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Keywords

Production networks, propagation of sector-level shocks, disaggregated macroeconomic models

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Beyond Domar Weights: A New Measure of Systemic Importance in Production Networks

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Abstract

We present a new measure of producers' aggregate importance in a production economy with input-output linkages. Unlike existing measures, which capture the impact of an isolated TFP shock to a sector on aggregate output, we quantify how a sector amplifies simultaneous shocks to all producers in the economy. In our context, a sector's systemic importance reflects its ability to i) directly impact final demand, ii) indirectly affect the production of downstream firms, and iii) amplify shocks originating in other industries. Notably, our measure encompasses and extends an existing notion of centrality in production networks: producers' Domar weight. Using US input-output data, we find that Domar weights underestimate sectors' systemic importance by $\approx$ 50%, on average, and the extent of underestimation increases with the Domar weight of the sector. Additionally, our measure reveals significant changes in key US industries' aggregate importance over time despite the relative stability of their Domar weights.

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1 Introduction

At least since Leontief (1936), economists have recognized the importance of systematically quantifying interrelationships between the producers of an economy.[1] More recently, studies have shown that these input-output linkages play a crucial role in amplifying microeconomic shocks into aggregate fluctuations (Acemoglu et al., 2012; Atalay, 2017, and Baqaee and Farhi, 2019). In addition to an industry's share of value added to GDP, its macroeconomic importance depends on how other sectors rely on it, directly or indirectly, for intermediate inputs. Moreover, existing measures of industries' systemic importance quantify the impact of an isolated idiosyncratic shock to a given sector on aggregate output (see, for example, Hulten, 1978, Liu, 2019, Baqaee and Farhi, 2020, and Bigio and La'O, 2020).[2] However, in reality, shocks rarely occur in isolation. An economy with millions of firms experiences multiple (idiosyncratic or common) disturbances simultaneously. As a result, the systemic importance of an industry extends beyond the isolated impact of a shock to itself. It also encompasses the industry's sensitivity to shocks originating from other sectors of the economy.

This paper presents a new measure of a sector's macroeconomic importance, which we refer to as *removal centrality*, that captures how an industry impacts real GDP by i) directly affecting the final consumption of its output, ii) indirectly affecting the production of firms that are directly or indirectly connected to it, and iii) transmitting simultaneous disruptions from other sectors of the economy to downstream producers and final demand. To demonstrate the intuition behind our measure, consider the mining and transportation industries in the US. Both industries account for about 1% of GDP and supply similar amounts of intermediate goods, valued at approximately USD 640 billion, to domestic producers. The results of Hulten (1978) imply that the systemic importance of these industries can be approximated by their gross sales as a share of GDP, or Domar weights, which are roughly equivalent for both sectors.

[1] We use the terms producers, sectors, industries, and firms interchangeably in this paper.

[2] While these papers employ different frameworks: efficient vs. inefficient economies (with taxes, markups or financial frictions) and differences in production technologies (Cobb-Douglas vs. CES), they share the common aim of characterizing the change in aggregate output due to an idiosyncratic disturbance to a microeconomic producer.

Our measure, however, offers a new perspective to sectoral importance. The transportation sector purchases nearly three times more intermediate inputs than the mining industry and is particularly dependent on goods and services from industries such as manufacturing and professional services. This higher reliance on intermediates makes the transportation industry more susceptible to upstream shocks, which it then transmits to downstream sectors and final consumers. On the other hand, the mining industry's lower dependence on intermediates makes it relatively more insulated against such shocks. Notably, removal centrality encapsulates sectors' Domar weight, and all the information contained in it, but also incorporates the greater systemic significance of the transportation industry in influencing aggregate volatility, given its position in the production network and its dependence on other sectors.

We measure a sector's removal centrality by first defining a counterfactual production network that omits the input-output linkages between the industry and its immediate customers, primary suppliers, and final users of its output.[3] Since productivity shocks propagate downstream through supply chains, severing these links prevents shocks from transmitting to sectors that directly or indirectly purchase inputs from the focal industry. We then compare the impact of simultaneous microeconomic TFP shocks on real GDP in the actual economy, which includes all input-output connections, with that of the hypothetical economy. The difference between the counterfactual and observed change in GDP yields the removal centrality of the sector in question.

A challenge in implementing our approach lies in characterizing the macroeconomic impact of shocks in the hypothetical economy. Severing input-output linkages leads to endogenous changes in the prices and quantities of traded inputs, which are not observable in the counterfactual scenario. A key contribution of this paper is to provide a nonparametric formula that allows us to quantify each sector's importance without having to characterize changes in prices and quantities in the counterfactual. Notably, only observable data on intermediate input

[3]Node and edge removal are standard techniques in graph theory for measuring the connectivity of vertices of a network (for a detailed overview of connectivity measures in the field of complex networks, see Estrada, 2012). Furthermore, node removal is also widely used for quantifying the *overall* stability of a network (see Albert et al. (2000) for an example of this approach).

purchases, nominal gross output, tax payments, and nominal GDP is necessary to estimate an industry's removal centrality. Since this information is available for most countries, our measure is readily computable.

Removal centrality can be decomposed into three distinct components, which we refer to as *direct*, *indirect*, and *supplier* effects. The direct effect captures the impact of a shock to the focal industry on real GDP via households' direct consumption of its output. The indirect effect, on the other hand, captures the spillover effect of the shock (to the focal industry) on final demand as it spreads to other industries. Lastly, the supplier effect quantifies how shocks to other industries in the economy transmit through the focal industry, affecting real GDP by influencing its downstream customers. Notably, the sum of a sector's direct and indirect effects corresponds to its Domar weight, a well-established measure of systemic significance (as in Hulten, 1978; Baqaee and Farhi, 2020).[4] The third component, the supplier effect, represents a sector's ability to transmit shocks originating elsewhere in the economy and distinguishes a sector's Domar weight from its removal centrality.[5]

Our measure of removal centrality is derived within the framework of a macroeconomic production network model in the spirit of Atalay (2017), Bernard and Moxnes (2018), Baqaee and Farhi (2019), and Carvalho et al. (2021). In the model, each sector produces output using a combination of labor and intermediate inputs and is subject to Hicks-neutral productivity shocks, which affect its own output and that of industries that purchase its intermediate goods. We capture relationships between industries via constant-elasticity-of-substitution (CES) aggregators of intermediate products. While goods may either be complements or substitutes

[4]In inefficient economies with one factor of production, distortion-adjusted Domar weights are sufficient to characterize the macroeconomic impact of isolated microeconomic TFP shocks (Baqaee and Farhi, 2020). Since our model includes taxes on sales and input purchases, we use the term "Domar weights" to refer to tax-adjusted sales shares. However, regardless of whether the economy is efficient or inefficient, removal centrality encapsulates the first-order macroeconomic impact of an idiosyncratic microeconomic TFP shock to the focal industry.

[5]Notably, a portion of the indirect effect of a supplier sector is included in the supplier effect of its customer as well. Thus, when aggregating across all industries, the sum of indirect effects is roughly equal to the sum of supplier effects. However, when analyzing the systemic importance of an industry in isolation, both indirect and supplier effects are relevant. In the aggregate, the two effects are approximately equivalent. See Section 2.3 for a detailed discussion on the relationship between indirect and supplier effects.

in our model, a sector's removal centrality does not depend on the value of the elasticity of substitution across intermediate or final goods. In this respect, our formulas align with the nonparametric results of much of the production networks literature, including Hulten (1978), Liu (2019), Baqaee and Farhi (2020), and Bigio and La'O (2020).

In an empirical application, we first estimate removal centrality for 393 sectors in the US in 2012 using the detailed input-output accounts provided by the Bureau of Economic Analysis (BEA). We show that Domar weights underestimate the systemic importance of an industry by 50%, on average, implying that supplier effects are quantitatively significant for most industries. We further demonstrate that the extent of underestimation increases with the Domar weight of a sector. This result is not immediately obvious as a sector's removal centrality and its Domar weight can coincide, even for industries with large Domar weights (say, for an upstream sector that uses no intermediate inputs for production).

We then compare the relationship between industries' removal centrality and their corresponding Domar weights between the years 1982 and 2012, using input-output data from the BEA. We find the relationship between sectors' Domar weights and removal centrality to be remarkably constant over time. That is, sectors with larger Domar weights have substantially greater supplier effects as well. Thus, as industries grow larger in size (as measured by their Domar weight), they also become increasingly reliant on inputs from other sectors in the economy, which increases their susceptibility to shocks originating elsewhere. Our findings reveal that industries such as petroleum refineries, oil & gas extraction, and electric power generation play a crucial role in transmitting and amplifying shocks from other sectors. As a result, these industries have a substantial capacity to drive aggregate fluctuations in the US.

Further, changes in a sectors' removal centrality need not coincide with changes in its Domar weight. Our analysis shows that key sectors of the US economy, such as motor vehicle production, electronic computer manufacturing, and retail trade experienced significant changes in their removal centrality over time, despite having relatively stable Domar weights. Take, for instance, the retail trade sector. Between 1982 and 1987, retail trade experienced a substantial decrease in its reliance on intermediates from other sectors, with its supplier effect declining from around 3.5% of GDP to less than 3% of GDP during this period. Surprisingly, however, the Domar weight of retail trade remained constant. Therefore, when assessing retail trade's systemic importance using our measure of removal centrality, we note a significant decline in its macroeconomic significance, which is not captured by its Domar weight. This discrepancy highlights the limitation of relying solely on the Domar weight of a sector to assess its overall importance and underscores the relevance of removal centrality.

Next, using data from the World Input-Output Database between 2000 and 2014, we identify the industries with the greatest removal centrality for six major economies: the United States, Great Britain, Japan, China, Germany, and Australia. For each economy, construction, real estate, public administration, and food & beverages are always among the top five sectors with the largest removal centrality. Notably, the magnitude of direct, indirect, and supplier effects vary significantly across countries and industries. Labor-intensive sectors, such as education and health services, have significant direct effects. This is because they rely less on intermediate inputs and provide services directly to end-consumers, limiting their ability to amplify shocks via indirect and supplier effects. Conversely, sectors like wholesale trade and electricity & gas supply derive most of their aggregate importance from the indirect effect, reflecting their critical role as input suppliers to other producers. Construction and food & beverages, on the other hand, have relatively large direct and supplier effects, which highlights their central role as both producers of final goods and consumers of intermediate goods.

Related literature. Our article relates to the literature on growth accounting and production networks. Hulten (1978) provided the economic rationale for using Domar aggregation to measure changes in aggregate TFP. Hulten's result was in contrast to Solow (1957), who used an aggregate production function and measured TFP growth as the residual change in output after accounting for the growth of factor inputs. Hulten's theorem has become a benchmark in the macroeconomic literature on production networks, demonstrating that in the presence of intermediate inputs, sales (rather than value-added) shares are the appropriate weights for aggregating microeconomic productivity changes.[6] Specifically, the theorem states that a producer's sales as a share of GDP (also called its Domar weight) is sufficient to capture the first-order macroeconomic impact of a microeconomic productivity shock to that producer. Relatedly, Acemoglu et al. (2012) demonstrate that Domar weights are linked to the economy's input-output network through the Leontief inverse, capturing each industry's direct and indirect dependencies on intermediate inputs from other sectors.[7] Our measure of systemic importance differs from that of Hulten (1978) and Acemoglu et al. (2012) as it captures a sector's ability to transmit and amplify a set of simultaneous shocks to all producers in the economy and impact final demand.

Since our key measure is derived in the context of an inefficient network model, we contribute to the growing literature on the propagation of shocks through input-output linkages in the presence of market imperfections. Some papers in this literature include Jones (2011, 2013); Bartelme and Gorodnichenko (2015); Caliendo et al. (2018); Liu (2019); Boehm and Oberfield (2020), and Boehm (2022). Bigio and La'O (2020) study the properties of inefficient (Cobb-Douglas) production networks with financial frictions, while Baqaee and Farhi (2020) study the impact of microeconomic productivity and factor supply shocks on aggregate output in a model with markups. We contribute to this literature by demonstrating analytically that taxes on intermediate input purchases and gross sales increase sectors' removal centrality relative to frictionless economies without such distortions.[8] We also show how to use our framework to quantify the importance of these distortions on sectors' removal centrality. Empirically, however, we find that sectoral taxes had a relatively limited impact on the amplification of microeconomic shocks in the United States between 1982 and 2012.

Our paper also relates to the recent macroeconomics literature that investigates

[6]See Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for a detailed overview of the production networks literature.

[7]Baqaee and Farhi (2019) build on the work of Hulten (1978) and Acemoglu et al. (2012), showing that nonlinearities in production have a significant impact on macroeconomic outcomes. While the first-order macroeconomic effect of a microeconomic shock to a sector is given by the sector's Domar weight, the second-order effect requires additional information, such as microeconomic elasticities of substitution and the degree of return to scale.

[8]Taxes on factor payments only affect aggregate volatility when there are factor supply shocks or factor-augmenting productivity shocks.

the role of input-output linkages in generating aggregate volatility, including Foerster et al. (2011), Acemoglu et al. (2012), di Giovanni et al. (2014), Acemoglu et al. (2017), Atalay (2017), Grassi (2017), Baqaee (2018), and Altinoglu (2021).[9] A common theme in this literature is that the interdependence of production through input-output linkages significantly amplifies aggregate volatility: small shocks cascade through supply chains resulting in larger fluctuations in output.[10] We contribute to this literature in three ways. First, we identify the key sectors that have the greatest impact on aggregate volatility using our measure of removal centrality and demonstrate that these are not necessarily the industries with the largest Domar weights. Second, to gain a deeper understanding of the key determinants of aggregate volatility, we decompose sectors' removal centrality into direct, indirect, and supplier effects. Our analysis underscores the critical role that supplier effects play in shaping aggregate fluctuations. For example, sectors such as construction and food & beverages have a great capacity to absorb shocks from other industries and propagate them to final demand. Lastly, we provide evidence that larger sectors tend to exhibit a higher level of dependence on intermediate inputs from other industries. This finding suggests that as sectors grow in size, they not only contribute more to aggregate fluctuations due to their increased share of GDP, but also because their ability to transmit shocks originating from other sectors increases.

The rest of the paper is structured as follows. In Section 2, we present the model and derive our measure of removal centrality. In Section 3, we use our framework to identify the industries that have a significant impact on the propagation of microeconomic shocks. Section 4 concludes. Proofs, a description of the data, and supplementary results appear in the Appendix.

[9]These papers build on earlier work in macroeconomics that studies aggregate volatility in multi-sector models, such as Long and Plosser (1983); Horvath (1998, 2000); Dupor (1999); Shea (2002). Other papers that model microeconomic behavior to shed light on macroeconomic phenomena include Durlauf (1993) and Jovanovic (1987). Studies such as Gabaix (2011) and Amiti and Weinstein (2018) focus on the role of the firm size distribution in shaping aggregate fluctuations. Finally, Elliott et al. (2022) and Carvalho et al. (2022) examine how supply chain complexity and bottlenecks contribute to macroeconomic fragility.

[10]The empirical networks literature examines the transmission of microeconomic shocks through input-output linkages using quasi-experiments. For instance, studies such as Barrot and Sauvagnat (2016), Boehm et al. (2019), and Carvalho et al. (2021) estimate the impact of natural disasters on output losses along firm-level supply chains, emphasizing the significance of indirect propagation.

2 Theoretical Framework

In this section, we set up a general equilibrium model in the spirit of Bernard and Moxnes (2018) and Baqaee and Farhi (2020) to derive our measure of removal centrality. We begin with a discussion of the intuition behind the measure.

Measuring a sector's removal centrality. Our measure of removal centrality μ_i quantifies industry i's capacity to amplify the effects of simultaneous microeconomic TFP shocks, including those affecting the sector itself and its direct or indirect suppliers. Given the interconnectedness of producers in the production network, an idiosyncratic TFP shock to a sector can have consequential effects on GDP not only through direct impacts on final consumption but also by disrupting other downstream sectors that rely on its goods for production. Furthermore, a sector can be influenced by shocks to its direct or indirect suppliers, which it can subsequently transmit down the supply chain. The measure μ_i captures all three aspects of shock propagation to measure sector i's macroeconomic importance.

In our model, productivity shocks propagate from upstream suppliers to downstream customers.[11] Industries that supply many intermediate goods to other producers have a greater impact on aggregate output than those that supply fewer inputs. Likewise, sectors that use a large volume of intermediates from a wide array of suppliers are susceptible to output disturbances originating in the suppliers' industries. To measure an industry's removal centrality μ_i, we compare the impact of a vector of idiosyncratic microeconomic TFP shocks on real GDP in two economies: the actual economy and a counterfactual economy. In the actual economy, all input-output linkages between the focal industry, its direct suppliers, immediate customers, and the household sector are intact, as illustrated in Panel A of Figure 1. In the counterfactual economy, depicted in Panel B of Figure 1, the links between the focal industry and all other economic agents are severed (shown using feint dotted arrows), preventing shocks to the focal industry from affecting final demand.

[11]Microeconomic shocks transmit downstream through price changes. For example, a decrease in productivity in an industry leads to an increase in the price of its output, causing a decline in demand from its direct and indirect customers. As a result, prices in the downstream sectors also rise, eventually reducing final demand.

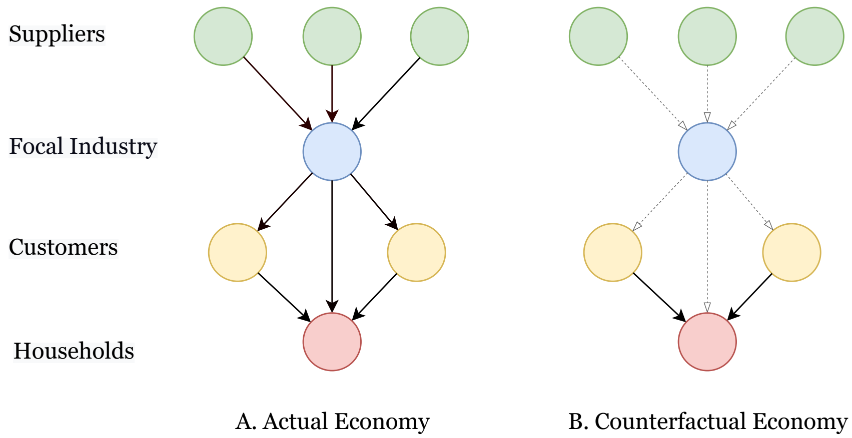


Figure 1: Illustration of Removal Centrality

Note: Red nodes represent households, green nodes represent suppliers, yellow nodes depict customer sectors, and the central blue node is the focal industry. Panel A shows the actual economy in which all input-output linkages are intact. Panel B depicts the counterfactual economy in which the focal sector's connections with its suppliers, customers, and the household sector are severed. In the counterfactual economy, feint dotted arrows represent the severed linkages. Black arrows capture the flow of goods.

Note that shocks to suppliers (green nodes) can still affect GDP in the counterfactual, but without affecting the focal industry, as long as these sectors supply goods to other industries or households. The difference between the change in GDP in the two economies, in response to the same vector of shocks, yields the removal centrality of the focal industry.

2.1 Model setup and equilibrium

We consider a static economy with N sectors that each produce one distinct product using some combination of labor and intermediate goods. The output of these sectors can either be consumed directly by households as final goods or used as an intermediate input by other sectors.

Aggregate output. Real GDP is the maximizer of a constant-elasticity-of-substitution (CES) aggregator of final consumption:

$$Y = \max_{\{c_i\}_{i=1}^{N}} \left(\sum_{i=1}^{N} \left(\omega_{\mathcal{D},i} c_i \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{subject to} \quad \sum_{i=1}^{N} p_i c_i = wL,$$

where Y is aggregate output, c_i is the final consumption of good i, $\omega_{\mathcal{D},i}$ is a sector-specific final demand shifter, σ is the elasticity of substitution, p_i is the price of i's output, w is the wage rate, and L is aggregate labor supply.

Producers. Sectors produce output using Cobb-Douglas technologies that combine labor and intermediate goods

$$y_i = A_i l_i^{\alpha_i} M_i^{1-\alpha_i},$$

where y_i is sector i's output, A_i is a Hicks-neutral productivity shifter, l_i is labor use, M_i is a bundle of intermediate goods used by i and $\alpha_i \in [0,1]$ is the importance of labor in i's production. Notably, we allow for heterogeneity in the initial level of productivity A_i across sectors.[12] Furthermore, M_i is a CES aggregator of intermediate goods from other sectors, defined as

$$M_i \equiv \left(\sum_{j=1}^{N} \left(\omega_{ij} x_{ij} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$

where x_{ij} is the quantity of good j used by sector i and θ is the elasticity of substitution between inputs. As in Bernard and Moxnes (2018), ω_{ij} is a sector-specific taste shifter that accounts for differences in production technologies between sectors at the initial equilibrium. We define an $N+1 \times N$ matrix $\boldsymbol{\omega} \equiv \left[\omega_{ij}\right]$, where an element $\omega_{ij} \geq 0$ captures the importance of good j in the production of good i at the initial equilibrium. If $\omega_{ij} = 0$, then sector i does not require the output of sector j to produce. The $N+1^{\text{th}}$ row of $\boldsymbol{\omega}$ contains the final demand shifters $\omega_{\mathcal{D},i}$, which capture households' reliance on final goods from each industry. Throughout the paper, we model the counterfactual economy by changing the parameters ω_{ij} and $\omega_{\mathcal{D},i}$.

Similar to Baqaee and Farhi (2020), Bigio and La'O (2020), and Liu (2019), we model distortions as taxes on sectors' sales and input use. Industry i's profits are therefore given by

[12] Additionally, our approach can accommodate factor-augmenting productivity shocks and shocks to factor supplies.

$$\pi_i = (1-\tau_{y,i})p_i y_i - (1+\tau_{L,i})wl_i - \sum_{j=1}^{N}(1+\tau_{x,ij})p_j x_{ij},$$

where $\tau_{y,i}$, $\tau_{L,i}$, and $\tau_{x,ij}$ denote taxes imposed on sector i's sales, labor use, and intermediate good purchases, respectively. The market-clearing conditions for goods $1 \le i \le N$ and labor are given by

$$y_i = c_i + \sum_{j=1}^{N} x_{ji} \quad \text{and} \quad L = \sum_{i=1}^{N} l_i = 1.$$

General equilibrium. Given taxes $\{\tau_{y,i}, \tau_{L,i}, \tau_{x,ij}\}$, productivities A_i, technology parameters ω_{ij}, and final demand shifters $\omega_{\mathcal{D},i}$, a general equilibrium is a set of prices $\{p_i, w\}$, input choices $\{l_i, x_{ij}\}$, outputs y_i and final demands c_i, such that: final demand maximizes the consumption aggregator subject to the budget constraint, producers maximize profits taking prices as given and the markets for labor and goods clear.

Input-output definitions. We now introduce some input-output notation that is central to our analysis. First, we define the $N \times N$ tax-adjusted equilibrium input-output matrix $\mathbf{\Omega} \equiv \left[\Omega_{ij}\right]$, where

$$\Omega_{ij} = \frac{(1+\tau_{x,ij})p_j x_{ij}}{(1-\tau_{y,i})p_i y_i}. \tag{1}$$

For brevity, we will refer to the matrix $\mathbf{\Omega}$ as the *input-output matrix* throughout the rest of the paper. Notably, Ω_{ij} captures the direct exposure of sector i to sector j, after accounting for the taxes $\tau_{y,i}$ and $\tau_{x,ij}$ that separate prices from marginal costs.[13] The first-order condition with respect to intermediate inputs from sector j (x_{ij}), implies

$$\Omega_{ij} = (1-\alpha_i)(1+\tau_{x,ij})^{1-\theta} p_j^{1-\theta} P_{M,i}^{\theta-1} \omega_{ij}^{\theta-1}, \tag{2}$$

[13]The input-output matrix $\mathbf{\Omega}$ is similar to the cost-based input-output matrix of Baqaee and Farhi (2020).

where $P_{M,i}$ is the price index associated with the intermediate goods bundle M_i.[14] The input-output parameter Ω_{ij} is therefore a function of the price of j's output relative to i's intermediate goods price index, $p_j/P_{M,i}$, sector i's productivity level A_i, the tax wedge $(1+\tau_{x,ij})$, the elasticity of substitution θ, the importance of labor in i's production α_i, and the technology parameter ω_{ij}.

As discussed above, to measure a sector's removal centrality, we construct a counterfactual economy by severing relationships between sectors. This is achieved by setting parameters of the matrix $\boldsymbol{\omega}$ to zero. For instance, setting $\omega_{ij}=0$ breaks the connection between sectors i and j. Thus, to measure the removal centrality of industry i, we set the i^{th} row and column of $\boldsymbol{\omega}$ (including the final demand parameter $\omega_{\mathcal{D},i}$) to zero. In other words, industry i ceases to exist under the counterfactual. Equation (2) implies that when $\omega_{ij}=0$, the corresponding entry in the input-output matrix, Ω_{ij}, is also equal to zero. However, it is important to note that severing the relationship between sectors i and j will have a ripple effect on other elements in the input-output matrix $\boldsymbol{\Omega}$, as it will alter the prices of all sectors' outputs. Since intermediate inputs can be either complements or substitutes, changes in the parameters ω_{ij} will prompt producers to adjust the mix of inputs they use, leading to price changes across all sectors. As a result, these relative price adjustments affect real GDP under the counterfactual network structure.[15]

Associated with the input-output matrix is the economy's Leontief inverse $\boldsymbol{\Psi} \equiv \left[\psi_{ij}\right]$, where

$$\boldsymbol{\Psi} \equiv (I-\boldsymbol{\Omega})^{-1} = I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + \ldots$$

Intuitively, a typical element of the Leontief inverse ψ_{ij} captures all direct and indirect ways that sector i uses sector j's output.[16] In this respect, the Leontief inverse

[14] Formally, $P_{M,i} \equiv \left(\sum_{j=1}^{N} \omega_{ij}^{\theta-1} \left[(1+\tau_{x,ij})p_j\right]^{1-\theta}\right)^{\frac{1}{1-\theta}}$.

[15] In the special case that all intermediate goods aggregators (M_i) are Cobb-Douglas ($\theta=1$), the input-output parameter Ω_{ij} is given by $\Omega_{ij}=(1-\alpha_i)\omega_{ij}$ for all i,j. This implies that shutting down links between producers will not impact other parameters in the input-output matrix $\boldsymbol{\Omega}$ when M_i is Cobb-Douglas for all i. However, when $\theta \neq 1$, the input-output matrix $\boldsymbol{\Omega}$ will respond endogenously to changes in the technology parameters ω_{ij}.

[16] See Carvalho and Tahbaz-Salehi (2019) for a more detailed discussion of the Leontief inverse matrix.

summarizes all production chains of any length. To see this, note that $(\mathbf{\Omega}^n)_{ij}$ measures the weighted sum of all paths of length n linking sector j to sector i through the production network.

Related to the matrix $\mathbf{\Omega}$ is the economy's *off-diagonal input output matrix* $\tilde{\mathbf{\Omega}}$, defined

$$\tilde{\mathbf{\Omega}} \equiv \mathbf{\Omega} - \text{diag}(\mathbf{\Omega}).$$

The diagonal of $\tilde{\mathbf{\Omega}}$ contains zeros, whereas all off-diagonal elements are identical to those of the input-output matrix $\mathbf{\Omega}$. Thus, the matrix $\tilde{\mathbf{\Omega}}$ captures all dependencies *between* sectors and omits producers' reliance on their own products. As we show in Section 2.2.1, the off-diagonal input-output matrix is required to measure industries' removal centrality.

Next, we denote the $N \times 1$ vector of final expenditure shares by $\mathbf{\Upsilon} \equiv [\Upsilon_i]$, where

$$\Upsilon_i = \frac{p_i c_i}{\sum_{j=1}^{N} p_j c_j} = \frac{p_i^{1-\sigma} \omega_{\mathcal{D},i}^{\sigma-1}}{\left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1}\right)}. \tag{3}$$

The denominator $\sum_{j=1}^{N} p_j c_j$ corresponds to nominal GDP.[17] Notably, Υ_i measures the direct exposure of the household to sector i. We highlight in Section 2.2.1 that the final expenditure share of an industry i sufficiently summarizes the *direct effect* of a shock to sector i on real GDP. Notably, equation (3) shows that shutting down the household's dependence on i's product (by setting $\omega_{\mathcal{D},i} = 0$) implies i's final expenditure share is equal to zero as well, $\Upsilon_i = 0$.

We also define an $N \times 1$ vector of tax-adjusted Domar weights, $\boldsymbol{\lambda} \equiv [\lambda_i]$, where

$$\lambda_i = \sum_{k=1}^{N} \Upsilon_k \psi_{ki}. \tag{4}$$

Throughout the rest of the paper we refer to λ_i as the *Domar weight* of sector i. Since λ_i implicitly embodies taxes on sales and intermediate goods, our Domar weights differ from those of Hulten (1978), which are defined for efficient economies. Notably, λ_i captures all direct and indirect ways the household uses goods from sector

[17] See Appendix A for the proof of equation (3).

i after accounting for taxes.[18]

The aggregate impact of sectoral shocks. Before defining the key measure of the paper, we must first characterize how real GDP responds to TFP shocks in the observed economy (which contains all linkages between sectors). For this, we use the central theorem of Baqaee and Farhi (2020), which characterizes the first-order macroeconomic impact of a microeconomic shock in the presence of distortions:

$$\frac{d \log Y}{d \log A_i} = \lambda_i. \tag{5}$$

Crucially, tax-adjusted Domar weights $\{\lambda_i\}_{i=1}^N$ are the correct statistics for determining the aggregate effect of productivity shocks $d \log A_i$ to each sector $i = 1, 2, ..., N$. The above equation is a variant of Hulten's (1978) theorem for economies with inefficient equilibria.[19] Equation (5) highlights why Domar weights are a measure of the aggregate importance of industries: in response to an infinitesimal change in sector i's productivity, real GDP will change by λ_i%. Since the Domar weight λ_i implicitly encompasses all direct and indirect paths from sector i to final demand (equation 4), it captures how a shock to i impacts real GDP by propagating to other producers in the production network and eventually to final consumers. However, while the effect of a shock to sector i on real GDP is given by its Domar weight, sector i's removal centrality μ_i captures how simultaneous shocks to *all* producers (including i) are amplified as they pass through i. Therefore, removal centrality encapsulates sectors' Domar weights and all the information included in these statistics, and serves as a complementary measure of systemic significance that is distinct from industries' Domar weights.

[18] Our Domar weights, defined in equation (4), are similar to the cost-based Domar weights of Baqaee and Farhi (2020).

[19] As is well-known in the production networks literature, in efficient economies, the first-order change in real GDP in response to a microeconomic productivity shock to sector i is given by Hulten's (1978) theorem, $\frac{d \log Y}{d \log A_i} = \frac{p_i y_i}{\text{GDP}}$. Notably, in our model, the Domar weights λ_i do not coincide with $\frac{p_i y_i}{\text{GDP}}$ due to the presence of tax wedges. See Baqaee and Farhi (2020) for a more detailed discussion on aggregation in inefficient economies.

2.2 Theoretical results

Having characterized the first-order change in real GDP in response to microeconomic productivity shocks, we now formally define our measure of removal centrality.

2.2.1 Deriving Sectors' Removal Centrality

We begin by defining the removal centrality of sector i, denoted by μ_i, which measures i's capacity to influence aggregate fluctuations by transmitting simultaneous idiosyncratic TFP shocks to final demand. To this end, we first construct a counterfactual production network $\boldsymbol{\omega}_i$ that omits the linkages between sector i and its direct suppliers and customers (including final consumers). Specifically, the i^{th} column and row of $\boldsymbol{\omega}_i$ contains zeros, while all other entries remain the same as in the observed network $\boldsymbol{\omega}$.[20]

We denote the first-order (relative) change in real GDP in response to a vector of productivity shocks under this counterfactual network structure by $\frac{\Delta\tilde{Y}_i}{\tilde{Y}_i}$. Removal centrality μ_i is then calculated as the difference between the aggregate effect of shocks in the observed and counterfactual economies.

Definition 1 (Removal Centrality). *The extent to which sector i propagates a vector of productivity shocks to final demand is given by*

$$\mu_i \equiv \frac{\Delta Y}{Y} - \frac{\Delta\tilde{Y}_i}{\tilde{Y}_i},$$

where $\frac{\Delta Y}{Y}$ *is the first-order change in real GDP in the actual economy, and* $\frac{\Delta\tilde{Y}_i}{\tilde{Y}_i}$ *is the first-order change in real GDP in the counterfactual economy defined by* $\boldsymbol{\omega}_i$*.*

The significance of sector i in transmitting simultaneous shocks to other producers and final consumers increases with the magnitude of μ_i. A higher value of μ_i indicates that removing sector i from the network would substantially impact real GDP, as numerous other producers and final consumers rely on its output. Con-

[20]This results in a change in network structure given by $\Delta\boldsymbol{\omega} = \boldsymbol{\omega}_i - \boldsymbol{\omega}$.

versely, a lower value of μ_i suggests that sector i is relatively less important as a producer and consumer of goods from a macroeconomic perspective.

A key challenge in estimating μ_i lies in characterizing the change in real GDP under the counterfactual scenario where sector i does not supply or use intermediate goods and does not sell final goods to households ($\frac{\Delta \tilde{Y}_i}{\tilde{Y}_i}$). In Proposition 1, we characterize μ_i for a vector of productivity shocks to all sectors of the economy. The resulting formula is the central object of our study.

Proposition 1. *The first-order macroeconomic importance of sector i in propagating a vector of idiosyncratic TFP shocks to final demand is given by*

$$\mu_i = \overbrace{\underbrace{\Upsilon_i}_{\textit{Direct effect}} + \underbrace{\boldsymbol{\lambda}'\boldsymbol{\Omega}_{(i)}}_{\textit{Indirect effect}}}^{\textit{Domar weight } \lambda_i} + \underbrace{\lambda_i \tilde{\boldsymbol{\Omega}}^{(i)} \mathbb{1}}_{\textit{Supplier effect}} , \tag{6}$$

where $\boldsymbol{\Omega}_{(i)}$ is the i^{th} column of the equilibrium input-output matrix, $\tilde{\boldsymbol{\Omega}}^{(i)}$ is the i^{th} row of the matrix $\tilde{\boldsymbol{\Omega}}$, $\mathbb{1}$ is an $N \times 1$ vector of ones, and Υ_i is the final expenditure share of sector i.

Proof. See Appendix A.

Equation (6) highlights that sector i's removal centrality can be decomposed into three distinct effects. The first term, the *direct effect* Υ_i, measures how a shock to sector i directly affects real GDP through households' consumption of final goods from i. The larger the value of Υ_i, the more significant sector i is as a producer of final goods and services. Next, the *indirect effect* measures the impact of a shock to sector i on real GDP by tracing how it spreads to other industries, influencing final demand indirectly. This effect, represented by $\boldsymbol{\lambda}'\boldsymbol{\Omega}_{(i)}$, is computed as the Domar-weighted sum of sector i's intermediate goods sales to the rest of the economy, with the vector $\boldsymbol{\Omega}_{(i)}$ containing elements from the i^{th} column of the input-output matrix $\boldsymbol{\Omega}$. By multiplying each element of $\boldsymbol{\Omega}_{(i)}$ with the corresponding customer sector's Domar weight, the indirect effect comprises all possible transmission paths from sector i to final demand, regardless of length. This can be seen in the alternate expression for the indirect effect: $\boldsymbol{\Upsilon}'\boldsymbol{\Omega}_{(i)} + \boldsymbol{\Upsilon}'\boldsymbol{\Omega}\boldsymbol{\Omega}_{(i)} + \boldsymbol{\Upsilon}'\boldsymbol{\Omega}^2\boldsymbol{\Omega}_{(i)} + \ldots$, where $\boldsymbol{\Upsilon}'\boldsymbol{\Omega}_{(i)}$ represents all transmission paths of length one, $\boldsymbol{\Upsilon}'\boldsymbol{\Omega}\boldsymbol{\Omega}_{(i)}$ denotes paths of length two, $\boldsymbol{\Upsilon}'\boldsymbol{\Omega}^2\boldsymbol{\Omega}_{(i)}$ captures paths of length three, and so on.

Before discussing the supplier effect of equation (6), we introduce Corollary 1, which shows how sector i's direct and indirect effects together constitute the first-order macroeconomic impact of a shock to sector i, which is given by i's Domar weight λ_i, as shown in equation (5).

Corollary 1. *The Domar weight of sector i is equivalent to the sum of i's direct and indirect effect*

$$\lambda_i = \underbrace{\Upsilon_i}_{\text{Direct effect}} + \underbrace{\boldsymbol{\lambda}'\boldsymbol{\Omega}_{(i)}}_{\text{Indirect effect}} .$$

Proof. First note that the sum of i's direct and indirect effect can be written as $\Upsilon_i + \sum_{k=1}^{N} \lambda_k \Omega_{ki}$. By equation (4), λ_k can be equivalently expressed as $\lambda_k = \sum_{m=1}^{N} \Upsilon_m \psi_{mk}$. Therefore, we can rewrite i's direct and indirect effect as $\Upsilon_i + \sum_{k=1}^{N} \sum_{m=1}^{N} \Upsilon_m \psi_{mk} \Omega_{ki}$, which, in matrix form is $\boldsymbol{\Upsilon}' + \boldsymbol{\Upsilon}'\boldsymbol{\Psi}\boldsymbol{\Omega}$. Rewriting this expression as $\boldsymbol{\Upsilon}'(I + \boldsymbol{\Psi}\boldsymbol{\Omega})$ and noting that $\boldsymbol{\Psi} = I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + ...$, we get $\boldsymbol{\Upsilon}'\left(I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + \boldsymbol{\Omega}^3 + ...\right)$. Once again using the result $\boldsymbol{\Psi} = I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + ...$, we can write $\boldsymbol{\Upsilon}'\left(I + \boldsymbol{\Omega} + \boldsymbol{\Omega}^2 + \boldsymbol{\Omega}^3 + ...\right) = \boldsymbol{\Upsilon}'\boldsymbol{\Psi}$, which is nothing but the Domar weight vector $\boldsymbol{\lambda}' = \boldsymbol{\Upsilon}'\boldsymbol{\Psi}$. ■

Corollary 1 provides an important insight into the direct and indirect effects of a sector's removal centrality μ_i. These effects (which are captured in the first two terms on the right-hand side of equation 6) sum to i's Domar weight, which is an alternative measure of a sector's systemic importance that has been widely studied in the literature (see, for example, Hulten, 1978, Acemoglu et al., 2012, Liu, 2019, Baqaee and Farhi, 2020, Bigio and La'O, 2020). Therefore, not only does Proposition 1 allow us to quantify the direct and indirect effect of an idiosyncratic TFP shock to sector i, but also provides a decomposition of i's Domar weight.

The final term on the right-hand side of equation (6), which we refer to as the *supplier effect*, distinguishes a sector's removal centrality μ_i from its Domar weight λ_i. The supplier effect quantifies how shocks to *all other* industries impact real GDP because sector i transmits these shocks to its direct and indirect customers and eventually to final demand. For example, negative shocks to i's primary, secondary, and higher-order suppliers reduce i's output because i relies either directly or indirectly on the products of the upstream sectors. Sector i, in turn, transmits these shocks to the household and also to its direct and indirect customers, affecting final consumption by reducing the output of all producers downstream of i. Importantly, shocks to sectors downstream of i may also affect direct or indirect suppliers of i, and hence pass through i. The supplier effect $\lambda_i \tilde{\mathbf{\Omega}}^{(i)} \mathbb{1}$ is calculated by interacting i's Domar weight with the elements of the i^{th} row of the matrix $\tilde{\mathbf{\Omega}}$, which captures the intensity with which i uses inputs from other industries. Specifically, if an industry j supplies inputs to i (i.e., $\tilde{\Omega}_{ij} \neq 0$), then sector i will transmit shocks that affect sector j and j's suppliers, suppliers' suppliers, and so on. As a result, i's supplier effect captures all possible ways through which i can transmit shocks to other sectors of the economy.

Finally, measuring removal centrality only requires *observed* data on sector-level intermediate input purchases, nominal gross output, final sales, and tax payments. This is because final expenditure shares Υ_i, Domar weights $\{\lambda_k\}_{k=1}^{N}$, and parameters of the input-output matrix $\mathbf{\Omega}$ (measured at the *initial* equilibrium) are sufficient to characterize the first-order change in real GDP in the actual $\left(\frac{\Delta Y}{Y}\right)$ and counterfactual economies $\left(\frac{\Delta \tilde{Y}_i}{\tilde{Y}_i}\right)$. While severing linkages between sector i and its customers, suppliers, and the household induce changes in prices and quantities of inputs traded at the microeconomic level, these endogenous changes are macroeconomically irrelevant to a first-order approximation, and only matter beyond the first-order. In Appendix D, we characterize i's removal centrality to a second-order approximation. Sector i's second-order removal centrality requires knowledge of changes in i's final expenditure share $(d \log \Upsilon_i)$, the Domar weights of all industries $(d \log \lambda_k)$, and the input-output parameters of i's suppliers $(d \log \Omega_{ik})$ and customers $(d \log \Omega_{ki})$. Since changes in these objects are unobservable under the counterfactual, we characterize them in terms of the elasticities of substitution in production θ and consumption σ, as well as the parameters of the production network at the initial equilibrium.

2.2.2 Illustrative Examples

In this section, we provide a deeper understanding of Proposition 1 by exploring the properties of three network structures. This exercise allows us to make empirical predictions about the relationship between sectors' removal centrality μ_i and their Domar weights λ_i.

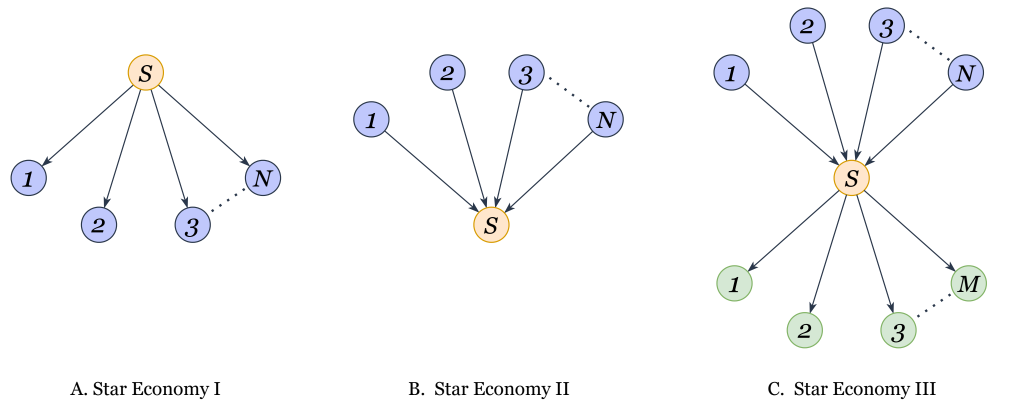


Figure 2: Example Network Structures

Note: This figure depicts three different network structures. Colored nodes represent different industries, whereas directed arrows depict the flow of intermediate goods between sectors. Orange nodes represent the "star" sector S. In Panel A, the star sector only uses labor to produce intermediate inputs and final goods. In Panel B, sector S uses labor and inputs from all other sectors to produce only final goods. In Panel C, the star sector S uses labor and intermediate inputs from the set of N purely upstream sectors (blue nodes) $1,2,...,N$ to produce final goods and intermediate inputs for the set of M purely downstream sectors $1,2,...,M$.

Consider the three "Star Economies" shown in Figure 2. In the figure, colored nodes represent different industries, and directed arrows depict the flow of intermediate goods between producers. In each panel, the orange node corresponds to the "star" sector denoted by S. In Panel A, the star sector uses only labor to produce intermediate inputs that are, in turn, used by all other industries $1,2,...,N$ (blue nodes), as well as final goods consumed by households. In Panel B, sector S uses labor and intermediates from all other industries to produce only final goods. Finally, in Panel C, sector S uses both labor and intermediate inputs from the set of N purely upstream sectors (blue nodes, denoted by $1,2,...,N$) to produce final goods and intermediate inputs for the set of M purely downstream sectors.[21]

[21] In symmetric networks, where all industries have equal interdependence for intermediate in-

Throughout our analysis of the network structures in Figure 2, we initially assume that all sectors are equally important in final demand, which implies a constant value of $\omega_{\mathcal{D},i}$ across all i. Moreover, we also assume that α_i is constant across all sectors that use intermediates in addition to labor, and that the importance of labor and intermediates for these sectors is the same. Finally, we impose a unitary elasticity of substitution in consumption and production, $\sigma = 1$ and $\theta = 1$, to simplify the expressions comparing the parameters μ and λ. We make these simplifying assumptions to isolate how the *structure* of each network generates differences in removal centralities and Domar weights across industries.

Star economy I. In the economy depicted in Figure 2, Panel A, the removal centrality of the star sector S is exactly equal to its Domar weight, or $\mu_S = \lambda_S$. However, for all other sectors $1, 2, ..., N$, their removal centrality exceeds their Domar weight. This is clear from the expressions for μ and λ:

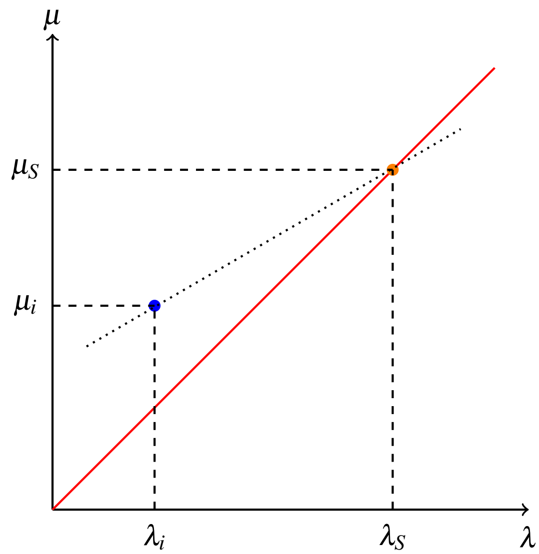
$$\mu_S = \lambda_S = \Upsilon_S \left(1 + \sum_{j \neq S}^{N} \Omega_{jS}\right) \quad \text{and} \quad \mu_i = \Upsilon_S (1 + \Omega_{iS}) > \lambda_i \;\; \text{for sectors } i \neq S.$$

Furthermore, under the production structure of Figure 2, Panel A, the removal centrality of the star sector S is always greater than that of all other industries. In other words, the star sector has the greatest ability to shape aggregate fluctuations, as it serves as the sole producer of intermediate inputs for all other sectors.

Figure 3 illustrates the relationship between μ and λ under the "Star Economy I" network structure. The red line is the 45-degree line, where $\mu = \lambda$. The orange point represents the star sector S, while the blue point represents all other sectors $i \neq S$. Finally, the faint dotted line visually approximates the trend between μ and λ.

The figure reveals two key insights: i) the relationship between μ and λ is positive since $\lambda_S > \lambda_i$ and $\mu_S > \mu_i$, and ii) the trend line has a slope less than one, since

puts, Domar weights and removal centralities are identical across sectors. However, in asymmetric networks like those in Figure 2, there are differences in Domar weights and removal centralities across industries. As our focus is on understanding the network properties that lead to variations in both λ and μ, we only discuss the asymmetric networks depicted in Figure 2.



Star Economy I: $\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$

Figure 3: Removal Centrality and Domar Weights (Star Economy I)

Note: This figure plots sectors' removal centrality μ against their Domar weight λ under the Star Economy I network structure shown in Figure 2. The orange point represents the star sector S, whereas the blue point represents all other industries $i \neq S$. Households' dependence on each industry's output is constant across all sectors, $\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$ for all $i \neq S$. The solid red line represents the 45-degree line, and the faint dotted line is a visual approximation of the trend in the relationship between μ and λ.

$\frac{\mu_S - \mu_i}{\lambda_S - \lambda_i} < 1$. In Section 3, we present empirical evidence that the degree to which Domar weights underestimate the systemic importance of an industry (as measured by μ) *increases* as the Domar weight of the sector increases. In other words, the slope of the trend line is estimated to be greater than one. Thus, the star economy depicted in Figure 2, Panel A, cannot account for this observed relationship. Our results suggest that the most important industries, as measured by removal centrality, are generally not upstream sectors that supply large quantities of intermediate inputs to other industries.

Star economy II. Next, consider the network in Panel B of Figure 2. In this economy, the star sector S uses, but does not supply, intermediate inputs to any other industry. Thus, the removal centrality of S is strictly greater than its Domar weight $\mu_S > \lambda_S$, while the removal centrality of all other sectors exactly equal their Domar weights $\mu_i = \lambda_i$ for all $i \neq S$. Proposition 1 implies

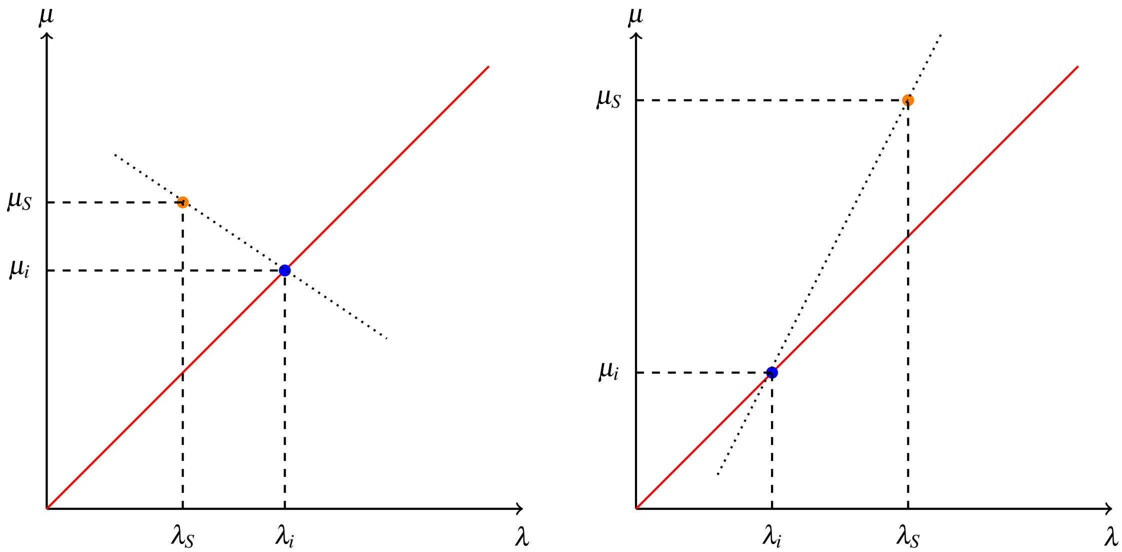
$$\mu_S = \Upsilon_S \left(1 + \sum_{j \neq S}^{N} \Omega_{Sj} \right) > \lambda_S \quad \text{and} \quad \mu_i = \lambda_i = \Upsilon_S \left(1 + \Omega_{Si}\right) \quad \text{for sectors } i \neq S.$$

The above expressions highlight that the removal centrality of the star sector S is greater than that of any other industry $i \neq S$. However, assuming households are equally reliant on the final products produced by each sector ($\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$ for all i), the Domar weight of S is *strictly less* than that of sectors $i \neq S$.[22] This is visually represented in Panel A of Figure 4, where again the orange point depicts the star sector, and the blue point represents a representative upstream sector. Thus, the network structure in Figure 2, Panel B, implies a negative relationship between sectors' removal centrality and Domar weights.

Panel B of Figure 4 depicts the condition under which a positive relationship between μ and λ exists in this network: the star sector S must not only use intermediate inputs but *also* be a more important producer of final goods than the other sectors, or $\omega_{\mathcal{D},S} >> \omega_{\mathcal{D},i}$.[23] When this condition is met, the Domar weight of S exceeds that of other industries, resulting in a positive relationship between λ and μ. Moreover, since $\frac{\mu_S - \mu_i}{\lambda_S - \lambda_i} > 1$, the trend line has a slope greater than one. Thus, in light of our empirical results of Section 3, Figure 4 suggests that industries with large removal centralities not only consume more intermediate goods relative to less influential sectors but may also play a critical role in supplying final goods.

[22] Formally, the Domar weight of S is $\lambda_S = \Upsilon_S$, whereas the Domar weight of an industry $i \neq S$ is $\lambda_i = \Upsilon_S \left(1 + \Omega_{Si}\right)$.

[23] This is because Υ_k is increasing in $\omega_{\mathcal{D},k}$ for all k. Thus, for a large value of $\omega_{\mathcal{D},S}$, where $\omega_{\mathcal{D},S} >> \omega_{\mathcal{D},i}$, the Domar weight of S is larger than the Domar weight of i, i.e., $\lambda_S = \Upsilon_S > \Upsilon_i \left(1 + \Omega_{Si}\right) = \lambda_i$. In this case, the *direct effect* of sector S is the largest of all industries, which implies that its Domar weight is also larger than that of other sectors.



Panel A. Star Economy II: $\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$ Panel B. Star Economy II: $\omega_{\mathcal{D},S} >> \omega_{\mathcal{D},i}$

Figure 4: Removal Centrality and Domar Weights (Star Economy II)

Note: This figure plots sectors' removal centrality μ against their Domar weight λ under the Star Economy II network structure shown in Figure 2. The orange point represents the star sector S, whereas the blue point represents all other industries $i \neq S$. In Panel A, households depend equally on the output produced by each industry $\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$ for all $i \neq S$, whereas in Panel B, the star sector is a more critical producer of final goods $\omega_{\mathcal{D},S} >> \omega_{\mathcal{D},i}$. In both panels, the faint dotted line provides a visual approximation of the trend in the relationship between μ and λ, whereas the solid red line is the 45-degree line.

Star economy III. We now turn to the economy shown in Panel C of Figure 2. The blue nodes (denoted by $1, 2, ..., N$) in the figure represent purely upstream sectors that supply intermediate inputs to the star sector S (represented by the orange node), while the green nodes $(1, 2, ..., M)$ depict purely downstream sectors that use inputs from S. In this economy, sector S always records a value of μ greater than every downstream *and* upstream sector. Specifically, proposition 1 implies that the removal centrality of S is given by

$$\mu_S = \Upsilon_S + \Upsilon_S \sum_{j=1}^{M} \Omega_{jS} + \Upsilon_S \sum_{j \neq S}^{N} \Omega_{Sj} + \Upsilon_S \left(\sum_{j=1}^{M} \Omega_{jS} \right) \sum_{j \neq S}^{N} \Omega_{Sj}, \tag{7}$$

whereas the removal centrality of a representative upstream sector U and downstream sector D is, respectively,

$$\mu_U = \Upsilon_S + \Upsilon_S \Omega_{SU} + \Upsilon_S \Omega_{SU} \sum_{j=1}^{M} \Omega_{jS}, \tag{8}$$

and

$$\mu_D = \Upsilon_S + \Upsilon_S \Omega_{DS}. \tag{9}$$

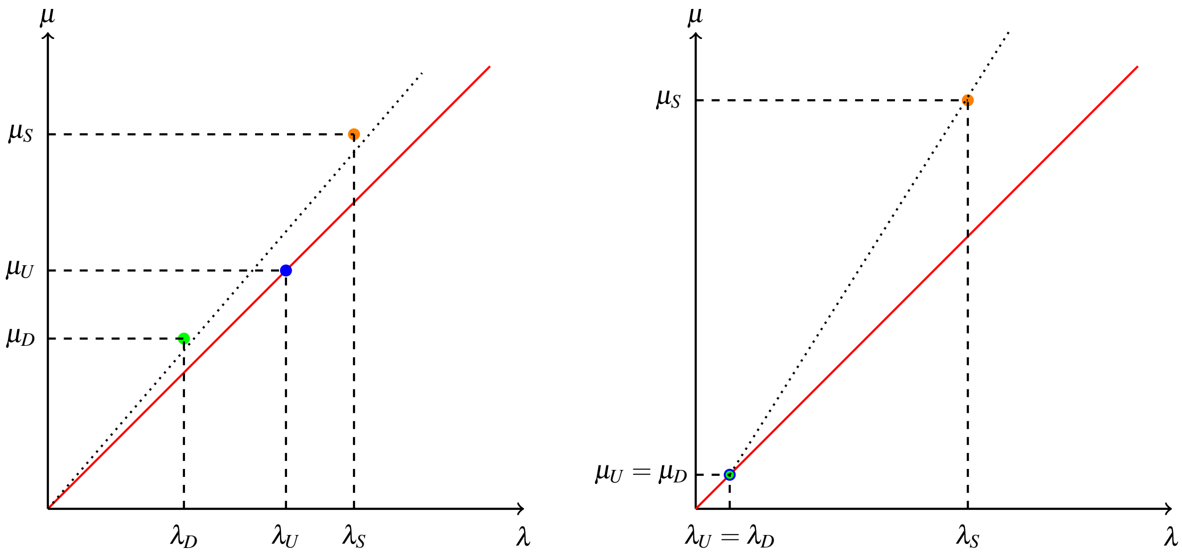
Thus, by comparing equations (7), (8), and (9), it is evident that i) $\mu_S > \mu_D$ and $\mu_S > \mu_U$, and ii) the removal centrality of upstream sectors U may be greater or less than that of downstream sectors D, depending on the relative number of upstream and downstream sectors.[24] Finally, the star sector S has the greatest Domar weight, where $\lambda_S > \lambda_U > \lambda_D$.[25]

Figure 5 shows the relationship between μ and λ as the star sector becomes increasingly important as a producer and consumer of *intermediate* goods. In each panel, the green point represents a downstream sector, the blue point is an upstream industry, and the orange point again represents the star sector. Panel A depicts a possible trend between μ and λ when there are few upstream and downstream sectors, where $\mu_U > \mu_D$. Crucially, the slope of the trend line in Panel A may not be greater than one since, i) upstream sectors always lie on the 45-degree line, ii) λ_D is less than λ_U, and iii) μ_U may be less or greater than μ_D. However, as the number of upstream and downstream industries increases ($N \to \infty$, $M \to \infty$), removal centralities and Domar weights of the upstream and downstream sectors approach zero, while both μ and λ remain greater than zero for the star sector, as shown in Panel B.[26]

[24] Specifically, $\mu_U > \mu_D$ if $M(1 - \alpha_S) > N - 1$.

[25] The Domar weight of S is given by $\lambda_S = \Upsilon_S + \Upsilon_S \sum_{j=1}^{M} \Omega_{jS}$, while the Domar weights of D and U are, respectively, $\lambda_D = \Upsilon_S$ and $\lambda_U = \Upsilon_S + \Upsilon_S \Omega_{SU} + \Upsilon_S \left(\sum_{j=1}^{M} \Omega_{jS} \right) \Omega_{SU}$. Although it is evident that $\lambda_S > \lambda_D$, it is not immediately clear that $\lambda_S > \lambda_U$. For λ_S to be strictly greater than λ_U, the condition $\sum_{j=1}^{M} \Omega_{jS} > \Omega_{SU} \left(1 - \Omega_{SU}\right)^{-1}$ must hold. Since the star sector and all downstream sectors are equally dependent on intermediates, $(1 - \alpha_j) = (1 - \alpha_S)$, this condition can be expressed as $\sum_{j=1}^{M} (1 - \alpha_S)\omega_{jS} > (1 - \alpha_S)\omega_{SU}(1 - (1 - \alpha_S)\omega_{SU})^{-1}$. Simplifying this expression yields $M > \frac{1}{N-(1-\alpha_S)}$, which is true whenever the number of upstream sectors is greater than one, $N > 1$.

[26] Formally, as $N, M \to \infty$, $\mu_S \to \frac{(1-\alpha_S)(1+(1-\alpha_S))}{2} > 0$ and $\lambda_S \to \frac{1-\alpha_S}{2} > 0$.



Panel A. Star Economy III: $\mu_U > \mu_D$

Panel B. Star Economy III: $N, M \to \infty$

Figure 5: Removal Centrality and Domar Weights (Star Economy III)

Note: Panel A plots sectors' removal centrality μ against their Domar weight λ under the Star Economy III network structure shown in Figure 2. The orange point represents the star sector S, the blue point represents a purely upstream industry U, and the green point depicts a purely downstream industry D. The household sector depends equally on the final goods produced by the sectors ($\omega_{\mathcal{D},S} = \omega_{\mathcal{D},i}$ for all i). The faint dotted line visually approximates the trend in the relationship between μ and λ, whereas the solid red line represents the 45-degree line. Panel B shows the trend as the number of upstream and downstream sectors increases.

In other words, the significance of sector S as both a user *and* producer of intermediate inputs determines whether the slope of the trend line is greater than one: the more important is S, the greater the slope. This point is significant in light of our empirical findings, where we observe the relationship between μ and λ to be similar to that of Figure 5, Panel B.

Anticipating our empirical results in Section 3, our analysis of the three network structures in Figure 2 indicates that sectors with high removal centralities i) exhibit a greater reliance on intermediates relative to sectors with lower values of μ, and ii) are also significantly more important producers of final or intermediate goods compared to other industries.

2.3 The Relationship Between Supplier Effects and Indirect Effects

We now highlight an important caveat of our measure of sectoral importance μ_i. Specifically, only while establishing (in isolation) the systemic importance of an industry to simultaneous disruptions in the economy do both indirect and supplier effects matter. In the aggregate, the two effects are (approximately) equivalent. In other words, what is classified as a supplier effect for one industry is also counted as part of the indirect effect of another industry. Equation (10) below shows that the sum of supplier effects across all industries approximately equals the sum of indirect effects (of sectors' removal centrality):

$$\underbrace{\mathbb{1}'\tilde{\mathbf{\Omega}}'\boldsymbol{\lambda}}_{\sum \textit{Supplier effect}} + \underbrace{\operatorname{tr}(\boldsymbol{\lambda} \circ \mathbf{\Omega})}_{\sum \textit{Own effect}} = \underbrace{\boldsymbol{\lambda}'\mathbf{\Omega}\mathbb{1}}_{\sum \textit{Indirect effect}}. \tag{10}$$

The term $\operatorname{tr}(\boldsymbol{\lambda} \circ \mathbf{\Omega})$ (Σ*Own effect*) reflects the impact of shocks to each sector on real GDP, which are attributed to the use of the sector's own intermediate inputs. This effect is therefore classified as an indirect effect. Importantly, the sum of supplier effects $\mathbb{1}'\tilde{\mathbf{\Omega}}'\boldsymbol{\lambda}$ and these "own effects" $\operatorname{tr}(\boldsymbol{\lambda} \circ \mathbf{\Omega})$ is equal to the sum of indirect effects $\boldsymbol{\lambda}'\mathbf{\Omega}\mathbb{1}$. Equation (10) thus shows that when aggregating across industries, supplier effects and indirect effects are (nearly) one and the same.

2.4 How Do Taxes Affect Sectors' Macroeconomic Importance?

Since we derive sectors' removal centrality within the framework of an inefficient production networks model, a natural question arises: what is the effect of distortions on the removal centrality of sectors? In our model, taxes increase sectors' macroeconomic importance μ_i relative to an economy without distortions. This result is formalized in Proposition 2 below.

Proposition 2. *Sector-level taxes on sales $\tau_{y,i}$ and intermediate purchases $\{\tau_{x,ik}\}_{k=1}^{N}$ contribute to the macroeconomic importance of a sector i via*

$$\zeta_i = \mu_i - \mu_i^F \geq 0, \tag{11}$$

where ζ_i is the contribution of taxes to μ_i and μ_i^F is the removal centrality of i in a frictionless economy without taxes.

Proof. See Appendix A.

Taxes on sales $\tau_{y,i}$ and intermediate inputs $\tau_{x,ij}$ have a compounding effect along supply chains, reducing the allocative efficiency of producers and amplifying the impact of microeconomic shocks on aggregate fluctuations. In equation (11), we demonstrate that distortions increase sectors' removal centrality compared to a frictionless economy without taxes.[27] The statistic ζ_i measures the contribution of taxes to the macroeconomic importance of sector i where μ_i^F represents i's removal centrality in a frictionless economy. In the Proof of Proposition 2 in Appendix A, we characterize ζ_i in terms of the parameters of the production network $\mathbf{\Omega}$, final expenditure shares $\mathbf{\Upsilon}$, and taxes on sales $\{\tau_{y,i}\}$ and intermediates $\{\tau_{x,ij}\}$ and show that ζ_i is always greater than or equal to zero.[28]

3 Taking the Model to the Data

In this section, we use the framework presented in Section 2 to estimate the removal centrality μ and Domar weight λ of approximately 450 sectors (on average) in the United States between 1982 and 2012. We then document the relationship between μ and λ and establish a connection between our empirical findings and

[27]Notably, taxes on factor payments $\tau_{L,i}$ only affect real GDP in the presence of labor supply shocks and play no role when there are TFP shocks.

[28]In Appendix C, we estimate the contribution of taxes to sectors' supplier effects using US input-output data from 1982 to 2012. Our analysis shows that taxes at the sector level have a minimal impact on the estimated values of supplier effects and removal centralities. While other distortions such as nominal rigidities, financial frictions, and market power may also affect the systemic importance of industries, we abstract from these distortions in our calculation of industries' removal centrality.

the discussion of the network structures presented in Section 2.2.2. Next, we identify sectors that have experienced substantial fluctuations in removal centrality, despite having relatively stable Domar weights. Finally, we study industries' removal centralities across six major economies: the United States, United Kingdom, Japan, China, Germany, and Australia, and decompose μ into direct, indirect, and supplier effects.

3.1 Comparing removal centrality with Domar weights

We begin by estimating removal centrality μ_i (as defined in equation 6) for 393 sectors in the US in 2012.[29] We use the detailed 2012 input-output table provided by the BEA to estimate $\mathbf{\Omega}$, which further yields $\mathbf{\Omega}_{(i)}$ and $\tilde{\mathbf{\Omega}}^{(i)}$. We use final demand data in the 2012 BEA Use table to estimate the final expenditure shares $\mathbf{\Upsilon}$. We then use $\mathbf{\Omega}$ and $\mathbf{\Upsilon}$ to estimate the Domar weights via $\boldsymbol{\lambda}' = \mathbf{\Upsilon}'(I-\mathbf{\Omega})^{-1}$. Figure 6 presents a comparison between sectors' removal centrality and Domar weights in 2012. The left figure (Panel A) compares these two measures of systemic influence for all 393 sectors in 2012. On the other hand, the right figure (Panel B) focuses specifically on the top 20 sectors (which accounted for 37% of GDP in 2012).

Panel A of Figure 6 yields three notable observations. Firstly, a positive relationship exists between sectors' removal centrality and Domar weights. This suggests that sectors with higher removal centrality typically also have larger Domar weights, signifying their greater systemic influence and highlighting significant asymmetries within the US production structure. Secondly, the removal centrality of a sector is generally distinct from its Domar weight, implying that these two measures capture different aspects of a sector's influence within the network. Finally, the removal centrality of a sector is typically greater than its Domar weight. This finding is expected, as removal centrality encompasses producers' Domar weight, providing a more comprehensive assessment of a sector's significance.

Interestingly, the figure highlights that the Domar weight of a sector tends to underestimate its systemic importance, and this underestimation becomes more pronounced as the Domar weight of the sector increases, as evidenced by the deviation from the 45-degree line. This finding is not immediately obvious since

[29] See Appendix B for a detailed discussion of the BEA input-output data.

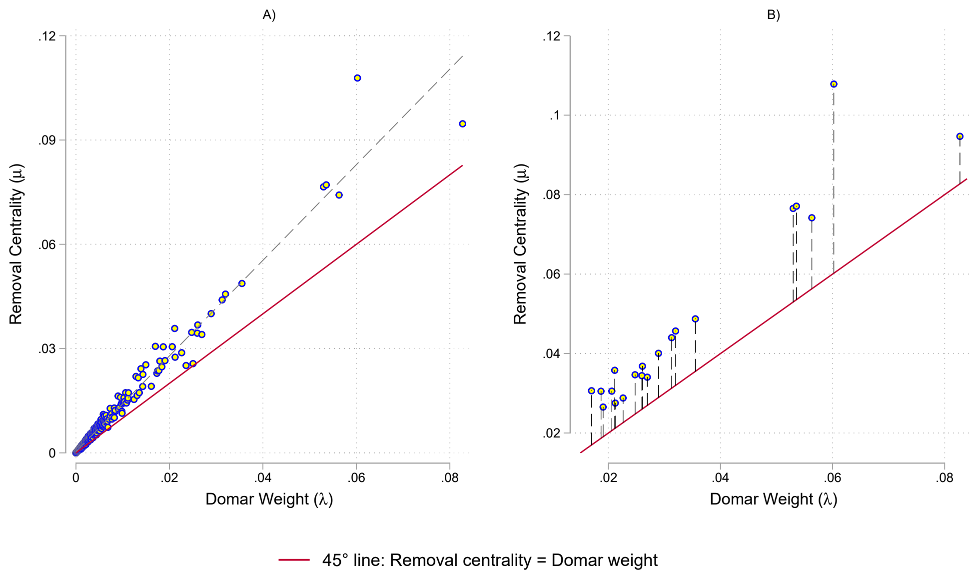


Figure 6: Removal Centrality and Domar Weights in the US (2012)

Note: This figure shows the relationship between sectors' removal centrality and Domar weights for 393 sectors in the US in 2012. Panel A displays data for all sectors in the detailed BEA input-output accounts, excluding government sectors. Panel B focuses on the top 20 sectors by removal centrality, which accounted for 37% of GDP in 2012.

removal centrality and Domar weights can coincide, even for sectors with a large Domar weight. This can occur when upstream sectors do not rely on intermediate inputs for production, as shown in Panel A of Figure 2 and Figure 3. However, the magnitude of this underestimation is significant and consequential. On average, Domar weights underestimate the systemic influence of a sector (under simultaneous shocks) by approximately 50% (with the median figure being similar).

As discussed in Section 2.2.2, our findings in Figure 6 reflect significant asymmetries in the US network structure (emphasized in Acemoglu et al., 2012). Panel B of Figure 6 uncovers two important observations. Firstly, highly influential sectors exhibit substantial supplier effects, indicating their greater reliance on inputs from other producers compared to less influential sectors. Consequently, they are positioned considerably above the 45-degree line.[30] Secondly, the most influen-

[30]These sectors include petroleum refineries, oil & gas extraction, and electric power generation, among others.

tial sectors typically fall into one of two categories: a) they are disproportionately significant producers of final goods (like food and beverage industries), or b) they serve as essential suppliers of intermediate inputs (like oil and gas), resulting in larger Domar weights. These findings suggest that the US network structure resembles the asymmetric star networks depicted in Panels B and C of Figure 2. In such networks, sectors with significant influence are typically located downstream in the supply chain *and* serve as critical suppliers of intermediates or final goods.

3.2 Removal centrality across time

Figure 7 compares US sectors' removal centrality and Domar weight between 1982 and 2012. Panel A plots all sectors (excluding government sectors) appearing in the detailed BEA input-output accounts from 1982 to 2012, along with a line-of-best-fit through the data for each year. The solid red line again represents the 45-degree line, or the point at which a sector's removal centrality coincides with its Domar weight.

The figure highlights the remarkable similarity in the relationship between removal centrality and Domar weights, as illustrated in Figure 6, over time. This is evidenced by the similar slope of the trend line for each year, suggesting that it is an empirical regularity that sectors with larger Domar weights also have significantly greater supplier effects. Moreover, the extent to which Domar weights (on average) underestimate the systemic significance of a sector ranges between approximately 40-50% over the years shown in Figure 7.

In Panel B of Figure 7, only the top 10 sectors with the largest removal centrality are plotted for the seven periods spanning from 1982 to 2012. The line of best fit in Panel B is based solely on the top 10 sectors with the greatest influence in each year. Despite some variation in the slope of the trend line, there is no systemic difference across the years. Our findings suggest that the relationship between removal centrality and Domar weights, as observed in Panel A of Figure 7, holds even when focusing exclusively on the most influential sectors. This suggests that as sectors grow larger in size, they become more prone to output disruptions originating in other industries.

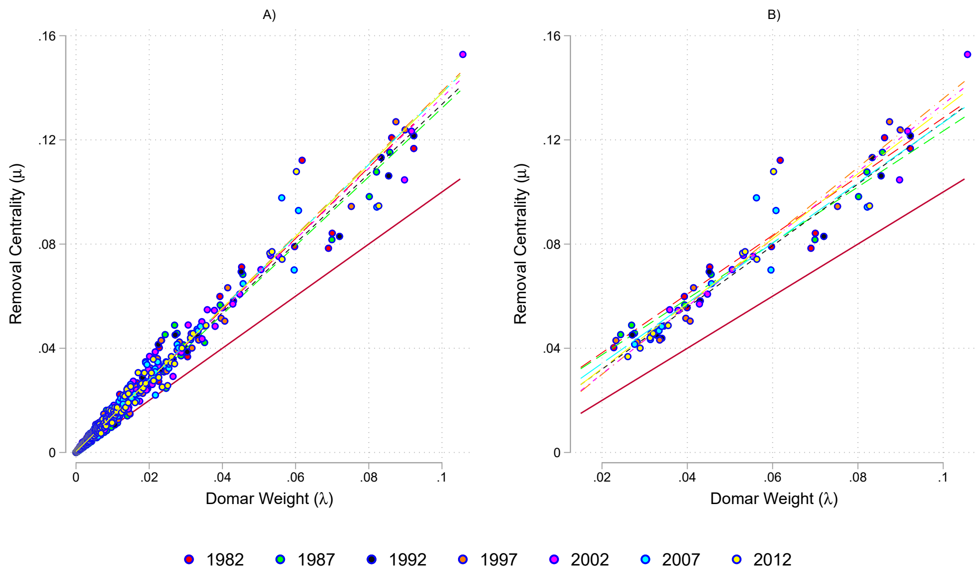


Figure 7: Removal Centrality and Domar Weights in the US (1982 - 2012)

Note: This figure shows the relationship between removal centrality and Domar weights in the US from 1982-2012. On average, there are approximately 450 sectors per year. Panel A displays data for all sectors in the detailed BEA input-output accounts, excluding government sectors. Panel B focuses on the top 10 sectors by removal centrality for each year displayed. The solid red line is the 45-degree line, representing the equality of Domar weights and removal centrality.

3.3 Tracking changes in supplier effects and Domar weights for specific sectors over time

While Figures 6 and 7 demonstrate a positive relationship between Domar weights and removal centrality, it is important to note that there is significant variation in how removal centralities and Domar weights of sectors change over time. To highlight this, Figure 8 compares supplier effects and Domar weights for select industries across time.[31] To ensure comparability of the BEA input-output codes, we divide the sample period into two intervals: 1982-1992 (Panel A) and 1997-2012 (Panel B).[32] The figure shows the four largest sectors (based on gross sales) where

[31] Here, we track supplier effects instead of removal centrality as the supplier effect distinguishes removal centrality from the Domar weight of a sector.

[32] The input-output industry codes between 1992 and 1997 are not consistent with each other, as the BEA revised its input-output classification system in 1997. Further details on the comparison of the 1992 and 1997 benchmark IO accounts of the BEA can be found at https://apps.bea.gov/

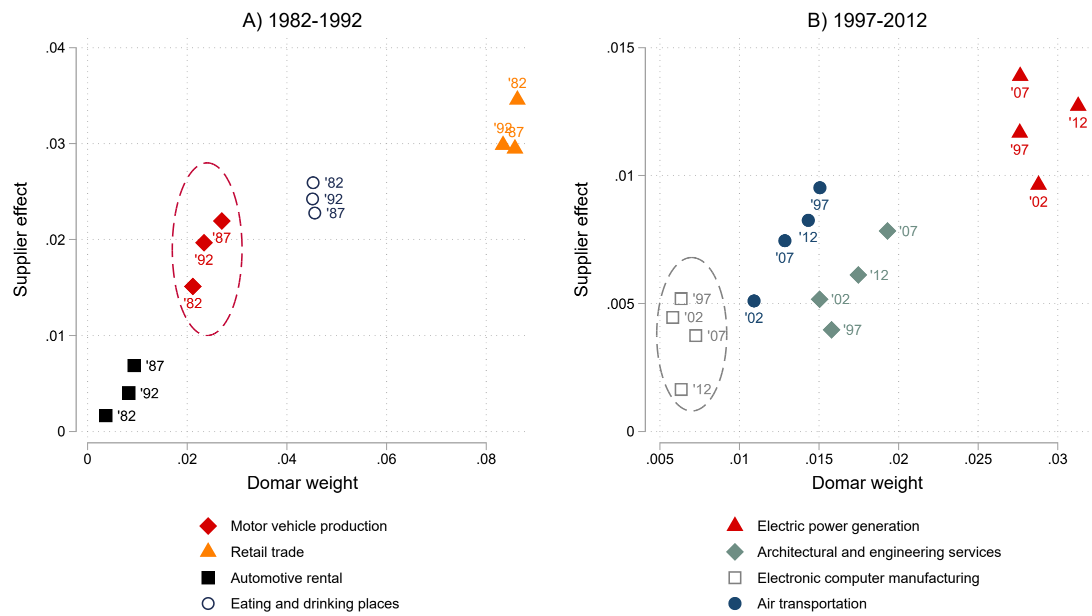


Figure 8: Tracking Changes in Supplier Effects and Domar Weights for Key Industries Over Time

Note: This figure illustrates the relationship between sectors' supplier effects and Domar weights at five-year intervals from 1982 to 1992 (Panel A) and from 1997 to 2012 (Panel B). For each sample period, the growth rate of supplier effects for each sector is at least twice as large (in absolute value) as the growth rates of their corresponding Domar weights. Supplier effects are calculated as in equation (6), and Domar weights are computed as in equation (4), both using the detailed input-output accounts of the BEA.

the absolute value of the ratio between the percentage change in supplier effects and the percentage change in Domar weights exceeded two during the respective period covered in each panel. For example, this ratio is approximately 27 for the "eating and drinking places" sector, reflecting the large change in its supplier effect relative to its Domar weight between 1982 and 1992.[33] The gross sales of the sectors depicted in Figure 8 equate to approximately 11% of GDP, on average, across all years.

The figure highlights that removal centrality can vary significantly across sec-

scb/pdf/2002/08August/0802_I-O_Benchmark.pdf.

[33]The aggregate sales of all sectors for which this ratio is greater than two amount to approximately 40% of GDP, on average, from 1982 to 2012.

tors over time, even when Domar weights remain relatively stable. For example, consider the case of motor vehicle production (represented by the red ellipse) in Panel A. Between 1982 and 1992, the industry experienced a 30% increase in its supplier effect (or an increase of around 0.5% of GDP), while its Domar weight remained almost unchanged. This finding may reflect the increase in the complexity of motor vehicle production over time. The number of electrical components and engine parts in motor vehicles grew substantially throughout the 1980s and 1990s (Fine et al., 1996), a trend that has continued to accelerate in the 21st century. By 2010, the average motor vehicle contained approximately 2,000 components, 30,000 parts, and 10 million lines of software code (MacDuffie and Fujimoto, 2010). Crucially, the increase in supplier effects implies that motor vehicle production became more susceptible to supply chain disruptions over time, which is not captured in the Domar weight of the industry.

In contrast, in Panel B, computer manufacturing (represented by the grey ellipse) experienced a 68% decline in its supplier effect, while its Domar weight slightly increased by 0.4% between 1997 and 2012. This finding reflects the rapid reduction in production costs in the computer manufacturing sector during the late 20th century, resulting from technological improvements in IT industries (Jorgenson, 2001). While input costs continued to fall throughout the mid-2000s, growth in final computer sales slowed substantially, leading to stability in the computer manufacturing sector's Domar weight between 1997 and 2012. The evolution of supplier effects in the computer manufacturing industry may also be explained by changes in market structure and increases in international competition during the 1990s. For example, in the mid-1990s, a growing portion of computer parts and components were imported from Asian markets, where production costs were significantly lower than in the US (Warnke, 1996). Taken together, the findings presented in Figure 8 suggest that relying solely on Domar weights provides only partial information about changes in sectors' systemic importance over time. This highlights the relevance of removal centrality as a complementary measure of the macroeconomic significance of industries.

3.4 Analyzing removal centrality for key sectors across countries

Next, using data from the World Input-Output Database (WIOD) from 2000 to 2014, we identify the industries with the greatest capacity to amplify microeconomic shocks for six major economies: the US, Great Britain, Japan, China, Germany, and Australia.[34] Figure 9 shows the top ten sectors (by removal centrality) for each of the countries. In the figure, each sector's removal centrality is captured by the total of the direct effect (orange bars), indirect effect (yellow bars), and supplier effect (blue bars). Overall, for each economy shown, construction, real estate, public administration, and food & beverages are always among the top five sectors in terms of their ability to influence aggregate output.[35]

Notably, there are significant differences in the size of each effect across countries and industries. Labor-intensive sectors like education and health services have substantial direct effects (accounting for $\approx 75\%$ of removal centrality, on average) due to their low dependency on intermediate inputs and direct provision of services to end-consumers. These attributes reduce their ability to amplify shocks, resulting in lower indirect and supplier effects compared to other sectors. On the other hand, the influence of sectors like wholesale trade and electricity & gas supply comes mostly from the indirect effect ($\approx 50\%$ of removal centrality for both industries), reflecting the importance of these sectors in providing key intermediate inputs for production for many other sectors in the economy. Finally, construction and food & beverages have proportionately large direct ($\approx 50\%$ for construction and $\approx 40\%$ for food & beverages) and supplier effects ($\approx 30\%$ for construction and

[34]See Appendix B for a detailed description of the WIOD data. These economies accounted for $\approx 57\%$ of nominal world GDP in 2021 (World Bank, 2023).

[35]We abstract from international trade when computing each sector's removal centrality. Baqaee and Farhi (2021) characterize how microeconomic shocks propagate through international input-output linkages and generalize Hulten's theorem to open economies. Other papers in the trade literature highlight the importance of input-output linkages as a mechanism for amplifying shocks and generating co-movements in business cycles across countries (see, for example, Caliendo and Parro, 2014, Chaney, 2014, di Giovanni et al., 2014, Redding and Rossi-Hansberg, 2017, di Giovanni et al., 2018, Auer et al., 2019, Antràs and de Gortari, 2020, and Kikkawa et al., forthcoming). A related literature emphasizes that diversified global value chains insulate economies against shocks, limiting aggregate volatility (Caselli et al., 2020; D'Aguanno et al., 2021, and Antràs, 2021). See Baldwin and Freeman (2022) for a detailed overview of the global supply chain literature and Bernard and Moxnes (2018) for a survey of the literature on production networks and international trade.

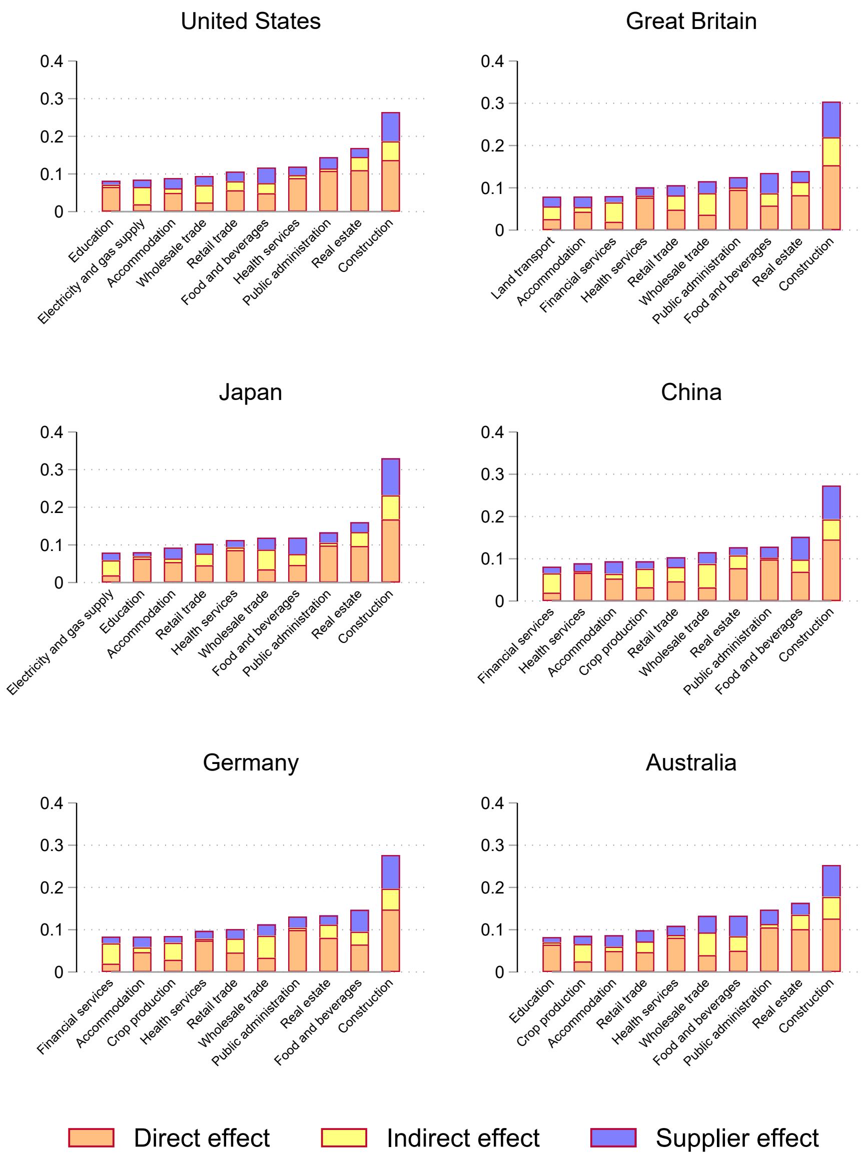


Figure 9: Removal Centrality by Country

Note: This figure presents estimates of removal centrality μ for six major economies. For each sector, removal centrality is measured as in equation (6), averaged over the years 2000 to 2014. The top 10 industries with the greatest removal centrality are shown for each economy. Orange bars capture the direct effects, whereas the yellow and blue bars represent the indirect and supplier effects, respectively. The data is from the World Input-Output Database (2016 Release).

$\approx$ 35% for food & beverages), attesting to their central role as both producers of final goods and consumers of intermediate goods. Hence, each effect represents a distinct mechanism with which a sector can influence aggregate outcomes, and each subcomponent explains a consequential proportion of removal centrality. For the countries and industries shown in Figure 9, direct effects account for about 50% of sectors' overall influence, on average, whereas indirect and supplier effects account for approximately 20% and 30%, respectively.

Our results demonstrate significant differences in the ability of industries to affect aggregate volatility through direct, indirect, and supplier effects. Moreover, our decomposition exercise indicates that the impact of a producer on the broader economy is heavily influenced by its location within the production network. The intensity with which producers use and supply intermediate inputs is a key determinant of their systemic importance.

4 Conclusion

We propose a novel measure of sectors' capacity to transmit simultaneous idiosyncratic microeconomic TFP shocks to all producers in the economy, thereby influencing both their immediate and indirect customers and, ultimately, final demand. Our measure, which we refer to as removal centrality, captures the industry's impact on real GDP through three key channels: i) directly affecting the final consumption of its output, ii) indirectly influencing the production of firms directly or indirectly connected to it, and iii) transmitting disruptions originating elsewhere in the economy.

Our approach is nonparametric, only requiring observable information on an industry's intermediate goods purchases, nominal gross sales, and tax payments. Notably, removal centrality encompasses and extends an existing notion of systemic importance in production networks: producers' Domar weights (or sales shares). In an empirical application for the US, we show that Domar weights underestimate the systemic importance of an industry by $\approx$ 40-50%, between 1982-2012. We find that the extent of underestimation increases with the Domar weight of a sector; a non-trivial result given that removal centrality and Domar weights can perfectly coincide, even for large sectors.

Additionally, we provide evidence of significant changes in the removal centrality of crucial sectors within the US economy over time, such as motor vehicle production and computer manufacturing, despite the relative stability of their Domar weights. Furthermore, we compare sectors' removal centrality across countries and identify industries such as construction, food & beverages, and real estate as powerful amplifiers of shocks between 2000 and 2014.

Our empirical findings are interpreted through the lens of an inefficient production network model. Though our model only includes one type of friction (taxes), it can be easily extended to incorporate other distortions such as financial frictions, market power, and nominal rigidities, as in Liu (2019); Baqaee and Farhi (2020); Bigio and La'O (2020) and Baqaee and Farhi (2022), highlighting the flexibility of our framework.

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Online Appendix A. Proofs

Proof of Proposition 1. The Lagrangean for the aggregator problem is

$$\mathcal{L} = \left(\sum_{j=1}^{N} \left(\omega_{\mathcal{D},j} c_j \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} + \eta \left(wL - \sum_{j=1}^{N} p_j c_j \right),$$

where η is the Lagrange multiplier. Optimization with respect to c_i yields

$$c_i = wLp_i^{-\sigma} \omega_{\mathcal{D},i}^{\sigma-1} \left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1} \right)^{-1}, \tag{12}$$

from which we derive an expression for Υ_i:

$$\Upsilon_i = \frac{p_i c_i}{wL} = \frac{p_i^{1-\sigma} \omega_{\mathcal{D},i}^{\sigma-1}}{\left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1} \right)}. \tag{13}$$

Substituting equation (12) into the consumption aggregator, yields

$$Y = wL \left(\sum_{j=1}^{N} \omega_{\mathcal{D},j}^{\sigma-1} p_i^{\sigma-1} \right)^{\frac{1}{\sigma-1}}. \tag{14}$$

Furthermore, total (log) differentiation of the consumption aggregator gives

$$d\log Y = \sum_{i=1}^{N} Y^{\frac{1-\sigma}{\sigma}} \left(\omega_{\mathcal{D},i} c_i \right)^{\frac{\sigma-1}{\sigma}} \left(d\log c_i + d\log \omega_{\mathcal{D},i} \right). \tag{15}$$

Substituting (12) and (14) into equation (15), yields

$$d\log Y = \sum_{i=1}^{N} \frac{p_i^{1-\sigma} \omega_{\mathcal{D},i}^{\sigma-1}}{\left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1} \right)} \left(d\log c_i + d\log \omega_{\mathcal{D},i} \right),$$

which can be rewritten using equation (13) as

$$d\log Y = \sum_{i=1}^{N} \Upsilon_i d\log c_i + \sum_{i=1}^{N} \Upsilon_i d\log \omega_{\mathcal{D},i}. \tag{16}$$

Total (log) differentiation of equation (12) implies that (16) can be written as

$$d\log Y = \sum_{i=1}^{N} \Upsilon_i \left(d\log w + d\log L - \sigma d\log p_i + (\sigma - 1) d\log \omega_{\mathcal{D},i} - d\log \left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1} \right) \right) + \sum_{i=1}^{N} \Upsilon_i d\log \omega_{\mathcal{D},i}. \quad (17)$$

Noting that $d\log \left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1} \right)$ simplifies to

$$d\log \left(\sum_{j=1}^{N} p_j^{1-\sigma} \omega_{\mathcal{D},j}^{\sigma-1} \right) = (1-\sigma) \left(\sum_{i=1}^{N} \Upsilon_i \left(d\log p_i - d\log \omega_{\mathcal{D},i} \right) \right),$$

equation (17) can be rewritten as

$$d\log Y = d\log w - \sum_{i=1}^{N} \Upsilon_i \left(d\log p_i - d\log \omega_{\mathcal{D},i} \right). \quad (18)$$

We now turn to producers' optimization problem to derive an expression for $d\log p_i$. The first-order conditions for labor and intermediate inputs imply

$$l_i = \alpha_i (1 - \tau_{y,i}) p_i y_i w^{-1} (1 + \tau_{L,i})^{-1}, \quad (19)$$

and

$$x_{ij} = (1 - \alpha_i)^{\theta} p_i^{\theta} y_i^{\theta} \omega_{ij}^{\theta-1} p_j^{-\theta} (1 - \tau_{y,i})^{\theta} (1 + \tau_{x,ij})^{-\theta} M_i^{1-\theta}. \quad (20)$$

From (19) and (20), we derive expressions for the labor expenditure shares Λ_i and input-output parameters Ω_{ij},

$$\Lambda_i = \frac{(1 + \tau_{L,i}) w l_i}{(1 - \tau_{y,i}) p_i y_i} = \alpha_i, \quad (21)$$

and,

$$\Omega_{ij} = (1 - \alpha_i)^{\theta} p_i^{\theta-1} y_i^{\theta-1} \omega_{ij}^{\theta-1} p_j^{1-\theta} (1 - \tau_{y,i})^{\theta-1} (1 + \tau_{x,ij})^{1-\theta} M_i^{1-\theta}. \quad (22)$$

Substituting (19) and (20) into sector i's production function, we derive the following expression for the price of good i,

$$p_i = A_i^{-1}(1-\tau_{y,i})^{-1}(1+\tau_{L,i})^{\alpha_i} w^{\alpha_i} \alpha_i^{-\alpha_i}(1-\alpha_i)^{\alpha_i - 1} P_{M,i}^{1-\alpha_i}, \tag{23}$$

where $P_{M,i} \equiv \left(\sum_{j=1}^{N} \omega_{ij}^{\theta-1}\left[(1+\tau_{x,ij})p_j\right]^{1-\theta}\right)^{\frac{1}{1-\theta}}$. Total differentiation of equation (23), yields

$$d\log p_i = \Lambda_i d\log w - d\log A_i - (\alpha_i - 1) d\log P_{M,i},$$

where $d\log P_{M,i}$ is given by

$$d\log P_{M,i} = (1-\alpha_i)^{-1}\sum_{j=1}^{N}\Omega_{ij} d\log \omega_{ij} + (1-\alpha_i)^{-1}\sum_{j=1}^{N}\Omega_{ij} d\log p_j.$$

Therefore, we can write

$$d\log p_i = \Lambda_i d\log w - d\log A_i + \sum_{j=1}^{N}\Omega_{ij} d\log \omega_{ij} + \sum_{j=1}^{N}\Omega_{ij} d\log p_j$$

which we can re-arrange to get

$$d\log p_i = \sum_{k=1}^{N}\psi_{ik}\Lambda_k d\log w - \sum_{k=1}^{N}\psi_{ik} d\log A_k - \sum_{k=1}^{N}\sum_{j=1}^{N}\psi_{ik}\Omega_{kj} d\log \omega_{kj}.$$

From the identity, $\mathbf{\Omega}\mathbb{1} + \mathbf{\Lambda} = \mathbb{1}$, it follows that $\mathbf{\Psi}'\mathbf{\Lambda} = \mathbb{1}$. Therefore, the above equation can be rewritten as

$$d\log p_i = d\log w - \sum_{k=1}^{N}\psi_{ik} d\log A_k - \sum_{k=1}^{N}\sum_{j=1}^{N}\psi_{ik}\Omega_{kj} d\log \omega_{kj}. \tag{24}$$

Substituting equation (24) into (18) yields

$$d\log Y = d\log w - \sum_{i=1}^{N}\Upsilon_i\left(d\log w - \sum_{k=1}^{N}\psi_{ik} d\log A_k - \sum_{k=1}^{N}\sum_{j=1}^{N}\psi_{ik}\Omega_{kj} d\log \omega_{kj} - d\log \omega_{\mathcal{D},i}\right),$$

which simplifies to

$$d\log Y = \sum_{i=1}^{N}\sum_{k=1}^{N}\Upsilon_i\psi_{ik}d\log A_k + \sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{j=1}^{N}\Upsilon_i\psi_{ik}\Omega_{kj}d\log\omega_{kj} + \sum_{i=1}^{N}\Upsilon_i d\log\omega_{\mathcal{D},i}.$$

Using the fact that $\lambda_k = \sum_{i=1}^{N}\Upsilon_i\psi_{ik}$, we can re-write the above equation as

$$d\log Y = \sum_{k=1}^{N}\lambda_k d\log A_k + \sum_{k=1}^{N}\sum_{j=1}^{N}\lambda_k\Omega_{kj}d\log\omega_{kj} + \sum_{i=1}^{N}\Upsilon_i d\log\omega_{\mathcal{D},i}. \tag{25}$$

Noting that, to a first-order approximation,

$$\frac{\Delta Y}{Y} \approx \sum_{i=1}^{N}\frac{d\log Y}{d\log A_i}\frac{\Delta A_i}{A_i},$$

and

$$\frac{\Delta\tilde{Y}_i}{\tilde{Y}_i} \approx \sum_{i=1}^{N}\frac{d\log Y}{d\log A_i}\frac{\Delta A_i}{A_i} + \sum_{k=1}^{N}\frac{d\log Y}{d\log\omega_{ki}}\frac{\Delta\omega_{ki}}{\omega_{ki}} + \sum_{k\neq i}^{N}\frac{d\log Y}{d\log\omega_{ik}}\frac{\Delta\omega_{ik}}{\omega_{ik}} + \frac{d\log Y}{d\log\omega_{\omega_{\mathcal{D},i}}}\frac{\Delta\omega_{\mathcal{D},i}}{\omega_{\mathcal{D},i}},$$

from equation (25), it follows that $\mu_i = \frac{\Delta Y}{Y} - \frac{\Delta\tilde{Y}_i}{\tilde{Y}_i}$ is given by

$$\mu_i = \sum_{i=1}^{N}\lambda_i\frac{\Delta A_i}{A_i} - \left(\sum_{i=1}^{N}\lambda_i\frac{\Delta A_i}{A_i} + \sum_{k=1}^{N}\lambda_k\Omega_{ki}\frac{\Delta\omega_{ki}}{\omega_{ki}} + \sum_{k\neq i}^{N}\lambda_i\Omega_{ik}\frac{\Delta\omega_{ik}}{\omega_{ik}} + \Upsilon_i\frac{\Delta\omega_{\mathcal{D},i}}{\omega_{\mathcal{D},i}}\right).$$

Therefore, since shutting down linkages implies $\frac{\Delta\omega_{ki}}{\omega_{ki}} = -1$ for all k, $\frac{\Delta\omega_{ik}}{\omega_{ik}} = -1$ for all $k \neq i$, and $\frac{\Delta\omega_{\mathcal{D},i}}{\omega_{\mathcal{D},i}} = -1$, we get

$$\mu_i = \Upsilon_i + \boldsymbol{\lambda}'\boldsymbol{\Omega}_{(i)} + \lambda_i\tilde{\boldsymbol{\Omega}}^{(i)}\mathbb{1}.$$

■

Proof of Proposition 2. Denote by $\boldsymbol{\Omega}^F$ the *frictionless* input-output matrix, with the ij^{th} element given by $\Omega^F_{ij} = \frac{p_j x_{ij}}{p_i y_i}$. The frictionless input-output matrix relates to the tax-adjusted input-output matrix $\boldsymbol{\Omega}$ via the identity

$$\mathbf{\Omega} = \mathbf{\Gamma} \circ \mathbf{\Omega}^F,$$

where $\mathbf{\Gamma}$ is an $N \times N$ matrix with ij^{th} element given by

$$\Gamma_{ij} = \frac{1 + \tau_{x,ij}}{1 - \tau_{y,i}}.$$

Noting that $\Gamma_{ij} \geq 1$ for all i, j since $0 \leq \tau_{x,ij} < 1$ and $0 \leq \tau_{y,i} < 1$, it implies that

$$\Omega_{ij} \geq \Omega^F_{ij},$$

for all i, j. Furthermore, the ij^{th} element of the Leontief inverse $\mathbf{\Psi} = \left(I - \mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)^{-1}$ can be written as

$$\psi_{ij} = 1 + \left(\frac{1 + \tau_{x,ij}}{1 - \tau_{y,i}}\right) \Omega^F_{ij} + \sum_{k=1}^{N} \left(\frac{1 + \tau_{x,ik}}{1 - \tau_{y,i}}\right) \left(\frac{1 + \tau_{x,kj}}{1 - \tau_{y,k}}\right) \Omega^F_{ik} \Omega^F_{kj} + \ldots$$

Noting that

$$\psi_{ij} \geq 1 + \Omega^F_{ij} + \sum_{k=1}^{N} \Omega^F_{ik} \Omega^F_{kj} + \ldots,$$

where the right-hand side of the above inequality is the ij^{th} element of the frictionless Leontief inverse, defined as $\left(I - \mathbf{\Omega}^F\right)^{-1}_{ij}$.

Defining by μ^F_i sector i's removal centrality in the frictionless economy, it follows that

$$\zeta_i = \mu_i - \mu^F_i \geq 0,$$

for all i, where

$$\zeta_i = \underbrace{\mathbf{\Upsilon}'\left(\left(I - \mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)^{-1}\left(\mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)_{(i)} - \left(I - \mathbf{\Omega}^F\right)^{-1}\mathbf{\Omega}^F_{(i)}\right)}_{\text{Impact of taxes on } i\text{'s indirect effect}}$$
$$+ \underbrace{\sum_{k=1}^{N} \Upsilon_k \left(\left(I - \mathbf{\Gamma} \circ \mathbf{\Omega}^F\right)^{-1}_{ki}\left(\mathbf{\Gamma} \circ \tilde{\mathbf{\Omega}}^F\right)^{(i)} \mathbb{1} - \left(I - \mathbf{\Omega}^F\right)^{-1}_{ki}\left(\tilde{\mathbf{\Omega}}^F\right)^{(i)} \mathbb{1}\right)}_{\text{Impact of taxes on } i\text{'s supplier effect}},$$

and where $(\mathbf{X})_{(i)}$ and $(\mathbf{X})^{(i)}$ denotes the i^{th} column and row of a matrix $\mathbf{X}$, respectively.

■

Online Appendix B. Data

BEA Input-Output Data

We use the detailed benchmark input-output accounts provided by the Bureau of Economic Analysis (BEA) from 1982 to 2012. These accounts, which are compiled every five years, offer comprehensive information on inter-industry relationships and the flow of goods and services in the economy. For our analysis, we use the commodity-by-industry Use table, assuming that each industry produces only one commodity (as in Baqaee and Farhi, 2020). We exclude the government, scrap, non-comparable imports, and used and secondhand goods sectors from our analysis. The number of industries included in the tables varies each year, ranging from 534 sectors in 1982 to 393 sectors in 2012.

To compute each entry of the US input-output table for each year, we divide the expenditure of industry i on commodity j by i's gross sales (net of sector-level taxes). We also calculate the final expenditure share of each sector i by summing all components of final demand in the detailed Use tables, excluding changes in private inventories, and dividing by nominal GDP. If any final demand share is negative, we set it equal to zero. Similarly, if any value in the equilibrium input-output matrix is negative, we set it to zero as well.

WIOD Input-Output Data

We use the 2016 release of the World Input-Output Database (WIOD) (see Timmer et al., 2015 for an overview of the WIOD data) for our cross-country analysis. The dataset contains information on gross output, value-added, factor compensation, tax payments, final expenditures, and intermediate input flows for 43 countries from 2000 to 2014. The WIOD data is disaggregated into 56 sectors based on the International Standard Industrial Classification Revision 4 (ISIC Rev. 4). The block-diagonal of each input-output table captures domestic intermediate input transactions for each country. In contrast, the off-diagonal relates to the flow of intermediates between countries. For our purposes, we focus solely on domestic transactions and abstract from international trade. We compute each entry of country c's input-output matrix $\mathbf{\Omega}_{ct} \equiv \left[\Omega_{ijct}\right]$ at time t by dividing sector i's nominal expenditure on sector j's product by sector i's total domestic nominal sales (net of taxes). Notably, we exclude each sector's spending on imported inputs, which ensures that factor compensation plus domestic intermediate input expenditure equals nominal gross output for each sector. We calculate tax rates τ_{ict} at the country-sector-year level as

$$\tau_{ict} = \frac{T_{ict}}{\sum_{k=1}^{N} p_{kct} y_{kct}},$$

where T_{ict} is the nominal value of taxes (less subsidies) paid by sector i in country c at year t, and $\sum_{k=1}^{N} p_{kct} y_{kct}$ is aggregate nominal gross output for country c. Therefore, a typical entry of country c's input-output matrix is computed as[36]

$$\Omega_{ijct} = \frac{p_{jct} x_{ijct}}{(1-\tau_{ict}) p_{ict} y_{ict}}.$$

Finally, we calculate industries' final expenditure shares Υ_{ict} as the sum of household and government final consumption expenditure plus gross fixed capital formation, all as a fraction of nominal GDP.

[36]Given that we do not have information on input-specific tax wedges $\tau_{x,ij}$, we do not include these when computing the input-output matrices.

Online Appendix C. The Effect of Taxes on Sectors' Importance

In the Proof of Proposition 2, we derived the following expression for the contribution of taxes to sector i's removal centrality, ζ_i:

$$\zeta_i = \underbrace{\boldsymbol{\Upsilon}' \left(\left(I - \boldsymbol{\Gamma} \circ \boldsymbol{\Omega}^F \right)^{-1} \left(\boldsymbol{\Gamma} \circ \boldsymbol{\Omega}^F \right)_{(i)} - \left(I - \boldsymbol{\Omega}^F \right)^{-1} \boldsymbol{\Omega}^F_{(i)} \right)}_{\text{Impact of taxes on } i\text{'s indirect effect}}$$
$$+ \underbrace{\sum_{k=1}^{N} \Upsilon_k \left(\left(I - \boldsymbol{\Gamma} \circ \boldsymbol{\Omega}^F \right)^{-1}_{ki} \left(\boldsymbol{\Gamma} \circ \tilde{\boldsymbol{\Omega}}^F \right)^{(i)} \mathbb{1} - \left(I - \boldsymbol{\Omega}^F \right)^{-1}_{ki} \left(\tilde{\boldsymbol{\Omega}}^F \right)^{(i)} \mathbb{1} \right)}_{\text{Impact of taxes on } i\text{'s supplier effect}}, \quad (26)$$

where $(\mathbf{X})_{(i)}$ and $(\mathbf{X})^{(i)}$ denotes the i^{th} column and row of a matrix $\mathbf{X}$, respectively.

In equation (26), the first term on the right-hand side quantifies the impact of sector-level sales and intermediates taxes on the indirect effect of sector i, while the second term measures the significance of these taxes for the supplier effect of sector i. Thus, in the absence of these frictions, the supplier effect of i is given by

$$\sum_{k=1}^{N} \Upsilon_k \left(I - \boldsymbol{\Omega}^F \right)^{-1}_{ki} \left(\tilde{\boldsymbol{\Omega}}^F \right)^{(i)} \mathbb{1} \quad (27)$$

and the importance of taxes for supplier effects is given by

$$\sum_{k=1}^{N} \Upsilon_k \left(\left(I - \boldsymbol{\Gamma} \circ \boldsymbol{\Omega}^F \right)^{-1}_{ki} \left(\boldsymbol{\Gamma} \circ \tilde{\boldsymbol{\Omega}}^F \right)^{(i)} \mathbb{1} - \left(I - \boldsymbol{\Omega}^F \right)^{-1}_{ki} \left(\tilde{\boldsymbol{\Omega}}^F \right)^{(i)} \mathbb{1} \right). \quad (28)$$

Figure 10 compares the impact of taxes on industries' supplier effects (equation 28), represented by yellow bars, for the top 10 sectors ranked by supplier effects. The analysis covers the years from 1982 to 2012, using the detailed BEA input-output accounts. In the figure, the orange bars represent the frictionless supplier effects of each industry, calculated using equation (27). The combined sum of the orange and yellow bars for each sector represents the total supplier effect, which includes the impact of taxes, for the sector listed on the horizontal axis.

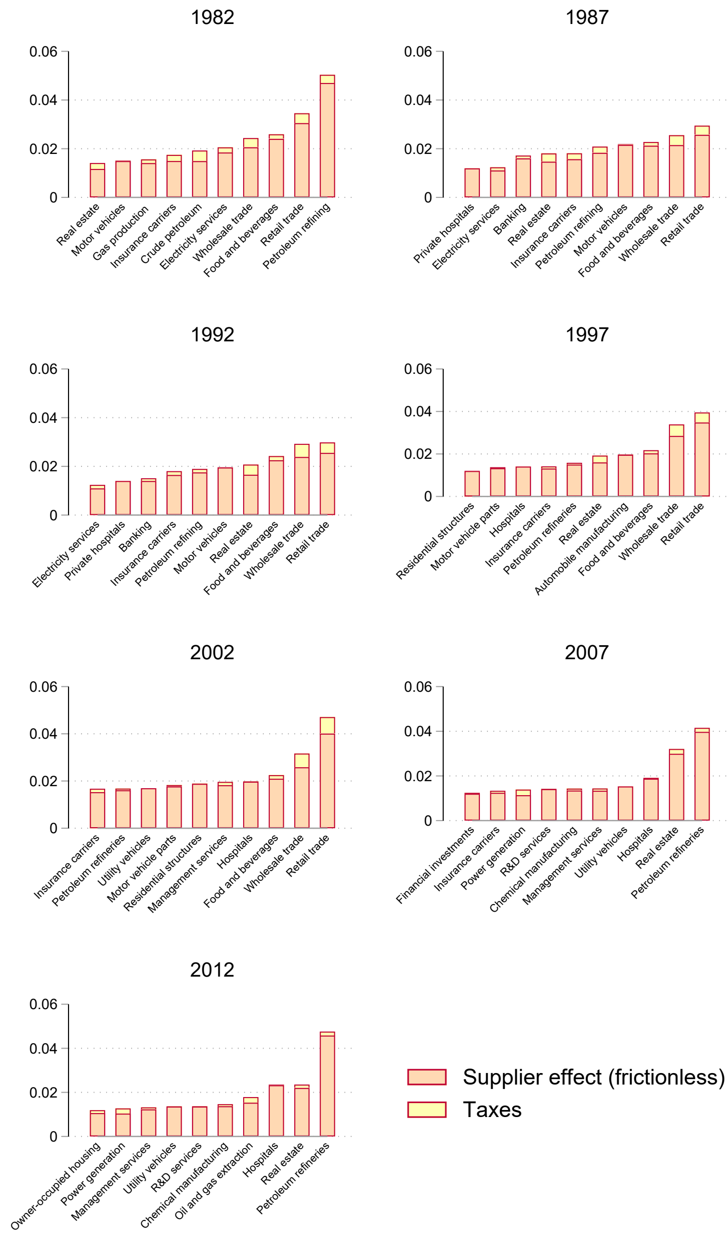

Figure 10: Contribution of Taxes to Sectors' Supplier Effects

Note: This figure presents estimates of the contribution of taxes to sectors' supplier effects from 1982 to 2012 using the detailed input-output accounts of the BEA. The top 10 sectors, ranked by supplier effects, are displayed for each year. The orange bars represent each sector's supplier effect in the absence of taxes, calculated using equation (27). The yellow bars capture the contribution of taxes to each sector's supplier effect, computed as in equation (28).

The figure shows that taxes only account for a small portion of the overall supplier effects of industries across all years. However, a few notable exceptions stand out. In 2002, taxes on retail and wholesale trade accounted for approximately 15% and 18%, respectively, of these industries' total supplier effects. Additionally, in 1982, taxes accounted for 23% of the supplier effect of crude petroleum production. Nonetheless, on average, taxes only account for around 4% of supplier effects across all industries and years in the BEA IO tables.

Online Appendix D. Second-Order Removal Centrality

We compute sectors' second-order removal centrality by characterizing the derivatives of Domar weights and input-output parameters. We begin by noting that changes in real GDP in the *actual* economy, to a second-order approximation, are given by

$$\frac{\Delta Y}{Y} \approx \sum_{i=1}^{N} \lambda_i \left(\frac{\Delta A_i}{A_i}\right) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{d\lambda_i}{d \log A_j} \left(\frac{\Delta A_i}{A_i}\right) \left(\frac{\Delta A_j}{A_j}\right), \tag{29}$$

where the first set of summands capture the first-order change in real GDP in response to the technology shocks $(\frac{\Delta A_1}{A_1}, \ldots, \frac{\Delta A_N}{A_N})$ and the second set of summands capture the second-order change in GDP. Next, note that the second-order change in real GDP in the *counterfactual* economy is given by

$$\frac{\Delta \tilde{Y}_i}{\tilde{Y}_i} \approx \underbrace{\sum_{i=1}^{N} \lambda_i \left(\frac{\Delta A_i}{A_i}\right)}_{\text{First-order technology effect}} + \underbrace{\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{d\lambda_i}{d\log A_j}\left(\frac{\Delta A_i}{A_i}\right)\left(\frac{\Delta A_j}{A_j}\right)}_{\text{Second-order technology effect}}$$

$$+ \underbrace{\Upsilon_i \left(\frac{\Delta \omega_{\mathcal{D},i}}{\omega_{\mathcal{D},i}}\right)}_{\text{First-order direct effect}} + \underbrace{\sum_{k=1}^{N} \lambda_k \Omega_{ki} \left(\frac{\Delta \omega_{ki}}{\omega_{ki}}\right)}_{\text{First-order indirect effect}} + \underbrace{\sum_{k\neq i}^{N} \lambda_i \Omega_{ik} \left(\frac{\Delta \omega_{ik}}{\omega_{ik}}\right)}_{\text{First-order supplier effect}}$$

$$+ \underbrace{\frac{1}{2}\sum_{k=1}^{N}\sum_{m=1}^{N} \frac{d(\lambda_k \Omega_{ki})}{d\log \omega_{mi}}\left(\frac{\Delta \omega_{ki}}{\omega_{ki}}\right)\left(\frac{\Delta \omega_{mi}}{\omega_{mi}}\right)}_{\text{Second-order indirect effect}}$$

$$+ \underbrace{\frac{1}{2}\frac{d\Upsilon_i}{d\log \omega_{\mathcal{D},i}}\left(\frac{\Delta \omega_{\mathcal{D},i}}{\omega_{\mathcal{D},i}}\right)^2}_{\text{Second-order direct effect}} + \underbrace{\frac{1}{2}\sum_{k\neq i}^{N}\sum_{m\neq i}^{N} \frac{d(\lambda_i \Omega_{ik})}{d\log \omega_{im}}\left(\frac{\Delta \omega_{ik}}{\omega_{ik}}\right)\left(\frac{\Delta \omega_{im}}{\omega_{im}}\right)}_{\text{Second-order supplier effect}}. \quad (30)$$

Noting that second-order removal centrality is defined as

$$\mu_i^{\text{2nd-order}} = \frac{\Delta Y}{Y} - \frac{\Delta \tilde{Y}_i}{\tilde{Y}_i}$$

we get

$$\mu_i^{\text{2nd-order}} = \mu_i^{\text{First-order}} - \underbrace{\frac{1}{2}\frac{d\Upsilon_i}{d\log \omega_{\mathcal{D},i}}\left(\frac{\Delta \omega_{\mathcal{D},i}}{\omega_{\mathcal{D},i}}\right)^2}_{\text{Second-order direct effect}}$$

$$- \underbrace{\frac{1}{2}\sum_{k=1}^{N}\sum_{m=1}^{N} \frac{d(\lambda_k \Omega_{ki})}{d\log \omega_{mi}}\left(\frac{\Delta \omega_{ki}}{\omega_{ki}}\right)\left(\frac{\Delta \omega_{mi}}{\omega_{mi}}\right)}_{\text{Second-order indirect effect}} - \underbrace{\frac{1}{2}\sum_{k\neq i}^{N}\sum_{m\neq i}^{N} \frac{d(\lambda_i \Omega_{ik})}{d\log \omega_{im}}\left(\frac{\Delta \omega_{ik}}{\omega_{ik}}\right)\left(\frac{\Delta \omega_{im}}{\omega_{im}}\right)}_{\text{Second-order supplier effect}}. \quad (31)$$

Therefore, to characterize equation (31), we must compute $\frac{d\Upsilon_i}{d\omega_{\mathcal{D},i}}$, $\frac{d(\lambda_k \Omega_{ki})}{d\log \omega_{mi}}$ and $\frac{d(\lambda_i \Omega_{ik})}{d\log \omega_{im}}$. We begin by noting that

$$\frac{d\Upsilon_i}{d\log \omega_{\mathcal{D},i}} = \frac{d\log \Upsilon_i}{d\log \omega_{\mathcal{D},i}}\Upsilon_i \quad (32)$$

where

$$\frac{d\log \Upsilon_i}{d\log \omega_{\mathcal{D},i}} = (\sigma - 1)(1 - \Upsilon_i). \tag{33}$$

Next, to characterize the second-order indirect effect, we need to compute

$$\frac{d(\lambda_k \Omega_{ki})}{d\log \omega_{mi}} = \frac{d\log \lambda_k}{d\log \omega_{mi}} \lambda_k \Omega_{ki} + \frac{d\log \Omega_{ki}}{d\log \omega_{mi}} \lambda_k \Omega_{ki}. \tag{34}$$

Firstly, the change in the input-output parameter Ω_{ki} in response to the change in ω_{mi} is given by

$$\frac{d\log \Omega_{ki}}{d\log \omega_{mi}} = (\theta - 1)\left(\psi_{im}\Omega_{mi} - \frac{1}{1-\Lambda_k} \sum_{j=1}^{N} \Omega_{kj} \psi_{jm} \Omega_{mi} + \frac{d\log \omega_{ki}}{d\log \omega_{mi}} \right). \tag{35}$$

Secondly, the change in sector k's Domar weight is given by

$$\frac{d\log \lambda_k}{d\log \omega_{mi}} = \frac{1}{\lambda_k} \sum_{l=1}^{N} \Upsilon_l \psi_{lk} \frac{d\log \Upsilon_l}{d\log \omega_{mi}} + \frac{1}{\lambda_k} \sum_{l=1}^{N} \sum_{s=1}^{N} \Omega_{ls} \lambda_l \psi_{sk} \frac{d\log \Omega_{ls}}{d\log \omega_{mi}}, \tag{36}$$

where,

$$\frac{d\log \Upsilon_l}{d\log \omega_{mi}} = (\sigma - 1)\left(\psi_{lm}\Omega_{mi} - \lambda_m \Omega_{mi}\right). \tag{37}$$

and

$$\frac{d\log \Omega_{ls}}{d\log \omega_{mi}} = (\theta - 1)\left(\psi_{sm}\Omega_{mi} - \frac{1}{1-\Lambda_l} \sum_{j=1}^{N} \Omega_{lj} \psi_{jm} \Omega_{mi} + \frac{d\log \omega_{ls}}{d\log \omega_{mi}} \right). \tag{38}$$

Together, equations (35), (36), (37) and (38) are sufficient to characterize the second-order indirect effect in equation (31).

Our last step is to characterize the second-order supplier effect. To this end, we must compute

$$\frac{d(\lambda_i \omega_{ik})}{d\log \omega_{im}} = \frac{d\lambda_i}{d\log \omega_{im}} \Omega_{ik} + \frac{d\Omega_{ik}}{d\log \omega_{im}} \lambda_i.$$

The change in the input-output parameter Ω_{ik} in response to a change in ω_{im} is given by

$$\frac{d\Omega_{ik}}{d\log\omega_{im}} = (\theta-1)\Omega_{ik}\left(\psi_{ki}\Omega_{im} - \frac{1}{1-\Lambda_i}\sum_{j=1}^{N}\Omega_{ij}\psi_{ji}\Omega_{im} + \frac{d\log\omega_{ik}}{d\log\omega_{im}}\right), \tag{39}$$

Next, we compute $\frac{d\lambda_i}{d\log\omega_{im}}$

$$\frac{d\lambda_i}{d\log\omega_{im}} = \sum_{l=1}^{N}\psi_{li}\frac{d\Upsilon_l}{d\log\omega_{mi}} + \sum_{l=1}^{N}\sum_{s=1}^{N}\lambda_l\psi_{si}\frac{d\Omega_{ls}}{d\log\omega_{im}}, \tag{40}$$

where

$$\frac{d\Upsilon_l}{d\log\omega_{im}} = (\sigma-1)\Upsilon_l\left(\psi_{li}\Omega_{im} - \lambda_i\Omega_{im}\right). \tag{41}$$

and

$$\frac{d\Omega_{ls}}{d\log\omega_{im}} = (\theta-1)\Omega_{ls}\left(\psi_{si}\Omega_{mi} - \frac{1}{1-\Lambda_l}\sum_{j=1}^{N}\Omega_{lj}\psi_{ji}\Omega_{im} + \frac{d\log\omega_{ls}}{d\log\omega_{im}}\right). \tag{42}$$

Together equations (32)–(42) characterize second-order removal centrality.