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A Suggested Framework for classifying the Modes of Cycle Research[1]

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1 Introduction

Conferences that we have recently attended have suggested to us that there is the potential for a great deal of confusion today in cycle research. This is caused by many different interpretations of what is meant by a "cycle" and how one goes about measuring it. In order to bring some order into this literature we feel that it is useful to provide some classification scheme which emphasizes this diversity. In doing so it is hoped that the scheme may foster a better understanding of the many modes of cycle research. In what follows we will be concerned with cycles in economic activity, but it is the case that the principles extend to cycles in any series. Nevertheless, it is useful to be able to concentrate upon a specific example that is probably the main object of most cycle research.

Our suggested framework involves distinguishing between the following issues:

I. What series is it that we are looking for cycles in?

II. How do we recognize and measure (describe) a cycle in the series once the previous decision has been made?

Section 2 of this paper develops the first of these issues in more detail while section 3 does the same for the second one. Section 4 then comments upon the implications of our classiciation scheme for cycle research.

2 The Underlying Series

For cycles in economic activity we need to first define that concept. Doing so will suggest an appropriate series to examine. Burns and Mitchell (1946) and the NBER are quite clear that their preferred definition involved Gross Domestic Product if one wished to investigate cycles at quarterly or annual frequencies.[1] For monthly cycles however the lack of GDP data generally means that a variety of monthly series must be combined in some way. For the moment we leave this technical issue of how to combine together a variety of series in order to measure economic activity and simply assume that it is

[1] See "The NBER's Recession Dating Procedure" at *http://www.nber.org/cycles/recessions.html*

quarterly economic activity, as represented by quarterly GDP, that we wish to investigate.[2]

Now we have only broadly answered the question of what series we wish to examine for a cycle. When one becomes more specific there are generally three responses in the literature.

A. The level of GDP (or a monotonic transformation such as the log). We will designate this as y_t and assume that it is the log of GDP.

B. The level of GDP less a permanent component P_t. We will designate this by $z_t = y_t - P_t$.

C. The growth rate in GDP. This could be either a quarterly, Δy_t , or annual, $\Delta_4 y_t$, growth rate.

The first quantity is chosen by the NBER and Burns and Mitchell and is sometimes referred to as the classical or business cycle. To operationalize the second one needs to take a stand on the form of the permanent component that is to be subtracted off. Many answers have been given to this, mostly under the guise of filtering operations e.g. the Hodrick-Prescott filter removes a permanent component from the series, leaving a transitory component z_t. But many other filters are in use e.g. Beveridge-Nelson(1991) , Band-Pass filter (Baxter and King (1999)) etc. Unfortunately, these filters often have associated with them the confusing terminology that the permanent component is a "trend" and the z_t is a "cycle". This arises from the term "stochastic trend" that is often used to describe an $I(1)$ series. To see why it is unhelpful in cycle research to call the permanent component a "trend", consider looking for a cycle in GDP if it followed a pure random walk $y_t = y_{t-1} + e_t$ .Then $P_t = y_t$, there is no transitory component and, adopting the terminology which equates transitory components and cycles, there can be no cycle. There are countries in the world like this i.e. which have no serial correlation in the growth rate of GDP. Yet we would rarely see the conclusion that there are no cycles in the economic activity of such countries. Consequently, if one removes a permanent component from a series, then the cycle in the series z_t is best given a special name, and generally it is referred to as a growth cycle.

[2]Of course we might be looking at cycles in a yearly measure of economic activity or even averages of activity over ten or twenty years. The latter measures would be the type of input needed to look at what were called Jugler and Krondratieff cycles.

It is worth observing here that quantities such as z_t are also often called "output gaps". The logic of this is that the permanent component is taken to represent supply side influences and the transitory components are demand side, so that an output gap attempts to separate out demand and supply in a simple fashion. There is no necessary connection with a cycle. There may be one in the output gap series but the latter itself is not a cycle.

Finally, the choice of activity variable in C means that we would be studying a cycle in growth rates as distinct from the growth cycle featured in B. However, this distinction is perhaps not a clear cut as it might first seem. One could always choose $P_t = y_{t-1}$ or $P_t = y_{t-4}$ in B and, with these choices of the permanent component, $z_t = \Delta y_t$ and $z_t = \Delta_4 y_t$ respectively as the transitory component.[3] Moreover, it is possible that z_t might actually be proportional to Δy_t if y_t followed processes that are empirically plausible and if the permanent component is measured by the Beveridge-Nelson approach. To see this suppose that the DGP for y_t is an $AR(1)$ in growth rates viz,

$$(\Delta y_t - \mu) = \rho (\Delta y_{t-1} - \mu) + \epsilon_t,$$

where $\epsilon_t \tilde{} i.i.d.N(0, \sigma^2)$ and $|\rho| < 1$. Then Morely (2002) showed that the exact Beveridge Nelson permanent component P_t^{BN} is given by

$$P_t^{BN} = y_t + \frac{\rho}{1-\rho} (\Delta y_t - \mu),$$

and so the transitory component will be

$$z_t^{BN} = -\frac{\rho}{1-\rho} (\Delta y_t - \mu).$$

Hence the cycle in z_t^{BN} will be the same as in Δy_t. Essentially the BN decomposition in this instance uses $y_{t-1} + \phi \Delta y_t$ as the permanent component, rather than y_{t-1}. Of course the quantity $y_{t-1} + \sum_{j=0}^{\infty} \phi_j \Delta y_{t-j}$ will also be a permanent component provided that the ϕ_j are square summable and this suggests that other ways of extracting the permanent component may actually have transitory components that are combinations of Δy_{t-j}..To get some feel for this possibility we know from King and Rebelo (1993) that the z_t which would arise from HP filtering y_t would in fact have the form

$$z_t^{HP} = \sum_{j=1}^{T} \omega_{jt} \Delta y_t.$$

[3] We thank Jan Jacobs for this observation.

Because the HP filtered is a two sided one with time varying weights we feel that it is useful to get some appreciation for what it does by the following experiment. First generate 60000 observations on a series for y_t, where the latter follows a pure random walk of the form $y_t = y_{t-1} + e_t$. Second, construct z_t^{HP} by applying the HP filter with $\lambda = 1600$. Finally regress this series upon $\Delta y_t, \Delta y_{t-1}, ...\Delta y_{t-4}$. Inspection of the estimates of the parameters atatched to the Δy_{t-j} the suggested that a good approximation would be $.47(.9)^j$. From tis result it seems that the value of z_t^{HP} can be expressed as a combination of current and lagged values of Δy_t with slowly declining weights. It is useful to contrast this outcome from the HP filter with the fact that $z_t^{BN} = \Delta y_t$ if y_t is a pure random walk, and this outcome makes it clear that the two ways of eliminating a permanent component will produce transitory components with very different serial correlation properties.

Becasue of the connection just described we will be able to shorten our discussion by treating C as a special case of B.

3 Three Ways of Describing the Cycle

Now all of the above simply defines what we are looking for a cycle in. The series y_t, z_t and $\Delta_j y_t$ are not cycles, even though sometimes people will refer to quantities such as z_t in this way. Although in some contexts it may be reasonable to use such loose language it becomes confusing to do so in many other instances.

There are three ways in the literature of describing what we mean by a cycle.

(i) Those looking for a *periodic cycle* focus upon statistical models of one of y_t, z_t and $\Delta_j y_t$. These statistical models generally involve cosine and sine waves in some fashion, although in the nonlinear literature one can have periodic "limit cycles".

(ii) Blinder and Fischer (1981, p 227) offered the definition of a cycle as "serially correlated deviations of output from trend" and

(iii) A different approach, and probably the most widespread given our tendency to present graphs of series when discussing cycles, is to recognize a cycle from the *turning points* in the series under investigation. These turning points are *peaks and troughs* and the periods between them are classified as expansions and contractions.

An early example of (i) is the "rocking horse cycle" in Frisch (1933), who asked whether the series under investigation had an AR(2) representation with complex roots.[4] But there are other representations that are now widely used which also have cosine and sine wave foundations e.g. the *local linear trend plus cycle* model in Harvey and Jaeger (1993) that is embodied in the STAMP program. The "rocking horse" view is also maintained in programs such as TREAMO-SEATS (see Kaiser and Maravall (2001)) and in books such as Arnold (2002). Underlying (i) therefore is a two- step strategy for extracting cycle information. First, fit (say) the cosine/sine model to whatever series in A-C is chosen for investigation. Second, ask the question whether there is evidence of a periodic cycle in the fitted model e.g. are the roots in the AR(2) complex? If accepted the period of the cycle is recorded. There can be problems with this strategy unless the representations are sufficiently general to allow for the possibility of no periodic cycles. Indeed, this is the case with Harvey's models which makes $z_t = \psi_t + \varepsilon_t$, where ε_t is $i.i.d.N(0, \sigma_\varepsilon^2)$ and ψ_t is governed by the following periodic process

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix} \tag{1}$$

where, $0 \leq \rho \leq 1$ to impose stationarity, λ is frequency in radians, κ_t and κ_t^* are mutually independent $i.i.d\ N(0, \sigma_\kappa^2)$ and $i.i.d\ N(0, \sigma_\kappa^2)$ respectively.Then ψ_t will be an ARMA(2,1) process with the AR(2) polynomial being $1 - 2\rho\cos\lambda\, L + \rho^2\cos^2\lambda\, L^2$ and inspection of this shows that its roots are always complex i.e. a periodic cycle is imposed upon the series, the only role of the data is to determine its period.

Blanchard and Fischer (1989) operationalized (ii) with the following refinement: "We then think of trends as that part of output which is due to permanent shocks....That part of output that comes from transitory shocks can be thought of as the cycle". Putting these together we might conclude that a cycle exists if there is serial correlation in the z_t, although they do not indicate how one would map the characteristics of any such cycle into the degree of correlation.

We will not spend much time on (ii) as it is too incomplete to be a useful definition of a cycle. Its fundamental weakness shows up when one recognizes how dependent it is upon the definition of the permanent component. Since there are many ways of constructing a permanent component each one will produce different degrees of serial correlation in z_t . Indeed, even the

[4]This view of the cycle is pushed in intermediate textbooks such as Dornbusch and Fischer ().

existence of a cycle may be dependent on this definition. To dramatize this, return to the simple example we had earlier where y_t followed a pure random walk, $y_t = y_{t-1} + e_t$, so that $P_t = y_t$. Now we can add any $I(0)$ series on to P_t and it will still be a permanent component. Suppose then that we write $y_t = P_t - \psi_t + \psi_t = P_t^* + z_t$, where $\psi_t = 0.777\psi_{t-1} - .0.066\psi_{t-1} + e_t$, $P_t^* = y_t - \psi_t$ and $z_t = \psi_t$. Now z_t is very close to what one would get as the residuals after a pure random walk was filtered with the Hodrick-Prescott filter on quarterly data using =1600 -see the formula in King and Rebelo (1993). So P_t^* is the Hodrick-Prescott estimate of the permanent component. Now notice what has happened. With the original definition of the permanent component (the Beveridge -Nelson decomposition) $z_t = 0$, and so there was no cycle using definition (ii). But, if one now chooses the Hodrick-Prescott filter, there will be serial correlation in z_t , and, under (ii), there is now a cycle. Clearly the existence of cycle shouldn't be dependent on the way we measure a permanent component, although the nature of it might be. Hence, because the definition is so vague, it is hard to see it as a serious candidate for cycle analysis.

3.1 Turning point based cycles

Under (iii) it is necessary to give some rules for locating the turning points. Because a turning point fundamentally involves the location of local maxima and minima in the chosen series, and that is known to involve a change in sign of the first derivative, it is natural in discrete time that the analogue of the derivative is taken to be the first difference of the series under examination. Thus the rules will involve studying quantities like Δy_t (when the series being investigated is y_t) and Δz_t (when the series being investigated is z_t). For the discussion that follows we will act as if we are looking for cycles in y_t but the methods are the same for any definition of economic activity.Now when we focus attention on such changes we may want to estimate the derivative using (say) an average of the y_t over some window around the point in time that is a candidate for being a turning point. Indeed this is exactly what the NBER methods of dating turning points effectively involve, with a peak in y_t at time t being detected by examining whether the following sequence holds

$$\{(\Delta_2 y_{t-1} > 0, \Delta y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\}$$

and with a trough being similarly defined.[5]But there would be other ways of doing this and, within the academic literature, we can find quite a few.

[5]We have taken some liberties here, The Bry-Boschan program that is used to date turning points in series is actually one based on monthly data inputs and works with the $sgn(\Delta y_{t\pm j})$, $j = 1,..,5$. In Harding and Pagan (2002) this was translated into a quarterly

Once the dating rules are applied, their output is a series of observations on a binary indicator S_t that takes the value unity when the period involves an expansion i.e. t is between a trough and a peak, and zero when it is a contraction. The S_t are then the basis for recognizing a cycle and measuring its characteristics (some of these e.g. amplitudes, will use both S_t and y_t).

Because it is the nature of Δy_t that tell us about the cycle in y_t we might proceed in a more parametric way than that described above. Hence we might first fit a parametric statistical model to yt and then use this model to perform turning point dating for us. That is what Hamilton's (1989) Markov Switching approach does. He first fits an MS model to Δy_t. In its simplest incarnation this has the form $\Delta y_t = a + b\xi_t + e_t$, where e_t is n.i.d. $(0, v)$ and ξ_t is a binary Markov process of latent states. After the model is fitted, the turning points in y_t are located using

$$\zeta_t = \Pr(\xi_t = 1|F_t) > .5],$$

where $\mathbf{1}[A]$ is the indicator function giving the value unity when the event A is true and zero when it is false, and $F_t = \{\Delta y_{t\pm j}\}_{j=\mathbf{0}}^{T}$. Clearly the ζ_t is a binary random variable and is the equivalent of the S_t coming from the NBER dating rules. Again the cycle should be described and measured by the behaviour of the binary random variable.[6]

Since $Pr(\xi_t = 1|F_t) > .5$ is based upon the DGP of Δy_t, Harding and Pagan use a linear approximation to this probability to show that Hamilton's approach detects turning points at t using weighted averages of $\Delta_j y_{t\pm j}$, $j = 1, \ldots, T$. This contrasts with NBER methods that use only local information i.e. $j = 1, 2$. Cast into non-parametric terms one might think of the weighting function used by Hamilton as like using a Gaussian rather than a uniform kernel (as the NBER procedure assigns zero weight outside the two period window). In Hamilton's procedure the weights are chosen according to the estimated model. The latter is both a strength and a weakness. It is a strength as the weights vary by data set, but it is also a weakness since one does not want to give too much weight to growth rates far away from the point t. There are well documented cases where the MS model has had to be changed in order to do this.

Finally we might ask what the relation between (i) and (iii) is. It is worth noting that the periodic cycle and turning point cycle approaches could be

variant, with five months becoming two quarters. It was shown that this rule reproduced NBER peaks and troughs quite closely, certainly closely enough for most research work.

[6] Often a mistake is made and the DGP of ξ_t rather than that of ζ_t is used to describe cycle characteristics. This particularly relates to calculations of durations of expansions and contractions where the transition probabilities for ξ_t rather than those of ζ_t are employed.

identical but, mostly, they are very different. There is little doubt that there is a widespread misconception about the equivalence as exemplified by the following quote from Arnold (2002, p4): "This view of business cycles as disturbed sine waves is also consistent with the NBER studies, which are concerned with characterizing the lengths, amplitudes and turning points of observed cycles". The fallacy in this reasoning can be seen most simply by looking at $y_t = \delta \cos(\lambda t) + u_t$, where u_t is some shock added on to reflect the fact that it is known that economic cycles will never be strictly cosine or sine waves, Then we have

$$\Delta y_t = \delta \Delta \cos(\lambda t) + \Delta u_t.$$

Now if we set $\Delta \cos(\lambda t)\ u_t = 0$, we would find that the turning points in $\delta \Delta \cos(\lambda t)$ would be found by looking at the sign changes in $\delta \Delta \cos(\lambda t)$ and that the duration of such a cycle would indeed be measured by λ. However the turning points in y_t depend of the sign of Δy_t i.e. whether $\delta \Delta \cos(\lambda t) + \Delta u_t > 0$. The two coincide only if $u_t = 0$ or is very small. Thus the period of the periodic cycle indicated by will not agree with that indicated by turning points. Indeed it is possible that there is no periodic cycle, although there will be a turning point cycle - the simplest example being when y_t is an $AR(1)$. Indeed Sargent (1979, p 240) simulated data from an AR(1) process and observed that "This illustrates how stochastic difference equations can generate processes that 'look like' they have business cycles even if their spectra do not have peaks...". To emphasize that the correspondence between turning point and periodic cycle characteristics is not close, we generated data from an AR(2), $y_t = 1.4y_{t-1} - 0.57y_{t-2} + e_t$, where e_t was $n.i.d.(0, 1)$. Then the duration of the periodic cycle is 23 quarters while the duration of the turning point cycle (using NBER rules) was 11 quarters.

4 Multivariate Issues

Sometimes more than one series is used to define economic activity. This may be because GDP is not available or because other measures may become available much more quickly. Two approaches have then been used to handle such multivariate information. In the first the series are combined together to produce a single measure of economic activity, whereupon the analysis proceeds as outlined earlier. Mostly the diversity encountered here involves how one should combine the multiple series together to produce a single one. Sometimes simple weighted averages are used. In other instances principal components of the series are constructed. More recently it has been popular

to regard economic activity as a latent factor and to then fit parametric models that would enable one to extract estimates of this factor e.g. Forni et al (2001).

An alternative method is to retain all the series but to define the cycle in terms of some multivariate measures. Thus the analogue of the periodic cycle methodology would be to compute the eigenvalues of a VAR system and to see if there are complex valued ones. The multivariate equivalent of the turning points methodology, when there is no reduction to a single measure of activity approach, is to aggregate the turning point information provided by all the individual cycles into a single set of turning points. This provides what the NBER refers to as the "reference cycle". Harding and Pagan (2003b) discuss the methods that the NBER employ to perform such an aggregation.

In all these cases one either ends up with a single series whose cycle is examined or with a single cycle if a multivariate approach is adopted. Thus, while the techniques used to perform these tasks will vary, the classification given above still serves to demarcate the various views on cycles and does not need to be extended.

5 Some Implications of the Above Classification

The fact that there are six possible combinations of the various choices means that, at any given conference and in published volumes on the cycle, we will have many of them being simultaneously represented. To illustrate this we note that A(iii) is what the NBER and the recently formed CEPR (London) European business cycle dating committee work with. In contrast, the COIN index of the European cycle, also published by CEPR, embodies a B(iii) philosophy but adopts a different choice of method for removing the permanent component. Many statistical models of the cycle using threshold autoregressions utilize B(iii), since the regime switching variable is the annual growth rate in the cycle variable. B(i) is represented in the work by Kaiser and Maravall (2001) and by Harvey and Jaeger (1993). A(i) is probably not favoured much anymore but it can be argued that this was Tinbergen's (1956) view, as seen from his attempt to find an AR(2) with complex roots in a variable representing the level of economic activity. Thus there is a great variety of definitions present in current cycle research and this can become very confusing since the researchers generally claim to be describing and measuring the business cycle, a term that seems best applied to A(iii), as that was its

historical connotation.

References

Arnold, L.G. *Business Cycle Theory* (Oxford University Press, Oxford, 2002)

Baxter, M. and R. King (1999), "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series," *Review of Economics and Statistics*, 81, pp. 575-593.

Blanchard, O.J. and S. Fischer (1989), *Lectures on Macroeconomics,* M.I.T. Press Cambridge, MA,

Blinder, A.S. and S. Fischer (1981), "Inventories, Rational Expectations and the Business Cycle", *Journal of Monetary Economics*, 8, 277-302.

Bry, G., Boschan, C., (1971), *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*, New York, NBER.

Burns, A.F., Mitchell, W.C., (1946), *Measuring Business Cycles*, New York, NBER.

Frisch, R. (1933), "Propogation and Impulse Problems in Dynamic Economics", in *Economic Essays in Honour of Gustav Cassel* (London, Allen and Unwin), 171-205.

Forni M., M. Hallin, M. Lippi and L. Reichlin (2001), "Coincident and Leading Indicators for the Euro Area", *The Economic Journal*, 111, 62-85.

Hamilton, J.D., (1989), "A New Approach to the Economic Analysis of Non-Stationary Times Series and the Business Cycle", *Econometrica*, 57, pp. 357-384.

Harding D., and A.R. Pagan, (2002), "Dissecting the Cycle: A Methodological Investigation", *Journal of Monetary Economics,* 49(2):365-381

Harding, D. and A. R. Pagan (2003a) "A Comparison of Two Business Cycle Dating Methods" *Journal of Economic Dynamics and Control,* 27, 1681-1690

Harding D., and A.R. Pagan, (2003b), "Synchronisation of Cycles", Available at http://econrsss.anu.edu.au/~arpagan/

Harvey, A.C., and A. Jaeger (1993). "Detrending, Stylized Facts and the Business Cycle", *Journal of Applied Econometrics,* 8,. pp 231-47.

King, R.G. and S.T. Rebelo (1993), "Low Frequency Filtering and Real Business Cycles", *Journal of Economic Dynamics and Control*, 17, 207-231.

Kaiser, R and A. Maravall (2001), "Measuring Business Cycles in Economic Time Series", *Lecture Notes in Statistics 154,* New York, Springer-Verlag

Morley, J.C. (2002), "A state-space approach to calculating the Beveridge-Nelson decomposition", *Economics Letters* 75, 123-127.

Morely, J.C. (2002), "Why are the Beveridge_Nelson and Unobserved Components Decompositions of GDP so Different", *The Review of Economics and Statistics*, Vol LXXV, May, No 2, pp235-242.

Pierce, D.A. (1979), Signal extraction error in nonstationary time series. *Annals of Statistics*, 7, pp. 1303-20.

Sargent, T.J. (1979), *Macroeconomic Theory* (Academic Press)

Tinbergen, J. (1956), *Business Cycles in the United Kingdom*, 1870-1914, (2nd edition, North-Holland, Amsterdam)