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Multivariate Decompositions and Seasonal Gender Employment

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1 Introduction

Over many decades, economists have sought to understand the drivers underlying the time series evolution of series of interest through the analysis of seasonally adjusted data. The notion that a series can be meaningfully decomposed into components that are not directly observed is, therefore, deeply embedded in empirical economic analyses.

An important strand of literature employs so-called unobserved components (UC) models to study trends and cycles as separate phenomena of economic interest. While conventional UC models, including those underlying seasonal adjustment, assume that the individual components are uncorrelated, a substantial literature questions this in the context of analyzing trend and cyclical movements in seasonally adjusted data; important contributions include Clark (1989), Morley, Nelson and Zivot (2003, henceforth MNZ), Morley (2007), Sinclair (2009), Dungey et al. (2013, 2015). Very recently, Hindrayanto, Jacobs, Osborn and Tian (2019, henceforth HJOT) extend the analysis to include a seasonal component, showing (in the univariate context) that the assumption of uncorrelated trend, cycle and seasonal components can be questionable.

Multivariate analysis of economic time series can throw important light on underlying economic phenomena, including trend and cyclical movements. In order to analyze such movements when they are potentially correlated with seasonality, a correlated multivariate UC model with a seasonal component is required. Building on HJOT, the present paper studies such a model, focusing on the case of quarterly data and examining sets of economic restrictions, including common trends and common cycles, that can be used to ensure identification. To our knowledge, no previous analysis has examined identification conditions for a correlated multivariate UC trend-cycle-seasonal model.

The model is applied to study seasonal gender employment in Australia. Gender issues in employment are a focus of interest for economists, including the gender wage gap (Blau and Kahn, 2017) and occupational differences (Preston and Whitehouse, 2004). Our analysis is distinctive, in examining quarterly aggregate employment by gender. A bivariate male/female model with a common cycle is preferred to both univariate correlated component and bivariate uncorrelated component specifications. This model evidences distinct gender-based seasonal patterns with seasonality declining over time for females and (if anything) increasing for males.

The paper proceeds as follows. Section 2 discusses multivariate correlated seasonal UC models. It is shown that while the general model is not identified, plausible economic restrictions can allow identification in the presence of non-zero correlations between trend, cycle and seasonal shocks. Section 3 then applies the approach to gender employment in Australia. The final section concludes.

2 Multivariate UC Models

This section describes the model and discusses its identification, including economically plausible restrictions that may apply.

2.1 Seasonal UC model

Many macroeconomic variables exhibit trend, cycle and seasonal characteristics. Hence, for an observed $k \times 1$ vector $\boldsymbol{Y}_t$, consider a multivariate UC model that explicitly recognizes these characteristics through the measurement equation

$$\boldsymbol{Y}_t = \boldsymbol{T}_t + \boldsymbol{C}_t + \boldsymbol{S}_t \tag{1}$$

in which the trend, cycle and seasonal components ($\boldsymbol{T}_t$, $\boldsymbol{C}_t$ and $\boldsymbol{S}_t$, respectively) are, in general, each $k \times 1$ vectors.

Following MNZ and many others, we assume that the trend for each variable is $I(1)$ and can be represented as a random walk with drift, so that

$$\boldsymbol{T}_t = \boldsymbol{T}_{t-1} + \boldsymbol{\beta} + \boldsymbol{\eta}_t, \tag{2}$$

where $\boldsymbol{\eta}_t = (\eta_{1t}, \ldots, \eta_{kt})'$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_k)$ and the $k \times k$ covariance matrix $E[\boldsymbol{\eta}_t \boldsymbol{\eta}_t'] = \boldsymbol{\Sigma}_{\boldsymbol{\eta}\boldsymbol{\eta}}$ is not *a priori* assumed to be diagonal. The multivariate cyclical component of (1) is represented by the AR processes

$$\boldsymbol{\Phi}(L)\boldsymbol{C}_t = \boldsymbol{\varepsilon}_t, \tag{3}$$

where $\boldsymbol{\Phi}(L)$ is a $k \times k$ matrix in the lag operator L, with $\boldsymbol{\Phi}(L) = \boldsymbol{I}_k - \boldsymbol{\Phi}_1 L - \ldots - \boldsymbol{\Phi}_p L^P$ ($\boldsymbol{I}_k$ being a $k \times k$ identity matrix) having all roots strictly outside the unit circle and, with $\boldsymbol{\varepsilon}_t$ defined in the obvious way, $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'] = \boldsymbol{\Sigma}_{\varepsilon\varepsilon}$. As usual in economic applications of multivariate UC models, such as Morley (2007), Sinclair (2009) or Ma and Wohar (2013), $\boldsymbol{\Phi}(L)$ is assumed diagonal with the cycle in each variable having the same univariate order p. Empirical analyses typically employ $p = 2$, since this can both adequately capture short-term nonseasonal movements in economic data while also allowing the parameters of the correlated UC trend-cycle model to be identified; see MNZ for the univariate case and Trenkler and Weber (2016), hereafter TW, for a multivariate analysis[1].

As in HJOT and many other papers, seasonality is modeled using the so-called "dummy variable" form

$$\Psi(L)\boldsymbol{S}_t = \boldsymbol{\omega}_t \tag{4}$$

where $\Psi(L)$ is the scalar annual summation polynomial over a year ($\Psi(L) = 1 + L + L^2 + L^3$ for quarterly data) and, in an obvious notation, $E[\boldsymbol{\omega}_t\boldsymbol{\omega}_t'] = \boldsymbol{\Sigma}_{\omega\omega}$.

To facilitate later discussion, stack the UC model disturbances of (2)-(4) to form the $3k \times 1$ vector $\boldsymbol{U}_t$ as

$$\boldsymbol{U}_t = [\boldsymbol{\eta}_t', \boldsymbol{\varepsilon}_t', \boldsymbol{\omega}_t']', \tag{5}$$

and define the $3k \times 3k$ covariance matrix

$$E[\boldsymbol{U}_t\boldsymbol{U}_t'] = \boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{\eta\eta} & \boldsymbol{\Sigma}_{\eta\varepsilon} & \boldsymbol{\Sigma}_{\eta\omega} \\ \boldsymbol{\Sigma}_{\eta\varepsilon}' & \Sigma_{\varepsilon\varepsilon} & \boldsymbol{\Sigma}_{\varepsilon\omega} \\ \boldsymbol{\Sigma}_{\eta\omega}' & \boldsymbol{\Sigma}_{\varepsilon\omega}' & \boldsymbol{\Sigma}_{\omega\omega} \end{bmatrix} \tag{6}$$

where, in an obvious notation,

$$E[\boldsymbol{\eta}_t\boldsymbol{\varepsilon}_t'] = \boldsymbol{\Sigma}_{\eta\varepsilon},\ E[\boldsymbol{\eta}_t\boldsymbol{\omega}_t'] = \boldsymbol{\Sigma}_{\eta\omega},\ E[\boldsymbol{\varepsilon}_t\boldsymbol{\omega}_t'] = \boldsymbol{\Sigma}_{\varepsilon\omega}. \tag{7}$$

Although the disturbances are possibly cross-correlated at t, they are assumed uncorrelated over time, so that

$$E[\boldsymbol{U}_{t_1}\boldsymbol{U}_{t_2}'] = \boldsymbol{0}, \qquad t_1 \neq t_2.$$

2.2 Reduced form

As a preliminary to identification, we consider the reduced form and autocovariances of the multivariate seasonal UC model for quarterly data[2]. It is straightforward to see that the system (1)-(4) implies the reduced form

$$\boldsymbol{\Phi}(L)\Delta_4\boldsymbol{Y}_t = \boldsymbol{\Phi}(1)\Psi(1)\boldsymbol{\beta} + \boldsymbol{\Phi}(L)\Psi(L)\boldsymbol{\eta}_t + \Delta_4\boldsymbol{\varepsilon}_t + \boldsymbol{\Phi}(L)\Delta_1\boldsymbol{\omega}_t \tag{8}$$

[1] With seasonal data, it is also important that the cycle component is not conflated with the seasonal component. A low order, such as $p = 2$ can be important for this purpose, especially for quarterly data.

[2] The expressions in this subsection can be easily generalized to monthly data, but the consequent identification issues are left for future research.

where $\Delta_4 = 1 - L^4$ is the annual difference and Δ_1 is the usual first difference. In this general model, each element of $\boldsymbol{Y}_t$ is seasonally integrated (see, for example, Ghysels and Osborn, 2001, Chapter 4), due to the presence of a zero frequency unit root in its trend component (2) and the full set of unit roots at seasonal frequencies through the nonstationary seasonal process of (4). Hence annual differencing is required to reduce each univariate process in $\boldsymbol{Y}_t$ to stationarity, but this does not rule out cointegration across the components of $\boldsymbol{Y}_t$.

To focus on the disturbances, define from (8)

$$\begin{aligned} \boldsymbol{Z}_t &= \boldsymbol{A}(L)\boldsymbol{\eta}_t + (1 - L^4)\boldsymbol{\varepsilon}_t + \boldsymbol{B}(L)\boldsymbol{\omega}_t \\ &= \boldsymbol{H}(L)\boldsymbol{U}_t \end{aligned} \tag{9}$$

where

$$\begin{aligned} \boldsymbol{A}(L) &= (1 + L + L^2 + L^3)\boldsymbol{\Phi}(L) = \boldsymbol{I}_k + \boldsymbol{A}_1 L + ... + \boldsymbol{A}_{p+3}L^{p+3}, \\ \boldsymbol{B}(L) &= (1 - L)\boldsymbol{\Phi}(L) = \boldsymbol{I}_k + \boldsymbol{B}_1 L + ... + \boldsymbol{B}_{p+1}L^{p+1} \end{aligned} \tag{10}$$

while $\boldsymbol{U}_t$ is defined in (5) and $\boldsymbol{H}(L)$ is the $k \times 3k$ matrix

$$\begin{aligned} \boldsymbol{H}(L) &\equiv \begin{bmatrix} \boldsymbol{A}(L) & (1 - L^4)\boldsymbol{I}_k & \boldsymbol{B}(L) \end{bmatrix} \\ &= \boldsymbol{H}_0 + \boldsymbol{H}_1 L + \boldsymbol{H}_2 L^2 + ... + \boldsymbol{H}_q H^q \end{aligned} \tag{11}$$

where $q = \max(p + 3, 4)$.

For the specific case of interest in our application, with $p = 2$ and quarterly data, then $q = 5$. Also noting that $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$ are diagonal and hence symmetric, it can easily be seen that

$$\begin{aligned} \boldsymbol{H}_0 &= \begin{bmatrix} \boldsymbol{I}_k & \boldsymbol{I}_k & \boldsymbol{I}_k \end{bmatrix}, \\ \boldsymbol{H}_1 &= \begin{bmatrix} \boldsymbol{A}_1 & 0 & \boldsymbol{B}_1 \end{bmatrix} = \begin{bmatrix} (\boldsymbol{I}_k - \boldsymbol{\Phi}_1) & 0 & -(\boldsymbol{I}_k + \boldsymbol{\Phi}_1) \end{bmatrix}, \\ \boldsymbol{H}_2 &= \begin{bmatrix} \boldsymbol{A}_2 & 0 & \boldsymbol{B}_2 \end{bmatrix} = \begin{bmatrix} (\boldsymbol{I}_k - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2) & 0 & (\boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2) \end{bmatrix}, \\ \boldsymbol{H}_3 &= \begin{bmatrix} \boldsymbol{A}_3 & 0 & \boldsymbol{B}_3 \end{bmatrix} = \begin{bmatrix} (\boldsymbol{I}_k - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2) & 0 & \boldsymbol{\Phi}_2 \end{bmatrix}, \\ \boldsymbol{H}_4 &= \begin{bmatrix} \boldsymbol{A}_4 & -\boldsymbol{I}_k & 0 \end{bmatrix} = \begin{bmatrix} -(\boldsymbol{\Phi}_1 + \boldsymbol{\Phi}_2) & -\boldsymbol{I}_k & 0 \end{bmatrix}, \\ \boldsymbol{H}_5 &= \begin{bmatrix} \boldsymbol{A}_5 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\Phi}_2 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Since $\boldsymbol{\Phi}(L)$ is of order p and $\boldsymbol{Z}_t$ is the sum of moving averages, the reduced form (8) is a VARMA(p, q) process, with $q = p + 3$ for $p > 0$. Ruling out the AR and MA polynominals in each equation $i = 1, ..., k$ having any factor in common[3], this VARMA process with diagonal $\boldsymbol{\Phi}(L)$ is identified (Dufour and Pelletier (2021, Theorem 3)). An immediate consequence is that the AR parameters in $\boldsymbol{\Phi}(L)$ are identified from the reduced form. Also, noting that $\Psi(L)$ is the (known) annual summation operator, the drift parameter vector $\boldsymbol{\beta}$ is identified through the reduced form intercept vector.

Therefore, the primary issue for identification (and discussed in the next subsection) concerns whether the elements of the covariance matrix (6) can be estimated given the values of $\boldsymbol{\Phi}(L)$ and $\boldsymbol{\beta}$. For this purpose, we consider the non-zero autocovariance matrices of $\boldsymbol{Z}_t$, namely

$$\boldsymbol{\Gamma}_\ell = \sum_{i=0}^{q-\ell} \boldsymbol{H}_{i+\ell} \Sigma \boldsymbol{H}_i' \qquad \ell = 0, 1, ..., q. \tag{12}$$

Using (6) and (11), (12) then yields the autocovariances of $\boldsymbol{Z}_t$ in terms of the elements of $\boldsymbol{\Sigma}$ and $\boldsymbol{\Phi}(L)$.

2.3 Covariance matrix identification

Identification proceeds by considering the relationship between the autocovariances of the moving average component of the reduced form and the covariance matrix (6) of the underlying model. MNZ show that $p \geq 2$ is sufficient for the identification of the univariate trend-cycle model, while TW generalize this result to the multivariate context. The addition of seasonality complicates identification, with HJOT showing not only that univariate models of the form (1)-(6) for quarterly data with $k = 1$ are under-identified for $p \leq 1$, but also that an additional disturbance covariance restriction is required for identification when $p = 2$.

Following the line of analysis used by the above authors, the previous subsection has already noted that $\boldsymbol{\Phi}(L)$ and $\boldsymbol{\beta}$ are identified from the multivariate ARMA reduced form. Since, from (10) and (11), the only unknowns in the matrices $\boldsymbol{H}_i$ $(i = 0, \ldots, q)$ are the AR coefficients of $\boldsymbol{\Phi}(L)$, these are also identified from the reduced form. Therefore, the autocovariances of $\boldsymbol{Z}_t$ defined by (12) can be used to provide information about the $3k(3k + 1)/2$ distinct elements of $\boldsymbol{\Sigma}$, effectively treating the other parameters as given. The order condition for identification then requires $\boldsymbol{\Gamma}_\ell$ of (12) for $\ell = 0, 1, \ldots, q$ to contain at least $3k(3k + 1)/2$ distinct elements.

[3]The AR polynomial $\phi_i(L)$ in the i^{th} equation will cancel in (8) when the corresponding cycle disturbance has zero variance. However, this implies the absence of a stochastic cycle component in the variable and hence $\phi_i(L)$ is not identified.

The $q+1$ non-null autocovariance matrices of (12) have $qk^2 + k(k+1)/2$ distinct elements, of which $k(k+1)/2$ are contributed by the contemporaneous covariance matrix $\boldsymbol{\Gamma}_0$. As discussed above, the VMA order q is a consequence of both the data frequency and cycle order p. For quarterly data and $p \leq 1$, $q = 4$ and hence the number of distinct autocovariance elements in $\boldsymbol{\Gamma}_\ell$ for $\ell = 0, ..., q$, namely $(9k^2+k)/2$, is less than the number of distinct elements of $\boldsymbol{\Sigma}$, $(9k^2+3k)/2$. Consequently, as for the univariate case, the parameters of the quarterly unrestricted correlated multivariate UC model with seasonality are not identified when $p \leq 1$. We therefore concentrate on the case $p = 2$, which is of interest for empirical as well as theoretical reasons.

With $p = 2$, the number of distinct elements in $\boldsymbol{\Gamma}_\ell$ of (12) for $\ell = 0, ..., 5$ is $5k^2 + k(k+1)/2$. It is easily seen that this can be written as $(11k^2 + k)/2 = (9k^2 + 3k)/2$ for $k = 1$ (the case discussed by HJOT) and $(11k^2 + k)/2 > (9k^2 + 3k)/2$ for $k > 1$. Therefore, the order condition for identification is satisfied for the correlated UC model for all values of k. However, the rank condition also needs to be satisfied and HJOT show that this fails in the univariate case.

Using a similar notation to TW, define $\boldsymbol{\gamma}_0^* = \text{vech}(\boldsymbol{\Gamma}_0)$, where the vech operator columnwise stacks the elements of $\boldsymbol{\Gamma}_0$ on and below the main diagonal into the $k(k+1)/2$ vector $\boldsymbol{\gamma}_0^*$, starting with the first column of $\boldsymbol{\Gamma}_0$ and with the elements of each subsequent column placed below the immediately preceding one. Also define the k^2 vectors $\boldsymbol{\gamma}_i = \text{vec}(\boldsymbol{\Gamma}_i)$, $i = 1, \ldots, 5$, where the conventional vec operator stacks all elements in the columns of the relevant matrix below each other. The vector $\boldsymbol{\gamma}^* = [\boldsymbol{\gamma}_0^{*\prime}, \boldsymbol{\gamma}_1', \boldsymbol{\gamma}_2', \boldsymbol{\gamma}_3', \boldsymbol{\gamma}_4', \boldsymbol{\gamma}_5']'$ then contains the $(11k^2 + k)/2$ distinct autocovariance elements for $\boldsymbol{Z}_t$ at lags $\ell = 0, \ldots, 5$. Similarly, define the vector $\boldsymbol{\sigma}^* = \text{vech}(\boldsymbol{\Sigma})$ containing the $(9k^2 + 3k)/2$ distinct elements of the component covariance matrix $\boldsymbol{\Sigma}$ and it is also possible to define a $(11k^2 + k)/2 \times (9k^2 + 3k)/2$ matrix $\boldsymbol{D}$ whose elements depend only on $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ to write the relationships as the system of equations

$$\boldsymbol{\gamma}^* = \boldsymbol{D}\boldsymbol{\sigma}^* \tag{13}$$

in which the elements of $\boldsymbol{\sigma}^*$ are unknown. Consequently, the rank condition for identification in the multivariate correlated UC model is that $\boldsymbol{D}$ has a rank of at least $(9k^2 + 3k)/2$.

HJOT show that a linear dependence exists between the autocovariances when $k = 1$ in the correlated seasonal UC model, and hence the rank condition for identification fails. A single covariance restriction on the component disturbances is then required for identification. In the multivariate case, although explicit expressions can be obtained for the elements of $\boldsymbol{D}$, these are substantially more complicated for the seasonal multivariate model than those presented by

TW for the trend-cycle model or HJOT in the univariate seasonal case.

Therefore, in order to check the rank condition relevant to our empirical analysis where $k = 2$, we construct the 23×21 matrix $\boldsymbol{D}$ for two sets of plausible values for $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$, namely the pairs of estimated values from the two sets univariate correlated UC models of Table 2 below, one pair assuming zero trend-seasonal correlations and the other assuming perfect negative trend-cycle correlations. In both cases the rank of $\boldsymbol{D}$ is 19 and hence, with 21 distinct elements in $\boldsymbol{\Sigma}$, the model is under-identified and at least two restrictions need to be imposed on this matrix for identification.

2.4 Restricted models

A conventional multivariate UC model, as used by Harvey (1989), among many others, allows the disturbances for a specific component to be correlated across variables, but imposes zero correlations across components. With trend, cycle and seasonal components, the covariance matrix of (6) is then block diagonal, with $6k^2$ zero restrictions thereby imposed on the $9k^2$ elements of $\boldsymbol{\Sigma}$. The discussion of the preceding subsection implies that this uncorrelated multivariate UC specification is over-identified.

Although some previous studies employing UC models (including Morley (2007), Ma and Wohar (2013), Clark (1989), Fleischman and Roberts (2011) and McElroy (2017)) employ restrictions across variables to improve efficiency of estimation, the inclusion of seasonality in the correlated UC model requires restrictions for identification. Rather than making a *priori* assumption on the appropriate restrictions, the application of the next section considers a range of models and judges the economic plausibility of the results *ex post*.

To be specific, for our bivariate model of male/female employment, for which $\boldsymbol{T}_t = (\tau_{1t}, \tau_{2t})'$, $\boldsymbol{C}_t = (c_{1t}, c_{2t})'$. $\boldsymbol{S}_t = (s_{1t}, s_{2t})'$, we consider:

1. Common trends, which imposes in (2)

$$\begin{aligned} \tau_{2t} &= d\tau_{1t} \\ &= d\tau_{1,t-1} + d\beta_1 + d\eta_{1t} \end{aligned}$$

   so that both the deterministic and stochastic trend components of τ_{2t} are the same scalar multiple d of τ_{1t}.

2. Common cycles, for which $c_{2t} = bc_{1t}$, implying that in (3) $\boldsymbol{\Phi}(L) = \boldsymbol{\Phi}(L)I_2$ and

$$\varepsilon_{2t} = b\varepsilon_{1t}. \tag{14}$$

3. Common seasonals, with $s_{2t} = as_{1t}$, so that

$$\omega_{2t} = a\omega_{1t}. \tag{15}$$

4. Perfectly correlated trend shock, which places no restriction on the drift parameters but imposes

$$\eta_{2t} = d\eta_{1t}. \tag{16}$$

5. Perfectly correlated cycle shock, in which no cross-equation restrictions are placed on the AR parameters, but the cycle shocks satisfy (14).

6. Same trend shock, which imposes the additional restriction $d = 1$ in the perfectly correlated trend shock model.

7. Same cycle shock, which imposes the restriction $b = 1$ in the perfectly correlated cycle shock model.

Of these specifications, models 1, 4 and 6 all imply cointegration between the two observed series. The common trends model is used by Morley (2007) and Ma and Wohar (2013), with the perfectly correlated trend shock specification relaxing the implied restriction across the stochastic and deterministic trends. Clearly, the same trend shock specification restricts the stochastic trends of the two series to be the same without restricting the deterministic trends. The common cycle specification of 2 is used by Clark (1989), Harvey and Trimbur (2003) and Fleischman and Roberts (2011), with the perfectly correlated cycle shock specification relaxing the restriction of identical AR coefficients across variables. While the studies just mentioned consider trend and cycle components, the reduced rank specification of seasonality employed by McElroy (2017) implies common seasonality in a bivariate context.

Table 1: Restrictions imposed in the estimated models below

	Uncorrelated	Common Components			Perfectly Corr. Shocks		Same Shock	
	Components	Trend	Cycle	Seasonality	Trend	Cycle	Trend	Cycle
Within component correlations		$\rho_{\eta_1\eta_2}=1$	$\rho_{\varepsilon_1\varepsilon_2}=1$	$\rho_{\omega_1\omega_2}=1$	$\rho_{\eta_1\eta_2}=1$	$\rho_{\varepsilon_1\varepsilon_2}=1$	$\rho_{\eta_1\eta_2}=1$	$\rho_{\varepsilon_1\varepsilon_2}=1$
Trend-cycle correlations	$\rho_{\eta_2\varepsilon_1}=\rho_{\eta_1\varepsilon_1}=0$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_1\varepsilon_2}=0$	$\rho_{\eta_2\varepsilon_1}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_1\varepsilon_2}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_2\varepsilon_1}$		$\rho_{\eta_2\varepsilon_1}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_1\varepsilon_2}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_2\varepsilon_1}$	$\rho_{\eta_2\varepsilon_1}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_1\varepsilon_2}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_2\varepsilon_1}$
Trend-seasonal correlations	$\rho_{\eta_2\omega_1}=\rho_{\eta_1\omega_1}=0$ $\rho_{\eta_2\omega_2}=\rho_{\eta_1\omega_2}=0$	$\rho_{\eta_2\omega_1}=\rho_{\eta_1\omega_1}$ $\rho_{\eta_2\omega_2}=\rho_{\eta_1\omega_2}$		$\rho_{\eta_1\omega_2}=\rho_{\eta_1\omega_1}$ $\rho_{\eta_2\omega_2}=\rho_{\eta_2\omega_1}$	$\rho_{\eta_2\omega_1}=\rho_{\eta_1\omega_1}$ $\rho_{\eta_2\omega_2}=\rho_{\eta_1\omega_2}$		$\rho_{\eta_2\omega_1}=\rho_{\eta_1\omega_1}$ $\rho_{\eta_2\omega_2}=\rho_{\eta_1\omega_2}$	
Cycle-seasonal correlations	$\rho_{\varepsilon_2\omega_1}=\rho_{\varepsilon_1\omega_1}=0$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_2}=0$		$\rho_{\varepsilon_2\omega_1}=\rho_{\varepsilon_1\omega_1}$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_2}$	$\rho_{\varepsilon_1\omega_2}=\rho_{\varepsilon_1\omega_1}$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_1}$		$\rho_{\varepsilon_2\omega_1}=\rho_{\varepsilon_1\omega_1}$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_2}$		$\rho_{\varepsilon_2\omega_1}=\rho_{\varepsilon_1\omega_1}$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_2}$
Other restrictions		$\frac{\beta_2}{\beta_1}=\frac{\sigma_{\eta_2}}{\sigma_{\eta_1}}$	$\phi_{21}=\phi_{11}$ $\phi_{22}=\phi_{12}$				$\sigma_{\eta_2}=\sigma_{\eta_1}$	$\sigma_{\varepsilon_2}=\sigma_{\varepsilon_1}$
Number of restrictions	12	6	7	5	5	5	6	6

Table 1 sets out the restrictions implied for each case considered, including the model where the only non-zero correlations are within components, which is the conventional bivariate UC model. The number of restrictions imposed by each specification is noted, with each substantially exceeding the minimum of two required for identification. Although a same seasonal shock imposing $a = 1$ in (15) could also be considered, this was not relevant for our empirical analysis, as discussed in the next section.

3 Gender employment in Australia

In their recent analysis, Birch and Preston (2020) highlight a number of gender-specific aspects of the Australian labour market. They note, in particular, that the female labour force participation rate reached an all-time high of 61.3% in 2019, only 10 percentage points below the male rate. In line with this, the current century has seen the participation rate for females increase more sharply than for males across all groups aged at 25 or more (Birch and Preston, 2020, Figure 1, p.346). These authors (see especially Table 3, p.349) also note important differences across industries in the gender composition of employees and in the proportion who work part-time.

Although it does not rule out cointegration, these findings suggest that the number of female and male employees in Australia have followed different (deterministic) trends over recent decades. Further, since more than twice as many females as males work part-time while many part-time workers in Australia are employed on a casual basis (Birch and Preston, 2020, pp.348-350), it is plausible that females employment may be more susceptible overall to cyclical and/or seasonal movements than that of males.

Recent studies relating to the US indicate that cyclical movements have gender-specific employment consequences, with both Hoynes, Miller and Schaller (2012) and Guisinger (2020) finding cyclical movements to be more marked for males than females. However, neither considers seasonal aspects of employment and, although Guisinger (2020) uses a correlated UC model as one of three decomposition techniques, the methods she applies are univariate.

In order to exploit communality across male and female employment, while not making the essentially arbitrary assumption that seasonal movements are uncorrelated with other time series characteristics, we apply a bivariate correlated UC model to examine the trend, cyclical and seasonal characteristics of male and female employment in Australia.

3.1 Preliminary analysis

Our data consists of the total number of employed persons (in thousands) by gender in Australia, provided by the Australian Bureau of Statistics[4]. We use quarterly data from 1986:Q3 to 2020:Q1, with the end-date avoiding issues arising from the Coronavirus pandemic. As usual, the original values are transformed by taking natural logarithms and, in order to more clearly show cyclical and seasonal movements, are multiplied by 100.

Figure 1 shows the key features of the series are the upward trends, with that for females steeper than for males[5], the downswings in both series during the early 1990s and the seasonality evident in quarter-to-quarter changes, especially for females.

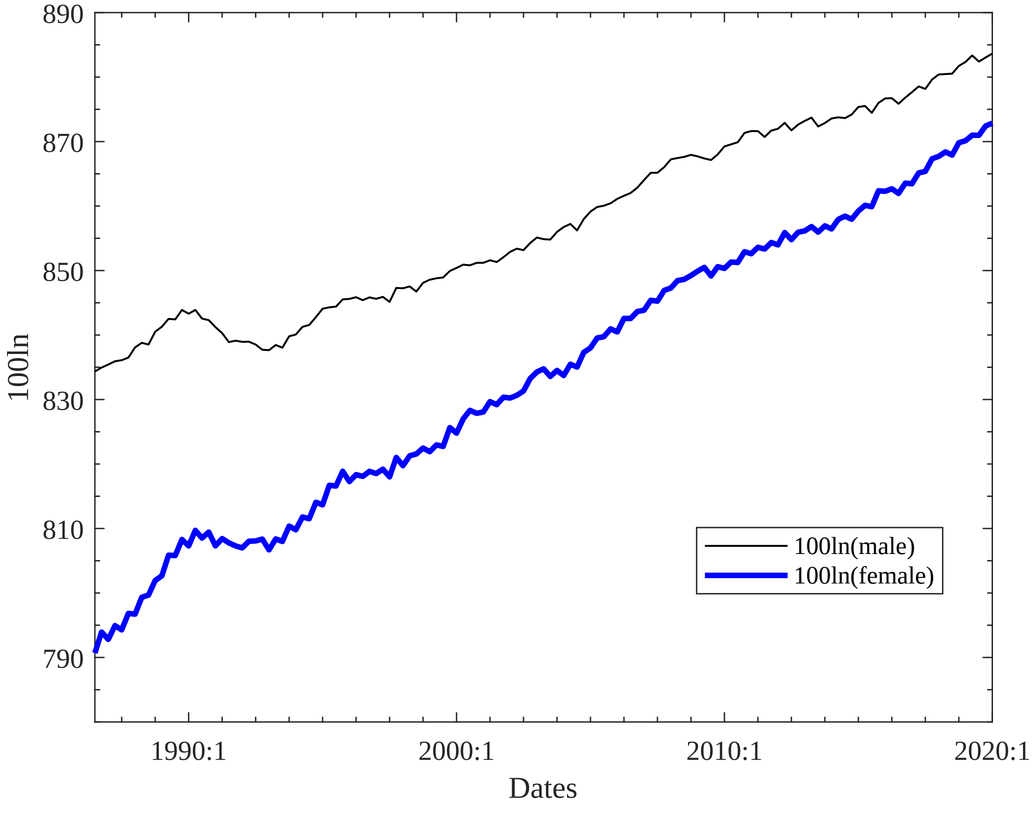

Figure 1: Male and female employment in Australia in natural logarithm times 100, from 1986:Q3 to 2020:Q1.

[4]The Australian labour force data can be downloaded at `https://www.abs.gov.au/statistics/labour/employment-and-unemployment/labour-force-australia-detailed-quarterly/feb-2020#data-download`

[5]The trends in Figure 1 suggest that female employment may be converging towards male employment. This possibility might be captured using the long-run convergence framework proposed by Dungey and Osborn (2020), but such an investigation is beyond the scope of the present paper.

As a preliminary to a bivariate model, Table 2 reports estimation results for two univariate UC models[6]. From HJOT we know that estimation of a correlated univariate UC trend-cycle-seasonal model requires the imposition of at least one covariance restriction. The first model for each series assumes zero correlation between trend and seasonal disturbances ($\rho_{\eta\omega} = 0$). Since estimation for both series delivers estimated trend-cycle correlations ($\rho_{\eta\varepsilon}$) very close to -1, this restriction is imposed in the second model estimated for each.

Table 2: Estimation results for univariate UC models for male and female employment in Australia 1986:Q3 to 2020:Q1

Parameter	Males		Females	
	$\rho_{\eta_1\omega_1} = 0$	$\rho_{\eta_1\varepsilon_1} = -1$	$\rho_{\eta_2\omega_2} = 0$	$\rho_{\eta_2\varepsilon_2} = -1$
Males:				
σ_{η_1}	1.043 (0.334)	1.068 (0.921)		
σ_{ε_1}	1.124 (0.397)	0.785 (1.072)		
σ_{ω_1}	0.019(0.010)	0.018 (0.010)		
$\rho_{\eta_1\varepsilon_1}$	-0.992 (0.014)	-1 (-)		
$\rho_{\eta_1\omega_1}$	0 (-)	-0.993 (0.043)		
$\rho_{\varepsilon_1\omega_1}$	-0.125 (0.112)	0.993 (0.043)		
Females:				
σ_{η_2}			0.823 (0.054)	0.969 (0.405)
σ_{ε_2}			0.215 (0.096)	0.402 (0.577)
σ_{ω_2}			0.040 (0.012)	0.041 (0.013)
$\rho_{\eta_2\varepsilon_2}$			-1.000 (0.000)	-1 (-)
$\rho_{\eta_2\omega_2}$			0 (-)	0.157 (0.741)
$\rho_{\varepsilon_2\omega_2}$			0.000 (0.004)	-0.157 (0.741)
Others:				
β_1	0.359 (0.083)	0.340 (0.110)		
β_2			0.619 (0.017)	0620 (0.094)
ϕ_{11}	0.588 (0.167)	1.308 (0.250)		
ϕ_{12}	0.136 (0.052)	-0.489 (0.398)		
ϕ_{21}			1.865 (0.008)	1.388 (0.113)
ϕ_{22}			-0.868 (0.003)	-0.732 (0.303)
Log Lik.	-114.669	-113.693	-159.331	-161.404
AIC	245.337	243.386	334.662	338.808
BIC	268.582	266.628	357.905	362.050

Note: Standard errors are shown in parentheses.

[6]The maximum likelihood estimation results of all the UC models in this paper are obtained using MATLAB, version R2019b, with the Econometrics Toolbox™ state-space functionality for building the UC models in state-space forms. The elements in the covariance matrices $\mathbf{\Sigma}$ are computed via nonlinear transformation of the parameters from the state-space forms, and the delta method is used for computing the standard errors of the estimated variances and correlations for component shocks.

Although some estimates for both series appear quite sensitive to the covariance restriction imposed, it should be borne in mind that the identification conditions for these models are only just satisfied. Identification requires the AR(2) coefficient to be non-zero and it is reassuring that the estimated values are generally significant at conventional levels, although the t-ratio in the second specification for male employment is only around 1.2. The estimated drift coefficients in Table 2 point to the steeper overall trend increase already noted for female employment compared with males.

According to AIC and BIC, the perfectly correlated trend-cycle model is preferred for male employment while the uncorrelated trend-seasonal specification is preferred for female employment. However, the estimated trend component (see Figure 2) for female employment with $\rho_{\eta\omega} = 0$ is implausible, with a 'hump' in the early 1990s. On the other hand, imposing the restriction $\rho_{\eta\varepsilon} = -1$ for female employment yields an estimated trend that closely tracks the actual data and leaves small cyclical fluctuations. The estimated components for male employment are, however, very similar from the two models.

We now turn to a bivariate analysis with the aim of exploiting information in both series to obtain a clearer representation of the underlying components.

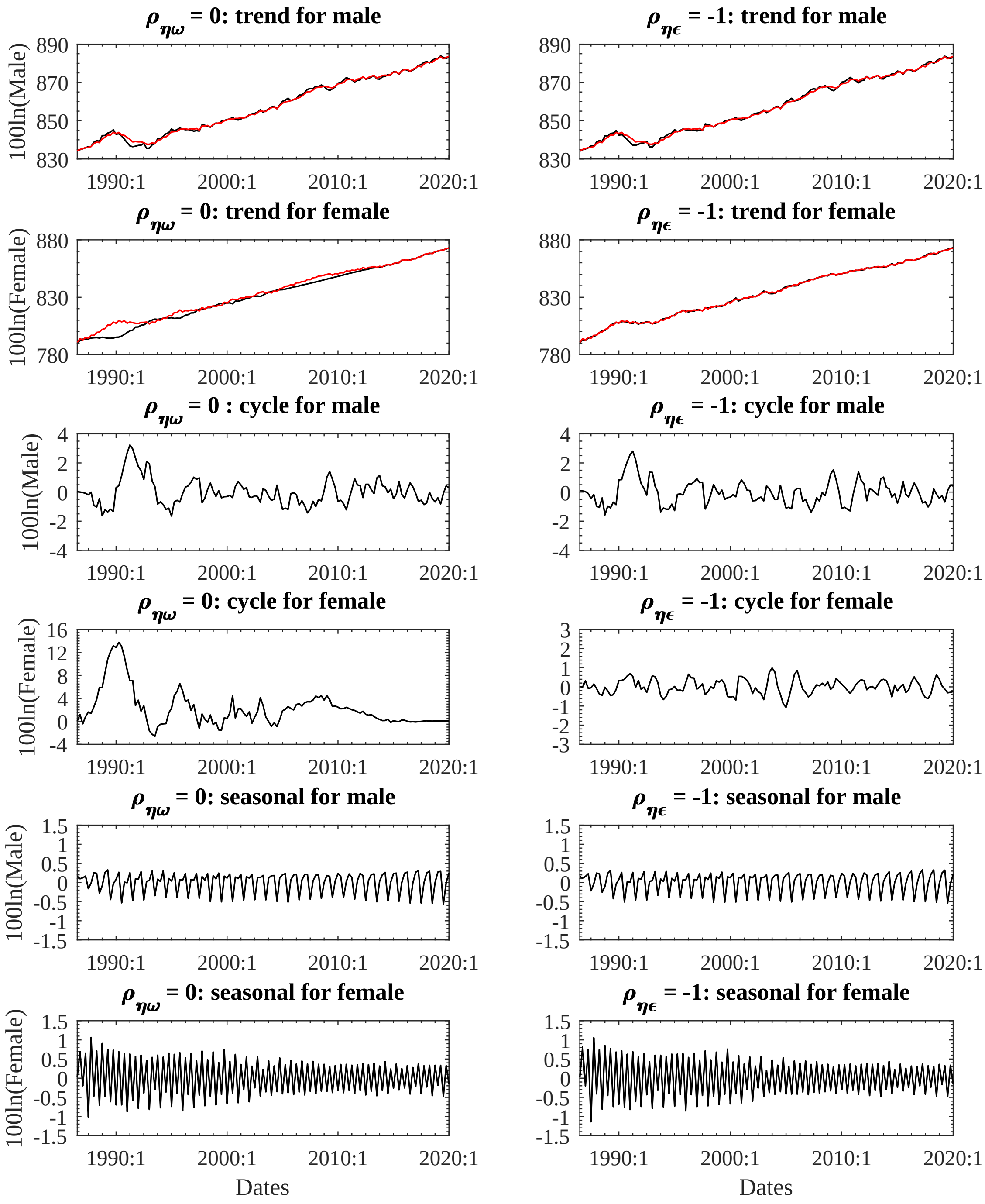

Figure 2: Estimated components of the univariate UC models for Australian male and female employments. The black lines are estimated components and the red lines are the employment data in natural logarithm times 100. The univariate model for the left column assumes zero correlation of trend and seasonal disturbances, i.e., $\rho_{\eta\omega} = 0$, and the univariate model for the right column restricts the correlation of trend and cycle disturbances to be -1, i.e., $\rho_{\eta\varepsilon} = -1$.

3.2 Bivariate analysis

Table 3 presents estimation results for bivariate seasonal UC models for employment by gender in Australia. Four specifications are included in the table, with the first being the standard uncorrelated model, which allows nonzero disturbance correlations across variables only within each component, so that all cross-component correlations are assumed to be zero. The other three models presented are the common trend, common cycle and common seasonal models which allow cross-component correlations to be nonzero but impose restrictions as discussed in subsection 2.4 and specified in Table 1. Results for the other specifications discussed there are presented in Appendix Table A.1.

Two models stand out in Table 3 in terms of the balance of goodness-of-fit and parameters estimated, namely in terms of information criteria values: these are the uncorrelated components and common cycle models, which are preferred by BIC and AIC respectively. Figure 3 compares the estimated components from these two models for the male and female employment series. Restricting the cross-component correlations to zero leads to the estimated trends for both series closely tracking the observed series, hence implying very small estimated cyclical variations. In contrast, the trend series extracted from the common cycle model are smooth and cyclical variation is evident. In particular, two downturns are detected during the 1990s, but with relatively little cyclical variation from early in the current century. These results for employment reflect the long period of growth experienced by the Australian economy since the 1990s[7]. Interestingly, and unlike results for the US (Hoynes, Miller and Schaller (2012), Guisinger (2020)), the estimate $\widehat{b} = 1.3$ for the common cycle model in Table 3 implies that cyclical variation in Australian employment is more marked for females than for men.

It is also notable that the bivariate common cycle model yields much smoother cyclical components than any of the univariate UC models in Figure 2, indicating the value of combining information available in the two series alongside a flexible covariance structure. Also note that some (albeit fairly subtle) differences can be seen in the estimated seasonal component for each series across the models of Figures 2 and 3.

[7] World Bank data (https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=AU) shows positive annual GDP growth for Australia in each year from 1991 to 2019.

Table 3: Estimation results for bivariate UC models with uncorrelated and common components for male and female employment in Australia from 1986:Q3 to 2020:Q1

Parameter	Uncorrelated	Common Trend	Common Cycle	Common Seasonal
Males:				
σ_{η_1}	0.544 (0.039)	0.631 (0.051)	0.489 (0.107)	0.741 (0.145)
σ_{ε_1}	0.025 (0.043)	0.296 (0.034)	0.041 (0.040)	0.268 (0.322)
σ_{ω_1}	0.015 (0.009)	0.020 (0.014)	0.011 (0.017)	0.016 (0.004)
$\rho_{\eta_1\varepsilon_1}$	0 (-)	-0.813 (0.058)	-0.170 (4.300)	-0.895 (0.173)
$\rho_{\eta_1\omega_1}$	0 (-)	-0.896 (0.245)	-0.719 (0.560)	-0.271 (0.821)
$\rho_{\varepsilon_1\omega_1}$	0 (-)	0.963 (0.189)	-0.399 (4.034)	0.599 (0.428)
Females:				
σ_{η_2}	0.635 (0.041)	$d \times \sigma_{\eta_1}$	0.681 (0.056)	1.459 (0.519)
σ_{ε_2}	0.138 (0.043)	0.831 (0.062)	$b \times \sigma_{\varepsilon_1}$	1.125 (0.508)
σ_{ω_2}	0.038 (0.011)	0.039 (0.011)	0.042 (0.025)	$a \times \sigma_{\omega_1}$
$\rho_{\eta_2\varepsilon_2}$	0 (-)	$\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_2\varepsilon_1}$	-0.917 (0.103)
$\rho_{\eta_2\omega_2}$	0 (-)	$\rho_{\eta_1\omega_2}$	0.519 (0.623)	$\rho_{\eta_2\omega_1}$
$\rho_{\varepsilon_2\omega_2}$	0 (-)	0.404 (1.413)	$\rho_{\varepsilon_1\omega_2}$	$\rho_{\varepsilon_2\omega_1}$
Cross-Series:				
$\rho_{\eta_1\eta_2}$	0.678 (0.056)	1 (-)	0.461 (0.207)	-0.478 (0.160)
$\rho_{\eta_1\varepsilon_2}$	0 (-)	-0.505(0.067)	$\rho_{\eta_1\varepsilon_1}$	0.785 (0.166)
$\rho_{\eta_1\omega_2}$	0 (-)	-0.556 (0.753)	-0.239 (0.865)	$\rho_{\eta_1\omega_1}$
$\rho_{\eta_2\varepsilon_1}$	0 (-)	$\rho_{\eta_1\varepsilon_1}$	-0.953 (1.256)	0.507 (0.280)
$\rho_{\eta_2\omega_1}$	0 (-)	$\rho_{\eta_1\omega_1}$	0.138 (1.121)	-0.191 (0.301)
$\rho_{\varepsilon_1\varepsilon_2}$	-0.995 (0.031)	0.911 (0.043)	1 (-)	-0.731 (0.249)
$\rho_{\varepsilon_1\omega_2}$	0 (-)	0.562 (1.122)	-0.658 (1.407)	$\rho_{\varepsilon_1\omega_1}$
$\rho_{\varepsilon_2\omega_1}$	0 (-)	0.795 (0.356)	$\rho_{\varepsilon_1\omega_1}$	0.067 (0.268)
$\rho_{\omega_1\omega_2}$	1.000 (0.001)	0.685 (0.244)	0.835 (0.445)	1 (-)
Others:				
β_1	0.366 (0.046)	0.386 (0.051)	0.345 (0.043)	0.386 (0.053)
β_2	0.605 (0.054)	$d \times \beta_1$	0.594 (0.061)	0.782 (0.142)
b			1.296 (1.134)	
a				2.269 (0.489)
d		0.948 (0.002)		
ϕ_{11}	0.181 (0.169)	1.655 (0.008)	1.872 (0.022)	0.037 (0.306)
ϕ_{12}	0.733 (0.154)	-0.713 (0.011)	-0.943 (0.029)	0.508 (0.171)
ϕ_{21}	-0.879 (0.313)	0.952 (0.012)		1.359 (0.013)
ϕ_{22}	-0.503 (0.312)	0.057 (0.012)		-0.354 (0.000)
Log Lik.	-257.512	-251.697	-248.193	-270.395
AIC	545.024	545.393	536.385	584.785
BIC	588.603	606.404	594.491	648.701

Note: The standard errors are included in parentheses.

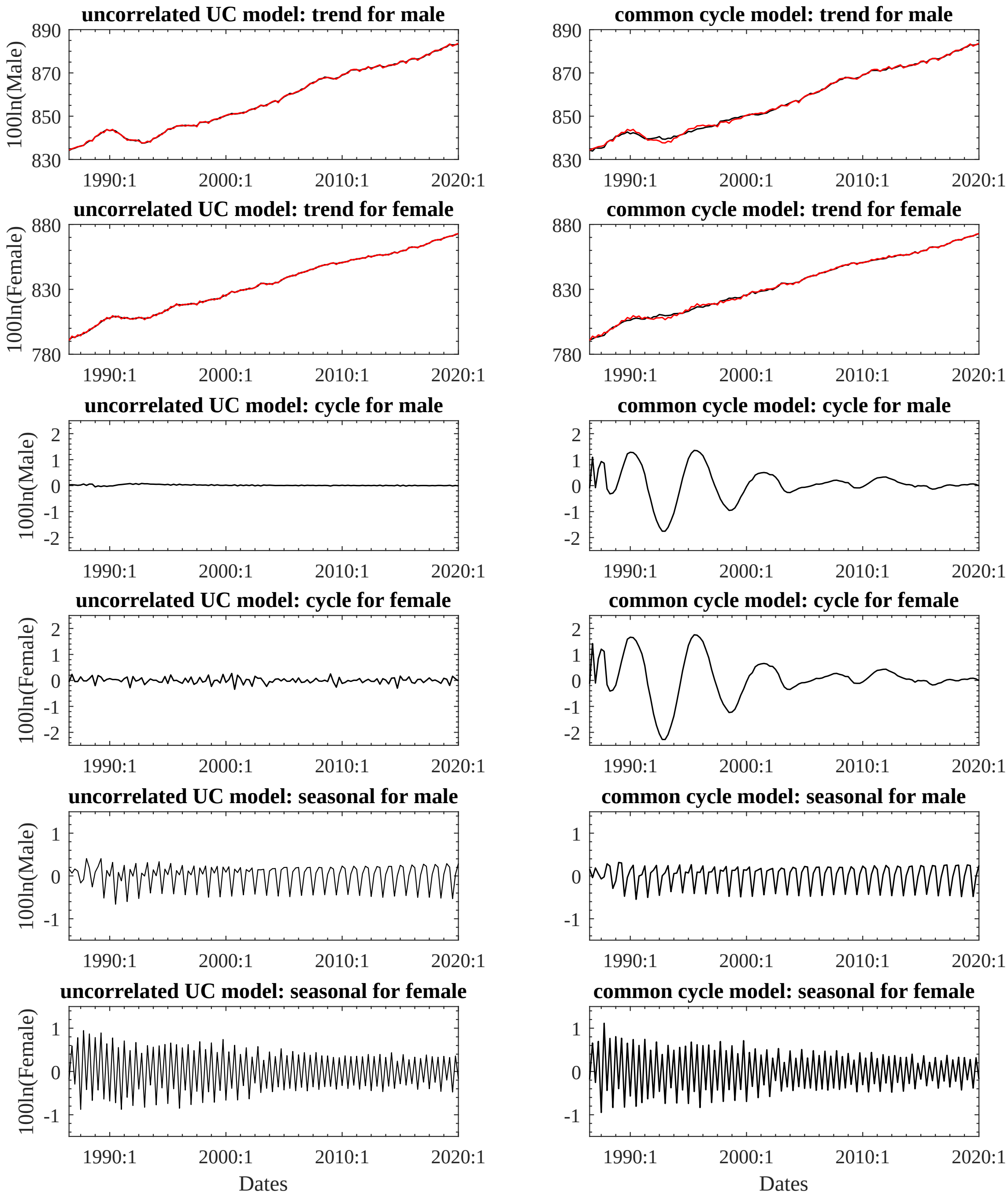

Figure 3: Comparison of the estimated trend, cycle and seasonal components for an uncorrelated UC model and a common cycle model for Australian male and female employment. The black lines are estimated components and the red lines are employment values in natural logarithm times 100.

Returning to the bivariate UC model estimates of Table 3, it is useful to discuss each model in turn. We have already commented above that male and female employment in Australia have exhibited different deterministic trends over time, and hence it is unsurprising that the common trend model is not a preferred specification. While the less restricted version of the perfectly correlated trend shock model in Appendix Table A.1 yields improved values for the information criteria, the common cycle model is still preferred to this specification and also to the same trend shock model. In the light of the estimated seasonal patterns for the two series across a range of specifications, it is also unsurprising that the common seasonal model leads to relatively poor information criteria values. Further, note that the common cycle model is preferred to the perfectly correlated cycle shock and the same cycle shock models (Table A.1).

Figure 4 zooms in on the estimated seasonal components for male and female employment from the bivariate UC model with a common cycle. The evolution of the seasonal patterns over time is interesting, with the extent of seasonality declining over time for females and (if anything) increasing for males. By the end of the sample period, the two seasonals are much more similar than they are in the early period. Nevertheless some gender differences remain even in the latter part of the sample.

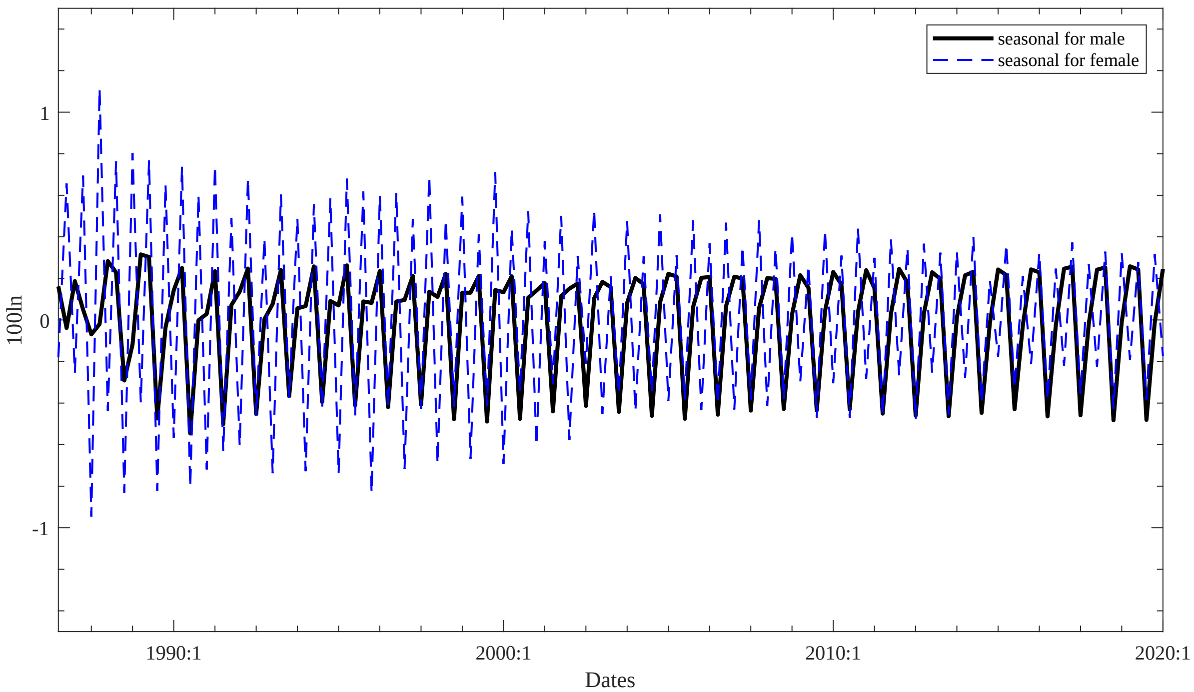

Figure 4: Estimated seasonal components for male and female employment using the bivariate UC model with a common cycle component.

The female pattern is of alternatively decline (in first and third quarters) and increase (second and fourth quarters). Although the pattern effectively repeats each two quarters, it is a little more marked in the second half of the year. The basic pattern for females effectively remains the same over time, but it has reduced substantially in magnitude since the 1990s. On the other hand, male employment follows an annual pattern, with positive seasonal contributions in the first and second quarters, followed by a third quarter decline and little seasonal contribution in the fourth quarter. While the relative magnitudes of the first and second quarter effects for male employment change a little over time, the third quarter effect is relatively constant from around 1990. It is interesting that the second and third quarter seasonal patterns are similar for males and females, including in magnitude, while those of the fourth and first quarters (the summer months in Australia) are quite different. This presumably reflects the different sectors and occupations in which males and females tend to be employed. Although further analysis to understand these differences is beyond the scope of the present paper, we note that both the Health Care and Social Assistance and the Education and Training sectors are female-dominated, with at least 70 percent of employees being female; indeed, gender segregation by industry and occupation has remained persistent in Australia over at least the last two decades[8].

In summary, the common cycle model produces plausible outcomes for trend, cycle and seasonal components for males and females and gives the lowest AIC value of all the bivariate models considered. The results suggest that males and females do not have the same seasonality. Explanations might be the sectors in which females work or that they prefer part-time to full-time.

3.3 Bivariate analysis for seasonally adjusted employment series

Here we investigate whether our finding above regarding the common cycle producing plausible outcomes for male and female employment also holds for seasonally adjusted data.

Economic time series are typically seasonally adjusted before being used in economic, econometric and policy analyses, where seasonality is defined as systematic, although not necessarily regular or unchanging, intrayear movement that is caused by climatic changes, timing of religious festivals, business practices, and expectations (Hylleberg 1986). Seasonal adjustment (SA) consists of the estimation of the seasonal component and, when applicable, also trading day and moving holiday effects, followed by their removal from the time series. The goal is usually to

[8]Further information can be found in the report "Gender segregation in Australia's workforce", published by the Australian Government's Workplace Gender Equality Agency, April 2019, available at https://www.wgea.gov.au/publications/gender-segregation-in-australias-workforce.

produce series whose movements are easier to analyze over consecutive time intervals and to compare to the movements of other series in order to detect co-movements (U.S. Census Bureau Basic Seasonal Adjustment Glossary; Wright 2013).

Several SA methods exist, but we use the industry standard Census X13ARIMA-SEATS (henceforth X13) here: the combination of X12-ARIMA and TRAMO-Seats (Department of Commerce Census Bureau `http://www.census.gov/srd/www/x13as/`).

An observed time series y_t can be decomposed into a trend-cycle y_t^{tc}, seasonal y_t^s, irregular y_t^i component, and deterministic effects due to the number of trading days y_t^{td}, and holidays y_t^h, such as Easter and Christmas (Ghysels and Osborn 2001, Section 4.2). Since our data are in logs, we apply the additive version of the decomposition. Ignoring the irregular and deterministic effects for convenience, this implies

$$y_t = y_t^{tc} + y_t^s, \qquad t = 1, \ldots, T. \tag{17}$$

The SA values, therefore, estimate the component y_t^{tc}. However, as noted in the Introduction, the X13ARIMA-SEATS process assumes the components are orthogonal[9].

Results in Table 4 and Figure 5 show that the common cycle model is also preferred for SA data over the correlated model that allows for a general covariance matrix. In particular, the common cycle model produces plausible outcomes for trend and cycle components for males and females, as seen in the right-hand panel of the figure. Indeed, these components are very similar to those obtained using data without seasonal adjustment.

Although the correlated model yields a marginally lower AIC value than the common cycle model, it produces an implausible estimated trend for male employment; see the middle panel of Figure 5. In particular, the trend lies above the observed values for most of the sample period, with the estimated cyclical component being negative. It can also be noted that the Figures in the Appendix give other examples of implausible estimated components, which may result from the imposition of inappropriate restrictions.

The two bottom left panels of Figure 5 show the seasonal patterns as detected by X13 in male and female employment. As found in our seasonal bivariate model above, males and females do not have the same seasonality. However, the seasonals obtained by X13 appear markedly different from those observed in Figures 2 and 3.

[9]We applied the additive procedure of the X13 procedure implemented in Eviews 9.

Table 4: Estimation results for univarate and bivariate UC models for seasonally adjusted male and female employment in Australia from 1986:Q3 to 2020:Q1

Parameter	Univariate model	Correlated bivariate model	Common cycle model
Males:			
σ_{η_1}	0.476 (0.035)	0.496 (0.097)	0.454 (0.110)
σ_{ε_1}	0.033 (0.021)	0.243 (0.032)	0.043 (0.048)
$\rho_{\eta_1\varepsilon_1}$	-1.00 (0.023)	-0.700 (0.089)	-0.136 (4.549)
Females:			
σ_{η_2}	0.693 (0.084)	1.227 (0.465)	0.675 (0.049)
σ_{ε_2}	0.025 (0.105)	0.835 (0.680)	$b \times \sigma_{\varepsilon_1}$
$\rho_{\eta_2\varepsilon_2}$	-1.00 (0.000)	-0.911 (0.035)	$\rho_{\eta_2\varepsilon_1}$
Cross-Series:			
$\rho_{\eta_1\eta_2}$		-0.227 (0.597)	0.430 (0.201)
$\rho_{\eta_1\varepsilon_2}$		0.597 (0.494)	$\rho_{\eta_1\varepsilon_1}$
$\rho_{\eta_2\varepsilon_1}$		0.854 (0.357)	-0.952 (1.331)
$\rho_{\varepsilon_1\varepsilon_2}$		-0.987 (0.092)	1 (-)
Others:			
β_1	0.346 (0.043)	0.382 (0.130)	0.344 (0.042)
β_2	0.605 (0.060)	0.691 (0.294)	0.592 (0.061)
b			1.303 (1.102)
ϕ_{11}	1.857 (0.004)	1.866 (0.030)	1.872 (0.023)
ϕ_{12}	-0.934 (0.004)	-0.868 (0.014)	-0.942 (0.033)
ϕ_{21}	1.865 (0.018)	1.283 (0.249)	
ϕ_{22}	-0.944 (0.002)	-0.403 (0.237)	
Log Lik.		-212.594	-217.605
AIC		457.187	457.210
BIC		503.672	489.168

Notes: The standard errors are included in parentheses.

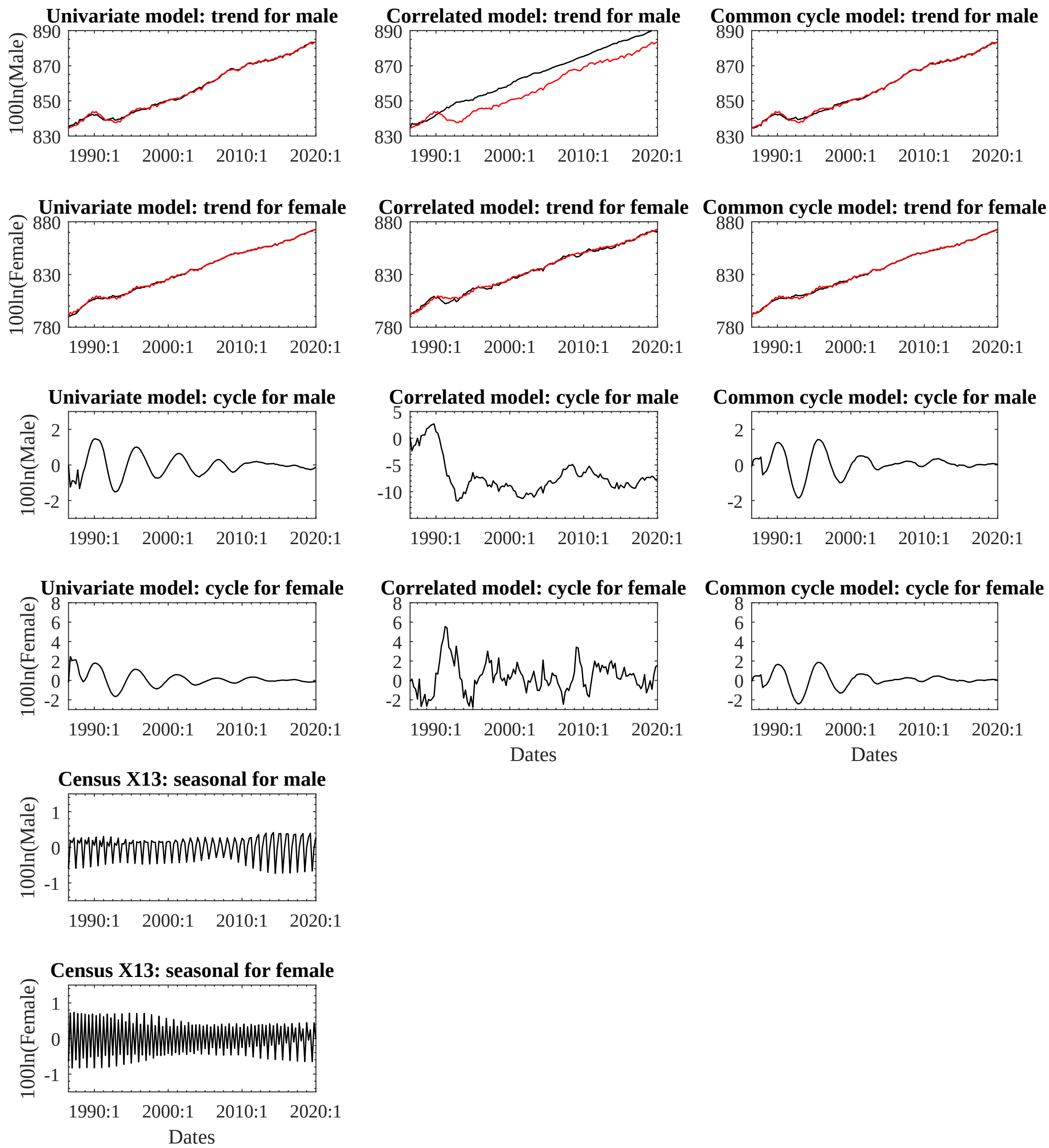

Figure 5: Estimated components by first using the Census X13 for seasonal adjustment and then decomposing trend and cycle based on the seasonally adjusted series. The black lines are estimated components and the red lines are employment values in natural logarithm times 100. All of the UC models allow correlation between trend and cycle disturbances to be estimated. The correlated model and common cycle model refer to bivariate UC models, with the former assuming different trend and cycle processes but the latter assuming a common cycle process shared between male and female employment.

4 Conclusion

Multivariate analysis of economic time series can throw important light on underlying economic phenomena, including trend, cyclical and seasonal movements. In order to analyze such movements when they are potentially correlated, a correlated multivariate unobserved components model is required. Although previously considered in a univariate context, to the best of our knowledge the present paper is the first to study identification conditions for a multivariate trend-cycle-seasonal model with correlated shocks. Although restrictions are required to deliver identification, we believe that forms of cross-equation restrictions that we study (including common trends, common cycles and common seasonality) are intuitive and allow the approach to be applied in a variety of real-world situations.

The model is applied to study seasonal aggregate employment by gender in Australia. Specifically, quarterly data is used to estimate a bivariate male/female model of employment. Although a formal comparison is undertaken with a range of specifications, including ones with common trend or common seasonality and the uncorrelated component model, the common cycle specification is preferred. Indeed, graphical and univariate analyses also point to a common cycle as the most plausible form of restriction to be imposed, with evidence of distinct gender-based trend and seasonal patterns. Although the use of seasonally adjusted data also supports the common cycle model, the seasonality implied by routine adjustment differs from that of our preferred bivariate trend-cycle-seasonal specification.

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A Additional estimation results

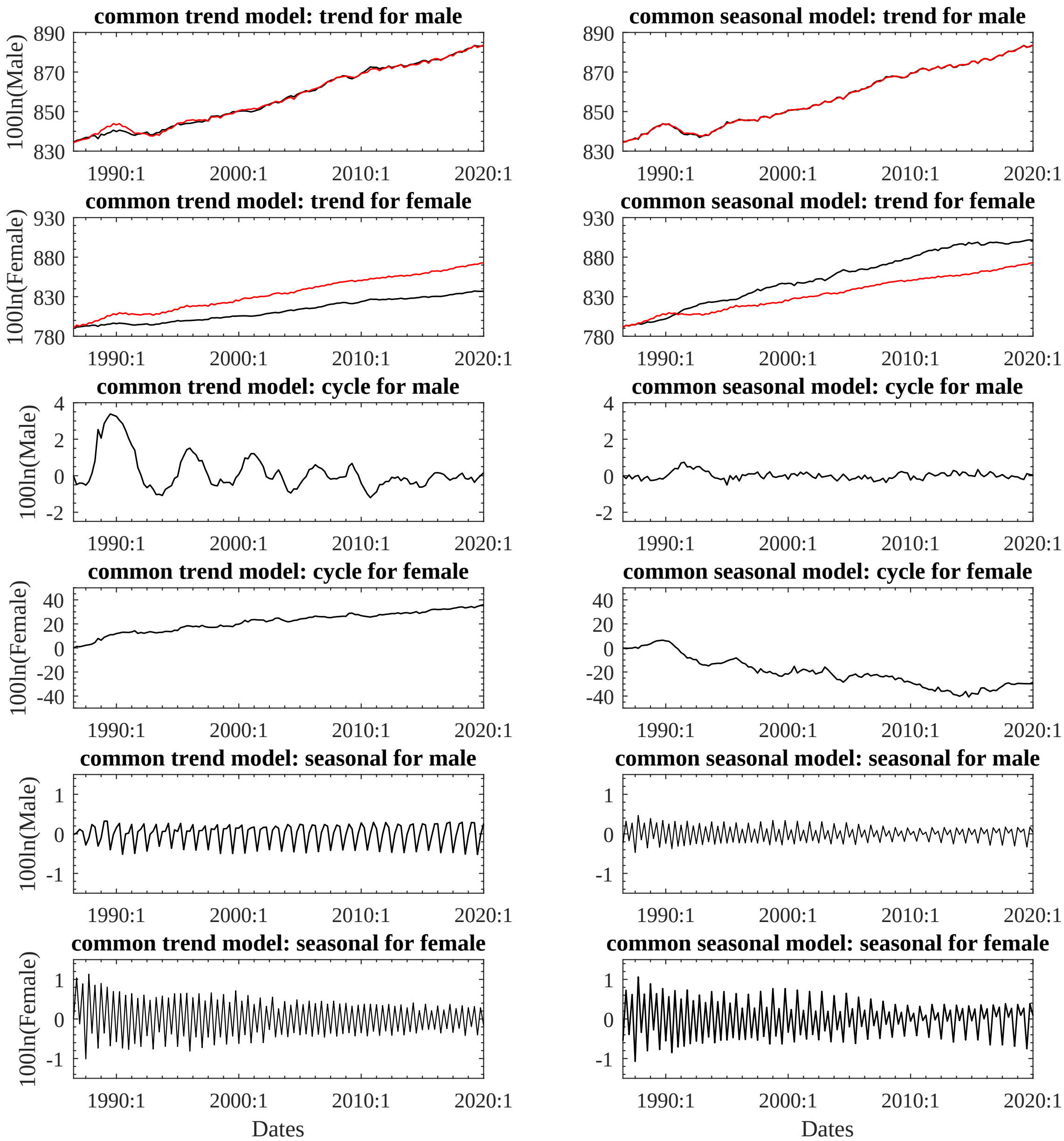

Figure A.1: A comparison of estimated trend, cycle and seasonal components for a common trend model and a common seasonal component model. The Black lines are estimated components and the red lines are employment values in natural logarithm times 100.

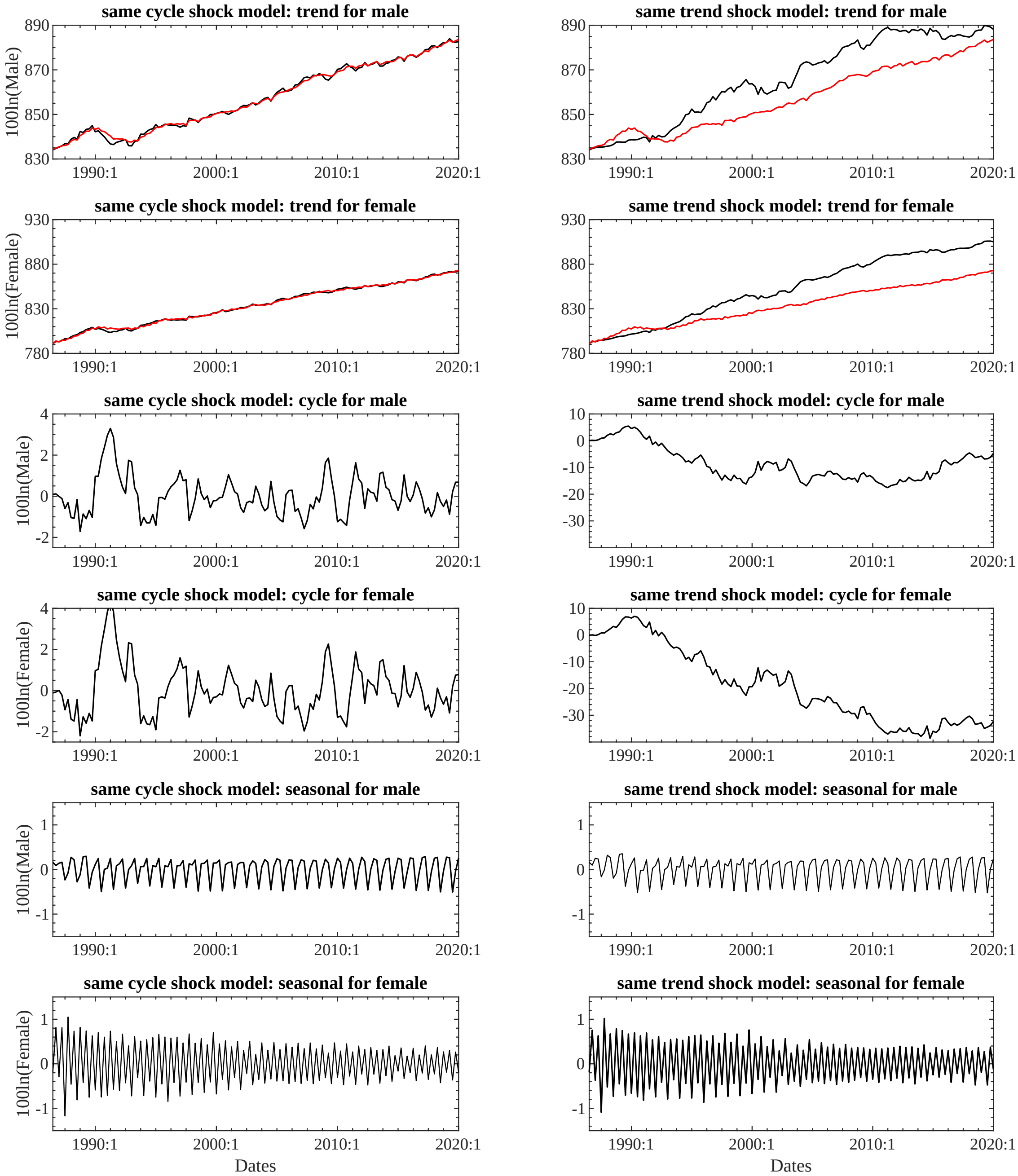

Figure A.2: A comparison of estimated trend, cycle and seasonal components for the model with the same cycle shock and the model with the same trend shock. The Black lines are estimated components and the red lines are employment values in natural logarithm times 100.

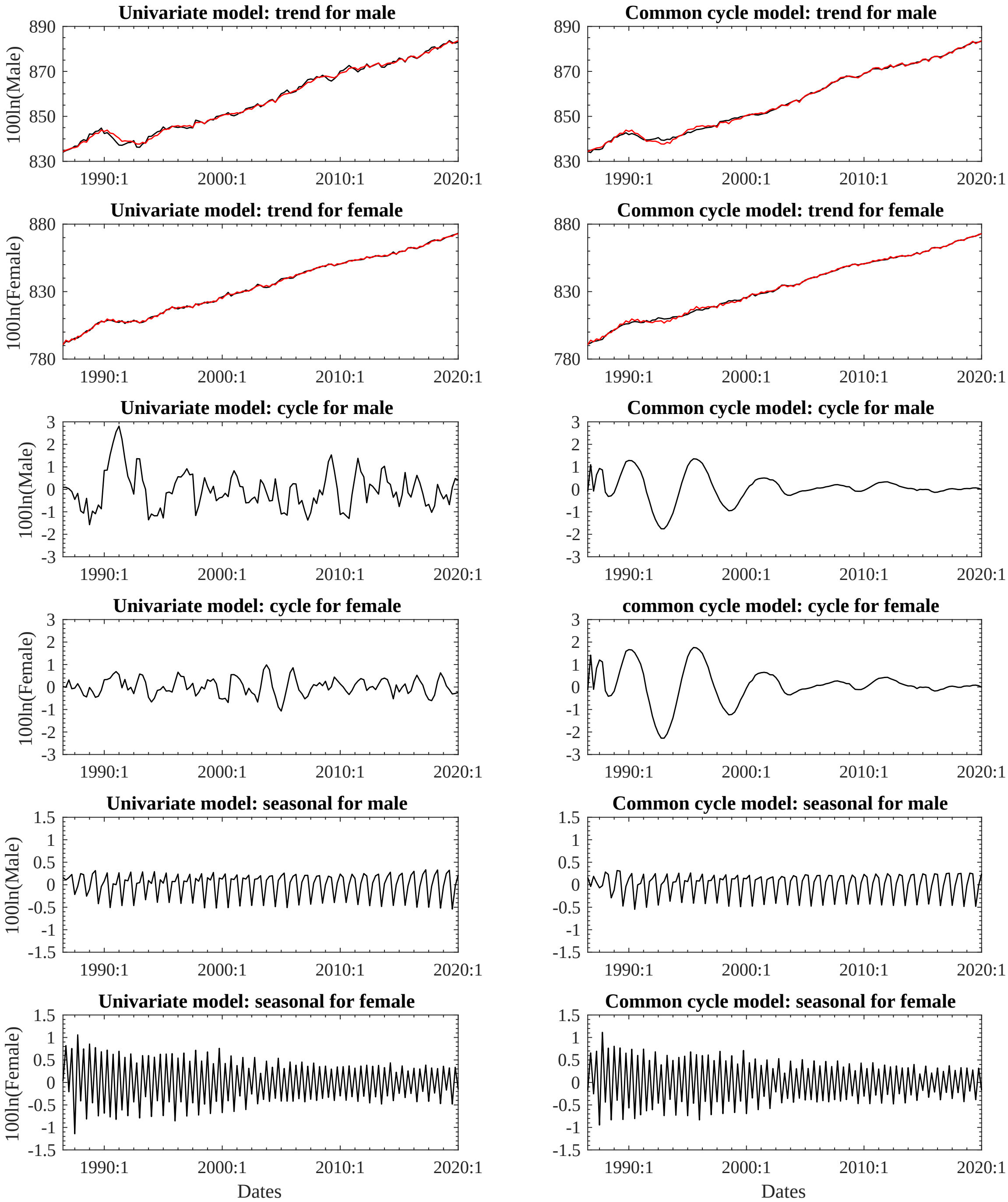

Figure A.3: A comparison of estimated trend, cycle and seasonal components between the univariate models and a bivariate model, in which male and female employment share a common cycle component. The Black lines are estimated components and the red lines are employment values in natural logarithm times 100. In the univariate model the correlation between trend and cycle shocks is assumed equal to -1.

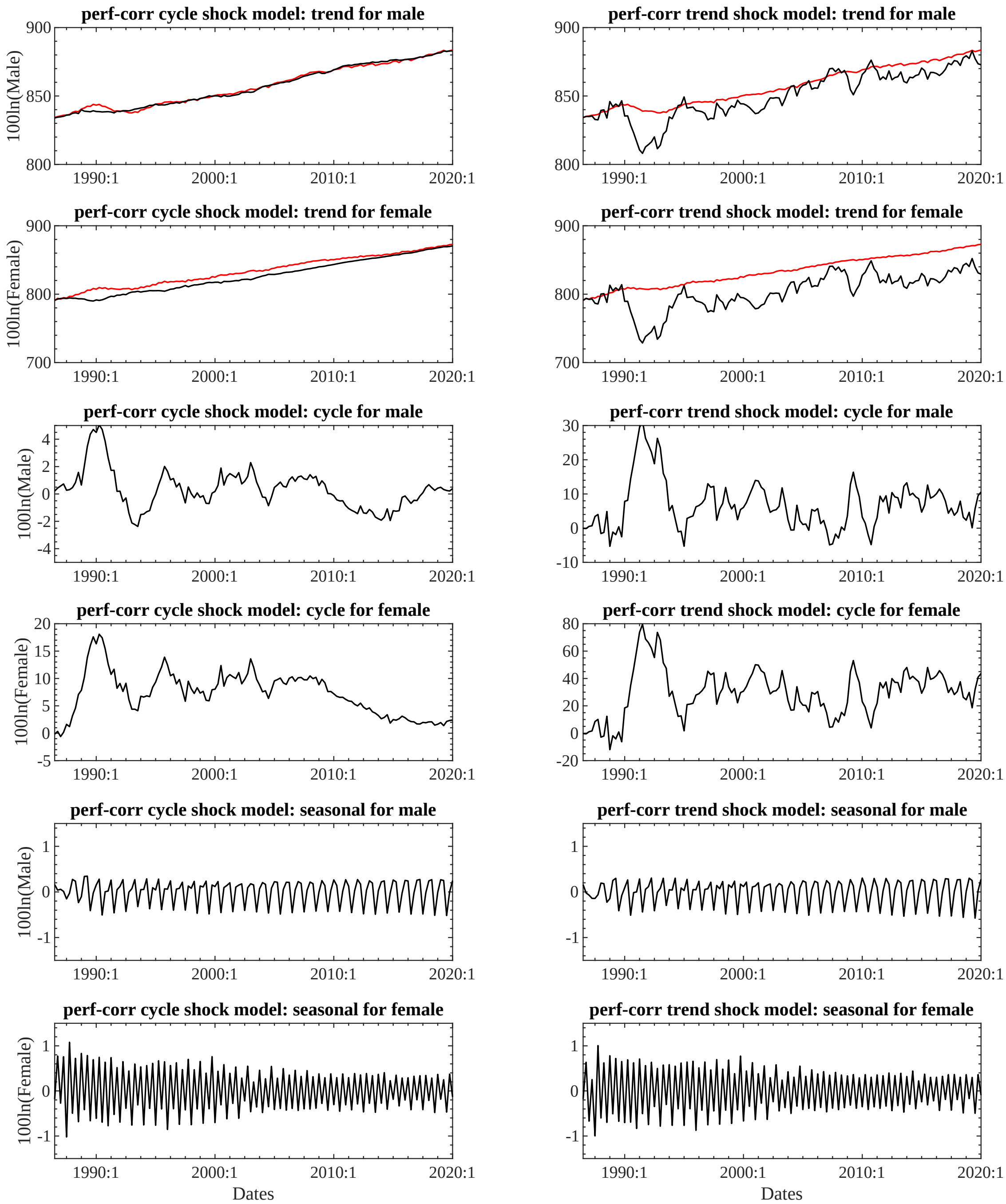

Figure A.4: A comparison of estimated trend, cycle and seasonal components between the bivariate model with perfectly correlated cycle shocks and the bivariate model with perfectly correlated trend shocks. The Black lines are estimated components and the red lines are employment values in natural logarithm times 100.

Table A.1: Estimation results of four other bivariate UC models for male and female employment in Australia from 1986:Q3 to 2020:Q1

Parameter	Same cycle shock	Same trend shock	Perf-corr cycle shock	Perf-corr trend shock
Males:				
σ_{η_1}	1.144 (0.323)	1.418 (0.409)	0.478 (0.076)	4.291 (0.121)
σ_{ε_1}	0.877 (0.362)	1.589 (0.272)	0.171 (0.207)	4.157 (0.125)
σ_{ω_1}	0.016 (0.013)	0.018 (0.013)	0.017 (0.227)	0.022 (0.012)
$\rho_{\eta_1\varepsilon_1}$	-0.992 (0.008)	-0.942 (0.018)	-0.155 (1.311)	-0.994 (0.001)
$\rho_{\eta_1\omega_1}$	-0.807 (0.672)	0.517 (0.641)	-0.468 (2.335)	-0.913 (0.242)
$\rho_{\varepsilon_1\omega_1}$	0.744 (0.754)	-0.701 (0.615)	-0.650 (0.535)	0.879 (0.278)
Females:				
σ_{η_2}	1.207 (0.309)	σ_{η_1}	1.045 (1.050)	$d \times \sigma_{\eta_1}$
σ_{ε_2}	σ_{ε_1}	1.045 (0.332)	$b \times \sigma_{\varepsilon_1}$	9.536 (0.048)
σ_{ω_2}	0.041 (0.013)	0.041 (0.015)	0.043 (0.043)	0.043 (0.014)
$\rho_{\eta_2\varepsilon_2}$	$\rho_{\eta_2\varepsilon_1}$	$\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_2\varepsilon_1}$	$\rho_{\eta_1\varepsilon_2}$
$\rho_{\eta_2\omega_2}$	0.030 (0.533)	$\rho_{\eta_1\omega_2}$	0.510 (1.286)	$\rho_{\eta_1\varepsilon_2}$
$\rho_{\varepsilon_2\omega_2}$	$\rho_{\varepsilon_1\omega_2}$	-0.041 (0.962)	$\rho_{\varepsilon_1\omega_2}$	0.435 (0.660)
Cross-Series:				
$\rho_{\eta_1\eta_2}$	0.866 (0.074)	1 (-)	0.385 (0.199)	1 (-)
$\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_1\varepsilon_1}$	-0.919 (0.086)	$\rho_{\eta_1\varepsilon_1}$	-0.999 (0.000)
$\rho_{\eta_1\omega_2}$	-0.464 (0.474)	0.053 (0.913)	0.072 (1.465)	-0.455 (0.649)
$\rho_{\eta_2\varepsilon_1}$	-0.912 (0.063)	$\rho_{\eta_1\varepsilon_1}$	-0.971 (0.281)	$\rho_{\eta_1\varepsilon_1}$
$\rho_{\eta_2\omega_1}$	-0.497 (0.898)	$\rho_{\eta_1\omega_1}$	0.495 (1.270)	$\rho_{\eta_1\omega_1}$
$\rho_{\varepsilon_1\varepsilon_2}$	1 (-)	0.998 (0.013)	1 (-)	0.997 (0.001)
$\rho_{\varepsilon_1\omega_2}$	0.353 (0.526)	-0.041 (0.947)	-0.527 (1.762)	0.473 (0.652)
$\rho_{\varepsilon_2\omega_1}$	$\rho_{\varepsilon_1\omega_1}$	-0.728 (0.646)	$\rho_{\varepsilon_1\omega_1}$	0.894 (0.266)
$\rho_{\omega_1\omega_2}$	0.744 (0.286)	0.580 (0.434)	0.751 (0.611)	0.630 (0.355)
Others:				
β_1	0.378 (0.082)	0.289 (0.113)	0.370 (0.081)	0.693 (0.061)
β_2	0.620 (0.086)	0.740 (0.497)	0.668 (0.062)	1.155 (0.121)
b			3.783 (3.330)	
d				2.147 (0.063)
ϕ_{11}	1.254 (0.101)	1.024 (0.002)	1.784 (0.070)	0.978 (0.009)
ϕ_{12}	-0.434 (0.127)	-0.034 (0.019)	-0.804 (0.029)	-0.018 (0.011)
ϕ_{21}	1.313 (0.100)	1.326 (0.001)	1.642 (0.012)	0.961 (0.001)
ϕ_{22}	-0.462 (0.109)	-0.322 (0.013)	-0.648 (0.008)	0.021 (0.003)
Log Lik.	-251.868	-252.705	-247.139	-247.142
AIC	545.735	547.409	538.279	538.285
BIC	606.746	608.42	602.195	602.201

Notes: The standard errors are included in parentheses.