbf{\Omega}}\equiv\mathbf{\Omega}-\operatorname{diag}(\mathbf{\Omega}).$$

The diagonal of $\tilde{\mathbf{\Omega}}$ contains zeros, whereas all off-diagonal elements are identical to those of the input-output matrix $\mathbf{\Omega}$. Thus, the matrix $\tilde{\mathbf{\Omega}}$ captures all dependencies

[19] Formally, $P_{M,i}\equiv\left(\sum_{j=1}^N\omega_{ij}^{\theta-1}\left[(1+\tau_{x,ij})p_j\right]^{1-\theta}\right)^{\frac{1}{1-\theta}}$.

[20] In the special case that all intermediate goods aggregators (M_i) are Cobb-Douglas ($\theta=1$), the input-output parameter Ω_{ij} is given by $\Omega_{ij}=(1-\alpha_i)\omega_{ij}$ for all i,j. This implies that a shock $d\log\omega_{ij}<0$ will not impact other parameters in the input-output matrix $\mathbf{\Omega}$ when M_i is Cobb-Douglas for all i. However, when $\theta\neq1$, the input-output matrix $\mathbf{\Omega}$ will respond endogenously to changes in the parameters ω_{ij}.

[21] See Carvalho and Tahbaz-Salehi (2019) for a more detailed discussion of the Leontief inverse matrix.

between sectors and omits producers' reliance on their own products. As we show in Section 2.1.1, the off-diagonal input-output matrix is required to measure industries' scale centrality.

Next, we denote the $N \times 1$ vector of final expenditure shares by $\boldsymbol{\Upsilon} \equiv [\Upsilon_i]$, where

$$\Upsilon_i = \frac{p_i c_i}{\sum_{j=1}^N p_j c_j} = \frac{p_i^{1-\sigma} \omega_{\mathcal{D},i}^{\sigma-1}}{\left(\sum_{j=1}^N p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1}\right)}. \tag{3}$$

The denominator $\sum_{j=1}^N p_j c_j$ corresponds to nominal GDP.[22] Notably, Υ_i measures the direct exposure of the household to sector i. We highlight in Section 2.1.1 that the final expenditure share of an industry i sufficiently summarizes the *direct effect* of a shock to sector i on real GDP. Notably, equation (3) shows that a shock to the household's dependence on i's product ($d \log \omega_{\mathcal{D},i} < 0$) implies i's final expenditure share (Υ_i) changes as well.

We also define an $N \times 1$ vector of tax-adjusted Domar weights, $\boldsymbol{\lambda} \equiv [\lambda_i]$, where

$$\lambda_i = \sum_{k=1}^N \Upsilon_k \psi_{ki}. \tag{4}$$

Throughout the rest of the paper we refer to λ_i as the *Domar weight* of sector i. Since λ_i implicitly embodies taxes on sales and intermediate goods, our Domar weights differ from those of Hulten (1978), which are defined for efficient economies. Notably, λ_i captures all direct and indirect ways the household uses goods from sector i after accounting for taxes.[23]

The aggregate impact of microeconomic TFP shocks. Before defining the key measure of the paper, we must first characterize how real GDP responds to TFP shocks. For this, we use the central theorem of Baqaee and Farhi (2020b), which characterizes the first-order macroeconomic impact of a microeconomic TFP shock in the presence of distortions:

$$\frac{d \log Y}{d \log A_i} = \lambda_i. \tag{5}$$

Crucially, tax-adjusted Domar weights $\{\lambda_i\}_{i=1}^N$ are the correct statistics for determin-

[22] See Appendix B for the proof of equation (3).

[23] Our Domar weights, defined in equation (4), are similar to the cost-based Domar weights of Baqaee and Farhi (2020b).

ing the aggregate effect of productivity shocks $d\log A_i$ to each sector $i = 1, 2, ..., N$. The above equation is a variant of Hulten's (1978) theorem for economies with inefficient equilibria.[24] Equation (5) highlights why Domar weights are a measure of the aggregate importance of industries: in response to an infinitesimal change in sector i's productivity, real GDP will change by λ_i%. Since the Domar weight λ_i implicitly encompasses all direct and indirect paths from sector i to final demand (equation 4), it captures how a supply shock to i impacts real GDP by propagating to other producers in the production network and eventually to final consumers. However, while the effect of a supply shock to sector i on real GDP is given by its Domar weight, sector i's scale centrality μ_i captures how shocks to i's input use and supply affect real GDP. Notably, scale centrality encapsulates sectors' Domar weights and all the information included in these statistics, and serves as a complementary measure of systemic significance that is distinct from industries' Domar weights.

2.1 Theoretical results

Having characterized the first-order change in real GDP in response to microeconomic productivity shocks, we now formally derive our measure of scale centrality.

2.1.1 Deriving Sectors' Scale Centrality

We first define the scale centrality of sector i, denoted by μ_i, which measures the macroeconomic impact (up to a first-order approximation) of uniform shocks to i's use and supply of goods.[25]

Definition 1. *The scale centrality of sector i is defined as*

$$\mathbb{1}'_{\mathcal{C}} \frac{d\log Y}{d\log \mathbf{X}_i} \mathbb{1}_{\mathcal{R}} \equiv \mu_i,$$

where $d\log \mathbf{X}_i$ represents a uniform change in i's use and supply of goods, and $\mathbb{1}_{\mathcal{C}}$ and $\mathbb{1}_{\mathcal{R}}$ are, respectively, $N+1 \times 1$ and $N \times 1$ vectors of ones.

[24]As is well-known in the production networks literature, in efficient economies, the first-order change in real GDP in response to a microeconomic productivity shock to sector i is given by Hulten's (1978) theorem, $\frac{d\log Y}{d\log A_i} = \frac{p_i y_i}{\mathrm{GDP}}$. Notably, in our model, the Domar weights λ_i do not coincide with $\frac{p_i y_i}{\mathrm{GDP}}$ due to the presence of tax wedges. See Baqaee and Farhi (2020b) for a more detailed discussion on aggregation in inefficient economies.

[25]Below, we characterize scale centrality under heterogeneous shocks.

The macroeconomic importance of sector i as a user of intermediate inputs and supplier of material and final goods increases with the magnitude of μ_i. A higher value of μ_i implies that a reduction in sector i's production and consumption of output would have a substantial impact on real GDP. This is because many other producers and final consumers in the network depend on sector i. Conversely, a lower value of μ_i suggests that sector i is relatively less important as a producer and consumer of goods from a macroeconomic perspective.

Theorem 1 characterizes μ_i in terms of observable input-output statistics (as measured at the initial equilibrium), permitting its computation. The resulting formula is the central object of our study.

Theorem 1. *The (first-order) scale centrality of sector i is given by*

$$\mu_i = \overbrace{\underbrace{\Upsilon_i}_{\text{Direct effect}} + \underbrace{\boldsymbol{\lambda}'\boldsymbol{\Omega}_{(i)}}_{\text{Indirect effect}}}^{\text{Domar weight } \lambda_i} + \underbrace{\lambda_i \tilde{\boldsymbol{\Omega}}^{(i)} \mathbb{1}_{\mathcal{R}}}_{\text{Supplier effect}}, \tag{6}$$

where $\boldsymbol{\Omega}_{(i)}$ is the i^{th} column of the equilibrium input-output matrix, $\tilde{\boldsymbol{\Omega}}^{(i)}$ is the i^{th} row of the matrix $\tilde{\boldsymbol{\Omega}}$, $\mathbb{1}_{\mathcal{R}}$ is an $N \times 1$ vector of ones, and Υ_i is the final expenditure share of sector i.

Proof. See Appendix B.

Equation (6) highlights that sector i's scale centrality can be decomposed into three distinct effects. The first term, the *direct effect* Υ_i, measures how a supply shock to sector i directly affects real GDP through households' consumption of final goods from i. The larger the value of Υ_i, the more significant sector i is as a producer of final goods and services. Next, the *indirect effect* measures the impact of the supply shock to sector i on real GDP by tracing how it spreads to other industries, influencing final demand indirectly. This effect, represented by $\boldsymbol{\lambda}'\boldsymbol{\Omega}_{(i)}$, is computed as the Domar-weighted sum of sector i's intermediate goods sales to the rest of the economy, with the vector $\boldsymbol{\Omega}_{(i)}$ containing elements from the i^{th} column of the input-output matrix $\boldsymbol{\Omega}$. By multiplying each element of $\boldsymbol{\Omega}_{(i)}$ with the corresponding customer sector's Domar weight, the indirect effect comprises all possible transmission paths from sector i to final demand, regardless of length. This can be seen in the alternate expression for the indirect effect: $\boldsymbol{\Upsilon}'\boldsymbol{\Omega}_{(i)} + \boldsymbol{\Upsilon}'\boldsymbol{\Omega}\boldsymbol{\Omega}_{(i)} + \boldsymbol{\Upsilon}'\boldsymbol{\Omega}^2\boldsymbol{\Omega}_{(i)} + \ldots$, where $\boldsymbol{\Upsilon}'\boldsymbol{\Omega}_{(i)}$ represents all transmission paths of length one, $\boldsymbol{\Upsilon}'\boldsymbol{\Omega}\boldsymbol{\Omega}_{(i)}$ denotes paths of length two, $\boldsymbol{\Upsilon}'\boldsymbol{\Omega}^2\boldsymbol{\Omega}_{(i)}$ captures paths of length three, and so on.

Before discussing the supplier effect of equation (6), we introduce Corollary 1, which shows how sector i's direct and indirect effects together constitute the first-order macroeconomic impact of a productivity shock to sector i, which is given by i's Domar weight λ_i, as shown in equation (5).[26]

Corollary 1. *The Domar weight of sector i is equivalent to the sum of i's direct and indirect effect*

$$\lambda_i = \underbrace{\Upsilon_i}_{\textit{Direct effect}} + \underbrace{\boldsymbol{\lambda}'\boldsymbol{\Omega}_{(i)}}_{\textit{Indirect effect}} .$$

Proof. First note that the sum of i's direct and indirect effect can be written as $\Upsilon_i + \sum_{k=1}^{N} \lambda_k \Omega_{ki}$. By equation (4), λ_k can be equivalently expressed as $\lambda_k = \sum_{m=1}^{N} \Upsilon_m \psi_{mk}$. Therefore, we can rewrite i's direct and indirect effect as $\Upsilon_i + \sum_{k=1}^{N} \sum_{m=1}^{N} \Upsilon_m \psi_{mk} \Omega_{ki}$, which, in matrix form is $\boldsymbol{\Upsilon}' + \boldsymbol{\Upsilon}'\boldsymbol{\Psi}\boldsymbol{\Omega}$. Rewriting this expression as $\boldsymbol{\Upsilon}'(I + \boldsymbol{\Psi}\boldsymbol{\Omega})$ and noting that $\boldsymbol{\Psi} = I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + ...$, we get $\boldsymbol{\Upsilon}'\left(I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + \boldsymbol{\Omega}^3 + ...\right)$. Once again using the result $\boldsymbol{\Psi} = I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + ...$, we can write $\boldsymbol{\Upsilon}'\left(I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + \boldsymbol{\Omega}^3 + ...\right) = \boldsymbol{\Upsilon}'\boldsymbol{\Psi}$, which is nothing but the Domar weight vector $\boldsymbol{\lambda}' = \boldsymbol{\Upsilon}'\boldsymbol{\Psi}$. ■

Corollary 1 provides an important insight into the direct and indirect effects of a sector's scale centrality μ_i. These effects (which are captured in the first two terms on the right-hand side of equation (6) sum to i's Domar weight, which is an alternative measure of a sector's systemic importance that has been widely studied in the literature (see, for example, Hulten, 1978, Acemoglu et al., 2012, Liu, 2019, Baqaee and Farhi, 2020b, Bigio and La'O, 2020). Therefore, not only does Theorem 1 allow us to quantify the direct and indirect effect of an idiosyncratic supply shock to sector i, but it also provides a decomposition of i's Domar weight.

The final term on the right-hand side of equation (6), which we refer to as the *supplier effect*, distinguishes a sector's scale centrality μ_i from its Domar weight λ_i. The supplier effect quantifies how idiosyncratic changes in the demand for inputs by producer i impact real GDP. For instance, a negative shock to i, $d \log \mathbf{X}_i < 0$, also affects the production of i's suppliers, as i's demand for intermediates decreases. These demand-side shocks propagate through the network, further affecting all producers downstream from i, either directly or indirectly, ultimately reducing aggregate output. As a result,

[26] Formally, a uniform change in the i^{th} column of $\mathbf{X}_i$ has a first-order aggregate impact that is isomorphic to a microeconomic productivity shock to i.

the decline in real GDP resulting from the shock to i exceeds λ_i, and includes the supplier effect as well. The supplier effect $\lambda_i \tilde{\mathbf{\Omega}}^{(i)} \mathbb{1}_{\mathcal{R}}$ is calculated by interacting i's Domar weight with the elements of the i^{th} row of the matrix $\tilde{\mathbf{\Omega}}$, which captures the intensity with which i uses inputs from other industries. Specifically, if an industry j supplies inputs to i (i.e., $\tilde{\Omega}_{ij} \neq 0$), then i's supplier effect will be nonzero. In an efficient economy, the supplier effect intuitively simplifies to the value of total input purchases by industry i as a share of GDP.

Measuring scale centrality. Computing scale centrality only requires *observed* data on sector-level intermediate input purchases, nominal gross output, final sales, and tax payments. This is because final expenditure shares Υ_i, Domar weights $\{\lambda_k\}_{k=1}^{N}$, and parameters of the input-output matrix $\mathbf{\Omega}$ (measured at the *initial* equilibrium) are sufficient to characterize the first-order change in real GDP in response to a shock to i's supply and use of inputs $d \log \mathbf{X}_i$. While the shock $d \log \mathbf{X}_i$ induces changes in prices and quantities of inputs traded at the microeconomic level, these endogenous changes are macroeconomically irrelevant to a first-order approximation and only matter beyond the first-order. In Appendix E, we characterize i's scale centrality up to a second-order approximation. Sector i's second-order scale centrality requires knowledge of changes in i's final expenditure share $(d \log \Upsilon_i)$, the Domar weights of all industries $(d \log \lambda_k)$, and the input-output parameters of i's suppliers $(d \log \Omega_{ik})$ and customers $(d \log \Omega_{ki})$. Specifically, we characterize these objects in terms of the elasticities of substitution in production θ and consumption σ, as well as the parameters of the production network at the initial equilibrium.

Heterogeneous shocks. Equation (6) assumes that the shock to sector i's use and supply of goods is uniform across all sectors. However, sectors may vary their demand for different inputs or prioritize customers differently in response to supply shocks. Corollary 2 characterizes sector i's scale centrality in the presence of heterogeneous supply and use shocks.

Corollary 2. *The scale centrality of sector i, in the presence of heterogeneous shocks to i's supply and use of goods, $\mu_i^{\mathcal{H}}$, is given by*

$$\mu_i^{\mathcal{H}} = \Upsilon_i d \log \omega_{\mathcal{D},i} + \sum_{k=1}^{N} \lambda_k \Omega_{ki} d \log \omega_{ki} + \sum_{k \neq i}^{N} \lambda_i \Omega_{ik} d \log \omega_{ik}. \tag{7}$$

Proof. See Appendix B.

Equation (7) generalizes Theorem 1 to economies with non-uniform shocks. The formula is useful for studying how intermediate input demand shocks propagate through the production network and influence real GDP: $\frac{d\log Y}{d\log \omega_{ik}} = \lambda_i \Omega_{ik}$. In efficient economies, $\lambda_i \Omega_{ik}$ is simply i's expenditure on inputs from sector k, as a share of nominal GDP, or $\frac{p_k x_{ik}}{\text{GDP}}$. In economies with inefficiencies, $\lambda_i \Omega_{ik}$ implicitly encompasses distortions (such as sales and intermediate goods taxes as in our model). Throughout the rest of the paper, we focus on scale centrality as defined in equation (6), but note that our framework can accommodate heterogeneous shocks.

2.1.2 Illustrative Examples

In this section, we provide a deeper understanding of Theorem 1 by exploring the properties of three network structures. This exercise allows us to make empirical predictions about the relationship between sectors' scale centrality μ_i (as characterized by equation 6) and their Domar weights λ_i.

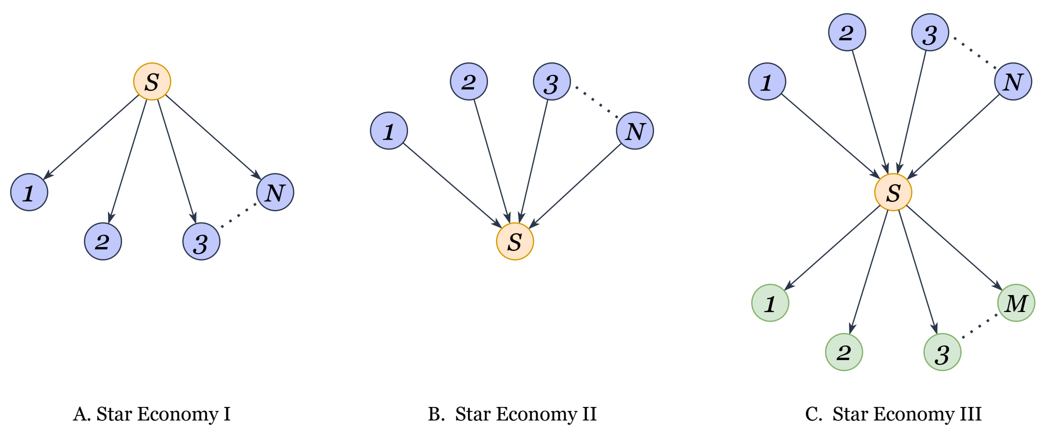


Figure 2: Example Network Structures

Note: This figure depicts three different network structures. Colored nodes represent different industries, whereas directed arrows depict the flow of intermediate goods between sectors. Orange nodes represent the "star" sector S. In Panel A, the star sector only uses labor to produce intermediate inputs and final goods. In Panel B, sector S uses labor and inputs from all other sectors to produce only final goods. In Panel C, the star sector S uses labor and intermediate inputs from the set of N purely upstream sectors (blue nodes) $1, 2, ..., N$ to produce final goods and intermediate inputs for the set of M purely downstream sectors $1, 2, ..., M$.

Consider the three "Star Economies" shown in Figure 2. In the figure, colored nodes represent different industries, and directed arrows depict the flow of intermediate goods between producers. In each panel, the orange node corresponds to the "star" sector denoted by S. In Panel A, the star sector uses only labor to produce intermediate inputs that are, in turn, used by all other industries $1, 2, ..., N$ (blue nodes), as well as final goods consumed by households. In Panel B, sector S uses labor and intermediates from all other industries to produce only final goods. Finally, in Panel C, sector S uses both labor and intermediate inputs from the set of N purely upstream sectors (blue nodes, denoted by $1, 2, ..., N$) to produce final goods and intermediate inputs for the set of M purely downstream sectors.[27]

Throughout our analysis of the network structures in Figure 2, we initially assume that all sectors are equally important in final demand, which implies a constant value of $\omega_{\mathcal{D},i}$ across all i (at the pre-shock equilibrium). Moreover, we also assume that α_i is constant across all sectors that use intermediates in addition to labor, and that the importance of labor and intermediates for these sectors is the same. Finally, we impose a unitary elasticity of substitution in consumption and production, $\sigma = 1$ and $\theta = 1$, to simplify the expressions comparing the parameters μ and λ. We make these simplifying assumptions to isolate how the *structure* of each network generates differences in scale centralities and Domar weights across industries.

Star economy I. In the economy depicted in Figure 2, Panel A, the scale centrality of the star sector S is exactly equal to its Domar weight, or $\mu_S = \lambda_S$. However, for all other sectors $1, 2, ..., N$, their scale centrality exceeds their Domar weight. This is clear from the expressions for μ and λ:

$$\mu_S = \lambda_S = \Upsilon_S \left(1 + \sum_{j \neq S}^{N} \Omega_{jS} \right) \quad \text{and} \quad \mu_i = \Upsilon_S \left(1 + \Omega_{iS} \right) > \lambda_i \;\; \text{for sectors } i \neq S.$$

Furthermore, under the production structure of Figure 2, Panel A, the scale centrality of the star sector S is always greater than that of all other industries. In other words, the star sector has the greatest ability to shape aggregate fluctuations, as it serves as the sole producer of intermediate inputs for all other sectors.

Figure 3 illustrates the relationship between μ and λ under the "Star Economy I" network structure. The red line is the 45-degree line, where $\mu = \lambda$. The orange point represents the star sector S, while the blue point represents all other sectors $i \neq S$. Fi-

[27] In symmetric networks, where all industries have equal interdependence for intermediate inputs, Domar weights and scale centralities are identical across sectors. However, in asymmetric networks like those in Figure 2, there are differences in Domar weights and scale centralities across industries. As our focus is on understanding the network properties that lead to variations in both λ and μ, we only discuss the asymmetric networks depicted in Figure 2.

nally, the faint dotted line visually approximates the trend between μ and λ.

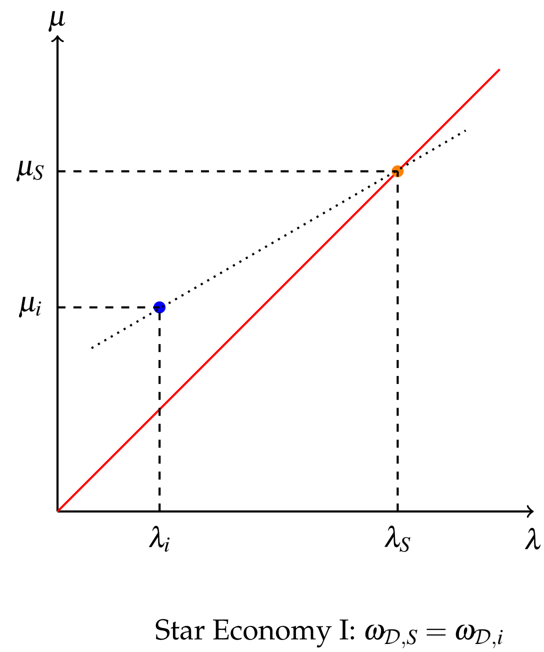


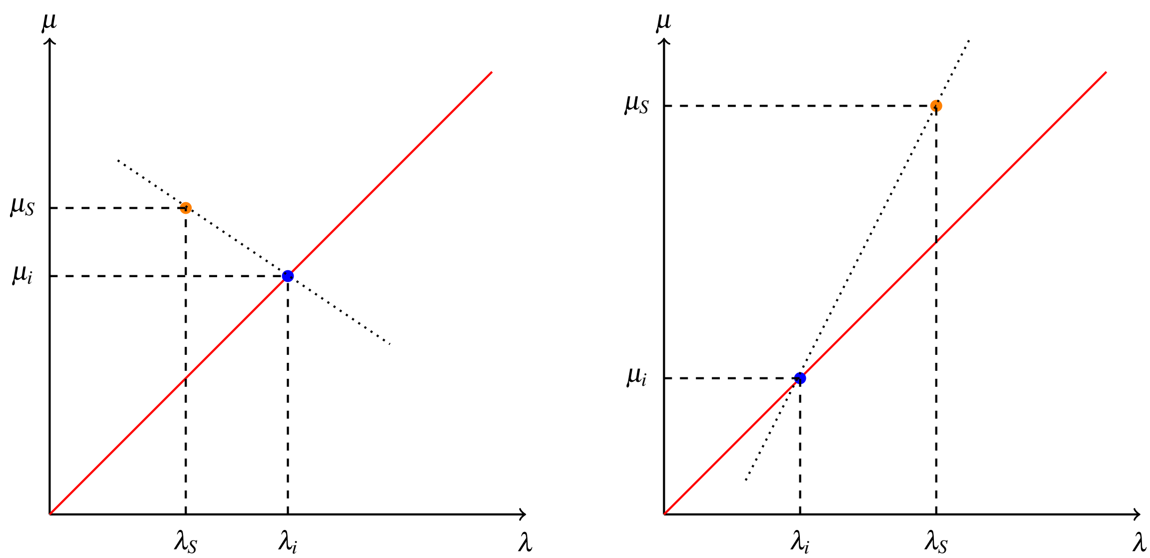
Figure 3: Scale Centrality and Domar Weights (Star Economy I)

Note: This figure plots sectors' scale centrality μ against their Domar weight λ under the Star Economy I network structure shown in Figure 2. The orange point represents the star sector S, whereas the blue point represents all other industries $i \neq S$. Households' dependence on each industry's output is the same across all sectors at the initial equilibrium, $\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$ for all $i \neq S$. The solid red line represents the 45-degree line, and the faint dotted line is a visual approximation of the trend in the relationship between μ and λ.

The figure reveals two key insights: i) the relationship between μ and λ is positive since $\lambda_S > \lambda_i$ and $\mu_S > \mu_i$, and ii) the trend line has a slope less than one since $\frac{\mu_S - \mu_i}{\lambda_S - \lambda_i} < 1$.[28] In Section 3, we present empirical evidence that the degree to which Domar weights underestimate the systemic importance of an industry (as measured by μ) *increases* as the Domar weight of the sector increases. In other words, the slope of the trend line is estimated to be greater than one. Thus, the star economy depicted in Figure 2, Panel A, cannot account for this observed relationship. Our results suggest that important industries, with large scale centrality, are generally *not* upstream sectors that produce using mostly labor and other factors of production but without intermediate inputs from other producers.

[28]These results hold even if S does not produce final goods. That is, even if the downstream sectors are more important in final demand than S, λ_S will always be greater than λ_i.

Star economy II. Next, consider the network in Panel B of Figure 2. In this economy, the star sector S uses, but does not supply, intermediate inputs to any other industry. Therefore, the scale centrality of S is strictly greater than its Domar weight $\mu_S > \lambda_S$, while the scale centrality of all other sectors exactly equal their Domar weights $\mu_i = \lambda_i$ for all $i \neq S$. Theorem 1 implies



Panel A. Star Economy II: $\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$

Panel B. Star Economy II: $\omega_{\mathcal{D},S} >> \omega_{\mathcal{D},i}$

Figure 4: Scale Centrality and Domar Weights (Star Economy II)

Note: This figure plots sectors' scale centrality μ against their Domar weight λ under the Star Economy II network structure shown in Figure 2. The orange point represents the star sector S, whereas the blue point represents all other industries $i \neq S$. In Panel A, households depend equally on the output produced by each industry (at the pre-shock equilibrium) $\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$ for all $i \neq S$, whereas in Panel B, the star sector is a more critical producer of final goods $\omega_{\mathcal{D},S} >> \omega_{\mathcal{D},i}$. In both panels, the faint dotted line provides a visual approximation of the trend in the relationship between μ and λ, whereas the solid red line is the 45-degree line.

$$\mu_S = \Upsilon_S \left(1 + \sum_{j \neq S}^{N} \Omega_{Sj}\right) > \lambda_S \quad \text{and} \quad \mu_i = \lambda_i = \Upsilon_S \left(1 + \Omega_{Si}\right) \quad \text{for sectors } i \neq S.$$

The above expressions highlight that the scale centrality of the star sector S is greater than that of any other industry $i \neq S$. However, assuming households are equally reliant on the final products produced by each sector at the initial equilibrium ($\omega_{\mathcal{D},S} =$

$\omega_{\mathcal{D},i}$ for all i), the Domar weight of S is *strictly less* than that of sectors $i \neq S$.[29] This is visually represented in Panel A of Figure 4, where again the orange point depicts the star sector, and the blue point represents a representative upstream sector. Thus, the network structure in Figure 2, Panel B, implies a negative relationship between sectors' scale centrality and Domar weights.

Panel B of Figure 4 depicts the condition under which a positive relationship between μ and λ exists in this network: the star sector S must not only use intermediate inputs but *also* be a more important producer of final goods than the other sectors, or $\omega_{\mathcal{D},S} >> \omega_{\mathcal{D},i}$.[30] When this condition is met, the Domar weight of S exceeds that of other industries, resulting in a positive relationship between λ and μ. Moreover, since $\frac{\mu_S-\mu_i}{\lambda_S-\lambda_i} > 1$, the trend line has a slope greater than one. In light of our empirical results of Section 3, Figure 4 suggests that industries with large scale centralities not only consume more intermediate goods relative to less influential sectors but may also play a critical role in supplying final goods.

Star economy III. We now turn to the economy shown in Panel C of Figure 2. The blue nodes (denoted by $1, 2, ..., N$) in the figure represent purely upstream sectors that supply intermediate inputs to the star sector S (represented by the orange node), while the green nodes $(1, 2, ..., M)$ depict purely downstream sectors that use inputs from S. In this economy, sector S always records a value of μ greater than every downstream *and* upstream sector. Specifically, Theorem 1 implies that the scale centrality of S is given by

$$\mu_S = \Upsilon_S + \Upsilon_S \sum_{j=1}^{M} \Omega_{jS} + \Upsilon_S \sum_{j \neq S}^{N} \Omega_{Sj} + \Upsilon_S \left(\sum_{j=1}^{M} \Omega_{jS} \right) \sum_{j \neq S}^{N} \Omega_{Sj}, \tag{8}$$

whereas the scale centrality of a representative upstream sector U and downstream sector D is, respectively,

$$\mu_U = \Upsilon_S + \Upsilon_S \Omega_{SU} + \Upsilon_S \Omega_{SU} \sum_{j=1}^{M} \Omega_{jS}, \tag{9}$$

[29] Formally, the Domar weight of S is $\lambda_S = \Upsilon_S$, whereas the Domar weight of an industry $i \neq S$ is $\lambda_i = \Upsilon_S (1 + \Omega_{Si})$.

[30] This is because Υ_k is increasing in $\omega_{\mathcal{D},k}$ for all k. For a large value of $\omega_{\mathcal{D},S}$, where $\omega_{\mathcal{D},S} >> \omega_{\mathcal{D},i}$, the Domar weight of S is greater than the Domar weight of i, i.e., $\lambda_S = \Upsilon_S > \Upsilon_i (1 + \Omega_{Si}) = \lambda_i$. In this case, the *direct effect* of sector S is the largest of all industries, which implies that its Domar weight is also larger than that of other sectors.

and

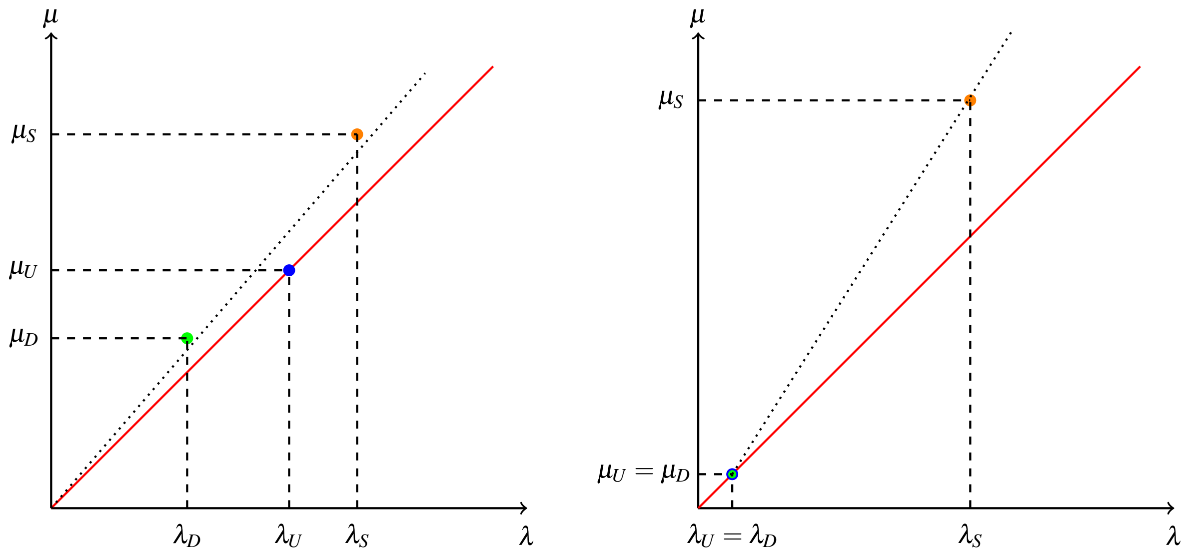
$$\mu_D = \Upsilon_S + \Upsilon_S \Omega_{DS}. \tag{10}$$

Comparing equations (8), (9), and (10), it is evident that μ_S is always larger than μ_U and μ_D. However, the scale centrality of upstream sectors U may be greater or less than that of downstream sectors D, depending on the relative number of upstream and downstream sectors.[31] As we show below, this implies that the trend between μ and λ depends crucially on the in-degree *and* out-degree (i.e., the number of upstream and downstream connections) of the star sector. Notably, however, as the number of upstream and downstream sectors increases, the slope of the trend line converges unambiguously to a value greater than one. Thus, economies populated with producers that purchase inputs from many upstream suppliers *and* supply intermediates to numerous downstream customers are likely to see μ increase more than one-for-one relative to λ. Indeed, in Section 3.1, we show this relationship exists for the US. Industries such as oil and gas extraction and petroleum refineries are disproportionate users and suppliers of intermediate inputs, and are thus positioned well above the 45-degree line.

While the rankings of μ are conditional on the characteristics of the star sector, the orderings of λ in the third star economy are unambiguous. Specifically, it is always the case that $\lambda_S > \lambda_U > \lambda_D$.[32] Figure 5 shows the relationship between μ and λ as the star sector becomes increasingly important as a producer and consumer of *intermediate* goods. In each panel, the green point represents a downstream sector, the blue point is an upstream industry, and the orange point again represents the star sector. Panel A depicts a possible trend between μ and λ when there are few upstream and downstream sectors, where $\mu_U > \mu_D$. As discussed above, the slope of the trend line in Panel A may not be greater than one since i) upstream sectors always lie on the 45-degree line, ii) λ_D is less than λ_U, and iii) μ_U may be less or greater than μ_D. However, as the number of upstream and downstream industries grow large, scale centralities and Domar weights of the upstream and downstream sectors approach zero, while both μ

[31] Specifically, $\mu_U > \mu_D$ if $M(1-\alpha_S) > N-1$.

[32] The Domar weight of S is given by $\lambda_S = \Upsilon_S + \Upsilon_S \sum_{j=1}^{M} \Omega_{jS}$, while the Domar weights of D and U are, respectively, $\lambda_D = \Upsilon_S$ and $\lambda_U = \Upsilon_S + \Upsilon_S \Omega_{SU} + \Upsilon_S \left(\sum_{j=1}^{M} \Omega_{jS}\right) \Omega_{SU}$. Although it is evident that $\lambda_S > \lambda_D$ and $\lambda_U > \lambda_D$, it is not immediately clear that $\lambda_S > \lambda_U$. For λ_S to be strictly greater than λ_U, the condition $\sum_{j=1}^{M} \Omega_{jS} > \Omega_{SU} \left(1-\Omega_{SU}\right)^{-1}$ must hold. Since the star sector and all downstream sectors are equally dependent on intermediates, $(1-\alpha_j) = (1-\alpha_S)$, this condition can be expressed as $\sum_{j=1}^{M} (1-\alpha_S)\omega_{jS} > (1-\alpha_S)\omega_{SU}(1-(1-\alpha_S)\omega_{SU})^{-1}$. Simplifying this expression yields $M > \frac{1}{N-(1-\alpha_S)}$, which is true whenever the number of upstream sectors is greater than one, $N > 1$.



Panel A. Star Economy III: $\mu_U > \mu_D$

Panel B. Star Economy III: $N, M \to \infty$

Figure 5: Scale Centrality and Domar Weights (Star Economy III)

Note: Panel A plots sectors' scale centrality μ against their Domar weight λ under the Star Economy III network structure shown in Figure 2. The orange point represents the star sector S, the blue point represents a purely upstream industry U, and the green point depicts a purely downstream industry D. The household sector depends equally on the final goods produced by the sectors ($\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$ for all i). The faint dotted line visually approximates the trend in the relationship between μ and λ, whereas the solid red line represents the 45-degree line. Panel B shows the trend as the number of upstream and downstream sectors increases.

and λ remain greater than zero for the star sector, as shown in Panel B.[33]

In other words, the significance of sector S as both a user *and* producer of intermediate inputs determines whether the slope of the trend line is greater than one: the more important is S, the greater the slope. This point is significant in light of our empirical findings, where we observe the relationship between μ and λ to be similar to that of Figure 5, Panel B.

Anticipating our empirical results in Section 3, our analysis of the three network structures in Figure 2 indicates that sectors with high scale centralities i) exhibit a greater reliance on intermediates relative to sectors with lower values of μ, and ii) are also significantly more important producers of final or intermediate goods compared to other industries.

[33] Formally, as $N, M \to \infty$, $\mu_S \to \frac{(1-\alpha_S)(1+(1-\alpha_S))}{2} > 0$ and $\lambda_S \to \frac{1-\alpha_S}{2} > 0$.

2.2 How Do Taxes Affect Sectors' Macroeconomic Importance?

Since we derive sectors' scale centrality within the framework of an inefficient production networks model, a natural question arises: what is the effect of distortions on the scale centrality of producers? In our model, taxes increase sectors' scale centrality μ_i relative to an economy without distortions. This result is formalized in Proposition 1 below.

Proposition 1. *Sector-level taxes on sales $\tau_{y,i}$ and intermediate purchases $\{\tau_{x,ik}\}_{k=1}^{N}$ contribute to the macroeconomic importance of a sector i via*

$$\zeta_i = \mu_i - \mu_i^F \geq 0, \tag{11}$$

where ζ_i is the contribution of taxes to μ_i and μ_i^F is the scale centrality of i in a frictionless economy without taxes.

Proof. See Appendix B.

Taxes on sales $\tau_{y,i}$ and intermediate inputs $\tau_{x,ij}$ have a compounding effect along supply chains, reducing the allocative efficiency of producers and amplifying the impact of microeconomic shocks on aggregate fluctuations. In equation (11), we demonstrate that distortions increase sectors' scale centrality compared to a frictionless economy without taxes.[34] The statistic ζ_i measures the contribution of taxes to the macroeconomic importance of sector i where μ_i^F represents i's scale centrality in a frictionless economy. In the Proof of Proposition 1 in Appendix B, we characterize ζ_i in terms of the parameters of the production network $\mathbf{\Omega}$, final expenditure shares $\mathbf{\Upsilon}$, and taxes on sales $\{\tau_{y,i}\}$ and intermediates $\{\tau_{x,ij}\}$ and show that ζ_i is always greater than or equal to zero.[35]

[34]Notably, taxes on factor payments $\tau_{L,i}$ only affect real GDP in the presence of labor supply shocks and play no role when there are shocks to sectors' input use and supply ($d \log \mathbf{X}_i$).

[35]In Appendix D, we estimate the contribution of taxes to sectors' supplier effects using US input-output data from 1982 to 2012. Our analysis shows that taxes at the sector level have a minimal impact on the estimated values of supplier effects and scale centralities. While other distortions such as nominal rigidities, financial frictions, and market power may also affect the systemic importance of industries, we abstract from these distortions in our calculation of industries' scale centrality.

3 Taking the Model to the Data

In this section, we use the framework presented in Section 2 to measure the scale centrality μ and Domar weight λ of approximately 450 sectors (on average) in the United States between 1982 and 2012. We then document the relationship between μ and λ and establish a connection between our empirical findings and the discussion of the network structures presented in Section 2.1.2. Next, we identify sectors that have experienced substantial fluctuations in scale centrality despite having relatively stable Domar weights. Finally, we study industries' scale centralities across six major economies: the United States, United Kingdom, Japan, China, Germany, and Australia, and decompose μ into direct, indirect, and supplier effects.

3.1 Comparing scale centrality with Domar weights

We begin by measuring scale centrality μ_i (as defined in equation 6) for all sectors in the US using the detailed input-output tables provided by the BEA between 1982 and 2012.[36] Specifically, we first calibrate $\mathbf{\Omega}$ to each year's input-output table, which further yields $\mathbf{\Omega}_{(i)}$ and $\tilde{\mathbf{\Omega}}^{(i)}$. We use final demand data in the BEA Use tables to estimate the final expenditure shares $\mathbf{\Upsilon}$. We then use $\mathbf{\Omega}$ and $\mathbf{\Upsilon}$ to compute the Domar weights via $\boldsymbol{\lambda}' = \mathbf{\Upsilon}'(I-\mathbf{\Omega})^{-1}$. Figure 6 compares sectors' scale centrality and Domar weights between 1982 and 2012. The red line is the 45-degree line, or the point at which a sector's scale centrality coincides with its Domar weight. Panel A plots all industries (excluding government sectors) appearing in the detailed BEA input-output accounts from 1982 to 2012, along with a line-of-best-fit through the data for each year. On the other hand, Panel B decomposes the scale centrality of three key sectors in 2012 into direct, indirect, and supplier effects.

Panel A of Figure 6 yields three notable observations. Firstly, a positive relationship exists between sectors' scale centrality and Domar weights. This suggests that sectors with higher scale centrality typically also have larger Domar weights, signifying their greater systemic influence and highlighting significant asymmetries within the US production structure. Secondly, the scale centrality of a sector is generally distinct from its Domar weight, implying that the two measures capture different aspects of a sector's influence within the network. Finally, the scale centrality of a sector is typically greater than its Domar weight. This finding is expected, as scale centrality

[36] The detailed input-output tables are only available at five-year intervals. See Appendix C for a discussion of the BEA data.

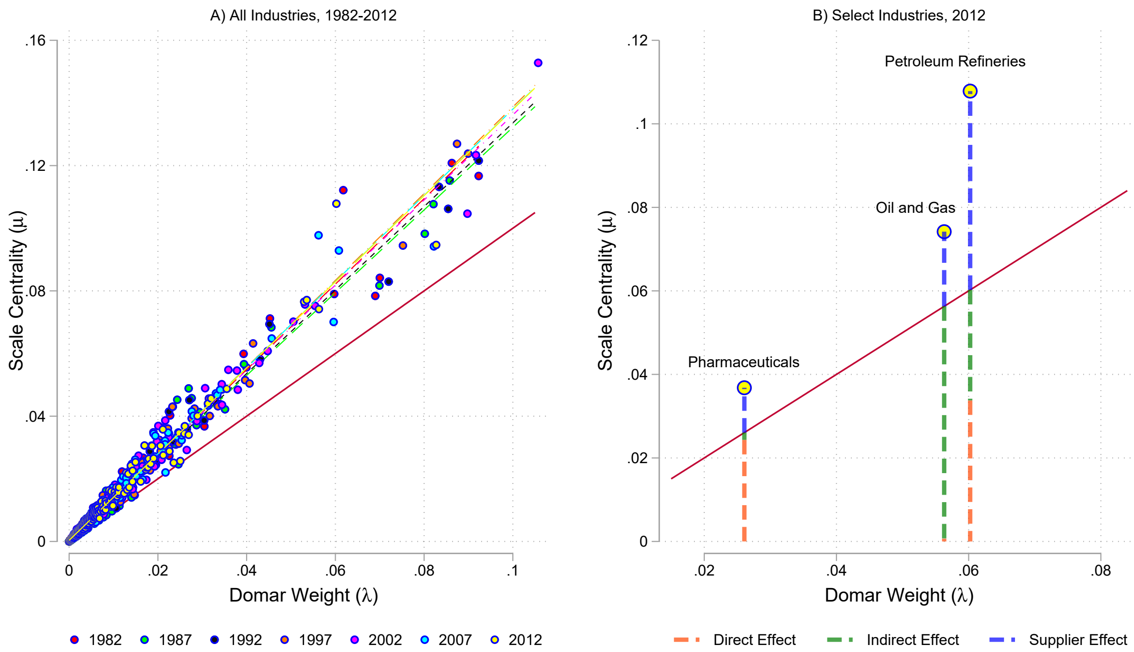


Figure 6: Scale Centrality and Domar Weights in the US (1982 - 2012)

Note: This figure shows the relationship between scale centrality and Domar weights in the US from 1982-2012. On average, there are approximately 450 sectors per year. The solid red line is the 45-degree line, representing the equality of Domar weights and scale centrality. Panel A displays data for all sectors in the detailed BEA input-output accounts, excluding government sectors. Panel B decomposes the scale centrality of three key industries in 2012 into direct, indirect, and supplier effects.

encompasses producers' Domar weight, providing a more comprehensive assessment of a sector's significance.

Importantly, the figure highlights that the Domar weight of a sector tends to underestimate its systemic importance, and this underestimation becomes more pronounced as the sector's Domar weight increases, as evidenced by the deviation from the 45-degree line. This finding is not immediately obvious since scale centrality and Domar weights can coincide, even for sectors with a large Domar weight. This can occur when upstream sectors do not rely on intermediate inputs for production, as shown in Panel A of Figure 2 and Figure 3. However, the magnitude of this underestimation is significant and consequential. On average, the extent to which Domar weights underestimate the systemic significance of a sector ranges between approximately 40-50% (with the median figure being similar), depending on the year.

Additionally, the figure also reveals a remarkable similarity in the relationship between scale centrality and Domar weights over time, as shown by the similar slope of the trend line for each year. It is thus an empirical regularity that sectors with larger Domar weights also have significantly greater supplier effects. This suggests that as sectors grow larger in size, they (in general) become increasingly reliant on intermediate inputs and, therefore, become even more important from a macroeconomic perspective.

As discussed in Section 2.1.2, our findings in Figure 6 reflect significant asymmetries in the US network structure (emphasized in Acemoglu et al., 2012). Panel B of Figure 6 decomposes the scale centrality of three key US sectors in 2012 (pharmaceuticals, oil and gas extraction, and petroleum refineries) into direct, indirect, and supplier effects. These sectors all record large scale centralities, and are consequently positioned well above the 45-degree line, albeit for different reasons. The figure highlights that most influential sectors typically fall into one of three categories: a) they are disproportionately significant producers of final goods (like pharmaceuticals) and hence have substantial direct effects, b) they serve as essential suppliers of intermediate inputs (like oil and gas), resulting in larger indirect effects, or c) they are important producers of both final and intermediate goods (like petroleum refineries). This suggests that the US network structure resembles the asymmetric star networks depicted in Panels B and C of Figure 2. In such networks, sectors with large scale centralities are typically located downstream in the supply chain *and* serve as critical suppliers of intermediate or final goods.

3.2 Tracking changes in supplier effects and Domar weights for specific sectors over time

While Figure 6 demonstrates a positive relationship between Domar weights and scale centrality, it is important to note that there is significant variation in how scale centralities and Domar weights of sectors change over time. To highlight this, Figure 7 compares supplier effects and Domar weights for select industries across time.[37] To ensure comparability of the BEA input-output codes, we divide the sample period into two intervals: 1982-1992 (Panel A) and 1997-2012 (Panel B).[38] The figure displays the

[37]Here, we track supplier effects instead of scale centrality as the supplier effect distinguishes scale centrality from the Domar weight of a sector.

[38]The input-output industry codes between 1992 and 1997 are not consistent with each other as the BEA revised its input-output classification system in 1997. Dividing the sample into the two periods, therefore, prevents fluctuations in supplier effects or Domar weights from being artificially driven by changes in BEA industry codes. Further details on the comparison of the 1992 and 1997 benchmark IO accounts of the BEA can be found at https://apps.bea.gov/scb/pdf/2002/08August/0802_I-O_

four largest sectors (by gross sales) in which the absolute value of the ratio between the percentage change in supplier effects and the percentage change in Domar weights exceeded two within the corresponding period covered in each panel. For example, this ratio is approximately 27 for the "eating and drinking places" sector, reflecting the large change in its supplier effect relative to its Domar weight between 1982 and 1992.[39] The gross sales of the sectors depicted in Figure 7 equate to approximately 11% of GDP, on average, across all years.

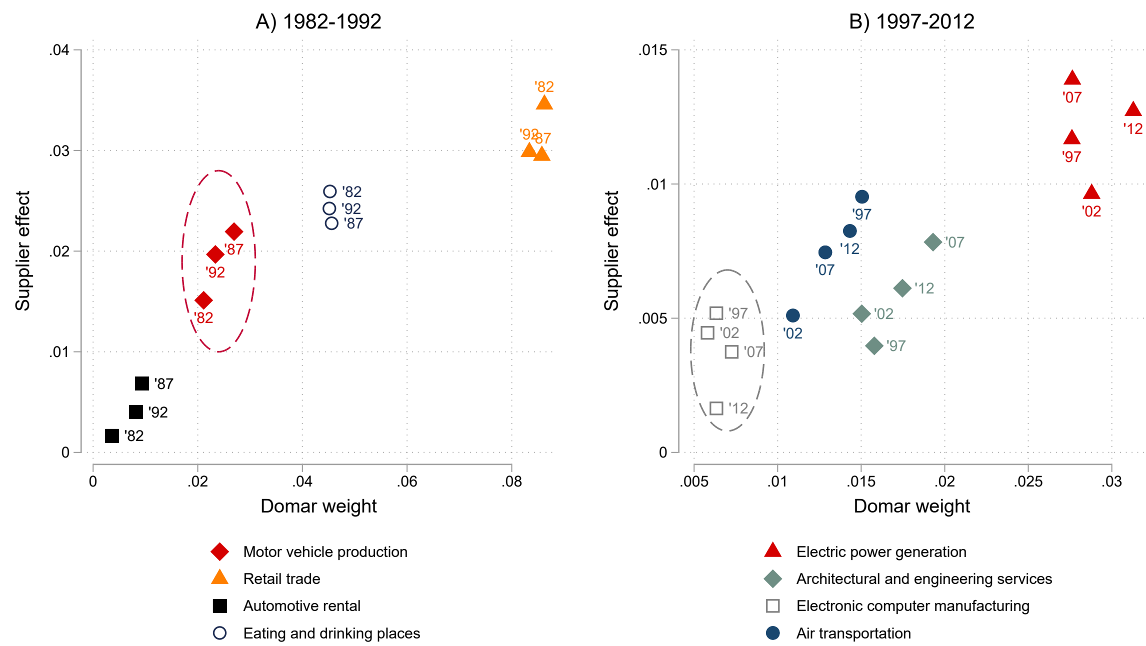


Figure 7: Tracking Changes in Supplier Effects and Domar Weights for Key Industries Over Time

Note: This figure illustrates the relationship between sectors' supplier effects and Domar weights at five-year intervals from 1982 to 1992 (Panel A) and from 1997 to 2012 (Panel B). For each sample period, the growth rate of supplier effects for each sector is at least twice as large (in absolute value) as the growth rates of their corresponding Domar weights. Supplier effects are calculated as in equation (6), and Domar weights are computed as in equation (4), both using the detailed input-output accounts of the BEA.

The figure highlights that scale centrality can vary significantly across sectors over time, even when Domar weights remain relatively stable. For example, consider the case of motor vehicle production (represented by the red ellipse) in Panel A. Between

Benchmark.pdf.

[39]The aggregate sales of all sectors for which this ratio is greater than two amount to approximately 40% of GDP, on average, from 1982 to 2012.

1982 and 1992, the industry experienced a 30% increase in its supplier effect (or an increase of around 0.5% of GDP), while its Domar weight remained almost unchanged. This finding may reflect the increase in the complexity of motor vehicle production over time. The number of electrical components and engine parts in motor vehicles grew substantially throughout the 1980s and 1990s (Fine et al., 1996), a trend that has continued to accelerate in the 21st century. By 2010, the average motor vehicle contained approximately 2,000 components, 30,000 parts, and 10 million lines of software code (MacDuffie and Fujimoto, 2010). Crucially, the increase in supplier effects implies that motor vehicle production became more systemically important over time, which is not captured in the Domar weight of the industry.

In contrast, in Panel B, computer manufacturing (represented by the grey ellipse) experienced a 68% decline in its supplier effect, while its Domar weight slightly increased by 0.4% between 1997 and 2012. This finding reflects the rapid reduction in production costs in the computer manufacturing sector during the late 20th century, resulting from technological improvements in IT industries (Jorgenson, 2001). While input costs continued to fall throughout the mid-2000s, growth in final computer sales slowed substantially, leading to stability in the computer manufacturing sector's Domar weight between 1997 and 2012. The evolution of supplier effects in the computer manufacturing industry may also be explained by changes in market structure and increases in international competition during the 1990s. For example, in the mid-1990s, a growing portion of computer parts and components were imported from Asian markets, where production costs were significantly lower than in the US (Warnke, 1996). Taken together, the findings presented in Figure 7 suggest that relying solely on Domar weights provides only partial information about changes in sectors' systemic importance over time. This highlights the relevance of scale centrality as a complementary measure of the macroeconomic significance of industries.

3.3 Analyzing scale centrality for key sectors across countries

Next, using data from the World Input-Output Database (WIOD) from 2000 to 2014, we identify the industries with the greatest scale centrality for six major economies: the US, Great Britain, Japan, China, Germany, and Australia.[40] Figure 8 shows the top ten sectors (by scale centrality) for each of the countries. In the figure, each sector's

[40] See Appendix C for a detailed description of the WIOD data. These economies accounted for $\approx$ 57% of nominal world GDP in 2021 (World Bank, 2023).

scale centrality is captured by the total of the direct effect (orange bars), indirect effect (yellow bars), and supplier effect (blue bars). Overall, for each economy shown, construction, real estate, public administration, and food & beverages are always among the top five sectors in terms of their ability to influence aggregate output.[41]

Notably, there are significant differences in the size of each effect across countries and industries. Labor-intensive sectors like education and health services have substantial direct effects (accounting for $\approx$ 75% of scale centrality, on average) due to their low dependency on intermediate inputs and direct provision of services to end-consumers. These attributes reduce their ability to shape aggregate fluctuations, resulting in lower indirect and supplier effects compared to other sectors. On the other hand, the influence of sectors like wholesale trade and electricity & gas supply comes mostly from the indirect effect ($\approx$ 50% of scale centrality for both industries), reflecting the importance of these sectors in providing key intermediate inputs for production for many other sectors in the economy. Finally, construction and food & beverages have proportionately large direct ($\approx$ 50% for construction and $\approx$ 40% for food & beverages) and supplier effects ($\approx$ 30% for construction and $\approx$ 35% for food & beverages), attesting to their central role as both producers of final goods and consumers of intermediate goods. Hence, each effect represents a distinct mechanism with which a sector can influence aggregate outcomes, and each subcomponent explains a consequential proportion of scale centrality. For the countries and industries shown in Figure 8, direct effects account for about 50% of sectors' overall influence, on average, whereas indirect and supplier effects account for approximately 20% and 30%, respectively.

Our results demonstrate significant differences in the ability of industries to affect aggregate volatility through direct, indirect, and supplier effects. Moreover, our decomposition exercise indicates that the impact of a producer on the broader economy is heavily influenced by its location within the production network. The intensity with which producers use and supply intermediate inputs is a key determinant of their sys-

[41] We abstract from international trade when computing each sector's scale centrality. Baqaee and Farhi (2021) characterize how microeconomic shocks propagate through international input-output linkages and generalize Hulten's theorem to open economies. Other papers in the trade literature highlight the importance of input-output linkages as a mechanism for amplifying shocks and generating co-movements in business cycles across countries (see, for example, Caliendo and Parro, 2014, Chaney, 2014, di Giovanni et al., 2014, Redding and Rossi-Hansberg, 2017, di Giovanni et al., 2018, Auer et al., 2019, Antràs and de Gortari, 2020, and Kikkawa et al., forthcoming). A related literature emphasizes that diversified global value chains insulate economies against shocks, limiting aggregate volatility (Caselli et al., 2020; D'Aguanno et al., 2021, and Antràs, 2021). See Baldwin and Freeman (2022) for a detailed overview of the global supply chain literature and Bernard and Moxnes (2018) for a survey of the literature on production networks and international trade.

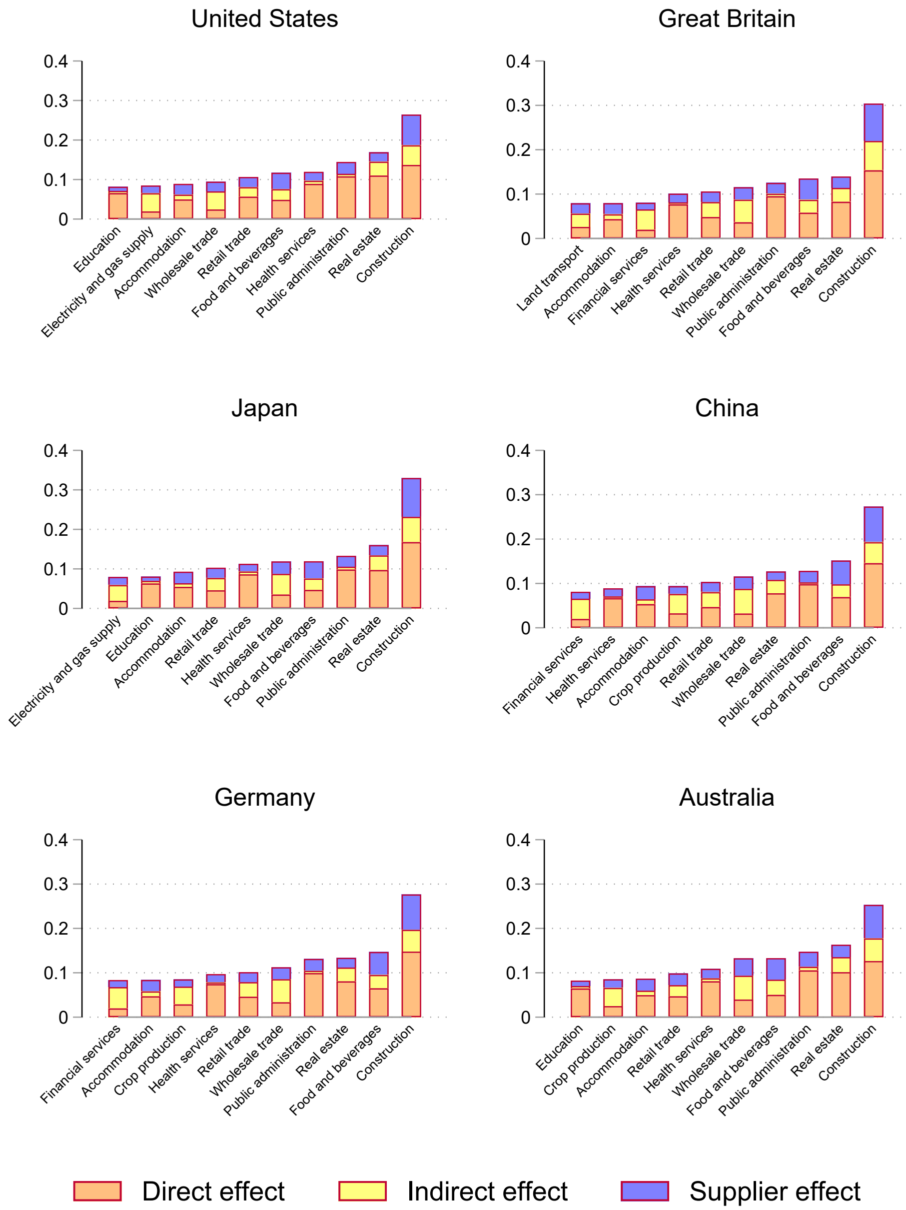


Figure 8: Scale Centrality by Country

Note: This figure presents estimates of scale centrality μ for six major economies. For each sector, scale centrality is measured as in equation (6), averaged over the years 2000 to 2014. The top 10 industries with the greatest scale centrality are shown for each economy. Orange bars capture the direct effects, whereas the yellow and blue bars represent the indirect and supplier effects, respectively. The data is from the World Input-Output Database (2016 Release).

temic importance.

4 Conclusion

We propose a novel measure of sectors' systemic importance in economies with production networks. Our measure, which we refer to as *scale centrality*, captures how shocks to a producer impact real GDP by i) directly affecting the final consumption of its output, ii) indirectly affecting the production of firms that are directly or indirectly connected to it, and iii) changing its demand for intermediate inputs from upstream suppliers.

Our approach is nonparametric, only requiring observable information on an industry's intermediate goods purchases, nominal gross sales, and tax payments. Notably, scale centrality encompasses and extends an existing notion of systemic importance in production networks: producers' Domar weights (or sales shares). In an empirical application for the US, we show that Domar weights underestimate the systemic importance of an industry by $\approx$ 40-50% between 1982-2012. We find that the extent of underestimation increases with the Domar weight of a sector; a non-trivial result given that scale centrality and Domar weights can perfectly coincide, even for large sectors.

Additionally, we provide evidence of significant changes in the scale centrality of crucial sectors within the US economy over time, such as motor vehicle production and computer manufacturing, despite the relative stability of their Domar weights. Furthermore, we compare sectors' scale centrality across countries and identify industries such as construction, food & beverages, and real estate as powerful amplifiers of shocks between 2000 and 2014.

Our empirical findings are interpreted through the lens of an inefficient production network model. Though our model only includes one type of friction (taxes), it can be easily extended to incorporate other distortions such as financial frictions, market power, and nominal rigidities, as in Liu (2019); Baqaee and Farhi (2020b); Bigio and La'O (2020) and Baqaee and Farhi (2022), highlighting the flexibility of our framework.

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Online Appendix to *Beyond Domar Weights: A New Measure of Systemic Importance in Production Networks*[1]

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This Appendix is organized as follows. Section A discusses the data and empirical strategy used to produce Figure 1 in the main text. Section B contains proofs of our key theoretical results. Section C provides a discussion of the data used in Section 3 of the paper. Section D discusses the role of taxes in the measurement of scale centrality. Section E presents the derivation of scale centrality up to a second-order approximation.

A Motivating Empirical Evidence

In this section, we discuss the data and identification strategy used in Figure 1 of the main text. For more detail on the data and our estimation strategy, see Bahal et al. (2023).

Firm-level financial data. Firms' financial data comes from Compustat's *North America Fundamentals Quarterly* database. We use quarterly information on firms' sales (in USD), number of employees, asset holdings, and income before depreciation, among other variables for all publicly listed firms in the US. We restrict the sample to firms headquartered in the US between 1978 and 2017.[2] We deflate firms' sales using the GDP price index from the Bureau of Economic Analysis so that sales growth (the dependent variable in our regressions) reflects firms' performance, not price dynamics.

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[2]Customer-supplier transactions data in Compustat's *Customer Segments* files is available from 1978 onwards.

All continuous variables are winsorized at the 1st and 99th percentiles. To reduce measurement error, we restrict the sample to firms that report financial information in calendar quarters (which is how the natural disaster data is reported). This reduces bias in our estimates by ensuring firms are not hit by natural disasters *after* they report their financial information.

Firm-level input-output linkages. We use Compustat's *Customer Segments* dataset for information on inter-firm relationships, the duration of each link, and customers' purchases from suppliers. The *Customer Segments* data has been used in Barrot and Sauvagnat (2016) and Bahal et al. (2023), among other studies. A key limitation of the data is that we only observe a fraction of each firm's suppliers. This is because suppliers are only required to report the identity of major customers that account for at least 10% of total annual sales. The reporting threshold likely biases our estimates against finding upstream or downstream propagation effects, as some observations in the control groups of suppliers and customers are actually treated (see Bahal et al., 2023 for more detail on this point). We assume input-output linkages are active for all quarters between the first and last time the connection is recorded in the *Customer Segments* data. We omit all links where suppliers and customers are within 300 kilometers of each other, allowing us to isolate the propagation effects from the direct effects of disasters on suppliers and customers.

Natural disaster data. We use the *Emergency Events Database* (EM-DAT) for information on all disaster events across our sample period. EM-DAT contains information on event duration, type of disaster, event name, affected regions (usually at the state level), and estimates of damages (in USD). A limitation of EM-DAT is that it does not contain county-level information. We complement the EM-DAT data with information from the Federal Emergency Management Agency's (FEMA) *Disaster Declarations* dataset, which contains information on the US counties affected by each disaster. Our resulting dataset contains county-level data for all US natural disasters between 1978 and 2017, total damage estimates, and each event's duration. Following Barrot and Sauvagnat (2016), we include all disasters with damages exceeding $1 billion (in 2017 USD) that lasted less than a month. Over our sample period, there were 52 major natural disasters.

Summary statistics. Table A.1 presents basic summary statistics for our subsamples of supplier (Panel A) and customer (Panel B) firms. Firms are included in the supplier (customer) sample from one year before being first reported to one year after last being reported as a supplier (customer). The supplier sample includes 5,665 distinct firms, generating 153,390 supplier-quarter observations. In contrast, the customer sample consists of 2,352 firms across 78,629 customer-quarter observations.

Table A.1: Descriptive Statistics

Panel A: Supplier Sample	Never Treated			Eventually Treated		
	Obs.	Mean	Std. Dev.	Obs.	Mean	Std. Dev.
Sales growth $(t-4,t)$	73,108	0.20	0.90	80,282	0.17	0.75
Total assets (Bil. USD)	73,108	0.70	2.22	80,282	1.42	3.76
Return on assets	73,108	0.03	0.29	80,282	0.06	0.26
Number of employees ('000s)	71,117	3.51	9.30	78,849	4.62	11.47
Age (Years)	73,108	37.87	17.97	80,282	38.73	19.89
Number of customers	73,108	0.98	0.96	80,282	1.51	1.40
Panel B: Customer Sample	Never Treated			Eventually Treated		
	Obs.	Mean	Std. Dev.	Obs.	Mean	Std. Dev.
Sales growth $(t-4,t)$	37,079	0.10	0.40	41,550	0.08	0.31
Total assets (Bil. USD)	37,079	3.02	6.09	41,550	13.02	21.95
Return on assets	37,079	0.11	0.15	41,550	0.14	0.10
Number of employees ('000s)	36,293	10.85	20.36	41,049	40.78	60.94
Age (Years)	37,079	41.19	17.75	41,550	43.96	16.15
Number of suppliers	37,079	0.73	0.96	41,550	4.28	9.36

Notes: The table presents summary statistics for the sample of supplier (Panel A) and customer (Panel B) firms in Compustat's *Customer Segments* files. Firms are included for every quarter from one year prior to when they first appear as a supplier (customer) and ending one year after they are last reported as a supplier (customer) in the *Customer Segments* dataset. The supplier sample comprises 5,665 distinct firms, creating 153,390 firm-quarter observations from 1978 to 2017. In Panel A, 'Never Treated' ('Eventually Treated') refers to those suppliers that did not have (had) at least one customer hit by a major natural disaster at some point in the sample period. The customer sample consists of 2,352 firms across 78,629 firm-quarter observations. In Panel B, 'Never Treated' ('Eventually Treated') refers to those firms for which a natural disaster did not (did) hit a supplier at any point in the sample period.

In Panel A, 'Never Treated' refers to those suppliers that at no point in the sample period had a customer affected by a major natural disaster, whereas 'Eventually treated' refers to those suppliers that at some point had an affected customer. The mean sales growth (measured over the previous four quarters) of never-treated and eventually-treated suppliers is comparable, at 20% and 17%, respectively. However, the median sales growth is much lower for both groups, at 3.8% and 4.3% for never- and eventually-treated suppliers, suggesting a right-skewed distribution. Affected suppliers tend to be slightly larger (in terms of total assets and number of employees) and more profitable (as measured by return on assets (ROA)). The average age of 'never-treated' suppliers is 37.9 years, while 'eventually-treated' suppliers have an average age of 38.7 years, highlighting the similar age profile of the two groups. Finally, treated suppliers have an average of 1.51 customers in a given quarter, whereas never-treated suppliers have 0.98 customers on average. This difference is expected since the probability of a supplier being affected by a disaster increases with a firm's number of customers.

In Panel B, 'Never Treated' refers to customers that at no point between 1978 and 2017 had a supplier affected by a disaster, and 'Eventually Treated' refers to firms with at least one supplier struck by a disaster. The mean sales growth of customers in these two groups is quite similar, at 10% and 8% for the never-treated and eventually-treated subgroups, respectively. However, treated customers are larger (as measured by total assets and number of employees), more profitable, older, and have more suppliers in a given quarter. These differences are expected due to the 10% reporting threshold, which results in larger customers being over-represented in the eventually-treated subgroup. Larger customers tend to have more suppliers, increasing the likelihood of having a supplier affected by a disaster in a given quarter. The differences in observable characteristics of treated and never-treated suppliers/customers highlight the need to control for these variables in our regressions.

Empirical models. We first establish that exogenous shocks (natural disasters) simultaneously affect firms' and their direct suppliers' sales growth. To this end, we estimate the following difference-in-differences specification using the supplier sample:

$$\begin{aligned}\text{Sales Growth}_{it,t-4} = \alpha &+ \sum_{j=-2}^{9} \beta_j \times \text{Disaster Strikes Firm}_{i,t-j} \\ &+ \sum_{j=-2}^{9} \eta_j \times \text{Disaster Strikes Customer}_{i,t-j} + \mathbf{X}_{i,t} + \tau_t + \gamma_i + \varepsilon_{i,t}. \quad \text{(A.1)}\end{aligned}$$

In equation (A.1), Sales Growth$_{it,t-4}$ represents the real sales growth of firm i in quarter t relative to the same quarter in the previous year. Disaster Strikes Firm$_{i,t-j}$ is a dummy variable that takes the value one if firm i was affected by a major natural disaster in quarter $t-j$ and zero otherwise. Similarly, Disaster Strikes Customer$_{i,t-j}$ is an indicator that takes the value one if at least one customer of firm i experienced a shock at time $t-j$. We include two leads and nine lags of these dummy variables to analyze sales dynamics before and after the shocks. The coefficients of interest are β_j and η_j, which respectively estimate the average change in the sales growth of firm i when a disaster directly strikes the firm or one of its customers at time $t-j$. $\mathbf{X}_{i,t}$ is a vector of time-varying controls, including firm age, number of employees, lagged values of ROA and total assets, a dummy variable that indicates if the firm has an above-median customer count, and state-decade and fiscal-quarter fixed effects. Equation (A.1) also includes year-quarter (τ_t) and firm (γ_i) fixed effects.

We also estimate the effect of natural disaster shocks on downstream firms' sales growth using the customer sample. For this, we estimate the following regression:

$$\begin{aligned}\text{Sales Growth}_{it,t-4} = \alpha &+ \sum_{j=-2}^{9} \kappa_j \times \text{Disaster Strikes Supplier}_{i,t-j} \\ &+ \sum_{j=-2}^{9} \mu_j \times \text{Disaster Strikes Firm}_{i,t-j} + \mathbf{X}_{i,t} + \tau_t + \gamma_i + \varepsilon_{i,t}, \quad \text{(A.2)}\end{aligned}$$

where Disaster Strikes Supplier$_{i,t-j}$ is a dummy that takes the value one if at least one supplier of customer firm i was hit by a natural disaster at time $t-j$ and zero otherwise. To avoid confounding propagation effects with direct effects of natural disasters on customers' sales growth, we also control for Disaster Strikes Firm$_{i,t-j}$. As above, we allow for two leads and nine lags of the key regressors to analyze propagation dynamics. All other variables in equation (A.2) are the same as in (A.1), except that instead of controlling for customer count in $\mathbf{X}_{i,t}$, we include a dummy that indicates whether firm i has an above-median *supplier* count. The coefficients of interest in (A.2) are the κ_j's, which estimate the average change in sales growth of downstream customer firms following a shock to at least one of their suppliers at quarter $t-j$. We cluster standard errors at the firm level in equations (A.1) and (A.2).

Results. Our results are summarized in Figure 1 of the main text. The left panel plots estimates of the impact of natural disasters on suppliers' sales growth (red line) as well as the direct effects of the shocks (black line) from two quarters before to nine quarters after a major natural disaster. The figure shows that the sales growth of directly affected firms decreases by 4.6pp two quarters following a shock, while shocked firms' immediate suppliers experience a decline of 3.6pp concurrently. Following this, sales growth of directly affected firms returns to pre-shock levels after eight quarters, while sales growth for upstream suppliers normalizes after nine quarters.

The right panel of Figure 1 shows customer firms' average change in sales growth following a shock to at least one supplier (orange line) from two quarters before to nine quarters after the shock. For reference, the right panel also includes the direct effect of natural disasters on the sales growth of directly affected firms (black line), which is the same as the left panel. The figure reveals that the propagation shocks to downstream firms occur with a lag. Specifically, the customers of shocked firms experience an average decline in sales growth of 3pp four quarters after a disruption and stay below pre-shock levels for a further four quarters. The delayed impact of the shock could be attributed to customers holding excess inventory, thereby preventing an immediate shortfall in production (Hendricks et al., 2009).[3]

Overall, our results from Figure 1 highlight two key points: 1) natural disasters have simultaneous effects on directly affected firms and their immediate suppliers, and 2) these disasters also impact the customers of affected firms, but with a substantial lag.

B Proofs

Proof of Theorem 1. The Lagrangean for the aggregator problem is

$$\mathcal{L} = \left(\sum_{j=1}^{N} \left(\omega_{\mathcal{D},j} c_j \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} + \eta \left(wL - \sum_{j=1}^{N} p_j c_j \right),$$

where η is the Lagrange multiplier. Optimization with respect to c_i yields

$$c_i = wL p_i^{-\sigma} \omega_{\mathcal{D},i}^{\sigma-1} \left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1} \right)^{-1}, \tag{B.1}$$

from which we derive an expression for Υ_i:

$$\Upsilon_i = \frac{p_i c_i}{wL} = \frac{p_i^{1-\sigma} \omega_{\mathcal{D},i}^{\sigma-1}}{\left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1} \right)}. \tag{B.2}$$

[3] Figure 1 also reveals that pre-existing trends do not account for the change in suppliers' and customers' sales growth, as treated and untreated firms exhibit no systematic difference in sales growth before a major natural disaster.

Substituting equation (B.1) into the consumption aggregator, yields

$$Y = wL\left(\sum_{j=1}^{N}\omega_{\mathcal{D},j}^{\sigma-1}p_i^{\sigma-1}\right)^{\frac{1}{\sigma-1}}. \tag{B.3}$$

Furthermore, total (log) differentiation of the consumption aggregator gives

$$d\log Y = \sum_{i=1}^{N} Y^{\frac{1-\sigma}{\sigma}}\left(\omega_{\mathcal{D},i}c_i\right)^{\frac{\sigma-1}{\sigma}}\left(d\log c_i + d\log\omega_{\mathcal{D},i}\right). \tag{B.4}$$

Substituting (B.1) and (B.3) into equation (B.4), yields

$$d\log Y = \sum_{i=1}^{N}\frac{p_i^{1-\sigma}\omega_{\mathcal{D},i}^{\sigma-1}}{\left(\sum_{j=1}^{N}p_j^{1-\sigma}\omega_{\mathcal{D},j}^{\sigma-1}\right)}\left(d\log c_i + d\log\omega_{\mathcal{D},i}\right),$$

which can be rewritten using equation (B.2) as

$$d\log Y = \sum_{i=1}^{N}\Upsilon_i d\log c_i + \sum_{i=1}^{N}\Upsilon_i d\log\omega_{\mathcal{D},i}. \tag{B.5}$$

Total (log) differentiation of equation (B.1) implies that (B.5) can be written as

$$\begin{aligned} d\log Y = \sum_{i=1}^{N}\Upsilon_i\left(d\log w + d\log L - \sigma d\log p_i + (\sigma-1)d\log\omega_{\mathcal{D},i} - d\log\left(\sum_{j=1}^{N}p_j^{1-\sigma}\omega_{\mathcal{D},j}^{\sigma-1}\right)\right) \\ + \sum_{i=1}^{N}\Upsilon_i d\log\omega_{\mathcal{D},i}. \end{aligned} \tag{B.6}$$

Noting that $d\log\left(\sum_{j=1}^{N}p_j^{1-\sigma}\omega_{\mathcal{D},j}^{\sigma-1}\right)$ simplifies to $(1-\sigma)\left(\sum_{i=1}^{N}\Upsilon_i\left(d\log p_i - d\log\omega_{\mathcal{D},i}\right)\right)$, equation (B.6) can be rewritten as

$$d\log Y = d\log w - \sum_{i=1}^{N}\Upsilon_i\left(d\log p_i - d\log\omega_{\mathcal{D},i}\right). \tag{B.7}$$

We now turn to producers' optimization problem to derive an expression for $d\log p_i$. The first-order conditions for labor and intermediate inputs imply

$$l_i = \alpha_i(1-\tau_{y,i})p_i y_i w^{-1}(1+\tau_{L,i})^{-1} \tag{B.8}$$

and

$$x_{ij} = (1-\alpha_i)^{\theta} p_i^{\theta} y_i^{\theta} \omega_{ij}^{\theta-1} p_j^{-\theta} (1-\tau_{y,i})^{\theta} (1+\tau_{x,ij})^{-\theta} M_i^{1-\theta}. \tag{B.9}$$

From (B.8) and (B.9), we derive expressions for the labor expenditure shares Λ_i and input-output parameters Ω_{ij}

$$\Lambda_i = \frac{(1+\tau_{L,i})wl_i}{(1-\tau_{y,i})p_i y_i} = \alpha_i \tag{B.10}$$

and,

$$\Omega_{ij} = (1-\alpha_i)^{\theta} p_i^{\theta-1} y_i^{\theta-1} \omega_{ij}^{\theta-1} p_j^{1-\theta} (1-\tau_{y,i})^{\theta-1} (1+\tau_{x,ij})^{1-\theta} M_i^{1-\theta}. \tag{B.11}$$

Substituting (B.8) and (B.9) into sector i's production function, we derive the following expression for the price of good i,

$$p_i = A_i^{-1} (1-\tau_{y,i})^{-1} (1+\tau_{L,i})^{\alpha_i} w^{\alpha_i} \alpha_i^{-\alpha_i} (1-\alpha_i)^{\alpha_i - 1} P_{M,i}^{1-\alpha_i}, \tag{B.12}$$

where $P_{M,i} \equiv \left(\sum_{j=1}^{N} \omega_{ij}^{\theta-1} \left[(1+\tau_{x,ij}) p_j \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}$. Total differentiation of equation (B.12), yields

$$d\log p_i = \Lambda_i d\log w - d\log A_i - (\alpha_i - 1) d\log P_{M,i},$$

where $d\log P_{M,i}$ is given by

$$d\log P_{M,i} = (1-\alpha_i)^{-1} \sum_{j=1}^{N} \Omega_{ij} d\log \omega_{ij} + (1-\alpha_i)^{-1} \sum_{j=1}^{N} \Omega_{ij} d\log p_j.$$

Therefore, we can write

$$d\log p_i = \Lambda_i d\log w - d\log A_i + \sum_{j=1}^{N} \Omega_{ij} d\log \omega_{ij} + \sum_{j=1}^{N} \Omega_{ij} d\log p_j,$$

which we can re-arrange to get

$$d\log p_i = \sum_{k=1}^{N} \psi_{ik} \Lambda_k d\log w - \sum_{k=1}^{N} \psi_{ik} d\log A_k - \sum_{k=1}^{N} \sum_{j=1}^{N} \psi_{ik} \Omega_{kj} d\log \omega_{kj}.$$

From the identity, $\mathbf{\Omega} \mathbb{1}_{\mathcal{R}} + \mathbf{\Lambda} = \mathbb{1}_{\mathcal{R}}$, it follows that $\mathbf{\Psi}' \mathbf{\Lambda} = \mathbb{1}_{\mathcal{R}}$ (with $\mathbb{1}_{\mathcal{R}}$ being an $N \times 1$

vector of ones). Therefore, the above equation can be rewritten as

$$d\log p_i = d\log w - \sum_{k=1}^{N} \psi_{ik} d\log A_k - \sum_{k=1}^{N}\sum_{j=1}^{N} \psi_{ik}\Omega_{kj} d\log \omega_{kj}. \tag{B.13}$$

Substituting equation (B.13) into (B.7) yields

$$d\log Y = d\log w - \sum_{i=1}^{N} \Upsilon_i \left(d\log w - \sum_{k=1}^{N} \psi_{ik} d\log A_k - \sum_{k=1}^{N}\sum_{j=1}^{N} \psi_{ik}\Omega_{kj} d\log \omega_{kj} - d\log \omega_{\mathcal{D},i} \right),$$

which simplifies to

$$d\log Y = \sum_{i=1}^{N}\sum_{k=1}^{N} \Upsilon_i \psi_{ik} d\log A_k + \sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{j=1}^{N} \Upsilon_i \psi_{ik}\Omega_{kj} d\log \omega_{kj} + \sum_{i=1}^{N} \Upsilon_i d\log \omega_{\mathcal{D},i}.$$

Using the fact that $\lambda_k = \sum_{i=1}^{N} \Upsilon_i \psi_{ik}$, we can re-write the above equation as

$$d\log Y = \sum_{k=1}^{N} \lambda_k d\log A_k + \sum_{k=1}^{N}\sum_{j=1}^{N} \lambda_k \Omega_{kj} d\log \omega_{kj} + \sum_{i=1}^{N} \Upsilon_i d\log \omega_{\mathcal{D},i}. \tag{B.14}$$

From the above equation, we derive

$$\frac{d\log Y}{d\log A_i} = \lambda_i,$$

which is equation (5). From equation (B.14), we derive i's scale centrality $\mu_i \equiv \mathbb{1}'_{\mathcal{C}} \frac{d\log Y}{d\log \mathbf{X}_i} \mathbb{1}_{\mathcal{R}}$

$$\mu_i = \sum_{k=1}^{N} \lambda_k \Omega_{ki} + \sum_{k\neq i}^{N} \lambda_i \Omega_{ik} + \Upsilon_i$$

or, in matrix form,

$$\mu_i = \Upsilon_i + \boldsymbol{\lambda}' \boldsymbol{\Omega}_{(i)} + \lambda_i \tilde{\boldsymbol{\Omega}}^{(i)} \mathbb{1}_{\mathcal{R}}.$$

■

Proof of Corollary 2. This follows from equation (B.14) in the Proof of Theorem 1.

■

Proof of Proposition 1. Denote by $\mathbf{\Omega}^F$ the *frictionless* input-output matrix, with the ij^{th} element given by $\Omega^F_{ij} = \frac{p_j x_{ij}}{p_i y_i}$. The frictionless input-output matrix relates to the tax-adjusted input-output matrix $\mathbf{\Omega}$ via the identity

$$\mathbf{\Omega} = \mathbf{\Gamma} \circ \mathbf{\Omega}^F,$$

where $\mathbf{\Gamma}$ is an $N \times N$ matrix with ij^{th} element given by

$$\Gamma_{ij} = \frac{1 + \tau_{x,ij}}{1 - \tau_{y,i}}.$$

Noting that $\Gamma_{ij} \geq 1$ for all i, j since $0 \leq \tau_{x,ij} < 1$ and $0 \leq \tau_{y,i} < 1$, it implies that

$$\Omega_{ij} \geq \Omega^F_{ij}$$

for all i, j. Furthermore, the ij^{th} element of the Leontief inverse $\mathbf{\Psi} = \left(I - \mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)^{-1}$ can be written as

$$\psi_{ij} = 1 + \left(\frac{1 + \tau_{x,ij}}{1 - \tau_{y,i}}\right) \Omega^F_{ij} + \sum_{k=1}^{N} \left(\frac{1 + \tau_{x,ik}}{1 - \tau_{y,i}}\right) \left(\frac{1 + \tau_{x,kj}}{1 - \tau_{y,k}}\right) \Omega^F_{ik} \Omega^F_{kj} + \ldots$$

Noting that

$$\psi_{ij} \geq 1 + \Omega^F_{ij} + \sum_{k=1}^{N} \Omega^F_{ik} \Omega^F_{kj} + \ldots,$$

where the right-hand side of the above inequality is the ij^{th} element of the frictionless Leontief inverse, defined as $\left(I - \mathbf{\Omega}^F\right)^{-1}_{ij}$.

Denoting by μ^F_i sector i's scale centrality in the frictionless economy, it follows that

$$\zeta_i = \mu_i - \mu^F_i \geq 0$$

for all i, where

$$\zeta_i = \underbrace{\Upsilon' \left(\left(I - \Gamma \circ \Omega^F \right)^{-1} \left(\Gamma \circ \Omega^F \right)_{(i)} - \left(I - \Omega^F \right)^{-1} \Omega^F_{(i)} \right)}_{\text{Impact of taxes on } i\text{'s indirect effect}}$$
$$+ \underbrace{\sum_{k=1}^{N} \Upsilon_k \left(\left(I - \Gamma \circ \Omega^F \right)^{-1}_{ki} \left(\Gamma \circ \tilde{\Omega}^F \right)^{(i)} \mathbb{1}_{\mathcal{R}} - \left(I - \Omega^F \right)^{-1}_{ki} \left(\tilde{\Omega}^F \right)^{(i)} \mathbb{1}_{\mathcal{R}} \right)}_{\text{Impact of taxes on } i\text{'s supplier effect}},$$

and where $(\mathbf{L})_{(i)}$ and $(\mathbf{L})^{(i)}$ denotes the i^{th} column and row of a matrix $\mathbf{L}$, respectively.

■

C Data for Quantitative Application

BEA Input-Output Data

We use the detailed benchmark input-output accounts provided by the Bureau of Economic Analysis (BEA) from 1982 to 2012. These accounts, which are compiled every five years, offer comprehensive information on inter-industry relationships and the flow of goods and services in the economy. For our analysis, we use the commodity-by-industry Use table, assuming that each industry produces only one commodity (as in Baqaee and Farhi, 2020b). We exclude the government, scrap, noncomparable imports, and used and secondhand goods sectors from our analysis. The number of industries included in the tables varies each year, ranging from 534 sectors in 1982 to 393 sectors in 2012.

To compute each entry of the US input-output table for each year, we divide the expenditure of industry i on commodity j by i's gross sales (net of sector-level taxes). We also calculate the final expenditure share of each sector i by summing all components of final demand in the detailed Use tables, excluding changes in private inventories, and dividing by nominal GDP. If any final demand share is negative, we set it equal to zero. Similarly, if any value in the equilibrium input-output matrix is negative, we set it to zero as well.

WIOD Input-Output Data

We use the 2016 release of the World Input-Output Database (WIOD) (see Timmer et al., 2015 for an overview of the WIOD data) for our cross-country analysis. The dataset contains information on gross output, value-added, factor compensation, tax payments, final expenditures, and intermediate input flows for 43 countries from 2000 to 2014. The WIOD data is disaggregated into 56 sectors based on the International Standard Industrial Classification Revision 4 (ISIC Rev. 4). The block-diagonal of each input-output table captures domestic intermediate input transactions for each country. In contrast, the off-diagonal relates to the flow of intermediates between countries. For our purposes, we focus solely on domestic transactions and abstract from international trade. We compute each entry of country c's input-output matrix $\boldsymbol{\Omega}_{ct} \equiv \left[\Omega_{ijct}\right]$ at time t by dividing sector i's nominal expenditure on sector j's product by sector i's total domestic nominal sales (net of taxes). Notably, we exclude each sector's spending on imported inputs, which ensures that factor compensation plus domestic intermediate input expenditure equals nominal gross output for each sector. We calculate tax rates τ_{ict} at the country-sector-year level as

$$\tau_{ict} = \frac{T_{ict}}{\sum_{k=1}^{N} p_{kct} y_{kct}},$$

where T_{ict} is the nominal value of taxes (less subsidies) paid by sector i in country c at year t, and $\sum_{k=1}^{N} p_{kct} y_{kct}$ is aggregate nominal gross output for country c. Therefore, a typical entry of country c's input-output matrix is computed as[4]

$$\Omega_{ijct} = \frac{p_{jct} x_{ijct}}{(1-\tau_{ict}) p_{ict} y_{ict}}.$$

Finally, we calculate industries' final expenditure shares Υ_{ict} as the sum of household and government final consumption expenditure plus gross fixed capital formation, all as a fraction of nominal GDP.

D The Effect of Taxes on Sectors' Importance

In the Proof of Proposition 1, we derived the following expression for the contribution of taxes to sector i's scale centrality, ζ_i:

[4] Given that we do not have information on input-specific tax wedges $\tau_{x,ij}$, we do not include these when computing the input-output matrices.

$$\zeta_i = \underbrace{\mathbf{\Upsilon}' \left(\left(I - \mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)^{-1} \left(\mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)_{(i)} - \left(I - \mathbf{\Omega}^F\right)^{-1} \mathbf{\Omega}^F_{(i)} \right)}_{\textit{Impact of taxes on i's indirect effect}}$$

$$+ \underbrace{\sum_{k=1}^{N} \Upsilon_k \left(\left(I - \mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)^{-1}_{ki} \left(\mathbf{\Gamma} \circ \tilde{\mathbf{\Omega}}^F\right)^{(i)} \mathbb{1}_{\mathcal{R}} - \left(I - \mathbf{\Omega}^F\right)^{-1}_{ki} \left(\tilde{\mathbf{\Omega}}^F\right)^{(i)} \mathbb{1}_{\mathcal{R}} \right)}_{\textit{Impact of taxes on i's supplier effect}}, \quad \text{(D.1)}$$

where $(\mathbf{L})_{(i)}$ and $(\mathbf{L})^{(i)}$ denotes the i^{th} column and row of a matrix $\mathbf{L}$, respectively.

In equation (D.1), the first term on the right-hand side quantifies the impact of sector-level sales and intermediates taxes on the indirect effect of sector i, while the second term measures the significance of these taxes for the supplier effect of sector i. Thus, in the absence of these frictions, the supplier effect of i is given by

$$\sum_{k=1}^{N} \Upsilon_k \left(I - \mathbf{\Omega}^F\right)^{-1}_{ki} \left(\tilde{\mathbf{\Omega}}^F\right)^{(i)} \mathbb{1}_{\mathcal{R}}, \quad \text{(D.2)}$$

and the importance of taxes for supplier effects is given by

$$\sum_{k=1}^{N} \Upsilon_k \left(\left(I - \mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)^{-1}_{ki} \left(\mathbf{\Gamma} \circ \tilde{\mathbf{\Omega}}^F\right)^{(i)} \mathbb{1}_{\mathcal{R}} - \left(I - \mathbf{\Omega}^F\right)^{-1}_{ki} \left(\tilde{\mathbf{\Omega}}^F\right)^{(i)} \mathbb{1}_{\mathcal{R}} \right). \quad \text{(D.3)}$$

Figure D.1 compares the impact of taxes on industries' supplier effects (equation D.3), represented by yellow bars, for the top 10 sectors ranked by supplier effects. The analysis covers the years from 1982 to 2012, using the detailed BEA input-output accounts. In the figure, the orange bars represent the frictionless supplier effects of each industry, calculated using equation (D.2). The combined sum of the orange and yellow bars for each sector represents the total supplier effect, which includes the impact of taxes, for the sector listed on the horizontal axis.

The figure shows that taxes only account for a small portion of the overall supplier effects of industries across all years. However, a few notable exceptions stand out. In 2002, taxes on retail and wholesale trade accounted for approximately 15% and 18%, respectively, of these industries' total supplier effects. Additionally, in 1982, taxes accounted for 23% of the supplier effect of crude petroleum production. Nonetheless, on average, taxes only account for around 4% of supplier effects across all industries and years in the BEA IO tables.

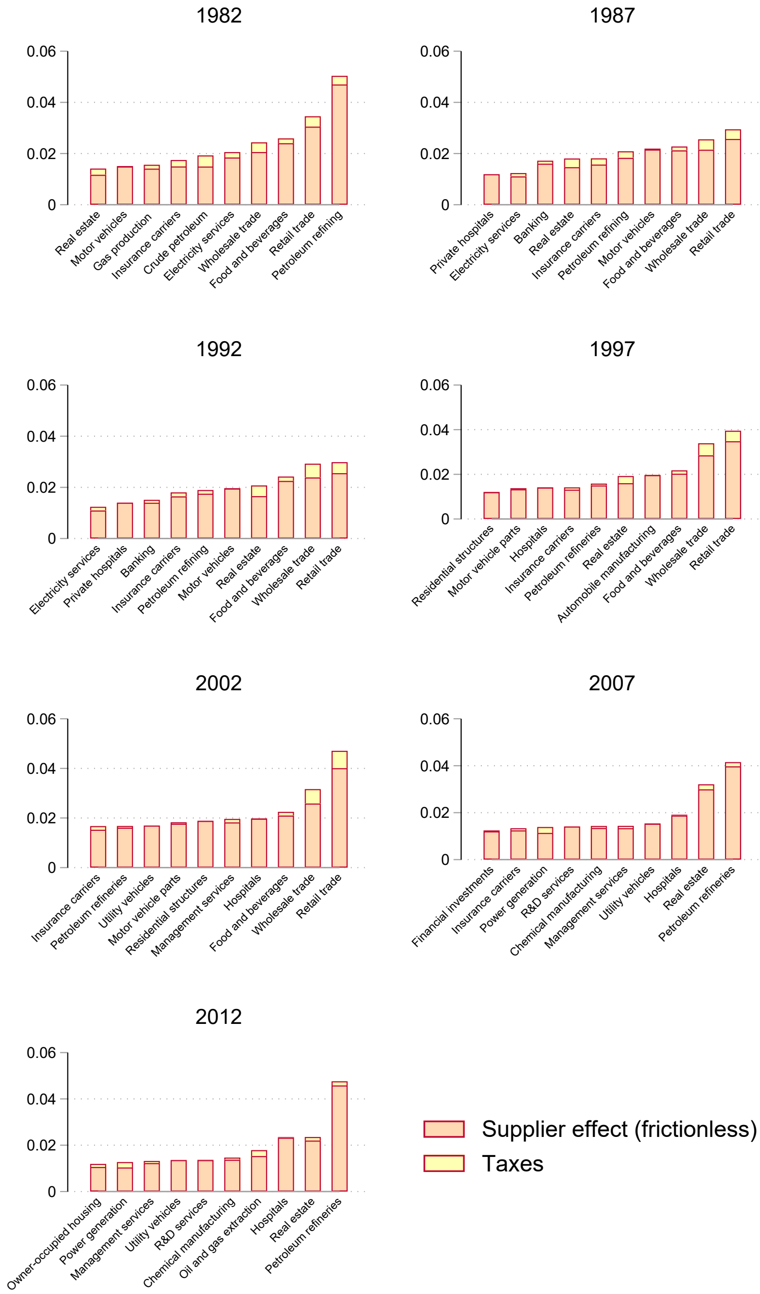

Figure D.1: Contribution of Taxes to Sectors' Supplier Effects

Note: This figure presents estimates of the contribution of taxes to sectors' supplier effects from 1982 to 2012 using the detailed input-output accounts of the BEA. The top 10 sectors, ranked by supplier effects, are displayed for each year. The orange bars represent each sector's supplier effect in the absence of taxes, calculated using equation (D.2). The yellow bars capture the contribution of taxes to each sector's supplier effect, computed as in equation (D.3).

E Second-Order Scale Centrality

We compute sectors' second-order scale centrality by characterizing the derivatives of Domar weights and input-output parameters. We begin by noting that the change in real GDP in response to productivity shocks, to a second-order approximation, is given by

$$\frac{\Delta Y}{Y} \approx \sum_{i=1}^{N} \lambda_i \left(\frac{\Delta A_i}{A_i}\right) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{d\lambda_i}{d\log A_j}\left(\frac{\Delta A_i}{A_i}\right)\left(\frac{\Delta A_j}{A_j}\right), \tag{E.1}$$

where the first set of summands capture the first-order change in real GDP in response to the technology shocks $(\frac{\Delta A_1}{A_1}, \ldots, \frac{\Delta A_N}{A_N})$ and the second set of summands capture the second-order change in GDP. Next, note that change in real GDP in response to shocks to i's supply and use of inputs, up to a second-order approximation, is given by

$$\begin{aligned}
\frac{\Delta Y_i}{Y_i} = \mu_i^{\text{2nd-order}} \approx \; & \underbrace{\Upsilon_i \left(\frac{\Delta \omega_{\mathcal{D},i}}{\omega_{\mathcal{D},i}}\right)}_{\text{First-order direct effect}} + \underbrace{\sum_{k=1}^{N} \lambda_k \Omega_{ki}\left(\frac{\Delta \omega_{ki}}{\omega_{ki}}\right)}_{\text{First-order indirect effect}} + \underbrace{\sum_{k\neq i}^{N} \lambda_i \Omega_{ik}\left(\frac{\Delta \omega_{ik}}{\omega_{ik}}\right)}_{\text{First-order supplier effect}} \\
& + \underbrace{\frac{1}{2}\sum_{k=1}^{N}\sum_{m=1}^{N} \frac{d(\lambda_k \Omega_{ki})}{d\log \omega_{mi}}\left(\frac{\Delta \omega_{ki}}{\omega_{ki}}\right)\left(\frac{\Delta \omega_{mi}}{\omega_{mi}}\right)}_{\text{Second-order indirect effect}} \\
& + \underbrace{\frac{1}{2}\frac{d\Upsilon_i}{d\log \omega_{\mathcal{D},i}}\left(\frac{\Delta \omega_{\mathcal{D},i}}{\omega_{\mathcal{D},i}}\right)^2}_{\text{Second-order direct effect}} + \underbrace{\frac{1}{2}\sum_{k\neq i}^{N}\sum_{m\neq i}^{N} \frac{d(\lambda_i \Omega_{ik})}{d\log \omega_{im}}\left(\frac{\Delta \omega_{ik}}{\omega_{ik}}\right)\left(\frac{\Delta \omega_{im}}{\omega_{im}}\right)}_{\text{Second-order supplier effect}}.
\end{aligned} \tag{E.2}$$

where $\frac{\Delta Y_i}{Y_i} = \mu_i^{\text{2nd-order}}$ is scale centrality of producer i under heterogeneous shocks, up to a second-order approximation. Therefore, to characterize equation (E.2), we must compute $\frac{d\Upsilon_i}{d\omega_{\mathcal{D},i}}$, $\frac{d(\lambda_k\Omega_{ki})}{d\log\omega_{mi}}$ and $\frac{d(\lambda_i\Omega_{ik})}{d\log\omega_{im}}$. We begin by noting that

$$\frac{d\Upsilon_i}{d\log \omega_{\mathcal{D},i}} = \frac{d\log \Upsilon_i}{d\log \omega_{\mathcal{D},i}}\Upsilon_i, \tag{E.3}$$

where

$$\frac{d\log \Upsilon_i}{d\log \omega_{\mathcal{D},i}} = (\sigma-1)(1-\Upsilon_i). \tag{E.4}$$

Next, to characterize the second-order indirect effect, we need to compute

$$\frac{d(\lambda_k \Omega_{ki})}{d\log \omega_{mi}} = \frac{d\log \lambda_k}{d\log \omega_{mi}} \lambda_k \Omega_{ki} + \frac{d\log \Omega_{ki}}{d\log \omega_{mi}} \lambda_k \Omega_{ki}. \tag{E.5}$$

Firstly, the change in the input-output parameter Ω_{ki} in response to the change in ω_{mi} is given by

$$\frac{d\log \Omega_{ki}}{d\log \omega_{mi}} = (\theta - 1) \left(\psi_{im}\Omega_{mi} - \frac{1}{1-\Lambda_k} \sum_{j=1}^{N} \Omega_{kj} \psi_{jm} \Omega_{mi} + \frac{d\log \omega_{ki}}{d\log \omega_{mi}} \right). \tag{E.6}$$

(See equation B.10 for the definition of Λ_k.) Secondly, the change in sector k's Domar weight is given by

$$\frac{d\log \lambda_k}{d\log \omega_{mi}} = \frac{1}{\lambda_k} \sum_{l=1}^{N} \Upsilon_l \psi_{lk} \frac{d\log \Upsilon_l}{d\log \omega_{mi}} + \frac{1}{\lambda_k} \sum_{l=1}^{N} \sum_{s=1}^{N} \Omega_{ls} \lambda_l \psi_{sk} \frac{d\log \Omega_{ls}}{d\log \omega_{mi}}, \tag{E.7}$$

where,

$$\frac{d\log \Upsilon_l}{d\log \omega_{mi}} = (\sigma - 1) \left(\psi_{lm}\Omega_{mi} - \lambda_m \Omega_{mi} \right) \tag{E.8}$$

and

$$\frac{d\log \Omega_{ls}}{d\log \omega_{mi}} = (\theta - 1) \left(\psi_{sm}\Omega_{mi} - \frac{1}{1-\Lambda_l} \sum_{j=1}^{N} \Omega_{lj} \psi_{jm} \Omega_{mi} + \frac{d\log \omega_{ls}}{d\log \omega_{mi}} \right). \tag{E.9}$$

Together, equations (E.6), (E.7), (E.8) and (E.9) are sufficient to characterize the second-order indirect effect in equation (E.2).

Our last step is to characterize the second-order supplier effect. To this end, we must compute

$$\frac{d(\lambda_i \omega_{ik})}{d\log \omega_{im}} = \frac{d\lambda_i}{d\log \omega_{im}} \Omega_{ik} + \frac{d\Omega_{ik}}{d\log \omega_{im}} \lambda_i.$$

The change in the input-output parameter Ω_{ik} in response to a change in ω_{im} is given by

$$\frac{d\Omega_{ik}}{d\log \omega_{im}} = (\theta - 1)\Omega_{ik} \left(\psi_{ki}\Omega_{im} - \frac{1}{1-\Lambda_i} \sum_{j=1}^{N} \Omega_{ij} \psi_{ji} \Omega_{im} + \frac{d\log \omega_{ik}}{d\log \omega_{im}} \right). \tag{E.10}$$

Next, we compute $\frac{d\lambda_i}{d\log\omega_{im}}$

$$\frac{d\lambda_i}{d\log\omega_{im}} = \sum_{l=1}^{N}\psi_{li}\frac{d\Upsilon_l}{d\log\omega_{mi}} + \sum_{l=1}^{N}\sum_{s=1}^{N}\lambda_l\psi_{si}\frac{d\Omega_{ls}}{d\log\omega_{im}}, \tag{E.11}$$

where

$$\frac{d\Upsilon_l}{d\log\omega_{im}} = (\sigma-1)\Upsilon_l\left(\psi_{li}\Omega_{im} - \lambda_i\Omega_{im}\right), \tag{E.12}$$

and

$$\frac{d\Omega_{ls}}{d\log\omega_{im}} = (\theta-1)\Omega_{ls}\left(\psi_{si}\Omega_{mi} - \frac{1}{1-\Lambda_l}\sum_{j=1}^{N}\Omega_{lj}\psi_{ji}\Omega_{im} + \frac{d\log\omega_{ls}}{d\log\omega_{im}}\right). \tag{E.13}$$

Together, equations (E.3)–(E.13) characterize second-order scale centrality.