

CENTRE FOR APPLIED MACROECONOMIC ANALYSIS

The Australian National University



CAMA Working Paper Series

February, 2009

WHY TAX CAPITAL?

Yili Chien
Purdue University

Junsang Lee
The Australian National University

CAMA Working Paper 5/2009
<http://cama.anu.edu.au>

Why Tax Capital?

Yili Chien*

Junsang Lee†

Purdue University

Australian National University

January 8, 2009

First Version: November 11, 2005

Abstract

We study optimal capital income taxation with a Ramsey problem and relate this optimal taxation problem to the question that has been asked in the asset pricing literature, which is why the risk free interest rate is too low. We show that the Ramsey planner chooses the optimal level of capital stock to be one that satisfies the modified golden rule in the steady state under some conditions. The conditions include sufficient government tax instruments and ability to issue bonds. We argue that the optimal capital level is different from that chosen in a competitive equilibrium unless the competitive equilibrium risk free interest rate is same as the time discount rate in the steady state. This difference in the choice of capital motivates imposing a positive capital income tax (or subsidy) on households to induce them to invest at the socially optimal amount. As examples, we investigate optimal capital taxation in a decentralized economy with limited commitment and one with private information. However, the result still holds in various types of economies with risk free interest rate that is too low.

JEL code: D86; E23; E44; E62.

Keywords: Capital taxation; Ramsey problem; Limited commitment; Private information.

*Department of Economics, Krannert School of Management, Purdue University, 403 West State Street, West Lafayette IN, 47906. Email: ychien@purdue.edu

†School of Economics, College of Business and Economics, The Australian National University, Acton, ACT 0200 Australia. Email: junsang.lee@anu.edu.au. The authors are grateful to their advisors, Andrew Atkeson, Harold Cole, and Hanno Lustig, for their help and encouragement. The authors thank for the financial support, CIGS from the Australian National University. All errors are ours.

1 Introduction

In the Ramsey literature on capital taxation, Chamley (1986) and Judd (1985) were the first to analytically argue for zero capital taxation in the long run. Subsequently, Chari and Kehoe (1999) have shown that the capital tax rate should be high initially and decrease to zero, and Atkeson, Chari and Kehoe (1999) show that the zero capital taxation result is robust to a wide range of the model assumptions. Finally, Lucas (1990) argues that for the U.S. economy there is a significant welfare gain to be realized in switching to this policy. In sum, the zero capital taxation argument suggests that the current capital stock in the U.S. economy is too low since the capital tax rate is too high, and that decreasing the tax rate can lead to large welfare gains.

We study optimal capital income taxation within a Ramsey model. As the above zero capital taxation papers showed, the Ramsey planner always chooses the optimal level of capital stock to be one that satisfies the following modified golden rule in the steady state,

$$1 = \beta [MP_{K^*} + 1 - \delta],$$

where β is a time discount factor and MP_K is a marginal product of capital.

However, in a competitive equilibrium of this model, where an agent takes the prices and the government's policies as given and makes a decision on capital investment, the level of capital stock in the steady state satisfies the standard intertemporal euler equation:

$$1 = \frac{1}{R} [MP_K + 1 - \delta],$$

where R is a gross risk free interest rate. Hence, the optimal capital level is different from that chosen by households unless the competitive equilibrium risk free interest rate is same as the time discount rate in the steady state. This difference in the choice of capital motivates imposing a positive capital income tax(or subsidy) on households to induce them to invest at the socially optimal amount.

$$1 = \frac{1}{R} [(1 - \tau_K) (MP_{K^*} - \delta) + 1]$$

This paper provides a generalized framework under which taxing capital income is optimal: any endogenous discrepancy between the government's discount factor and that of the private sector will support such a result. A subsequent work by Albanesi and Armenter (2007) also studies optimal taxation in various environments by using Ramsey problems.

We relate the question why tax capital to the question that the asset pricing literature has asked, which is why market risk-free interest rate is too low. Those asset pricing models that provide too

low a risk-free market interest rate would imply a positive optimal capital tax. In Section 6 and 7, we provide two example economies where the risk-free interest rate is too low and hence, the capital income should be taxed in the steady state.

This paper is organized as follows. Section 2 describes the model economy. Section 3 characterizes and define the competitive equilibrium. Section 4 solves a Ramsey problem. Section 5 discuss the optimal capital income tax in steady state. In Section 6 and 7, we show that in both economy with limited commitment and with private information, our result holds. Section 8 concludes.

2 Environment

Time is discrete. There is a continuum of agents of measure one. Each agent starts off with an initial asset holding, a_0 , and an initial idiosyncratic shock, s_0 . The initial joint distribution over (a_0, s_0) is given by Φ_0 . In this paper, we do not consider the aggregate shock. The only event that each household faces is an idiosyncratic stochastic shock. When we consider the specific example of the model, the actual idiosyncratic stochastic shock will be specified. Each event s_i takes values on a discrete grid $S \equiv \{s_1, \dots, s_i, \dots, s_I\}$. The shock $\{s\}$ follows a Markov process, with a transition probability $\pi(s'|s)$. We denote s^t as a history of realization of the shock:

$$s^t = (s_0, s_1, \dots, s_{t-1}, s_t)$$

There is a single, non-storable, consumption good. A household's preferences are described by the expected value of the sum of discounted utilities of consumption and labor streams:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t | s_0) U(c_t, l_t)$$

where c_t, l_t denote consumption and labor respectively. β is the discount factor. The functions $U(\cdot)$ is assumed to be bounded, continuously differentiable, strictly increasing, and strictly concave.

The output Y_t is produced by aggregate labor and capital input with a single technology that exhibits constant returns to scale:

$$Y_t = F(K_t, L_t)$$

where $F(\cdot, \cdot)$ is the market production function, and K_t and L_t denote aggregate capital and aggregate labor inputs respectively. Assume that $F(\cdot, \cdot)$ is homogeneous of degree one, and twice

continuously differentiable.

The output can be used in consumption, government spending or capital investment. The resource constraint is

$$C_t + G_t + K_{t+1} = Y_t + (1 - \delta)K_t$$

3 Competitive Equilibrium

3.1 Household's Problem

At this stage of the model, we will leave the specific details of the household problem aside. In period zero, taking the sequence of after-tax wages \bar{w}_t and market interest rates R_t as given, a household chooses consumption and labor sequences $\{c_t, l_t\}_{t=0}^{\infty}$ that maximize the agent's life time utility subject to her constraints ¹. Once we solve this household's maximization problem, then the indirect utility function of a household with initial states (a_0, s_0) can be written as

$$W_0(a_0, s_0, \{\bar{w}_t\}_{t=0}^{\infty}, \{R_t\}_{t=0}^{\infty}), \quad (1)$$

which is a function of the initial state, and sequences of prices.

3.2 Firms' Problem

Firms operate market production technology through a market production function, $F(K_t, L_t)$. At period 0, taking a sequence of pre-tax wage rates $\{w_t\}$, market interest rates $\{R_t\}$, and corporate profit taxes τ_K as given, a firm chooses a sequence of capital stocks K_{t+1} and labor demand L_t in order to maximize the discounted after-tax profit function:

$$\max_{\{K_{t+1}, L_t\}} \sum_t \prod_{s=1}^t \frac{1}{R_{s-1}} [(1 - \tau_{K,t}) \varphi_t - I_t + \delta \tau_{K,t} K_t]$$

subject to

$$\begin{aligned} \varphi_t &= F(K_t, L_t) - w_t L_t \\ K_{t+1} &= (1 - \delta) K_t + I_t \end{aligned}$$

¹One of the example could be the budget constraint and households might also have constraints on asset trading which depending on the market structure.

where φ_t is a corporate profit. Note that it is firms that must pay the capital income tax, which is imposed on the income paid to the physical capital.

The firms' problem yields the following first order conditions:

$$\begin{aligned} w_t &= F_{L,t} \\ 1 &= q_t [(F_{K,t+1} - \delta)(1 - \tau_{K,t+1}) + 1] \end{aligned} \quad (2)$$

where $q_t \equiv \frac{1}{R_t}$ is an intertemporal price. Based on these optimal conditions, firms make decisions on labor demand and capital investment.

3.3 The Government

As in standard Ramsey problems, we assume that the government levies linear tax on capital and labor and issues new government bonds in order to finance an endogenous government spending and an outstanding government debt.²

Government expenditure is composed of government consumption G_t and debt payments $(r_t^b + 1)B_t$. r_t^b is the return of one period bond held from period $t - 1$ to t . Government revenue consists of taxes on market labor income and capital income, respectively labeled $\tau_{K,t}$ and $\tau_{L,t}$. Additionally, the government can finance its expenditures by issuing new debt B_{t+1} . Hence, the government constraint is as follows:

$$\tau_{K,t}r_tK_t + \tau_{L,t}w_tL_t + B_{t+1} = (r_t^b + 1)B_t + G_t$$

3.4 Competitive Equilibrium

Definition 1 *A competitive equilibrium is a given initial condition K_0 , a sequence of allocations $\{c_t(a_0, s^t), l_t(a_0, s^t), K_t\}$, a sequence of prices $\{w_t, R_t\}$, and a sequence of policies $\{\tau_{L,t}, \tau_{K,t}, B_t, G_t\}$*

²We can introduce an exogenous government spending, $e_{g,t}$ in our economy with following government budget constraint,

$$\tau_{K,t}r_tK_t + \tau_{L,t}w_tL_t + B_{t+1} = (r_t^b + 1)B_t + G_t + e_{g,t}$$

,and the following resource constraint,

$$C_t + K_{t+1} + G_t + e_{g,t} = F(K_t, L_t) + (1 - \delta)K_t$$

such that the household problem is solved for each (a_0, s_0) , the firm problem is solved, the government budget constraint is satisfied for all periods and the markets clear.

4 Ramsey Problem

Definition 2 Given K_0 , the Ramsey problem is to choose a competitive equilibrium that maximizes following social welfare function:

$$\int W_0(a_0, s_0, \{\bar{w}_t\}_{t=0}^\infty, \{R_t\}_{t=0}^\infty) d\Phi_0 + \sum_{t=0}^{\infty} \beta^t U(G_t)$$

Following Aiyagari (1996), we formulate a dual approach Ramsey problem as if the government can pick the sequences of $\{\bar{w}_t, R_t, G_t, K_{t+1}\}$. First, the social welfare function is the sum of the integration of all household's life time utility over the initial distribution Φ_0 and the discounted utility from government consumptions, so this social welfare function depends on the sequence of $\{\bar{w}_t, R_t, G_t\}$. Second, by choosing the sequence of R_t, G_t and K_{t+1} together with equation (2), the capital tax sequence can be decided. In addition, the labor tax sequence is decided by $\bar{w}_t = (1 - \tau_{L,t})w_t$. Note that the optimal choices of consumption, labor and the household's budget constraint have embedded in the indirect utility function expressed in equation (1). Hence, the government's optimal tax problem is to choose sequences of $\{\bar{w}_t, G_t, K_{t+1}\}$ and a sequence $\{R_t\}$ that is consistent with the competitive equilibrium such that social welfare is maximized.

$$\max_{\{\bar{w}_t, R_t, G_t, K_{t+1}\}} \int W_0(a_0, s_0, \{\bar{w}_t\}_{t=0}^\infty, \{R_t\}_{t=0}^\infty) d\Phi_0 + \sum_{t=0}^{\infty} \beta^t U(G_t)$$

subject to

$$C_t + K_{t+1} + G_t = F(K_t, L_t) + (1 - \delta)K_t \quad (3)$$

Equation (3) is the resource constraint and must hold for all periods. Notice that as is generally the case in Ramsey problems we exclude the government budget constraint, since household consumption choices satisfy the household's budget constraints, which together with the resource constraints imply the government budget constraints.

The Lagrangian is written as

$$L_G = \min_{\lambda_t} \max_{\{\bar{w}_t, R_t, G_t, K_{t+1}\}} \int W_0(s_0, \{\bar{w}_t\}_{t=0}^\infty, \{R_t\}_{t=0}^\infty) d\Phi_0 + \sum_{t=0}^{\infty} \beta^t U(G_t) \\ + \sum_{t=0}^{\infty} \beta^t \lambda_t (F(K_t, L_t) + (1 - \delta)K_t - C_t - K_{t+1} - G_t)$$

where λ_t is the Lagrange multiplier for the resource constraints.

Then, first order conditions with respect to G_t and K_{t+1} implies

$$U'(G_t) = \beta U'(G_{t+1}) [MP_{K,t+1} + 1 - \delta] \quad (4)$$

Proposition 1 $\frac{1}{\beta} = MP_K + 1 - \delta$ in steady state.

Proof. The result follows directly from the steady-state version of the following Euler equation in the Ramsey planner's problem.

$$-U'(G_t) + \beta U'(G_{t+1}) [MP_{K,t+1} + 1 - \delta] = 0$$

■

Proposition 1 states that in the steady state pre-tax capital return, $MP_K - \delta$, must equal the rate of time preference, $\frac{1}{\beta} - 1$, and it characterizes the optimal level of capital stock in the economy. It shows that the Ramsey government would like to implement the capital stock that satisfies equation (4) regardless of the market structures. The key elements we required here are as follows. First, there should be a competitive equilibrium such that the household's life time indirect utility is a function of all future after-tax prices. Second, the government has the ability to issue bond and choose the whole sequences of after-tax prices by levying tax on each market every period. Third, there has to be an endogenous component in the government expenditure. Lastly, we need to assume that there exists a steady state. In the next section, we show that as long as the pre-tax capital return is different from the time preference rate, then there is a room for capital tax or subsidy.

5 Optimal Capital Tax

This section explains how we compute the steady state capital tax rate. Proposition 1 provides the planner's Euler equation, based on which the planner chooses the optimal level of capital.

Equation (2) is the firm's Euler equation, based on which firms make the capital investment decisions in competitive equilibrium.

We choose the steady state capital tax rate such that these two equations are consistent and equivalent to each other.

$$\begin{aligned} 1 &= q [(1 - \tau_K) (MP_K - \delta) + 1] \\ &\equiv \beta [MP_K + 1 - \delta] \end{aligned}$$

For the private sector to achieve the optimal capital level in the competitive equilibrium, the optimal capital tax rate should be the following:

$$\tau_K = 1 - \frac{1/q - 1}{1/\beta - 1} \quad (5)$$

Proposition 2 *Positive capital tax is optimal if and only if $\beta < q$ in steady state.*

Proof. *The result follows directly from Equation (5) ■*

Recall the dual approach in the Ramsey problem. The government chooses the after-tax wage rate and the interest rate. As long as the government chooses an interest rate that is consistent with the competitive equilibrium, it will always result in the modified golden rule in steady state, regardless of frictions in the competitive equilibrium.

Proposition 2 shows that this Ramsey problem yields the same outcome (5) of positive capital taxation as long as the market interest rate is lower than time preference rate in steady state. This result is general: as long as a risk-free market interest rate is too low compared to the time discount rate, then the optimal capital taxation in steady state is positive and could be high. Now the question we ask why tax capital is actually equivalent to the question that the asset pricing literature has asked and tried to answer, which is why risk-free interest rate is too low. In the next section, we have two example economies where the risk-free interest rate is too low, and hence the capital income should be taxed in the steady state.

6 Economy with Limited Commitment

In this section, we study a decentralized economy with limited commitment. And we show the market risk-free interest rate is lower than time discount rate in the steady state and hence the optimal capital tax rate is positive in the steady state.

The basic economic environment is same as we introduced before in the earlier section. We will be more specific in a stochastic structure and we add some features if needed.

In each period, each agent is endowed with one unit of time and derives a utility from consumption, c_t , and leisure, $1 - l_t$, where l_t is a labor supply. We assume that the utility function is separable in consumption and leisure:

$$U(c_t, l_t) = u(c_t) + v(1 - l_t)$$

where $u(\cdot)$ and $v(\cdot)$ are assumed to be bounded, continuously differentiable, strictly increasing and strictly concave.

6.1 Enforcement Technology

Following Kehoe and Levine (1993) and Kocherlakota (1996), this literature commonly assumes that when households default on their existing debts, they are excluded from financial markets forever, and have their assets seized at the time of default. This punishment forces households to consume only their wage income forever, which implies, in turn, that the per-period utility that a household obtains after default is

$$U_{aut}(s_t, \bar{w}_t) \equiv \max_{l_t} u(\bar{w}_t s_t l_t) + v(1 - l_t),$$

where \bar{w}_t is an after-tax wage rate. We can thus define an autarky value at period t as

$$V_{aut,t}(s_t, \{\bar{w}_\tau\}_{\tau=t}^\infty) \equiv \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \succeq s^t} \pi(s^\tau | s^t) U_{aut}(s_\tau, \bar{w}_\tau).$$

This is the autarky value that a household receives from period t afterward if it defaults at period t .

To keep households from defaulting, we need to ensure that the expected utility of staying in the risk-sharing pool is greater than or equal to the value of defaulting for each possible s^t . Hence, the enforcement constraints can be written as follows:

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau \succeq s^t} \beta^{\tau-t} \pi(s^\tau | s^t) [u(c_\tau) + v(1 - l_\tau)] \geq \sum_{\tau=t}^{\infty} \sum_{s^\tau \succeq s^t} \beta^{\tau-t} \pi(s^\tau | s^t) U_{aut}(s_\tau, \bar{w}_\tau) \text{ for } \forall s^t$$

In other words, if this constraints are satisfied in all states and all periods, households do not wish to exercise their default option.

6.2 Competitive Equilibrium

In this section, we describe details of the household's problem. Firm's problem, financial intermediaries and government are the same as in Section 3. We will define this competitive equilibrium and characterize it.

6.2.1 Household (a_0, s_0) 's Problem

Taking the sequence of after-tax wages \bar{w}_t and market interest rates R_t as given, a household trades history-contingent consumption claims $\{c_t\}$ and makes labor allocation decisions $\{l_t\}$ subject to both a lifetime budget constraint and a sequence of enforcement constraints, one for each history:

$$W_0(a_0, s_0, \{\bar{w}_t\}_{t=0}^{\infty}, \{R_t\}_{t=0}^{\infty}) \equiv \max_{\{c_t, l_t\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) [u(c_t) + v(1 - l_t)],$$

subject to

$$\sum_{t=0}^{\infty} \sum_{s^t} \prod_{s=0}^t \frac{1}{R_{s-1}} \pi(s^t | s_0) [\bar{w}_t s_t l_t - c_t] \geq -a_0 \quad (6)$$

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau \succeq s^t} \beta^{\tau-t} \pi(s^\tau | s^t) [u(c_\tau) + v(1 - l_\tau)] - \sum_{\tau=t}^{\infty} \sum_{s^\tau \succeq s^t} \beta^{\tau-t} \pi(s^\tau | s^t) U_{aut}(s_t, \bar{w}_t) \geq 0 \text{ for } \forall s^t, t \geq 0, \quad (7)$$

where $\frac{1}{R_{-1}} = 1$. The equation (6) is the present value life time budget constraint and the equation (7) is the enforcement constraint.

6.2.2 Characterizing Equilibrium Prices and Allocations

Household (a_0, s_0) Let the Lagrangian multipliers for constraints (6) and (7) be θ and $\beta^t \pi(s^t) \mu_t(s^t)$ respectively. Cumulative multipliers³, $\zeta_t(s^t)$, can be defined to make the problem recursive:

$$\zeta_t(s^t) = 1 + \sum_{s^r \succeq s^t} \mu_r(s^r),$$

where s^r is a subsequent history of s^t . Rewriting cumulative multipliers recursively yields:

$$\begin{aligned} \zeta_t(s^t) &= \zeta_{t-1}(s^{t-1}) + \mu_t(s^t), \\ \zeta_0(s_0) &= 1 \end{aligned}$$

³We formulate the Lagrangian by using the cumulative multiplier as in Marcet and Marimon (1999)

Note that $\{\zeta_t(s^t)\}$ is a non-decreasing stochastic process.

The Lagrangian can be written as:

$$L(a_0, s_0, \{\bar{w}_t\}_{t=0}^\infty, \{R_t\}_{t=0}^\infty) = \min_{\theta, \xi_t} \max_{c_t, l_t} \sum_t \sum_{s^t} \beta^t \pi(s^t) [\zeta_t(a_0, s^t) [u(c_t) + v(1 - l_t)] + (1 - \zeta_t(a_0, s^t)) U_{aut}(\bar{w}_t, s_t)] \\ + \theta(a_0, s^t) \left[\sum_t \sum_{s^t} \prod_{s=1}^t \frac{1}{R_{s-1}} \pi(s^t) [\bar{w}_t s_t l_t - c_t] - a_0 \right]$$

The first-order condition with respect to c_t is

$$\beta^t \zeta_t(a_0, s^t) u_{c,t} = \theta(a_0, s^t) \prod_{s=1}^t \frac{1}{R_{s-1}} \\ \beta^t \xi_t(a_0, s^t) u_{c,t} = \prod_{s=1}^t \frac{1}{R_{s-1}} \quad (8)$$

where $\xi_t(a_0, s^t) \equiv \frac{\zeta_t(a_0, s^t)}{\theta(a_0, s^t)}$. R_t is the market interest rate faced by all households. ξ_t is a summary statistic of a household's history. It measures how severely and how many times the household has been constrained in history. Therefore, the equation (8) implies that a household's consumption is history dependent, and that a household should have a higher consumption level if it has a higher ξ_t .

The first-order condition implies that the ratio of marginal utilities in consecutive nodes (s^t, s^{t+1}) satisfies the following restriction:

$$\frac{1}{R_t} = q_t \\ = \beta \frac{u_{c,t+1} \xi_{t+1}}{u_{c,t} \xi_t} \\ = \max_i \beta \frac{u_{c,t+1}}{u_{c,t}}$$

The intertemporal price, q_t , is equal to the maximum intertemporal marginal rate of substitution (IMRS) across all households. This can be clearly seen by the fact that only an unconstrained household can engage in arbitrage when its IMRS is smaller than the state price of consumption in a particular state of the world. If the price of consumption in that state were larger than IMRS, the household can short a contract that delivers one unit of consumption in that state, or sell a contingent claim, at a price q_t that would increase its overall utility.

The first-order condition with respect to l_t is as usual:

$$v_{l,t} = u_{c,t} \bar{w}_t s_t, \text{ for } \forall t.$$

This first-order condition implies that the marginal disutility from supplying labor is equal to the marginal utility from the effective wage income.

In addition, aggregate consumption C_t , aggregate market efficient labor supply L_t , and aggregate home production output H_t can be written as follows:

$$C_t = \sum_{s^t} \int \pi(s^t | s_0) c_t(a_0, s^t) d\Phi_0$$

$$L_t = \sum_{s^t} \int \pi(s^t | s_0) s_t l(a_0, s^t) d\Phi_0$$

The resource constraint for this economy can now be written as

$$C_t + G_t + K_{t+1} = F(K_t, L_t) + (1 - \delta) K_t, \quad \text{for } \forall t \geq 0.$$

6.2.3 Competitive Equilibrium

Definition 3 *A competitive equilibrium is a given initial condition K_0 , a sequence of allocations $\{c_t(a_0, s^t), l_t(a_0, s^t), K_t\}$, a sequence of prices $\{w_t, R_t\}$, and a sequence of policies $\{\tau_{L,t}, \tau_{K,t}, B_t, G_t\}$ such that the household problem is solved for each (a_0, s_0) , the firm problem is solved, the government budget constraint is satisfied for all periods and the markets clear.*

6.3 Steady State Analysis

Aggregate Consumption Allocation The aggregate steady state consumption C can be derived from the steady state resource constraint:

$$C = F(K, L) - \delta K - G \tag{9}$$

while aggregate consumption is allocated across agents:

$$C = \sum_{s^t} \int \pi(s^t) c_t(a_0, s^t) d\Phi_0 \tag{10}$$

Household Consumption Allocation From the equation (8), the steady state intertemporal price can be obtained as follows;

$$q = \beta \frac{u_{c,t+1}(a_0, s^{t+1})}{u_{c,t}(a_0, s^t)} \frac{\xi_{t+1}(a_0, s^{t+1})}{\xi_t(a_0, s^t)} \tag{11}$$

When a household does not switch to a state with a binding enforcement constraint, i.e., $\xi_{t+1}(a_0, s^{t+1}) = \xi_t(a_0, s^t) + \mu_t(a_0, s^t) = \xi_t(a_0, s^t)$, the equation (11) becomes

$$\beta G_{MU,t}^{UN} = q \quad (12)$$

where $G_{MU,t+1}^{UN}$ is the growth rate of the unconstrained household's marginal utility from period t to period $t + 1$.

On the other hands, a household does switch to a state with a binding enforcement constraint, i.e., $\xi_{t+1}(a_0, s^{t+1}) \geq \xi_t(a_0, s^t)$, the equation (11) becomes

$$\beta G_{MU,t}^{CON} < q \quad (13)$$

where $G_{MU,t}^{CON}$ is the growth rate of the constrained household's marginal utility from period t to period $t + 1$.

The equations (12) and (13) imply that the growth rate of the constrained household's consumption is higher than the growth rate of the unconstrained consumption, together with the assumption that utility function is strictly concave.

Proposition 3 $\beta < q$ in steady state, if and only if full-risk sharing is not feasible.

Proof.

The equations (12) and (13) show that

$$G_{MU,t}^{CON} < G_{MU,t}^{UN},$$

and this implies that

$$G_{c,t}^{CON} > G_{c,t}^{UN},$$

where $G_{c,t}^{CON}$, $G_{c,t}^{UN}$ are the growth rate of the constrained household's consumption from period t to period $t + 1$ and that of the unconstrained household's consumption from period t to period $t + 1$ respectively.

In steady state, the average growth rate of consumption is constant at 1 as in equation (10). Therefore it must be the case that

$$G_{c,t}^{CON} > 1, \text{ and } G_{c,t}^{UN} < 1$$

This suggest that the growth rate of the unconstrained household's marginal utility is greater than 1.

$$G_{MU,t}^{UN} > 1$$

Along with the equation (12), this prove that the time discount factor is smaller than the intertemporal price in the steady state.

$$\beta < q$$

■

Optimal Capital Tax Rate Proposition 2 implies that the optimal capital tax rate in the long run should be positive in the decentralized economy with limited commitment. The tax rate can be obtained:

$$\tau_K = 1 - \frac{1/q - 1}{1/\beta - 1} > 0$$

7 Economy with Private Information

In this section, we study a decentralized economy with a private information. We closely follow an economy in Atkeson and Lucas (1995). Each period, each household finds a job opportunity with a positive probability and fails to find such an opportunity. In order to insure against unemployment risk, the risk-averse households sign an insurance contract with a zero profit insurance company.

Each period, each household has a job experience denoted as $s_t \in \{0, 1\}$. We use $s_t = 0$ to indicate no job opportunity at period t and $s_t = 1$ to indicate his having a job opportunity at period t . The probability of having a job opportunity is given by $\pi(1)$ and this probability follows an i.i.d. A household who find a job can work $l_t \in [0, 1]$ units of time. We assume that this job opportunity s_t is private information and so is household's labor supply of a household, l_t . We only observe household's report about his job experience $z_t \in \{0, 1\}$. We denote their history of reported job opportunity as $z^t = (z_0, z_1, \dots, z_t)$.

We assume that the utility function is separable in consumption and labor supply and linear in labor supply.

$$U(c_t, l_t) = u(c_t) - vl_t$$

where v is a constant and the assumptions for the utility function are the same as in the previous section. Let $C(u)$ be the inverse of the flow utility function $u(c)$. We assume that C is continuously differentiable, strictly increasing, and strictly convex.

The insurance contract is an allocation sequence specifying each household's labor supply and consumption for each period $t \geq 1$. The contract offered by an insurance company to a household with an initial asset holding, a_0 is

$$\sigma = \{x_t(a_0, z^t), l_t(a_0, z^t)\}_{t=1}^{\infty}, \quad (14)$$

where $x_t(a_0, z^t)$ is a level of current flow utility from consumption.

Given an allocation σ , a household chooses a strategy for reporting job opportunities to maximize the discounted expected utility he obtains under that allocation. This strategy is denoted as $z = \{z_t(s^t)\}_{t=1}^{\infty}$, where $s^t = (s_1, \dots, s_t)$, $s_t \in \{0, 1\}$ for all $t \geq 1$, denotes the household's true job experience.

A household's initial discount utility can be written as a function of a_0, σ , and z as

$$U(a_0, \sigma, z) = \sum_{t=1}^{\infty} \sum_{s^t} \pi(s^t) [x_t(a_0, z^t) - v l_t(a_0, z^t)].$$

Let $z^* = \{z_t^*(s^t)\}_{t=1}^{\infty}$, where $z_t^*(s^t) = s_t$ for all $t \geq 1$, denote the truthful reporting strategy. We use the notation $U_t(a_0, \sigma, z^*, s^{t-1})$ to denote the discounted utility from period t on received under the allocation σ by a household who had his initial asset holding a_0 , who has reported employment history s^{t-1} up to period t , and who uses the truthful reporting strategy z^* .

We impose constraints on allocations. The first is incentive compatibility:

$$U(a_0, \sigma, z^*) \geq U(a_0, \sigma, z), \quad (15)$$

for all a_0 and all reporting strategies z . The second is a lower bound on the discounted expected utility that a household can receive after any history of shocks:

$$U_t(a_0, \sigma, z^*, s^{t-1}) \geq \underline{\omega} \quad (16)$$

for all $t \geq 1$, all s^{t-1} . We need this lower bound constraint in order to guarantee a non-degenerate stationary distribution. Green (1987), Thomas and Worrall (1990) and Atkeson and Lucas (1995) showed that without an exogenously imposed lower bound, the promised utility would spread out forever over time and no long run stationary distribution exists.

7.1 Competitive Equilibrium

In this section, we formulate optimal contract problem and define an competitive equilibrium since the details about firm's problem, and government are the same as in the previous section.

7.1.1 Optimal Insurance Contract Problem

We assume that there is a perfect competition among insurance companies. The best contract that insurance companies can offer is chosen to maximize the household's expected life time utility.

$$W_0(a_0, \{\bar{w}_t\}_{t=1}^\infty, \{R_t\}_{t=1}^\infty) \equiv \max_{\sigma} \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \pi(s^t) [x_t(a_0, z^t) - vl_t(a_0, z^t)]$$

subject to

$$\begin{aligned} \sum_{t=1}^{\infty} \sum_{s^t} \prod_{r=1}^t \frac{1}{R_{r-1}} \pi(s^t) [\bar{w}_t z_t l_t(a_0, z^t) - C(x_t(a_0, z^t))] &\geq -a_0, \\ U(a_0, \sigma, z^*) &\geq U(a_0, \sigma, z), \\ U_t(a_0, \sigma, z^*, s^{t-1}) &\geq \underline{\omega}, \end{aligned}$$

where \bar{w}_t is an after-tax wage rate, $(1 - \tau_{L,t}) w_t$.

7.1.2 Characterizing Equilibrium prices and Allocations

In order to characterize the optimal contract, we formulate a dual problem with a promise keeping constraint which requires that σ delivers the promised expected utility for period $t + 1$ onward. First, we define an initial promised utility as

$$\omega_0 = U(a_0, \sigma, z^*), \quad (17)$$

and then the promise keeping constraint can be written as

$$\omega_{t-1}(a_0, z^{t-1}) = \sum_{s^t} \pi(s^t) [x_t(a_0, z^{t-1}, z_t) - vl_t(a_0, z^{t-1}, z_t) + \beta \omega_t(a_0, z^{t-1}, z_t)] \quad (18)$$

for all a_0 and all t , where $\omega_{t-1}(a_0, z^{t-1})$ is the promised expected utility for period t onward.

The incentive constraint can be written with the promised expected utility as follows;

$$x_t(a_0, z^{t-1}, 1) - vl_t(a_0, z^{t-1}, 1) + \beta \omega_t(a_0, z^{t-1}, 1) \geq x_t(a_0, z^{t-1}, 0) + \beta \omega_t(a_0, z^{t-1}, 0) \quad (19)$$

for all t and for all z^{t-1} .

The lower bound constraint becomes

$$\omega_t(a_0, z^t) \geq \underline{\omega} \text{ for all } z^t. \quad (20)$$

The dual problem can be written as

$$V(a_0, \{\bar{w}_t\}_{t=1}^\infty, \{R_t\}_{t=1}^\infty) \equiv \min_{\{l_t(a_0, z^t), x_t(a_0, z^t), \omega_t(a_0, z^t)\}_{t=1}^\infty} \sum_{t=1}^\infty \sum_{s^t} \prod_{r=1}^t \frac{1}{R_{r-1}} \pi(s^t) [C(x_t(a_0, z^t)) - \bar{w}_t z_t l_t(a_0, z^t)]$$

subject to (18), (19) and (20).

For simplicity, we rewrite the dual problem in terms of the following Bellman equation.

$$V(\omega_{t-1}, \{R_\tau\}_{\tau=t}^\infty, \{\bar{w}_\tau\}_{\tau=t}^\infty) = \min_{x_t, l_t, \omega_t} E_t \left[C(x_t(\omega_{t-1}, z_t)) - \bar{w}_t z_t l_t(\omega_{t-1}, z_t) + \frac{1}{R_t} V(\omega_t, \{R_\tau\}_{\tau=t+1}^\infty, \{\bar{w}_\tau\}_{\tau=t+1}^\infty) \right]$$

subject to

$$\omega_{t-1} = E_t [x_t(\omega_{t-1}, z_t) - v l_t(\omega_{t-1}, z_t) + \beta \omega_t(\omega_{t-1}, z_t)], \quad (21)$$

$$x_t(\omega_{t-1}, 1) - v l_t(\omega_{t-1}, 1) + \beta \omega_t(\omega_{t-1}, 1) \geq x_t(\omega_{t-1}, 0) + \beta \omega_t(\omega_{t-1}, 0), \quad (22)$$

$$\omega_t(\omega_{t-1}, z_t) \geq \underline{\omega}, \text{ for } \forall z_t, \quad (23)$$

w_0 given,

for all $t \geq 1$.

Lemma 1 *Incentive compatibility constraint (22) have to hold in equality at optimum.*

Proof. *Suppose it is not the case, we can always increase $l_t(\omega_{t-1}, 1)$ to lower the value of the the objective function. ■*

By using Lemma 1, the two constraints (18) and (19) can be replaced by the equalities

$$x_t(\omega_{t-1}, 0) = \omega_{t-1} - \beta \omega_t(\omega_{t-1}, 0) \quad (24)$$

$$x_t(\omega_{t-1}, 1) = \omega_{t-1} - \beta \omega_t(\omega_{t-1}, 1) + v l_t(\omega_{t-1}, 1) \quad (25)$$

Using the equations (24) and (25), we can derive the first order conditions. The first order condition with respect to $\omega_t(\omega_{t-1}, s_t)$ is given as

$$C'(x_t(\omega_{t-1}, s_t)) \beta \leq \frac{1}{R_t} V'(\omega_t(\omega_{t-1}, s_t), \{R_\tau\}_{\tau=t+1}^\infty, \{\bar{w}_\tau\}_{\tau=t+1}^\infty), \text{ for all } t \text{ and all } s_t, \quad (26)$$

where (26) holds with equality when $\omega_t(\omega_{t-1}, z_t) \geq \underline{\omega}$. The envelope condition is given by

$$\frac{dV'(\omega_{t-1}, \{R_\tau\}_{\tau=t}^\infty, \{\bar{w}_\tau\}_{\tau=t}^\infty)}{d\omega_{t-1}} = E_t [C'(x_t(\omega_{t-1}, s_t))], \text{ for all } t \quad (27)$$

And the first order condition with respect to $l_t(\omega_{t-1}, 1)$ can be obtained as follows:

$$\begin{aligned} C'(x_t(\omega_{t-1}, 1)) &= \frac{\bar{w}_t}{v}, \text{ if } l_t(\omega_{t-1}, 1) \in (0, 1), \\ C'(x_t(\omega_{t-1}, 1)) &\leq \frac{\bar{w}_t}{v}, \text{ if } l_t(\omega_{t-1}, 1) = 1, \\ C'(x_t(\omega_{t-1}, 1)) &\geq \frac{\bar{w}_t}{v}, \text{ if } l_t(\omega_{t-1}, 1) = 0. \end{aligned} \quad (28)$$

Then the first order condition (26) together with the envelope condition (27) implies

$$V'(\omega_{t-1}) \leq \frac{q_t}{\beta} E_t[V'(\omega_t)]. \quad (29)$$

where (29) holds with equality when $\omega_t(\omega_{t-1}, z_t) \geq \underline{\omega}$.

7.1.3 Competitive Equilibrium

Definition 4 *A competitive equilibrium is a given initial condition K_0 , a sequence of allocations $\{c_t(a_0, s^t), l_t(a_0, s^t), K_t\}$, a sequence of prices $\{\omega_t, R_t\}$, and a sequence of policies $\{\tau_{L,t}, \tau_{K,t}, B_t, G_t\}$ such that the optimal contract problem is solved for each $\omega_0(a_0)$, the firm problem is solved, the government budget constraint is satisfied for all periods and the markets clear.*

7.2 Steady State Analysis

We again assume that there exists the steady state, in which all the aggregate variables stay constant. We characterize household's utility entitlement and his labor supply in steady state. Then we show that the market risk-free interest rate is lower than the time discount factor in steady state.

Lemma 2 *Define $\omega^0 \in [\underline{\omega}, \infty)$ to be such that $C'(\omega^0 - \beta\omega_t(\omega^0, 1)) = \frac{\bar{w}}{v}$ and suppose $\omega_t(\omega_{t-1}, s_t) \geq \underline{\omega}$, Then*

- i) $\omega_t(\omega_{t-1}, 1) > \omega_t(\omega_{t-1}, 0)$, for all t and all $\omega_{t-1} \in [\underline{\omega}, \omega^0)$, and
- ii) $\omega_t(\omega_{t-1}, 1) = \omega_t(\omega_{t-1}, 0)$, for all t and all $\omega_{t-1} \in [\omega^0, \infty)$.

Proof. *We use a contradiction to prove the result i) in Lemma 2. Suppose $\omega_t(\omega_{t-1}, 1) \leq \omega_t(\omega_{t-1}, 0)$, then since $V'(\cdot)$ is an increasing function,*

$$V'(\omega_t(\omega_{t-1}, 1)) \leq V'(\omega_t(\omega_{t-1}, 0)).$$

By the equation (26),

$$C'(\omega_{t-1} - \beta\omega_t(\omega_{t-1}, 1) + vl_t(\omega_{t-1}, 1)) \leq C'(\omega_{t-1} - \beta\omega_t(\omega_{t-1}, 0)).$$

Since $C'(\cdot)$ is also strictly increasing,

$$\beta\omega_t(\omega_{t-1}, 1) - vl_t(\omega_{t-1}, 1) \geq \beta\omega_t(\omega_{t-1}, 0).$$

Then since $l_t(\omega_{t-1}, 1) > 0$ for $\omega_{t-1} \in [\underline{\omega}, \omega^0)$, this equation implies that

$$\omega_t(\omega_{t-1}, 1) > \omega_t(\omega_{t-1}, 0).$$

It is a contradiction.

The result ii) in Lemma 2 follows from Lemma 4 in Atkeson and Lucas (1995). No one works, $l_t(\omega_{t-1}, 1) = 0$, if his utility entitlement is higher than or equal to a certain threshold, $\omega_{t-1} \geq \omega^0$. Suppose $\omega_t(\omega_{t-1}, 1) \neq \omega_t(\omega_{t-1}, 0)$, for all $\omega_{t-1} \geq \omega^0$, then equation (26) implies contradiction. ■

An optimal employment contract requires that the promised future utility that household receives when he reports the unemployment is lower than the utility that he receives when he reports the employment if household's utility entitlement ω_{t-1} is lower than a certain threshold, ω^0 . We can understand that the insurance company must spread out promises to future utility, since otherwise it would be impossible to provide any insurance in the form of contingent payment today. The equation (26), together with (27) shows how the insurance company balances the delivery of the utility today as compared to future utilities.

Household's labor supply decision is made based on equation (28). However, for household's utility entitlements $\omega_{t-1} \geq \omega^0$, the marginal disutility of work is larger than that household's after-tax wage income on the job measured in utility. Thus no household with such high utility entitlements works and there is no incentive problem. Therefore the insurance company does not spread out the promised future utility any more, i.e., $\omega_t(\omega_{t-1}, 1) = \omega_t(\omega_{t-1}, 0)$.

Lemma 3 *If $q < \beta$, there exists $k > 0$ such that $\omega_t(\omega_{t-1}, 1) \geq \omega_{t-1} + k$, for all $\omega_{t-1} \in [\underline{\omega}, \infty)$.*

Proof. *If $q < \beta$, then the equation (29) implies that*

$$V'(\omega_{t-1}) < \pi(1)V'(\omega_t(\omega_{t-1}, 1)) + (1 - \pi(1))V'(\omega_t(\omega_{t-1}, 0)). \quad (30)$$

Suppose $\omega_t(\omega_{t-1}, 1) \leq \omega_{t-1}$, then the equation (30) implies

$$V'(\omega_{t-1}) < \pi(1)V'(\omega_{t-1}) + (1 - \pi(1))V'(\omega_t(\omega_{t-1}, 0)).$$

And we can obtain

$$\omega_{t-1} \leq \omega_t(\omega_{t-1}, 0),$$

since $V'(\cdot)$ is an increasing function. Lemma 2 implies the contradiction and this contradiction suggests that

$$\begin{aligned} \omega_{t-1} &< \omega_t(\omega_{t-1}, 1), \quad \text{for all } \omega_{t-1} \in [\underline{\omega}, \omega^0) \\ \omega_{t-1} &\leq \omega_t(\omega_{t-1}, 1), \quad \text{for all } \omega_{t-1} \in [\omega^0, \infty). \end{aligned} \quad (31)$$

For $\omega_{t-1} \in [\omega^0, \infty)$, if $q < b$, then by the equation (26), (27), and (31),

$$\begin{aligned} C'(\omega_{t-1} - \beta\omega_t(\omega_{t-1}, 1)) &< C'(\omega_t(\omega_{t-1}, 1) - \beta\omega_{t+1}(\omega_t, 1)), \\ \omega_{t-1} - \beta\omega_t(\omega_{t-1}, 1) &< \omega_t(\omega_{t-1}, 1) - \beta\omega_{t+1}(\omega_t, 1), \\ &\leq \omega_t(\omega_{t-1}, 1) - \beta\omega_t(\omega_{t-1}, 1). \end{aligned}$$

The last inequality follows the equation (31). Hence,

$$\omega_{t-1} < \omega_t(\omega_{t-1}, 1), \quad \text{for all } \omega_{t-1} \in [\omega^0, \infty).$$

■

Proposition 4 *In the steady state of this economy, $\beta < q$.*

Proof. We use contradiction to prove this proposition and the proof is divided into two cases: $q < \beta$ and $q = \beta$. The proof is an application of Atkeson and Lucas (1995).

Part 1. We want to study the process generated by $(\pi(1), \omega_t(\omega_{t-1}, s_t))$ on the set $S = [\underline{\omega}, \bar{\omega}]$, where $\bar{\omega} \in [\omega^0, \infty)$. Let λ be any probability measure on the Borel sets \mathbf{S} of S , and define Markov operator P_q by

$$(P_q\lambda)(A) = \pi(1) \int_{\omega_t(\omega_{t-1}, 1) \in A} d\lambda + (1 - \pi(1)) \int_{\omega_t(\omega_{t-1}, 0) \in A} d\lambda,$$

for any $A \in \mathbf{S}$.

Let λ_ω be the probability measure that concentrates mass on the point ω . We show that if $q < \beta$, there exist $N \geq 1$ and $\varepsilon > 0$ such that $(P_q^N \lambda_\omega)(\bar{\omega}) \geq \varepsilon$, for all $\omega \in S$. By Lemma 4, if we choose N large enough so that $\underline{\omega} + (N - 1)k > \bar{\omega}$, then the probability of passing from point $\underline{\omega}$ to point $\bar{\omega}$ is at

least $\pi(1)^N$. Since $\omega_t(\omega_{t-1}, 1)$ is non-decreasing in ω_{t-1} , this transition to $\bar{\omega}$ is at least as probable from any other point in S . Thus letting $\varepsilon = \pi(1)^N$, this proves that the Markov process under study satisfies the hypothesis of Theorem 11.12 in Stokey et al. (1989) is complete. The probability of transiting from any other point in the state space to $\bar{\omega}$ is strictly positive and the point $\bar{\omega}$ is an absorbing state. Since for $\bar{\omega} \in [\omega^0, \infty)$, no one works from equation (28) but a positive fraction of them consumes, the steady state with $q < \beta$ is not feasible.

Part 2. Suppose $q = \beta$, by Lemma 8 and the proof of Lemma 9, part 2 in Atkeson and Lucas (1995), the point ω_0 is an absorbing state and the unique invariant distribution is concentrated at the point ω^0 . At this point, no one works but a positive fraction of them consumes. Therefore, the steady state with $q = \beta$ is not feasible. ■

Atkeson and Lucas (1995) provides the proof of existing an invariant distribution in this economy when $q > \beta$. Bohacek (2005) also show that the risk-free market interest rate is lower than the agent's discount rate in the economy with private information.

Proposition 4 confirms that in a private information economy the market clearing risk-free interest rate is less than the households' time preference rate. We argue that the optimal tax rate on capital income in the long run is positive together with Proposition 2.

$$\tau_K = 1 - \frac{1/q - 1}{1/\beta - 1} > 0.$$

8 Conclusion

We study optimal capital income taxation with a Ramsey problem. We show that Ramsey planner chooses the optimal level of capital stock to be the one that satisfies the modified golden rule in the steady state under some conditions. The conditions include sufficient government tax instruments and ability to issue bonds. We argue that the optimal capital level is different from that chosen in a competitive equilibrium unless the competitive equilibrium risk free interest rate is same as the time discount rate in the steady state. This difference in the choice of capital motivates imposing a positive capital income tax (or subsidy) on agents in the equilibrium to induce them to invest at the socially optimal amount. As examples, we investigate the optimal capital taxation in a decentralized economy with limited commitment and one with private information. However, the result still holds in various types of economies with too low risk free interest rates.

The key elements we require in this paper are as follows. First, there should be a competitive equilibrium such that the household's lifetime indirect utility is a function of all future after-tax prices. Second, the government has the ability to issue bond and choose the whole sequences of after-tax prices by levying tax on each market every period. Third, there has to be an endogenous component in the government expenditure. Lastly, we need to assume that there exists a steady state.

This paper provides a generalized framework under which taxing capital income is a good idea. We relate this framework of optimal capital taxation to the asset pricing application. We show any competitive equilibrium which yields too low market risk-free interest rate can support that taxing capital income is a good idea.

References

- [1] AIYAGARI, S. R. (1995): "Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting," *Journal of Political Economy*, 103, 1158-1175.
- [2] ALBANESI, S. AND R. ARMENTER (2007): "Intertemporal Distortions in the Second Best," NBER Working Paper No. 13629.
- [3] ALBANESI, S. AND C. SLEET (2006): "Dynamic Optimal Taxation with Private Information," *Review of Economic Studies*, 73(1),1-30.
- [4] ALBANESI, S. AND S. YELTEKIN (2005): "Optimal Taxation with Endogenously Incomplete Debt Markets," *Journal of Economic Theory*, 127(1), 36-73.
- [5] ALVAREZ, F.,AND U. JERMANN (2000): "Efficiency, Equilibrium, and Asset Pricing with Risk of Default," *Econometrica*, 68(4), 755-798.
- [6] ————— (2001): "Quantitative Asset Pricing Implications of Endogenous Solvency Constraints," *Review of Financial Studies*, 14 1117-1152.
- [7] ATKESON, A., V.V. CHARI and P. KEHOE (1999): "Taxing Capital Income: A Bad Idea?," *Federal Reserve Bank of Minneapolis Quarterly Review*, 23, 3-17.
- [8] ATKESON, A., AND R. E. LUCAS (1992): "On Efficient Distribution with Private Information," *Review of Economic Studies*, 59, 427-453.

- [9] ————— (1995): "Efficiency and Equality in a Simple Model of Unemployment Insurance," *Journal of Economic Theory*, 66, 64-85.
- [10] BOHACEK, R., (2005): "Capital Accumulation in Private Information Economies," *B.E. Journal of Macroeconomics*, 5, 1172-1195.
- [11] CARCELES-POVEDA, E., AND A. ABRAHAM (2006): "Endogenous Trading Constraints with Incomplete Asset Markets," University of Rochester, Working Paper.
- [12] CHAMLEY, C. (1986): "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, 607-622.
- [13] CHARI, V. V., AND P. KEHOE (1999): "Optimal Fiscal and Monetary Policy," in *Handbook of Macroeconomics*, ed. Taylor, J., and Woodford, M., New York: Elsevier.
- [14] CHIEN, Y., AND J. LEE (2006): "Optimal Capital Taxation under Lack of Commitment," Australian National University Working Paper.
- [15] CONSTANTINIDES, G. M., AND D. DUFFIE (1996): "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy*, 104, 219-240.
- [16] DA COSTA, C. AND I. WERNNING (2007): "On the Optimality of the Friedman Rule with Heterogeneous Agents and Non-linear Income Taxation," MIT Working Paper.
- [17] GOLOSOV, M., N. KOCHERLAKOTA, AND A. TSYVINSKI (2003): "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, 70 (3), 569-587.
- [18] HUGGETT, M.,(1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of Economic Dynamics and Control* Vol.17, 5-6, 953-969.
- [19] JUDD, K., (1985): "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 28, 59-83.
- [20] KEHOE, P. J., AND F. PERRI (2002): "International Business Cycles with Endogenous Incomplete Markets," *Econometrica* 70,907-928.
- [21] ————— (2004): "Competitive Equilibria with Limited Enforcement," *Journal of Economic Theory*, 119 (1), 184-206.

- [22] KEHOE, T., AND D. K. LEVINE (1993): "Debt Constraint Asset Markets," *Review of Economic Studies*, 60, 856-888.
- [23] ————— (2001): "Liquidity constrained markets versus debt constrained markets," *Econometrica*, 69 (3), 575-598.
- [24] KOCHERLAKOTA, N. (1996): "The equity premium: It's still a puzzle," *Journal of Economic Literature*, Vol.34, p42-71.
- [25] ————— (1996): "Implications of Efficient Risk Sharing without Commitment," *Review of Economic Studies*, 63, 595-610.
- [26] KRUEGER, D. (1999): "Risk Sharing in Economies with Incomplete Markets," Ph.D. thesis, University of Minnesota.
- [27] KRUEGER, D. AND F. PERRI (1999): "Risk Sharing: Private Insurance Markets or Redistributive Taxes?," *Federal Reserve Bank of Minneapolis Staff Report* 262.
- [28] KRUSELL, P., AND J. A. SMITH (1997): "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomics Dynamics* 1(2), 387-422.
- [29] ————— (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 6, 867-896.
- [30] LJUNGQVIST L. AND T. SARGENT (2004): *Recursive Macroeconomic Theory*, MIT Press, Cambridge, Mass.
- [31] LUCAS, R. E. (1990): "Supply-Side Economics: An Analytical Review." *Oxford Econ. Papers* 42, 293-316.
- [32] LUSTIG, H. (2007): "The Market Price of Aggregate Risk and the Wealth Distribution," Working Paper, UCLA.
- [33] MARCET, A., AND R. MARIMON (1998): "Recursive Contracts," Working Paper Univeraitat Pompeu Fabra.
- [34] MEHRA, R. AND E. C. PRESCOTT (1985): "The equity premium: A puzzle," *Journal of Monetary Economics* 15, 145-165.

- [35] SEPPÄLÄ, J. I. (1999): "Asset Prices and Business Cycles Under Limited Commitment," UIUC Working Paper.
- [36] SHIN, Y.(2005): "Ramsey Meets Bewley: Optimal Government Financing with Incomplete Markets," University of Wisconsin at Madison, Working Paper.
- [37] STOKEY N., R. E. LUCAS AND E. C. PRESCOTT (1989): Recursive Methods in Economic Dynamics. Harvard University Press., Cambridge, Mass.
- [38] STORESLETTEN, K., C. TELMER, AND A, YARON (2004): "Cyclical Dynamics of Idiosyncratic Labor Income Risk," Journal of Political Economy, 12(103), 695-717.