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ABSTRACT: A structural discounted cash flow (DCF) model shows that the underlying sources of earnings growth generate very different growth paths and equity values than assumed in traditional DCF calculations. Moreover, the structural DCF model can assess the impact of exogenous factors on valuation, uncovering new costs of deflation or high inflation among other results. These findings have important implications for researchers, policy makers, and practitioners.

Keywords: valuation; growth; oligopoly; inflation

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EARNINGS VALUATION AND SOURCES OF GROWTH

1. Introduction

The value of an equity or other financial assets is commonly imputed according to the anticipated cash flows associated with the asset, discounted at some rate. Applications of this approach typically impose the assumption of a constant geometric growth rate of cash flows over time (Williams, 1938; Gordon, 1962; Shaffer, 2006). In practice, however, a firm's nominal cash flows are the result of a complex interaction among several factors including exogenous market demand, production technology, competition, inflation, and the firm's strategic management decisions. The purpose of this paper is to show that even a simple incorporation of such distinct factors implies a time path of cash flows that cannot be represented, or even closely approximated, by a constant geometric growth rate.

On the one hand, it is possible to solve for a virtual or "as-if" growth rate that replicates the value of equity implied by the correct structural approach. On the other hand, that virtual growth rate does not match the actual short-term growth rate of cash flows, implying that any naive calibration of the Gordon (1962) growth model will yield erroneous valuations. These results demonstrate that discounted cash flow (DCF) methods of imputing equity value need to be adjusted to reflect the more complex and realistic growth path based on underlying structural conditions of demand, technology, and other sources of earnings growth. Because academic researchers and practitioners alike continue to rely on DCF methods as a fundamental tool for valuation (Claus and Thomas, 2001; Cornell, 2001; Fama and French, 2002, 2006; O'Brien, 2003; Mukherjee et al., 2004; Dittmann et al., 2004; Oded and Michel, 2007), the analysis presented here has important practical ramifications.

Many complications confront the application of DCF methods to realistic situations. Research over the past two decades has focused mainly on the important extension of allowing for stochastic variation in growth rates and discount rates. Standard dynamic asset pricing models, however, typically embody the assumption that nominal cash flows follow a geometric Brownian motion process, a stochastic analogue to the deterministic geometric growth process assumed in

Gordon (1962) and elsewhere. This paper pursues a different line of development, unpacking the “black box” of earnings growth and exploring the effect on valuation and growth rates of recognizing the basic factors that underlie earnings, as identified in the separate literature on industrial organization. The resulting analysis may be termed a “structural DCF” model.

I show that the discrepancies between the correct structural model and traditional DCF calculations are large, indicating that the separate components need to be explicitly represented in any accurate application of DCF analysis. A further benefit of modeling the separate sources of earnings growth is an ability to assess the effects on value of exogenous changes in factors such as market structure, taxes, and inflation, as well as an ability to characterize the effects of endogenous choices of the firm such as investment in cost-saving technology or R&D. To illustrate, several of these factors are explicitly analyzed using the structural DCF framework. The conclusion identifies several other topics that the expanded framework could usefully address in future research.

This paper is not the first to combine elements from the theory of the firm and industrial organization with financial models to derive important new results. Subramanyam and Thomadakis (1980), Booth (1981, 1991), and others have shown that market structure and production technology can affect a firm’s cost of capital, thus not only extending the known range of relevant factors but also providing new ways of empirically estimating the cost of capital for nontraded firms or divisions of a firm. O’Brien (2008) has shown that a firm’s operating beta is affected by its product demand elasticities and production cost structure. Recent empirical studies have established that market concentration affects stock returns (Hou and Robinson, 2006) and patterns of investment (Akdogu and MacKay, 2008), while production costs affect the sensitivity of equity values to changes in uncertainty (Haushalter et al., 2002). This paper appears to be the first to extend that line of inquiry toward the question of earnings growth and valuation.

2. Model

I employ the simplest standard model of the firm that is capable of incorporating the key features under scrutiny: market growth, cost trends, and strategic interaction among firms. Market growth is represented as a steadily growing intercept of the market inverse demand function, reflecting some combination of growing numbers of consumers, increasing real per capita income, price inflation, expansion of the firm into new markets or, in the case of a new product, diffusion of awareness and acceptance by consumers.¹ The model also allows for negative growth, as in declining industries. Cost trends may be upward, due to inflation or to an exhaustible, essential input; or downward, due to cost-saving technological progress, learning by doing, or economies of scale as the firm grows. Comparison of these typical factors suggests that unit costs may increase at a slower rate than demand in many cases. It is generically unlikely that cost and demand grow at the same rate over the long run.

In practice, any of the relevant parameters may fluctuate over time, including growth rates of demand, cost changes, and the discount rate. Here I abstract from stochastic changes in order to adhere as closely as possible to the standard DCF assumptions while isolating the role of allowing demand and costs to follow distinct growth paths. Practitioners using DCF methods often compensate for stochastic fluctuations in parameters by incorporating a subjective risk premium in the discount rate, and that approach can be implemented in any calibration of the model here. The contribution of this paper in that regard is to permit a more precise calculation of future cash flows using minimal additional information, thereby reducing somewhat the need to rely on subjective risk premia or subjective adjustments to assumed growth rates of cash flows.

The degree of competition or strategic interaction among firms affects the level of earnings in each period. However, if the degree of competition is stable over time, it will not alter the growth path of earnings. Accordingly, the particular degree of competition represented in the model is not critical

¹ In practice, the slope of the demand curve may also change over time. This feature can also be represented within the structural DCF model, but would further complicate the calculations without altering the key insights.

for the purpose of characterizing the growth path. Here I employ the standard model of Cournot-Nash oligopoly (Novshek, 1980; Johnson and Myatt, 2006). The Cournot outcome has the appeal of being subgame perfect (Friedman, 1971) and meta-consistent (Daughety, 1985), may be interpreted as the post-defection phase of a supergame trigger strategy (Friedman, 1971), and can arise when firms first choose production capacity and then compete in price (Kreps and Scheinkman, 1983; Moreno and Ubeda, 2006). When firms produce outputs that are net substitutes, as is typical of rival firms in the same industry, the Cournot quantity-setting strategy has also been shown to dominate the Bertrand price-setting strategy (Singh and Vives, 1984; Cheng, 1985); Tasnádi (2006) presents further results along these lines. Thus, the Cournot assumption is not only analytically convenient but also reasonably robust as a plausible characterization of strategic interaction in various markets.

Consider a fixed number n of firms producing a homogeneous output at marginal cost $c_t = c(1 + i)^t$, where i represents a constant annual rate of change of marginal cost.² Inflation would suggest $i > 0$ while a steady rate of cost-saving innovation could provide an offsetting influence toward $i < 0$. The market inverse demand function is linear, $P_t = a_t - b \sum_{j=1}^n x_{jt} = a(1 + g)^t - b \sum_{j=1}^n x_{jt}$ where each firm j produces an output quantity x_{jt} in period t , and g is the annual rate of change of the demand intercept. A positive value of g would be expected under any combination of price inflation, a growing population of consumers, and growing per capita income; a declining industry would exhibit $g < 0$. Homogeneous firms imply a symmetric equilibrium in which $x_{jt} = x_t$ for all j , so $P_t = a(1 + g)^t - bnx_t$. In general, there is no reason to expect $i = g$.

In the absence of fixed costs, each firm's profit function is given by:

$$(1) \quad \pi_t = x_t(P_t - c_t) = x_t[a(1 + g)^t - b \sum_{j=1}^n x_{jt} - c(1 + i)^t].$$

² It is straightforward to assume some rate of entry by new firms, but would greatly complicate the calculations without altering the key insights. Non-geometric growth of earnings results from as few as two separately growing variables, and adding a third growing variable would contribute nothing to this qualitative outcome.

In a Cournot-Nash equilibrium, each firm produces the output quantity in each period that maximizes this expression, under the assumption that each rival firm likewise selects its optimal output level. Standard calculations give the respective output quantity, equilibrium price, and profit level as:

$$(2) \quad x_t = [a(1 + g)^t - c(1 + i)^t] / [b(n + 1)]$$

$$(3) \quad P_t = [a(1 + g)^t + nc(1 + i)^t] / (n + 1)$$

$$(4) \quad \pi_t = [a(1 + g)^t - c(1 + i)^t]^2 / [b(n + 1)^2]$$

where firm subscripts are suppressed for convenience. These calculations are generally valid for $g \geq i$ and $a \geq c$ to meet standard nonnegativity requirements on x_t ; to avoid the degenerate outcome $x_t = 0$, we henceforth focus on the case where $g > i$ and $a > c$.³

Equation (4) demonstrates that profit grows over time as a function of both g and i . While the manner of strategic interaction (here assumed to be Cournot) affects the way in which the number of firms enters the equilibrium profit function, and thus affects the equilibrium level of profit in each period, the strategic interaction does not directly alter the rate of growth of profits for a given industry structure.⁴ Also note that equation (4) can represent the monopoly outcome when $n = 1$.

Before proceeding to the main task of evaluating the future stream of discounted profits, I note certain properties of output and prices over time in this model. Because the output level x_t in equation (2) is a nonlinear function of g and i , the model indicates that output will not follow a geometric growth path, a point that has important implications for aggregate growth theory. The period-to-period growth rate of output from equation (2), which equals $[a(1 + g)^{t+1} - c(1 + i)^{t+1}] / [a(1 + g)^t - c(1 + i)^t]$

³ Note that this requirement is slightly stronger than necessary because x_t would remain positive if either $g > i$ and $a = c$, or else $g = i$ and $a > c$.

⁴ For example, the joint monopoly profit of a given firm has the same numerator as equation (4) while the denominator is $4bn$.

$+ g)^t - c(1 + i)^t] - 1$, is positive whenever output levels are positive.⁵ This endogenous output growth is driven by the exogenous growth in nominal inverse demand (market prices). Because welfare is an increasing function of output, this output growth relationship implies that rising prices, even if driven in part by macroeconomic inflation, can cause growth of output and hence of welfare, apart from other costs of inflation not modeled here. This point provides added support for, and complementary insight to, previous research on positive optimal inflation as in Marty and Chaloupka (1988), Dutta and Kapur (1998), Heak and Kumar (2005), and others.⁶ A subsequent section will explore a different aspect of inflation, namely the impact of exogenous inflation on the equilibrium valuation of the firm's equity.

Similarly, equation (3) indicates that prices will generally change over time. The rate of growth of equilibrium prices over consecutive periods is calculated from equation (3) as:

$$(5) \quad (P_{t+1} / P_t) - 1 = [a(1 + g)^{t+1} + nc(1 + i)^{t+1}] / [a(1 + g)^t + nc(1 + i)^t] - 1$$

which is positive whenever prices and g are positive and i is nonnegative (or not too negative). Thus, P_t will grow with demand even without cost inflation and even if the underlying demand growth results from purely noninflationary sources such as the growth of population.⁷ In fact, equation (5) can exhibit positive price growth even if marginal costs fall somewhat over time or even if both demand and costs share some common component of exogenous macroeconomic price deflation, as

⁵ In the unlikely case where $i = g$, this expression reduces to g .

⁶ It is important to emphasize that this paper does not focus on inflation as a macroeconomic or monetary phenomenon. Accordingly, the model abstracts from the money supply or from intrinsic costs of inflation.

⁷ The specific form of price growth results from the assumption that demand growth is reflected purely in the intercept of the inverse demand function. If demand growth entails a declining value of b (flattening of the demand curve over time), prices would still grow in general but the growth path could not be easily characterized analytically. If population growth is the only source of demand growth, a rising intercept would be associated with some dispersion of tastes across the growing population, as a strict replication of identical consumers would flatten the demand curve while leaving the reservation price unchanged.

long as real factors more than offset the nominal decline in inverse demand. Note that, in general, the right-hand side of equation (5) varies as a function of t .⁸ We may interpret equation (5) as demonstrating a microeconomic foundation for the observation that some price inflation may be a normal characteristic of markets that are growing in real terms, rather than merely a byproduct of expansionary monetary policy.

The discounted present value of profits in this model is given as:

$$(6) \quad PV = \sum_{t=0}^{\infty} \pi_t / (1 + r)^t$$

where r is the annual rate of discount, π_t is given by equation (4), and I assume that profits are realized at the beginning of each period.⁹ As shown in the appendix, this expression equals:

$$(7) \quad PV = (1 + r)[a^2 / (r - 2g - g^2) - 2ac / (r - g - i - gi) + c^2 / (r - 2i - i^2)] / [b(n + 1)^2]$$

which is a complicated function of both g and i , as well as of the other parameters.¹⁰ Validity of equation (7) requires the bracketed expression to be positive, which is satisfied for $r > g(2 + g)$ in conjunction with the requirements noted above ($g > i$ and $a > c$) to ensure positive output levels.

The primary conceptual advantage of equation (7) over traditional DCF calculations is that this expression characterizes an *equilibrium* growth path under the given conditions of cost, demand, and strategic interaction. If dividends are paid as a constant fraction α of each period's profit, the discounted present value of dividends will equal αPV from equation (7).¹¹ In the remainder of the

⁸ In the unlikely case where $i = g$, the right-hand side of equation (5) reduces to g and prices would grow at the same rate as unit costs.

⁹ In what follows, similar but more complex expressions would result from an alternate assumption that profits are realized at the end of each period, or in the middle of each period, or in any other prescribed pattern. Without loss of generality, we select this assumption for simplicity and clarity.

¹⁰ In the unlikely case where $i = g$, PV reduces to $(1 + r)(a - c)^2 / [b(n + 1)^2(r - 2g - g^2)]$.

¹¹ A different payout policy could be incorporated, but would greatly complicate the calculations without altering the key insights.

analysis, I focus for simplicity on PV, keeping in mind the necessary adjustment by α to interpret the results with respect to equity valuation.

3. Properties of the Model and Comparison with Traditional Analysis

Basic properties of the DCF value in this model are easily established. Most relevant to our focus, PV responds differently to g , i , and r :

$$(7a) \quad \partial PV / \partial g = 2a(1+r)[a(1+g)/(r-2g-g^2)^2 - c(1+i)/(r-g-i-gi)^2] / [b(n+1)^2]$$

$$(7b) \quad \partial PV / \partial i = -2c(1+r)[a(1+g)/(r-g-i-gi)^2 - c(1+i)/(r-2i-i^2)^2] / [b(n+1)^2]$$

$$(7c) \quad \partial PV / \partial r = PV / (1+r) - (1+r)[a^2 / (r-2g-g^2)^2 - 2ac / (r-g-i-gi)^2 \\ + c^2 / (r-2i-i^2)^2] / [b(n+1)^2]$$

where the last expression corresponds to a non-elasticity form of equity duration, or response of present value to changes in the discount rate (see for example Boquist et al., 1975). As shown in the appendix, under the parameter restrictions needed to ensure positive output levels in equation (2) as noted above, equation (7a) is positive, while (7b) and (7c) are negative.

Before calibrating the results to assess the magnitude of distortion that would result from erroneously assuming a simple geometric growth path of cash flows, I next perform two related exercises. First, I solve for the virtual or “as-if” growth rate g^* that equates the value given by (7) to that implied by the growth at rate g^* of the initial profit level π_0 given by equation (4) at time 0. Then I calculate the actual growth rate of profit from period 0 to period 1, using equation (4), and calculate the discounted present value of future profits under the false assumption that profits growth at that rate forever.

The first step begins by noting that the discounted present value of a stream of profits growing at rate g^* is given by:

$$(8) \quad PV^* = \pi_0 \sum_{j=1}^n [(1 + g^*) / (1 + r)]^j = \pi_0 (1 + r) / (r - g^*)$$

(see the appendix). Setting this expression equal to equation (6), substituting $\pi_0 = (a - c)^2 / [b(n + 1)^2]$ from equation (4) evaluated at $t = 0$, and solving for g^* , we find:

$$(9) \quad g^* = r - (a - c)^2 / [a^2 / (r - 2g - g^2) - 2ac / (r - g - i - gi) + c^2 / (r - 2i - i^2)].$$

This result establishes that a single growth rate can be found that yields the correct valuation, but that growth rate is a complicated function of the underlying parameters, including not only g and i but also r as well as the demand and cost parameters a and c .¹²

To compare this g^* with the actual initial growth rate of profit, I next calculate the latter rate over the interval from $t = 0$ to $t = 1$ using equation (4). The initial growth rate g_0 is measured as $1 + g_0 = \pi_1 / \pi_0$ which reduces to:

$$(10) \quad g_0 = [a(1 + g) - c(1 + i)]^2 / (a - c)^2 - 1.$$

From a comparison of equations (9) and (10), it is apparent that $g^* \neq g_0$. Like g^* , g_0 is a function of g , i , a , and c . However, g_0 is independent of r . The discrepancy between g^* and g_0 implies that an empirical measure of g_0 , if extrapolated into the indefinite future, will yield erroneous values of future profits and hence an erroneous valuation of equity.

We can assess the distortion of the traditional DCF calculations in several ways. First, Table 1 displays equity values for several values of g , i , and r under the assumption that all profits are paid out as dividends, along with the corresponding values of g^* and g_0 .¹³ In all of these cases, the myopic

¹² In the unlikely case where $i = g$, g^* reduces to $g(2 + g)$.

¹³ As noted above, modifying the dividend payout ratio would merely scale the equity value by the same proportion.

growth rate exceeds the virtual average growth rate of earnings, leading to an overstatement of true value when the myopic growth rate is applied in the traditional DCF calculation (Gordon growth model).¹⁴ Because the myopic growth rate is the current growth rate of earnings actually observed by investors and analysts (or, similarly, the short-run projected growth rate of earnings), it is the value that practitioners and researchers would use in any application or calibration here. When the growth rates of demand and marginal cost are similar, the valuation error is relatively small. But when those rates diverge, the traditional DCF calculation can overstate the true value by an order of magnitude.

In the table, the largest discrepancy occurs when demand is growing but marginal cost is declining over time, describing an industry such as computers where technological progress yields a steady trend of cost reductions. In the worst case, the standard calculation overstates the true equity value by more than *11 times*. In general, faster demand growth exacerbates the valuation error as a function of the discrepancy between the myopic and virtual average growth rates, owing to the sensitivity of exponential growth paths to small changes in larger growth rates.¹⁵ This pattern may suggest one factor historically underlying asset price bubbles in high-tech sectors such as computers and Internet businesses, where the combination of declining cost and growing demand renders the standard DCF calculations the most overoptimistic.

As an additional comparison, Figure 1 depicts the growth path of actual profit, the growth path corresponding to g^* , and the growth path corresponding to g_0 for one combination of parameters. It is apparent that after 50 years, the use of either g^* or g_0 in a simple geometric growth trend can lead to errors on the order of 50% in projected annual profit, with g_0 overstating true profits and g^*

¹⁴ Different ranges of i are shown for each value of g because positive output quantities require $g > i$ while, for sufficiently small values of i , the myopic growth rate exceeds r , yielding meaningless negative values for the last column.

¹⁵ Note that, in the first row of Table 1 where $g = 0.005$, the myopic growth rate is 1.55 times the virtual average growth rate, while the traditionally calculated PV is 2.50 times the true PV; whereas in the first row of the case where $g = 0.010$, the myopic growth rate is only 1.46 times the virtual average growth rate, yet the traditionally calculated PV is 11.67 times the true PV.

understating them throughout this time frame. (The growth path calculated using g^* crosses the true growth path beyond year 240, not shown in the graph.) Such large errors are clearly unacceptable for purposes of equity valuation, investment management, assessment of mergers and acquisitions, or other typical applications of DCF. These errors point to the need to take explicit account of individual factors underlying cash flows when using DCF methods.

These numerical examples also illustrate some instructive properties of the solution when $g > i$. First, the annual growth rate of earnings declines over time in our model. For the parameter values selected in Figure 1, profits grow by 6.09 percent from year 0 to year 1, but by only 4.51 percent from year 100 to year 101. This pattern results because of the unequal growth rates of costs and prices. Figure 2 depicts the time path of year-to-year profit growth rates for the same set of parameter values. A declining growth rate is realistic for many firms and suggests that the model, despite its simplicity, does capture some empirically relevant aspects of firm growth. Notably, this outcome also demonstrates that profits can – at least in the short run – grow at a faster rate than $g - i$ (that is, faster than demand growth, and faster than the sum of demand growth and cost reductions). This pattern arises because strategic interactions among oligopolistic firms cause optimal adjustments in output quantities in response to changes in costs, an effect that traditional DCF calculations cannot capture.

4. Impact of Selected Factors

Besides improving the potential precision of valuation and growth projections, another contribution of the analysis here is to permit a better understanding of the role of specific factors in determining a stock's value or P/E ratio, an important generic advantage of analytical models of valuation (Leibowitz and Kogelman, 1990). The model provides a direct method of evaluating the impact on stock prices of revised expectations concerning future demand growth, cost trends, and entry or exit of rival firms. The traditional DCF model cannot address these important issues, nor can its more sophisticated stochastic extensions. In this section, I briefly address the impact of four distinct factors (market structure, taxes, exogenous inflation, and R&D expenditures) and point out

the relationship between equity value as derived in the structural DCF model and consumer surplus, which can be used to facilitate social welfare analysis.

Although the linear demand and cost conditions assumed in the model are unrealistically simple, they provide a clear framework for illustrating the effects of the key underlying drivers of cash flow. At the same time, they can be pragmatically viewed both as a first-order approximation to more general functional forms and as a starting point for empirically driven refinements in functional form. The linearity assumed here is important because it demonstrates that even the simplest possible decomposition of earnings growth into separate demand and cost factors will yield a more complex growth path and equity valuation than can be accommodated within the traditional DCF model that assumes a constant rate of earnings growth. The assumption of static Cournot-Nash equilibrium can also be modified if empirical information suggests a different equilibrium concept as more appropriate in particular applications, but it was noted above that an alternate assumption about strategic interaction would affect only the levels and not the growth path of earnings.

Market Structure

The simplest effect to address in the new framework is the impact of market structure on each firm's valuation. As the number of firms increases from $n - 1$ to n in a symmetric Cournot oligopoly, the ratio of values of each incumbent firm is calculated from equation (7) as:

$$(11) \quad PV_n/PV_{n-1} = n^2/(n + 1)^2$$

which is equal to $1 + (\Delta PV / PV) / \Delta n$. Therefore, the semi-elasticity of PV with respect to n , $(\Delta PV / PV) / \Delta n$, is $-(2n + 1) / (n + 1)^2 < 0$. This is the proportional amount by which the entry of a single firm changes the equity value of each incumbent firm. The reason for the decline in value is the increased competition provided by the additional firm, under the assumption that aggregate demand and production costs are not altered by entry. The expression shown, albeit simple and direct, cannot be derived in the traditional DCF framework.

Taxes

Taxes provide another illustration of the structural DCF model's ability to assess the impact of exogenous factors on equity valuation and growth paths. I evaluate two types of taxes within the model, a corporate income tax versus an additive unit tax (specific tax). Any form of tax will reduce the level of earnings by some proportion, but that proportion can change over time for certain types of taxes and will thus alter the discounted present value of future earnings by different amounts. The structural DCF model is able to show this distinction, unlike the traditional DCF model.

A corporate income tax at a constant proportional rate α will reduce the firm's PV to $PV(1 - \alpha)$ without altering the level or time path of other factors such as cost, demand, output, or price. The same effect on value can be shown in the traditional DCF model. By contrast, an additive unit tax drives a wedge β_t between the firm's cost of production and the consumer's purchase price.¹⁶ If the unit tax grows over time at the rate i , then we can use the original inverse demand function to characterize market demand, where P_t now represents the consumer's outlay (price plus tax).¹⁷ The firm's profit in each period is then a slight modification of equation (1), becoming $\pi_t = x_t(P_t - \beta_t - c_t)$. In solving for the firm's optimal output level, growth path, and equilibrium value, we can thus combine $\beta_t + c_t$ and express the firm's equivalent cost as $c_t + \beta_t = c(1 + i)^t(1 + \beta_0 / c)$. The remaining calculations of equations (2) through (7) are then unchanged except for that substitution:

$$(2') \quad x_t = [a(1 + g)^t - (\beta + c)(1 + i)^t] / [b(n + 1)]$$

$$(3') \quad P_t = [a(1 + g)^t - n(\beta + c)(1 + i)^t] / [b(n + 1)]$$

$$(4') \quad \pi_t = [a(1 + g)^t - (\beta + c)(1 + i)^t]^2 / [b(n + 1)^2]$$

¹⁶ A sales tax, calculated as a proportion of the price paid, is an ad valorem tax that can also be addressed in the structural DCF model, but is algebraically more complicated. In the rare situation where the tax is a fixed proportion α of c , then we can substitute $c(1 + \alpha)$ for c in the calculations above and the analysis is straightforward.

¹⁷ Any different growth rate, including zero growth of the tax, can also be accommodated in the structural DCF model, but with greater algebraic complexity.

$$(5') \quad (P_{t+1} / P_t) - 1 = [a(1 + g)^{t+1} + n(\beta + c)(1 + i)^{t+1}] / [a(1 + g)^t + n(\beta + c)(1 + i)^t] - 1$$

$$(7') \quad PV = (1 + r)[a^2 / (r - 2g - g^2) - 2a(\beta + c) / (r - g - i - gi) + (\beta + c)^2 / (r - 2i - i^2)] / [b(n + 1)^2].$$

Several observations are in order. First, the firm alters its chosen output level in each period in response to the unit tax (equation (2')), in contrast to the situation with a corporate income tax:

$$(12) \quad \partial x_t / \partial \beta = -(1 + i)^t / [b(n + 1)].$$

Second, the altered output level causes a different level (equation (3')) and growth path (equation (5')) of output prices, via the market inverse demand relationship. Third, the effect of the unit tax on per-period net earnings (equation (4')) and overall present value (equation (7')) is not a fixed proportion of untaxed net earnings $\alpha x_t(P_t - \beta_t - c)$. Rather, the fraction of earnings reduced by the tax varies systematically over time so that the effect of the unit tax on present value is a nonlinear function of the tax rate:

$$(13) \quad \partial PV / \partial \beta = [2(1 + r) / b(n + 1)^2][c / (r - 2i - i^2) - a / (r - g - i - gi)].$$

This nonlinearity results from a combination of three effects: (a) the unit tax reduces earnings by the amount $\beta_t x_t$, (b) x_t itself varies in response to β_t , and (c) the output price varies in response to x_t . It is striking that this degree of complexity results from the simplest, most linear type of structural DCF model, and cannot be represented even as a reasonable approximation within the traditional DCF model.

Inflation

As noted above, growth of demand can reflect a combination of demographic, real, and inflationary factors, while cost trends can reflect both technological and inflationary components. While growing demand was shown to induce endogenous increases in equilibrium prices, an exogenous macroeconomic component of inflation typically exists as well. Here I extend the

structural DCF model to address the latter factor by decomposing growth into a common, exogenous inflation rate versus all other (“real”) factors, using the exact form of Fisher’s equation (Mishkin and Eakins, 2006, p. 53, footnote 5). This exercise illustrates the contribution of the model in characterizing the response of equity valuation to exogenous inflation as a function of market structure and other parameters, complementing the implications of equation (5) to enrich our understanding of how inflation can have diverse impacts on firms in different industries.

If we extend the analysis to consider g and i as exogenous *real* growth rates of demand and cost, respectively, and introduce a separate exogenous rate of inflation f , then demand grows at the nominal rate $g + f + gf$, cost grows at the nominal rate $i + f + if$, and the nominal discount rate becomes $r + f + rf$.¹⁸ Following calculations similar to those underlying equation (7), we find that the DCF value becomes:

$$(14) \quad PV = [(1 + r + f + rf) / b (n + 1)^2] \{ a^2 / [r + f + rf - 2(g + f + fg) - (g + f + fg)^2] \\ - 2ac / [r + f + rf - (g + f + fg) - (i + f + if) - (g + f + fg)(i + f + if)] \\ + c^2 / [r + f + rf - 2(i + f + if) - (i + f + if)^2]$$

It is straightforward but tedious to verify that the sensitivity of PV to inflation, $\partial PV / \partial f$, corresponds to a polynomial of degree 17 in f , a fact that has several implications. First, except on a set of measure zero, $\partial PV / \partial f \neq 0$, meaning that the equity value is sensitive to the general rate of inflation. Second, while PV might possibly be maximized or minimized for certain rates of inflation, as a function of the other parameter values, there is no analytical solution to the associated first-order condition $\partial PV / \partial f = 0$. Third, the high order of the polynomial implies multiple roots; but because not all of those roots are necessarily real or distinct, even a numerical evaluation will not reliably identify a global optimum. Fourth, because of interdependences between the rate of inflation and

¹⁸ This exercise can be motivated by the empirical finding that nominal discount rates and nominal cash flows are correlated with inflation rates, suggesting that real growth rates and discount rates tend to be more stable than nominal growth rates and discount rates (Leibowitz et al., 1989; Hamelink et al., 2002).

other industry-specific or firm-specific parameters in equation (14), no single rate of inflation will optimally benefit all firms in general, and any change in the rate of inflation may benefit some firms while harming others.

Therefore, I proceed by numerically evaluating equation (14) within a policy-relevant range of inflation rates. One important aspect of the model is that it allows us to assess the impact of deflation (negative inflation) on equity values, thus contributing to a question that has seen a resurgence of interest among researchers and policy makers (Williamson, 1987; Bernanke, 2002, 2003; Greenspan, 2002; Baig, 2003; Sinclair, 2003; Stern, 2003; Humphrey, 2004; Palley, 2008). Table 2 indicates that, for the selected values of parameters on the levels and growth rates of cost and demand, nominal present value is an increasing function of the nominal inflation rate when inflation is low or slightly negative, then fluctuates drastically and nonmonotonically for higher rates of inflation.¹⁹ Within the monotonic region, the present value of earnings is less sensitive to small changes in inflation at lower or negative rates of deflation; that is, present value is a convex function of inflation.²⁰ These findings suggest an additional reason why optimal monetary policy might wish to promote a small positive rate of inflation, rather than zero or negative rates, even without relying on conditions such as sticky prices or a lower bound on nominal interest rates that have often been assumed in such debates. At the same time, the nonmonotonic region at somewhat higher inflation rates suggests a previously unrecognized cost of higher long-run inflation, as the validity of even a structural DCF approach to asset valuation breaks down, rendering an accurate valuation virtually impossible under known methods.

¹⁹ Negative present values in the table indicate combinations of parameter values for which the model provides invalid valuations. It is important to note that such values present a serious problem for investors and analysts concerned with asset valuation: if a meaningful present value of discounted cash flow cannot be computed, but the asset is nevertheless being traded, how are investors to determine an appropriate price for the asset?

²⁰ These calculations were repeated for alternate values of c and i , not shown in the table, with no qualitative changes in the findings.

R&D

As another brief illustration of the model's ability to analyze the impact of individual factors on equity value, consider a form of research and development (R&D) expenditure in which a firm can choose to spend in each period a constant (steady-state) increment of marginal cost c to obtain a subsequent reduction in the growth rate i of marginal cost. That is, the firm can select its preferred point along an exogenous tradeoff between the *level* and *growth rate* of marginal cost. Applying this scenario to equation (7), we obtain:

$$(15) \quad \partial PV / \partial c = [2(1+r) / b(n+1)^2][c / (r-2i-i^2) - a / (r-g-i-gi)] \text{ and}$$

$$(16) \quad \partial PV / \partial i = [2c(1+r) / b(n+1)^2][c(1+i) / (r-2i-i^2) - a(1+g) / (r-g-i-gi)^2].$$

These calculations allow us to identify the condition under which R&D expenditure is value-neutral, such that the beneficial impact on value from the slower growth of marginal cost exactly balances the reduction in value due to the direct R&D expenditure.²¹ The condition for value-neutral R&D expenditure is $dPV = dc \partial PV / \partial c + di \partial PV / \partial i = 0$ which implies, using the implicit function theorem in conjunction with equations (15) and (16),

$$(17) \quad di / dc = \{c(r-g-i-gi) - a(r-2i-i^2)\}(r-2i-i^2)(r-g-i-gi)/c / [a(1+g)(r-2i-i^2)^2 - c(1+i)(r-g-i-gi)^2].$$

The actual tradeoff di / dc available to firms will generally vary with the level of R&D expenditure, as well as with other aspects of the production technology facing the individual firm or industry, and is typically stochastic in any case. For a given set of parameter values, it may or may not be possible to set the expenditure at a level that satisfies equation (17).²²

²¹ Note that the conditions represented in equations (15) and (16) implicitly assume that all firms in the symmetric Cournot oligopoly will choose to adopt the same level of R&D expenditure, and that market prices of output adjust in response to those expenditures. For initially identical firms, these conditions will be true in equilibrium.

²² Eberhart et al. (2004) and others have documented positive stock market reactions to increased

Consumer Surplus

A different type of insight afforded by the structural DCF model is a relationship between the value of a firm's equity and the consumer surplus contributed by the associated industry, which is useful for public policy regarding growth, competition, technology, and other factors. Simple calculations give consumer surplus for period t in this model as $CS_t = n^2 \pi_t / 2$ where π_t is given by equation (4), implying that consumer surplus grows at the same rate as profit in each period. The discounted present value of the entire future stream of consumer surplus is $DCS = n^2 PV / 2$ where PV is given by equation (7), while the discounted present value of total surplus is $PV (n + n^2 / 2)$.²³ Thus, both consumer surplus and total surplus are simple functions of equity value and the number of firms, quantities that are readily observable and potentially influenced by various policy actions such as taxes, antitrust restrictions, R&D subsidies, etc.

5. Conclusion

This paper has introduced a structural DCF model to illustrate the impact of key drivers of cash flow on the growth path and valuation of future earnings. It was shown that explicit recognition of these factors predicts very different patterns of growth than assumed in traditional DCF calculations, including an endogenous upward trend in output prices driven by exogenous demand growth independently of monetary policy or general macroeconomic inflation. In addition, calibrating the traditional DCF model using currently observed growth rates can significantly overstate true equity values, especially in firms with growing demand and declining costs, suggesting a previously unrecognized element underlying historical Internet and other high-tech asset price bubbles. The structural DCF model can also assess the effects of specific factors on growth paths and valuation, in contrast to the traditional model or its stochastic extensions. Among the effects

R&D expenditures in large, multiyear samples of firms.

²³ The convenient linearity of surplus in PV is driven by the assumed linearity of demand and cost.

illustrated here, the impact of exogenous inflation on valuation is perhaps the most striking, suggesting a previously unrecognized cost of high inflation as well as a new reason for targeting a small positive (rather than zero or negative) rate of inflation.

The simple framework presented here can be usefully applied or expanded in several ways. A partial list of natural possibilities might include general equilibrium analysis, stochastic factors, nonlinear dynamics, and aggregation to address macroeconomic growth paths and welfare. I elaborate briefly on each point in turn.

Future research could endogenize the assumed trends in demand and cost as the outcome of a general equilibrium model, to assess the realism of a geometric growth path of those variables. A general equilibrium model would also permit individual firms, workers, and consumers to be aggregated up to explore equilibrium macroeconomic growth paths and social welfare with more precision than is possible without explicit microeconomic foundations. As noted above, even the simplest form of the model enables analysis of the growth path and comparative statics of consumer surplus and total welfare.

Extending the structural approach to stochastic dynamic asset pricing models would comprise another important direction for future research. The primary modeling challenge in this case would be to identify the appropriate way to introduce randomness. In principle, all of the structural components may involve stochastic factors, including demand, cost, and the discount rate at a minimum. Such a three-factor model might prove analytically intractable, raising the question of which source of randomness is the most important in practice.

The effect of chaotic industry dynamics could be assessed in contrast to a steady-state growth path, as in Matsumoto and Nonaka (2006). The probability of failure of a given firm could be incorporated into its valuation, as in Shaffer (2006). Other aspects of structural change could also be enriched beyond the simple comparative statics presented above. The number of firms in the model could be allowed to vary over time, reflecting both entry of new firms and exit due to failure or acquisition. The modeling task here would be to identify the empirically appropriate assumptions

regarding the pattern of net entry, which may vary across applications. If entry is endogenous, the key factors influencing entry would require careful modeling and might include such elements as sunk costs, financing constraints, regulatory barriers to entry or consolidation, and informational opacity. A deeper model here would recognize the long-term endogeneity of financing and regulatory policies.

Because, as noted above, output levels also do not follow a simple geometric growth path in this model, another important direction of extension is to assess the implications for aggregate growth theory of looking inside the black box of factors affecting growth. Similarly, price inflation emerges as an outcome of this model without any assumption about monetary policy, casting a new perspective on the issue of the extent to which inflation can be contained by purely macroeconomic policies, either monetary or fiscal. A related issue would explore the interactions between microeconomic (industry-specific) and macroeconomic policy in terms of the resulting endogenous path of inflation. That is, if policy makers identify some optimal rate of inflation, is monetary policy the best tool to foster that outcome, or is some combination of monetary and industrial policy superior in that regard?

Appendix

A1. Derivation of Equity Value

The value of an infinite stream of discounted cash flows starting in period 0 corresponds to a geometric series of the general form $V = A \sum_{t=0}^{\infty} \alpha^t$ which, as shown by standard calculations, equals $A / (1 - \alpha)$. In each application, the specific assumptions about growth rates and discount rates will determine the form of the terms A and α . In our model, the discounted profit in each period is derived from equation (4) as $\pi_t = [a(1 + g)^t - c(1 + i)^t]^2 / [b(n + 1)^2 (1 + r)^t] = [a^2 / b(n + 1)^2][(1 + g)^2 / (1 + r)]^t - [2ac / b(n + 1)^2][(1 + g)(1 + i) / (1 + r)]^t + [c^2 / b(n + 1)^2][(1 + i)^2 / (1 + r)]^t$. Hence, the sum of discounted earnings over all periods is:

$$(A1) \quad PV = [a^2 / b(n + 1)^2] \sum_{t=0}^{\infty} [(1 + g)^2 / (1 + r)]^t \\ - [2ac / b(n + 1)^2] \sum_{t=0}^{\infty} [(1 + g)(1 + i) / (1 + r)]^t \\ + [c^2 / b(n + 1)^2] \sum_{t=0}^{\infty} [(1 + i)^2 / (1 + r)]^t$$

which takes the form $PV = \sum_{i=1}^3 A_i \sum_{t=0}^{\infty} \alpha_i^t = \sum_{i=1}^3 A_i / (1 - \alpha_i)$ whenever each of the three infinite series is convergent (that is, sums to a finite value). Substitution of the respective terms in (A1) into the variables A_i and α_i yields equation (7).

A2. Signs of Comparative Statics

The sign of equation (7a) is that of the bracketed expression in its numerator, which can be rewritten as:

$$(A2) \quad [a(r - g - gi)^2 - c(1 + i)(r - 2g - g^2)] / [(r - 2g - g^2)^2(r - g - gi)^2].$$

This expression has the same sign as its numerator, $a(r - g - gi)^2 - c(1 + i)(r - 2g - g^2)^2$. To sign this expression, use the parameter requirements to ensure positive output levels in equation (2) as noted in the text: $a > c$ and $g > i$. Then $r - g - gi > r - g - g^2 > r - 2g - g^2$ so $a(r - g - gi)^2 - c(1 + i)(r - 2g - g^2)^2 > [a - c(1 + i)](r - g - gi)^2 > (a - c)(r - g - gi)^2 > 0$.

The sign of equation (7b) is the opposite of that of the bracketed expression in its numerator, which can be rewritten as:

$$(A3) \quad [a(1 + g)(r - 2i - i^2)^2 - c(1 + i)(r - g - i - gi)^2] / [(r - g - i - gi)^2(r - 2i - i^2)^2].$$

The expression in (A3) has the same sign as its numerator, which we establish under the maintained requirements $g > i$ and $a > c$. First, $g > i$ implies $r - 2i - i^2 > r - i - g - gi$. Thus, the numerator in (A3) exceeds $[a(1 + g) - c(1 + i)](r - g - i - gi)^2 > (a - c)(1 + g)(r - g - i - gi)^2 > 0$ since $a > c$. Therefore, equation (7b) is negative under the parameter requirements.

To sign equation (7c), note that π_t is independent of r in equation (4). Thus, using equation (6), we see that $\partial PV / \partial r = -\sum t\pi_t / (1 + r)^{t-1}$. Since each term under the summation is positive, we conclude that $\partial PV / \partial r < 0$.

Table 1
Calculated Value of Equity under Alternative Growth Rates

Growth rate of demand, g	Annual rate of change of marginal cost, i	True PV from equation (7)	Virtual average growth rate g^* from equation (9)	Myopic growth rate g_0 from equation (10)	Imputed PV, erroneously assuming constant growth rate g_0
0.005	-0.010	4865	0.0260	0.0404	12,153
	-0.005	4291	0.0228	0.0302	5900
	0.000	3637	0.0179	0.0201	3902
	0.002	3355	0.0153	0.0161	3438
0.010	-0.004	7023	0.0334	0.0486	81,929
	-0.002	6662	0.0325	0.0445	21,151
	0.000	6274	0.0314	0.0404	12,153
	0.002	5857	0.0301	0.0363	8531
	0.004	5409	0.0285	0.0323	6575
0.015	0.006	10,530	0.0390	0.0486	81,929
	0.008	9676	0.0380	0.0445	21,151
	0.010	8721	0.0368	0.0404	12,153
0.020	0.016	21,073	0.0447	0.0486	81,929
	0.017	19,175	0.0442	0.0465	33,612
	0.018	17,050	0.0436	0.0445	21,151

Note: These calculations assume that all earnings are paid out as dividends; a positive retention rate would reduce the value of equity proportionately. Values of other parameters in these calculations were held fixed at $n = 2$, $a = 20$, $b = 0.1$, $c = 10$, and $r = 0.05$. In the rightmost column, representing the traditional DCF calculation assuming a constant geometric growth rate of earnings calibrated from the current observed growth rate, the PV imputed from g_0 is calculated as the product of π_0 from equation (4) evaluated at $t=0$, times $(1 + r) / (r - g_0)$. Positive output requires $g > i$ while the last column would be negative if i is too small.

Table 2
Value of Equity with Inflation

Rate of Inflation	Value of Equity (Equation (14))
-1%	4267
-0.5%	5095
0	6274
0.5%	8058
1%	10,999
1.5%	16,521
2%	29,453
2.5%	79,149
3%	-706,098
3.5%	-172,992
4%	1,135,536
4.5%	79,802
5%	4.65×10^{19}
5.5%	-11,131
6%	-3920

Note: These calculations assume that all earnings are paid out as dividends; a positive retention rate would reduce the value of equity proportionately. Values of other parameters in these calculations were held fixed at $n = 2$, $a = 20$, $b = 0.1$, $c = 10$, $g = 0.01$, $i = 0$, and $r = 0.05$. Negative values indicate combinations of parameters for which the model is invalid; investors would be challenged to estimate sensible stock prices in those cases. Similar patterns of equity values also resulted from alternate values of c and i , not reported in the table.

Figure 1: Profit Growth ($a=20$, $b=.1$, $c=10$, $r=.05$, $g=.02$, $i=.01$, $n=2$)

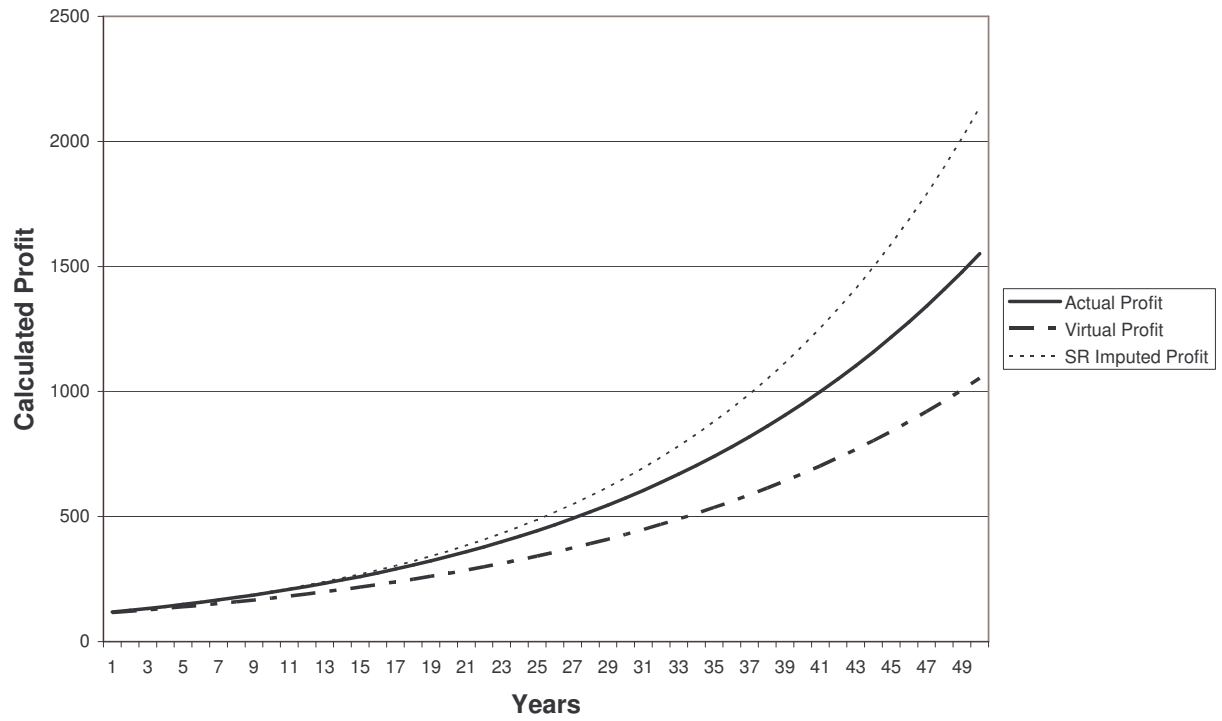
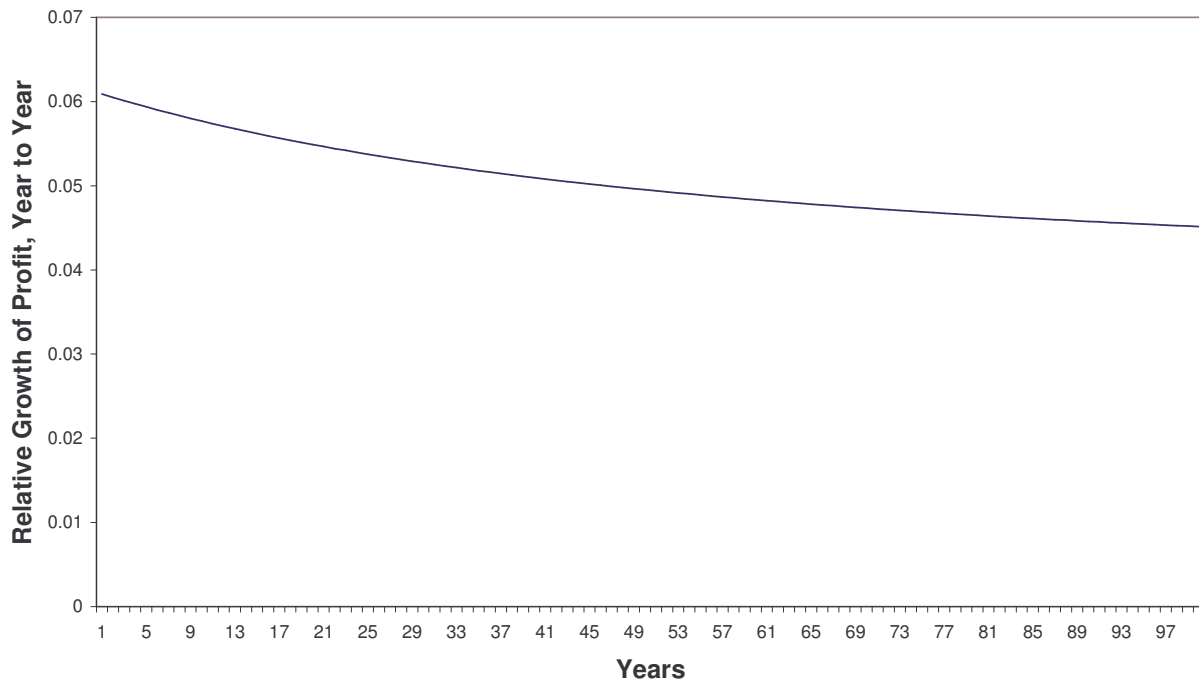


Figure 2: Annual Growth Rate of Profit over Time ($a=20$, $c=10$, $g=0.02$, $i=0.01$)



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