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Wealth and income inequality in the long run

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Keywords

Income inequality, wealth inequality, top shares, saving rates, cointegration, error correction

JEL Classification

D31, E21, E25, N32, N34

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This paper analyses the joint long-run evolution of wealth and income inequality. We show that top wealth and income shares were cointegrated over the past century in France and the US. We rationalise this finding using a two-agent version of the Solow growth model. In this framework, the co-movement of top wealth and income shares is determined by the relative saving rate at the top, i.e. the ratio of the saving rate of rich individuals to the aggregate saving rate. The cointegration finding suggests that relative saving rates at the top are fairly stable over time, thus explaining the tight co-movement between top wealth and income shares over the past century.

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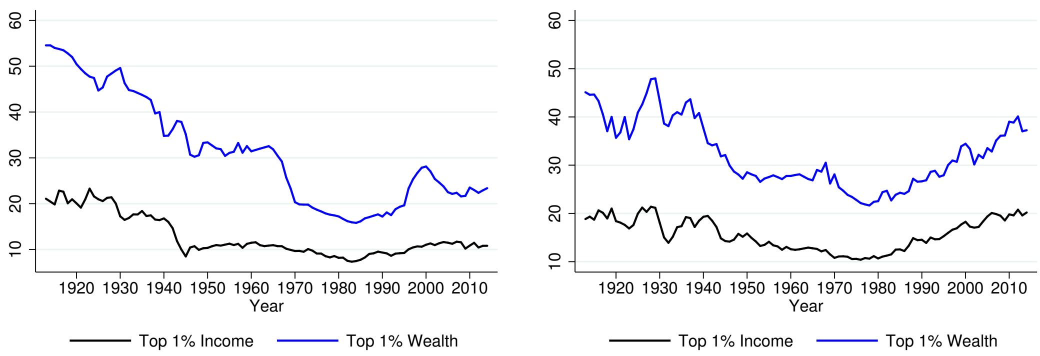
1 Introduction

Inequality has risen dramatically in many countries in recent decades, sparking considerable research interest (Alvaredo et al., 2013; Piketty, 2014). A particularly prominent example is the US, where the top 1% income share almost doubled since the 1980s, to reach around 19% in the 2010s. A similar development is observable for wealth inequality: the top 1% wealth share increased from 23% to 36% in the same time span. Such a co-movement in wealth and income inequality is not limited to recent decades and historically far from uncommon. To illustrate this argument, Figure 1 shows a remarkably uniform evolution of top 1% wealth and income shares over around 100 years in the US and in France. Despite severe data issues, this observation equally applies to various other countries around the world (Alvaredo et al., 2017).

Figure 1: Historical evolution of inequality

(a) France **(b) US**



Note: Top 1% wealth and income shares. Data from the World Inequality Database.

The remarkable co-movement of top income and wealth shares across time raises intriguing questions: Why do wealth and income inequality co-move so strongly over the long-run? Is the co-movement spurious, or is it the result of some underlying economic mechanism? And if so, what is that mechanism? This paper provides answers to these question by analyzing the joint long-run evolution of wealth and income inequality both theoretically and empirically.

To shed light on the determinants of the dynamic relationship between wealth and income shares, we first develop a two-agent Solow model. In this framework, one agent is representative of the "rich" and the other of the rest of the population. Each agent has its own (different) saving rate. A key outcome of the model is the relative saving rate (RSR in the following) of the top, defined as the ratio of the saving rate of the rich agent to the aggregate saving rate. We derive two theoretical results. First, in the short run, the change in the top wealth share depends on the RSR and the distance of the top wealth share from the top income share. In essence, an error correction mechanism exists between the wealth and income inequality: if the top wealth share is "too high" relative to the top income share, it drops (and vice versa when it is too low). Second, in the long run, the ratio of the top wealth to the income share is equal to the RSR. Put differently, the model implies a steady state in which the top wealth share is equal to the top income share multiplied by the RSR. This notion of the RSR translating "flow inequality" to "stock inequality" is quite general and not limited to this specific framework.

Using these theoretical insights, we assess that strong co-movements of top wealth and income shares require stability of the RSR. In other words, if the saving rate of the rich (relative to the aggregate saving rate) is stable, top income and wealth shares are expected to follow each other closely and feature a stable long-run relationship. From an empirical perspective, this implies that estimates of the "long-run" RSR are obtainable by regressing the wealth share on the income share. This procedure constitutes a transparent, simple and new method to infer long-run (relative) saving rates. The opposite case in which the RSR is strongly time-varying and unstable generates distinct dynamics of wealth and income shares, such that no stable long-run relationship exists. Appropriate econometric techniques can detect such instability and provide insights about potential structural breaks in saving rates.

Our empirical contribution is an analysis of the long-run relationship between wealth and income inequality based on a century of data for France and the US. For our analysis, we use wealth and income shares of the top 1% and the top 10% for the period 1913-2014. Various empirical tests yield strong evidence for a long-run relationship between top 1% income and wealth shares in both the US and France, in particular for cointegration. For the top 10%, the evidence of long-run stability is much weaker. Using appropriate cointegration techniques, we estimate that the (very) long-run saving rate of the top 1% is twice as high as the aggregate saving rate in the US (RSR = 2.0), and somewhat higher in France (RSR = 2.2).

Investigating the stability of the long-run relationship over time, we find evidence of structural breaks in France around WWII and in 1968 close to the end of the so-called Trentes Glorieuses – thirty years of high growth after WWII in France, extensively discussed in Piketty (2014). For the US, we detect a structural break around 1983, contemporaneous with significant shifts in tax policy induced by amid the "Reagonomics" era. For both countries, sub-period estimations imply a decline in the RSR around the structural breaks (2.9 to 2.1 in France, 2.1 to 1.9 in the US). From a long-term perspective, the existence of infrequent structural breaks implies that RSRs are fairly stable over decades. As a result, wealth and income inequality necessarily track each other closely over long periods of time. The novel finding that RSRs are relatively stable over long periods forms an explanation for the remarkable co-movement of wealth and income shares observed empirically.

As a final contribution, we perform counterfactual simulations of the evolution of the top 1% wealth shares. Using our error correction estimates to conduct out-of-sample forecasts starting at the structural break dates, we find that wealth concentration would be higher today absent the structural decline in RSRs. The drop in RSRs in both France and the US in the second half of the 20th century thus implies that the top 1% wealth and income shares are closer together today than they were in the first half of the 20th century.

Our paper is related to a number of strands in the inequality literature. Much of this literature investigate the determinants of (steady state) income and wealth distributions separately. For example, Quadrini and Ríos-Rull (2015), Hubmer et al. (2020), De Nardi and Fella (2017) and Benhabib and Bisin (2018) analyze the mechanisms and assumptions under which canonical macroeconomic models such as the Bewley-Huggett-Aiyagari framework generate wealth distributions in line with empirical observations, in particular fat tails at the top. In contrast to this literature, our paper is interested in understanding the co-movement of wealth and income inequality over long periods of time. Closely related to our paper, Saez and Zucman (2016) and Garbinti et al. (2020) document long-term dynamics of wealth and income inequality in the US and in France, respectively. Both papers furthermore derive a dynamic relationship between top wealth share, top income share and saving rates using national accounting. In contrast, we show that a long-run relationship between wealth and income shares arises in a standard Solow-type framework, investigate its long-run stability empirically and provide estimates of long-run RSRs.

The rest of the paper is organized as follows. In Section 2, we present the theoretical framework. In Section, 3 we discuss its implications regarding estimations of the RSR. Section 4 discusses the data used in our application. In Section 5, we test for the existence of a long-run relationship between wealth and income shares. Section 6 presents the estimated relative saving rates, analyzes their evolution and the implications for the historical evolution of top wealth shares. Section 7 concludes.

2 A neoclassical growth model with two agents

Why do wealth and income shares co-move? To answer this question, we derive the short-run and long-run dynamic relationship between wealth and income shares based on a macroeconomic growth model. We show that the relative saving rate (RSR) is the key variable explaining the co-movement of wealth and income share in the short and in the long run. In particular, the model implies that wealth and income shares are linked by an error correction relationship, governed by the RSR.

The framework is a neoclassical growth model building on Solow (1956). We introduce one important modification: instead of one representative agent, we consider two agents which each represent a homogeneous group of people (e.g. the top 1% wealthiest individuals and the bottom 99 %). From now on, we hence use "agent" and "group" interchangeably. Each group has its own saving rate, owns a fraction of the aggregate capital stock and earns a fraction of total income.

In the following, we focus on the equations and definitions needed to derive our main result. A detailed exposition of the model and associated derivations can be found in the Appendix. For each group (indexed by $i = 1, 2$), physical capital K_t^i accumulates according to

$$K_{t+1}^i = s^i Y_t^i + (1 - \delta) K_t^i \qquad i = 1, 2 \tag{1}$$

where s^i and Y_t^i denote the saving rate and income of group i, while δ is the depreciation rate of capital. The aggregate saving rate s_t is defined as the weighted average of the two saving rates

$$s_t \equiv s^1 \frac{Y_t^1}{Y_t} + s^2 \frac{Y_t^2}{Y_t} \tag{2}$$

where Y_t is aggregate output, the sum of income across the two groups. Likewise, aggregate capital K_t is the sum of capital owned by the two groups. As capital is the only form of wealth in this model, wealth and income shares are defined by:

$$sh^i_{W,t} \equiv \frac{K^i_t}{K_t} \qquad i = 1,2 \tag{3}$$

$$sh^i_{Y,t} \equiv \frac{Y^i_t}{Y_t} \qquad i = 1,2 \tag{4}$$

The law of motion for the wealth share of group i can be obtained by combining the individual capital accumulation from Equation (1) with the definition of the aggregate capital stock and aggregate output. This yields (as shown in the Appendix)

$$\Delta sh^i_{W,t+1} = -\frac{S_t}{K_{t+1}}(sh^i_{W,t} - \widetilde{s}^i_t \, sh^i_{Y,t}) \tag{5}$$

where $\widetilde{s}^i_t = \frac{s^i}{s_t}$ is the *relative* saving rate of group i (at time t) and $S_t \equiv s_t Y_t$ denotes aggregate saving. Equation (5) relates the change in the wealth share of group i to current wealth and income shares and the *relative* saving rate. This dynamic short-run relationship describes the evolution of wealth inequality along the transition path to the long-run steady state (characterized in more detail in the Appendix) and constitutes an error correction mechanism: if, away from the steady state, the wealth share is "too high" relative to the income share (i.e. $sh^i_{W,t} > \widetilde{s}^i_t \, sh^i_{Y,t}$), it will drop (and vice versa when it is too low).

Turning to the long-run, it can be shown that the model features a steady state (see the Appendix). From an aggregate perspective, this steady state is very similar to the canonical representative agent version. Importantly for our analysis, however, the steady state of the two-agent version implies that wealth and income shares are constant. Intuitively, the existence of a steady state implies that the RSR is stable over the long-run, thereby guaranteeing that wealth and income concentration converge to a long-run equilibrium. Using this information in Equation (5) (by setting the left-hand side to zero and dropping time subscripts to denote steady state values) yields:

$$sh^i_W = \widetilde{s}^i \, sh^i_Y \tag{6}$$

Equation (6) displays the long-run relationship between wealth and income shares of a given group of the population, governed by the RSR of that particular group. To fix intuition, suppose that both groups had the same saving rate (i.e. $s^1 = s^2$) such that the RSR in this economy is equal to one. Equation (6) then implies that wealth shares equal income shares. However, if one group (for example the wealthy) is characterized by a higher (lower) saving rate, its RSR is above (below) unity and the corresponding wealth share is higher (lower) than the income share in the long-run. Put differently, in the long-run, the steady state RSR is the factor which translates "flow inequality" into "stock inequality".[1] We hence conclude that a two-agent Solow growth model implies a long-run relationship between wealth and income shares, linked by the long-run RSR.

[1]In the model that we consider here, the return on capital is deterministic and homogeneous across groups. Introducing stochastic and potentially heterogeneous returns would allow to differentiate between *net* saving (excluding capital gains) and *gross* saving (including capital gains) following Fagereng et al. (2019). However, the basic notion conveyed in Equation (6) would carry over subject to an appropriate redefinition of the RSR.

3 Estimating long-run relative saving rates

The theoretical results obtained in the two-agent Solow growth model suggest that three important questions concerning the dynamics of inequality can be answered by employing appropriate empirical analyses using historical data on top wealth and income shares. First, is the RSR at the top of the wealth distribution *stable* over time empirically? Stability of the RSR at the top would imply that the long-run relationship between top wealth and income shares is stable, such that wealth and income inequality display substantial co-movement across time. Second, *over which period* is the RSR at the top stable, if at all? This provides insights about the historical evolution of inequality. Third, provided some stability, *how large* is the long-run RSR at the top? The economic magnitude of the RSR at the top is informative about the long-run factor with which top income shares have to be multiplied to obtain the wealth share.

All of these questions can be answered by *estimating* the long-run RSR and investigating its stability over time using appropriate econometric techniques. Following the theoretical framework, estimates of the long-run RSR rate of a given fractile f of the population (i.e. the top 1%) can be obtained by two econometric models for the relationship between wealth and income shares. The first is an error correction model of the form

$$\Delta sh^f_{W,t} = -\alpha_W \left(sh^f_{W,t-1} - \widetilde{s}^f sh^f_{Y,t-1} \right) + \varepsilon_t \tag{7}$$

which follows from Equation (5). In Equation (7), the long-run relationship between wealth share and income share is the term in brackets, governed by the long-run RSR. The coefficient α_W captures the speed of short-run adjustments of the wealth share in response to imbalances in the long-run relationship. The error term ε_t contains short-term fluctuations that occur empirically but are not captured by the Solow model. The second econometric model is a level relationship based on the long-run steady state relationship obtained in the Solow model (Equation 6):

$$sh^f_{W,t} = \widetilde{s}^f \, sh^f_{Y,t} + \varepsilon_t \tag{8}$$

While these econometric models are suggested by the Solow model, it is important to note that the theory is (inherently) silent about associated econometric techniques to estimate the long-run RSR. The appropriate empirical methodology depends fundamentally on the order of integration, which needs to be determined *based on the data.* In particular, there are two different relevant cases. If both wealth and income shares are I(0) in the data and feature a long-run relationship, standard estimation techniques such as OLS are appropriate. In contrast, if both shares are I(1), spurious regressions need to be ruled out by testing for cointegration as a particular form of a long-run relationship and cointegration estimators should be employed. In both cases, a positive indication of a stable long-run relationship is a necessary prerequisite to estimate the long-run RSR. The stability can be investigated by subsequent structural break tests.

In terms of estimators, the error correction model in Equation (7) may be operationalized by an Autoregressive Distributed Lag (ARDL) model in conditional error correction form:

$$\Delta sh^f_{W,t} = -\alpha_W \left(sh^f_{W,t-1} - \widetilde{s}^f sh^f_{Y,t-1} \right) + \sum_{i=1}^{p-1} \gamma_{W,i} \Delta sh_{W,t-1} + \sum_{i=1}^{q-1} \gamma_{Y,i} \Delta sh_{Y,t-1} + \varepsilon_t \tag{9}$$

The rationale of the ARDL model is to include further lags to soak up short-run fluctuations. A further advantage of the ARDL model is that it serves as a basis for the bounds testing methodology proposed by Pesaran et al. (2001). This test for a long-run relationship and the associated estimation can be used irrespective of the order of integration.

Regarding the level relationship, the simple OLS estimator of the RSR in Equation (8) is appropriate (and super-consistent) only if a) wealth and income shares are I(1), b) they are cointegrated, and c) income shares are uncorrelated with the error term. While a) and b) might hold, depending on the data, c) is likely violated empirically. Hence, OLS estimates likely feature non-Gaussian asymptotically biased and asymmetric distributions in our context, invalidating standard inference. If a) and b) hold, this suggests using a set of three alternative estimators which have been specifically designed to overcome this issue: the canonical cointegration regression (CCR) approach proposed by Park (1992), the Fully-Modified OLS (FMOLS) by Phillips and Hansen (1990) and Dynamic OLS (DOLS) as described by Saikkonen (1992) and Stock and Watson (1993).[2] All of these estimators feature asymptotically unbiased and fully efficient normal asymptotics, allowing for standard Wald tests using asymptotic χ-squared statistical inference.

Finally, the theoretically-derived error correction model may also be incorporated in a broader Vector Autoregressive Model (VECM), which captures the dynamics of the income share as well:

$$\begin{pmatrix} \Delta sh^f_{W,t} \\ \Delta sh^f_{Y,t} \end{pmatrix} = \begin{pmatrix} -\alpha_W \\ -\alpha_Y \end{pmatrix} \begin{pmatrix} 1 & -\widetilde{s}^f \end{pmatrix} \begin{pmatrix} sh^f_{W,t-1} \\ sh^f_{Y,t-1} \end{pmatrix} + \sum_{i=1}^{p-1} \begin{pmatrix} \gamma_{WW,i} & \gamma_{WY,i} \\ \gamma_{YW,i} & \gamma_{YY,i} \end{pmatrix} \begin{pmatrix} \Delta sh^f_{W,t-i} \\ \Delta sh^f_{Y,t-i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{W,t} \\ \varepsilon_{Y,t}. \end{pmatrix} \tag{10}$$

While the second equation has no specific theoretical foundation, it allows to account for the contemporaneous feedback of wealth concentration on income concentration.

4 Data

To showcase our method to estimate long-run RSRs, we provide evidence on the co-movement of wealth and income shares for France and the US. We use data from the World Inequality Database (WID). We focus on France and the US because of excellent data coverage; annual data on wealth and income shares for both of these countries is available almost extensively for the 20th century. To generate long time series without gaps, we use linear interpolation to deal with a few missing observations within the samples.[3] The resulting sample covers 1913-2014 for both countries. We use wealth shares of the top 1% and the top 10% based on net personal

[2]CCR and FMOLS rely on semi-parametric corrections to eliminate the correlation between the income share and ε_t. The DOLS approach involves adding additional leads and lags of sh_{Y_t} to soak up the long-run correlation between innovations to income shares and ε_t.

[3]For France, income shares are interpolated for 1913/1914 (based on earlier data) and wealth shares in 1928/1934/1961/1963. For the US, income shares are interpolated for 1963 and 1965.

wealth, defined as the sum of non-financial and financial assets owned by an individual minus its personal debt.[4] We use the shares of pre-tax national income accruing to the top 1% and top 10%.[5]

The data for the top 1% is equivalent to the time series displayed in Figure 1. As argued in the introduction, the most striking observation emerging from this graph is the remarkably close long-run co-movement of top wealth and income shares. This applies to both the top 1% and the top 10% (see Figure A1). Raw visual inspection of the data is thus in line with our theoretical model implying a long-run relationship between top shares. However, as the co-movement is naturally not perfectly uniform, empirical statements about the evolution of inequality and the RSR at the top require a more detailed econometric analysis.

As discussed in Section 3, the appropriate econometric methodology to estimate RSRs depends crucially on the data used for this purpose, in particular their order of integration. A set of unit root tests suggests that the WID data is I(1), as shown in Table A1: the standard (augmented) test by Dickey and Fuller (1979) as well as the tests by Phillips and Perron (1988) and Elliott et al. (1996) fail to reject the null hypothesis of a unit root in both wealth and income shares for the top 1 and top 10% in both France and the US. We also test for unit roots in the presence of possible structural breaks in the underlying time series using the methodology proposed by Perron and Vogelsang (1992a, 1992b, 1998). This procedure also fails to reject unit roots in all cases. Finally, the test proposed by Kwiatkowski et al. (1992) rejects stationarity. Overall, these tests thus unanimously favor treating wealth and income shares as being I(1) for our data and hence suggest that cointegration techniques are appropriate.

5 The stability of the long-run relationship between wealth and income inequality

In this section, we investigate the stability of the long-run relationship between wealth and income inequality using cointegration tests and complementary structural break tests. Given that the unit root tests suggest I(1) for our data, wealth and income shares need to be *cointegrated* to rule out spurious regressions. In other words, cointegration tests are appropriate to evaluate whether a stable long-run relationship between wealth and income shares exists. Based on the theoretical framework, such tests are equivalent to assessing the long-run stability of the RSR.

5.1 Cointegration tests

We employ a variety of cointegration tests that cover a wide range of alternative hypotheses.[6]

[4] The wealth data in WID for France is from Garbinti et al. (2020), US data stems from Saez and Zucman (2016). For further details on the exact definitions, concepts and calculation methods underlying the income and wealth shares see these papers and Alvaredo et al. (2016).

[5] The income data for France is from Garbinti et al. (2018), for the US from Piketty et al. (2018).

[6] The most well-known tests by Engle and Granger (1987) and Phillips and Ouliaris (1990) test the null hypothesis of no cointegration against the alternative of cointegration by performing unit root tests on OLS residuals. The procedure proposed by Gregory and Hansen (1996) explicitly tests the null hypothesis of no cointegration against the alternative of cointegration with a possible regime shift. In contrast, the methodology by Hansen (1992) tests for OLS parameter instability and features a null hypothesis of cointegration. Finally, the bounds test by Pesaran et al. (2001) is based on an ARDL(p,q) model and works for both I(0) and I(1) variables, thereby constituting a more general test for a long-run relationship in levels.

Table 1: Cointegration tests 1913-2014

	Top 1%		Top 10%	
	France	US	France	US
H0: No Level Relationship				
Pesaran-Shin-Smith (2001)				
I(1) F-stat	7.05***	4.99**	3.60*	2.58
I(0) F-stat	7.05***	4.99***	3.60**	2.58*
I(1) t-stat	−3.25**	−3.09**	−2.05	−2.26
I(0) t-stat	−3.25***	−3.09***	−2.05*	−2.26**
H0: No Cointegration				
Engle-Granger (1987) t-stat	−3.49***	−2.93**	−2.62*	−1.40
Engle-Granger (1987) z-stat	−24.28***	−17.01**	−14.09*	−4.76
Phillips-Ouliaris (1990) t-stat	−2.87**	−2.98**	−2.22	−1.45
Phillips-Ouliaris (1990) z-stat	−15.32**	−15.37**	−9.37	−4.97
Gregory-Hansen (1996) t-stat	−5.73***	−5.34**	−5.19**	−4.89*
Gregory-Hansen (1996) z-stat	−5.26**	−4.84*	−5.20**	−4.70*
H0: Cointegration				
Hansen (1992) Lc-stat	0.08	0.20	0.10	0.09

Note: Cointegration tests are based on FMOLS regressions (without constant), using a prewhitened Bartlett kernel with Newey-West automatic bandwidth to calculate the long-run variance. The Pesaran et al. (2001) test indicates a (long-run) level relationship if both F-stat and t-stat are statistically significant. Critical values used for this test are based on Kripfganz and Schneider (2020). Stars indicate * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 1 shows the results of these tests over the whole sample 1913-2014. For the top 1%, all tests provide strong evidence for cointegration between wealth and income shares for both France and the US. The results for the top 10% are less uniform, as the conventional cointegration tests fail to reject the null hypothesis at satisfactory confidence levels. Interestingly, however, the test by Gregory and Hansen (1996) rejects the null hypothesis in both countries for the top 10% against the alternative of cointegration with potential structural breaks, and the procedure by Hansen (1992) fails to reject the null hypothesis of cointegration. We interpret these results as suggesting that the long-run RSRs of the top 10% are not stable over the whole sample.

5.2 Structural break tests

In interpreting the results above, it is important to note that cointegration tests do not rule out structural breaks within the period under investigation. However, a visual inspection of the data displayed in Figure A1 is clearly suggestive of the possibility of structural breaks: While the distance between income and wealth concentration (which can be seen as a rule-of-thumb measure of the steady state RSR) is quite stable over time, it is by no means perfectly constant, for both the top 1% and the top 10%. Structural breaks in long-run RSRs at the top would constitute important determinants for our economic understanding of the joint evolution of wealth and inequality over the past 100 years. In the following, we hence perform structural break tests to gain further insights about the stability of the long-run relationship between wealth and income inequality.

Our preferred method is the sequential test proposed by Kejriwal and Perron (2010), an I(1)-variant of the more well-known procedure by Bai and Perron (1998) which allows to test for multiple breakpoints at unknown dates. We also employ two more standard structural break tests: The Quandt-Andrews supremum Wald test as proposed by Quandt (1960), Andrews (1993) and Andrews and Ploberger (1994) and the CUSUM test following Brown et al. (1975) and Ploberger and Kramer (1992). While these tests allow to test for a single unknown breakpoint only and are primarily derived for the stationary context, we believe they constitute reasonable, straightforward and complementary cross-checks. Table 2 shows the results of the structural break tests.

Table 2: Structural break tests

	France		US	
Top 1%				
Kejriwal-Perron (2010) Sequential Test	22.8***	1968	30.9***	1983
	31.7***	1943		
Quandt-Andrews (1993) Supremum Wald Test	149.1***	1968	73.9***	1983
Ploberger-Kraemer (1992) Cusum Test	3.1***	1968	3.1***	1982
Top 10%				
Kejriwal-Perron (2010) Sequential Test	16.6***	1968	22.0***	1984
	45.7***	1942		
Quandt-Andrews (1993) Supremum Wald Test	243.5***	1968	450.0***	1983
Ploberger-Kraemer (1992) Cusum Test	2.9***	1968	4.2***	1983

Note: The table shows the respective test-statistics – F-statistic, Wald-statistic and cusum-statistic, respectively – alongside the estimated structural break date. Stars indicate * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

All tests suggest the presence of structural breaks in the long-run relationship between wealth and income shares in both countries. For France, the test by Kejriwal and Perron (2010) detects two structural breaks, occurring in 1943 and 1968 for the top 1%. In the US, the sequential test procedure implies one break only in 1983. These break dates are almost identical to the ones obtained by the more standard structural break tests and those for the top 10%. In other words, the structural break tests suggest that RSRs at the top are not *entirely stable* over the very long-run (i.e. 100 years), for both the top 1% and the top 10%. In turn, this provides evidence that long-run RSRs may have changed around 1943 and 1968 in France, and around 1983 in the US. Historically, these identified break dates are quite interesting. Related to the first break in 1943, it should not be surprising to find a break in saving behaviour at the top in France during WWII. Similarly, 1968 is a significant year in French history characterized by massive social unrest and economic upheaval. The year 1983 represents an interesting year in US economic history associated with Reagonomics and the Reagan tax cuts. We discuss these break dates in Section 6 in more detail against the backdrop of our estimates.

5.3 Cointegration tests within subperiods

Following the results of the structural break tests, we split our sample at the estimated break dates. This yields three subsamples for France and two subsamples for the US, given by 1913-1942, 1943-1967, 1968-2014 and 1913-1982, 1983-2014 for the top 1%. The corresponding subsamples for the top 10% are almost identical. Using these subsamples, we perform the same cointegration tests as before. The result are shown in Table 3.

Table 3: Subsample cointegration tests

Top 1%	France				US		
	1913-2014	1913-1942	1943-1967	1968-2014	1913-2014	1913-1982	1983-2014
H0: No Level Relationship							
Pesaran-Shin-Smith (2001) I(1) F-stat	7.05***	4.26*	6.37**	8.23***	4.99**	10.41***	15.37***
Pesaran-Shin-Smith (2001) I(1) t-stat	−3.25**	−2.39*	−3.00**	−4.06***	−3.09**	−4.39***	−5.14***
H0: No Cointegration							
Engle-Granger (1987) t-stat	−3.49***	−2.95**	−4.20***	−3.71***	−2.93**	−3.30**	−3.84***
Engle-Granger (1987) z-stat	−24.28***	−17.04**	−23.02***	−59.89***	−17.01**	−21.58***	−78.90***
Phillips-Ouliaris (1990) t-stat	−2.87**	−2.67*	−2.33	−2.98**	−2.98**	−3.20**	2.93**
Phillips-Ouliaris (1990) z-stat	−15.32**	−11.13*	−8.65	−13.81*	−15.37**	−17.57**	−13.67**
H0: Cointegration							
Hansen (1992) Lc-stat	0.08	0.07	0.25	0.06	0.20	0.04	0.07

Top 10%	France				US		
	1913-2014	1913-1941	1942-1967	1968-2014	1913-2014	1913-1983	1984-2014
H0: No Level Relationship							
Pesaran-Shin-Smith (2001) I(1) F-stat	3.60*	6.27**	2.25	10.14***	2.58	1.09	4.70**
Pesaran-Shin-Smith (2001) I(1) t-stat	−2.05	−1.28	−1.92	−4.45***	−2.26	−0.36	−2.11
H0: No Cointegration							
Engle-Granger (1987) t-stat	−2.62*	−2.93**	−2.73*	−3.02**	−1.40	−2.57*	−1.66
Engle-Granger (1987) z-stat	−14.09*	−14.00**	−13.15**	−33.65***	−4.76	−13.77*	−5.43
Phillips-Ouliaris (1990) t-stat	−2.22	−3.06**	−2.07	−2.99**	−1.45	−2.52*	−2.09
Phillips-Ouliaris (1990) z-stat	−9.37	−15.06**	−8.39	−13.51*	−4.97	−12.01*	−8.52
H0: Cointegration							
Hansen (1992) Lc-stat	0.10	0.06	0.31	0.02	0.09	0.42*	0.20

Note: Cointegration tests are based on FMOLS regressions (without constant), using a prewhitened Bartlett kernel with Newey-West automatic bandwidth to calculate the long-run variance. The Pesaran et al. (2001) test indicates cointegration if both F-stat and t-stat are statistically significant. Critical values for this test are based on Kripfganz and Schneider (2020). Stars indicate * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Overall, the cointegration tests for the top 1% provide strong support for cointegration within the subsamples, i.e. *episodic* cointegration for both France and the US. This finding is remarkable given the relatively small samples sizes implied by the break dates. In economic terms, this suggests that RSRs at the top within the subsamples are stable for the top 1%. These results also reconcile the findings from the cointegration tests over the whole sample and the break tests: RSRs of the top 1% are not stable over the whole sample, but stable enough such that the relationship over the very long run is comparably stable. Overall, the pre-testing procedure thus implies that the data for the top 1% supports a long-run relationship between wealth and income shares. As such, given the theoretical frameworks, this implies that corresponding estimations of the relationship between wealth and income shares reveal insights about the long-run RSR.

Regarding the top 10%, the results are more mixed. In some subsamples, the relationship between wealth and income shares is stable and the tests indicate cointegration. This is especially the case in France (for the period before WW II and the period after 1968), but to a way lesser extent in the US. We may interpret this as evidence that RSRs of the top 10% in France are comparably stable (within the subsamples), whereas such evidence is more limited for the US. One possible, admittedly speculative, interpretation for the stark difference between the top 1% and top 10% results is that the top 1% might be a more homogeneous group over time characterized by a more stable saving behaviour.

6 Estimated relative saving rates at the top

We now present and discuss our estimates of the RSRs at the top for both France and the US. Given the pre-estimation tests, we are mainly interested in (and confident about) the estimates for the top 1% in the respective sub-samples. For completeness, we also report and discuss estimates for the entire sample and the top 10%, although these should be viewed with caution.

6.1 Estimates

Table 4 shows the estimated RSRs based on the estimators discussed in Section 3. For the sake of illustration, let us focus on the whole sample first, ignoring the structural break results. For 1913-2014, all estimators yield remarkably similar estimates for the RSRs at or above 2 for the top 1% in both countries. These results are statistically significantly different from unity in all cases at high confidence levels. Put differently, these findings suggest that the long-run RSR of the top 1% is *twice or even slightly more than twice as high* as the aggregate saving rate in both countries. Interestingly, the estimates for France are a bit higher than the ones for the US (2.24 vs. 2.00 based on the ARDL). For the top 10%, the estimated RSRs are uniformly lower in both countries (1.63 in France, 1.73 in the US based on the ARDL). While the stability of these RSRs is called into question by the pre-estimation tests, it is interesting and encouraging to note that they are in line with existing studies showing that saving propensity increases along the distribution (Quadrini, 1999; Dynan et al., 2004; De Nardi et al., 2010; De Nardi and Fella, 2017; Fagereng et al., 2019).

Turning to the sub-samples, all estimators suggest a decline of the RSR at the top after the (most recent) structural break in all cases. For France, the ARDL estimate for the period 1943-1967 is 2.85, whereas the 1968-2014 RSR at the top is estimated to be 2.14. For the US, the results indicate a similar, albeit smaller decrease of RSRs at the top after 1983, from 2.09 in the period before to 1.88 based on the ARDL in the period after. For all cases, the estimated decline of RSRs at the top around the (most) recent structural break date is statistically significant based on Wald tests.

Overall, our results provide evidence for a structural decline of relative saving rates at the top. This suggests that the long-run relationship between wealth and income shares changed in both countries following the most recent structural break dates, 1968 in France and 1983 in the US – noteworthy years in each country's history. Against the backdrop of the estimated break dates, one might hence speculate that the significant economic events in both countries caused a structural change in active saving behavior at the top. In France, 1968 was characterized by massive social unrest and economic upheaval, primarily originating from anti-capitalism protests by students. In May 1968, the economy of France was effectively halted given demonstrations, strikes and factory occupations. As such, these events, often considered a key turning point in the history of France, might have affected saving rates at the top to a considerable extent. Similarly, for the US, the estimated break date coincides with Reaganomics, i.e. the massive fiscal reforms initiated by US President Reagan in the 1980s. Key ingredients of the reforms were lower income and capital gains taxes as well as reductions in government regulations and government spending. These substantial changes to the economic rules of the game may have

Table 4: Estimated relative saving rates $\widehat{s}^f$

Top 1%	France				US		
	1913-2014	1913-1942	1943-1967	1968-2014	1913-2014	1913-1982	1983-2014
ARDL	2.24***	2.17***	2.85***	2.14***	2.00***	2.09***	1.88***
	(0.12)	(0.20)	(0.03)	(0.06)	(0.08)	(0.08)	(0.03)
FMOLS	2.43***	2.42***	2.90***	2.13***	2.07***	2.15***	1.88***
	(0.12)	(0.07)	(0.05)	(0.06)	(0.06)	(0.07)	(0.03)
DOLS	2.41***	2.39***	2.95***	2.13***	2.05***	2.14***	1.88***
	(0.11)	(0.10)	(0.05)	(0.06)	(0.09)	(0.07)	(0.02)
CCR	2.40***	2.41***	2.92***	2.13***	2.03***	2.13***	1.91***
	(0.12)	(0.07)	(0.05)	(0.06)	(0.06)	(0.07)	(0.03)
OLS	2.44***	2.42***	2.99***	2.13***	2.05***	2.14***	1.86***
	(0.09)	(0.08)	(0.05)	(0.11)	(0.08)	(0.05)	(0.02)
VECM	2.34***	2.34***	2.96***	2.13***	2.02***	2.13***	1.87***
	(0.10)	(0.07)	(0.04)	(0.05)	(0.06)	(0.06)	(0.03)

Top 10%	France				US		
	1913-2014	1913-1941	1942-1967	1968-2014	1913-2014	1913-1983	1984-2014
ARDL	1.63***	1.54***	1.93***	1.65***	1.73***	1.69***	1.64***
	(0.08)	(0.10)	(0.08)	(0.01)	(0.07)	(0.31)	(0.04)
FMOLS	1.76***	1.72***	1.98***	1.65***	1.79***	1.84***	1.59***
	(0.06)	(0.02)	(0.04)	(0.02)	(0.06)	(0.04)	(0.03)
DOLS	1.75***	1.71***	1.98***	1.65***	1.76***	1.84***	1.59***
	(0.06)	(0.02)	(0.06)	(0.03)	(0.10)	(0.05)	(0.02)
CCR	1.75***	1.72***	1.97***	1.64***	1.75***	1.84***	1.60***
	(0.06)	(0.02)	(0.04)	(0.02)	(0.06)	(0.04)	(0.03)
OLS	1.76***	1.72***	1.99***	1.66***	1.75***	1.83***	1.59***
	(0.06)	(0.02)	(0.04)	(0.02)	(0.09)	(0.04)	(0.02)
VECM	1.74***	1.64***	1.99***	1.65***	1.72***	1.83***	1.52***
	(0.06)	(0.03)	(0.04)	(0.01)	(0.06)	(0.04)	(0.04)

Note: Standard errors in parentheses. Stars indicate * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ for H_0: $\widetilde{s}_g^f = 1$. Standard errors for ARDL, FMOLS, DOLS, CCR and OLS are heteroskedasticity- and autocorrelation-consistent (HAC), long-run variances are calculated using prewhitened Bartlett kernels with Newey-West automatic bandwidth.

altered saving behavior at the top. While our method is inherently silent about the precise economic channels and mechanisms, our results nevertheless provide the basis for interesting historical speculation and may constitute worthwhile avenues for future research.

Interestingly, the decline in the RSR at the top coincides with a decrease in the *aggregate* saving rate in both countries. In France, the aggregate private saving rate averaged 16.4% over the period 1950-1967, compared to 15.6% in the period 1968-2014.[7] In the US, the aggregate private saving rate declined from an average of 11.4% over the period 1959-1982 to 6.2% over the period 1983-2014. The drop in the US in the early 1980s was so remarkable that an earlier literature aimed to investigate the reasons behind this drop (see among others Bosworth et al., 1991; Gokhale et al., 1996; Attanasio, 1998; Parker, 1999). This literature was mainly concerned with identifying the group of people which reduced saving the most, but was however seriously hampered by the limited prevailing microdata. Browning and Lusardi (1996) provide an early overview of this literature and list eleven(!) different possible explanations while concluding that

[7] These averages are calculated using the personal saving rate, U.S. Bureau of Economic Analysis, Personal Saving Rate [PSAVERT], retrieved from FRED, Federal Reserve Bank of St. Louis; `https://fred.stlouisfed.org/series/PSAVERT`, July 5, 2018. For France, they rely on the series Taux d'épargne des ménages, retrieved from INSEE, `https://www.insee.fr/fr/statistiques/2830268`, July 5, 2018. The drop in the French saving rate is most remarkable from the 1980s onwards; the average saving rate was 14.3 % over the period 1980-2014. A similar result can also be seen from the data from Piketty (2011) and Piketty et al. (2018), as shown.

“*[t]he variety of proposed explanations is per se an indication that there exists little consensus on what underlies the decline in saving rates.*” While this literature could not reach a definitive conclusion, our results are in line with more recent findings of Juster et al. (2005). Based on PSID microdata, they provide evidence that the RSR of stock owners versus non-stock owners dropped throughout the 1980s.[8] As stock ownership is mostly concentrated for households at the top of the wealth distribution, our findings of a declining RSR at the top fits their findings and provides new evidence on why the aggregate saving rate dropped so much from the early 1980s onward.

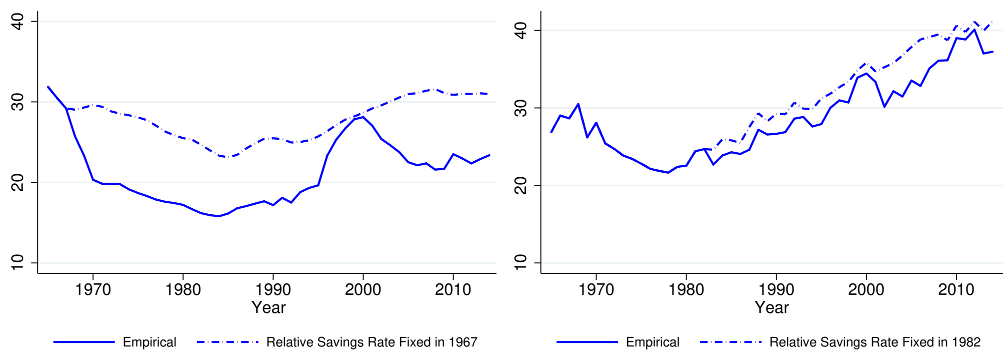
6.2 Declining relative saving rates and the evolution of wealth inequality

How important is the drop of the RSR to understand the evolution of wealth inequality? In this section, we perform counterfactual simulations of wealth shares to gauge the extent to which the estimated decrease in RSRs affected wealth inequality.

In particular, we use the ARDL estimates of RSRs from the beginning of our sample in 1913 until the structural breaks in 1968 for France and 1983 for the US, respectively, as reported in Table 4. We then use the corresponding estimates of the long-run RSRs at the top after the structural break to perform out-of-sample model simulations, using the empirically observed income shares. In other words, the counterfactual scenario assumes that RSRs at the top stayed constant at the 1968-level in France and the 1983-level in the US, and simulates the evolution of wealth shares for the observed development of income shares. Obviously, this is not a structural exercise – any change in the wealth share should also affect the income share. Because these second round effects are excluded here by construction, the effects on the wealth shares are likely underestimated. Figure 2 shows the resulting counterfactual evolution of wealth inequality.

Figure 2: Wealth shares for counterfactual relative saving rates

(a) France Top 1% **(b) US Top 1%**



Note: Out-of-sample prediction about the evolution of wealth inequality according to the ARDL results from 1913 until the respective break (FR: 1968, US: 1983), using the estimated long-run RSR at the top prior to the break.

As seen in the left panel, the estimated decrease of RSRs at the top in 1968 contributed substantially to the historical evolution of wealth inequality in France. Absent the estimated

[8]Following their findings, the average saving rate for stock owners dropped from 13.2% in 1984-1989 to 8.6% in 1989-1994, while those for non-stock owners barely moved from 7.7% to 7.6% (see Table 2 in Juster et al., 2005). This corresponds to a decline of the RSR from 1.71 to 1.13.

structural decline in RSRs, the share of wealth accruing to the top 1% would be higher today relative to the empirical observation, at 31.0% (compared to 23.4%). While the rise in income inequality in France was comparably subdued in recent decades (see Figure 1), the substantial decline in RSRs after 1968 also attenuated the associated upward pressure on wealth inequality. This is in contrast to the US, where the estimated RSRs declined less after 1983. As a result, the counterfactual simulations produce wealth inequality dynamics largely in line with the observed increasing profile for the US (41.2% compared to 37.2% in 2014), suggesting a much smaller role of the structural break in saving rates in recent decades. These results suggests that the observed increase in wealth inequality in the US seems to be largely attributable to the rise in income inequality. In contrast, the subdued wealth inequality rise in France appears to be a joint result of lower RSRs at the top and modestly increasing income inequality.

The results from these counterfactual simulations reiterate one key finding of the theoretical framework: income inequality and RSRs are important drivers of the evolution of wealth inequality, both in the short and in the long run. As RSRs at the top are comparably stable over time and larger than one, a given rise in income concentration leads to a relatively rapid increase in wealth inequality. In the long-run, the increase in wealth shares is around twice as large as the increase in income shares. In terms of policy implications, we may thus conclude that policies aimed at reducing wealth inequality should directly tackle income inequality first. Alternatively, policies could be aimed at decreasing RSRs at the top, for example by providing saving incentives via the tax system to the bottom of the distribution. However, RSRs at the top appear comparably stable over many decades, despite numerous tax reforms occurring in the meantime. This suggests that policy leverage regarding RSRs at the top might be limited.

7 Conclusion

According to our analysis, the strong co-movement of wealth and income shares over a century is anything but spurious. An extended Solow-type model with two agents implies a dynamic error correction relationship between wealth and income shares. This relationship is governed by the RSR as the key underlying economic mechanism. Metaphorically speaking, the RSR is a rope that ties wealth and income shares together.

We provide empirical tests and estimations of the long-run relationship between wealth and income inequality. We find that wealth and income shares for the top 1% over the period 1913-2014 are cointegrated in France and the US. This novel result implies that RSRs at the top are fairly stable over multiple decades. Our results suggest that the saving rate of the top 1% is around twice as high as the average saving rate.

Our results further indicate that the imaginary rope has become somewhat shorter in the second half of the 20th century. In particular, we find evidence for a structural decline in RSRs at the top after 1968 in France and 1983 in the US. This result is consistent with an observed drop in the aggregate saving rate and with microeconomic evidence of a decline in saving rates of stockholders by Juster et al. (2005). By construction, our analysis is unable to provide a deeper explanation of why saving rates at the top declined. However, our results provide the basis for interesting historical discussions and may constitute worthwhile avenues for future research.

References

Alvaredo, F., Atkinson, A., Chancel, L., Piketty, T., Saez, E., and Zucman, G. (2016). Distributional National Accounts (DINA) Guidelines : Concepts and Methods used in WID.world. Working Papers 201602, World Inequality Lab.

Alvaredo, F., Atkinson, A. B., Piketty, T., and Saez, E. (2013). The Top 1 Percent in International and Historical Perspective. *Journal of Economic Perspectives*, 27(3):3–20.

Alvaredo, F., Chancel, L., Piketty, T., Saez, E., and Zucman, G. (2017). Global Inequality Dynamics: New Findings from WID.world. *American Economic Review*, 107(5):404–09.

Andrews, D. W. K. (1993). Tests for Parameter Instability and Structural Change with Unknown Change Point. *Econometrica*, 61(4):821–856.

Andrews, D. W. K. and Ploberger, W. (1994). Optimal Tests When a Nuisance Parameter Is Present Only under the Alternative. *Econometrica*, 62(6):1383–1414.

Attanasio, O. (1998). Cohort Analysis of Saving Behavior by U.S. Households. *The Journal of Human Resources*, 33(3):575–609.

Bai, J. and Perron, P. (1998). Estimating and Testing Linear Models with Multiple Structural Changes. *Econometrica*, 66(1):47–78.

Benhabib, J. and Bisin, A. (2018). Skewed Wealth Distributions: Theory and Empirics. *Journal of Economic Literature*, 56(4):1261–1291.

Bosworth, B., Burtless, G., and Sabelhaus, J. (1991). The Decline in Saving: Evidence from Household Surveys. *Brookings Papers on Economic Activity*, I:183–256.

Brown, R. L., Durbin, J., and Evans, J. M. (1975). Techniques for Testing the Constancy of Regression Relationships over Time. *Journal of the Royal Statistical Society. Series B (Methodological)*, 37(2):149–192.

Browning, M. and Lusardi, A. (1996). Household Saving: Micro Theories and Micro Facts. *Journal of Economic Literature*, 34(4):1797–1855.

De Nardi, M. and Fella, G. (2017). Saving and Wealth Inequality. *Review of Economic Dynamics*, 26:280–300.

De Nardi, M., French, E., and Jones, J. B. (2010). Why Do the Elderly Save? The Role of Medical Expenses. *Journal of Political Economy*, 118(1):39–75.

Dickey, D. A. and Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *Journal of the American Statistical Association*, 74(366):427–431.

Dynan, K. E., Skinner, J., and Zeldes, S. P. (2004). Do the Rich Save More? *Journal of Political Economy*, 112(2):397–444.

Elliott, G., Rothenberg, T. J., and Stock, J. H. (1996). Efficient Tests for an Autoregressive Unit Root. *Econometrica*, 64(4):813–836.

Engle, R. F. and Granger, C. W. J. (1987). Co-integration and Error Correction: Representation, Estimation, and Testing. *Econometrica*, 55(2):251–276.

Fagereng, A., Holm, M. B., Moll, B., and Natvik, G. (2019). Saving Behavior Across the Wealth Distribution: The Importance of Capital Gains. Working Paper 26588, National Bureau of Economic Research.

Garbinti, B., Goupille-Lebret, J., and Piketty, T. (2018). Income Inequality in France, 1900-2014: Evidence from Distributional National Accounts (DINA). *Journal of Public Economics*, 162(C):63–77.

Garbinti, B., Goupille-Lebret, J., and Piketty, T. (2020). Accounting for Wealth-Inequality Dynamics: Methods, Estimates, and Simulations for France. *Journal of the European Economic Association*, 19(1):620–663.

Gokhale, J., Kotlikoff, L., and Sabelhaus, J. (1996). Understanding the Postwar Decline in U.S. Saving: A Cohort Analysis. *Brookings papers on Economic Activity*, I:315–407.

Gregory, A. W. and Hansen, B. E. (1996). Residual-based tests for cointegration in models with regime shifts. *Journal of Econometrics*, 70(1):99–126.

Hansen, B. E. (1992). Tests for Parameter Instability in Regressions with I(1) Processes. *Journal of Business & Economic Statistics*, 10(3):321–335.

Hubmer, J., Krusell, P., and Jr., A. A. S. (2020). Sources of US Wealth Inequality: Past, Present, and Future. In *NBER Macroeconomics Annual 2020, volume 35*, NBER Chapters, pages 391–455. National Bureau of Economic Research, Inc.

Juster, T., Lupton, J., Smith, J., and Stafford, F. (2005). The decline in household saving and the wealth effect. *The Review of Economics and Statistics*, 87(4):20–27.

Kejriwal, M. and Perron, P. (2010). Testing for Multiple Structural Changes in Cointegrated Regression Models. *Journal of Business & Economic Statistics*, 28(4):503–522.

Kripfganz, S. and Schneider, D. C. (2020). Response Surface Regressions for Critical Value Bounds and Approximate p-values in Equilibrium Correction Models. *Oxford Bulletin of Economics and Statistics*, 82(6):1456–1481.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54(1-3):159–178.

Park, J. Y. (1992). Canonical Cointegrating Regressions. *Econometrica*, 60(1):119–143.

Parker, J. (1999). Spendthrift in America? On two decades of decline in the U.S. saving rate. *NBER Macroeconomics Annual*, 14:317–370.

Perron, P. and Vogelsang, T. J. (1992a). Nonstationarity and Level Shifts with an Application to Purchasing Power Parity. *Journal of Business & Economic Statistics*, 10(3):301–320.

Perron, P. and Vogelsang, T. J. (1992b). Testing for a Unit Root in a Time Series with a Changing Mean: Corrections and Extensions. *Journal of Business & Economic Statistics*, 10(4):467–470.

Pesaran, M. H., Shin, Y., and Smith, R. J. (2001). Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics*, 16(3):289–326.

Phillips, P. C. B. and Hansen, B. E. (1990). Statistical Inference in Instrumental Variables Regression with I(1) Processes. *Review of Economic Studies*, 57(1):99–125.

Phillips, P. C. B. and Ouliaris, S. (1990). Asymptotic Properties of Residual Based Tests for Cointegration. *Econometrica*, 58(1):165–193.

Phillips, P. C. B. and Perron, P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrika*, 75(2):335–346.

Piketty, T. (2011). On the Long-Run Evolution of Inheritance: France 1820–2050. *The Quarterly Journal of Economics*, 126(3):1071–1131.

Piketty, T. (2014). *Capital in the Twenty-First Century*. Harvard University Press.

Piketty, T., Saez, E., and Zucman, G. (2018). Distributional National Accounts: Methods and Estimates for the United States. *The Quarterly Journal of Economics*, 133(2):553–609.

Ploberger, W. and Kramer, W. (1992). The CUSUM Test with OLS Residuals. *Econometrica*, 60(2):271–285.

Quadrini, V. (1999). The Importance Of Entrepreneurship For Wealth Concentration And Mobility. *Review of Income and Wealth*, 45(1):1–19.

Quadrini, V. and Ríos-Rull, J.-V. (2015). Chapter 14 - Inequality in Macroeconomics. In Atkinson, A. B. and Bourguignon, F., editors, *Handbook of Income Distribution*, volume 2 of *Handbook of Income Distribution*, pages 1229 – 1302. Elsevier.

Quandt, R. E. (1960). Tests of the Hypothesis that a Linear Regression System Obeys Two Separate Regimes. *Journal of the American Statistical Association*, 55(290):324–330.

Saez, E. and Zucman, G. (2016). Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data. *The Quarterly Journal of Economics*, 131(2):519–578.

Saikkonen, P. (1992). Estimation and Testing of Cointegrated Systems by an Autoregressive Approximation. *Econometric Theory*, 8(1):1–27.

Solow, R. (1956). A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1):65–94.

Stock, J. and Watson, M. (1993). A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems. *Econometrica*, 61(4):783–820.

Vogelsang, T. J. and Perron, P. (1998). Additional Tests for a Unit Root Allowing for a Break in the Trend Function at an Unknown Time. *International Economic Review*, 39(4):1073–1100.

Appendix

A Solow model

This section presents a neoclassical growth model with two agents. The goal of the following exposition is to show that rather standard neoclassical growth model assumptions quite naturally imply a steady state within this framework. In this steady state, the wealth to income share ratio is equal to the relative savings rate, which is one of the key results shown in the main part. We adapt an otherwise standard Solow growth model in discrete time and introduce two representative agents instead of one. The idea is to have the population split into two homogeneous groups, each with their own saving rate. Each of the groups owns a fraction of the capital stock. In the Solow model, aggregate wealth is the capital stock.

A.1 Assumptions

We start from the standard neoclassical growth model assumptions:

1. Aggregate output follows a Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \tag{A1}$$

2. Constant exogenous population growth and technology growth :

$$A_{t+1} = (1+g)A_t \tag{A2}$$

$$L_{t+1} = (1+n)L_t \tag{A3}$$

3. Price taking and competitive equilibrium: the wage is equal to the marginal product of labour and the rental rate is equal to the marginal product of capital. Using the Cobb-Douglas assumption:

$$\frac{\delta F(K_t, A_t L_t)}{\delta L_t} = (1-\alpha)K_t^{\alpha}(A_t)^{1-\alpha}L_t^{-\alpha} = (1-\alpha)\frac{Y_t}{L_t} = w_t \tag{A4}$$

$$\frac{\delta F(K_t, A_t L_t)}{\delta K_t} = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha} = \alpha\frac{Y_t}{K_t} = r_t \tag{A5}$$

The standard Solow growth model features only one representative household with exogenous saving rate s. To make the Solow model suitable to answer questions about the distribution, we assume two representative households, each with their own saving rate.

4. We consider two homogeneous groups in the population. Each group can be represented by a representative agent. Group 1, denoted by L_{1t} has a share of a_1 of the population. Group 2, denoted by L_{2t} has a share of $1 - a_1$ of the population. Group 1 owns one part of the capital stock, i.e K_{1t}, group 2 owns the other part, K_{2t}.

$$\frac{L_{1t}}{L_t} = a_1 \tag{A6}$$

$$\frac{L_{2t}}{L_t} = 1 - a_1 \tag{A7}$$

$$K_t = K_{1t} + K_{2t} \tag{A8}$$

5. Given price taking and competitive equilibrium the wage rate and rental rate are the same for everyone. Total income for group 1 and group 2 are given by:

$$Y_{1t} = w_t L_{1t} + r_t K_{1t} \tag{A9}$$

$$Y_{2t} = w_t L_{2t} + r_t K_{2t} \tag{A10}$$

$$Y_t = Y_{1t} + Y_{2t} \tag{A11}$$

6. The laws of motion of K_{1t} and K_{2t} are determined by the savings rate of the respective group and the depreciation rate δ. Group 1 has a saving rate of s_1, and group 2 has a saving rate of s_2. Without loss of generality, we assume $s_1 > s_2$.

$$K_{1t+1} = s_1 Y_{1t} + (1 - \delta) K_{1t} \tag{A12}$$

$$K_{2t+2} = s_2 Y_{2t} + (1 - \delta) K_{2t} \tag{A13}$$

7. Initial capital stocks at $t = 0$, population and technology are given

$$K_{10}, K_{20}, L_0, A_0 \, given \tag{A14}$$

The Assumptions (A1)-(A14) present a standard neoclassical growth model setup, with the only exception that there are two representative agents instead of one. Intuitively, it is clear that the agent with the higher savings rate will accumulate a larger fraction of the capital stock. Below, we infer what happens in steady state to the relative saving rate, wealth share and income share of the two groups.

A.2 The aggregate steady state

Under Assumptions (A8), (A11), (A12) and (A13), the aggregate law of motion of the capital stock is:

$$K_{t+1} = s_{nt} Y_t + (1 - \delta) K_t \tag{A15}$$

with

$$s_{nt} \equiv s_1 \frac{Y_{1t}}{Y_t} + s_2 \frac{Y_{2t}}{Y_t} \tag{A16}$$

the aggregate saving rate. The aggregate saving rate is the weighted average of the two saving rates, where the income shares are the weights.

We will use small-cap letters to denote variables in intensive form, i.e. as a ratio of the variable to the product of technology and labour (i.e. effective units of labour):

Define capital per effective unit of labour:

$$k_t \equiv \frac{K_t}{A_t L_t} \tag{A17}$$

Define output per effective unit of labour:

$$y_t \equiv \frac{Y_t}{A_t L_t} \tag{A18}$$

Define income of group 1 per effective unit of labour:

$$y_{1t} = \frac{Y_{1t}}{A_t L_t} \tag{A19}$$

and similarly for y_{2t}, k_{1t} and k_{2t}.

The production function in intensive form is (combine assumption (A1) and definitions (A17) and (A18)):

$$y_t = k_t^\alpha \tag{A20}$$

Using the intensive form (A17) and assumptions (A2) and (A3), the law of motion of the aggregate capital stock (A15) be written as

$$(1+g)(1+n)k_{t+1} = s_{nt} y_t + (1-\delta)k_t \tag{A21}$$

and using the production function in intensive form (A20):

$$(1+g)(1+n)k_{t+1} = s_{nt} k_t^\alpha + (1-\delta)k_t \tag{A22}$$

This law of motion of the capital stock per effective unit of labour is almost identical to the standard Solow model (with Cobb-Douglas production function). It differs in the fact that whereas the aggregate saving rate is constant in the Solow growth model, here the aggregate saving rate is a function of time. The aggregate saving rate will move as the income shares $\frac{Y_{1t}}{Y_t}$ and $\frac{Y_{2t}}{Y_t}$ move. However, in steady state, these shares will be constant and so also the aggregate savings rate will be constant.

Assume that there is a steady state aggregate saving rate s_n^* (we show below this indeed exists). If we replace s_{nt} with the steady state value s_n^* in (A22), we get the standard Solow model law of motion of the capital stock per effective worker. This implies a steady state of capital per effective worker k^*.

$$k^* = \left(\frac{s_n^*}{(1+g)(1+n)-(1-\delta)}\right)^{\frac{1}{(1-\alpha)}} \tag{A23}$$

A.3 Steady state wealth and income shares

Using intensive form notation, we can write the capital accumulation equations of groups 1 and 2, (A12) and (A13), as:

$$(1+g)(1+n)k_{1t+1} = s_1 y_{1t} + (1-\delta)k_{1t} \tag{A24}$$

$$(1+g)(1+n)k_{2t+1} = s_2 y_{2t} + (1-\delta)k_{2t} \tag{A25}$$

Imagine again the steady state values k_1^*, k_2^*, y_1^*, y_2^*. If these values exist, they have to obey the following equations:

$$(1+g)(1+n)k_1^* = s_1 y_1^* + (1-\delta)k_1^* \tag{A26}$$

$$(1+g)(1+n)k_2^* = s_2 y_2^* + (1-\delta)k_2^* \tag{A27}$$

These imply that:

$$k_1^* = \frac{s_1 y_1^*}{(1+g)(1+n)-(1-\delta)} \tag{A28}$$

and

$$k_1^* + k_2^* = \frac{s_1 y_1^* + s_2 y{*}_2}{(1+g)(1+n)-(1-\delta)} \tag{A29}$$

Which implies

$$\frac{k_1^*}{k_1^*+k_2^*} = \frac{s_1 y_1^*}{s_1 y_1^* + s_2 y_2^*} \tag{A30}$$

and

$$\frac{k_1^*}{k_1^*+k_2^*} = \frac{s_1 \frac{y_1^*}{y_1^*+y_2^*}}{s_1 \frac{y_1^*}{y_1^*+y_2^*} + s_2 \frac{y_2^*}{y_1^*+y_2^*}} \tag{A31}$$

It is easy to see that $s_1 \frac{y_1^*}{y_1^*+y_2^*} + s_2 \frac{y_2^*}{y_1^*+y_2^*}$ is the steady state aggregate saving rate, s_n^*. In consequence, in the steady state, the wealth share to income share ratio of group 1 is equal to the relative saving rate of group 1:

$$\frac{\frac{k_1^*}{k_1^*+k_2^*}}{\frac{y_1^*}{y_1^*+y_2^*}} = \frac{s_1}{s_n^*} \tag{A32}$$

A.4 Wealth share law of motion

An alternative way of deriving the steady state result (A32) is via the law of motion of the wealth share of group 1. We start from the law of motion of the capital stock owned by group 1 (A12) and divide both sides by the aggregate capital stock K_{t+1}:

$$\frac{K_{1t+1}}{K_{t+1}} = s_1 \frac{Y_{1t}}{K_{t+1}} + (1-\delta)\frac{K_{1t}}{K_{t+1}} \tag{A33}$$

Use the law of motion of the aggregate capital stock (A15) to substitute out K_{t+1} and denote the wealth share of group 1 $\frac{K_{1t+1}}{K_{t+1}}$ as $sh^1_{W_{t+1}}$ yields

$$sh^1_{W_{t+1}} = s_1 \frac{Y_{1t}}{s_{nt}Y_t + (1-\delta)K_t} + (1-\delta)\frac{K_{1t}}{s_{nt}Y_t + (1-\delta)K_t}. \tag{A34}$$

Denote the income share of group 1 $\frac{Y_{1t}}{Y_t}$ as $sh^1_{Y_t}$. We get:

$$sh^1_{W_{t+1}} = s_1 \frac{sh^1_{Y_t}}{s_{nt} + (1-\delta)K_t/Y_t} + (1-\delta)\frac{sh^1_{W_t}}{s_{nt}Y_t/K_t + (1-\delta)} \tag{A35}$$

Rearrange:

$$sh^1_{W_{t+1}} = s_1 \frac{sh^1_{Y_t}Y_t/K_t}{s_{nt}Y_t/K_t + (1-\delta)} + (1-\delta)\frac{sh^1_{W_t}}{s_{nt}Y_t/K_t + (1-\delta)} \tag{A36}$$

Multiply out denominator:

$$sh^1_{W_{t+1}}(s_{nt}Y_t/K_t + (1-\delta)) = s_1.sh^1_{Y_t}.Y_t/K_t + (1-\delta)sh^1_{W_t} \tag{A37}$$

Rearrange:

$$(1-\delta)sh^1_{W_{t+1}} - (1-\delta)sh^1_{W_t} = s_1.sh^1_{Y_t}.Y_t/K_t - sh^1_{W_{t+1}}.(s_{nt}Y_t/K_t) \tag{A38}$$

Rearrange:

$$(1-\delta)\Delta sh^1_{W_{t+1}} = (s_{nt}Y_t/K_t)(\frac{s_1}{s_{nt}}sh^1_{Y_t} - sh^1_{W_{t+1}}) \tag{A39}$$

Use $sh^1_{W_{t+1}} = \Delta sh^1_{W_{t+1}} + sh^1_{W_t}$:

$$(1-\delta)\Delta sh^1_{W_{t+1}} = (s_{nt}Y_t/K_t)(\frac{s_1}{s_{nt}}sh^1_{Y_t} - \Delta sh^1_{W_{t+1}} - sh^1_{W_t}) \tag{A40}$$

Rearrange:

$$((1-\delta) + (s_{nt}Y_t/K_t))\Delta sh^1_{W_{t+1}} = (s_{nt}Y_t/K_t)(\frac{s_1}{s_{nt}}sh^1_{Y_t} - sh^1_{W_t}) \tag{A41}$$

Denote aggregate saving as $S_t = s_{nt}Y_t$:

$$\Delta sh^1_{W_{t+1}} = \frac{S_t/K_t}{(1-\delta) + S_t/K_t}(\frac{s_1}{s_{nt}}sh^1_{Y_t} - sh^1_{W_t}) \tag{A42}$$

or

$$\Delta sh^1_{W_{t+1}} = \frac{S_t}{K_{t+1}}(\frac{s_1}{s_{nt}}sh^1_{Y_t} - sh^1_{W_t}) \tag{A43}$$

In steady state we have that the wealth share stays constant, i.e. $\Delta sh^1_{W_{t+1}} = 0$. As a consequence, we again obtain the result of (A32): in steady state, the ratio of the wealth to income share equals the relative saving ratio.

A.5 Steady state wealth, income and saving rate

We now show how to find the steady state values of y_1^*, y_2^* and k_1^*, k_2^* and s_n^*. The steady state aggregate saving rate s_n^* depends on the steady state weights of the two saving rates. These weights depend on the steady state income per effective worker, y_1^*, y_2^*, so we first solve for those. We first write total income of group 1 (A9) in intensive form

$$\frac{Y_{1t}}{A_t L_t} = w_t \frac{L_{1t}}{A_t L_t} + r_t \frac{K_{1t}}{A_t L_t} \tag{A44}$$

and use assumptions (A4) and (A5)

$$\frac{Y_{1t}}{A_t L_t} = (1-\alpha)\frac{Y_t}{L_t}\frac{L_{1t}}{A_t L_t} + \alpha \frac{Y_t}{K_t}\frac{K_{1t}}{A_t L_t} \tag{A45}$$

and assumptions (A6) and (A7). Now consider again the steady state values y_1^*, y^*, k_1^*, k^*. These have to obey:

$$y_1^* = (1-\alpha)a_1 y^* + \alpha \frac{k_1^*}{k^*} y^* \tag{A46}$$

$$k^* = k_1^* + k_2^* \tag{A47}$$

$$y^* = y_1^* + y_2^* \tag{A48}$$

Rearranging (A46) and combine with (A31):

$$\frac{y_1^*}{y^*} - (1-\alpha)a_1 = \alpha \frac{s_1 \frac{y_1^*}{y_1^*+y_2^*}}{s_1 \frac{y_1^*}{y_1^*+y_2^*} + s_2 \frac{y_2^*}{y_1^*+y_2^*}} \tag{A49}$$

use $y^* = y_1^* + y_2^*$:

$$\frac{y_1^*}{y^*} - (1-\alpha)a_1 = \alpha \frac{s_1 \frac{y_1^*}{y^*}}{s_1 \frac{y_1^*}{y^*} + s_2 \frac{y_2^*}{y^*}} \tag{A50}$$

use $\frac{y_1^*+y_2^*}{y^*} = 1$:

$$\frac{y_1^*}{y^*} - (1-\alpha)a_1 = \alpha \frac{s_1 \frac{y_1^*}{y^*}}{(s_1 - s_2)\frac{y_1^*}{y^*} + s_2} \tag{A51}$$

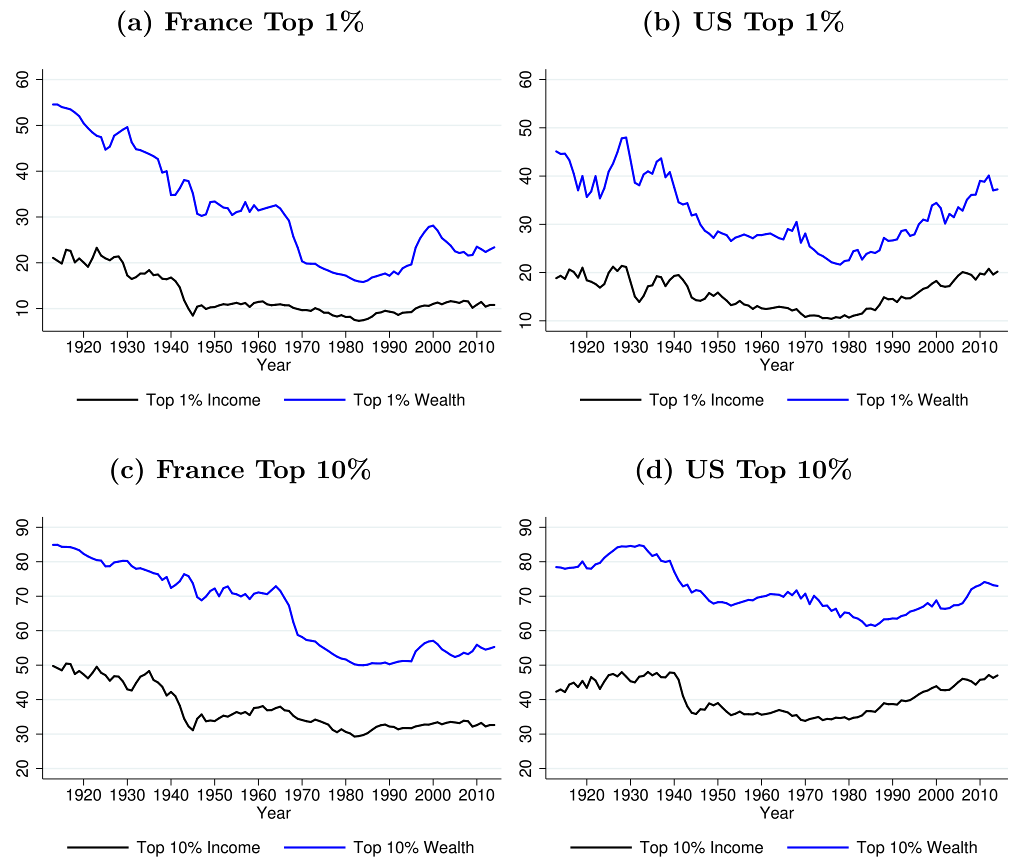
Rearrange:

$$(s_1 - s_2)(\frac{y_1^*}{y^*})^2 - (s_2 - (1-\alpha)a_1(s_1 - s_2) - s_1\alpha)\frac{y_1^*}{y^*} - (1-\alpha)a_1 s_2 = 0 \tag{A52}$$

The positive root of this quadratic equation determines the steady state value of $\frac{y_1^*}{y^*}$. The other steady state values are then $\frac{y_2^*}{y^*} = 1 - \frac{y_1^*}{y^*}$, $\frac{k_1^*}{k^*}$ follows from (A49) and similarly $\frac{k_2^*}{k^*} = 1 - \frac{k_1^*}{k^*}$. Equation (A32) then determines s_n^*. Equation (A23), determines k^*.

B Empirical analysis

Figure A1: Historical evolution of top shares



Note: Top 1% and top 10% shares for pre-tax national income and net personal wealth. Income shares are measured using the population income distribution, wealth shares are measured using the population wealth distribution. Data from the World Inequality Database.

Table A1: Unit root tests

		Top 1%		Top 10%	
		France	US	France	US
sh_W	*H0: Unit Root, I(1)*				
	ADF	0.27	0.32	0.55	0.60
	PP	0.29	0.32	0.58	0.58
	ERS	0.98	0.38	0.90	0.36
	PV	0.74	0.70	0.12	0.34
	H0: Stationarity, I(0)				
	KPSS	1.05***	0.56**	1.13***	0.78***
sh_Y	*H0: Unit Root, I(1)*				
	ADF	0.40	0.45	0.35	0.74
	PP	0.36	0.67	0.35	0.74
	ERS	0.69	0.17	0.33	0.77
	PV	0.11	0.95	0.81	0.28
	H0: Stationarity, I(0)				
	KPSS	0.88***	0.35*	0.98***	0.55**

Note: The upper (lower) panel shows unit root tests for the share of wealth sh_W (share of income sh_Y). The table reports p-values for the tests by Dickey and Fuller (1979) (ADF), Phillips and Perron (1988) (PP), Elliott et al. (1996) (ERS) and Perron and Vogelsang (PV). For the test by Kwiatkowski et al. (1992) (KPSS), the LM-statistic is shown, with stars indicating * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.