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Monetary and Macroprudential Policy Interactions in a Model of the European Union

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Keywords

Monetary policy, macro-prudential policy, policy coordination

JEL Classification

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Monetary and Macroprudential Policy Interactions in a Model of the European Union*

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Abstract

We use the two-country euro-area model developed by Quint and Rabanal (2014) to study policymaking in the European Monetary Union (EMU). We focus on strategic interactions: 1) between an EMU-level monetary authority and an EMU-level macro-prudential authority, and; 2) between an EMU-level monetary authority and regional macro-prudential authorities. In the former, price stability and financial stability are pursued at the EMU level, while in the latter each macro-prudential authority adopts region-specific objectives. We compare cooperative and non-cooperative equilibria in simultaneous-move and leadership environments, each obtained assuming discretionary policymaking. Further, we assess the effects on policy performance of assigning shared objectives across policymakers and of altering the relative importance attached to different policy objectives. In the three-policymaker setting, we find that regional macro-prudential policymakers play an important role in achieveing regional stability.

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1 Introduction

This paper studies the strategic interaction between monetary and macroprudential policies in the context of a two region international business cycle model of the Euro-area. During the European debt crisis that emerged from the global financial crisis, banks and financial institutions across different countries and regions in the European Monetary Union (EMU) faced different balance-sheet pressures and liquidity concerns. As a result, governments were forced to respond at the national level to rescue troubled financial institutions, which in turn put upward pressure on debt burdens in countries such as Greece, Ireland, Italy, Portugal and Spain, and has led to increased macroeconomic divergence with countries such as France and Germany. Since regional imbalances across the banking system could not be addressed through conventional monetary policy, greater focus was placed on macroprudential policy.

To better understand the interaction between monetary policy and macroprudential policy, and the effect this interaction has on different regions of Europe, we study policy interactions using the two-region Dynamic Stochastic General Equilibrium (DSGE) model that Quint and Rabanal (2014) estimated for the Euro-area. Their framework allows us to analyze the current policy landscape in the European Monetary Union (EMU), where the European Central Bank (ECB) is charged as the single monetary authority with the task to maintain price stability, while at the same time its area-wide macroprudential responsibilities are shared with national-level macroprudential authorities. This multilayer-feature of policy-setting makes cooperation across the different authorities a challenging task. We investigate the consequences for macroeconomic stability when cooperation does not occur, and identify cases, such as the pursuit of regional or country-specific objectives, in which it might not even be desirable for authorities to cooperate.

We perform our analysis in a setup where each policy maker is assumed to minimize a loss function, and behave as a strategic player in a dynamic game that is either cooperative or non-cooperative. We study simultaneous-move and leadership settings, allowing us to distinguish between various timings of the different players' moves. In the first setting, we consider an EMU-wide monetary authority and an EMU-wide macroprudential authority, each pursuing area-wide objectives, and thereby ignoring regional imbalances and heterogeneity across the euro-area. This is a two-player setting that is similar in spirit to the closed-economy analyses performed by De Paoli and Paustian (2017), Angelini et al. (2012), Bean et al. (2010), Beau et al. (2014), Darracq Paries et al. (2011), Gelain and Ilbas (2017), and others. In the second setting, we assume that macroprudential duties are performed at the regional level[1],

[1] While in reality the ECB and national authorities have shared competences, the role of the former is limited to imposing stricter capital requirements as foreseen in the EU legislations. Allowing for only national prudential authorities however simplifies the analysis without too much loss of realism.

where one authority is in charge of macroprudential policy in the euro-area core and another authority is in change in the euro-area periphery, each pursuing regional objectives, while the ECB remains in charge of area-wide price stability.

In the two-policymaker setting, we find that the gains from either the ECB or the macroprudential regulator having a first-mover, or leadership, advantage are somewhat limited. In fact, when monetary policy and macroprudential policy are conducted at the area-wide level, the most favorable outcome is achieved under cooperation where both policies are formulated and conducted simultaneously. The lack of any substantial leadership advantage indicates either that the two policies operate through distinct channels or that there is considerable alignment between their differing objectives. Our finding that there was little leadership advantage remained robust when we varied the objective functions, suggesting that it is due to the policies operating through distinct channels. For both simultaneous-move and leadership settings, we find that successful stabilization of area-wide objectives comes at the cost of highly volatile credit-to-GDP ratios in both the core and the periphery (although the credit-to-GDP ratio is very stable in the union as a whole). Focusing on non-cooperative policies, but assigning area-wide real GDP growth as a common objective leads to non-cooperation outperforming cooperation, but it does not remove the excessive volatility in the regional-level credit-to-GDP ratios.

The regional-level volatility that emerges when policy is conducted at the area-wide level suggests there may be some advantages to having macroprudential policy conducted at the regional level. To explore this issue we introduce regional-level macroprudential policymakers and move to a three-policymaker setting. In the three-policymaker setting, we consider cooperation and non-cooperation with the latter allowing each macroprudential regulator to focus on their own region-specific objectives. For standard objectives that focus on factors such as inflation, real GDP growth, and the credit-to-GDP ratio, we find that non-cooperation out-performs cooperation. We continue to find very little leadership advantage when monetary policy has a leadership role. Overall, we found that delegating macroprudential policy to regional macroprudential regulators damped the volatility of the credit-to-GDP ratios in the core and the periphery, while having little impact on area-wide outcomes for output growth and inflation.

A notable aspect of our analysis is the presence of three policymakers whose cooperative and non-cooperative interactions are analyzed. Allowing three policymakers with separate instruments and separate objectives to interact yields a broader and more realistic policy analysis within the context of a monetary union. Moreover, within the non-cooperative setting, we make the further distinction between simultaneous-move and leadership equilibria under optimal discretionary policy. In this respect, our analysis differs from most existing literature. In particular, to our knowledge, other papers that employ a similar, two-country, setup to analyze policy interactions in a monetary union, such as Brzoza-Brzezina et al. (2015) and Quint and Rabanal (2014), do not consider and explore the effects of strategic interactions between more than two policymakers. Although Dehmej and Gambacorta (2019) and Sergeyev (2016) consider three policy makers in a monetary union, their results are analytical in nature and they only consider the setting in which policymakers set policy simultaneously. The latter paper, in addition, focuses on the optimization of a single (i.e. cooperation) welfare-based loss function under Ramsey policy, while we also consider distinct loss functions for each policy maker in order to investigate non-cooperative equilibria. The approach of optimizing a single welfare function is also followed by Palek and Schwanenbeck (2019), who compare discretion to simple rules, as well as by Rubio (2014), Rubio and Carrasco-Gallego (2016) and Agénor et al. (2021), whose focus is on optimized simple rules. In all the aforementioned papers, the decision-making environment in a non-cooperative setting is one in which policymakers move simultaneously. Although a leader-follower environment setup is considered by Poutineau and Vermandel (2017), in their analysis all three policy makers employ optimized simple rules supported by a welfare-based loss function, while we assign each policy maker their own loss function (allowing us to consider non-cooperation) where each objective function is intended to reflect their respective mandates. Moreover, our focus is on discretionary policymaking and not on optimized simple rules.

This paper is organized as follows. Section 2 summarizes the economic model, which is based on Quint and Rabanal (2014). Section 3 explains the monetary and macroprudential policy framework adopted throughout the paper. Section 4 presents the results of the strategic interactions between monetary and macroprudential policy at the area-wide level, while section 5 presents the results when the interactions occur between area-wide monetary policy and regional macroprudential policies. Finally, section 6 concludes.

2 The modeling framework

Our analysis is conducted using the model developed and estimated in Quint and Rabanal (2014). Briefly, the Quint and Rabanal (2014) model is a two-region model of Europe, with the regions corresponding to the "core" and the "periphery", respectively. The core consists of France and Germany while the periphery contains Greece, Ireland, Italy, Portugal, and Spain. Each region has sectors that produce nondurables, which can be consumed or traded, and durables, which cannot be traded and are accumulated (subject to an adjustment cost) to augment the housing stock. The durables and nondurables sectors are monopolistically competitive, consisting of intermediate-good producers who set their price subject to a Calvo-based (Calvo, 1983) price rigidity and inflation indexation (Christiano, Eichenbaum and Evans, 2005).

On the demand side, each region is populated by two types of households: borrowers and savers, where borrowers are more impatient than savers, characterized by a lower discount factor. Both types of household obtain utility by consuming non-durable goods and housing services, and obtain disutility by supplying labor, working in both sectors to produce durables and nondurables. However, there is a reallocation cost associated with labor moving from one sector to the other. For both borrowers and savers, the consumption of non-durable goods exhibits external habit formation. The nominal wage is assumed to be perfectly flexible.

With regard to the financial sector, both the core and the periphery contain financial intermediaries. These intermediaries receive the savings of the patient households and lend them to the impatient households. Excess savings in one region can be transferred to the other region through the purchase of one-period nominal bonds from an international intermediary. Impatient households, or borrowers, can purchase one-period nominal loans from their regional financial intermediary, using the value of their housing stock as collateral. However, the quality of housing as collateral is subject to an idiosyncratic shock and this creates some risk that a borrower may not repay their loan. If the value of the housing stock falls below the loan repayment, then the borrower will default. In instances that default occurs, the borrower pays the house's collateral value, which is split between the financial intermediary and a debt collection agency, and keeps their house. The revenues generated by the debt collectors are returned (lump-sum) to savers. Although the regional financial intermediaries are risk neutral, this is not the case for the international financial intermediary, which charges a risk premium (related to the region's debt-to-GDP ratio) on regional borrowing.

While the model is large, with multiple regions, sectors, and agents, we use it because it is estimated, because it provides roles for both monetary policy and macroprudential policy, and because it allows the effects of these policies on the core and the periphery to be examined. We highlight some key aspects of the model below and describe it in more detail in Appendix A; interested readers should consult Quint and Rabanal (2014).

2.1 Key shocks and policy channels

Risk shocks are an important aspect of the model. Borrowers face an idiosyncratic shock to the value of their collateral (their housing stock) and are more likely to default on their loan when the idiosyncratic shock is small and their collateral has less value. The standard deviation of each idiosyncratic shock is stochastic and assumed to follow an autoregressive process. An increase in the standard deviation affects the volatility, but not the mean of the shock to collateral quality, and is termed a risk shock. When a risk shock occurs collateral-quality becomes more variable making it more likely that realized collateral quality will be below the default threshold, which increases the default rate and raises the credit spread. The increased credit spread adversely effects the balance sheet of financial intermediaries, which has ongoing adverse effects on bank lending. One way to think about these risk shocks is that they raise the loan-to-value ratio for borrowers. Other shocks that raise the loan-to-value ratio have similar effects on the default rate, the credit spread, and bank lending.

The policy instrument for the monetary authority is assumed to be the bank deposit rate, which, because monetary policy is conducted at the area-wide level, means financial intermediaries in both regions pay the same rate on deposits. A monetary policy tightening has the effect of raising the deposit rate, which encourages saving by the patient households and discourages borrowing by the impatient households. As such, tighter monetary policy has the usual damping effect on consumption spending. However, the higher deposit rate also raises the cost to intermediaries of providing loans, which gets passed on to borrowers through a higher lending rate and can lower the value of housing collateral. All else constant, a higher lending rate raises the loan-to-value ratio, increases the default rate and the credit spread.

Macroprudential policy is assumed to operate through a variable lending fraction, with tighter macroprudential policy associated with a lower lending fraction. In this model, when the macroprudential authority lowers the lending fraction, perhaps by requiring financial intermediaries to increase their reserves or to raise their loan-loss provisions, financial intermediaries must restrict their lending, which leads to a higher lending rate and an increase in the credit spread, and has an adverse effect on the demand for goods by borrowers, particularly.

3 Monetary and macroprudential policy framework

In addition to private agents, the model is inhabited by a monetary authority and by a macroprudential authority (or two authorities, for some specifications) and is one in which the policies of each policy institution can have important economic effects. Recognizing the model's size and complicated structure, we approach the decision problems faced by the monetary authority and the macroprudential authority in terms of constrained optimization problems in which each authority chooses its policy to optimize an objective function, subject to constraints, some of which are forward-looking. We take the "primal" approach and express objectives in terms of loss functions that view unfavorably volatility in key macroeconomic aggregates, such as inflation, output growth and the credit-to-GDP ratio. We take the approach of specifying policy objectives in terms of loss functions for a couple of reasons. First, policy objectives involving targets for inflation and credit-to-GDP ratios appear in legislation and/or policy mandates, unlike objectives expressed (directly) in terms of welfare, and can be readily captured through loss functions. Second, exercises such as Schmitt-Grohe and Uribe (2007) showed that welfare-based objectives can often lead to extreme policies, ones in which policy coefficients need to be artificially constrained.[2]

3.1 Objectives and instruments of the monetary authority

The central bank operates at the area-wide level. Its objectives are captured by the following intertemporal loss function:

$$L_t^{CB} = (1-\beta)\, E_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda_y (y_{t+i} - y_{t-1+i})^2 + \lambda_r (r_{t+i} - r_{t-1+i})^2\right], \tag{1}$$

where $0 < \beta < 1$ denotes the central bank's discount factor and E_t denotes the mathematical expectation operator conditional on information available at time t. With π_t, y_t, and r_t representing annual inflation, output, and the deposit rate (the monetary policy instrument) at the area-wide level, respectively, the central bank is assumed to set the deposit rate in order to stabilize area-wide annual inflation and area-wide real output growth, without creating large changes in the deposit rate. The weight on the inflation target is normalized to one, hence the weights on output growth, $\lambda_y > 0$, and on the change in the nominal interest rate, λ_r, indicate the importance assigned to stabilizing these variables relative to stabilizing inflation.

As is well-known, in the limit as $\beta \uparrow 1$, equation (1) converges to:

$$L_t^{CB} = Var(\pi_t) + \lambda_y Var(\Delta y_t) + \lambda_r Var(\Delta r_t), \tag{2}$$

where $Var(\pi_t)$, for example, denotes the unconditional variance of annual inflation. The central bank conducts policy to minimize equation (2) under discretion, subject to restrictions that come from the structural model.

[2]Further, although it is possible to construct second-order accurate measures of welfare for each household-type in each region of the model, exactly how to perform the aggregation is unclear. The method of Negishi (1960) is often invoked, but it cannot be used here as it requires the welfare theorems to hold.

3.2 Objectives and instruments of the macroprudential authority

With regard to the macroprudential loss function we consider two distinct cases. In the first case, macroprudential policy is conducted at the area-wide level and the macroprudential authority (or regulator) has the area-wide loss function:

$$L_t^{MP} = (1-\beta)\, E_t \sum_{i=0}^{\infty} \beta^i \left[\delta_{cr} (cr_{t+i})^2 + \lambda_\eta (\eta_{t+i} - \eta_{t-1+i})^2 \right], \tag{3}$$

where cr_t represents credit-to-GDP ratio. Following Quint and Rabanal (2014), the regulator sets the macroprudential instrument, η_t, which influences credit spreads by affecting the fraction of funds that financial intermediaries are able to lend. We assume that the area-wide macroprudential authority has a double mandate. Specifically, it is tasked with stabilizing both the area-wide credit-to-GDP ratio, cr_t, without making large changes in the lending fraction, so the weights on these two objectives, δ_{cr} and δ_η, respectively, are positive. As earlier, we consider the limiting case where $\beta \uparrow 1$ so that the macroprudential authority conducts policy under discretion by choosing η_t to minimize:

$$L_t^{MP} = \delta_{cr} Var\left(cr_t\right) + \delta_\eta Var\left(\Delta \eta_t\right), \tag{4}$$

subject to restrictions reflected in the structural model.

In the second case, we assume that each region—the core and the periphery—has its own macroprudential authority, or regulator, and that each regulator seeks to stabilize its own region's credit-to-GDP ratio and to smooth its own lending fraction. For this case, the macroprudential loss functions for the core and periphery are, respectively:

$$L_t^{MP,c} = (1-\beta)\, E_t \sum_{i=0}^{\infty} \beta^i \left[\delta_{cr^c} (cr_{t+i}^c)^2 + \delta_{\eta^c} (\eta_{t+i}^c - \eta_{t-1+i}^c)^2 \right], \tag{5}$$

$$L_t^{MP,p} = (1-\beta)\, E_t \sum_{i=0}^{\infty} \beta^i \left[\delta_{cr^p} (cr_{t+i}^p)^2 + \delta_{\eta^p} (\eta_{t+i}^p - \eta_{t-1+i}^p)^2 \right], \tag{6}$$

where we distinguish between the macroprudential instruments for core and the periphery, η_t^c and η_t^p, respectively. Again, in the limit as $\beta \uparrow 1$, these loss functions converge to:

$$L_t^{MP,c} = \delta_{cr^c} Var\left(cr_t^c\right) + \delta_{\eta^c} Var\left(\Delta \eta_t^c\right), \tag{7}$$

$$L_t^{MP,p} = \delta_{cr^p} Var\left(cr_t^p\right) + \delta_{\eta^p} Var\left(\Delta \eta_t^p\right). \tag{8}$$

3.3 Strategic interaction between monetary and macroprudential policy

We consider optimal discretionary policies under both cooperation and non-cooperation. Cooperative policies are the outcome of shared policy objectives while non-cooperative policies allow the central bank and the macroprudential regulator to have distinct policy objectives. For the non-cooperative case, we consider three timing environments. The first of these timing environments is where both policymakers move simultaneously whereas the second and third environments correspond to those where either the monetary authority or the macroprudential authority has leadership or first-mover advantage, i.e., moves first within the period. These three discretionary policies are compared to an optimal benchmark, which is described by the optimal commitment policy under cooperation. For the cooperative case, there is no first-mover advantage so we consider the single environment where the policymakers choose simultaneously. Clearly, the interaction between monetary policy and macroprudential policy will depend on whether macroprudential policy operates at the regional level or the area-wide level, and we consider each in turn below.

3.3.1 Cooperation

In the case of a area-wide macroprudential authority, the policy problem involves two "players", a area-wide monetary authority and a area-wide macroprudential authority, whose policies can potentially interact. When these two policymakers cooperate, their respective loss functions, equations (2) and (4), are combined into a single joint loss function, which is given by:

$$L_t^{coop} = Var\left(\pi_t\right) + \lambda_y Var\left(\Delta y_t\right) + \lambda_r Var\left(\Delta r_t\right) + \delta_{cr} Var\left(cr_t\right) + \delta_\eta Var\left(\Delta \eta_t\right). \qquad (9)$$

The resulting decision problem can be interpreted two ways: as a problem with one policy maker that has two policy instruments (i.e. the interest rate, r_t, and the lending fraction, η_t) or as a problem with two policymakers having one policy instrument each. For this application, we prefer the second interpretation, but both interpretations produce the same results and each decision problem boils down to a standard discretionary problem and can be solved using standard methods (Dennis, 2007).

3.3.2 Non-cooperation

Non-cooperation differs from cooperation in as much as the two policymakers do not share a common objective function. As mentioned above, when policymakers do not cooperative the timing with which decisions are made can be important and it can matter whether one policy maker has a leadership advantage relative to the other. For this reason, with non-cooperation we consider the case where the two policymakers move simultaneously as well as cases where one policy maker (the leader) has a first-mover advantage with respect to the other policy maker (the follower). In these cases, the instrument of the leader is chosen first and the follower sets its instrument taking the leader's policy into account. With this timing structure, the leader can predict the follower's reaction and exploit this reaction when setting its own policy. We consider both monetary leadership and macroprudential leadership. In each of these three non-cooperative cases, the monetary authority and the macroprudential regulator move separately, with the monetary authority choosing r_t to minimize equation (2) and the macroprudential regulator choosing η_t to minimize equation (4), each subject to constraints imposed by the model equations.

3.3.3 Regional macroprudential policies

When macroprudential policy is formulated at the regional level the model has two macroprudential authorities, one conducting macroprudential policy in the core and the other in the periphery. Because there are two macroprudential authorities and a area-wide central bank, the model contains three policymakers with each choosing its policy optimally under discretion. With three policymakers all optimizing, the range of strategic environments that could be considered proliferate. For the cooperative case, the (cooperative) loss function is given as the sum of their respective loss functions, i.e., the sum of equations (2), (7) and (8):

$$\begin{aligned} L_t^{CB} + L_t^{MP,c} + L_t^{MP,p} &= Var\left(\pi_t\right) + \lambda_y Var\left(\Delta y_t\right) + \lambda_r Var\left(\Delta r_t\right) \\ &\quad + \delta_{cr^c} Var\left(cr_t^c\right) + \delta_{\eta^c} Var\left(\Delta \eta_t^c\right) \\ &\quad + \delta_{cr^p} Var\left(cr_t^p\right) + \delta_{\eta^p} Var\left(\Delta \eta_t^p\right). \end{aligned} \tag{10}$$

As above, this cooperative loss function is minimized as a standard discretionary problem with the three instruments, r_t, η_t^c, and η_t^p chosen simultaneously.

In addition to the simultaneous-move cooperative case, we also consider the leadership case where the area-wide central bank has a first-mover advantage with respect to the two macroprudential authorities, who are assumed to choose their policies simultaneously with each other, but following the central bank. Finally, we also consider the non-cooperative case where the central bank and the core and periphery macroprudential authorities have differing policy objectives, governed by equations (2), (7) and (8), respectively.

4 Monetary-macroprudential interactions at the area-wide level

In this section we focus on area-wide policymaking and explore the interactions between monetary policy and macroprudential policy for a range of cooperative and non-cooperative environments. We begin by comparing optimal discretionary policy to the optimal commitment policy and to an estimated Taylor-rule policy. The optimal commitment policy serves as a useful benchmark because departures by the discretionary policy from the commitment outcome represent sub-optimality due to time-inconsistency. Similarly, differences between the discretionary policy and the Taylor-rule outcomes can reflect important differences between the suggested policy and estimated policy behavior.

4.1 Taylor rule vs. optimal cooperative policies

Table 1 presents summary statistics for four different policy regimes. The first regime is one where monetary policy is conducted according to estimated Taylor rule and macroprudential policy is implemented by a constant lending fraction. This estimated Taylor rule is the rule that was obtained by Quint and Rabanal (2014) when estimating the whole model. The second regime has monetary policy again conducted according to the estimated Taylor rule while macroprudential policy is conducted optimally under commitment. The third and fourth regimes have both monetary policy and macroprudential policy set cooperatively with decisions made simultaneously, but under commitment and discretion, respectively. For the cases where policymakers are cooperating, we assume that the shared loss function is given by equation (9) with $\lambda_y = 1$, $\lambda_r = 0.5$, $\delta_{cr} = 1$, and $\delta_\eta = 0.5$; we also use this parameterization to evaluate loss for all four regimes.

To interpret the results, we begin by comparing the first and second regimes in which monetary policy follows a Taylor rule. When macroprudential policy is implemented via a constant lending fraction the outcome is a high value for the loss function brought about by an extremely volatile credit-to-GDP ratio, volatility that arises because when the lending fraction does not respond to shocks their impact is felt directly on the level of lending that banks can undertake. Allowing the lending fraction to be set endogenously, as per the second of the four regimes, leads to a drastic decline in both the volatility of the credit-to-GDP ratio and the level of loss, but interestingly, it also leads to a small increase in the volatility of inflation.

Looking now at the cooperative policies for which both monetary policy and macroprudential policy are chosen optimally we see that allowing monetary policy to be set optimally—whether under commitment or discretion—leads to big declines in volatility, especially for the credit-to-GDP ratio and annual inflation. Importantly, these EMU-wide declines in inflation and credit-to-GDP volatility are achieved without generating excessive volatility in either the nominal interest rate or the lending fraction. Because the main differences between regimes 2 and 3 rests in how monetary policy is conducted, these big declines in volatility speak to the importance of having monetary policy set optimally.

Comparing the commitment and discretionary policies (regimes 3 and 4), the difference between them that stands out is the higher inflation volatility and lower output growth volatility under commitment. This result might be a bit surprising from the viewpoint of models without financial frictions and in which monetary policy only is used for macro-stabilization, where the discretionary stabilization bias leads to output being over-stabilized. However, in the presence of an additional macroprudential policy maker, and with time-inconsistency affecting more than just the trade-off between inflation and output volatility, higher inflation volatility under commitment can arise when policy promises, or forward guidance, is directed more toward stabilizing the credit-to-GDP ratio than towards stabilizing inflation.[3] Interestingly, no stable discretionary solution could be found when monetary policy only was used to minimize equation (9) (with macroprudential policy set to deliver a constant lending fraction).

Table 1: **Some benchmark cooperative solutions**

	Estimated Taylor rule		Optimal cooperative policy	
	Constant lending fraction	Optimal lending fraction	Commitment	Discretion
σ^2_π	1.123	1.276	0.159	0.081
$\sigma^2_{\Delta y}$	1.973	1.744	1.545	1.757
$\sigma^2_{\Delta r}$	0.305	0.280	0.162	0.287
σ^2_{cr}	82.647	0.430	0.027	0.080
$\sigma^2_{\Delta\eta}$	0.000	0.188	0.163	0.178
L^{coop}	85.896	3.683	1.892	2.150

Note: The table reports the variances and the unconditional losses under the estimated Taylor rule (and absent macroprudential policy) and under optimal cooperative policies (commitment and discretion) where the EMU prudential regulator and the central bank jointly minimize equation (9), and the case where macroprudential policy only acts optimally to minimize equation (9). Where policy is determined optimally, and for evaluating policy losses, we set $\lambda_y = 1$, $\lambda_r = 0.5$, $\delta_{cr} = 1$, and $\delta_\eta = 0.5$.

The following two figures compare the impulse responses to a risk shock in the periphery and the core, respectively, under the estimated Taylor rule (with constant lending fraction) and the optimal cooperative policies (commitment and discretion) for the parameterization of the joint loss function (9) considered above. First of all, the spillover of the shock from the periphery to the core is qualitatively similar in magnitude to the spillover of the shock from the core to the periphery, even though the risk shock in the periphery is twice as volatile as that of the core. The figures also reveal only small differences in the responses under commitment and discretion, suggesting that time-inconsistency considerations do not seem to cause discretionary policy to respond with great inefficiency to risk shocks. Looking at the differences between the optimal cooperative policies and the Taylor rule policy, Figure 1

[3] With a lower weight assigned to stabilizing the credit-to-GDP ratio in the shared loss function, the usual stabilization bias associated with discretionary policymaking reasserts itself.

shows that the absence of macroprudential policy results in a more accommodative interest rate than the optimal polices, which is needed to address the larger fall in the EMU-wide credit-to-GDP ratio and the deeper recession at the EMU level. In contrast to the optimal cooperative policies—where the shock creates a boom in the core—under the Taylor rule policy the core is also hit by a recession. With both the core and the periphery experiencing a recession, the Taylor rule's policy response is the appropriate one (qualitatively) for both regions.

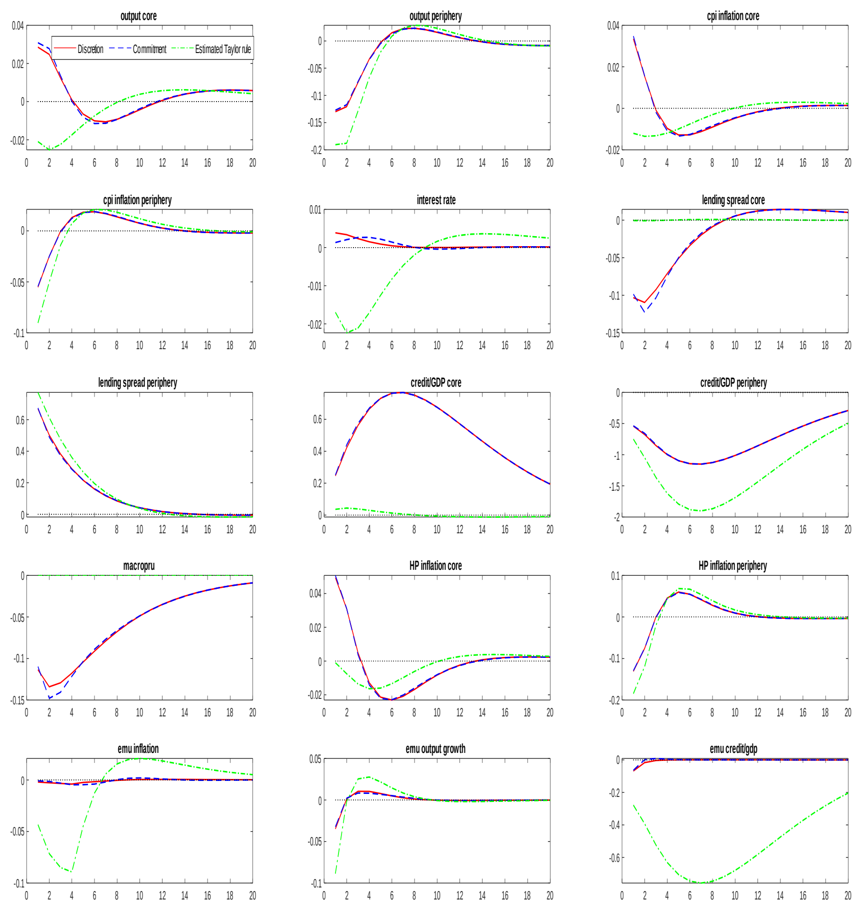

Figure 1. Responses to a risk shock in the periphery: estimated rule vs. cooperative policies (commitment vs. discretion)

Note: The figure plots the responses to a risk shock in the periphery under the estimated Taylor rule and the optimal cooperative policies (commitment and discretion) for the loss function, equation (9), with the following weights: $\lambda_y = 1$, $\lambda_r = 0.5$, $\delta_{cr} = 1$, $\delta_\Delta = 0.5$.

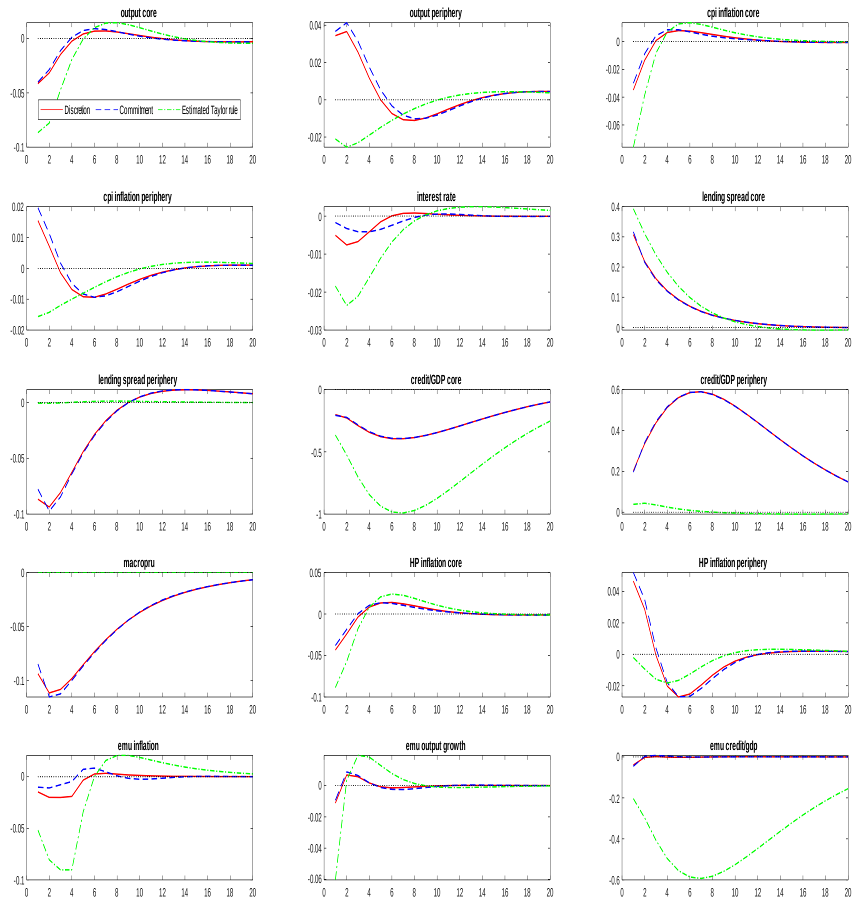

Figure 2. Responses to a risk shock in the core: estimated rule vs. cooperative policies (commitment vs. discretion)

Note: The figure plots the responses to a risk shock in the core under the estimated Taylor rule and the optimal cooperative policies (commitment and discretion) for the loss function, equation (9), with the following weights: $\lambda_y = 1$, $\lambda_r = 0.5$, $\delta_{cr} = 1$, $\delta_\eta = 0.5$.

4.2 Non-cooperation

In the case of cooperation, when monetary policy is set optimally under either commitment or discretion there is no advantage to a policy maker from being either a leader or a follower; both situations give the same outcomes as simultaneous choice. But this is not necessarily so for the non-cooperative case where policymakers do not share the same policy objectives. To investigate non-cooperative policy outcomes, we assign distinct loss functions to the monetary authority and the prudential regulator and thereby assess the extent to which it matters whether one or other policy maker is the leader or the follower. We assume that policymakers act under discretion.

Table 2 reports the unconditional variances and the losses under cooperative and non-cooperative equilibria. As before the cooperative environment assumes that the two policymakers share equation (9) as their loss function and the results shown are consistent with those in Table 1. For the non-cooperative policies we assume that the central bank minimizes the loss function (2) with $\lambda_y = 1$, $\lambda_r = 0.5$ and that the macroprudential regulator minimizes equation (4) with $\delta_{cr} = 1$, $\delta_\eta = 0.5$. A key feature of this assignment is that when added together the combined loss function has the same structure as the cooperative loss function. With each policy maker having distinct policy objectives, we consider timing environments in which both policymakers move simultaneously and in which either the central bank or the macroprudential authority has a first-mover advantage. Qualitatively, two results in Table 2 stand out. The first result is that the differences between the cooperative outcomes and the non-cooperative outcomes are relatively small. This result most likely arises because there is considerable alignment, or co-movement, across the policy objectives, with the non-cooperative objectives formed from the goals in the cooperative objective. The second result is that among the non-cooperative policies there is little difference between the leadership cases and the simultaneous move case. This similarity in equilibrium outcomes across the non-cooperative policies appears to be due to monetary policy and macroprudential policy operating through distinct channels, leading to relatively little policy interaction.

Table 2: **Comparing cooperation and non-cooperation**

	Cooperation	Non-cooperation		
		Simultaneous move	Monetary leadership	Prudential leadership
σ^2_π	0.081	0.100	0.097	0.100
$\sigma^2_{\Delta y}$	1.757	1.834	1.822	1.832
$\sigma^2_{\Delta r}$	0.287	0.349	0.386	0.350
σ^2_{cr}	0.080	0.033	0.033	0.030
$\sigma^2_{\Delta \eta}$	0.178	0.076	0.076	0.081
L^{CB}	1.981	2.108	2.111	2.107
L^{MP}	0.168	0.071	0.071	0.071
L^{coop}	2.150*	2.179	2.182	2.177
L^c	37.990	37.882	37.882	37.882
L^p	85.271	85.201	85.201	85.198

Note: The table reports the variances and the unconditional losses under alternative cooperative and non-cooperative schemes. For cooperation the shared policy objective has the form of equation (9) with $\lambda_y = 1$, $\lambda_r = 0.5$, $\delta_{cr} = 1$, and $\delta_\eta = 0.5$. In the non-cooperative cases, the prudential regulator minimizes equation (4) with $\delta_{cr} = 1$, $\delta_\eta = 0.5$ and the central bank minimizes equation (2) with $\lambda_y = 1$, $\lambda_r = 0.5$.

Overall, the cooperative solution slightly outperforms the non-cooperative solutions, with its success driven primarily by greater stability of the monetary policy objectives. From the individual policymakers' perspectives, column two shows that the central bank clearly gains from cooperation. In contrast, the macroprudential regulator would fare better with less cooperation, as non-cooperation allows the prudential regulator to better stabilize the credit-to-GDP ratio with smaller changes in the lending fraction. These findings remain valid when a lower weight on output growth is assigned in the central bank's loss function ($\lambda_y = 0.5$), and when the weight on both smoothing coefficients is increased ($\lambda_r = \delta_\eta = 1$).

While the policy objectives all relate to area-wide outcomes, we also report in Table 2 the corresponding losses for the core and the periphery. What we see is that although union-wide policy objectives delivers stability at the area-wide level, this stability masks considerable volatility at the regional level. In particular, where the credit-to-GDP ratio is very stable at the area-wide level the underlying credit-to-GDP ratios in the core and the periphery are highly volatile. This finding suggests that, although some area-wide stability might be sacrificed, macroprudential policies directed at stabilizing regional variables might drastically reduce regional-level volatility.

Figure 3 plots the impulse responses for a positive risk shock in the periphery for the same parameterization of the loss function weights (2) and (4) as used in Table 2, i.e., $\lambda_y = 1$, $\lambda_r = 0.5$, $\delta_{cr} = 1$, and $\delta_\eta = 0.5$. The figure compares the cooperative and non-cooperative environments where policymakers move simultaneously[4] and shows the consequences of non-cooperation among the monetary and the prudential authorities when the economy is hit by a positive risk shock in the periphery. The risk shock increases the lending-deposit spread in the periphery, which lowers credit and house prices in the periphery, leading to a recession in the periphery. Because the area-wide credit-to-GDP ratio decreases, the area-wide macroprudential regulator loosens, raising the lending fraction, in order to stimulate lending. The lower credit standards adopted at the area-wide level, however, leads to a boom in economic activity and to higher inflation in the core. Under cooperation, the central bank increases slightly the interest rate in order to contain area-wide inflation. Under non-cooperation, the regional macroprudential authorities are less constrained than they are under cooperation and the task of stabilising inflation falls solely on the monetary authority, and, as a result, area-wide inflation is slightly higher, while the regional-level variables are largely unaffected.

[4] We leave out the leadership solution from the comparison as the qualitative differences with the simultaneous move solution are small.

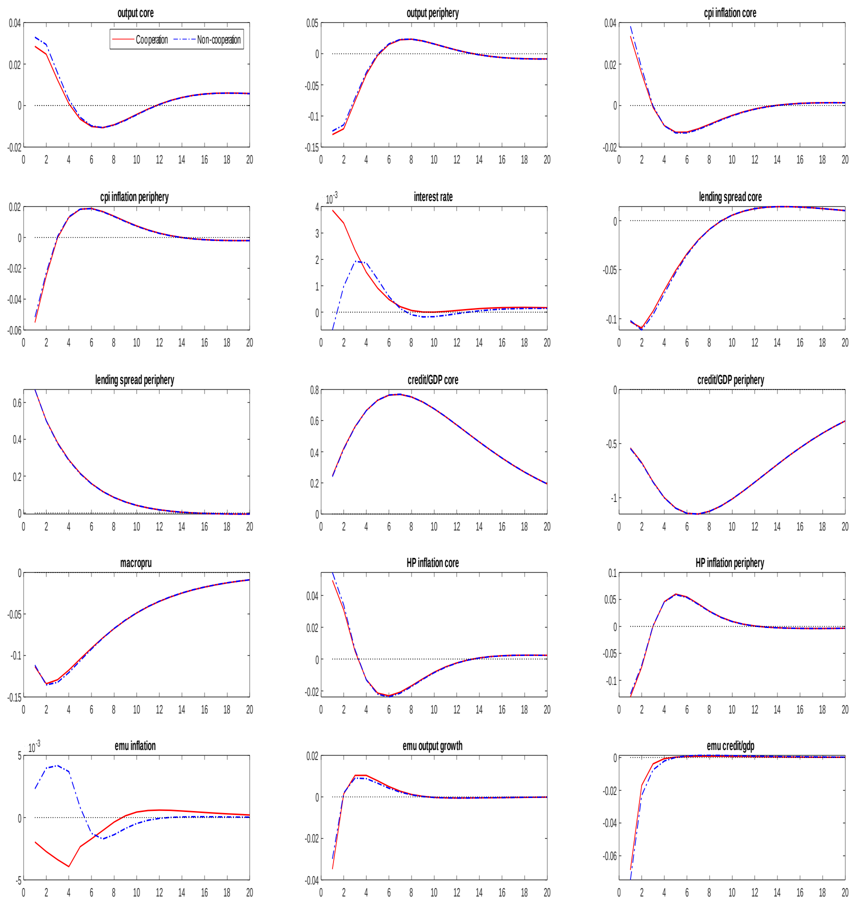

Figure 3. Responses to a risk shock in the periphery: cooperation vs. non-cooperation

Note: The figure plots the impulse responses to a risk shock in the periphery under cooperation and non-cooperation for the following parameterizations of the loss function weights: $\lambda_y = 1$ and $\lambda_r = 0.5$ in equation (2) and $\delta_{cr} = 1$, $\delta_\eta = 0.5$ in equation (4)

Figure 4 shows the impulse responses to a area-wide technology shock. Both the core and the periphery react in a similar way to the shock, with output rising and inflation falling. The central bank responds to the decrease in area-wide inflation by lowering the interest rate. How the macroprudential regulator responds depends on the cooperative environment. For non-cooperation the macroprudential regulator reacts to a higher credit-to-GDP ratio by lowering the lending fraction and thereby raising lending spreads in both regions. Under cooperation, however, macroprudential policy loosens, in line with monetary policy's accommodative move, which leads to lower lending spreads and higher credit-to-GDP ratios in both regions under cooperation than under non-cooperation. As a result, the area-wide credit-to-GDP ratio increases and takes longer to return to steady state.

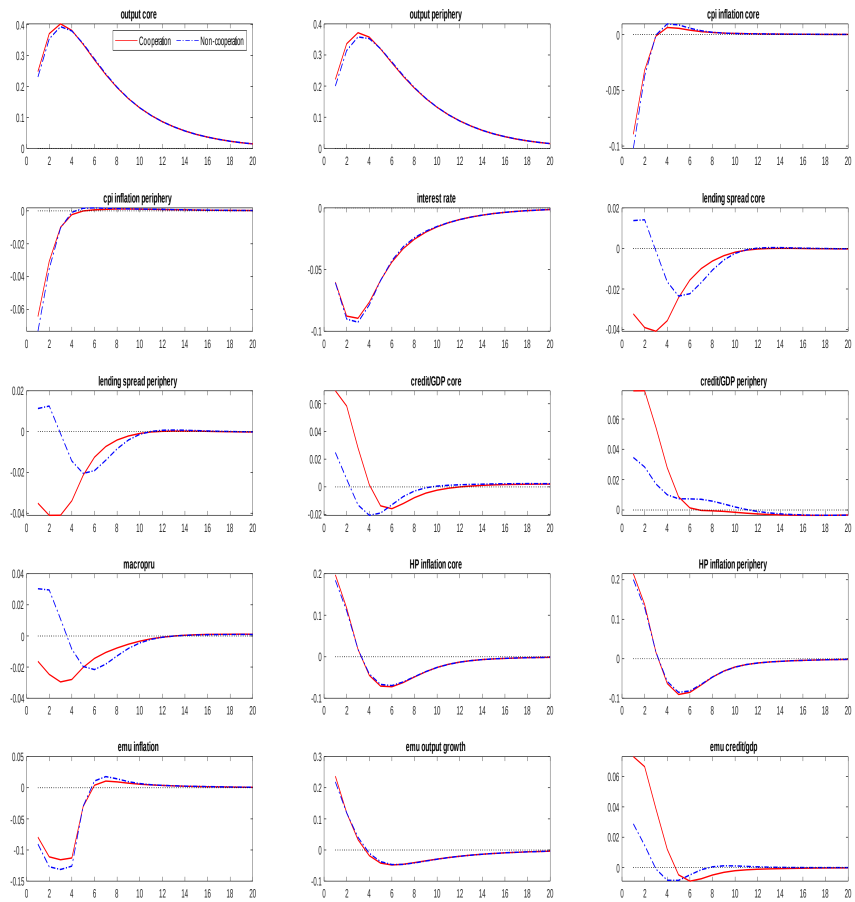

Figure 4. Responses to an EMU-wide technology shock: cooperation vs. non-cooperation

Note: The figure plots the impulse responses following a positive area-wide technology shock under cooperation and non-cooperation with the loss function weights given by $\lambda_y = 1$ and $\lambda_r = 0.5$ in equation (2) and by $\delta_{cr} = 1$, $\delta_\eta = 0.5$ in equation (4)

Our analysis of non-cooperative policy thus far has assumed that the monetary policy objective and the macroprudential objective are distinct, having no overlap. In Table 3, we consider the case where area-wide output growth is assigned as a common objective to both policymakers. Hence, the individual loss functions have one objective in common, and with each policy maker assigning equal weight to that objective, i.e., $\lambda_y = \delta_y = 0.5$. These weights imply that the central bank cares relatively less than previously about output growth, but, at the same time, macroprudential policy is now charged with stabilizing output growth, such that the total weight assigned to growth across the two policymakers remains equal to 1.0. Of central interest is whether the shared objective closes the gap between cooperation and non-cooperation, or even allows the non-cooperative policy environment to be preferred. The results in Table 3 show that assigning a common objective (with the current parameterization of the model) leaves the central bank best off under the cooperative scheme. However, this is not the case for the macroprudential policy maker, who gains most from macroprudential leadership by trading off slightly higher output volatility with more stable macroprudential objectives. Indeed, the gain to the macroprudential policy maker is such that the combined loss is now smaller under non-cooperation than it is under cooperation, suggesting that there can be advantages to allowing the two policymakers not to cooperate provided their incentives are guided by a common objective.

Table 3: **Comparing cooperation and non-cooperation: output as common goal**

	Cooperation	Non-cooperation		
		Simultaneous move	Monetary leadership	Prudential leadership
σ^2_π	0.081	0.081	0.078	0.080
$\sigma^2_{\Delta y}$	1.757	1.846	1.843	1.863
$\sigma^2_{\Delta r}$	0.287	0.233	0.243	0.235
σ^2_{cr}	0.080	0.040	0.039	0.033
$\sigma^2_{\Delta\eta}$	0.1780	0.116	0.115	0.105
L^{CB}	1.103	1.121	1.121	1.129
L^{MP}	1.047	1.021	1.018	1.017
L^{coop}	2.150	2.142	2.139*	2.146
L^c	37.990	37.921	37.920	37.910
L^p	85.271	85.204	85.203	85.200

Note: The table reports the variances and the unconditional losses under the alternative coordination schemes, where area-wide output growth is assigned as a objective to both policymakers. Hence, the area-wide regulator minimizes loss function (4) with $\delta_{cr} = 1$, $\delta_\eta = 0.5$, and $\delta_y = 0.5$, which assigns weight to area-wide output growth, and the central bank minimizes loss function (2) with $\lambda_y = 0.5$ and $\lambda_r = 0.5$.

As an alternative, in Table 4 we assign the area-wide credit-to-GDP ratio as a common objective to the two policymakers, setting $\lambda_{cr} = \delta_{cr} = 0.5$. In this case the central bank has an additional objective, while the macroprudential regulator assigns relatively less importance (with the weight declining from 1.0 to 0.5) to its main objective. Importantly, although now assigned as an objective to both policymakers, the overall importance of the credit-to-GDP ratio in the joint loss function remains unchanged, with its combined weight still equaling 1.0. The preferred cooperation scheme for the individual policymakers is in line with the case where output growth was assigned as a common objective, i.e., non-cooperation yields the lowest loss for the macroprudential regulator, while the central bank fares best under cooperation. However, the gains from non-cooperation received by the macroprudential regulator are now insufficient to overcome the greater loss received by the central bank, with cooperation remaining the preferred scheme.

Table 4: **Comparing cooperation and non-cooperation: EMU-wide credit-to-gdp as common goal**

	Cooperation	Non-cooperation		
		Simultaneous move	Monetary leadership	Prudential leadership
σ^2_π	0.081	0.097	0.094	0.097
$\sigma^2_{\Delta y}$	1.757	1.862	1.845	1.858
$\sigma^2_{\Delta r}$	0.287	0.336	0.368	0.337
σ^2_{cr}	0.080	0.063	0.063	0.058
$\sigma^2_{\Delta \eta}$	0.1780	0.046	0.048	0.050
L^{CB}	2.021	2.159	2.155	2.153
L^{MP}	0.129	0.054	0.055	0.054
L^{coop}	2.150*	2.213	2.210	2.210
L^c	37.990	37.877	37.879	37.877
L^p	85.271	85.233	85.236	85.226

Note: The table reports the variances and the unconditional losses under the alternative cooperative schemes, where the area-wide credit-to-GDP ratio is assigned as a common objective to both policymakers. Hence, the central bank minimizes loss function (2) with $\lambda_y = 1$, $\lambda_r = 0.5$, and $\lambda_{cr} = 0.5$, which assigns weight to the area-wide credit-to-GDP ratio, and the area-wide macroprudential regulator minimizes loss function (4) with $\delta_{cr} = 0.5$ and $\delta_\eta = 0.5$.

4.3 A robustness exercise

In this section we consider an alternative loss function for the macroprudential regulator, one that depends on the credit spread, as in Cecchetti and Kohler (2014), rather than on the credit-to-GDP ratio. Table 5 shows the simulation results for the case where the area-wide credit-to-GDP ratio is replaced by the area-wide average spread between the lending and deposit rate in the macroprudential policymaker's loss function. The results are reported assuming policies are chosen simultaneously under discretion, for cooperation and non-cooperation, respectively.

Looking at Table 5, the results and conclusions are qualitatively in line with those reported in Table 2. Specifically, the cooperative scheme delivers outcomes that are preferred to the non-cooperative scheme and the central bank benefits from cooperation while the prudential regulator prefers non-cooperation.[5]

Table 6 reports the results for different weights in the loss function. In a first case (regime 1), we assign lower weight to the policy instruments in each policymaker's respective

[5] The non-cooperative environments with leadership generate outcomes that are very similar to the simultaneous move case and are not reported.

Table 5: **Comparing cooperation and non-cooperation: alternative measures of financial stability**

	Cooperation	Non-cooperation
σ^2_π	2.028	2.200
$\sigma^2_{\Delta y}$	2.281	2.388
$\sigma^2_{\Delta r}$	0.915	1.301
σ^2_{sp}	0.043	0.012
$\sigma^2_{\Delta\eta}$	0.067	0.051
L^{CB}	4.766*	5.238
L^{MP}	0.080	0.037*
L^{coop}	4.843	5.275

Note: The table reports the variances and the unconditional losses when the policies are chosen simultaneously, where the central bank minimizes loss function (2) with $\lambda_y = 1$ and $\lambda_r = 0.5$ and the macroprudential regulator minimizes loss function (4) with $\delta_{sp} = 1$ and $\delta_\eta = 0.5$, where the spread instead of the credit-to-GDP ratio is used as financial objective in the latter's loss function.

Table 6: **Comparing cooperation and non-cooperation: loss function parameters**

	Cooperation	Non-cooperation	Cooperation	Non-cooperation
	Regime 1		Regime 2	
σ^2_π	0.079	0.077	0.041	0.045
$\sigma^2_{\Delta y}$	1.634	1.738	1.813	1.871
$\sigma^2_{\Delta r}$	0.625	0.764	0.360	0.389
σ^2_{cr}	0.057	0.011	0.030	0.010
$\sigma^2_{\Delta\eta}$	0.327	0.123	0.165	0.115
L^{CB}	1.838*	1.968	1.019*	1.059
L^{MP}	0.122	0.036*	0.063	0.033*
L^{coop}	1.960	2.004	1.082	1.092

Note: The table reports the variances and the unconditional losses under the alternative cooperative schemes. The first regime has the central bank minimizing loss function (2) with $\lambda_y = 1$ and $\lambda_r = 0.2$ and the macroprudential regulator minimizing loss function (4) with $\delta_{cr} = 1$ and $\delta_\eta = 0.2$. The second regime has the central bank minimizing loss function (2) with $\lambda_y = 0.5$ and $\lambda_r = 0.2$ and the macroprudential regulator minimizing loss function (4) with $\delta_{cr} = 1$ and $\delta_\eta = 0.2$.

loss functions. Not surprisingly, this leads to more volatile instruments, but also allows policymakers to better stabilize their remaining objectives. In the second case (regime 2), we lower the weight on the output component in the central bank's loss function, leading to a hierarchical ordering of the central bank's objectives where the inflation objective receives the highest relative weight. This weighting of the central bank's objectives is in line with the inflation targeting literature that gives primacy to inflation in the central bank's loss function. Although inflation is more stable in this case, at the expense of greater output volatility, the qualitative results remain consistent with Table 2. Specifically, in both experiments outlined in Table 6, the finding that the central bank gains more from cooperation and that the macroprudential regulator gains more from the non-cooperative setting remains.

4.4 Optimal assignment of objectives/weights under non-cooperation

In this section we assume that society's loss function is represented by the benchmark cooperative loss function adopted in the previous simulations:

$$L_t^{society} = Var(\pi_t) + \lambda_y Var(\Delta y_t) + \lambda_r Var(\Delta r_t) + \delta_{cr} Var(cr_t) + \delta_\eta Var(\Delta \eta_t) \qquad (11)$$

where $\lambda_y = 1$, $\lambda_r = 0.5$, $\delta_{cr} = 1$ and $\delta_\eta = 0.5$. Society minimizes equation (11) with respect to the policy instruments r_t and η_t (with the model providing constraints). The outcome of this optimization serves as a benchmark that is used to help determine appropriate loss functions for the central bank and the macroprudential regulator of the non-cooperative case. Following Debortoli et al. (2019) and Gelain and Ilbas (2017), the objectives and the weights in the two individual loss functions (under non-cooperation) are chosen to match or improve upon the benchmark cooperative policy. We find that the following assignment of loss functions does a reasonably good job of approximating the outcomes achieved by the benchmark cooperative policy:

$$L_t^{CB} = Var(\pi_t) + \lambda_y Var(\Delta y_t) + \lambda_r Var(\Delta r_t),$$

where $\lambda_y = \frac{2}{3}$ and $\lambda_r = \frac{1}{3}$, and:

$$L_t^{MP} = \delta_{cr} Var(cr_t) + \delta_y Var(\Delta y_t) + \delta_\eta Var(\Delta \eta_t),$$

where $\delta_{cr} = \frac{3}{2}$, $\delta_y = 1$, and $\delta_\eta = \frac{1}{2}$. The non-cooperative solution, with these loss functions assigned to monetary policy and macroprudential policy, respectively, yields a total loss of 2.127 (evaluated using the benchmark loss, equation (11)), compared to a total loss of 2.150 obtained by the cooperative solution. Looking at the loss function that gets given to the central bank, we find that the delegated loss function assigns relatively higher weight to inflation than equation (11) in order to improve on the cooperative outcome. Greater weight on inflation stabilization ensures better control over inflation stabilization under non-cooperation, with some of the (relative) importance of stabilizing output growth being passed to the macroprudential regulator. As a result, both policymakers have a common objective, i.e., output growth, that helps to outperform on the cooperative outcome. With output growth an objective in the macroprudential regulator's loss function, the delegated loss function places greater weight than society on the credit-to-GDP ratio, helping to ensure that the credit-to-GDP ration remains well-stabilized under non-cooperation.

5 Regional macroprudential policymakers

Previous sections have had a single authority setting macroprudential policy for both the core and the periphery. We now assume that macroprudential policy is conducted at the regional level, i.e., that there is a macroprudential regulator in the core and another in the periphery, and that each is charged with safeguarding financial stability in its own region. Their focus on their own region sees these prudential authorities caring only indirectly about outcomes in the other region and about outcomes at the EMU-wide level. As previously, a single central bank remains in charge of conducting monetary policy at EMU-wide level. Because there are now three players in the model, a much wider range of strategic-interaction schemes can be considered than could be for the two-player case. However, we focus on what we believe are the three most interesting cases: the cooperative solution, the non-cooperative solution where all policymakers choose simultaneously, and the case where the central bank has a first-mover advantage with respect to the two macroprudential regulators (monetary leadership).[6]

Table 7 reports the results for the case where the central bank minimizes its usual loss function, equation (2), and the macroprudential regulators in the core and the periphery minimize their region-specific loss functions, equations (7) and (8), respectively. The loss functions for the macroprudential authorities include the regional credit-to-GDP ratio and smoothing of the regional lending fraction. In the cooperative solution the shared loss function is given by the sum of the loss functions across the three policymakers. The results indicate that, although the central bank prefers the cooperative environment, both regional macroprudential authorities prefer outcomes under the non-cooperative solutions (with no preference across the two alternative non-cooperative schemes). Interestingly, in

[6] We did not consider the case where the two regional macroprudential authorities have a first-mover advantage with respect to the area-wide monetary authority, partly because it did not seem to be the most interesting/relevant case, but also because we found little by way of first-mover advantage in any of the scenarios we did consider.

the absence of commitment, the shared loss function, $L^{society} = L^{CB+MP,c+MP,p}$, is lower in the simultaneous move non-cooperative solution than it is in the cooperative solution, suggesting that some level of regional-based macroprudential policymaking can be desirable.

Table 7: **Comparing coperation and non-cooperation: three policymakers**

	Cooperation	Non-cooperation	
		Simultaneous move	Monetary leadership
σ^2_{π}	0.082	0.100	0.097
$\sigma^2_{\Delta y}$	1.817	1.830	1.817
$\sigma^2_{\Delta r}$	0.328	0.353	0.392
$\sigma^2_{cr^c}$	0.081	0.048	0.048
$\sigma^2_{\Delta \eta^c}$	0.142	0.118	0.118
$\sigma^2_{c^p}$	0.064	0.061	0.062
$\sigma^2_{\Delta \eta^p}$	0.199	0.180	0.181
L^{CB}	2.062	2.107	2.110
$L^{MP,c}$	0.153	0.107	0.107
$L^{MP,p}$	0.163	0.152	0.152
$L^{society}$	2.378	2.366*	2.370

Note: The table reports variances and the unconditional losses under the alternative non-cooperative schemes, where the central bank the minimizes loss function (2) with $\lambda_y = 1$ and $\lambda_r = 0.5$, the core's macroprudential regulator minimizes equation (7) with $\delta_{cr^c} = 1$ and $\delta_{\eta^c} = 0.5$, and the periphery's macroprudential regulator minimizes equation (8) with $\delta_{cr^p} = 1$ and $\delta_{\eta^p} = 0.5$.

Figure 5 plots the responses to a risk shock in the periphery under the cooperative and non-cooperative simultaneous-move policy schemes considered in Table 7.[7] Unlike the case where there was a common macroprudential regulator at the area-wide level—where this risk shock causes a boom in the core—, when we allow separate macroprudential regulators to respond to regional variables this is no longer the case. The reason is that macroprudential policy in the periphery loosens in response to the recession created by the shock, while macroprudential policy in the core counteracts the immediate spillover effect of the shock by increasing the lending-deposit spread, which has a restraining effect on the credit-to-GDP ratio and on output in the core. As a result, inflation decreases slightly in the core, which lowers area-wide inflation, to which monetary policy reacts by lowering the nominal interest rate. The net result of the opposing regional macroprudential policies is that the shock has a more synchronized effect across the core and the periphery.

[7] We do not consider the leadership outcome in the comparison since the responses under monetary leadership are similar to those under simultaneous move.

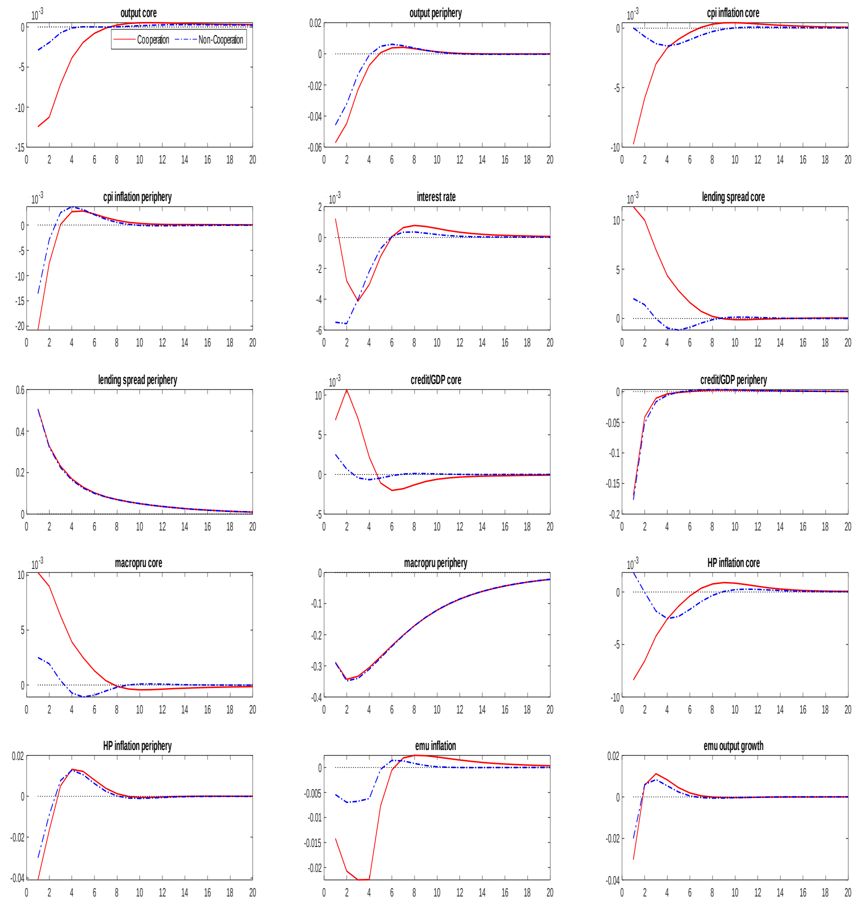

Figure 5. Responses to a risk shock in the periphery

Note: The figure plots the impulse responses to a risk shock in the periphery under alternative interaction schemes (cooperation and non-cooperation) for the following parameterizations of loss function weights: $\lambda_y = 1$ and $\lambda_r = 0.5$ in equation (2), $\delta_{cr^c} = 1$ and $\delta_{\eta^c} = 0.5$ in equation (7), and $\delta_{cr^p} = 1$ and $\delta_{\eta^p} = 0.5$ in equation (8).

We saw above that when the risk shock hits only the periphery, the scope for conducting independent macroprudential policy leads to diverging policy responses. The picture, however, changes somewhat when we consider an area-wide technology shock. Because both economies are affected similarly by the shock, the regional macroprudential authorities respond in a similar way. Therefore, the effects of the shock and the policy responses resemble closely those obtained when macroprudential policy is operated at the EMU-level. Hence, the added value from allowing independent macroprudential policies mainly arises from the possibility of responding more efficiently to the spillover effects arising from region-specific shocks.

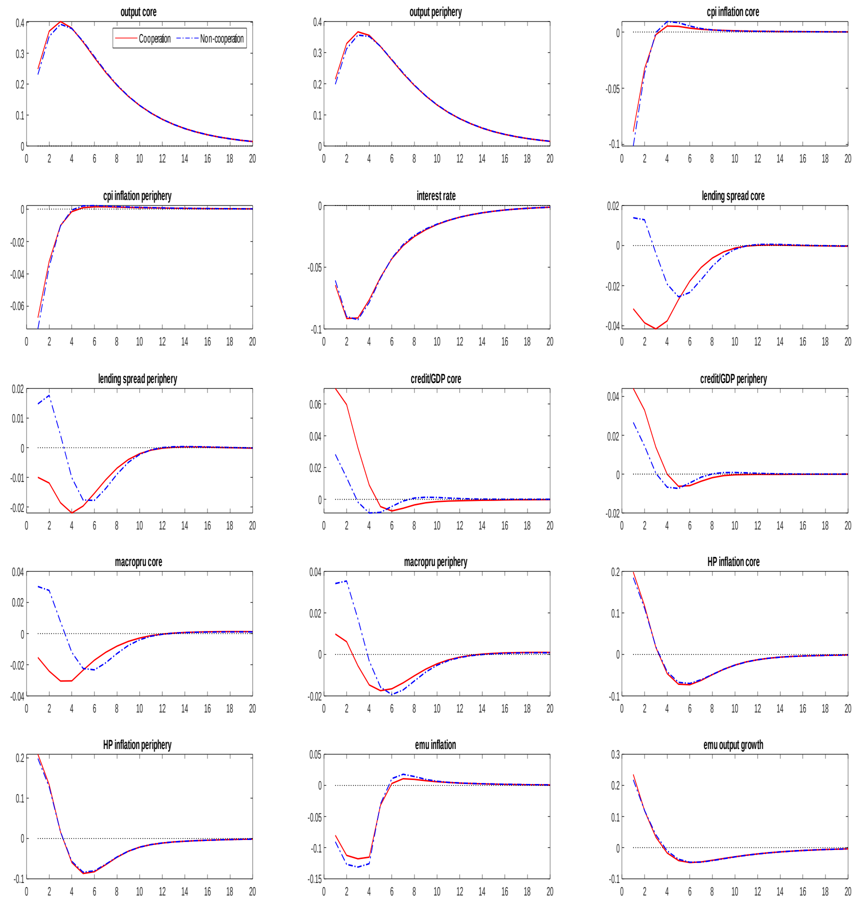

Figure 6. Responses to an area-wide technology shock

Note: The figure plots the impulse responses to a common area-wide technology shock under alternative interaction schemes (cooperation and non-cooperation) for the following parameterizations of loss function weights: $\lambda_y = 1$, $\lambda_r = 0.5$ in equation (2), $\delta_{cr^c} = 1$, $\delta_{\eta^c} = 0.5$ in equation (7), and $\delta_{cr^p} = 1$, $\delta_{\eta^p} = 0.5$ in equation (8).

In Table 8, the regional macroprudential authorities are assumed to receive, in addition to their existing objectives described in equations (7) and (8), the objective of stabilizing output growth in their respective region. Moreover, we assume that the ECB places weight on stabilizing the region-specific output growth variables, instead of area-wide output growth. We see in Table 8 that introducing this common objective between the ECB and the regional macroprudential authorities does not alter the earlier finding that society prefers the non-cooperative solution (monetary leadership this time) to the cooperative solution.

Table 8: **Comparing cooperation and non-cooperation: three policymakers with additional objectives**

	Cooperation	Non-cooperation	
		Simultaneous move	Monetary leadership
σ^2_{π}	0.081	0.079	0.075
$\sigma^2_{\Delta y^c}$	2.035	2.109	2.104
$\sigma^2_{\Delta y^p}$	1.915	1.979	1.974
$\sigma^2_{\Delta r}$	0.326	0.239	0.251
$\sigma^2_{cr^c}$	0.087	0.054	0.053
$\sigma^2_{\Delta \eta^c}$	0.138	0.124	0.124
$\sigma^2_{cr^p}$	0.067	0.061	0.061
$\sigma^2_{\Delta \eta^p}$	0.194	0.183	0.183
L^{CB}	1.238	1.227	1.226
$L^{MP,c}$	0.766	0.748	0.746
$L^{MP,p}$	0.547	0.548	0.547
$L^{society}$	2.551	2.523	2.519*

Note: The table reports the variances and the unconditional losses under the alternative non-cooperative schemes, where the central bank the minimizes loss function (2) with $\lambda_{y^c} = 0.3$, $\lambda_{y^p} = 0.2$, $\lambda_r = 0.5$, the prudential regulator in the core minimizes equation (7) with $\delta_{cr^c} = 1$, $\delta_{\eta^c} = 0.5$, $\delta_{y^c} = 0.3$, and the periphery's regulator minimizes equation (8) with $\delta_{cr^p} = 1$, $\delta_{y^p} = 0.2$, $\delta_{\eta^p} = 0.5$.

We also considered the case where area-wide real output growth is assigned as a common objective to all three policymakers. Doing so did not change significantly the story that emerged above for the case where the three policymakers had separate objectives (Table 7). Overall, separating the loss functions and allowing each policy maker to act independently seems to be a better strategy than assigning common area-wide objectives because it compensates to a certain degree for the regions not having independent monetary policies.

6 Conclusion

We use an estimated DSGE model of the euro-area to study the interaction between monetary policy and macroprudential policy in a monetary union. The model is one in which monetary policy and macroprudential policy interact through the behavior of savers and borrowers and through the balance sheets of financial intermediaries, with the economic effects of tighter monetary policy potentially offset through looser macroprudential policy. An essential and defining feature of the model is that it contains two regions—a core and a periphery—which allows the effects of area-wide and region-specific policies and shocks on each region to be explored. We assume that policymakers behave purposefully, but do not have access to a commitment technology, so that monetary policy and macroprudential policy are each formulated to be optimal under discretion. With policymakers optimizing, we consider a broad range of cooperative and non-cooperative decision problems, examine the effects of leadership, and compare the implications of having two region-focused rather than a single area-wide-focused macroprudential regulator.

Using the optimal commitment policy as one benchmark we show that the discretionary stabilization bias leads to inefficient responses to shocks, but in a way that is quite different from models that focus only on monetary policy. With time-inconsistency also affecting the macroprudential regulator, inflation volatility is actually lower under discretion than commitment, a result that arises as policymakers use policy promises to instead secure greater stability in the credit-to-GDP ratio, the nominal interest rate, and the lending fraction. When monetary policy and macroprudential policy are both conducted at the area-wide level, the preferred decision-making environment had policymakers cooperating and choosing their policies simultaneously. However, although area-wide policies were able to successfully stabilize area-wide variables, we showed that this stability masked considerable underlying volatility at the regional level. In particular, the credit-to-GDP ratio remained highly volatile in both the core and the periphery while the area-wide credit-to-GDP ratio was quite stable. Focusing on non-cooperative policies, but assigning area-wide real GDP growth as a common objective led to non-cooperation out-performing cooperation, but it did not remove the excessive regional-level volatility in the credit-to-GDP ratio.

Introducing regional-level macroprudential regulators we found that non-cooperation performed slightly better than cooperation. Importantly, with macroprudential policy conducted at the regional level the excessive regional-level volatility in the credit-to-GDP ratio was eliminated, but at the expense of slightly higher volatility in area-wide inflation and output growth.

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Appendix A - Model overview

This appendix provides a brief overview of the model; readers are encouraged to consult Quint and Rabanal (2014) for the model's complete description. The model has two regions, however we will focus on just a single region below while noting that the decision problems for the second region are symmetric to those of the first. In each region there are two types of households — savers and borrowers — and two types of producers — nondurables-producers and housing-producers. Both savers and borrowers have external habits regarding nondurables' consumption and both types of firm produce in a monopolistically competitive market and are subject to a Calvo-style price rigidity. Nondurable goods can be traded between regions while housing goods cannot. Intermediation between savers and borrowers takes place through credit markets, facilitated by financial intermediaries. The financial intermediaries are risk neutral and extend loans to borrowers that are financed by domestic and foreign savings, the latter of which are subject to a country risk premium.

6.1 Households

The domestic economy consists of two types of households: savers and borrowers. The fraction of each type of household is fixed, and we denote using $\lambda \in (0,1)$ the fraction of households that are savers.

6.1.1 Savers

The decision problem for the representative saver is to choose $\left\{C_t^s, D_t^s, S_t^s, I_t^s, L_t^{s,C}, L_t^{s,D}\right\}_{t=0}^{\infty}$ to maximize:

$$E_0\left[\sum_{t=0}^{\infty}\beta^{s,t}\left(\gamma\xi_t^C\log\left(C_t^s-\varepsilon^s C_{t-1}^s\right)+(1-\gamma)\,\xi_t^D\log\left(D_t^s\right)-\frac{\left(L_t^s\right)^{1+\varphi}}{1+\varphi}\right)\right]$$

subject to the budget constraint

$$P_t^C C_t^s + P_t^D D_t^s + S_t^s = R_{t-1} S_{t-1}^s + W_t^C L_t^{s,C} + W_t^D L_t^{s,D} + \Pi_t,$$

the time constraint:

$$L_t^s = L_t^{s,C} + L_t^{s,D},$$

and the law-of-motion for housing

$$D_t^s = (1-\delta) D_{t-1}^s + \left(1 - \left(\frac{I_{t-1}^s}{I_{t-2}^s} - 1\right)^2\right) I_{t-1}^s.$$

6.1.2 Borrowers

Borrowers use loans in addition to income to finance their consumption and housing investment. Depending on the realization of an idiosyncratic shock (which exhibits stochastic volatility), a borrower may default on their loan. The decision problem for the representative borrower is to choose $\left\{C_t^b, D_t^b, S_t^b, I_t^b, L_t^{b,C}, L_t^{b,D}\right\}_{t=0}^{\infty}$ to maximize:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^{b,t} \left(\gamma \xi_t^C \log\left(C_t^b - \varepsilon^b C_{t-1}^b\right) + (1-\gamma) \xi_t^D \log\left(D_t^b\right) - \frac{\left(L_t^b\right)^{1+\varphi}}{1+\varphi} \right) \right]$$

subject to the budget constraint

$$P_t^C C_t^b + P_t^D \left[I_t^b + G\left(\overline{\omega}_{t-1}^p, \sigma_{\omega,t-1}\right) D_t^b \right] + \left[1 - F\left(\overline{\omega}_{t-1}^p, \sigma_{\omega,t-1}\right)\right] R_{t-1}^L S_{t-1}^b = S_t^b + W_t^C L_t^{b,C} + W_t^D L_t^{b,D},$$

the time constraint:

$$L_t^b = L_t^{b,C} + L_t^{b,D},$$

and the law-of-motion for housing

$$D_t^b = (1-\delta) D_{t-1}^b + \left(1 - \left(\frac{I_{t-1}^b}{I_{t-2}^b} - 1\right)^2\right) I_{t-1}^b.$$

Unlike savers, borrowers, in the event that they do not default, pay the rate of interest R_t^L on their loans, and the default whenever their realized idiosyncratic shock is below the threshold, $\overline{\omega}_t^p$, which is given by:

$$\overline{\omega}_t^p = \frac{R_t^L S_t^b}{P_{t+1}^D D_{t+1}^b}.$$

The function $F\left(\overline{\omega}_{t-1}^p, \sigma_{\omega,t-1}\right)$ represents the fraction of loans that are defaulted on while $G\left(\overline{\omega}_{t-1}^p, \sigma_{\omega,t-1}\right)$ represents the fraction of the housing stock that is colleterial for loans that are in default, and that must be repurchased by borrowers.

6.2 Firms

There are two final goods in the model: a nondurable good that can be consumed and a housing good that provides housing services. Each of these final goods is produced using as inputs the goods manufactured by intermediate-good producers. Intermediate-good producers are monopolistically competitive and their price is subject to a Calvo-style (Calvo, 1983) price rigidity.

6.2.1 Intermediate–good producers

Intermediate goods are produced in both the nondurable sector and the housing sector. These intermediate goods are sold to final good-producers in a monopolistically competitive market where they are aggregated into bundles and sold to households. Intermediate-good producers in both sectors employ only labour and produce according to the technologies:

$$\begin{aligned} Y_t^C &= A_t Z_t^C L_t^C(h), \\ Y_t^D &= A_t Z_t^D L_t^D(h), \end{aligned}$$

where $h \in [0, n]$ indexes a particular intermediate-good producer and n represents the faction of intermediate-good producers that are domestic. Subject to their production technology, and the demand curve they face for their product,

$$\begin{aligned} Y_t^C(h) &= \left(\frac{P_t^H(h)}{P_t^H}\right)^{-\sigma_C} Y_t^C, \\ Y_t^D(h) &= \left(\frac{P_t^D(h)}{P_t^D}\right)^{-\sigma_D} Y_t^D, \end{aligned}$$

respectively, each intermediate-good producer chooses the price at which to sell their good in order to maximize their equity-value, subject to a Calvo-style price rigidity with inflation indexation.

Aggregate prices are defined by:

$$\begin{aligned} P_t^H &= \left[\frac{1}{n}\int_0^n \left(P_t^H(h)\right)^{1-\sigma_C} dh\right]^{\frac{1}{1-\sigma_C}}, \\ P_t^D &= \left[\frac{1}{n}\int_0^n \left(P_t^D(h)\right)^{1-\sigma_D} dh\right]^{\frac{1}{1-\sigma_D}}. \end{aligned}$$

With the aggregate price for nondurables given by:

$$P_t^C = \left[\tau\left(P_t^H\right)^{1-\iota_C} + (1-\tau)\left(P_t^F\right)^{1-\iota_C}\right]^{\frac{1}{1-\iota_C}},$$

where P_t^F represents the price of imported nondurables.

6.3 Financial intermediation

Financial intermediaries take deposits from savers and extend load to borrowers; they pay the interest rate R_t on deposits and charge the rate R_t^L on loans, which are secured against the value of the borrower's housing. Financial intermediaries are risk neutral, and therefore require the expected return on a loan, accounting for default, to equal the deposit rate. Because default can occur, $R_t^L > R_t$. In the financial intermediation sector, macroprudential policy operates by affecting the fraction of its liabilities that a bank can lend. Aggregating across the sector, the balance sheet in the home country is:

$$n\lambda \frac{1}{\eta_t} (S_t - B_t) = n (1 - \lambda) S_t^b,$$

where B_t represent claims on domestic intermediaries by the foreign country, and, recall, λ represents the fraction of households that are savers, and n denotes the relative size of the domestic country.

Financial intermediation also occurs between the domestic a foreign country, subject to a country risk premium:

$$R_t^* = R_t + \vartheta_t \left[e^{\left(\kappa_b \left(\frac{B_t}{P_t^C Y^C}\right)\right)} - 1 \right],$$

where ϑ_t denotes a country risk-premium shock.

Appendix B - Solution procedure for two players

With two regions, two productive sectors, and two types of households, along with regional and international financial intermediaries and a host of real and nominal rigidities, the model is large, containing over 100 equations. To solve for the optimal discretionary policies we draw on the solution methods developed by Dennis (2007) which can be applied to larger models because they do not require the model to be put in a state space form. De Paoli and Paustain (2013) extended the methods in Dennis (2007) to two-player settings, and the solution methods we employ in this paper are straightforward extensions of that work.

In what follows we describe the solution for both the simultaneous move and the leader-follower cases, both of which work with the equations summarizing the model expressed in the form

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2 E_t\mathbf{y}_{t+1} + \mathbf{A}_{31}\mathbf{x}_{1t} + \mathbf{A}_{32}\mathbf{x}_{2t} + \mathbf{A}_{41}E_t\mathbf{x}_{1t+1} + \mathbf{A}_{42}E_t\mathbf{x}_{2t+1} + \mathbf{A}_5\mathbf{v}_t, \quad (12)$$

where $\mathbf{y}_t$ is a vector of endogenous variables, $\mathbf{x}_t$ is the vector of policy instruments for one policymaker, $\widetilde{\mathbf{x}}_t$ is the vector of policy instruments of the other policymaker, $\mathbf{v}_t \sim i.i.d.[\mathbf{0}, \mathbf{\Omega}]$ is a vector of stochastic disturbances, and the matrices $\mathbf{A}_0$—$\mathbf{A}_5$ contain the model's parameters.

For convenience we will label the policymakers 1 and 2, then the loss functions for the two policymakers are assumed to be given by

$$Loss_1 = E_0\left[\sum_{t=0}^{\infty}\beta_1^t\left(\mathbf{y}_t^{'}\mathbf{W}_1\mathbf{y}_t+\mathbf{x}_{1t}^{'}\mathbf{Q}_{11}\mathbf{x}_{1t}+\mathbf{x}_{2t}^{'}\mathbf{Q}_{12}\mathbf{x}_{2t}\right)\right], \tag{13}$$

$$Loss_2 = E_0\left[\sum_{t=0}^{\infty}\beta_2^t\left(\mathbf{y}_t^{'}\mathbf{W}_2\mathbf{y}_t+\mathbf{x}_{1t}^{'}\mathbf{Q}_{21}\mathbf{x}_{1t}+\mathbf{x}_{2t}^{'}\mathbf{Q}_{22}\mathbf{x}_{2t}\right)\right], \tag{14}$$

where the $\mathbf{W}$ and $\mathbf{Q}$ matrices (symmetric and positive semi-definite) contain the policy preferences of the two policymakers. In the case where the two policymakers cooperate the loss function parameters must satisfy $\beta_1 = \beta_2$, $\mathbf{W}_1 = \mathbf{W}_2$, $\mathbf{Q}_{11} = \mathbf{Q}_{21}$, and $\mathbf{Q}_{12} = \mathbf{Q}_{22}$. The two discount factors, β_1 and β_2, lie between 0 and 1. Note that equations (24) and (25) differ from De Paoli and Paustain (2013) because they allow each policymaker's loss to depend on both policymaker's policy instruments, not just their own. Although the solution methods described below are closely related to those presented in De Paoli and Paustain (2013), we present them below, partly for completeness and partly because our use of different loss functions leads to different updating equations.

B1 - Simultaneous move

Employing results from Dennis (2007), the conjectured solution is

$$\mathbf{y}_t = \mathbf{H}_1\mathbf{y}_{t-1}+\mathbf{H}_2\mathbf{v}_t, \tag{15}$$

$$\mathbf{x}_{1t} = \mathbf{F}_1\mathbf{y}_{t-1}+\mathbf{F}_2\mathbf{v}_t, \tag{16}$$

$$\mathbf{x}_{2t} = \mathbf{G}_1\mathbf{y}_{t-1}+\mathbf{G}_2\mathbf{v}_t, \tag{17}$$

allowing the constraints (equation 12) to we written as

$$\mathbf{D}\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1}+\mathbf{A}_{31}\mathbf{x}_{1t}+\mathbf{A}_{32}\mathbf{x}_{2t}+\mathbf{A}_5\mathbf{v}_t, \tag{18}$$

where

$$\mathbf{D} = \mathbf{A}_0-\mathbf{A}_2\mathbf{H}_1-\mathbf{A}_{41}\mathbf{F}_1-\mathbf{A}_{42}\widetilde{\mathbf{F}}_1. \tag{19}$$

After a few substitutions and exploiting the properties of convergent geometric series we obtain

$$\begin{aligned} Loss_1 = {} & \left(\mathbf{A}_1\mathbf{y}_{t-1}+\mathbf{A}_{31}\mathbf{x}_{1t}+\mathbf{A}_{32}\mathbf{x}_{2t}+\mathbf{A}_5\mathbf{v}_t\right)^{'}\mathbf{D}^{'-1}\mathbf{P}_1\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1}+\mathbf{A}_{31}\mathbf{x}_{1t}+\mathbf{A}_{32}\mathbf{x}_{2t}+\mathbf{A}_5\mathbf{v}_t\right) \\ & +\mathbf{x}_{1t}^{'}\mathbf{Q}_{11}\mathbf{x}_{1t}+\mathbf{x}_{2t}^{'}\mathbf{Q}_{12}\mathbf{x}_{2t}+\frac{\beta_1}{1-\beta_1}tr\left[\left(\mathbf{F}_2^{'}\mathbf{Q}_{11}\mathbf{F}_2+\mathbf{G}_2^{'}\mathbf{Q}_{12}\mathbf{G}_2+\mathbf{H}_2\mathbf{P}_1\mathbf{H}_2\right)\mathbf{\Omega}\right], \end{aligned} \tag{20}$$

$$\begin{aligned} Loss_2 = {} & \left(\mathbf{A}_1\mathbf{y}_{t-1}+\mathbf{A}_{31}\mathbf{x}_{1t}+\mathbf{A}_{32}\mathbf{x}_{2t}+\mathbf{A}_5\mathbf{v}_t\right)^{'}\mathbf{D}^{'-1}\mathbf{P}_2\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1}+\mathbf{A}_{31}\mathbf{x}_{1t}+\mathbf{A}_{32}\mathbf{x}_{2t}+\mathbf{A}_5\mathbf{v}_t\right) \\ & +\mathbf{x}_{1t}^{'}\mathbf{Q}_{21}\mathbf{x}_{1t}+\mathbf{x}_{2t}^{'}\mathbf{Q}_{22}\mathbf{x}_{2t}+\frac{\beta_2}{1-\beta_2}tr\left[\left(\mathbf{F}_2^{'}\mathbf{Q}_{21}\mathbf{F}_2+\mathbf{G}_2^{'}\mathbf{Q}_{22}\mathbf{G}_2+\mathbf{H}_2\mathbf{P}_2\mathbf{H}_2\right)\mathbf{\Omega}\right], \end{aligned} \tag{21}$$

where

$$\mathbf{P}_1 = \mathbf{W}_1 + \beta_1 \left(\mathbf{F}_1' \mathbf{Q}_{11} \mathbf{F}_1 + \mathbf{G}_1' \mathbf{Q}_{12} \mathbf{G}_1 + \mathbf{H}_1' \mathbf{P}_1 \mathbf{H}_1 \right), \quad (22)$$

$$\mathbf{P}_2 = \mathbf{W}_2 + \beta_2 \left(\mathbf{F}_1' \mathbf{Q}_{21} \mathbf{F}_1 + \mathbf{G}_1' \mathbf{Q}_{22} \mathbf{G}_1 + \mathbf{H}_1' \mathbf{P}_2 \mathbf{H}_1 \right). \quad (23)$$

Differentiating $Loss_1$ with respect to $\mathbf{x}_{1t}$ and $Loss_2$ with respect to $\mathbf{x}_{2t}$ gives the first order conditions

$$\frac{\partial Loss_1}{\partial \mathbf{x}_{1t}} = \mathbf{A}_{31}' \mathbf{D}'^{-1} \mathbf{P}_1 \mathbf{D}^{-1} (\mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{A}_{31} \mathbf{x}_{1t} + \mathbf{A}_{32} \mathbf{x}_{2t} + \mathbf{A}_5 \mathbf{v}_t) + \mathbf{Q}_{11} \mathbf{x}_{1t} = 0, \quad (24)$$

$$\frac{\partial Loss_2}{\partial \mathbf{x}_{2t}} = \mathbf{A}_{32}' \mathbf{D}'^{-1} \mathbf{P}_2 \mathbf{D}^{-1} (\mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{A}_{31} \mathbf{x}_{1t} + \mathbf{A}_{32} \mathbf{x}_{2t} + \mathbf{A}_5 \mathbf{v}_t) + \mathbf{Q}_{22} \mathbf{x}_{2t} = 0. \quad (25)$$

The simultaneous move solution can now be obtained using the following iterative scheme

1. Initialize $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{G}_1$, and $\mathbf{G}_2$.

2. Compute $\mathbf{D}$ using equation (19), $\mathbf{P}_1$ using equation (22) and $\mathbf{P}_2$ using equation (23).

3. Update $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{G}_1$, and $\mathbf{G}_2$ according to

$$\begin{aligned}
\mathbf{F}_1 &= -\left(\mathbf{Q}_{11} + \mathbf{A}_{31}' \mathbf{D}'^{-1} \mathbf{P}_1 \mathbf{D}^{-1} \mathbf{A}_{31} \right)^{-1} \mathbf{A}_{31}' \mathbf{D}'^{-1} \mathbf{P}_1 \mathbf{D}^{-1} (\mathbf{A}_1 + \mathbf{A}_{32} \mathbf{G}_1), \\
\mathbf{F}_2 &= -\left(\mathbf{Q}_1 + \mathbf{A}_{31}' \mathbf{D}'^{-1} \mathbf{P}_1 \mathbf{D}^{-1} \mathbf{A}_{31} \right)^{-1} \mathbf{A}_{31}' \mathbf{D}'^{-1} \mathbf{P}_1 \mathbf{D}^{-1} (\mathbf{A}_5 + \mathbf{A}_{32} \mathbf{G}_2), \\
\mathbf{G}_1 &= -\left(\mathbf{Q}_{22} + \mathbf{A}_{32}' \mathbf{D}'^{-1} \mathbf{P}_2 \mathbf{D}^{-1} \mathbf{A}_{32} \right)^{-1} \mathbf{A}_{32}' \mathbf{D}'^{-1} \mathbf{P}_2 \mathbf{D}^{-1} (\mathbf{A}_1 + \mathbf{A}_{31} \mathbf{F}_1), \\
\mathbf{G}_2 &= -\left(\mathbf{Q}_{22} + \mathbf{A}_{32}' \mathbf{D}'^{-1} \mathbf{P}_2 \mathbf{D}^{-1} \mathbf{A}_{32} \right)^{-1} \mathbf{A}_{32}' \mathbf{D}'^{-1} \mathbf{P}_2 \mathbf{D}^{-1} (\mathbf{A}_5 + \mathbf{A}_{31} \mathbf{F}_2), \\
\mathbf{H}_1 &= \mathbf{D}^{-1} (\mathbf{A}_1 + \mathbf{A}_{31} \mathbf{F}_1 + \mathbf{A}_{32} \mathbf{G}_1), \\
\mathbf{H}_2 &= \mathbf{D}^{-1} (\mathbf{A}_5 + \mathbf{A}_{31} \mathbf{F}_2 + \mathbf{A}_{32} \mathbf{G}_2).
\end{aligned}$$

4. Iterate over steps 2—4 until convergence.

B2 - Leader-follower

For the leader-follower case, to ease exposition let us designate policymaker 1 as the leader and policymaker 2 as the follower. Then the conjectured reaction function for the follower takes the form

$$\mathbf{x}_{2t} = \mathbf{G}_1 \mathbf{y}_{t-1} + \mathbf{G}_2 \mathbf{v}_t + \mathbf{L} \mathbf{x}_{1t}, \quad (26)$$

with the remainder of the conjectured solution continuing to be given by equations (15) and (16). With this reaction function, policymaker 1 can take the behavior (reaction) of policymaker 2 into account when formulating policy.

The solution procedure now precedes as before. Substituting the conjectured solution into the constraints gives equation (18), but where now $\mathbf{D}$ is given by

$$\mathbf{D} = \mathbf{A}_0 - \mathbf{A}_2\mathbf{H}_1 - \mathbf{A}_{41}\mathbf{F}_1 - \mathbf{A}_{42}\mathbf{G}_1 - \mathbf{A}_{42}\mathbf{L}\mathbf{F}_1. \quad (27)$$

Then the loss functions for the two policymakers are given by

$$Loss_1 = \mathbf{y}_t^{'}\mathbf{P}_1\mathbf{y}_t + \mathbf{x}_{1t}^{'}\mathbf{Q}_{11}\mathbf{x}_{1t} + \mathbf{x}_{2t}^{'}\mathbf{Q}_{12}\mathbf{x}_{2t} + \frac{\beta_1}{1-\beta_1} tr\left[\left(\mathbf{F}_2^{'}\mathbf{Q}_{11}\mathbf{F}_2 + (\mathbf{G}_2 + \mathbf{L}\mathbf{F}_2)^{'}\mathbf{Q}_{12}(\mathbf{G}_2 + \mathbf{L}\mathbf{F}_2) + \mathbf{H}_2\mathbf{P}\right.\right.$$

$$Loss_2 = \mathbf{y}_t^{'}\mathbf{P}_2\mathbf{y}_t + \mathbf{x}_{1t}^{'}\mathbf{Q}_{21}\mathbf{x}_{1t} + \mathbf{x}_{2t}^{'}\mathbf{Q}_{22}\mathbf{x}_{2t} + \frac{\beta_2}{1-\beta_2} tr\left[\left(\mathbf{F}_2^{'}\mathbf{Q}_{21}\mathbf{F}_2 + (\mathbf{G}_2 + \mathbf{L}\mathbf{F}_2)^{'}\mathbf{Q}_{22}(\mathbf{G}_2 + \mathbf{L}\mathbf{F}_2) + \mathbf{H}_2\mathbf{P}\right.\right.$$

where

$$\mathbf{P}_1 = \mathbf{W}_1 + \beta_1\left(\mathbf{F}_1^{'}\mathbf{Q}_{11}\mathbf{F}_1 + \mathbf{G}_1^{'}\mathbf{Q}_{12}\mathbf{G}_1 + \mathbf{H}_1^{'}\mathbf{P}_1\mathbf{H}_1\right), \quad (30)$$

$$\mathbf{P}_2 = \mathbf{W}_2 + \beta_2\left(\mathbf{F}_1^{'}\mathbf{Q}_{21}\mathbf{F}_1 + (\mathbf{G}_1 + \mathbf{L}\mathbf{F}_1)^{'}\mathbf{Q}_{22}(\mathbf{G}_1 + \mathbf{L}\mathbf{F}_1) + \mathbf{H}_1^{'}\mathbf{P}_2\mathbf{H}_1\right). \quad (31)$$

After substituting equations (18) and (26) into the two loss functions, differentiating them with respect to $\mathbf{x}_{1t}$ and $\mathbf{x}_{2t}$, respectively, gives

$$\frac{\partial Loss_1}{\partial \mathbf{x}_{1t}} : (\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L})^{'}\mathbf{D}^{'-1}\mathbf{P}_1\mathbf{D}^{-1}\left((\mathbf{A}_1 + \mathbf{A}_{32}\mathbf{G}_1)\mathbf{y}_{t-1} + (\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L})\mathbf{x}_{1t} + (\mathbf{A}_5 + \mathbf{A}_{32}\mathbf{G}_2)\mathbf{v}_t\right)$$

$$: +\mathbf{Q}_{11}\mathbf{x}_{1t} + \mathbf{L}^{'}\mathbf{Q}_{12}(\mathbf{G}_1\mathbf{y}_{t-1} + \mathbf{G}_2\mathbf{v}_t + \mathbf{L}\mathbf{x}_{1t}) = 0, \quad (32)$$

$$\frac{\partial Loss_2}{\partial \mathbf{x}_{2t}} = \mathbf{A}_{32}^{'}\mathbf{D}^{'-1}\mathbf{P}_2\mathbf{D}^{-1}(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_{31}\mathbf{x}_t + \mathbf{A}_{32}\mathbf{x}_{2t} + \mathbf{A}_5\mathbf{v}_t) + \mathbf{Q}_{22}\mathbf{x}_{2t} = 0. \quad (33)$$

The leader-follower solution can now be obtained using the following iterative scheme

1. Initialize $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{G}_1$, $\mathbf{G}_2$ and $\mathbf{L}$.

2. Compute $\mathbf{D}$ using equation (27), $\mathbf{P}_1$ using equation (30) and $\mathbf{P}_2$ using equation (31).

3. Update $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{G}_1$, $\mathbf{G}_2$ and $\mathbf{L}$ according to

$$\mathbf{F}_1 = -\left(\mathbf{Q}_{11} + \mathbf{L}^{'}\mathbf{Q}_{12}\mathbf{L} + (\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L})^{'}\mathbf{D}^{'-1}\mathbf{P}_1\mathbf{D}^{-1}(\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L})\right)^{-1} \times \left((\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L})^{'}\mathbf{D}^{'-1}\mathbf{P}_1\mathbf{D}^{-1}(\mathbf{A}_1 + \mathbf{A}_{32}\mathbf{G}_1) + \mathbf{L}^{'}\mathbf{Q}_{12}\mathbf{G}_1\right),$$

$$\mathbf{F}_2 = -\left(\mathbf{Q}_{11} + \mathbf{L}^{'}\mathbf{Q}_{12}\mathbf{L} + (\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L})^{'}\mathbf{D}^{'-1}\mathbf{P}_1\mathbf{D}^{-1}(\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L})\right)^{-1} \times \left((\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L})^{'}\mathbf{D}^{'-1}\mathbf{P}_1\mathbf{D}^{-1}(\mathbf{A}_5 + \mathbf{A}_{32}\mathbf{G}_2) + \mathbf{L}^{'}\mathbf{Q}_2\mathbf{G}_2\right),$$

$$\mathbf{G}_1 = -\left(\mathbf{Q}_{22} + \mathbf{A}_{32}^{'}\mathbf{D}^{'-1}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{32}\right)^{-1}\mathbf{A}_{32}^{'}\mathbf{D}^{'-1}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_1,$$

$$\mathbf{G}_2 = -\left(\mathbf{Q}_{22} + \mathbf{A}_{32}^{'}\mathbf{D}^{'-1}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{32}\right)^{-1}\mathbf{A}_{32}^{'}\mathbf{D}^{'-1}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_5,$$

$$\mathbf{H}_1 = \mathbf{D}^{-1}(\mathbf{A}_1 + \mathbf{A}_{31}\mathbf{F}_1 + \mathbf{A}_{32}\mathbf{G}_1 + \mathbf{A}_{32}\mathbf{L}\mathbf{F}_1),$$

$$\mathbf{H}_2 = \mathbf{D}^{-1}(\mathbf{A}_5 + \mathbf{A}_{31}\mathbf{F}_2 + \mathbf{A}_{32}\mathbf{G}_2 + \mathbf{A}_{32}\mathbf{L}\mathbf{F}_2),$$

$$\mathbf{L} = -(\mathbf{Q}_{22} + \mathbf{A}_{32}^{'}\mathbf{D}^{'-1}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{32})^{-1}\mathbf{A}_{32}^{'}\mathbf{D}^{'-1}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{31}.$$

4. Iterate over steps 2—4 until convergence.

7 Appendix C - Solution procedure for three players

In this appendix we present our algorith to solve the models in the three-player cases. Specifically, we present the solution methods for the cases where all three policymakers set their instruments simultaneously and where their is a single leader and two followers, with the two followers choosing simultaneously. For the three player cases, the equations summarizing the model are expressed in the form:

$$\begin{aligned}\mathbf{A}_0\mathbf{y}_t &= \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2 E_t\mathbf{y}_{t+1} + \mathbf{A}_{31}\mathbf{x}_{1t} + \mathbf{A}_{32}\mathbf{x}_{2t} + \mathbf{A}_{33}\mathbf{x}_{3t} \\ &\quad + \mathbf{A}_{41}E_t\mathbf{x}_{1t+1} + \mathbf{A}_{42}E_t\mathbf{x}_{2t+1} + \mathbf{A}_{43}E_t\mathbf{x}_{3t+1} + \mathbf{A}_5\mathbf{v}_t, \end{aligned} \tag{34}$$

while the policy objectives for the three players are:

$$Loss_1 = E_0\left[\sum_{t-0}^{\infty}\beta_1^t\left(\mathbf{y}_t'\mathbf{W}_1\mathbf{y}_t + \mathbf{x}_{1t}'\mathbf{Q}_{11}\mathbf{x}_{1t} + \mathbf{x}_{2t}'\mathbf{Q}_{12}\mathbf{x}_{2t} + \mathbf{x}_{3t}'\mathbf{Q}_{13}\mathbf{x}_{3t}\right)\right], \tag{35}$$

$$Loss_2 = E_0\left[\sum_{t-0}^{\infty}\beta_2^t\left(\mathbf{y}_t'\mathbf{W}_2\mathbf{y}_t + \mathbf{x}_{1t}'\mathbf{Q}_{21}\mathbf{x}_{1t} + \mathbf{x}_{2t}'\mathbf{Q}_{22}\mathbf{x}_{2t} + \mathbf{x}_{3t}'\mathbf{Q}_{23}\mathbf{x}_{3t}\right)\right], \tag{36}$$

$$Loss_3 = E_0\left[\sum_{t-0}^{\infty}\beta_3^t\left(\mathbf{y}_t'\mathbf{W}_3\mathbf{y}_t + \mathbf{x}_{1t}'\mathbf{Q}_{31}\mathbf{x}_{1t} + \mathbf{x}_{2t}'\mathbf{Q}_{32}\mathbf{x}_{2t} + \mathbf{x}_{3t}'\mathbf{Q}_{33}\mathbf{x}_{3t}\right)\right], \tag{37}$$

for players 1, 2, and 3, respectively.

7.1 C1 - Simultaneous move

With the three policymakers choosing simultaneously, the conjectured solution is:

$$\begin{aligned}\mathbf{y}_t &= \mathbf{H}_1\mathbf{y}_{t-1} + \mathbf{H}_2\mathbf{v}_t, \\ \mathbf{x}_{1t} &= \mathbf{F}_1\mathbf{y}_{t-1} + \mathbf{F}_2\mathbf{v}_t, \\ \mathbf{x}_{2t} &= \mathbf{G}_1\mathbf{y}_{t-1} + \mathbf{G}_2\mathbf{v}_t, \\ \mathbf{x}_{3t} &= \mathbf{K}_1\mathbf{y}_{t-1} + \mathbf{K}_2\mathbf{v}_t.\end{aligned}$$

Now we have:

$$\mathbf{D} = \mathbf{A}_0 - \mathbf{A}_2\mathbf{H}_1 - \mathbf{A}_{41}\mathbf{F}_1 - \mathbf{A}_{42}\mathbf{G}_1 - \mathbf{A}_{43}\mathbf{K}_1,$$

$$
\begin{aligned}
\mathbf{P}_1 &= \mathbf{W}_1 + \beta\left(\mathbf{H}_1'\mathbf{P}_1\mathbf{H}_1 + \mathbf{F}_1'\mathbf{Q}_{11}\mathbf{F}_1 + \mathbf{G}_1'\mathbf{Q}_{12}\mathbf{G}_1 + \mathbf{K}_1'\mathbf{Q}_{13}\mathbf{K}_1\right), \\
\mathbf{P}_2 &= \mathbf{W}_2 + \beta\left(\mathbf{H}_1'\mathbf{P}_2\mathbf{H}_1 + \mathbf{F}_1'\mathbf{Q}_{21}\mathbf{F}_1 + \mathbf{G}_1'\mathbf{Q}_{22}\mathbf{G}_1 + \mathbf{K}_1'\mathbf{Q}_{23}\mathbf{K}_1\right), \\
\mathbf{P}_3 &= \mathbf{W}_3 + \beta\left(\mathbf{H}_1'\mathbf{P}_3\mathbf{H}_1 + \mathbf{F}_1'\mathbf{Q}_{31}\mathbf{F}_1 + \mathbf{G}_1'\mathbf{Q}_{32}\mathbf{G}_1 + \mathbf{K}_1'\mathbf{Q}_{33}\mathbf{K}_1\right), \\
d_1 &= \frac{\beta}{1-\beta} tr\left[\left(\mathbf{H}_2'\mathbf{P}_1\mathbf{H}_2 + \mathbf{F}_2'\mathbf{Q}_{11}\mathbf{F}_2 + \mathbf{G}_2'\mathbf{Q}_{12}\mathbf{G}_2 + \mathbf{K}_2'\mathbf{Q}_{13}\mathbf{K}_2\right)\mathbf{\Omega}\right], \\
d_2 &= \frac{\beta}{1-\beta} tr\left[\left(\mathbf{H}_2'\mathbf{P}_2\mathbf{H}_2 + \mathbf{F}_2'\mathbf{Q}_{21}\mathbf{F}_2 + \mathbf{G}_2'\mathbf{Q}_{22}\mathbf{G}_2 + \mathbf{K}_2'\mathbf{Q}_{23}\mathbf{K}_2\right)\mathbf{\Omega}\right], \\
d_3 &= \frac{\beta}{1-\beta} tr\left[\left(\mathbf{H}_2'\mathbf{P}_3\mathbf{H}_2 + \mathbf{F}_2'\mathbf{Q}_{31}\mathbf{F}_2 + \mathbf{G}_2'\mathbf{Q}_{32}\mathbf{G}_2 + \mathbf{K}_2'\mathbf{Q}_{33}\mathbf{K}_2\right)\mathbf{\Omega}\right],
\end{aligned}
$$

and the derivatives of the three loss functions are:

$$
\begin{aligned}
\frac{\partial Loss_1}{\partial \mathbf{x}_{1t}} &= \mathbf{A}_{31}'\mathbf{D}^{-1\prime}\mathbf{P}_1\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_{31}\mathbf{x}_{1t} + \mathbf{A}_{32}\mathbf{x}_{2t} + \mathbf{A}_{33}\mathbf{x}_{3t} + \mathbf{A}_5\mathbf{v}_t\right) + \mathbf{Q}_{11}\mathbf{x}_{1t} = \mathbf{0}, \\
\frac{\partial Loss_2}{\partial \mathbf{x}_{2t}} &= \mathbf{A}_{32}'\mathbf{D}^{-1\prime}\mathbf{P}_2\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_{31}\mathbf{x}_{1t} + \mathbf{A}_{32}\mathbf{x}_{2t} + \mathbf{A}_{33}\mathbf{x}_{3t} + \mathbf{A}_5\mathbf{v}_t\right) + \mathbf{Q}_{22}\mathbf{x}_{2t} = \mathbf{0}, \\
\frac{\partial Loss_3}{\partial \mathbf{x}_{3t}} &= \mathbf{A}_{33}'\mathbf{D}^{-1\prime}\mathbf{P}_3\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_{31}\mathbf{x}_{1t} + \mathbf{A}_{32}\mathbf{x}_{2t} + \mathbf{A}_{33}\mathbf{x}_{3t} + \mathbf{A}_5\mathbf{v}_t\right) + \mathbf{Q}_{33}\mathbf{x}_{3t} = \mathbf{0}.
\end{aligned}
$$

It follows that the updating equations are:

$$
\begin{aligned}
\mathbf{F}_1 &= -\left(\mathbf{Q}_{11} + \mathbf{A}_{31}'\mathbf{D}^{-1\prime}\mathbf{P}_1\mathbf{D}^{-1}\mathbf{A}_{31}\right)^{-1}\mathbf{A}_{31}'\mathbf{D}^{-1\prime}\mathbf{P}_1\mathbf{D}^{-1}\left(\mathbf{A}_1 + \mathbf{A}_{32}\mathbf{G}_1 + \mathbf{A}_{33}\mathbf{K}_1\right), \\
\mathbf{F}_2 &= -\left(\mathbf{Q}_{11} + \mathbf{A}_{31}'\mathbf{D}^{-1\prime}\mathbf{P}_1\mathbf{D}^{-1}\mathbf{A}_{31}\right)^{-1}\mathbf{A}_{31}'\mathbf{D}^{-1\prime}\mathbf{P}_1\mathbf{D}^{-1}\left(\mathbf{A}_5 + \mathbf{A}_{32}\mathbf{G}_2 + \mathbf{A}_{33}\mathbf{K}_2\right), \\
\mathbf{G}_1 &= -\left(\mathbf{Q}_{22} + \mathbf{A}_{32}'\mathbf{D}^{-1\prime}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{32}\right)^{-1}\mathbf{A}_{32}'\mathbf{D}^{-1\prime}\mathbf{P}_2\mathbf{D}^{-1}\left(\mathbf{A}_1 + \mathbf{A}_{31}\mathbf{F}_1 + \mathbf{A}_{33}\mathbf{K}_1\right), \\
\mathbf{G}_2 &= -\left(\mathbf{Q}_{22} + \mathbf{A}_{32}'\mathbf{D}^{-1\prime}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{32}\right)^{-1}\mathbf{A}_{32}'\mathbf{D}^{-1\prime}\mathbf{P}_2\mathbf{D}^{-1}\left(\mathbf{A}_5 + \mathbf{A}_{31}\mathbf{F}_2 + \mathbf{A}_{33}\mathbf{K}_2\right), \\
\mathbf{K}_1 &= -\left(\mathbf{Q}_{33} + \mathbf{A}_{33}'\mathbf{D}^{-1\prime}\mathbf{P}_3\mathbf{D}^{-1}\mathbf{A}_{33}\right)^{-1}\mathbf{A}_{33}'\mathbf{D}^{-1\prime}\mathbf{P}_3\mathbf{D}^{-1}\left(\mathbf{A}_1 + \mathbf{A}_{31}\mathbf{F}_1 + \mathbf{A}_{32}\mathbf{G}_1\right), \\
\mathbf{K}_2 &= -\left(\mathbf{Q}_{33} + \mathbf{A}_{33}'\mathbf{D}^{-1\prime}\mathbf{P}_3\mathbf{D}^{-1}\mathbf{A}_{33}\right)^{-1}\mathbf{A}_{33}'\mathbf{D}^{-1\prime}\mathbf{P}_3\mathbf{D}^{-1}\left(\mathbf{A}_5 + \mathbf{A}_{31}\mathbf{F}_2 + \mathbf{A}_{32}\mathbf{G}_2\right), \\
\mathbf{H}_1 &= \mathbf{D}^{-1}\left[\mathbf{A}_1 + \mathbf{A}_{31}\mathbf{F}_1 + \mathbf{A}_{32}\mathbf{G}_1 + \mathbf{A}_{33}\mathbf{K}_1\right] \\
\mathbf{H}_2 &= \mathbf{D}^{-1}\left[\mathbf{A}_5 + \mathbf{A}_{31}\mathbf{F}_2 + \mathbf{A}_{32}\mathbf{G}_2 + \mathbf{A}_{33}\mathbf{K}_2\right].
\end{aligned}
$$

7.2 C2 - One leader and two followers

For this case we will assume that player one is the leader and players two and three are the followers; the two followers choose simultaneously. The conjectured solution is:

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{H}_1\mathbf{y}_{t-1} + \mathbf{H}_2\mathbf{v}_t, \\
\mathbf{x}_{1t} &= \mathbf{F}_1\mathbf{y}_{t-1} + \mathbf{F}_2\mathbf{v}_t, \\
\mathbf{x}_{2t} &= \mathbf{G}_1\mathbf{y}_{t-1} + \mathbf{G}_2\mathbf{v}_t + \mathbf{L}_{21}\mathbf{x}_{1t}, \\
\mathbf{x}_{3t} &= \mathbf{K}_1\mathbf{y}_{t-1} + \mathbf{K}_2\mathbf{v}_t + \mathbf{L}_{31}\mathbf{x}_{1t}.
\end{aligned}$$

Now we have:

$$\mathbf{D} = \mathbf{A}_0 - \mathbf{A}_2\mathbf{H}_1 - \mathbf{A}_{41}\mathbf{F}_1 - \mathbf{A}_{42}\left(\mathbf{G}_1 + \mathbf{L}_{21}\mathbf{F}_1\right) - \mathbf{A}_{43}\left(\mathbf{K}_1 + \mathbf{L}_{31}\mathbf{F}_1\right),$$

$$\begin{aligned}
\mathbf{P}_1 &= \mathbf{W}_1 + \beta\left(\begin{array}{c} \mathbf{H}_1'\mathbf{P}_1\mathbf{H}_1 + \mathbf{F}_1'\mathbf{Q}_{11}\mathbf{F}_1 + \left(\mathbf{G}_1 + \mathbf{L}_{21}\mathbf{F}_1\right)'\mathbf{Q}_{12}\left(\mathbf{G}_1 + \mathbf{L}_{21}\mathbf{F}_1\right) \\ +\left(\mathbf{K}_1 + \mathbf{L}_{31}\mathbf{F}_1\right)'\mathbf{Q}_{13}\left(\mathbf{K}_1 + \mathbf{L}_{31}\mathbf{F}_1\right) \end{array}\right), \\
\mathbf{P}_2 &= \mathbf{W}_2 + \beta\left(\begin{array}{c} \mathbf{H}_1'\mathbf{P}_2\mathbf{H}_1 + \mathbf{F}_1'\mathbf{Q}_{21}\mathbf{F}_1 + \left(\mathbf{G}_1 + \mathbf{L}_{21}\mathbf{F}_1\right)'\mathbf{Q}_{22}\left(\mathbf{G}_1 + \mathbf{L}_{21}\mathbf{F}_1\right) \\ +\left(\mathbf{K}_1 + \mathbf{L}_{31}\mathbf{F}_1\right)'\mathbf{Q}_{23}\left(\mathbf{K}_1 + \mathbf{L}_{31}\mathbf{F}_1\right) \end{array}\right), \\
\mathbf{P}_3 &= \mathbf{W}_3 + \beta\left(\begin{array}{c} \mathbf{H}_1'\mathbf{P}_3\mathbf{H}_1 + \mathbf{F}_1'\mathbf{Q}_{31}\mathbf{F}_1 + \left(\mathbf{G}_1 + \mathbf{L}_{21}\mathbf{F}_1\right)'\mathbf{Q}_{32}\left(\mathbf{G}_1 + \mathbf{L}_{21}\mathbf{F}_1\right) \\ +\left(\mathbf{K}_1 + \mathbf{L}_{31}\mathbf{F}_1\right)'\mathbf{Q}_{33}\left(\mathbf{K}_1 + \mathbf{L}_{31}\mathbf{F}_1\right) \end{array}\right), \\
d_1 &= \frac{\beta}{1-\beta}tr\left[\left(\begin{array}{c} \mathbf{H}_2'\mathbf{P}_1\mathbf{H}_2 + \mathbf{F}_2'\mathbf{Q}_{11}\mathbf{F}_2 + \left(\mathbf{G}_2 + \mathbf{L}_{21}\mathbf{F}_2\right)'\mathbf{Q}_{12}\left(\mathbf{G}_2 + \mathbf{L}_{21}\mathbf{F}_2\right) \\ +\left(\mathbf{K}_2 + \mathbf{L}_{31}\mathbf{F}_2\right)'\mathbf{Q}_{13}\left(\mathbf{K}_2 + \mathbf{L}_{31}\mathbf{F}_2\right) \end{array}\right)\mathbf{\Omega}\right], \\
d_2 &= \frac{\beta}{1-\beta}tr\left[\left(\begin{array}{c} \mathbf{H}_2'\mathbf{P}_2\mathbf{H}_2 + \mathbf{F}_2'\mathbf{Q}_{21}\mathbf{F}_2 + \left(\mathbf{G}_2 + \mathbf{L}_{21}\mathbf{F}_2\right)'\mathbf{Q}_{22}\left(\mathbf{G}_2 + \mathbf{L}_{21}\mathbf{F}_2\right) \\ +\left(\mathbf{K}_2 + \mathbf{L}_{31}\mathbf{F}_2\right)'\mathbf{Q}_{23}\left(\mathbf{K}_2 + \mathbf{L}_{31}\mathbf{F}_2\right) \end{array}\right)\mathbf{\Omega}\right], \\
d_3 &= \frac{\beta}{1-\beta}tr\left[\left(\begin{array}{c} \mathbf{H}_2'\mathbf{P}_3\mathbf{H}_2 + \mathbf{F}_2'\mathbf{Q}_{31}\mathbf{F}_2 + \left(\mathbf{G}_2 + \mathbf{L}_{21}\mathbf{F}_2\right)'\mathbf{Q}_{32}\left(\mathbf{G}_2 + \mathbf{L}_{21}\mathbf{F}_2\right) \\ +\left(\mathbf{K}_2 + \mathbf{L}_{31}\mathbf{F}_2\right)'\mathbf{Q}_{33}\left(\mathbf{K}_2 + \mathbf{L}_{31}\mathbf{F}_2 +\right) \end{array}\right)\mathbf{\Omega}\right].
\end{aligned}$$

and the derivatives of the loss functions are:

$$\begin{aligned}
\frac{\partial Loss_1}{\partial \mathbf{x}_{1t}} &= \left(\mathbf{A}_{31} + \mathbf{A}_{32}\mathbf{L}_{21} + \mathbf{A}_{33}\mathbf{L}_{31}\right)'\mathbf{D}^{-1'}\mathbf{P}_1\mathbf{y}_t + \mathbf{Q}_{11}\mathbf{x}_{1t} + \mathbf{L}_{21}'\mathbf{Q}_{12}\mathbf{x}_{2t} + \mathbf{L}_{31}'\mathbf{Q}_{13}\mathbf{x}_{3t} = \mathbf{0}, \\
\frac{\partial Loss_2}{\partial \mathbf{x}_{2t}} &= \mathbf{A}_{32}'\mathbf{D}^{-1'}\mathbf{P}_2\mathbf{y}_t + \mathbf{Q}_{22}\mathbf{x}_{2t} = \mathbf{0}, \\
\frac{\partial Loss_3}{\partial \mathbf{x}_{3t}} &= \mathbf{A}_{33}'\mathbf{D}^{-1'}\mathbf{P}_3\mathbf{y}_t + \mathbf{Q}_{33}\mathbf{x}_{3t} = \mathbf{0}.
\end{aligned}$$

These first-order conditions give rise to the updating equations:

$$
\begin{aligned}
\mathbf{F}_1 &= -\left[\begin{array}{c} \left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}+\mathbf{A}_{33}\mathbf{L}_{31}\right)' \mathbf{D}^{-1'}\mathbf{P}_1\mathbf{D}^{-1}\left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}+\mathbf{A}_{33}\mathbf{L}_{31}\right) \\ +\mathbf{Q}_{11}+\mathbf{L}_{21}'\mathbf{Q}_{12}\mathbf{L}_{21}+\mathbf{L}_{31}'\mathbf{Q}_{13}\mathbf{L}_{31} \end{array}\right]^{-1} \\
&\quad\times\left[\begin{array}{c} \left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}+\mathbf{A}_{33}\mathbf{L}_{31}\right)' \mathbf{D}^{-1'}\mathbf{P}_1\mathbf{D}^{-1}\left(\mathbf{A}_{1}+\mathbf{A}_{32}\mathbf{G}_{1}+\mathbf{A}_{33}\mathbf{K}_{1}\right) \\ +\mathbf{L}_{21}'\mathbf{Q}_{12}\mathbf{G}_{1}+\mathbf{L}_{31}'\mathbf{Q}_{13}\mathbf{K}_{1} \end{array}\right], \\
\mathbf{F}_2 &= -\left[\begin{array}{c} \left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}+\mathbf{A}_{33}\mathbf{L}_{31}\right)' \mathbf{D}^{-1'}\mathbf{P}_1\mathbf{D}^{-1}\left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}+\mathbf{A}_{33}\mathbf{L}_{31}\right) \\ +\mathbf{Q}_{11}+\mathbf{L}_{21}'\mathbf{Q}_{12}\mathbf{L}_{21}+\mathbf{L}_{31}'\mathbf{Q}_{13}\mathbf{L}_{31} \end{array}\right]^{-1} \\
&\quad\times\left[\begin{array}{c} \left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}+\mathbf{A}_{33}\mathbf{L}_{31}\right)' \mathbf{D}^{-1'}\mathbf{P}_1\mathbf{D}^{-1}\left(\mathbf{A}_{5}+\mathbf{A}_{32}\mathbf{G}_{2}+\mathbf{A}_{33}\mathbf{K}_{2}\right) \\ +\mathbf{L}_{21}'\mathbf{Q}_{12}\mathbf{G}_{2}+\mathbf{L}_{31}'\mathbf{Q}_{13}\mathbf{K}_{2} \end{array}\right], \\
\mathbf{G}_1 &= -\left(\mathbf{Q}_{22}+\mathbf{A}_{32}'\mathbf{D}^{-1'}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{32}\right)^{-1}\mathbf{A}_{32}'\mathbf{D}^{-1'}\mathbf{P}_2\mathbf{D}^{-1}\left(\mathbf{A}_1+\mathbf{A}_{33}\mathbf{K}_1\right) \\
\mathbf{G}_2 &= -\left(\mathbf{Q}_{22}+\mathbf{A}_{32}'\mathbf{D}^{-1'}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{32}\right)^{-1}\mathbf{A}_{32}'\mathbf{D}^{-1'}\mathbf{P}_2\mathbf{D}^{-1}\left(\mathbf{A}_5+\mathbf{A}_{33}\mathbf{K}_2\right) \\
\mathbf{L}_{21} &= -\left(\mathbf{Q}_{22}+\mathbf{A}_{32}'\mathbf{D}^{-1'}\mathbf{P}_2\mathbf{D}^{-1}\mathbf{A}_{32}\right)^{-1}\mathbf{A}_{32}'\mathbf{D}^{-1'}\mathbf{P}_2\mathbf{D}^{-1}\left(\mathbf{A}_{31}+\mathbf{A}_{33}\mathbf{L}_{31}\right) \\
\mathbf{K}_1 &= -\left(\mathbf{Q}_{33}+\mathbf{A}_{33}'\mathbf{D}^{-1'}\mathbf{P}_3\mathbf{D}^{-1}\mathbf{A}_{33}\right)^{-1}\mathbf{A}_{33}'\mathbf{D}^{-1'}\mathbf{P}_3\mathbf{D}^{-1}\left(\mathbf{A}_1+\mathbf{A}_{32}\mathbf{G}_1\right) \\
\mathbf{K}_2 &= -\left(\mathbf{Q}_{33}+\mathbf{A}_{33}'\mathbf{D}^{-1'}\mathbf{P}_3\mathbf{D}^{-1}\mathbf{A}_{33}\right)^{-1}\mathbf{A}_{33}'\mathbf{D}^{-1'}\mathbf{P}_3\mathbf{D}^{-1}\left(\mathbf{A}_5+\mathbf{A}_{32}\mathbf{G}_2\right) \\
\mathbf{L}_{31} &= -\left(\mathbf{Q}_{33}+\mathbf{A}_{33}'\mathbf{D}^{-1'}\mathbf{P}_3\mathbf{D}^{-1}\mathbf{A}_{33}\right)^{-1}\mathbf{A}_{33}'\mathbf{D}^{-1'}\mathbf{P}_3\mathbf{D}^{-1}\left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}\right) \\
\mathbf{H}_1 &= \mathbf{D}^{-1}\left[\mathbf{A}_1+\mathbf{A}_{32}\mathbf{G}_1+\mathbf{A}_{33}\mathbf{K}_1+\left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}+\mathbf{A}_{33}\mathbf{L}_{31}\right)\mathbf{F}_1\right], \\
\mathbf{H}_2 &= \mathbf{D}^{-1}\left[\mathbf{A}_5+\mathbf{A}_{32}\mathbf{G}_2+\mathbf{A}_{33}\mathbf{K}_2+\left(\mathbf{A}_{31}+\mathbf{A}_{32}\mathbf{L}_{21}+\mathbf{A}_{33}\mathbf{L}_{31}\right)\mathbf{F}_2\right].
\end{aligned}
$$

Once convergence is achieved the state-contingent decision rules for players 2 and 3 are computed from:

$$
\begin{aligned}
\mathbf{x}_{2t} &= \left(\mathbf{G}_1+\mathbf{L}_{21}\mathbf{F}_1\right)\mathbf{y}_{t-1}+\left(\mathbf{G}_2+\mathbf{L}_{21}\mathbf{F}_2\right)\mathbf{v}_t, \\
\mathbf{x}_{3t} &= \left(\mathbf{K}_1+\mathbf{L}_{31}\mathbf{F}_1\right)\mathbf{y}_{t-1}+\left(\mathbf{K}_2+\mathbf{L}_{31}\mathbf{F}_2\right)\mathbf{v}_t.
\end{aligned}
$$