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# Optimal Parental Leave Subsidization with Endogenous Fertility and Growth


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## Abstract


In a life-cycle dynastic family model with endogenous fertility, labor-leisure, and accumulations of human and physical capital, this study examines the growth and welfare effects of parental leave subsidization when there is human capital externality. Compared with the social optimum, such externality causes higher fertility and less parental time and expenditure inputs in child human capital development, and thus lower growth and welfare in the laissez-faire equilibrium. Parental leave subsidization financed by a lump-sum tax (PLS_LS) promotes economic growth and welfare by improving the quantity-quality trade-off of children. There exists an optimal rate of parental leave subsidy but it cannot achieve the social optimum. Parental leave subsidization financed by a labor income tax (PLS_LI) increases the parental time input in child human capital and economic growth. It may improve welfare despite the distortionary effects of labor income taxes in exacerbating the problems of excessive fertility and under-investment of parental expenditure in child human capital. By calibrating the laissez-faire model economy to the U.S. data, our quantitative results show that for an empirically plausible degree of human capital externalities, the optimal parental leave subsidy under PLS_LI implies a fully-covered leave duration of 8.7 weeks per parent, which increases the annual growth rate of output per worker by 0.3 percentage points and welfare by 0.02 percent from the laissez-faire equilibrium.

**Keywords**

Parental leave, Labor income tax, Fertility, Human capital, Welfare, Growth

**JEL Classification**

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# Optimal Parental Leave Subsidization with Endogenous Fertility and Growth♠

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In a life-cycle dynastic family model with endogenous fertility, labor-leisure, and accumulations of human and physical capital, this study examines the growth and welfare effects of parental leave subsidization when there is human capital externality. Compared with the social optimum, such externality causes higher fertility and less parental time and expenditure inputs in child human capital development, and thus lower growth and welfare in the laissez-faire equilibrium. Parental leave subsidization financed by a lump-sum tax (PLS_LS) promotes economic growth and welfare by improving the quantity-quality trade-off of children. There exists an optimal rate of parental leave subsidy but it cannot achieve the social optimum. Parental leave subsidization financed by a labor income tax (PLS_LI) increases the parental time input in child human capital and economic growth. It may improve welfare despite the distortionary effects of labor income taxes in exacerbating the problems of excessive fertility and under-investment of parental expenditure in child human capital. By calibrating the laissez-faire model economy to the U.S. data, our quantitative results show that for an empirically plausible degree of human capital externalities, the optimal parental leave subsidy under PLS_LI implies a fully-covered leave duration of 8.7 weeks per parent, which increases the annual growth rate of output per worker by 0.3 percentage points and welfare by 0.02 percent from the laissez-faire equilibrium.

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# 1. Introduction

Human capital is not only a central factor of output production but has also been well-recognised as an engine of economic growth. Since children are the future builders of all nations, their human capital development crucially determines a nation's future economic performance. Among many factors that can affect human capital formation, parent time input is an extremely crucial factor in child human capital development (e.g., Ruhm, 2004; Del Boca et al., 2014; Fiorini and Keane, 2014; Heckman and Cunha, 2007).

However, how much time should parents invest in their children? Furthermore, should, or how much should, the government subsidize parental time investment? In reality, the availability and generosity of paid parental leave such as the duration of leave and the rate of income support varies considerably across countries. For example, leave with full-payment rates available for mothers varies from 0 weeks in the U.S. to 84.4 weeks in Estonia (OECD Family Database).[1]

To gain insight into these questions, this study develops and calibrates a life-cycle dynastic family model with endogenous fertility and human capital accumulation to examine the growth and welfare effects of parental time subsidization and the optimal parental time subsidization, both analytically and quantitatively. Our basic model features an economy with overlapping generations of three-period-lived agents. In their first period of life, they develop human capital and do not make any decisions. In their second period of life, they make decisions regarding work and leisure time, consumption, savings, the number of children to have, and expenditure and time spent on each child's human capital development. In their third period of life, they retire and decide upon consumption and transfers to each child. The human capital of a child is specified as a function of parental time and expenditure inputs, parental human capital, and the economy-wide average human capital, that is, there are externalities in human capital accumulation.[2] The preference of an infinitely-lived dynastic family is defined as a discounted sum of utilities derived from consumption levels of the old and young parents, leisure of young parent and the number of children in the family. This feature allows for a

[1] Under the most basic form of parental leave, parents are entitled to take leave to take care of a newborn child. However, in many countries, parental leave extends beyond the basic entitlement, so mothers or fathers can take some time off work to take care of an infant or older child. In this study, we use the term "parental time subsidization" and "parental leave subsidization" interchangeably to refer to paid parental leave policies to care for and nurture children.

[2] The notion of human capital externalities is old in economics (Marshall, 1890), and has received a great deal of attention by economists especially when endogenous growth theories emerge (e.g., Nelson and Phelps, 1966; Lucas, 1988; Romer, 1990; Tamura, 1991). Many empirical studies find evidence of human capital externalities through channels such as ethnic groups, neighbourhoods, workplaces, or schools, see, e.g., Borjas (1992, 1994, 1995), Rauch (1993), Moretti (2004a, b), Mas and Moretti (2009), Choi (2011), and Wantchekon et al. (2015).

direct comparison of the social optimum and the competitive equilibrium based on the same welfare function which takes into account utilities of all generations, and hence facilitates the analysis of optimal parental leave subsidization. The production side is characterized by a constant-return-to-scale production function that transforms physical capital, labor and human capital into a single final good. We show that the model globally converges to a balanced growth path which features a constant physical capital to effective labor ratio.

In the presence of human capital externalities, parents under-invest their time and expenditure inputs in children's human capital. As a result, the growth rate of human capital and output per worker is lower than the socially optimal level. Furthermore, through the trade-off between the quantity and quality of children (Becker and Lewis, 1973; Willis, 1973; Becker, 1991; Galor and Weil, 2000; Hazan and Berdugo, 2002; Moav, 2005; Li et al., 2008; Fernihough, 2017), fertility in the laissez-faire economy is too high compared with the socially optimal level. A higher fertility rate also means that parents have less time available for their own leisure and labor as having more children requires parents to spend more time raising them. Also, compared with the social optimum, both young-age and old-age consumption are higher. In short, the laissez-faire equilibrium is not socially optimal or efficient.

We then analyse if parental time subsidization can mitigate the efficiency losses that occur in the laissez-faire system. We interpret such subsidization as capturing paid parental leave provided by governments to parents to care for newborn and young children, a benefit that is available in most advanced economies. To better understand the efficiency gain emerging from parental leave subsidization and the efficiency loss emerging from distortionary taxation required to finance it, we first consider parental leave subsidization financed by lump-sum taxation (PLS_LS), and then parental leave subsidization financed by labor income taxation (PLS_LI) as commonly observed in reality.

We show analytically that PLS_LS increases parental time investment per child by lowering the marginal cost of investing time in a child and reduces fertility by increasing the marginal time cost of having a child. Through the quantity-quality trade-off of children, PLS_LS also increases parent's expenditure on a child.[3] Since parental time investment in child human capital is higher now, PLS_LS reduces parental leisure and labor supply. PLS_LS also lowers old-age consumption. Overall, PLS_LS can mitigate the efficiency losses arising from human capital externalities and raises the balanced steady-state growth rate of output per

[3] We normalize all the quantity variables by output per worker to detrend the model. So, the expenditure input here and other quantity variables to be discussed below are all relative to output per worker.

worker. However, it cannot achieve social optimum when fertility, labor and leisure choices are all endogenous. We also show that an optimal leave subsidy rate exists such that PLS_LS increases welfare until the leave subsidy rate reaches this optimal level. The effects of PLS_LI on parental time investment in child human capital, leisure and labor supply are similar to those of PLS_LS. However, differing from PLS_LS, PLS_LI increases fertility and parental consumption but reduces parental expenditure per child. Hence, there are opposing welfare effects of PLS_LI. Welfare is higher when PLS_LI raises parental time input in child human capital, but welfare is lower when distortions associated with income tax exacerbate the problems of excessive fertility and parental consumption and under-investment in expenditure input in child human capital. In terms of growth effect, we show that PLS_LI leads to a higher growth rate of output per worker under a sufficient condition.

To evaluate the quantitative effects of parental leave subsidization, in particular, the optimal PLS_LI and its welfare and growth effect, we calibrate the laissez-faire model to the U.S. economy. The U.S. makes for an interesting case study as it is currently the only developed country that does not subsidize parental leave at the national level. [4] The quantitative exercise also aims to provide a policy recommendation for the U.S. to improve its current parental leave policy to achieve better outcomes.

In our calibration, we first choose an empirically plausible degree of externality in the human capital production function based on the estimated range in Borjas (1995). We then calibrate other parameters to match observed data moments in the U.S. data (American Time Use Survey, Panel Study of Income Dynamics, and others) that capture time use of households in childcare, leisure and work, parental expenditure on children's human capital development, consumption in young and old age, and so on. We compute the optimal parental subsidy rate under PLS_LS and PLS_LI, respectively, and convert it into a corresponding parental leave duration that is fully covered. The implied optimal leave duration is 8.7 (14.3) weeks per child for a single parent for PLS_LI (PLS_LS), which raises the annual growth rate of output per worker by 0.3 (0.7) percentage points and welfare by 0.024 (0.058) percent from the laissez-faire equilibrium. The leave entitlement implied by the optimal PLS_LI is comparable to the practice in some European countries like Belgium, Netherland and Spain.

To the best of our knowledge, this study is the first that examines the growth and welfare effects of parental leave subsidization as well as the optimal parental leave

[4] In the US, some people have access to paid leave through their employers or because they live in states that have state paid leave policies. However, there is no access to paid leave at the national level.

subsidization in the presence of human capital externalities both analytically and quantitatively. Our study contributes to the literature in several ways.

First, our study contributes to a limited theoretical and quantitative literature that analyzes optimal parental leave policies (e.g., Bastani et al., 2019; Barigozzi et al., 2018) and the effects of parental leave policies (e.g., Bernal and Fruttero, 2008; Erosa et al., 2010; Del Rey et al., 2017; Del Rey et al., 2021). Among this literature, most studies focus on the effects of parental leave policies on labor market outcomes such as wages, employment and career choice (Bernal and Fruttero, 2008; Erosa et al., 2010; Del Rey et al., 2017; Bastani et al., 2018; Barigozzi et al., 2018; Del Rey et al., 2021). For example, Bastani et al. (2019) provide a theoretical justification for parental leave mandates to alleviate the under-provision of workplace flexibility and the gender wage gap in the labor market. Barigozzi et al. (2018) theoretically show that parental leave reduces welfare by exacerbating the negative externality generated by social norms concerning childcare activities and career choices. Erosa et al. (2010) quantitatively find that paid parental leave leads to welfare losses in a search and matching model since parental leave leads to inefficient matches in the labor market and encourages too much leave taking by fertile females. Our paper differs from the above literature in two ways: first, we focus on the effects of parental leave subsidization on child human capital development and the subsequent effects on growth and welfare; and second, we consider human capital externalities as the source of inefficiency and characterize the optimal level of parental leave subsidization both analytically and quantitatively.

Second, our study complements studies that analyse the effects of parental time investments on child human capital. For example, Bernal and Fruttero (2008) find a positive effect of parental leave on human capital quantitatively using a model of marriage and divorce with labor market frictions. Gahramanov et al. (2020) conduct a quantitative analysis of parental time investment on children's skill formation and find that parental time subsidisation reduces fertility and has an uncertain effect on the labor supply of the primary caretaker. Youderian (2019), focusing on the evaluation of education policies with a parental leave policy, finds quantitatively that education subsidization is more effective than parental leave subsidization in promoting human capital and welfare. In contrast to our study, the above literature uses a partial equilibrium setup (Gahramanov et al., 2020) and abstract from endogenous fertility (Bernal and Fruttero, 2008; Youderian, 2019) and endogenous growth (Bernal and Fruttero, 2008; Youderian, 2019; Gahramanov et al., 2020).

Third, our study also contributes to the literature on public investment in human capital and economic growth. There is considerable literature on the effects of education subsidization or public education on economic growth (Glomm and Ravikumar, 1992, 1998; Zhang, 1996; Benabou, 1996, 2002; de la Croix and Doepke, 2004). However, this literature does not pay much attention to the influence of parental time subsidization on human capital development and economic growth. Our finding of positive impacts of parental time subsidization on economic growth and welfare suggests that the subsidy aspect of parental time deserves more attention in this literature. This is in line with empirical evidence which finds that parental time input is an extremely important factor in child human capital development (Ruhm, 2004; Del Boca et al., 2014) and that monetary expenditure is less beneficial than time input in the cognitive development of children particularly in regard to young children (Del Boca et al., 2014).

Additionally, our theoretical and quantitative results concerning the effects of parental leave on fertility and child human capital are consistent with the empirical evidence. Carneiro et al. (2015) study and compare the effect of a change in parental leave entitlements in Norway and find that when parental leave entitlements are very low, similar to the current situation in the U.S., longer entitlements with parental leave subsidization can increase parent time spent with children and, thus, child human capital development. Using Austrian data, Lalive and Zweimuller (2009) find that increasing the parental leave duration increases fertility, as predicted by our study. Furthermore, the welfare implication of parental leave in our study agrees with the empirical evidence which shows that short to moderate periods of parental leave can increase economic efficiency (Ruhm and Teague, 1997).

The rest of the paper proceeds as follows: section 2 describes the basic model without parental leave subsidization, i.e., the laissez-faire economy; section 3 characterizes the laissez-faire equilibrium, the social optimum, and the equilibria with PLS_LS and PLS_LI; section 4 describes the calibration; section 5 discusses the quantitative results; and the final section offers the conclusion of the study.

## 2. The Basic Model

We first describe the basic model without parental leave subsidization. The model economy is inhabited by overlapping generations of a large number of identical agents who live for three periods. In their first period of life, these agents develop human capital and do not make any decisions. In their second period of life, they make decisions regarding work and leisure time, consumption, savings, expenditure and time spent on their children's human capital development, and the number of children to have. In their third period of life, they retire and decide upon consumption and transfers to their children. The mass of the working generation in period $t$ is denoted by $L_t$.

The preferences of the coexisting old parent and young working parent in a family are assumed to be identical and are defined over the consumption levels of the old and young parents, $C_{o,t}$ and $C_{y,t}$ respectively, the young parent's leisure time, $Z_t$, and the number of children, $N_t$, in the family. Therefore, the preference of the infinitely-lived dynastic family is given by

(1) $U_o = \sum_{t=0}^{\infty} \alpha^t \{\beta \ln C_{o,t} + \alpha[\ln C_{y,t} + \gamma \ln Z_t + \rho \ln N_t]\}$,

where $\alpha \in (0,1)$ and $\beta \in (0,1)$ are discount factors across generations and life stages, $\gamma > 0$ is the weight on utility from leisure, and $\rho > 0$ is the weight on utility from the number of children.[5] The assumption of having average utility of children in the parental preference follows Mill (1848) and Razin and Ben-Zion (1975), and the assumption of identical preferences shared by the old and the young avoids generational conflicts and can serve as the Millian social welfare function.

Each young parent in period $t$ devotes one unit of time endowment to four activities: (i) a fixed time spent on looking after each child $v \in (0,1)$, and hence a total amount of time of $vN_t$; (ii) a variable time spent on looking after each child $e_t \in (0,1)$ which is a choice of the young parent and enters a child's human capital formation (to be described later), and hence a total amount of time of $e_t N_t$; (iii) leisure $Z_t$; and (iv) working time $(1 - vN_t - e_t N_t - Z_t)$.[6] Each young parent earns wage income $(1 - vN_t - e_t N_t - Z_t)W_t H_t$, where $W_t$ is the wage rate per unit of effective labor, and $H_t$ is the young parent's human capital. A young parent in period $t$ receives transfers from her parent if $B_t > 0$ or provides transfers to her parent if $B_t < 0$. The young parent spends her wage income and transfers from her parent on young-age consumption $C_{y,t}$, retirement savings $S_t$, and resources spent on each child's human capital development $Q_t$. An old parent in period $t$ spends her savings plus interest income on old-age consumption $C_{o,t}$ and transfers to her children $B_t$.

In the present model, children are consumption goods as the number of children enters the utility function. The model also allows for altruism of young parents towards their children as well as towards their parents that induces private intergenerational transfers (transfers to the

[5] The model is an extension of Zhang and Zhang (2007) and Yew and Zhang (2009, 2013) to include leisure of the young parent in the preferences to explore the growth and welfare implications of parental time subsidization.
[6] In our calibration, we interpret $e$ as parental time raising a child when the child is younger, while $v$ as that when the child is elder. See Section 4 for a detailed discussion on this interpretation.

children or the elderly). When the altruism toward parents is sufficiently strong, transfers from the young parent to the old parent can be operative (i.e., $B_t < 0$) so that children are investment goods as well.[7]

The budget constraints of old and young parents can be written as follows:

(2) $C_{o,t} = S_{t-1}R_t - B_t N_{t-1}$,

(3) $C_{y,t} = B_t + (1 - vN_t - e_t N_t - Z_t)W_t H_t - S_t - Q_t N_t$,

where $R_t$ is the gross interest rate on savings.

The production function of the single final good is given by

(4) $Y_t = DK_t{}^{\theta}[L_t l_t H_t]^{1-\theta},\ D > 0,\ \theta \in (0,1)$

where $K_t$ is the aggregate stock of physical capital, $H_t$ is human capital, $l_t$ is labor time per worker, and $L_t$ is total number of workers. In per worker terms, $y_t = Dk_t{}^{\theta}(l_t H_t)^{1-\theta}$, where $y_t = Y_t/L_t$ and $k_t = K_t/L_t$ denote output per worker and capital per worker, respectively. As one period in this model corresponds to 30 years, assuming that both physical capital and human capital depreciate fully within one period is reasonable and can help obtain closed form solutions. Factors earn their marginal products, and the price of the sole final good is normalized to unity. Denoting $\mu_t = K_t/(L_t l_t H_t)$ as the physical capital-effective labor ratio, the wage rate per unit of effective labor and the real interest rate are then respectively given by:

(5) $W_t = (1 - \theta)D\mu_t^{\theta}$,

(6) $R_t = \theta D\mu_t^{\theta-1}$.

The market clearing conditions for the labour and capital markets are given by:

(7) $l_t = 1 - vN_t - e_t N_t - Z_t$,

(8) $K_{t+1} = L_t S_t$.

The working population evolves according to $L_{t+1} = L_t N_t$. Then the resource feasibility constraint is given by

(9) $C_{y,t} = Dk_t{}^{\theta}((1 - vN_t - e_t N_t - Z_t)H_t)^{1-\theta} - k_{t+1}N_t - Q_t N_t - C_{o,t}/N_{t-1}$.

The human capital of a child, $H_{t+1}$, is given by

(10) $H_{t+1} = AQ_t^{\delta} e_t^{\varepsilon}\left(H_t^{\zeta}\bar{H}_t^{1-\zeta}\right)^{1-\delta},\ \ A > 0, 0 < \varepsilon, \delta < 1, 0 < \zeta \leq 1$,

[7] Ioannides and Kan (2000) and Raut and Tran (2005) find supporting evidence for two-sided altruism using U.S. and Indonesian data, respectively.

where $Q_t$ is parental expenditure on a child's human capital formation, $e_t$ is the parental time input in a child's human capital development, $H_t$ is the human capital of the child's parent, and $\overline{H}_t$ is the average human capital in the economy.[8] Parameters $\delta$ and $\varepsilon$ indicate the weights of parental expenditure and parental time in human capital technology, respectively, and $\zeta$ determines the degree of human capital externalities with a lower value indicating a higher degree of externalities. In equilibrium, $H_t = \overline{H}_t$ by symmetry.

## 3. Equilibrium

We first characterize the competitive equilibrium in a laissez-faire system and compare it with the social optimum. We then add parental leave subsidization financed by lump-sum taxation and labor income taxation into the model and characterize the competitive equilibrium. All proofs are provided in Appendix A.

### *3.1. Laissez-faire System*

The problem of a dynastic family is to maximize utility in (1) subject to budget constraints (2) and (3), and the human capital technology (10), taking the average human capital and prices as given. The budget constraints (2) and (3) can be combined into a single budget constraint:

(11) $C_{y,t} = (S_{t-1}R_t - C_{o,t})/N_{t-1} + (1 - vN_t - e_tN_t - Z_t)W_tH_t - S_t - Q_tN_t.$

By using the combined budget constraint and the human capital technology for substitution, this problem can be rewritten as follows:

$$\max_{\{C_{o,t},S_t,Q_t,H_{t+1},e_t,Z_t,N_t\}} U_o = \sum_{t=0}^{\infty}\{\alpha^t[\beta\ln C_{o,t} + \alpha\ln((S_{t-1}R_t - C_{o,t})/N_{t-1} + (1 - vN_t - e_tN_t - Z_t)W_tH_t - S_t - Q_tN_t) + \alpha\gamma\ln Z_t + \alpha\rho\ln N_t] + \lambda_t[AQ_t^{\delta}e_t^{\varepsilon}\left(H_t^{\zeta}\overline{H}_t^{1-\zeta}\right)^{1-\delta} - H_{t+1}]\},$$

where $\lambda_t$ is the multiplier for the human capital technology. The first-order conditions for $t \geq 0$ are given as follows.

The family equates the marginal rate of substitution between old parent consumption and young parent consumption to their relative prices:

(12) $\frac{\beta C_{y,t}}{\alpha C_{o,t}} = \frac{1}{N_{t-1}}.$

[8] Any possible effect of the fixed parental time $v$ on a child's human capital development is captured in the constant parameter $A$ in equation (10).

Equating the marginal rate of substitution between young-age consumption across generations to their relative prices yields:

(13) $$\frac{C_{y,t+1}}{\alpha C_{y,t}} = \frac{\theta y_{t+1}}{S_t},$$

after using $R_{t+1}/N_t = \theta y_{t+1}/S_t$ for substitution.

The marginal rate of substitution between young parent's consumption and expenditure on a child's human capital equals their relative prices:

(14) $$\frac{Q_t C_{y,t+1}}{\alpha C_{y,t}[\delta(1-\theta)y_{t+1}+\zeta(1-\delta)Q_{t+1}N_{t+1}]} = \frac{1}{N_t},$$

after using $l_{t+1}W_{t+1}H_{t+1} = (1-\theta)y_{t+1}$ for substitution. When human capital externalities exist ($\zeta < 1$), the marginal utility from spending on a child's human capital is lower. As a consequence, parental expenditure on a child's human capital, $Q_t$, is lower than the socially optimal level. When human capital externalities exist, the young parent fails to recognize the positive effect of average human capital on the child's human capital.

The marginal rate of substitution between parental time input in a child's human capital and labor equals their relative prices:

(15) $$\frac{\varepsilon Q_t(1-vN_t-e_tN_t-Z_t)}{\delta e_t(1-\theta)y_t} = 1.$$

Since under-spending on a child's human capital formation reduces the marginal utility of the parental time input in a child's human capital, the parental time input in a child's human capital, $e_t$, is also lower than the socially optimal level when human capital externalities exist.

Equating the marginal rate of substitution between fertility and young parent consumption to their relative prices gives:

(16) $$\frac{\rho C_{y,t}}{N_t} = \frac{(v+e_t)(1-\theta)y_t}{(1-vN_t-e_tN_t-Z_t)} + Q_t + \frac{\alpha C_{y,t}}{C_{y,t+1}N_t}\left[\theta y_{t+1} - \frac{C_{o,t+1}}{N_t}\right].$$

The three terms of the relative prices are the time cost of a child, the amount of expenditure spent on a child, and the transfer cost of a child via budget constraint (2), respectively. When parental expenditure and time input in child human capital are too low, the expenditure and time costs of a child are also too low. Thus, fertility, $N_t$, will be too high compared to the socially optimal level.

The marginal rate of substitution between young parent leisure and consumption equals their relative prices:

(17) $$\frac{\gamma C_{y,t}}{Z_t} = \frac{(1-\theta)y_t}{(1-vN_t-e_tN_t-Z_t)},$$

after using $W_tH_t = (1-\theta)y_t/(1-vN_t-e_tN_t-Z_t)$ for substitution. When fertility is higher, the relative prices in (17) are higher. This implies that leisure, $Z_t$, is too low compared to the socially optimal level when human capital externalities exist. Moreover, when fertility is too high, labor is also too low, since raising a child is time-intensive.

**Definition 1.** Given an initial state $(N_{-1}, K_0, H_0)$, a competitive equilibrium in the laissez-faire economy is a sequence of allocations $\{B_t, C_{y,t}, C_{o,t}, N_t, S_t, e_t, Q_t, Z_t, Y_t, K_{t+1}, H_{t+1}\}_{t=0}^{\infty}$ and prices $\{R_t, W_t\}_{t=0}^{\infty}$ such that: (i) taking the average human capital and prices as given, firms and households optimize and their solutions satisfy the budget constraints (2) and (3), technologies (4) and (10), optimal conditions (5)–(6) and (12)–(17), and the transversality conditions (given in Appendix A); (ii) all markets clear with $K_{t+1} = L_t S_t$ and per worker labor input $l_t$ being equal to $1 - vN_t - e_t N_t - Z_t$; and (iii) consistency holds with $H_t = \overline{H}_t$.

With the log utility, the Cobb–Douglas functions for both human capital and production technologies, and full depreciation of physical and human capital within one period, it is expected that fertility, time allocations, and the proportional allocations of output will be constant over time given any initial state. Therefore, by letting the fraction of output per worker spent on item $X_t$ be a time-invariant lower-case variable $x_t = X_t / y_t$, where $y_t = Y_t / L_t$ and $X_t = B_t, C_{y,t}, C_{o,t}, S_t, Q_t$, we transform the variables in the first-order conditions and budget constraints into their relative ratios to output per worker, thus obtaining the following constant allocation rules:[9]

(18) $b = \theta - \frac{\beta c_y}{\alpha}$,

(19) $s = \alpha\theta$,

(20) $c_o = \frac{c_y \beta N}{\alpha}$ (for $t > 0$),

(21) $c_{o,0} = \frac{c_y \beta N_{-1}}{\alpha}$ (for $t = 0$),

(22) $c_y = \frac{\alpha[(1-\alpha\theta)(1-\alpha\zeta(1-\delta))-\alpha\delta(1-\theta)]}{(\alpha+\beta)(1-\alpha\zeta(1-\delta))}$,

(23) $q = \frac{\alpha\delta(1-\theta)}{N(1-\alpha\zeta(1-\delta))}$,

(24) $e = \frac{\alpha\varepsilon(\alpha+\beta)(1-\theta)}{N N_d^{LF}}$,

(25) $Z = \frac{\gamma\alpha[(1-\alpha\theta)(1-\alpha\zeta(1-\delta))-\alpha\delta(1-\theta)]}{N_d^{LF}}$,

[9] The transformed budget constraints take the following forms: $c_y = b + (1-\theta) - s - qN$, $c_o = N(\theta - b)$ for $t > 0$ and $c_{o,0} = N_{-1}(\theta - b)$ for a predetermined $N_{-1}$.

(26) $N = \frac{N_n^{LF}}{vN_d^{LF}}$,

where

$$N_n^{LF} = \alpha\{(\rho+\beta)[(1-\alpha\theta)\big(1-\alpha\zeta(1-\delta)\big)-\alpha\delta(1-\theta)]-(\alpha+\beta)\times [(1-\alpha\zeta(1-\delta))\theta+(1-\theta)(\delta+\varepsilon)]\},$$

$$N_d^{LF} = [\alpha+\beta+\alpha(\rho+\beta+\gamma)][(1-\alpha\theta)\big(1-\alpha\zeta(1-\delta)\big)-\alpha\delta(1-\theta)]- \theta(\alpha+\beta)(1-\alpha\zeta(1-\delta)).$$

The inverse relationship between the ratio of parental expenditure on child human capital to output per worker, $q$, and the number of children, $N$, given by (23), and the inverse relationship between the parental time input in child human capital, $e$, and the number of children, $N$, given by (24), reflects the quantity–quality trade-off of children.

The above solutions satisfy all the equilibrium conditions, including budget constraints, first-order conditions, and market clearing conditions in all periods. Therefore, they are valid solutions at all times on the entire equilibrium path. However, because of the presence of non-convexity in the form of $B_{t+1}N_t$ or $Q_tN_t$ or $e_tN_t$ in the budget constraints (2) and (3), the feasible set of households is not a convex set. As shown by Zhang (1995), Zhang et al. (2001), and Yew and Zhang (2009), a sufficient condition for the solution to be optimal is a sufficiently strong taste parameter for the number of children ($\rho$) relative to the taste parameter for the welfare of children ($\alpha$) such that an interior solution for fertility exists. The following lemma establishes a sufficient condition under which the model has an interior solution for fertility and hence for other choice variables as well.

**Lemma 1.** *There exists a unique equilibrium interior solution* $(N, c_y, s, q, e, Z, c_o, c_{o,0})$ *if the taste for the number of children is strong enough such that*

$$\rho > \underline{\rho}^{LF} \equiv \frac{(\alpha+\beta)[(1-\alpha\zeta(1-\delta))\theta+(1-\theta)(\delta+\varepsilon)]-\beta[(1-\alpha\theta)(1-\alpha\zeta(1-\delta))-\alpha\delta(1-\theta)]}{(1-\alpha\theta)(1-\alpha\zeta(1-\delta))-\alpha\delta(1-\theta)}.$$

*Moreover,* $b > 0$ *if the discount factor* $\alpha$ *is large enough and the externality* $\zeta$ *is weak enough.*

We now show that the model globally converges to a unique balanced growth path. To see this, we first track down the entire dynamic equilibrium path starting from any initial levels of physical capital $K_0$ and human capital $H_0$, and any predetermined fertility rate $N_{-1}$. By substituting $y_t = D{\mu_t}^{\theta} l_t H_t$ into the solutions for $H_{t+1}$ and $k_{t+1}$ and using (8) and (10), we obtain the evolution equation for the physical capital-effective labor ratio

(27) $\mu_{t+1} = \frac{sD^{1-\delta}\mu_t{}^{\Gamma}}{A(ql)^{\delta}Ne^{\varepsilon}}$.

Since $0 < \Gamma \equiv \theta(1-\delta) < 1$, $\mu_t$ globally converges to a constant

(28) $\mu_{\infty} = \left\{\frac{sD^{1-\delta}}{AN(ql)^{\delta}e^{\varepsilon}}\right\}^{\frac{1}{1-\Gamma}}$.

Using (27) and (28), the following lemma establishes the global convergence of the model toward a balanced growth path.

**Lemma 2.** *Starting from any initial levels of physical capital $K_0$ and human capital $H_0$, and any predetermined fertility rate $N_{-1}$, the economy globally converges toward its balanced growth path.*

To obtain an analytical expression for the balanced steady-state growth rate of output per worker, note that combining the production function (4) and human capital technology (10) gives:

(29) $\frac{H_{t+1}}{H_t} = A(Dql)^{\delta}e^{\varepsilon}\mu_t{}^{\theta\delta}$.

As $\mu_t$ converges to $\mu_{\infty}$, given by (28), $H_{t+1}/H_t$ converges to $1 + g_{\infty}$, where the balanced steady-state growth rate of output per worker $g_{\infty}$ is obtained as:

(30) $g_{\infty} = \left\{A^{1-\theta}D^{\delta}(ql)^{\delta(1-\theta)}e^{\varepsilon(1-\theta)}\left(\frac{s}{N}\right)^{\theta\delta}\right\}^{\frac{1}{1-\Gamma}} - 1$.

The steady-state growth rate positively depends on the productivity parameters $(A, D)$, the ratio of parental expenditure on child human capital to output per worker $(q)$, labor input $(l)$, the parental time input in child human capital $(e)$, and the saving rate $(s)$, while it negatively depends on fertility $(N)$.[10]

With the full characterization of the equilibrium path for the model, we can now solve for the level of welfare. Using the solutions for $(c_y, s, b, q, e, Z, N, c_o, c_{o,0})$ and the sequence $\{\ln y_t\}_{t=0}^{\infty}$, given an initial state $(N_{-1}, H_o, k_o)$, we can obtain

(31) $U_o = \beta\ln\left(c_{o,0}y_0\right) +$

$$\alpha\sum_{t=0}^{\infty}\alpha^t[\ln c_{y,t} + \ln y_t + \gamma\ln Z_t + \rho\ln N_t + \beta\ln c_{o,t+1} + \beta\ln y_{t+1}]$$

[10] The productivity parameters $A$ and $D$ in (30) are assumed to be sufficiently large. Thus, steady-state growth exists.

$$= \Psi_0 + \Psi_1 \ln c_y + \Psi_2 \ln l + \Psi_3 \ln N + \Psi_4 \ln q + \Psi_5 \ln e + \Psi_6 \ln s + \Psi_7 \ln Z$$

where $\Psi_0$ is a constant that consists of the initial state $(N_{-1}, H_o, k_o)$ and the productivity parameters $(A, D)$, and $\Psi_i$, $i = 1,2, ..7$, are positive constants consisting of the model parameters and their expressions are given in Appendix A. Note that the utility level in (31) is bounded above with $0 < \alpha < 1$.

Now that we have solved the model, we can examine the effects of human capital externalities $(\zeta < 1)$ on the endogenous variables. Proposition 1 establishes these effects.

**Proposition 1.** *Larger human capital externalities (i.e., lower $\zeta$) (i) do not affect the savings rate $(s)$, but lead to (ii) lower ratio of parental expenditure on a child's human capital to output per worker $(q)$, (iii) lower parental time input in child human capital $(e)$, (iv) lower leisure $(Z)$, (v) lower labor supply $(l)$, (vi) higher young-age consumption relative to output per worker $(c_y)$, (vii) higher old-age consumption relative to output per worker $(c_o)$, and (viii) higher fertility $(N)$. Results (i), (ii), (iii), (v) and (viii) suggest that larger human capital externalities lead to a lower balanced steady-state growth rate of output per worker $(g_\infty)$.*

The results in Proposition 1 imply that in the laissez-faire economy with human capital externalities, the ratio of parental expenditure on a child's human capital to output per worker $(q)$, the parental time input in child human capital $(e)$, leisure $(Z)$, labor supply $(l)$ and the growth rate of output per worker $(g_\infty)$ are too low compared with their counterparts in the social planner economy; whereas the fraction of output spent on young-age consumption $(c_y)$, the fraction of output spent on old-age consumption $(c_o)$ and fertility $(N)$ are too high compared with their counterparts in the social planner economy. The larger the human capital externalities, the larger the efficiency losses are in the laissez-faire economy. In terms of the welfare effect, intuitively, the efficiency losses arising from human capital externalities lead to a lower welfare level in the laissez-faire economy relative to the social optimum. However, it is complicated to show this effect analytically and hence we will examine it in the quantitative exercise.

## *3.2. Social Optimum*

In reference to Mill (1848), the social planner chooses a sequence $\{C_{o,t}, C_{y,t}, Q_t, e_t, Z_t, N_t, k_{t+1}, H_{t+1}, \overline{H}_{t+1}\}_{t=0}^{\infty}$ to maximize the utility in (1) subject to the resource feasibility constraint (9) and the human capital production function (10), given an initial state $(H_0, k_0, N_{-1})$. Differing from individual households, the social planner internalizes the human capital externalities by setting $H_{t+1} = \overline{H}_{t+1}$.

The optimal conditions for $C_{o,t}, k_{t+1}, N_t, e_t$, and $Z_t$ are the same as those in (12), (13), and (15)-(17) for the laissez-faire economy. The first-order conditions with respect to $Q_t$, $H_{t+1}$, and $\overline{H}_{t+1}$ are combined to yield the optimal condition for parental expenditure input in child human capital:

$$\frac{Q_t c_{y,t+1}}{\alpha c_{y,t}[\delta(1-\theta)y_{t+1}+(1-\delta)Q_{t+1}N_{t+1}]} = \frac{1}{N_t},$$

which is a special case of (14) with $\zeta = 1$. Therefore, the social planner's solution for $(c_y, q, e, Z, N)$ is a special case of the laissez-faire solution with $\zeta = 1$.

To ensure the existence of a solution to the social planner's problem, it is assumed that the taste for the number of children $\rho$ is sufficiently strong relative to the taste for the welfare of children $\alpha$:

**Assumption 1**. $\rho > \underline{\rho}^{SP} \equiv \frac{(\alpha+\beta)[(1-\alpha(1-\delta))\theta+(1-\theta)(\delta+\varepsilon)]-\beta(1-\alpha)(1-\alpha\theta(1-\delta))}{(1-\alpha)(1-\alpha\theta(1-\delta))}$.

When Assumption 1 holds, fertility is positive and there is a unique solution to the planner's problem. Furthermore, Assumption 1 ensures the condition in Lemma 1 holds such that the laissez-faire economy has a unique equilibrium.

As the social planner can internalize the external effect of average human capital in the economy, which is equivalent to setting $\zeta = 1$ in the laissez-faire economy, by Proposition 1 $(c_y, c_o, N)$ are higher while $(q, e, Z, l)$ are lower in the laissez-faire equilibrium than those implied by the social planner's solution.

The socially optimal growth rate of output per worker and welfare level can be obtained from (30) and (31), respectively, by substituting the socially optimal allocations $(c_y, l, N, q, e, Z, s, c_o, c_{o,0})$ into (30) and (31). The growth rate of human capital (or that of output per worker) is higher in the social planner's solution than in the laissez-faire equilibrium, the ratio of parental expenditure on child human capital to output per worker ($q$), labor input ($l$), the parental time input in child human capital ($e$) are higher while fertility is lower compared with those in the laissez-faire. Therefore, starting with the laissez-faire situation, if parental leave subsidization can mitigate the efficiency losses that occur in the laissez-faire system, then parental leave subsidization may help increase the growth rate of output per worker and the welfare level toward the socially optimal levels. Hence, next section considers parental leave subsidization financed by lump-sum taxation, and is followed by parental leave subsidization financed by labor income taxation. Comparing the results under these two leave policies is useful to identify efficiency losses arising from the distortionary effects of labor income taxation.

### *3.3. Parental Leave Subsidization Financed by Lump-sum Taxes (PLS_LS)*

This section considers a parental time or parental leave subsidization policy financed by a lump-sum tax.[11] Let $T_t > 0$ denote the amount of lump-sum taxes and $p_t \in (0,1)$ denote the parental leave subsidy rate. Then, the budget constraint in young adulthood becomes

(32) $$C_{y,t} = B_t + (1 - vN_t - e_tN_t - Z_t)W_tH_t - T_t - S_t - Q_tN_t + \; p_te_tN_tW_t\bar{H}_t,$$

where $p_te_tN_tW_t\bar{H}_t$ is the amount of parental leave subsidization which is determined by the parental leave subsidy rate $p_t$, the total parental time input in children human capital development $e_tN_t$, and the average effective wage in the economy $W_t\bar{H}_t$.[12]

The government balanced budget constraint is given by

$$\bar{T}_t = p_t\bar{e}_t\bar{N}_tW_t\bar{H}_t,$$

where the bar above each variable indicates its average level within the economy.

The problem of a dynastic family is to maximize utility in (1) subject to the budget constraints (2) and (32), and the human capital technology in (10) given the average human capital, prices, lump-sum taxes, $T_t$, and the leave subsidy rate, $p_t$. For $t \geq 0$, the first-order conditions with respect to $C_{o,t}$, $S_t$, $H_{t+1}$ and $Z_t$ are the same as those in the laissez-faire economy. The first-order conditions with respect to $e_t$ and $N_t$ are given as follows.

The marginal rate of substitution between the parental time input in child human capital and labor equals their relative prices:

$$\frac{\varepsilon Q_t(1-vN_t-e_tN_t-Z_t)}{\delta e_t(1-p_t)(1-\theta)y_t} = 1.$$

The parental leave subsidy $p_t$ tends to increase the parental time input in child human capital by increasing the marginal utility of the parental time input in child human capital. Hence,

[11] In this study, parental leave subsidization refers to paid parental leave policies to care for and nurture children such as leave around childbirth and leave for parental involvement in childcare activities.

[12] If the parental leave subsidy rate depends on a parent's effective wage, the positive growth and welfare effects will be stronger in this case than that if the subsidy rate depends on the average effective wage in the economy. To be more conservative about the welfare gains of parental leave subsidization, we assume the parental leave subsidy rate depends on the average effective wage, which is in line with leave policies in some countries such as Australia.

parental leave subsidization financed by lump-sum taxation (PLS_LS) can mitigate under-investing of parental time in child human capital that occurs in the laissez-faire system.

Equating the marginal rate of substitution between fertility and the young-parent's consumption to their relative prices gives:

$$\frac{\rho C_{y,t}}{N_t} = \frac{[v+e_t(1-p_t)](1-\theta)y_t}{(1-vN_t-e_tN_t-Z_t)} + Q_t + \frac{\alpha C_{y,t}}{C_{y,t+1}N_t}\left[\theta y_{t+1} - \frac{C_{o,t+1}}{N_t}\right].$$

The parental leave subsidy $p_t$ tends to increase the time costs of having a child because the parental time input in child human capital with the leave subsidy, $e_t(1-p_t)$, is higher. Therefore, $p_t$ tends to reduce the fertility rate.

Given a time-invariant parental leave subsidy rate and the corresponding lump-sum tax, the constant allocation rules for $b$, $s$, $c_o$ and $c_{o,0}$ are the same as those in $(18)-(21)$, respectively. The constant allocation rules for $c_y$, $q$, $e$, $Z$, $N$ and $l$ are given in Appendix A.

The effects of PLS_LS are summarized in Proposition 2.

**Proposition 2.** *Parental leave subsidization financed by lump-sum taxation (p) has (i) positive effects on the parental time input in child human capital* $(e)$, *the ratio of parental expenditure on a child's human capital to output per worker* $(q)$, *and the balanced steady-state growth rate of output per worker* $(g_\infty)$*; (ii) negative effects on fertility* $(N)$, *leisure* $(Z)$, *labor supply* $(l)$, *and old-age consumption relative to output per worker* $(c_o)$*; and (iii) no effect on young-age consumption relative to output per worker* $(c_y)$, *the ratio of transfers per child to output per worker* $(b)$, *and the saving rate* $(s)$.

Proposition 2 suggests that when PLS_LS increases the marginal utility of parental time input in child human capital, young parent tends to spend more time on a child's human capital but less time on leisure and labor supply. Additionally, higher parental time input in child human capital also leads to higher parental expenditure on child human capital relative to output per worker and a lower fertility rate through the quantity-quality trade-off of children. The balanced steady-state growth rate of per worker output in (30) increases with PLS_LS, as PLS_LS increases parental time input in child human capital, reduces fertility, and does not affect other variables in (30).[13]

The next proposition summarizes the welfare effect of PLS_LS.

[13] The product of the ratio of parental expenditure on child human capital to output per worker and labor input is given by $ql = v\alpha[\delta(1-\theta)]^2/\langle\{\delta[(\rho+\beta)c_y - qN - \alpha\theta] - \varepsilon qN\}[1-\alpha\zeta(1-\delta)]\rangle$, which is independent of $p$.

**Proposition 3.** *When parental leave subsidization is financed by lump-sum taxation, the optimal parental leave subsidy rate,* $p^*$, *exists and it is positive and unique:*

$$p^* = \frac{\alpha(1-\zeta)(1-\delta)[(\alpha+\beta)(1-\theta-\alpha\theta)+\alpha(1-\alpha\theta)(\rho+\beta+\gamma)]}{p_d} > 0,$$

*where* $p_d$ *is a positive constant when Assumption 1 holds. In addition,* $p^*$ *is zero when human capital externalities are absent* ($\zeta = 1$), *and is higher when the degree of human capital externalities is higher (lower* $\zeta$*).*

The results stated above suggest that PLS_LS leads to higher welfare and a higher growth rate of output per worker than under the laissez-faire system. Because of the non-distortionary nature of lump-sum taxation, its positive effects on welfare and growth are unambiguous. However, parental leave subsidization is typically financed by distortionary taxes in practice. Hence, we consider parental leave subsidization financed by labor income taxation in next section to further explore the growth and welfare effects of parental leave subsidization in presence of the distortionary effects of labor income taxes.

### *3.4. Parental Leave Subsidization Financed by Labor Income Taxes (PLS_LI)*

Let $\tau_t \in (0,1)$ denote the labor income tax rate imposed on young parents or workers.[14] Then, the budget constraint in young adulthood becomes

(33) $$C_{y,t} = B_t + (1-\tau_t)(1 - vN_t - e_tN_t - Z_t)W_tH_t - S_t - Q_tN_t + p_te_tN_tW_t\overline{H}_t.$$

And the government balanced budget constraint is given by

$$\tau_t(1 - v\overline{N}_t - \bar{e}_t\overline{N}_t - \bar{Z}_t)W_t\overline{H}_t = p_t\bar{e}_t\overline{N}_tW_t\overline{H}_t,$$

where the bar above each variable indicates its average level within the economy.

The problem of a dynastic family is to maximize utility in (1) subject to the budget constraints (2) and (33) and the human capital technology in (10), given the average human capital, prices, the labor income tax rate, $\tau_t$, and the leave subsidy rate, $p_t$. For $t \geq 0$, the first-order conditions with respect to $C_{o,t}$ and $S_t$ are the same as those in the laissez-faire economy. Using the government balanced budget constraint $p_t = \tau_t(1 - v\overline{N}_t - \bar{e}_t\overline{N}_t - \bar{Z}_t)/\bar{e}_t\overline{N}_t$ for substitution, the first-order conditions with respect to $H_{t+1}$, $Z_t$, $e_t$ and $N_t$ are given as follows.

[14] We expect to obtain the same qualitative results if the government requires firms to pay for the labor income taxes to finance parental leave subsidization as the after-tax effective wage rate received by workers will be lower in the competitive labor markets regardless of who pays for the taxes.

The marginal rate of substitution between the young parent's consumption and expenditure on a child's human capital equals their relative prices:

$$\frac{Q_t C_{y,t+1}}{\alpha C_{y,t}[(1-\tau_{t+1})\delta(1-\theta)y_{t+1}+\zeta(1-\delta)Q_{t+1}N_{t+1}]} = \frac{1}{N_t}.$$

By reducing the after-tax return to child human capital investment, the labor income tax $\tau_{t+1}$ reduces the marginal utility of expenditure on child human capital. As a result, PLS_LI tends to reduce parental expenditure on child human capital and increase the young-parent's consumption. These distortionary effects are absent in PLS_LS.

The marginal rate of substitution between leisure and consumption at young age equals their relative prices:

$$\frac{\gamma C_{y,t}}{Z_t} = \frac{(1-\tau_t)(1-\theta)y_t}{(1-vN_t-e_tN_t-Z_t)}.$$

The labor income tax $\tau_t$ increases leisure by reducing the relative price of leisure, holding the parental time spent on children fixed. However, leisure tends to fall when parental leave subsidies increase the parental time spent on children. Consequently, labor income taxes may cause the level of leisure to be higher than that in PLS_LS.

Equating the marginal rate of substitution between fertility and the young-parent's consumption to their relative prices gives:

$$\frac{\rho C_{y,t}}{N_t} = \frac{(1-\tau_t)(v+e_t)(1-\theta)y_t}{(1-vN_t-e_tN_t-Z_t)} + Q_t - \frac{\tau_t(1-\theta)y_t}{N_t} + \frac{\alpha C_{y,t}}{C_{y,t+1}N_t}\left[\theta y_{t+1} - \frac{C_{o,t+1}}{N_t}\right].$$

PLS_LI tends to reduce the costs of raising a child because an increase in the labor income tax rate reduces the time cost of a child (the first term on the right-hand side) and increases the amount of leave subsidies received by a young parent when the number of children increases (the third term on the right-hand side). Therefore, labor income taxes exacerbate the problem of excessive fertility in the laissez-faire equilibrium by increasing the fertility rate which can be avoided when lump-sum taxes are available.

The marginal rate of substitution between the parental time input in child human capital and labor equals their relative prices:

$$\frac{\varepsilon Q_t(1-vN_t-e_tN_t-Z_t)}{\delta e_t(1-\tau_t)(1-\theta)y_t} + \frac{\tau_t(1-vN_t-e_tN_t-Z_t)}{N_te_t(1-\tau_t)} = 1.$$

PLS_LI tends to increase the parental time input in child human capital by increasing the marginal utility of the parental time input in child human capital. This effect is similar to that in PLS_LS. Additionally, labor income taxes reduce the marginal utility of labor, and hence, labor income taxes exacerbate the problem of under-supply of labor in the laissez-faire equilibrium. This suggests that labor time tends to be lower in PLS_LI than PLS_LS and the laissez-faire.

Given a time-invariant labor income tax rate and a parental leave subsidy rate, the constant allocation rules for $b$, $s$, $c_o$, and $c_{o,0}$ are the same as those in (18)−(21), respectively. The constant allocation rules for $c_y$, $q$, $e$, $Z$ and $N$ are given in Appendix A.

The effects of PLS_LI are summarized in Proposition 4.

**Proposition 4.** *Parental leave subsidization financed by labor income taxation ($\tau$) has (i) positive effects on young-age consumption relative to output per worker ($c_y$), old-age consumption relative to output per worker ($c_o$), fertility ($N$), the parental time input in child human capital ($e$); (ii) negative effects on the ratio of parental expenditure on a child's human capital to output per worker ($q$), leisure ($Z$), labor supply ($l$), and the ratio of transfers per child to output per worker ($b$); (iii) a positive effect on the balanced steady-state growth rate of per worker income ($g_\infty$) if a sufficient condition holds; and (iv) no effect on the saving rate ($s$).*

Proposition 4 suggests that PLS_LI reduces efficiency loss from human capital externalities by increasing the parental time input in child human capital, but at the same time PLS_LI increases efficiency loss from human capital externalities by reducing the ratio of parental expenditure on a child's human capital to output per worker, leisure and labor supply, and increasing young-age consumption and old-age consumption relative to output per worker, and fertility. Despite the distortionary effects of labor income taxes, PLS_LI increases the balanced steady-state growth rate of output per worker if a sufficient condition holds. We verify this sufficient condition numerically and find that it holds for $\zeta$ ranging from 0 to 1. This result suggests that the positive effect of PLS_LI on the growth rate is robust to various calibrations of the model, as discussed in detail in the quantitative section.

By substituting the equilibrium solutions for ($c_y, s, b, q, e, Z, N, c_o, c_{o,0}$) given in Appendix A into (34), we can obtain the welfare level for this economy given any plausible labor income tax rate ($\tau$) that is needed to finance parental leave subsidization. As in PLS_LS, we expect an optimal parental leave subsidy rate and the corresponding optimal labor income tax rate ($\tau$) exist in PLS_LI, because Proposition 4 shows that $\tau$ has both positive and negative effects on the welfare level given by (31). Since analytically solving for the optimal rate of $\tau$ is complicated, we solve for the optimal rate of $\tau$ numerically in Section 5.2.

By comparing the results for PLS_LS reported in Proposition 2 with the results for PLS_LI reported in Proposition 4, we can identify the distortionary effects of labor income taxation. First, fertility ($N$) under PLS_LI is higher than that under PLS_LS because labor income taxation reduces the time cost of raising a child, and thus leads to higher fertility. Second, the ratio of parental expenditure on a child's human capital to output per worker ($q$) under PLS_LI is lower than that under PLS_LS because labor income taxation increases fertility, and thus reduces $q$ through the quantity-quality trade-off of children. The same result and intuition apply to the parental time input in child human capital ($e$), that is, $e$ is lower under PLS_LI than under PLS_LS. Third, higher fertility rate also implies higher old-age consumption relative to output per worker ($c_o$) (see Equation (20)) under PLS_LI than under PLS_LS. Fourth, labor income taxation reduces labor time, and so labor time is lower in PLS_LI than PLS_LS. Fifth, lower $e$, lower $q$, lower $l$ and higher $N$ in PLS_LI imply that the growth rate of output per worker in PLS_LI is lower than that in PLS_LS. Therefore, the welfare level in PLS_LI tends to be lower than in PLS_LS. In our quantitative analysis below, we confirm this is the case and quantitatively measure the magnitude of the efficiency loss due to labor income taxation.

## 4. Calibration

We calibrate the baseline model, i.e., the laissez-faire model, to match a selected set of moments in the U.S. data, given that parental leave entitlements in the U.S. are very low (Carneiro et al., 2015). Subsection 4.1 briefly describes the calibration procedure and Subsection 4.2 describes the construction of the data moments used in the calibration. The targeted moments and parameter values obtained are reported in Table 1 and Table 2, respectively.

### *4.1. Calibration procedure*

The parameters that need to be calibrated include: the preference parameters, $\alpha$, $\beta$, $\gamma$, and $\rho$ in the utility function; parameters in the production function $\theta$ and $D$; parameters in the human capital accumulation equation, $\delta$, $\varepsilon$, $\zeta$, and $A$; and the fixed parental time spent on a child, $\nu$.

The income share of capital $\theta$, and $\nu$ can be directly calibrated to match their empirical counterparts. Given $\theta$, $\nu$ and a value of $\zeta$, the values of $\alpha$, $\beta$, $\gamma$, $\rho$, $\delta$ and $\varepsilon$ can be uniquely and jointly determined to match the empirical counterparts of $c_o$, $c_y$, $q$, $e$, $Z$, and $N$, according to Equations (20) and (22)-(26). The parameter $\zeta$ indicates human capital externality in the production function of child human capital. To calibrate this parameter, we note that Borjas (1995) estimates a range of 0.18 to 0.3 for the elasticity of a person's human capital with respect to the average human capital of the father's generation, which corresponds to $(1-\zeta)(1-\delta)$ in our model. Hence, we consider the middle point of this range and choose the baseline value of $\zeta$ such that $(1-\zeta)(1-\delta)=0.24$. Estimates of Choi (2011) imply a much higher value of 0.44 for $(1-\zeta)(1-\delta)$. In a later section a robustness analysis is performed in which we consider alternative values of $(1-\zeta)(1-\delta)$ and re-calibrate the parameters. Finally, based on Equation (29), the value of the constant $A^{1-\theta}D^{\delta}$ can be obtained by matching the empirical counterpart of $g_{\infty}$.[15]

Next, we describe how we construct the empirical counterparts of θ, ν, $c_o$, $c_y$, $q$, $e$, $Z$, and $N$, and $g_{\infty}$ in the U.S. data. The empirical counterparts of these parameters or variables are straightforward except for $e$ and ν. We interpret $e$ as the fraction of a young parent's time spent on a child when the child is young (0-6 years old), while the fixed parameter ν as the fraction of time allocated when the child is elder (7 to 17 years old). This interpretation is motivated by a few considerations. First, as documented in Ruhm (2004) and Del Boca et al. (2014), parental time input in children in their early childhood is particularly important for children's human capital development. Our formulation that $e$ enters the child human capital accumulation equation is in line with this empirical evidence. Second, the amount of time spent with younger children before they reach the compulsory school age is more of a choice to parents.[16] Some parents choose to look after their young children by themselves while others choose to use childcare services. For children of school age, the amount of time parents spent with them tends to be much shorter and less variable across families.[17] Third, this interpretation allows us to connect parental time subsidization considered in the model with parental leave policies in practice, the majority of which concern maternity or paternity leave for looking after newborns and younger children.

## *4.2. Data moments*

This subsection describes the targeted data moments that we use to calibrate the parameters in the baseline calibration. The observed values of the data moments for the U.S. economy are presented in Table 1. We collect data from two micro-data sources for the U.S. economy: American Time Use Survey (ATUS) and American Panel Study of Income Dynamics (PSID).

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[15] We do not need separate values for the scale parameters *A* and *D*. The constant $A^{1-\theta}D^{\delta}$ is sufficient.

[16] For the majority of states in the U.S., age of required school attendance starts at 6 years old. See Table 5.1 (available at https://nces.ed.gov/programs/statereform/tab5_1.asp ) which reports compulsory school attendance laws in different states in the U.S..

[17] According to American Time Use Survey, the mean and standard deviation of the number of hours parents spent with children of age 7 to 17 are both lower than that of parents spent with children of age 0 to 6.

In addition to these two datasets, we also collect education expenditure on children from Lino et al. (2017). Finally, data for annual growth rate of GDP per capita and share of labor compensation in GDP are collected from World Bank and Federal Reserve Economic Data (FRED), respectively.

Unless otherwise stated, the data moments are obtained by taking the average of the annual figures for years between 2011 and 2015. The main reason we choose these years is that data for education expenditure on children are only available for these years.

One period in the model is set to 30 years, so young age and old age in the model correspond to ages 30-59 and 60-89. In both micro-data sets we consider two age groups. The young parent age group includes households with at least one child under 18 years of age. The reference person for each household in this age group is required to be between age 30 and 59 and work full time. The old age group includes households who do not have any child below 18 years of age and whose reference persons are retired and between age 60 and 89.

[Table 1 goes here]

**Fertility rate.** We use PSID to construct the average number of children and the average income of a household in the young parent age group and in the old age group, as well as the average consumption expenditure of a household for both age groups. PSID Family Level is a bi-annual dataset, so it provides us data for 2011, 2013 and 2015. The average number of children of a young household (typically with two parents) over these three years is 1.9162, implying that the empirical counterpart of the number of children per young parent, $N$, is 1.9162/2=0.9581.

**Expenditure allocation**. Recall that in the model $c_y$ refers to consumption expenditure of a young parent (which excludes education expenditure on children, $Nq$) relative to output per worker, while $c_o$ refers to consumption expenditure of an old parent relative to output per worker. Their empirical counterparts can be defined as the average household consumption expenditure of a young or old household divided by income per young household. Household consumption expenditure in the PSID dataset includes the following ten categories: food, transportation, education, childcare, health care, household repairs, household furnishing, clothing, trips and other recreation. [18] The average consumption expenditure of an old

[18] The education category includes expenditure on education for children and parents, so we do not use this data information to calculate expenditure on children's education by a young household.

household is obtained by averaging the sum of all ten categories of consumption expenditures across all households in the old age group. The average consumption expenditure of a young household is obtained by subtracting the average expenditure on childcare and education and health for household children, which is separately obtained from Lino et al. (2017) (see below for details), from the sum of the ten categories of household consumption expenditure. This gives us an average annual consumption expenditure of $61425.73 and $36460.22 for a household in the young and old age group, respectively. The income per young household is obtained by dividing total household income by the number of young households in PSID. To be consistent with our model, we use labor income as the household income for a young household while total income for an old household. The average annual labor income of a young household and average total income of an old household are $95134.61 and $53864.21, respectively. And in our PSID sample, on average 68.73 and 31.27 percent of households belong to the young and old age group, respectively. Then income per young household is calculated as $119644.945 ((95134.61*0.6873 +53864.21*0.3127)/0.6873). The ratio of consumption expenditure of a young and old household to output per worker is therefore given by 0.5246 (62770.43 /119644.945) and 0.3047 (36460.22 /119644.945), respectively.

To obtain the empirical counterpart of $q$, we use data on expenses for raising a child reported in Lino et al. (2017) which are based on Consumer Expenditure Survey between 2011 and 2015. According to this report child-rearing expenses for one child from birth through age 17 in a two-child, middle-income, married-couple family is $233610 in 2015 U.S. dollar. Moreover, 25 percent of this total child-rearing expenses is expenditure on childcare and education, and health care. The childcare and education expenditures include day care tuition and supplies, babysitting, elementary and high school tuition, books, fees, and supplies. The health care expenditures include expenses on medical and dental services, prescription drugs and medical supplies that are not covered by insurance and health insurance premiums paid by an employer or other organization. We believe these expenses are the most relevant expenditure input from parents for human capital development of children. Given that one period in our model is 30 years, the average annual expenditure on one child's education and childcare, and health care of a young household is calculated as $233610*0.25/30=$1946.8. Normalizing this expenditure by the income per young household figure obtained above gives the empirical counterpart of $q$, 0.0163. Also, we need to subtract this expenditure multiplied by the average number of children per young household from total household consumption expenditure in calculating the average consumption expenditure of a young household, as mentioned above.

**Time allocation**. We use ATUS to construct the average time allocation on leisure, work and children for a household in the young parent age group. Leisure includes time spent on socializing, relaxing, leisure sport, exercise and recreation and work includes time allocated to work and work-related activities as well as travel related to work. Time spend on/with children includes the following six categories: 1) physical care for household children, 2) looking after household children as a primary activity, 3) caring for and helping household children, 4) all activities related to children's health (e.g. providing/obtaining medical care), 5) reading to/with children and 6) playing/doing hobbies with children. These categories cover all time spent on/with children for physical care and education-related activities.

To calculate the time allocated to young children (i.e. time input in young children's human capital) and time allocated to older children, we split the sample of young parent to two subsamples. The first subsample includes young parents whose youngest child is between 0 and 6 (young parents with young children, hereafter). The second subsample includes young parents whose youngest child is between 7 and 17 (young parents with elder children). On average a young parent with young children spends 1.3709 hours per day on the six categories of time spent on/with children. We multiply this number by 365*6 (there are 6 years between age 0 to age 6) to approximate the total amount of time spent by a young parent with young children. On average a young parent with elder children spends 0.595 hours per day on the six categories of time spent on/with children. We then multiply this number by 365*11 years to approximate the total amount of time spent by a young parent with elder children.

In a similar way, we calculate the total amount of time spent on leisure and work for a young parent with young children and for a young parent with elder children. We sum up the total amount of time spent on children, leisure, and work to obtain the total number of hours for a young parent with young children and for a young parent with elder children. We then sum up the total number of hours for both young parent subgroups to obtain the total amount of time of a young parent, as the two subgroups capture the two stages of a young parent in raising a child from age 0 to age 17.

The empirical counterpart of $Z$ is obtained by dividing the sum of leisure time for both young parent subgroups by the total amount of time of a young parent, $Z = 0.3313$. Dividing the total amount of time spent by a young parent with young children (elder children) by the total amount of time of a young parent gives us a young parent's fraction of time spent on children in their early childhood (later childhood). Normalising these fractions by the average number of children per young parent, i.e., the $N$ value obtained earlier, we get the empirical counterparts of $e$ and ν: $e = 0.0471$ and ν=0.0327. These numbers highlight that on average a young parent tends to spend more time on a child when the child is under age 6 than when the child is elder.

**Growth rate and labor share**. Finally, the average annual growth rate of GDP per capita for the U.S. economy between 2011 and 2015 is 1.46 percent. Given that one period in our model is 30 years, we transform this annual figure to a one period growth rate of $(1 + 0.0146)^{30} - 1$. The average income share of labor compensation for the U.S. economy between 2011 and 2015 is 0.5941, implying a $\theta$ value of 0.4059.

### *4.3. Parameter values*

Given the targeted data moments described above and following the calibration procedure described earlier, we obtain the calibrated values of parameters reported in Table 2, referred to as the baseline calibration hereafter. For parameters in the utility function, the discount factor across generations is α=0.3778, implying an annual discount factor of 0.9669. The taste parameter for old age consumption is β=0.2337, suggesting that the weight households put on old age consumption relative to young age consumption is β/α=0.62. Moreover, the taste parameter for the number of children is ρ=0.2451 and the taste parameter for leisure in the utility function is γ=0.6474.

For parameters in the human capital accumulation equation, the weight of parental expenditure on child human capital (which can be interpreted as physical capital input in the production of human capital) is $\delta = 0.0510$, which is smaller than the weight of physical capital in the production of the final goods ($\theta = 0.4059$). This reflects the fact that the production of human capital is less physical capital (or more human capital) intensive than the production of the final goods. The calibrated value of $\zeta$ is 0.7471, not too far below a value of 1 which indicates no externality. This suggests that the degree of human capital externality is not very high in our baseline calibration. We consider alternative calibrations with higher or lower degree of human capital externality in Subsection 5.4. The weight of parental time input in the production of child human capital, $\varepsilon$, is 0.1479. As $\varepsilon$ is larger than $\delta$, this result suggests that parent's time input is more important than expenditure input in child human capital development. This finding is consistent with the empirical finding of Del Boca et al. (2014), where they examine children's human capital development and find that the parent time input is more important than the expenditure or physical input for younger children. Finally, the constant that consists of productivity parameters $A$ and $D$, $A^{1-\theta}D^{\delta}$, is 2.0428. It is straightforward to verify that Assumption 1 holds under the baseline calibration.

[Table 2 goes here]

## 5. Quantitative Results

In this section, we compare the laissez-faire equilibrium, the social optimum, and the equilibria with parental leave subsidy financed by lump-sum tax and labor income tax. Any difference in welfare and growth across these scenarios is of policy importance. Table 3 reports the equilibrium outcomes based on the baseline calibration given in Table 2 for four different scenarios: (1) the laissez-faire, LF, which is the benchmark model, (2) the social optimum, SP, (3) the economy with optimal leave subsidy financed by lump-sum tax, PLS_LS, and (4) the economy with optimal leave subsidy financed by labor income tax, PLS_LI.

We define the welfare gain over the laissez-faire using the notion of the consumption equivalent variation, which is defined as the percentage change in consumption in every period $(\Delta^i)$ such that the welfare in the laissez-faire economy ($U_0^{LF}$) reaches the welfare in economy $i$ ($U_0^i$), where $i$ refers to the social optimum (SP), or the economy with parental leave subsidy financed by lump-sum tax or by labor income tax (PLS_LS or PLS_LI). That is,

$$U_0^i = U_0^{LF} + \sum_{t=0}^{\infty} \alpha^t (\beta + \alpha) \ln(1 + \Delta^i),$$

and thus

$$\Delta^i = exp\left[\frac{(U_0^i - U_0^{LF})(1-\alpha)}{\alpha+\beta}\right] - 1.$$

The welfare gains $\Delta^i$ over the laissez-faire outcome are reported in Table 3 as well.

### *5.1. Comparison between LF and SP*

First, we compare the benchmark laissez-faire equilibrium with the social optimum. The quantitative results in Table 3 are consistent with the qualitative results established in Proposition 1. In particular, fertility ($N$) and old-age consumption relative to output per worker ($c_o$) are substantially higher, while parental expenditure on child human capital relative to output per worker ($q$) and the parental time input in a child's early childhood ($e$) are substantially lower in the laissez-faire compared with the social optimum, suggesting that the presence of human capital externalities causes these variables to deviate from their socially optimal levels substantially. As a result, the annual growth rate of output per worker ($g_a$) is also much lower in the laissez-faire, 1.46 percent versus 1.81 percent in social optimum. Quantitative differences in other variables are less significant.

The welfare gain of the social optimum over the laissez-faire outcome is 0.25 percent, suggesting that human capital externalities cause a welfare loss in the laissez-faire. According to Equation (31), this welfare loss arises from much lower levels of $q$ and $e$, and slightly lower leisure $(Z)$ and labor $(l)$ in the laissez-faire, despite that there is welfare gain from higher levels of fertility ($N$) and consumption ($c_y$ and $c_o$) in the laissez-faire.

[Table 3 goes here]

### *5.2. Optimal leave subsidy with lump-sum taxes*

Next, we compare the model with optimal leave subsidy financed by lump-sum taxes, PLS_LS, with the laissez-faire, LF. We first solve for the optimal leave subsidy $p$, which is chosen to maximize the welfare $U_o$, or equivalently, given by the expression in Proposition 3. The optimal leave subsidy rate and the corresponding lump-sum tax needed relative to output per worker are given by $p^* = 0.1332$ and $T^*/y$ =0.0073, respectively.

Under the optimal subsidy rate, the total subsidy a worker receives for her parental time on one child as a fraction of income per worker is given by $p_t\bar{e}_t W_t \bar{H}_t/\bar{y}_t$=$p_t\bar{e}_t(1-\theta)/\bar{l}_t$, which is 0.73 percent. This optimal subsidy rate can be converted into an optimal leave period that is fully covered. Suppose a worker is entitled to take a one-year parental leave for every child with full replacement rate, which corresponds to the worker receiving a subsidy of one year's labor income for each child she has. Such a subsidy as a fraction of income per worker over 30 years can be approximated by a young household's annual labor income divided by the income per young household over 30 years, which is about 2.65 percent in the data. So, in the model, the optimal subsidy rate corresponds to an optimal fully-covered parental leave of (0.73/2.65)(52), i.e., 14.3 weeks.

In this model, the annual growth rate of output per worker is 1.53 percent, 0.07 percentage points higher than in the laissez-faire economy. The welfare gain over the laissez-faire case is 0.058 percent. This welfare gain arises mainly from a substantial increase in the parental time input in a child's early childhood ($e$), despite a fall in fertility ($N$). The effects of the optimal leave subsidy on other variables are also consistent with Proposition 2, but less significant quantitatively. Note that the welfare gain is much lower than the welfare gain implied by the social optimum. This result suggests that the social optimum outcome cannot be achieved even with a non-distortionary lump-sum tax. In other words, parental leave subsidization alone cannot correct all the distortions caused by human capital externality with endogenous fertility, labor, and leisure choices.

### *5.3. Optimal leave subsidy with labor income taxes*

When the leave subsidy is financed by labor income taxes, the optimal leave subsidy rate, again chosen to maximize $U_o$, is solved as $p^* = 0.0872$, which is 34.5 percent lower than the optimal subsidy rate in the PLS_LS economy. The corresponding labor income tax rate $\tau$, found by equating the government budget constraint, is given by $\tau^* = 0.0073$. Similar to the above description, this optimal leave subsidy can be converted into a fully-covered parental leave period of 8.7 weeks. This is 39 percent shorter than the optimal leave duration implied by the optimal leave subsidy financed by a lump-sum tax.

Consistent with Proposition 4, the optimal leave subsidy financed by a labor income tax leads to a significant increase in the parental time input ($e$) of 7.2 percent, and slight increases in fertility ($N$), and consumption in young and old-age relative to income per worker ($c_y$ and $c_o$) from their laissez-faire levels. These effects tend to increase welfare. However, the leave subsidy also slightly reduces parental expenditure on a child's human capital ($q$) and labor supply ($l$), which tend to reduce welfare. The overall effect is a small welfare gain of 0.024 percent over the laissez-faire outcome.

This welfare gain is 58 percent lower than the welfare gain in the lump-sum tax case. This lower welfare gain reflects the distortionary effects of labor income taxation that is used to finance the parental leave subsidy. Compared with the lump-sum tax case, the increase in the parental time input ($e$) over laissez-faire is 53 percent smaller while the decrease in labor input over laissez-faire is slightly larger. Furthermore, there is a slight decrease in the parental expenditure on a child's human capital ($q$). All these effects contribute to a smaller welfare gain, although there are small increases in fertility ($N$) and the consumption ratios ($c_y$ and $c_o$).

Figure 1 shows the welfare gain of the leave subsidy models over the laissez-faire outcome for a range of fully-covered parental leave periods.[19] In both the labor income tax and lump-sum tax cases, there is a hump-shaped relationship between the leave duration and welfare gain. Hence, an increase in parental leave duration, whether it's financed by a labor income tax or a lump-sum tax, improves welfare until a certain level of leave duration (8.7

[19] To obtain this figure, we consider a vector of parental leave subsidy rate ranging from 0.001 to 0.3. For each subsidy rate, we compute the equilibrium of each subsidy model and find the welfare gain and the implied fully-covered parental leave duration corresponding to the subsidy rate. Then we plot the vector of welfare gains against the vector of parental leave periods.

weeks in labor income tax case and 14.3 weeks in lump-sum tax case). Above this level, an increase in the leave duration reduces the welfare gain from parental leave subsidy. We observe a welfare loss for a leave duration above 17.8 (30.5) weeks in the labor income (lump-sum) tax case. It is also clear from the figure that welfare gains are smaller with labor income tax than with lump-sum tax at all levels of leave duration, giving rise to a smaller optimal leave duration in the labor income tax case.

[Figure 1 goes here]

Finally, we look at the quantitative effect of optimal leave subsidy financed by a labor income tax on the growth rate of output per worker. Proposition 4 establishes a sufficient condition for a positive effect of leave subsidy on the growth rate of output per worker, as given in Equation (A17) in Appendix A. We have verified that this condition holds under our baseline calibration. To check the robustness of this condition, we also consider a vector of $\zeta$ values varying between its domain [0,1], i.e., we consider a wide range of degrees of human capital externality. For each $\zeta$ value, we re-calibrate the parameters (following the procedure described in Section 4.1) and re-compute the equilibrium, and check whether the sufficient condition holds. We find that this condition holds for $\zeta$ ranging from 0 to 1, suggesting that the positive effect of leave subsidy on the growth rate is robust to alternative calibrations of the model.

Consistent with this theoretical prediction, the annual growth rate of output per worker in the subsidy model with labor income tax is 0.03 percentage points higher than in the laissez-faire economy, reaching a level of 1.49 percent. The increase in the growth rate is 57 percent lower than in the subsidy model with lump-sum tax. For the range of parental leave periods in Figure 1, we also calculate the percentage change in growth rates relative to the laissez-faire level corresponding to each leave duration. Unlike the hump-shaped effect on welfare, the increase in the growth rate of output per worker increases monotonically with the length of the leave duration, as displayed in Figure 2 in Appendix B. This monotonically increasing relationship stems from the positive effect of leave subsidy on the growth rate of output per worker as established in Proposition 4.

[Figure 2 goes here]

### 5.4. Alternative calibrations

In this subsection we examine how the results change when we change the degree of externality in the human capital production function. Specifically, we consider two alternative calibrations by varying the degree of human capital externalities while still targeting the same data moments as reported in Table 1. Recall that Borjas (1995) provides a range between 0.18 and 0.3 for the elasticity of human capital with respect to average human capital of parents' generation, which is captured by $(1-\zeta)(1-\delta)$ in our model. A more recent work by Choi (2011) implies a value of 0.44 for $(1-\zeta)(1-\delta)$. Hence, we consider the minimum and maximum value of this range in the two alternative calibrations, that is, we let $(1-\zeta)(1-\delta)=0.18$ in one calibration and $(1-\zeta)(1-\delta)=0.44$ in another, and recalibrate the parameters following the same procedure as described in Section 4.1.

The change in the value of $(1-\zeta)(1-\delta)$ only leads to changes in the calibrated values of the parameters in the human capital production function, $\varepsilon$, $\zeta$, and $\delta$. Table 4 reports the recalibrated values of these parameters in the two alternative calibrations. Similar to the baseline calibration (column (2) in Table 4), we still have $\delta$ much smaller than $\theta$ and $\varepsilon$ greater than $\delta$ in both alternative calibrations. However, a higher value of $(1-\zeta)(1-\delta)$ implies a higher degree of human capital externality, i.e., a lower value of $\zeta$, as well as higher weights of parental expenditure and time inputs in the production of human capital, i.e., higher $\delta$ and $\varepsilon$ values. This is because a higher value of $(1-\zeta)(1-\delta)$ implies a lower weight of parental human capital in the production of human capital, $\zeta(1-\delta)$, which thereby requires the weights of parental expenditure and time inputs, i.e., $\delta$ and $\varepsilon$, to be larger to induce the same levels of parental expenditure $q$ and parental time $e$ as given in the targeted data moments.

Next, we discuss how the results vary under the two alternative calibrations. Table 5 reports the equilibrium solutions for $(1-\zeta)(1-\delta)=0.18$ (top panel) and for $(1-\zeta)(1-\delta)=0.44$ (bottom panel). These results are qualitatively similar to the baseline results reported in Table 3. When the degree of human capital externality is higher ($(1-\zeta)(1-\delta)=0.44$), the welfare gains of the social optimum and the optimal leave subsidy models, as well as the increases in the growth rates from the laissez-faire outcome, are all significantly higher than under the baseline calibration.

In particular, with a higher degree of externality, the optimal parental leave subsidy rate in the model with labor income tax is 0.1448, 66 percent higher than the optimal rate in the baseline calibration. The corresponding optimal parental leave is 15.3 weeks, almost doubling the level in the baseline calibration. This higher subsidy rate leads to significantly higher fertility ($N$) and parental time input ($e$), contributing to a welfare gain of 0.077 percent which is three times larger than in the baseline calibration. On the contrary, when the degree of externality is lower ($(1-\zeta)(1-\delta)=0.18$), the optimal leave subsidy rate in the labor income tax model is smaller than in the baseline calibration, and the corresponding optimal leave duration is 6.6 weeks, two weeks shorter than implied by the baseline calibration.

The upper panel of Figure 2 in Appendix B depicts the welfare gains of the subsidy model with labor income tax for a range of parental leave duration under the baseline calibration and the two alternative calibrations. It is clear that the welfare gains are higher at all levels of parental leave duration when the degree of externality is the highest (red dashdot line). The lower panel of Figure 2 shows the percentage changes in the growth rate of output per worker from the corresponding laissez-faire level under the three different calibrations. Note that the percentage increase in growth rate increases monotonically with the duration of parental leave, confirming that the parental leave subsidy raises the growth rate monotonically. For a given level of parental leave duration (i.e., a given level of leave subsidy rate), the increase in the growth rate is the highest when the degree of externality is the highest.

[Tables 4 and 5 go here]

### *5.5. Leave policies among OECD countries*

This subsection summarizes the parental leave policies for a group of OECD countries. This allows us to compare our quantitative results for the U.S. economy with other countries. In this comparison, we focus on the results for the optimal leave duration when it is financed by labor income tax.

Parental leave policies for early childhood vary substantially across countries. In general, there are four types of leave policy for early childhood: maternity leave, paternity leave, parental leave and home care leave.[20] Given that this paper does not distinguish between these types of leave, we look at the total leave available to mothers and fathers for the group of OECD countries. Another point to consider is that parental leave policies across countries vary in leave duration and average payment rate (for details see OECD, 2019). To make the comparison across the countries easier, here we report full-rate equivalent leave duration (weeks) which is obtained by multiplying the duration of leave in weeks by the payment rate (i.e., payment relative to average earnings) received by the claimant over the duration of the leave. Table 6 reports full-rate equivalent leave (in weeks) per child available to parents for a group of OECD countries in 2018.

[20] For the definition of each type of leave, see the report published by OECD (2019).

[Table 6 goes here]

As can be seen from Table 6, there are substantial variations in the full-rate equivalent parental leave duration across the countries. The U.S. is the only country in the sample that does not offer statutory entitlement for paid leave on a national basis. This is equivalent to the laissez-faire economy in our baseline calibration. Ireland has the second lowest paid leave entitlements in this sample, 7.5 full-rate equivalent weeks.[21] In addition to the U.S. and Ireland, six other countries provide a leave entitlement below14 weeks which is the minimum duration of maternity leave recommended by International Labor Organization. At the other extreme, nine countries provide full-rate equivalent leave longer than a year. For example, Estonia and Hungary provide 86.4 and 69.2 weeks of full-rate equivalent leave per child, respectively.

Our quantitative analysis shows that for the U.S. economy, the optimal leave duration when it is financed by distortionary labor income tax is 8.7 weeks per child for a single parent, implying a full-rate equivalent leave entitlement of 17.4 weeks per child for a household. Such a leave entitlement is comparable to the practice in some European countries like Belgium, Netherland and Spain. Our alternative calibrations suggest a minimum full-rate equivalent leave duration of 14 weeks and a maximum of 32 weeks for both parents for the U.S. economy (see Table 5). This range appears reasonable, considering that about 40 percent of the OECD countries fall in this range.

Our results suggest that for the U.S. economy, introducing a parental leave entitlement of 17.4 weeks for a household would raise the annual growth rate of output per worker by 0.03 percentage points and a welfare gain of 0.024 percent over the laissez-faire outcome. However, as shown in Figure 2, a leave duration that is longer than 35.6 weeks would reduce welfare. This result suggests that the extremely long parental leave duration among some OECD countries could be welfare reducing for the entire society.

## 6. Conclusion

This study examines the growth and welfare effects of parental leave subsidization and explores the optimal rates of parental leave subsidization financed by lump-sum taxation (PLS_LS) and labor income taxation (PLS_LI) in a life-cycle dynastic family model featuring endogenous fertility, labor, and accumulation of physical and human capital in presence of human capital

[21] This is mainly due to a low average payment rate of 26.7% in Ireland, despite that it provides relatively long maternity leave entitlements, 26 weeks for mothers and 2 weeks for fathers.

externalities. The model is first solved analytically and then quantitative policy exercises for the U.S. economy are conducted.

In line with the economic intuition, our results indicate that PLS_LS produces higher growth rate and welfare level than PLS_LI, but more importantly, our results reveal the potential for PLS_LI to perform better than the laissez-faire outcome despite the distortionary effects of labor income taxation. Specifically, we find that PLS_LI promotes the growth rate of output per worker and improves welfare until parental leave subsidy reaches an optimal level. The important mechanism through which PLS_LI can promote growth and welfare is the parental time input in child human capital which increases with PLS_LI. Given the empirical evidence of human capital externalities and the importance of parental time inputs on child human capital development, these results highlight the importance of parental leave subsidization in practice.

Our quantitative results based on the U.S. observations suggest that, for an empirically plausible degree of externalities, the optimal parental leave subsidy under PLS_LI implies a fully-covered leave duration of 8.7 weeks per parent, which increases the annual growth rate of output per worker by 0.03 percentage points and welfare by 0.024 percent from the laissez-faire equilibrium. These results are relevant for the U.S. given that it is currently the only developed country that does not subsidize parental leave at the national level. The Family and Medical Leave Act in the U.S. only provides certain workers (not all workers) with 12 weeks of unpaid leave to care for a new child. Hence, our quantitative policy exercises suggest that the U.S. may benefit in the long-term by subsidizing parental leave and expanding the coverage of parental leave given that its parental leave entitlements are still very low at the national level.

The findings of our study may also shed light on parental leave policy for other economies. As developing countries typically have lower standards for parental leave provisions, inadequate parental care for young children, lower levels of human capital and lower growth rates of output per capita in comparison to developed countries, our results support the implementation of subsidized parental leave policy as a way to improve the economic performance in developing countries. In addition, developed economies such as Estonia and Hungary that currently provide very generous parental leave subsidization or allow a long leave duration may benefit in the long-term by reducing parental leave subsidization and the leave duration. Indeed, based on a comprehensive review by Rossin-Slater (2017), extensions on existing leave policies in developed economies have little impact on child welfare, while shorter leave programs improve child welfare. The empirical evidence by Ruhm and Teague (1997) also shows that short to moderate periods of parental leave can increase economic efficiency (Ruhm and Teague, 1997).

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## Appendix A: Proofs and Additional Equations

**The transversality conditions.** Define period utility as $F_t$:

$$F_t \equiv \beta \ln C_{o,t} + \alpha \ln[(S_{t-1}R_t - C_{o,t})/N_{t-1} + (1 - vN_t - e_tN_t - Z_t)W_tH_t$$
$$-S_t - \left(H_{t+1}A^{-1}e_t^{-\varepsilon}\left(H_t^{\zeta}\bar{H}_t^{1-\zeta}\right)^{\delta-1}\right)^{\frac{1}{\delta}}N_t] + \alpha\gamma \ln Z_t + \alpha\rho \ln N_t.$$

Then, the transversality conditions are given by

$$\lim_{t\to\infty} \alpha^t \frac{\partial F_t}{\partial N_{t-1}} N_{t-1} = 0,$$

$$\lim_{t\to\infty} \alpha^t \frac{\partial F_t}{\partial H_t} H_t = 0,$$

$$\lim_{t\to\infty} \alpha^t \frac{\partial F_t}{\partial S_{t-1}} S_{t-1} = 0.$$

Utilizing the first-order conditions given in the text, these equations can be written as:

$$\lim_{t\to\infty} \alpha^t \left(\frac{\rho}{N_{t-1}} - \frac{(v+e_{t-1})(1-\theta)y_{t-1}}{C_{y,t-1}(1-vN_{t-1}-e_{t-1}N_{t-1}-Z_{t-1})} - \frac{Q_{t-1}}{C_{y,t-1}}\right) N_{t-1} = 0,$$

$$\lim_{t\to\infty} \alpha^t \left(\frac{\alpha(1-\theta)y_t}{C_{y,t}H_t} + \frac{\alpha\zeta(1-\delta)Q_tN_t}{\delta C_{y,t}H_t}\right) H_t = 0,$$

$$\frac{\alpha}{C_{y,0}} \lim_{t\to\infty} S_t \prod_{j=1}^{t} \left(\frac{N_{j-1}}{R_j}\right) = 0.$$

**Proof of Lemma 1.** It is easy to verify that if $\rho > \underline{\rho}^{LF}$ (as defined in the lemma), then the numerator of $N$ is positive, that is $N_n^{LF} > 0$, according to the solution for fertility in (26). Moreover, $N = N_n^{LF}/(vN_d^{LF}) > 0$ under $\rho > \underline{\rho}^{LF}$ because $N_d^{LF}$ is larger than $N_n^{LF}$. Therefore, when $N > 0$, $q > 0$ in (23), and $e > 0$ in (24). As $(1 - \alpha\theta)(1 - \alpha\zeta(1 - \delta)) - \alpha\delta(1 - \theta) > 0$, $c_y > 0$ in (22), and $Z > 0$ in (25). Then, $c_o > 0$ in (20), and $c_{o,0} > 0$ in (21). Clearly, $s > 0$ in (19).

To obtain the conditions for $b > 0$ in (18), its signing part is defined as follows:

$$b(\alpha) = \theta(\alpha + \beta)(1 - \alpha\zeta(1 - \delta)) - \beta[(1 - \alpha\theta)(1 - \alpha\zeta(1 - \delta)) - \alpha\delta(1 - \theta)].$$

Note that $b(0) < 0$ and $b'(\alpha) > 0$. Moreover, when $\alpha \to 1$ and $\zeta \to 1$, $b \to \theta\delta(1 + \beta) > 0$. Consequently, if $\alpha$ and $\zeta$ are large enough, transfers from an old parent to a grown child tend to be positive ($b > 0$). Note that $b < 0$ means intergenerational transfers from a grown child to an old parent in this model. The results of this study remain qualitatively the same regardless of the direction of intergenerational transfers. Note that the solution for each of these variables is unique under the stated conditions.

The remaining task is to argue for the optimality, which is built on the following facts: (i) as the log utility excludes corner solutions, any solution for fertility, leisure, or consumption must be strictly positive; (ii) all choice variables lie in closed and bounded sets: $(c_y, s, q, e, Z, c_o, c_{o,0})$ are in [0,1], $b$ in [-1,1], and $N$ in [0, $(1-Z)/(v+e)$]; (iii) the utility function $U_t$ is continuous for the interior values of the choice variables $(c_y, c_o, Z, N)$; and (iv) the utility level $U_t$ is bounded above under $\alpha < 1$, as shown in (31). By (i)–(iv), at least one optimum can be found. From both (i) and the uniqueness of the interior solution, the optimum must correspond to this unique solution. Q.E.D.

**Proof of Lemma 2.** Using (27) and (28) and taking log, we have

(A1) $\quad \ln\mu_{t+1} = (1-\Gamma)\ln\mu_\infty + \Gamma\ln\mu_t.$

By solving the log-linear first-order difference equation (A1), we have

(A2) $\quad \ln\mu_t = \Gamma^t\ln\mu_0 + (1-\Gamma^t)\ln\mu_\infty.$

With the solution in (A2), we can now solve for the log-linear first-order difference equation for human capital per worker:

(A3) $\quad \ln H_t = \ln H_0 + \left(\frac{\theta\delta}{1-\Gamma}\right)(1-\Gamma^t)\ln\mu_0 + \theta\delta\left(\frac{\Gamma^t-1}{1-\Gamma} + t\right)\ln\mu_\infty + t\ln[A(Dql)^\delta e^\varepsilon].$

From the solutions in (A2) and (A3), we can also solve for $\ln y_t$:

$$\ln y_t = \ln(Dl) + \theta\ln\mu_t + \ln H_t.$$

Clearly, the economy globally converges toward its balanced growth path since $0 < \Gamma \equiv \theta(1-\delta) < 1$. Q.E.D.

**Proof of Proposition 1.** First, it can be easily observed from (19) and (22) that lower $\zeta$ does not affect $s$ but increases $c_y$, respectively.

To see how $\zeta$ affects $N$, differentiate $N$ in (26) with respect to $\zeta$ which yields

$$\frac{\partial N}{\partial\zeta} = \frac{-\alpha^2(1-\delta)(1-\theta)(\alpha+\beta)\{\alpha\delta[(1-\theta)(\rho+\beta)+\gamma]+(\alpha+\beta)[(1-\theta)(\delta+\varepsilon)-\alpha\theta\varepsilon]+\alpha\varepsilon(1-\alpha\theta)(\rho+\beta+\gamma)\}}{v(N_d^{LF})^2} < 0.$$

So, lower $\zeta$ increases $N$.

The result $\partial N/\partial\zeta < 0$ implies lower $\zeta$ decreases $q$ in (23), i.e., $\partial q/\partial\zeta > 0$.

Since lower $\zeta$ increases $c_y$ and $N$, lower $\zeta$ increases $c_o$ in (20).

To see how $\zeta$ affects $e$, i differentiate $e$ in (24) with respect to $\zeta$ which yields

$$\frac{\partial e}{\partial\zeta} = \frac{-v\alpha\varepsilon(\alpha+\beta)(1-\theta)}{(N_n^{LF})^2}\frac{\partial N_n^{LF}}{\partial\zeta} > 0,$$

where under Assumption 1,

$$\frac{\partial N_n^{LF}}{\partial \zeta} = -\alpha^2(1-\delta)[(\rho+\beta)(1-\alpha\theta)-(\alpha+\beta)\theta] < 0.$$

Thus, lower $\zeta$ decreases $e$.

Differentiate $Z$ in (25) with respect to $\zeta$ yields

$$\frac{\partial Z}{\partial \zeta} = \frac{\gamma\alpha^3\delta(\alpha+\beta)(1-\theta)(1-\delta)\theta}{(N_d^{LF})^2} > 0,$$

and hence lower $\zeta$ decreases $Z$.

Since $l = 1 - vN - eN - Z$, which can be written as

$$l = \frac{(1-\theta)(1-\alpha\zeta(1-\delta))(\alpha+\beta)}{N_d^{LF}},$$

we can show that lower $\zeta$ decreases $l$:

$$\frac{\partial l}{\partial \zeta} = \frac{(1-\theta)^2\alpha^2\delta(\alpha+\beta)(1-\delta)[\alpha+\beta+\alpha(\rho+\beta+\gamma)]}{(N_d^{LF})^2} > 0.$$

The above results that show $\partial s/\partial \zeta = 0, \partial q/\partial \zeta > 0$, $\partial e/\partial \zeta > 0$, $\partial l/\partial \zeta > 0$ and $\partial N/\partial \zeta < 0$ imply that lower $\zeta$ decreases the balanced steady-state growth rate of per worker output ($g_\infty$) in (30), i.e., $\partial g_\infty/\partial \zeta > 0$. Q.E.D.

**Equations $\Psi_i$, $i = 1, 2, .., 7$, that characterize the welfare level.** The welfare level is given by (31):

$$U_o = \Psi_0 + \Psi_1 \ln c_y + \Psi_2 \ln l + \Psi_3 \ln N + \Psi_4 \ln q + \Psi_5 \ln e + \Psi_6 \ln s + \Psi_7 \ln Z,$$

where $\Psi_0$ is a constant that consists of the initial state $(N_{-1}, H_o, k_o)$ and the productivity parameters $(A, D)$. Each $\Psi_i$, $i = 1,2,..7$, is given as follows:

$$\Psi_1 \equiv \frac{\alpha+\beta}{1-\alpha} > 0,$$

$$\Psi_2 \equiv \frac{(\alpha+\beta)(1-\theta)[1-\alpha(1-\delta)]}{(1-\alpha)^2[1-\alpha\theta(1-\delta)]} > 0,$$

$$\Psi_3 \equiv \frac{\alpha\{(\rho+\beta)(1-\alpha)[1-\alpha\theta(1-\delta)]-\theta[1-\alpha(1-\delta)](\alpha+\beta)\}}{(1-\alpha)^2[1-\alpha\theta(1-\delta)]} > 0,$$ [22]

$$\Psi_4 \equiv \frac{\delta\alpha(\alpha+\beta)(1-\theta)}{(1-\alpha)^2[1-\alpha\theta(1-\delta)]} > 0,$$

$$\Psi_5 \equiv \frac{\varepsilon\alpha(\alpha+\beta)(1-\theta)}{(1-\alpha)^2[1-\alpha\theta(1-\delta)]} > 0,$$

$$\Psi_6 \equiv \frac{\alpha\theta(\alpha+\beta)[1-\alpha(1-\delta)]}{(1-\alpha)^2[1-\alpha\theta(1-\delta)]} > 0,$$

$$\Psi_7 \equiv \frac{\alpha\gamma}{1-\alpha} > 0.$$

[22] $\Psi_3 > 0$ under Assumption 1 (see section 3.2).

**The constant allocation solutions with PLS_LS.** From the first-order conditions given in Section 3.3, the constant allocation rules for ($c_y$, $q$, $e$, $Z$, $N$, $l$) are given below:

(A4) $c_y = \frac{\alpha(1-\alpha\theta-qN)}{(\alpha+\beta)}$,

(A5) $q = \frac{\alpha\delta(1-\theta)}{N(1-\alpha\zeta(1-\delta))}$,

(A6) $e = \frac{v\varepsilon qN}{(1-p)\{\delta[(\rho+\beta)c_y-qN-\alpha\theta]-\varepsilon qN\}}$,

(A7) $N = \frac{(1-p)\{\delta[(\rho+\beta)c_y-qN-\alpha\theta]-\varepsilon qN\}}{v\{(1-p)\delta[(\rho+\beta+\gamma)c_y-qN-\alpha\theta+1-\theta]+p\varepsilon qN\}}$,

(A8) $Z = \frac{(1-p)\delta\gamma c_y}{\{(1-p)\delta[(\rho+\beta+\gamma)c_y-qN-\alpha\theta+1-\theta]+p\varepsilon qN\}}$.

By substituting the solutions for $e, N$ and $Z$ given above into $l = 1-(v+e)N-Z$, the solution for labor supply is:

(A9) $l = \frac{(1-p)\delta(1-\theta)}{\{(1-p)\delta[(\rho+\beta+\gamma)c_y-qN-\alpha\theta+1-\theta]+p\varepsilon qN\}}$.

**Proof of Proposition 2.** The effects of PLS_LS on $(q, e, N, Z, l, c_o, c_y, b, s, g_\infty)$ can be observed directly by using the solutions given in (A4)-(A9) and Equations (18)-(20) and (30). Q.E.D.

**Proof of Proposition 3.** The welfare level can be obtained from (31) by substituting the equilibrium allocations under PLS_LS $(c_y, l, N, q, e, Z, s, c_o, c_{o,0})$ into (31). The welfare level as a function of parental leave subsidization ($p$), through labor supply ($l$), fertility ($N$), the ratio of parental expenditure on child human capital to output per worker ($q$), parental time input in child human capital ($e$), and leisure ($Z$) is given by

$$G(p) = (\Psi_2+\Psi_3+\Psi_7-\Psi_4)\ln\left(\frac{1-p}{(1-p)\delta[(\rho+\beta+\gamma)c_y-qN-\alpha\theta+1-\theta]+p\varepsilon qN}\right) - \Psi_5\ln(1-p)$$

where it can be shown that $(\Psi_2+\Psi_3+\Psi_7-\Psi_4) > 0$.

By differentiating $G(p)$ with respect to $p$, the following can be obtained

$$\frac{\partial G(p)}{\partial p} = 0 \Rightarrow p^* = \frac{\alpha(1-\zeta)(1-\delta)[(\alpha+\beta)(1-\theta-\alpha\theta)+\alpha(1-\alpha\theta)(\rho+\beta+\gamma)]}{p_d},$$

where

$$p_d = [\alpha(\rho+\beta+\gamma)+(\alpha+\beta)][(1-\alpha\theta)\big(1-\alpha\zeta(1-\delta)\big)-\alpha\delta(1-\theta)] - (\alpha+\beta)\big[\big(1-\alpha\zeta(1-\delta)\big)\theta+\alpha\varepsilon(1-\theta)\big] > 0,$$

when Assumption 1 holds. It can be shown that $\partial G(p)/\partial p > 0$ when $p = 0$ and $\partial^2 G(p)/\partial p^2 < 0$ at $p = p^*$. Therefore, the optimal parental leave subsidization exists, and is positive and unique. Q.E.D.

**The constant allocation solutions with PLS_LI.** From the first-order conditions given in Section 3.4, the constant allocation rules for $(c_y, q, e, Z, N)$ are given below:

(A10) $c_y = \frac{\alpha(1-\alpha\theta-qN)}{(\alpha+\beta)}$,

(A11) $q = \frac{\alpha\delta(1-\theta)(1-\tau)}{N(1-\alpha\zeta(1-\delta))}$,

(A12) $e = \frac{v(\alpha+\beta)(1-\theta)\{[1-\alpha\zeta(1-\delta)]\tau+\alpha\varepsilon(1-\tau)\}}{\alpha\{[1-\alpha\zeta(1-\delta)]f_4-(1-\theta)(1-\tau)f_1\}}$,

(A13) $N = \frac{\alpha\{[1-\alpha\zeta(1-\delta)]f_4-(1-\theta)(1-\tau)f_1\}}{v\{[1-\alpha\zeta(1-\delta)]f_2-(1-\theta)(1-\tau)f_3\}}$,

(A14) $Z = \frac{\alpha\gamma\{[1-\alpha\zeta(1-\delta)](1-\alpha\theta)-\alpha\delta(1-\theta)(1-\tau)\}}{\{[1-\alpha\zeta(1-\delta)]f_2-(1-\theta)(1-\tau)f_3\}}$,

where

$$f_1 \equiv [(\delta+\varepsilon)(\alpha+\beta)+\alpha\delta(\rho+\beta)] > 0,$$
$$f_2 \equiv [\alpha(\rho+\beta+\gamma)(1-\alpha\theta)+(\alpha+\beta)(1-\theta-\alpha\theta)] > 0,$$
$$f_3 \equiv \alpha\delta[\beta+\alpha(1+\rho+\beta+\gamma)] > 0,$$
$$f_4 \equiv [(\rho+\beta)(1-\alpha\theta)-(\alpha+\beta)\theta] > 0,$$

where $[1-\alpha\zeta(1-\delta)]f_4-(1-\theta)(1-\tau)f_1 > 0$ for any given $\tau \in [0,1)$ because when $\tau = 0$, $[1-\alpha\zeta(1-\delta)]f_4-(1-\theta)f_1 = N_n^{LF}$ which is the numerator of the number of children given by Equation (26) in the laissez-faire economy. Note that we can obtain the solution for labor supply, $l = 1-(v+e)N-Z$, by substituting the solutions for $e, N$ and $Z$ given above into $l = 1-(v+e)N-Z$.

**Proof of Proposition 4.** First, it is easy to see $\tau$ does not affect the saving rate $(s)$ in (19). From (A11), $qN$ is negatively affected by parental leave subsidization financed by labor income taxation $\tau$ and, thus, the fraction of output spent on young-age consumption $(c_y)$ in (A10) increases with $\tau$, and the ratio of transfers per child to output per worker $(b)$ in (18) decreases with $\tau$.

By differentiating the solution for fertility $(N)$ in (A13) with respect to $\tau$ and following cancellation, the effect of $\tau$ on $N$ is negative as shown below:

(A15) $\frac{\partial N}{\partial\tau} = \frac{\alpha(1-\theta)[1-\alpha\zeta(1-\delta)](f_1 f_2+f_3 f_4)}{v\{[1-\alpha\zeta(1-\delta)]f_2-(1-\theta)(1-\tau)f_3\}^2} > 0.$

Since the fraction of output spent on old-age consumption $(c_o)$ in (20) is positively related to $c_y$ and $N$, the effect of $\tau$ on $c_o$ is positive. Additionally, the negative effect of $\tau$ on $N$ implies that the effect of $\tau$ on the ratio of parental expenditure on a child's human capital to output per worker $(q)$ is negative.

By differentiating the solution for the parental time input in child human capital $(e)$ in (A12) with respect to $\tau$ and following cancellation, the effect of $\tau$ on $e$ is positive as shown below:

(A16) $\frac{\partial e}{\partial \tau} = \frac{v(\alpha+\beta)(1-\theta)[1-\alpha\zeta(1-\delta)]\{[1-\alpha\zeta(1-\delta)-\alpha\varepsilon]f_4-(1-\theta)f_1\}}{\alpha\{[1-\alpha\zeta(1-\delta)]f_4-(1-\theta)(1-\tau)f_1\}^2} > 0.$

The effects of $\tau$ on leisure $(Z)$ in (A14) and labor supply $(l = 1-(v+e)N-Z)$ are given, respectively, by:

$$\frac{\partial Z}{\partial \tau} = \frac{-\alpha^2\delta\gamma\theta(1-\theta)(\alpha+\beta)[1-\alpha\zeta(1-\delta)]}{\{[1-\alpha\zeta(1-\delta)]f_2-(1-\theta)(1-\tau)f_3\}^2} < 0,$$

$$\frac{\partial l}{\partial \tau} = \frac{-(1-\theta)(\alpha+\beta)[1-\alpha\zeta(1-\delta)]^2 f_2}{\{[1-\alpha\zeta(1-\delta)]f_2-(1-\theta)(1-\tau)f_3\}^2} < 0.$$

Using the balanced steady-state growth rate of per worker income $g_\infty$ given in (30), a sufficient condition for $\partial g_\infty/\partial\tau > 0$ is when the effect of $\partial e/\partial\tau > 0$ is sufficiently larger than the effects of $\partial q/\partial\tau < 0$, $\partial l/\partial\tau < 0$ and $\partial N/\partial\tau > 0$ in absolute terms, i.e.,

(A17) $\frac{\varepsilon(1-\theta)}{e}\frac{\partial e}{\partial \tau} - \frac{\delta\theta}{N}\frac{\partial N}{\partial \tau} + \frac{\delta(1-\theta)}{ql}\frac{\partial(ql)}{\partial \tau} > 0,$

where the expressions for $\partial N/\partial\tau$ and $\partial e/\partial\tau$ are given by Equations (A15) and (A16), respectively, and

$$\frac{\partial(ql)}{\partial \tau} = \frac{-v\delta(\alpha+\beta)(1-\theta)^2(1-\tau)\{2[1-\alpha\zeta(1-\delta)]f_4-(1-\theta)(1-\tau)f_1\}}{\{[1-\alpha\zeta(1-\delta)]f_4-(1-\theta)(1-\tau)f_1\}^2} < 0,$$

as $[1-\alpha\zeta(1-\delta)]f_4-(1-\theta)(1-\tau)f_1 > 0$. Q.E.D.

# Appendix B: Tables and Figures

## Table 1. Targeted data moments

| Description and notation | Value |
|---|---|
| Number of children per young parent ($N$) | 0.9594 |
| Ratio of consumption expenditure of a young household to income per young household ($c_y$) | 0.5134 |
| Ratio of consumption expenditure of an old household to income per young household ($c_o$) | 0.3047 |
| Ratio of expenditure on child human capital to income per young household ($q$) | 0.0163 |
| Fraction of time allocated on a child when the child is young (0-6 years old) for a young parent ($e$) | 0.0471 |
| Fraction of time allocated on a child when the child is elder (7-17 years old) for a young parent ($v$) | 0.0327 |
| Fraction of time allocated on leisure for a young parent ($Z$) | 0.3313 |
| Externality in the human capital production $(1-\zeta)(1-\delta)$ | 0.2400 |
| Annual growth rate of real GDP per capita ($g_a$) | 1.4569 |
| Income share of labor $(1-\theta)$ | 0.5941 |

*Source:* The data source for $v$,$e$ and $Z$ is ATUS. The data source for N, $c_y$ and $c_o$ is PSID. Data for q is collected from Lino et al. (2017). Data moment for the externality of human capital in the production function is collected from Borjas (1995). Data source for annual GDP per capita growth and the share of labor compensation in GDP are from World Bank and FRED, respectively.

## Table 2. Baseline calibration

| | |
|---|---|
| **Utility** | |
| Discounting factor | $\alpha = 0.3778$ |
| Taste parameter for old-age consumption | $\beta = 0.2337$ |
| Taste parameter for number of children | $\rho = 0.2451$ |
| Taste parameter for leisure | $\gamma = 0.6474$ |
| **Production of final output** | |
| Income share of capital | $\theta = 0.4059$ |
| **Production of human capital** | |
| Weight of parental expenditure input | $\delta = 0.0510$ |
| Weight of parental time input | $\varepsilon = 0.1479$ |
| Degree of externality | $\zeta = 0.7471$ |
| Productivity parameters | $A^{1-\theta}D^{\delta} = 2.0428$ |

**Table 3. A comparison of equilibria**

| Variables | Laissez-faire | Social optimum | Optimal PLS_LS | Optimal PLS_LI |
|---|---|---|---|---|
| $N$ | 0.9594 | 0.6792 | 0.9528 | 0.9738 |
| $q$ | 0.0163 | 0.0263 | 0.0164 | 0.0159 |
| $e$ | 0.0471 | 0.0762 | 0.0543 | 0.0505 |
| $Z$ | 0.3313 | 0.3317 | 0.3290 | 0.3313 |
| $l$ | 0.5921 | 0.5944 | 0.5881 | 0.5877 |
| $c_y$ | 0.5134 | 0.5120 | 0.5134 | 0.5135 |
| $c_o$ | 0.3047 | 0.2152 | 0.3026 | 0.3093 |
| $g_a$ | 0.0146 | 0.0181 | 0.0153 | 0.0149 |
| $\Delta(\%)$ | - | 0.254 | 0.058 | 0.024 |
| $p$ | - | - | 0.1332 | 0.0872 |
| $OLD$ | - | - | 14.3 | 8.7 |

Note: $g_a$ is the annual growth rate, and $OLD$ denotes the optimal leave duration in weeks. The corresponding optimal lump-sum tax as a fraction of output per worker and labor income tax rate are $T/y = 0.0073$ and $\tau = 0.0073$, respectively. The saving rate $s$ and the ratio of bequest to output per worker $b$ are the same across equilibria, with $s = 0.1534$, and $b = 0.0883$.

**Table 4. Alternative calibrations**

| | (1)<br>$(1-\zeta)(1-\delta)=0.18$ | (2)<br>$(1-\zeta)(1-\delta)=0.24$ | (3)<br>$(1-\zeta)(1-\delta)=0.44$ |
|---|---|---|---|
| $\delta$ | 0.0494 | 0.0510 | 0.0564 |
| $\varepsilon$ | 0.1432 | 0.1479 | 0.1635 |
| $\zeta$ | 0.8106 | 0.7471 | 0.5337 |
| $A^{1-\theta}D^{\delta}$ | 2.0134 | 2.0428 | 2.1440 |

Note: We only report re-calibrated values of parameters that are different from the benchmark calibration in Table 2. Column (2) is the baseline case.

**Table 5. Equilibrium solutions under alternative calibrations**

| | Laissez-faire | Social optimum | Optimal PLS_LS | Optimal PLS_LI |
|---|---|---|---|---|
| | | $(1-\zeta)(1-\delta)=0.18$ | | |
| $N$ | 0.9594 | 0.7493 | 0.9544 | 0.9703 |
| $q$ | 0.0163 | 0.0231 | 0.0164 | 0.0160 |
| $e$ | 0.0471 | 0.0669 | 0.0525 | 0.0497 |
| $Z$ | 0.3313 | 0.3316 | 0.3296 | 0.3313 |
| $l$ | 0.5921 | 0.5938 | 0.5891 | 0.5888 |
| $c_y$ | 0.5134 | 0.5124 | 0.5134 | 0.5135 |
| $c_o$ | 0.3047 | 0.2375 | 0.3031 | 0.3082 |
| $g_a$ | 0.0146 | 0.0171 | 0.0151 | 0.0148 |
| $\Delta(\%)$ | - | 0.141 | 0.033 | 0.014 |
| $p$ | - | - | 0.1031 | 0.067 |
| $OLD$ | - | - | 10.7 | 6.6 |
| | | $(1-\zeta)(1-\delta)=0.44$ | | |
| | Laissez-faire | Social optimum | Optimal PLS_LS | Optimal PLS_LI |
| $N$ | 0.9594 | 0.4458 | 0.9473 | 0.9850 |
| $q$ | 0.0163 | 0.0441 | 0.0165 | 0.0157 |
| $e$ | 0.0471 | 0.1284 | 0.0604 | 0.0531 |
| $Z$ | 0.3313 | 0.3320 | 0.3271 | 0.3313 |
| $l$ | 0.5921 | 0.5962 | 0.5847 | 0.5843 |
| $c_y$ | 0.5134 | 0.5109 | 0.5134 | 0.5135 |
| $c_o$ | 0.3047 | 0.1409 | 0.3008 | 0.3129 |
| $g_a$ | 0.0146 | 0.0228 | 0.0159 | 0.0151 |
| $\Delta(\%)$ | - | 0.922 | 0.188 | 0.077 |
| $p$ | - | - | 0.2207 | 0.1448 |
| $OLD$ | - | - | 26.6 | 15.3 |

Note: $g_a$ refers to annual growth rate of output per worker. In all cases, $s=\alpha\theta=0.1534$, and $b=0.0883$.

**Table 6: Paid leave entitlements per child available to parents in 2018**

| Country | Full-rate equivalent leave | Country | Full-rate equivalent leave |
|---|---|---|---|
| Australia | 8.6 | Korea | 40.6 |
| Austria | 55.9 | Latvia | 52.8 |
| Belgium | 18.0 | Lithuania | 66.0 |
| Canada | 26.6 | Luxembourg | 45.3 |
| Chile | 31.0 | Mexico | 13.0 |
| Czech Republic | 47.5 | Netherlands | 16.4 |
| Denmark | 27.6 | New Zealand | 8.4 |
| Estonia | 86.4 | Norway | 52.4 |
| Finland | 46.0 | Poland | 43.6 |
| France | 23.4 | Portugal | 32.9 |
| Germany | 48.3 | Slovak Rep. | 53.1 |
| Greece | 21.7 | Slovenia | 52.3 |
| Hungary | 69.2 | Spain | 20.3 |
| Iceland | 26.6 | Sweden | 45.4 |
| Ireland | 7.5 | Switzerland | 8.2 |
| Israel | 15.0 | Turkey | 11.7 |
| Italy | 26.0 | UK | 12.1 |
| Japan | 66.1 | United States | 0.0 |

Note: Here we report the sum of full-rate equivalent (weeks) available to mothers and fathers (refer to Tables PF2.1.A and PF2.1.B in OECD (2019), respectively). Paid leaves for fathers tend to be far shorter than paid leaves available to mothers.

**Figure 1. Welfare gain over the Laissez-faire for different parental leave duration**

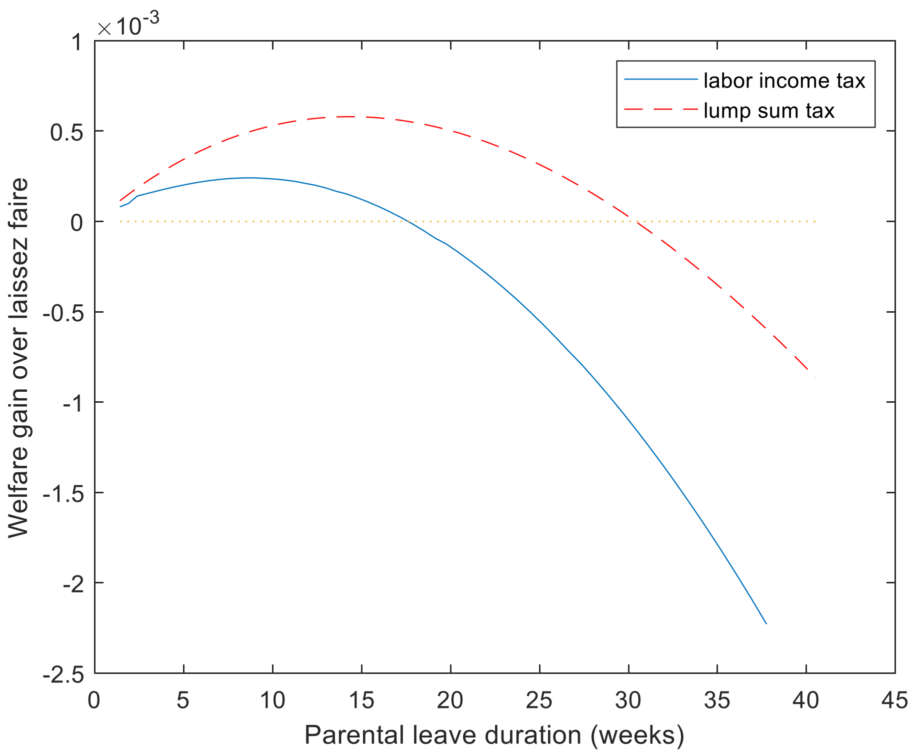
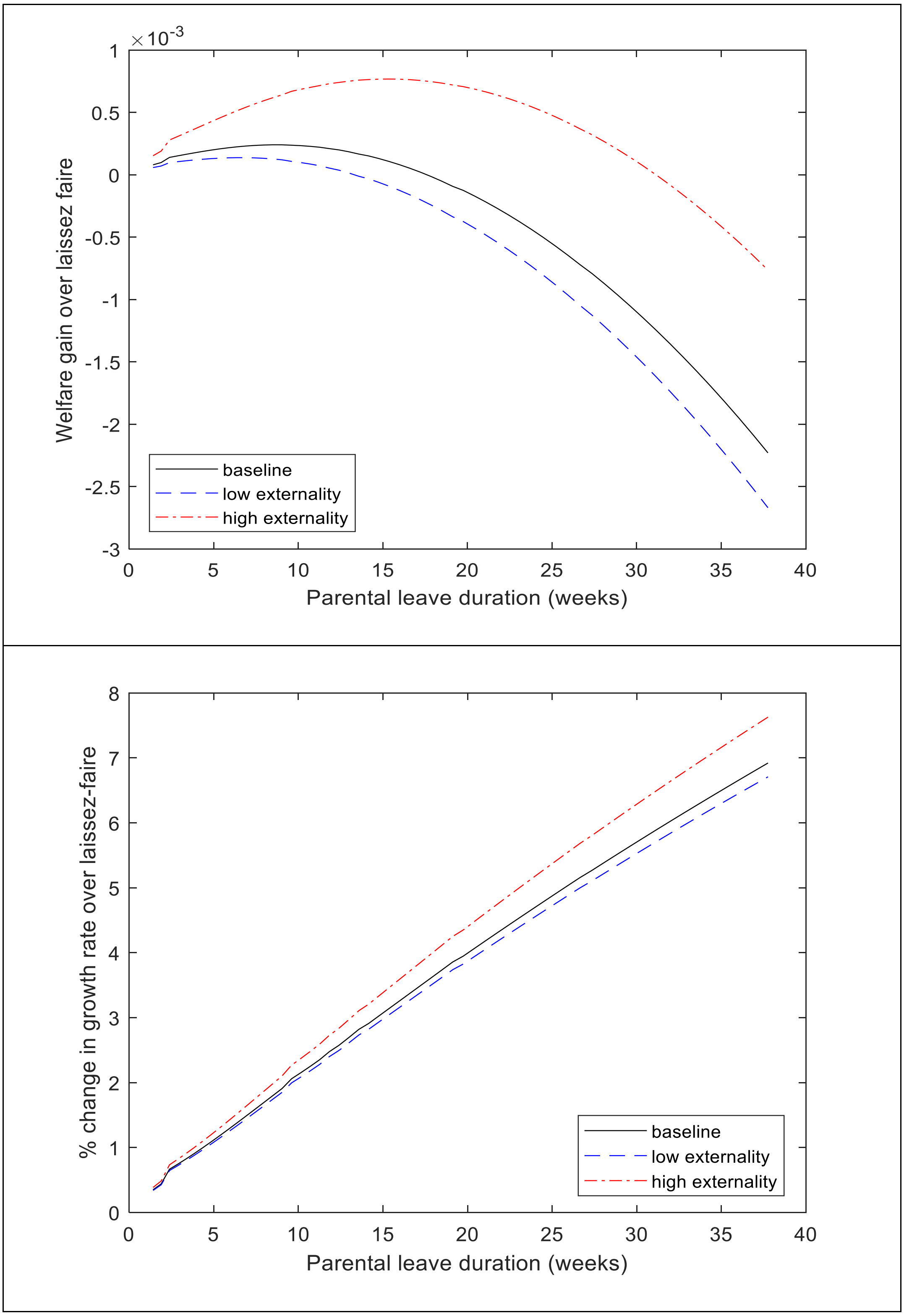


**Figure 2: Welfare gain and percentage change in growth rate over laissez-faire for alternative calibrations**




**Notes:** In both plots, the solid black lines refer to the baseline calibration, $(1-\zeta)(1-\delta) = 0.24$, the blue dashed lines refer to the calibration with low human capital externality, $(1-\zeta)(1-\delta) = 0.18$, the red dashed lines refer to the calibration with high human capital externality, $(1-\zeta)(1-\delta) = 0.44$.