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Relative risk aversion and the transmission of financial crises

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Abstract

We study how investor behaviour affects the transmission of financial crises. If investors exhibit decreasing relative risk aversion, then negative wealth shocks increase the risk premium required to hold risky assets. We integrate this into a second generation model of currency crises which allows for a competitiveness effect and for contagion through changes in fundamentals. The investor behaviour can lead to the transmission of financial crises even in the absence of the competitiveness effect, and makes multiple equilibria more likely. The possible stabilization effects of capital controls and a Tobin tax on the international transmission of financial crises are also studied.

Keywords: Financial crises, contagion, wealth effects, international asset pricing, relative risk aversion, capital controls, Tobin tax.

JEL Classification: D91, F31, F32, G11, G15.

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1 Introduction

One of the main features of the financial and currency crises of the nineties was their tendency to spread across countries. The Mexican devaluation of 1994 affected other Latin American countries (the Tequila effect); the currency crisis in Thailand of July 1997 spread across East Asia; several months after the Russian crisis of August 1998, the Brazilian crisis ended up with the floatation of Real. This is by no means a feature of past crises. The subprime mortgage crisis originated in the United States during the Summer of 2007 hit financial institutions all over the world.

We extend a second generation model of currency crises (Jeanne (1997)) by combining it with international portfolio choice and show how contagion arises naturally as time-varying relative risk aversion and wealth effects are considered. In this model the risk premium required by investors and the currency devaluation probability are endogenous and mutually dependent. In fact, once a currency crisis occurs in country a the wealth of investors is hit and they will demand higher risk premia on assets issued by country b even when the fundamentals of the latter are unchanged. This happens simply because investors become more risk averse. In turn, the higher risk premia will increase the coverage ratio and make debt servicing more difficult in country b. Since the government's loss function depends on the deficit growth, the larger service on debt will undermine its commitment to keep the fixed exchange rate, hence weakening its credibility, raising the devaluation probability, and, eventually, triggering a currency crisis. Through this mechanism portfolio decisions are an independent source of contagion.

In our model it is *relative* risk aversion that is crucial in establishing a link between wealth losses and attitude towards risk, and hence, risk premium. If the coefficient of relative risk aversion is decreasing, then a wealth loss increases the risk premium that risk averse investors require to hold risky assets. This is intuitively appealing as for large financial investors the percentage loss of wealth should be more important than the absolute loss in wealth: a ten million dollar loss will have different effects if it is ten rather than one percent of the portfolio. This distinguishes our paper from others in the literature which have relied on decreasing *absolute* risk aversion to explain wealth effects and transmission of financial crises (Broner *et al.* (2006), Goldstein and Pauzner (2004), Kyle and Xiong (2001)). As it has been noted in the literature, external habit formation (Campbell and Cochrane (1999)) also generates the same predictions as assuming varying relative risk aversion.

Further, we show that by assuming the government's preferences as sensitive to changes in international trade competitiveness the model can accommodate simultaneously for contagion through trade in addition to financial linkages. This encompasses the usual distinction between trade and finance in the theoretical literature and reconciles it with the increasing empirical evidence suggesting that both channels interact in the spread of crises across countries' borders (see Broner *et al.* (2006), Glick and Rose (1999), Kaminsky and Reinhart (2000), and Van Rijckeghem and Weder (2001, 2003)). These explanations are typically considered as “fundamental based” explanations. “Pure contagion” occurs when the crisis transmission is not related to changes in fundamentals. In this case, contagion is due to a change in investor behaviour (see Allen and Gale (2000) and Masson (1999)). In our framework, contagion occurs in a frictionless environment, where both the government and investors behave rationally and information is fully available (see Chari and Kehoe (2004) and Goldstein and Pauzner (2004) for information based explanations). Our model encompasses both fundamental based explanations - financial and trade links - as well as pure contagion based on self-fulfilling expectations and multiple equilibria. An important implication is that it is the very nature of free international capital movements that introduces the risk of financial contagion. To see whether restrictions on such movements can reduce contagion we consider the effect of capital controls and Tobin taxes, i.e. taxes levied on foreign exchange transactions. We find that introducing no-short-selling constraints reduces the risk premium and hence the probability of the crisis happening. However a Tobin tax has no effect on either.

The nonlinear relationship between the risk premium and the probability of devaluation and thus the interaction between governments and investors makes self-fulfilling expectations and multiple equilibria possible under certain conditions on fundamentals. In particular, multiple equilibria are possible for better fundamentals than in Jeanne (1997). See the Appendix for details.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses policy measures commonly regarded to as stabilizing devices. Section 4 concludes. The Appendix contains the details on how the region with multiple equilibria is affected by portfolio choice.

2 The model

There are N identical price-taking international investors and M countries. The time horizon is infinite. The representative international investor maximizes the period utility flow which depends on current consumption:

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\} \tag{1}$$

where E_t is the expectation operator conditional on information available at date t, $\beta \in (0, 1)$ is the constant subjective time-preference factor, $u(\cdot)$ is the period utility function which is assumed to be twice-continuously differentiable, strictly increasing, and strictly concave, i.e. $u'(\cdot) > 0$, $u''(\cdot) < 0$, and C_s is real consumption on period s. The price of the consumption good is normalized to 1.

Each country may issue either dollar-denominated and local currency-denominated debt. Superscript ‘$*$’ denotes dollar-denominated variables. The period-by-period budget constraint is given by:

$$B^{*f}_{s+1} + \sum_{m=1}^{M} x^{*m}_{s+1} B^{*m}_s + \sum_{m=1}^{M} x^m_{s+1} \frac{B^m_s}{e^m_s} = (1 + r^{*f}_s) B^{*f}_s + \sum_{m=1}^{M} x^{*m}_s \left(I^{*m}_s + B^{*m}_s\right) + \sum_{m=1}^{M} x^m_s \left(\frac{I^m_s + B^m_s}{e^m_s}\right) - C_s \quad (2)$$

where B^{*f}_s is the real net risk-free (dollar-denominated) bond purchase at time $s-1$, x^{*m}_s and x^m_s are respectively the fractional shares of country m's dollar- and local currency-denominated debt purchased by the agent in period $s-1$, B^{*m}_s and B^m_s denote respectively the date s real market value of country m's dollar- and local currency-denominated debt, r^{*f}_s is the net real interest rate on the risk-free bond B^{*f}_s between period $s-1$ and s, I^{*m}_s and I^m_s are the coupons paid on country m's securities at time s, and e^m_s is time s spot exchange rate (price of dollars in terms of country m's currency). Equation (2) expresses the link between period s saving and period $s+1$ financial wealth. One can think of B^{*f}_s as the net purchase of a United States Treasury bill.

Maximizing the utility function (1) subject to the constraints (2) with respect to x^{*m}_{s+1}, x^m_{s+1}, and B^{*f}_{s+1}, gives the following Euler equations:

$$u'(C_s) B^{*m}_s = \beta E_s \left\{ u'(C_{s+1}) \left(I^{*m}_{s+1} + B^{*m}_{s+1}\right) \right\} \quad (3)$$

$$u'(C_s) \frac{B^m_s}{e^m_s} = \beta E_s \left\{ u'(C_{s+1}) \left(\frac{I^m_{s+1} + B^m_{s+1}}{e^m_{s+1}} \right) \right\} \quad (4)$$

and

$$u'(C_s) = (1 + r^{*f}_{s+1}) \beta E_s \left[u'(C_{s+1}) \right] \quad (5)$$

Define the ex post net real rates of return on country m's dollar- and local currency-denominated risky bond as:

$$r^{*m}_{t+1} \equiv \frac{I^{*m}_{t+1}}{B^{*m}_t} + \frac{B^{*m}_{t+1} - B^{*m}_t}{B^{*m}_t}; \text{ and } r^m_{t+1} \equiv \frac{I^m_{t+1}}{B^m_t} + \frac{B^m_{t+1} - B^m_t}{B^m_t}$$

Therefore, from (3), recalling that $E(XY) = Cov(X,Y) + E(X)E(Y)$, we obtain:

$$u'(C_s) = \beta Cov \left\{ u'(C_{s+1}), (1 + r^{*m}_{s+1}) \right\} + \beta E_s \left[u'(C_{s+1}) \right] E_s (1 + r^{*m}_{s+1}) \quad (6)$$

Dividing both sides by $u'(C_s)$, using (5) to substitute out $\beta E_s u'(C_{s+1})/u'(C_s)$, and rearranging, we obtain, for $s = t$:

$$E_t(1 + r^{*m}_{t+1}) - (1 + r^{*f}_{t+1}) = -(1 + r^{*f}_{t+1}) Cov \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)}, r^{*m}_{t+1} \right\} \tag{7}$$

Equation (7) says that, given the concavity assumptions of the period utility function, the risk premium on asset m depends positively on the covariance of the asset's return with consumption growth (Obstfeld and Rogoff (1996), Ch. 6).

Following the same steps, from (4) we obtain:

$$E_t \left[\frac{e^m_t}{e^m_{t+1}} (1 + r^m_{t+1}) \right] - (1 + r^{*f}_{t+1}) = -(1 + r^{*f}_{t+1}) Cov \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)}, r^{*m}_{t+1} \right\} \tag{8}$$

where we assume that:

$$Cov \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)}, r^{*m}_{s+1} \right\} = Cov \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)}, \frac{e^m_t}{e^m_{t+1}} (1 + r^m_{t+1}) \right\}$$

so that the risk premium is invariant across assets, either dollar- or local currency denominated, issued by the same country. The following proposition states a version of the Uncovered Interest Parity (UIP) condition under risk aversion.

Proposition 1 *The local currency-denominated asset must yield a premium over the riskless rate of return to compensate the investor's risk aversion and the exchange rate risk.*

Proof. Let π^m_{t+1} be the probability of a devaluation of country m's currency occurring at time $t+1$, and $\Delta e = \ln \left(e^m_{t+1}/e^m_t \right)$ the proportional, time-invariant, extent of such devaluation, where Δ is the difference operator. Hence, equating the l.h.s. of (7) and (8) and taking logarithms, yields:

$$E_t(r^m_{t+1}) = E_t(r^{*m}_{t+1}) + \pi^m_{t+1} \Delta e$$

where $r^m_{t+1} \approx \ln \left(1 + r^m_{t+1} \right)$. Adding and subtracting r^{*f}_{t+1} on the r.h.s. of the equation above, gives:

$$E_t(r^m_{t+1}) = r^{*f}_{t+1} + \rho^m_{t+1} + \pi^m_{t+1} \Delta e \tag{9}$$

where $\rho^m_{t+1} \equiv E_t(r^{*m}_{t+1}) - r^{*f}_{t+1}$ is the risk premium given by equation (7). ■

From equation (7) we derive the key relationship between current consumption and the risk premium. This depends on the coefficient of relative risk aversion. In particular, if this coefficient is decreasing, then the risk premium is negatively related to current consumption.

Proposition 2 *If the coefficient of relative risk aversion is decreasing (in current consumption), then the risk premium is negatively related to consumption:*

$$E_t(r_{t+1}^{*m}) - r_{t+1}^{*f} \approx (1 + r_{t+1}^{*f})\beta \left(\frac{-C_t u''(C_t)}{u'(C_t)} \right) Cov \left\{ \frac{C_{t+1}}{C_t}, r_{t+1}^{*m} \right\}$$

Proof. Define the following function:

$$G(C_{t+1}, r_{t+1}^{*m}) \equiv \frac{\beta u'(C_{t+1})}{u'(C_t)} \left[r_{t+1}^{*m} - E\left(r_{t+1}^{*m} \right) \right].$$

A second order Taylor expansion at the points $C_{t+1} = C_t$ and $r_{t+1}^{*m} = E_t\left(r_{t+1}^{*m}\right)$ of the function above yields:

$$\begin{aligned} G(C_{t+1}, r_{t+1}^{*m}) \quad &\approx \quad \beta \left[r_{t+1}^{*m} - E_t\left(r_{t+1}^{*m} \right) \right] + \frac{\beta u''(C_t)}{u'(C_t)} \cdot \\ &\qquad [C_{t+1} - C_t] \left[r_{t+1}^{*m} - E_t\left(r_{t+1}^{*m} \right) \right]. \end{aligned} \tag{10}$$

Taking conditional expectations of both sides of (10) and substituting out in (7) gives:

$$E_t(r_{t+1}^{*m}) - r_{t+1}^{*f} \approx (1 + r_{t+1}^{*f})\beta \left[\frac{-C_t u''(C_t)}{u'(C_t)} \right] Cov \left\{ \frac{C_{t+1}}{C_t}, r_{t+1}^{*m} \right\}. \tag{11}$$

■

Thus, if the coefficient of relative risk aversion, $-C_t u''(C_t)/u'(C_t)$, is decreasing then a fall in consumption will increase the time-varying local curvature of utility function and, hence, the risk premium required on the country m's asset. Note that what is important is the percentage change in consumption and not the absolute change. Hence, the same change in consumption will have different effects depending on how large it is relative to the level of consumption. This is unlike Goldstein and Pauzner (2004) where decreasing absolute risk aversion is assumed and hence the absolute change in consumption is important. From now on we maintain the assumption that the coefficient of relative risk aversion is decreasing.

2.1 Linkages through portfolio choice

The result derived in Proposition 2 enables us to show that the probability of a currency crisis in one country influences the risk premium through a wealth effect. This is shown by a log-linear approximation of the budget constraint (see Campbell (1993)). The approximation establishes that unexpected changes in the portfolio return lead to changes in consumption. This in turn affects the risk premium.

Consider that the representative international investor's dynamic budget constraint (equation (2)) can be alternatively written as:

$$W_t^* = \left(W_{t-1}^* - C_{t-1}\right)(1 + r_t^{*w}) \tag{12}$$

where W_t^* denotes total real wealth (denominated in dollars) and $(1 + r_t^{*w})$ is defined to be the gross real return on wealth invested from period $t-1$ to period t. Given international portfolio diversification, the ex post gross return can be decomposed as follows:

$$(1 + r_t^{*w}) = q_t^{*f}(1 + r_t^{*f}) + \sum_{m=1}^{M} q_t^{*m}(1 + r_t^{*m}) + \sum_{m=1}^{M} q_t^m \frac{e_{t-1}^m}{e_t^m}(1 + r_t^m) \tag{13}$$

where q_t^{*f} is the proportion of wealth invested in the risk-free bond and q_t^{*m} and q_t^m are, respectively, the proportions of wealth invested in country m's dollar- and local currency-denominated assets at time $t-1$, implying that $q_t^{*f} + \sum_{m=1}^{M} q_t^{*m} + \sum_{m=1}^{M} q_t^m = 1$.

Taking logarithms of expectations of both sides of (13) gives:

$$\begin{aligned} E_{t-1}\left(r_t^{*w}\right) \quad \approx \quad & \log\{q_t^{*f}(1 + r_t^{*f}) + \sum_{m=1}^{M} q_t^{*m} \exp\left[E_{t-1}\left(r_t^{*m}\right)\right] \\ & + \sum_{m=1}^{M} q_t^m \exp\left[E_{t-1}\left(r_t^m\right) - \pi_t^m \Delta e\right]\} \end{aligned} \tag{14}$$

Proposition 3 *An unexpected decrease (resp. increase) in wealth through an unexpected fall (resp. rise) in the portfolio rate of return leads to a decrease (resp. increase) in current consumption. This in turn increases (resp. decreases) the risk premium.*

Proof. Dividing (12) by W_{t-1}^* and taking logarithms, we obtain:

$$\Delta w_t^* \approx r_t^{*w} + \log\left[1 - \exp(c_{t-1} - w_{t-1}^*)\right] \tag{15}$$

where $r_{t+1}^{*w} \approx \ln\left(1 + r_{t+1}^{*w}\right)$.

Taking a first-order Taylor expansion around the mean $(\overline{c} - \overline{w}^*)$ of the second term on the r.h.s. of (15) we get the following approximation to the budget constraint (12):

$$\Delta w_t^* \approx r_t^{*w} + k + \left(1 - \frac{1}{\eta}\right)(c_{t-1} - w_{t-1}^*) \tag{16}$$

where

$$\begin{aligned} k &= \log(1-\exp(\overline{c}-\overline{w}^*)) - \left(1-\frac{1}{\eta}\right)(\overline{c}-\overline{w}^*), \\ \left(1-\frac{1}{\eta}\right) &= -\frac{-\exp(\overline{c}-\overline{w}^*)}{1-\exp(\overline{c}-\overline{w}^*)}, \\ \text{and } \eta &= 1-\exp(\overline{c}-\overline{w}^*). \end{aligned}$$

Next, consider the equality:

$$\Delta w_t^* = \Delta c_t + (c_{t-1} - w_{t-1}^*) - (c_t - w_t^*) \tag{17}$$

Equating the l.h.s. of (16) and (17), solving forward the resulting difference equation in $c_{t-1} - w_{t-1}^*$, assuming that $\lim_{j\longrightarrow\infty} \eta^j \left(c_{t+j} - w_{t+j}^*\right) = 0$, and taking expectations at time $t-1$ we obtain:

$$c_{t-1} - w_{t-1}^* = E_{t-1} \sum_{j=1}^{\infty} \eta^j (r_{t-1+j}^{*w} - \Delta c_{t-1+j}) + \frac{\eta k}{1-\eta} \tag{18}$$

Finally, substitute out equation (18) into (16) and (17) to obtain:

$$\begin{aligned} c_t - E_{t-1} c_t &= (E_t - E_{t-1}) \sum_{j=0}^{\infty} \eta^j r_{t+j}^{*w} \\ &\quad - (E_t - E_{t-1}) \sum_{j=1}^{\infty} \eta^j \Delta c_{t+j} \end{aligned} \tag{19}$$

Assuming that $(E_t - E_{t-1})\Delta c_{t+j} = 0$, for $j = 1, ..., \infty$, equation (19) simplifies to:

$$c_t - E_{t-1} c_t = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \eta^j r_{t+j}^{*w} \tag{20}$$

Paraphrasing Campbell (1993), equation (20) indicates that an unexpected decrease in consumption today must be determined by an unexpected reduction of return on wealth today or by news that future returns will be lower.

The decrease in current consumption implies an increase in the risk premium by Proposition 2. ■

Equation (20) provides the link between the risk premium and the probability of devaluation of country m's currency. An increase in the probability of devaluation of country m's currency at time $t+1$ will decrease the expected return on total wealth $E_t\left(r_{t+1}^{*w}\right)$ through equation (14), which in turn through a decrease in future returns will cause an unexpected decrease in consumption, as implied by equation (20). This in turn will increase the risk premium required by the investor through equation (11). Notably, the increase in the risk premium will involve all assets since the recession state will affect the investor's attitude towards risk.

2.2 Risk aversion and currency crises

We now extend a second generation model of currency crises (see Jeanne (1997) and Bratsiotis and Robinson (2004)) to allow for the portfolio choice by international investors. This will generate a process through which the risk premium affects the probability of devaluation. Thus, we show how the investor's portfolio choice can in itself be a source of transmission of financial crises.[1] In what follows contagion is implicitly defined as an increase in the probability of a crisis in one country given a crisis is occurring in another country. The literature reviewed in the introduction emphasizes that trade linkages, both bilateral and with third markets, are among the main channels of contagion. We include these effects as well by modelling the policymaker as an optimizing agent whose loss function is affected by unemployment, the deficit, and the trade balance (see Masson (1999) for a model with a non-optimizing policymaker).

Suppose, without loss of generality, that the world consists of three countries, the US, the emerging market a, and the emerging market b. We assume that the emerging markets trade balance as a proportion to nominal GDP evolves as follows:

$$tb_t^i = \overline{tb}^i + \varsigma RER_t^i, \quad i = a, b \tag{21}$$

where $tb_t^i = \widehat{TB}_t^i / P_t^i Y_t^i$, $\widehat{TB}_t^i$ is the nominal value of the trade balance, $\overline{tb}^i$ is a constant, and RER is the (log) real effective exchange rate, which gives a weight w^i on country j and $(1 - w^i)$ on the US, with $i, j = a, b$, $i \neq j$:

$$RER_t^i = \ln(e_t^i) - (p_t^i - p_t^*) - w^i \left[\ln(e_t^j) - (p_t^j - p_t^*) \right] \tag{22}$$

where e_t^i and e_t^j are the country i's and j's nominal exchange rates *vis-à-vis* the US dollar.

Equation (22) shows that a devaluation of country b's currency *vis-à-vis* the US dollar (that is, an increasing competitiveness of country b on the US market) decreases RER_t^a, that is the general competitiveness of country a. We can refer to this as to a competitiveness effect. Assuming for sake of simplicity that $\overline{tb}^i = 0$ for $i = a, b$, we can approximate the change in the trade balance tb_t for country a by the total differential:

$$\begin{aligned} \Delta tb_{t+1}^a &= \varsigma \Delta RER_{t+1}^a \\ &= \varsigma \left[\left(\pi_{t+1}^a - 1 \right) \Delta e^a - w^a (\pi_{t+1}^b - 1) \Delta e^b \right] \end{aligned}$$

[1] While the risk premium is derived in an infinitely lived agents framework, the following analysis focuses on a given two times period. This is consistent with the entire class of models of currency crises.

where we have assumed that a relative Purchasing Power Parity (between each single emerging market's currency and the US dollar) holds, that the Uncovered Interest Parity holds in the form derived in Proposition 1, that US prices are normalized to unity, so that $\Delta p^i_{t+1} = \Delta e^i$, where $\Delta p^i_{t+1} = \ln(P^i_{t+1}/P^i_t) \approx \Delta P^i_{t+1}/P^i_t$, and P^i_t is the domestic price level, for $i = a, b$, and where π^b_{t+1} is the probability of devaluation of country b's currency at time $t+1$.

Suppose now that the government pegs country a's currency to the US dollar. As long as the country's debt is partly denominated in local currency, the government faces a policy dilemma as it has an incentive to devalue in order to reduce the cost of debt. It will take any decisions about either maintaining the peg or not by minimizing the following quadratic loss function:[2]

$$L^a_{t+1} = \left(u^a_{t+1}\right)^2 + \left(\Delta d^a_{t+1}\right)^2 - (\Delta tb^a_{t+1})^2 + \delta\Gamma^a_{t+1} \tag{23}$$

where u^a_{t+1} is the unemployment rate, Δd^a_{t+1} is the growth in government real debt proportional to GDP, Δtb^a_{t+1} is the trade balance proportional to GDP, δ is a dummy variable which is equal to 1 if devaluation occurs and 0 if the peg is maintained, and Γ^a_{t+1} denotes the exogenous cost of devaluing.

We assume that the dynamics of unemployment is determined by an expectations-augmented Phillips curve:

$$u^a_{t+1} = \theta u^a_t - \zeta(\Delta e^a - E_t \Delta e^a).$$

As there is a devaluation with probability π^a_{t+1}, we see that $E_t \Delta e^a = \pi^a_{t+1}\Delta e^a$. Thus, we have:

$$u^a_{t+1} = \theta u^a_t - \zeta(\Delta e^a - \pi^a_{t+1}\Delta e^a) \tag{24}$$

with $\zeta > 0$ and $0 < \theta < 1$. We make the further assumptions that the rate of growth of real GDP is zero and that $r^a_{t+1} = E_t(r^a_{t+1}) + \epsilon_{r^a,t+1}$and $r^*_{t+1} = E_t(r^*_{t+1}) + \epsilon_{r^*,t+1}$ where $\epsilon_i \sim$i.i.d.$(0, \sigma_{\epsilon_i})$, for $i = r^a, r^*$ are stochastic shocks.

To derive the deficit, we consider the budget constraint of the consolidated government (including the central bank) as a proportion of GDP:

$$\begin{aligned}\frac{\Delta \widehat{B}^a_{t+1} + e^a_t(\Delta \widehat{B}^{*a}_{t+1} - \Delta \widehat{F}^{*a}_{t+1})}{P^a_t Y^a_t} &= \frac{(\widehat{G}^a_t - \widehat{T}^a_t)}{P^a_t Y^a_t} + r^a_{t+1}\frac{\widehat{B}^a_t}{P^a_t Y^a_t} \\ &\quad + r^*_{t+1} e^a_t \frac{(\widehat{B}^{*a}_t - \widehat{F}^{*a}_t)}{P^a_t Y^a_t}\end{aligned} \tag{25}$$

where $\widehat{B}^a_t$ and $\widehat{B}^{*a}_t$ are the country's currency-denominated and US dollar-denominated stocks of bonds respectively, $\widehat{F}^a_t$ is the amount of official foreign reserves, $\widehat{G}^a_t$ and $\widehat{T}^a_t$ are the government expenditure and taxes, $P^a_t Y^a_t$ is the country's nominal GDP, and $\widehat{X}$ indicates a nominal variable, i.e. $\widehat{X} = PX$. Denoting

[2] The present formulation follows Jeanne (1997) and Bratsiotis and Robinson (2004). These studies stress the role played by a country's debt in triggering some of the crises of the nineties. Following Masson (1999) it includes also the trade balance as an argument.

the country's proportional debt as $d_t^a = D_t^a / Y_t^a = (\widehat{B}_t^a + e_t^a(\widehat{B}_t^{*a} - \widehat{F}_t^{*a}))/P_t^a Y_t^a$, then the change in d^a can be approximated by the total differential:

$$\Delta d_{t+1}^a = \frac{\Delta \widehat{B}_{t+1}^a + e_t^a(\Delta \widehat{B}_{t+1}^{*a} - \Delta \widehat{F}_{t+1}^{*a})}{P_t^a Y_t^a} - \left(\Delta p_{t+1}^a\right) d_t^a - e_t^a(f_t^{*a} - b_t^{*a})\Delta e^a \quad (26)$$

where $f_t^{*a} = \widehat{F}_t^{*a} / (P_t^a Y_t^a)$ and $b_t^{*a} = \widehat{B}_t^{*a} / (P_t^a Y_t^a)$. Substituting equation (25) into (26), recalling that Purchasing Power Parity holds, and rearranging, we obtain:

$$\Delta d_{t+1}^a = \overline{g}_t^a + \rho_{t+1}^a d_t^a - (\Delta e^a - \pi_{t+1}^a \Delta e^a)(d_t^a + z_t^a) \quad (27)$$

where $\overline{g}^a = g_t^a - \tau_t^a + (r_{t+1}^{a*f} + \epsilon_{r^a,t+1}) d_t^a + (\epsilon_{r^a,t+1} - \epsilon_{r^*,t+1}) z_t^a$, $z_t^a = e_t^a(f_t^{a*} - b_t^{a*})$, $g_t^a = \widehat{G}_t^a / (P_t^a Y_t^a)$, and $\tau_t^a = \widehat{T}_t^a / (P_t^a Y_t^a)$. From equation (27) we see how an unexpected devaluation decreases the cost of government debt by reducing the debt service and increasing the returns on net foreign assets. It is also evident that a rise in the risk premium increases the government deficit by driving up the expected return on debt.

Proposition 4 *The probability of devaluation of a country's currency is increasing in the risk premium and in the probability of devaluation of its trading partners.*

Proof. Given:

$$\begin{aligned}\left[(\Delta tb_{t+1}^a)^d\right]^2 - \left[(\Delta tb_{t+1}^a)^f\right]^2 \quad = & \\ & \varsigma^2[(\Delta e^a)^2 - 2(\Delta e^a)^2 \pi_{t+1}^a \\ & +2w^a \Delta e^a \Delta e^b \left(\pi_{t+1}^b - 1\right)]\end{aligned}$$

the net benefit of maintaining the peg is given by the difference between the loss functions corresponding to the devaluation, L_{t+1}^{ad}, and to the fixed peg hypothesis, L_{t+1}^{af}, respectively:

$$\begin{aligned}\overline{V}_{t+1}^a &= L_{t+1}^{ad} - L_{t+1}^{af} \\ &= \left[\left(u_{t+1}^a\right)^d\right]^2 + \left[\left(\Delta d_{t+1}^a\right)^d\right]^2 - \left[(\Delta tb_{t+1}^a)^d\right]^2 + \Gamma_{t+1}^a \\ &\quad - \left[\left(u_{t+1}^a\right)^f\right]^2 - \left[\left(\Delta d_{t+1}^a\right)^f\right]^2 + \left[(\Delta tb_{t+1}^a)^f\right]^2 \\ &= \Gamma_{t+1}^a + (\Delta e^a)^2 \left\{\left[\zeta^2 + (d_t^a + z_t^a)\right]^2 - \varsigma^2\right\} \\ &\quad -2\Delta e^a \left[\theta\zeta u_{t+1}^a + \overline{g}_t^a (d_t^a + z_t^a) + \varsigma^2 w^a \Delta e^b \left(\pi_{t+1}^b - 1\right)\right] \\ &\quad -2\Delta e^a \rho_{t+1}^a d_t^a (d_t^a + z_t^a) \\ &\quad -2(\Delta e^a)^2 \left\{\left[\zeta^2 + (d_t^a + z_t^a)\right]^2 - \varsigma^2\right\} \pi_{t+1}^a \\ &= V_{t+1}^a - \varphi^a \rho_{t+1}^a - \alpha^a \pi_{t+1}^a \qquad (28)\end{aligned}$$

where

$$V_{t+1}^{a} = \Gamma_{t+1}^{a} + \frac{1}{2}\alpha^{a} - 2\Delta e^{a}\left[\theta\zeta u_{t+1}^{a} + \overline{g}_{t}^{a}\left(d_{t}^{a} + z_{t}^{a}\right) + \varsigma^{2} w^{a} \Delta e^{b}\left(\pi_{t+1}^{b} - 1\right)\right]$$

is the gross benefit of maintaining the peg,

$$\varphi^{a} = 2d_{t}^{a}\left(d_{t}^{a} + z_{t}^{a}\right)\Delta e^{a}$$

and

$$\alpha^{a} = 2\left(\Delta e^{a}\right)^{2}\left\{\left[\zeta^{2} + \left(d_{t}^{a} + z_{t}^{a}\right)\right]^{2} - \varsigma^{2}\right\}$$

Equation (28) says that the net benefit of keeping the peg depends not only on the fundamentals, summarized by the term V_{t+1}^{a}, but also on the credibility of the government's commitment to it, indicated by π_{t+1}^{a}, and on the risk aversion of the international investor, as captured by the risk premium term ρ_{t+1}^{a}. Given the macroeconomic conditions, a lower credibility, as implied by a higher π_{t+1}^{a}, or a higher risk aversion, as implied by a higher ρ_{t+1}^{a}, reduces the benefit of the peg.

The decision rule of the policymaker is derived optimally, taking into account the probability of devaluation π_{t+1}^{a} formulated by the private international investor, that is:

$$\phi_{t+1}^{a} = E_{t}\left(V_{t+1}^{a}\right) \tag{29}$$

and:

$$V_{t+1}^{a} - \phi_{t+1}^{a} = \epsilon_{t+1}^{a} \sim \text{i.i.d.}N(0, \sigma_{\epsilon^{a}}) \tag{30}$$

The variable ϕ_{t+1}^{a} summarizes the exogenous fundamentals affecting the probability of devaluation at time $t+1$ in country a.

The international investor, in turn, formulates rational expectations about the probability of devaluation as depending on the net benefit of the peg becoming negative, i.e.:

$$\pi_{t+1}^{a} = \Pr[\overline{V}_{t+1}^{a} < 0]$$

Using equations $(28)-(30)$, the probability of devaluation may be rewritten as:

$$\pi_{t+1}^{a} = \Pr[\epsilon_{t+1}^{a} < \alpha^{a}\pi_{t+1}^{a} + \varphi^{a}\rho_{t+1}^{a} - \phi_{t+1}^{a}]$$

or:

$$\pi_{t+1}^{a} = F[\alpha^{a}\pi_{t+1}^{a} + \varphi^{a}\rho_{t+1}^{a} - \phi_{t+1}^{a}] \tag{31}$$

where $F(\cdot)$ is the cumulative distribution function of $f(\cdot)$, with the latter being the density function of ϵ_{t+1}^{a}.

Furthermore, the probability of devaluation in country a depends on the probability of devaluation in b, π_{t}^{b}, through the further channel represented by the trade balance and competitiveness effects, since $\pi_{t+1}^{a} = \Pr[\overline{V}_{t+1}^{a} < 0]$ will increase with the probability of devaluation of country b's currency through a change in its real exchange rate. ■

Equation (31) implies that contagious currency crises with self-fulfilling expectations can arise from the endogeneity of the risk premium itself. In fact, the devaluation probability of country a, for example, depends on itself and on the risk premium on country a's asset, with the latter depending, in turn, on the probability of devaluation of country a's and of all other countries in the investors' portfolios, as discussed above.

To get the intuition of the model, consider a representative investor holding risky assets issued by two emerging market countries as well as a risk-free bond issued by a developed country (US is a reasonable assumption). The investor will require a risk premium on the risky assets to hold them. The risk premium is inversely related to unexpected changes in consumption, which in turn depends on the devaluation probability of emerging markets' currencies. Hence, an increase in the devaluation probability of country a's currency will decrease the consumption of the investor who will in turn ask for a higher risk premium on all assets, including those of country b. This will raise the cost of debt service in country b and hence will reduce the government's net benefit of maintaining the peg.[3] Thus, the devaluation probability of country b's currency will raise through equation (31), which in turn will widen the risk premium on country b's asset, and so forth, thus feeding a self-fulfilling expectation process. Hence, the international transmission of financial crises relies on portfolio re-balancing driven by wealth effects, and is magnified by self-fulfilling expectations. Finally, the wealth effect acts differently on the probability of devaluation according to the amount of debt accumulated by the emerging country, as equation (28)-(31) show.

Such a model can provide a theoretical framework useful to explain some of the empirical evidence reported in the introduction. For example, a commercial bank holding claims issued by a number of countries in the same region (e.g. East Asia) may be hit by the devaluation of one of its debtors' currency and see its financial wealth reduced. This may lead the bank to reassess the risk premium required on the bonds issued by all other countries as a consequence of the increase in effective risk aversion. The spreading of the currency crisis to the whole region is thus triggered by an investors' sentiment shift. This can spread internationally if international investors holding a geographically diversified portfolio are induced to a re-allocation of their wealth as it is hit by a currency devaluation, consistently with the strong correlation of emerging markets sovereign spreads documented by Baig and Goldfajn (2001). Whether this will in fact happen or not will depend on how large the loss is in percentage terms of the portfolio, as the behaviour is dependent on the coefficient of relative

[3] The expected cost of debt service in real terms is given by equation (9) which for country b is:

$$E_t(r^b_{t+1}) = r^{*f}_{t+1} + \rho^b_{t+1} + \pi^b_{t+1}\Delta e^b.$$

Following a crisis in country a, this cost will increase even if the expected devaluation is zero ($\pi^b_{t+1} = 0$) because of the higher risk premium ρ^b_{t+1}.

risk aversion. The same model can help explain why other financial crises did not spread from the origin country, as it happened for Argentina in 2001-2002[4]. In that case, the sharp decline in the correlation between emerging markets' sovereign spreads and stock indexes can be the result of a previous re-allocation of the international investors' portfolios away from Argentinean assets triggered by frequent signals of instability over the months preceding December 2001, the conventional starting date of the Argentinean crisis. Thus even though investors would have suffered losses, these would be small relative to the portfolio. Hence, the effect on the risk premium would be small.

Moreover, the specification of the two countries' trade balance as dependent on the real exchange rate makes the devaluation expectations depend on trade competitiveness. This framework can give a complete picture of how contagion works. In fact, a portfolio effect, a competitiveness effect, or both can increase the probability of devaluation in one country as a consequence of the expected devaluation in another country. The link between the expectations of devaluation in the two countries is provided by the inclusion of the risk premium and by trade competitiveness on a third market. Finally, the crisis can also be triggered simultaneously in the two countries by a common global shock captured by the rate of interest of a risk-free bond issued by the industrial country. The empirical evidence provided by, for example, Glick and Rose (1999), according to which contagion tends to occur as a consequence of trade links and competitiveness effects, is consistent with the theoretical model here presented.

Notice that adding the trade competitiveness channel makes this model capable of explaining the regional complexion of contagion through a further mechanism since trade linkages tend to be stronger at a regional level.

Some other papers also study changes in risk aversion and wealth effects as possible sources of contagion. Kyle and Xiong (2001), Goldstein and Pauzner (2004), and Broner *et al.* (2006) are the main examples. Our model, however, differs from these studies. First we explain contagion in the framework of a second generation model of currency crises, thus extending a tool which has been extensively used to understand the crisis episodes of the nineties. Second, our model allows simultaneously for all the channels of contagion, namely trade, financial links and self-fulfilling expectations with multiple equilibria, thus reconciling the most important explanations always considered separately in previous literature. Third, in our model we do not use heterogeneous agents or information frictions. Fourth, this model does not focus exclusively on currency or financial crises, but rather explains the relation between the two. Finally and most importantly, these papers rely on decreasing absolute risk aversion. Thus, a given loss will have a similar effect no matter how large or small it is relative to the portfolio. Our paper brings out the crucial role of decreasing relative risk aversion. Thus for understanding the transmission of financial crises through the wealth effect it is important to know how large the losses are relative to the portfolio.

[4]The lack of contagious effects from the Argentinean crisis has been documented, among others, by the IMF (2002) and Boschi (2005).

An interesting and policy relevant implication of this model is that the currency composition of debt matters for contagion. In fact, a currency devaluation only affects investors' wealth if their portfolios include assets denominated in local currency (see equations (14) and (19)) . If this is not the case, than currency devaluations and the risk premium are decoupled. On the other hand, if international portfolios include either dollar and local currency-denominated assets, currency and financial crises are linked: an increase in the devaluation probability can induce investors not to roll over existing investments and thus trigger a solvency crisis.

Jeanne (1997) illustrates the conditions under which an equation similar to (31) may have multiple solutions. Such conditions require the fundamentals to lie within a certain interval $\left(\underline{\phi}, \overline{\phi}\right)$. Following Jeanne's line of argument, in the Appendix we show how the introduction of a risk premium modifies the Jeanne's model affecting the conditions for multiple equilibria and the range of fundamentals. We find that an increase in the probability of devaluation shifts to the right the interval of fundamentals within which multiple equilibria may arise. This makes more difficult for the emerging country to avoid dropping in the multiple equilibria range by running very good fundamentals. Conversely, running bad fundamentals increases the chance that equation (31) holds with a unique equilibrium characterized by a high crisis probability. Notice that if α and φ are small enough, then

$$\frac{\partial\overline{\phi}}{\partial\overline{\pi}}, \frac{\partial\underline{\phi}}{\partial\underline{\pi}} < 0$$

and an increase in the probability of devaluation makes the region for multiple equilibria shift to the left. We see, then, that taking into account investors' risk aversion makes sound macroeconomic policies more compelling to the policymaker.

3 Policy measures to avoid crises and contagion

Recently, a number of authors have invoked some sort of restrictions on capital mobility in order to stabilize the international financial system. For example, Krugman (1999) argues that countries which cannot adopt either currency unions or free floating exchange rates should limit capital flows. Stiglitz (1999) endorses the same view with special reference to developing countries. Eichengreen *et al.* (1995) argue that as real markets adjust sluggishly to shocks, a second best can be achieved by "throwing sand in the wheels" of international finance through a global foreign exchange transaction tax in the spirit of Tobin (1978).

These views were broadly discussed in the aftermath of the frequent currency crises of the 1990s. In this Section we analyse the effects of such restrictions on the probability of transmission of crises in the context of our model.

3.1 Capital controls

The present set up cannot accommodate for features of capital controls such as the effects on the composition of inflows or their effectiveness and timing. Thus we model this restriction by introducing a no-short-selling constraint on either dollar- and local currency-denominated risky assets held by private investors:

$$x_s^{*m}, x_s^m \geq 0 \tag{32}$$

In order to simplify notation and make the argument clearer, we assume that the marginal effect on expected utility of relaxing the nonnegative constraints (32) is invariant across different currency-denominated assets issued by the same country, and that the market value of dollar-denominated debt is equal to the market value of local currency-denominated debt converted to dollars, i.e. $B_s^{*m} = B_s^m / e_s^m$. As shown below, these assumptions allow for the same effect of capital controls on country m's assets denominated in different currencies.

Proposition 5 *The introduction of a no-short-selling constraint on risky assets reduces the probability of currency devaluation.*

Proof. When the constraints (32) are added to the model, the Euler equations (3) and (4) change as follows:

$$u'(C_s)B_s^{*m} = \beta E_s \left\{ u'(C_{s+1}) \left(I_{s+1}^{*m} + B_{s+1}^{*m} \right) \right\} + v_{s+1}^m \tag{33}$$

$$u'(C_s)\frac{B_s^m}{e_s^m} = \beta E_s \left\{ u'(C_{s+1}) \left(\frac{I_{s+1}^m + B_{s+1}^m}{e_{s+1}^m} \right) \right\} + v_{s+1}^m \tag{34}$$

where v_{s+1}^m is the Lagrange multiplier associated with constraint (2) at time $s+1$, denoting the increase in expected lifetime utility that would result if the current constraint were relaxed by one unit. It is assumed to be equal across assets issued by the same country. Going again through the steps discussed in Section 2 leads to the following expression for the risk premium on dollar-denominated risky assets under capital controls, for $s = t$:

$$\begin{aligned} E_t(1 + r_{t+1}^{*m}) - (1 + r_{t+1}^{*f}) &= -(1 + r_{t+1}^{*f}) Cov \left\{ \frac{\beta u'(C_{t+1})}{u'(C_t)}, r_{t+1}^{*m} \right\} \\ &\quad - \frac{v_{t+1}^m (1 + r_{t+1}^{*f})}{B_t^{*m} u'(C_t - X_t)} \\ &= \rho_{t+1}^m - \Theta_{t+1}^m \end{aligned} \tag{35}$$

where ρ_{t+1}^m is the risk premium we derived in Section 2 under free capital mobility and

$$\Theta_{t+1}^m = \frac{v_{s+1}^m (1 + r_{t+1}^{*f})}{B_t^{*m} u'(C_t)}.$$

An analogous expression can be obtained for the risk premium on local currency-denominated risky assets:

$$\begin{aligned} E_t\left[\frac{e_t^m}{e_{t+1}^m}(1+r_{t+1}^m)\right]-(1+r_{t+1}^{*f}) &= -(1+r_{t+1}^{*f})Cov\left\{\frac{\beta u'(C_{t+1})}{u'(C_t)},r_{t+1}^{*m}\right\} \\ &\quad -\frac{e_s^m v_{s+1}^m(1+r_{t+1}^{*f})}{B_t^m u'(C_t)} \\ &= \rho_{t+1}^m-\Theta_{t+1}^m \end{aligned} \quad (36)$$

where the third equality derives from the assumptions that the risk premium under free capital mobility ρ_{t+1}^m is equal across country m's assets and that $B_s^{*m}=B_s^m/e_s^m$.

The new expression for the probability of devaluation of country m's currency under capital controls, $\overline{\pi}_{t+1}^m$, is, therefore:

$$\overline{\pi}_{t+1}^m=F[\alpha\overline{\pi}_{t+1}^m+\varphi(\rho_{t+1}^m-\Theta_{t+1}^m)-\phi_{t+1}^m]<\pi_{t+1}^m \quad (31')$$

since $F[\cdot]$ is an increasing function. ■

Proposition 5 implies that the introduction of capital controls in the form of administrative measures can help stabilize the international financial system through a reduction of the risk premium and, in turn, a reduction of the probability of currency devaluation.

3.2 Tobin tax

We now apply our model to answer the following question: do indirect capital controls in the form of a Tobin tax, i.e. a foreign exchange transactions tax, reduce the probability of contagion of financial crises?

Proposition 6 *The introduction of a Tobin tax does not affect the risk premium demanded on risky assets, and thus does not affect the probability of devaluation.*

Proof. Remembering that the budget constraint is expressed in dollars, levying a tax with rate τ^m on each foreign exchange transactions modifies the international investor's budget constraint (2) as follows:

$$\begin{aligned} B_{s+1}^{*f}+\sum_{m=1}^{M}x_{s+1}^{*m}B_s^{*m}+\sum_{m=1}^{M}x_{s+1}^{m}(1-\tau^m)\frac{B_s^m}{e_s^m} &= (1+r_s^{*f})B_s^{*f}+ \\ &\quad +\sum_{m=1}^{M}x_s^{*m}\left(I_s^{*m}+B_s^{*m}\right) \\ &\quad \sum_{m=1}^{M}x_s^{m}\left[\frac{(1-\tau^m)\left(I_s^m+B_s^m\right)}{e_s^m}\right] \\ &\quad -C_s \end{aligned} \quad (37)$$

and the FOCs (3) and (4) remain unchanged since:

$$\frac{e^m_{s+1}(1-\tau^m)\left(I^m_{s+1}+B^m_{s+1}\right)}{e^m_s(1-\tau^m)B^m_s}=\frac{e^m_{s+1}\left(I^m_{s+1}+B^m_{s+1}\right)}{e^m_s B^m_s}.$$

■

Proposition 6 implies that levying a tax on foreign exchange transactions leaves unchanged the equilibrium returns on the risky asset and, thus, the representative investor's equilibrium behaviour. Therefore the risk premium and, in turn, the probability of currency devaluation and transmission of crises remain unchanged.

Interestingly enough, this result is different from Cordella (2003) who asserts that in a "bank run" model controls in the form of a tax on short-term capital inflows can increase expected returns by preventing bank runs. This, the argument goes, may lead to an increase of gross investments in the emerging market. Conversely, in the international asset pricing framework here presented the same kind of tax does not affect rates of return implying that the marginal investment decisions are unchanged.

Advocates of the Tobin tax as a means of stabilization of international financial markets argue that it should be universally and uniformly levied in order to be effective (Eichengreen *et al.* 1995). Again, in our framework a Tobin tax levied universally on all countries' securities and with a uniform rate is ineffective.

4 Conclusion

The existing literature presents an unsatisfactory partition of explanations of contagion between theories based on fundamentals and theories based on the investor behaviour. The model presented in this paper extends a standard second generation model of currency crises (Jeanne (1997)) by adding the international portfolio choice by risk averse investors. This framework allows us to nest both the main sources of contagion of financial crises, and introduces another dimension of non-linearity increasing the likelihood of self-fulfilling equilibria. It shows that financial crises can be transmitted across seemingly unrelated countries (e.g. Russia and Brazil) through the risk attitudes of international investors. Thus, to understand financial crises it is not sufficient to look at the countries in question, but also at the portfolios of international investors. Here what is important are not the magnitude of absolute losses but the losses of investors relative to their portfolios. International business cycle considerations through the wealth effects can also play a role in the incidence of financial crises. The model also suggests that bond spreads, in the event of a financial crisis, would change in emerging markets depending on the pattern of portfolio holdings of international investors. The model can help better understand the transmission of crises across markets which do not seem to be directly related to each other by emphasizing the role of capital flows and thus, integrating international trade and finance considerations. In preventing the transmission of crises a no-short-selling constraint by introducing frictions in international capital flows can help reduce the risk premium and thus, the instability originating from the self-fulfilling expectations of rational investors. However, a Tobin tax has no effect on the transmission of financial crises.

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6 Appendix: The range of fundamentals with multiple equilibria

Propositions 2 - 4 show that the risk premium is a function of the probability of devaluation, i.e.: $\rho_t = h(\pi_t)$. Hence, equation (31) can be rewritten as follows:

$$\pi = F[\alpha\pi + \varphi h(\pi) - \phi] \tag{38}$$

where time subscripts and country superscripts are omitted for simplicity. We assume that the density function $f(\cdot)$ is continuous, symmetric (i.e. $\forall\epsilon$, $f(-\epsilon) = f(\epsilon)$), and strictly increasing (decreasing) in $(-\infty, 0)$ $((0, \infty))$ respectively, that is it reaches its maximum at zero, where $\alpha\pi_0 + \varphi h(\pi_0) = \phi$. The possible multiplicity of equilibria arises from the fact that both sides of equation (38) are increasing with π. Figure (1) shows the case of a unique equilibrium and figure (2) the case of multiple equilibria.[5]

[Figure (1) about here]

[5] We are grateful to Paul Masson for providing us with the Gauss codes of the graphs of his 1999 paper.

[Figure (2) about here]

The 45° line plots the l.h.s. of equation (38), while the curve $C_\phi \equiv F[\alpha\pi + \varphi h(\pi) - \phi]$ plots the r.h.s. Equation (38) is satisfied at the intersections of the 45° line with C_ϕ. Following Jeanne (1997), if at its maximum the slope of C_ϕ is smaller than 1, i.e. $\frac{dC_\phi}{d\pi} < 1$, then it is such everywhere. Hence π is uniquely determined by ϕ and strictly decreasing with it. There may be multiple equilibria if $\frac{dC_\phi}{d\pi} > 1$ at its maximum. If this necessary condition holds, then there are two critical values of the fundamentals $\underline{\phi} < \overline{\phi}$ such that: if $\phi < \underline{\phi}$ or $\phi > \overline{\phi}$, the devaluation probability π is uniquely determined by (and strictly decreasing with) the fundamental ϕ. Therefore, the further condition for having multiple equilibria is that the fundamentals ϕ lie in the range $\left(\underline{\phi}, \overline{\phi}\right)$. The critical values of fundamentals $\underline{\phi}$ and $\overline{\phi}$ are identified by the tangency points between the 45° line and C_ϕ (see Figure (3)).

[Figure (3) about here]

Whether these tangency points lie to the right or to the left of the point of maximum value of the pdf $f(\cdot)$ cannot be determined analytically. As shown below, in fact, whether the value of π at which C_ϕ is tangent to the 45° line lies to the right or to the left of π_0 depends on the sign of $h''(\pi)$ which depends on the sign of $f'(\cdot)$ which in turn depends on whether π is on the right or on the left of π_0. Fortunately enough, it can be shown easily that the following results are independent of where the tangency points lie with respect to π_0. Therefore, we will make the further simplifying assumptions that $\overline{\pi} > \pi_0$, where $\overline{\pi}$ is the probability of devaluation corresponding to $\overline{\phi}$, and $\underline{\pi} < \pi_0$, with $\underline{\pi}$ defined correspondingly. This also allows an easier comparison to the derivation in Jeanne (1997).

We will now determine the upper bound $\overline{\phi}$ and lower bound $\underline{\phi}$ of the range of fundamentals within which multiple equilibria may arise.

Upper bound $\overline{\phi}$ The tangency conditions between the function $C_{\overline{\phi}} = F[\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi}]$ and the 45° line are:

$$\overline{\pi} = F[\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi}] \tag{39}$$

$$1 = f[\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi}] \cdot [\alpha + \varphi h'(\overline{\pi})] \tag{40}$$

From equation (40), recalling that by assumption $\overline{\pi} > \pi_0$, we have:

$$\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi} = f^{-1}\left[\frac{1}{\alpha + \varphi h'(\overline{\pi})}\right] \tag{41}$$

where $f^{-1}(\cdot) : [0, f(0)] \to \Re^{+}$ denotes the inverse function of $f(\cdot)$ that takes positive values.

Substituting out $\overline{\pi}$ and rearranging yields:

$$\begin{aligned}\overline{\phi} &= \alpha F[\alpha\overline{\pi}+\varphi h(\overline{\pi})-\overline{\phi}]+\varphi h\left\{F[\alpha\overline{\pi}+\varphi h(\overline{\pi})-\overline{\phi}]\right\} \\ &\quad -f^{-1}\left[\frac{1}{\alpha+\varphi h'(\overline{\pi})}\right] \qquad (42)\end{aligned}$$

Finally, substituting out $\alpha\overline{\pi}+\varphi g(\overline{\pi})-\overline{\phi}$ given by equation (41) into equation (42), we obtain:

$$\begin{aligned}\overline{\phi} &= \alpha F\left\{f^{-1}\left[\frac{1}{\alpha+\varphi h'(\overline{\pi})}\right]\right\}+\varphi h\left(F\left\{f^{-1}\left[\frac{1}{\alpha+\varphi h'(\overline{\pi})}\right]\right\}\right) \\ &\quad -f^{-1}\left[\frac{1}{\alpha+\varphi h'(\overline{\pi})}\right] \qquad (43)\end{aligned}$$

Proposition 7 *If α and φ are large enough, the upper bound of the range of fundamentals for which there are multiple equilibria increases with the probability of devaluation.*

Proof. In order to prove this proposition we need to know how $h(\cdot)$ and $h'(\cdot)$ change with $\overline{\pi}$, i.e. we need to know the sign of $h'(\overline{\pi})$ and of $h''(\overline{\pi})$. We know from Proposition 3 that $h'(\overline{\pi})>0$, $\forall\overline{\pi}$: this is because an increase in the probability of devaluation decreases the expected rate of return on assets which in turn makes the risk premium go up through a wealth effect. From equations (39) and (41) we have:

$$\overline{\pi}=F\left(f^{-1}\left\{L\left[h'(\overline{\pi})\right]\right\}\right) \qquad (44)$$

where:

$$L\left[h'(\overline{\pi})\right]=\frac{1}{\alpha+\varphi h'(\overline{\pi})} \qquad (45)$$

We can sign $h''(\overline{\pi})$ applying the Implicit Function Theorem (IFT) to equation (44). Define:

$$G\left[\overline{\pi},h'(\overline{\pi})\right]\equiv\overline{\pi}-F\left(f^{-1}\left\{L\left[h'(\overline{\pi})\right]\right\}\right)$$

Then:

$$\frac{dh'(\overline{\pi})}{d\overline{\pi}}=-\frac{\partial G/\partial\overline{\pi}}{\partial G/\partial h'(\overline{\pi})}$$

where:

$$\partial G/\partial\overline{\pi}=1$$

and

$$\partial G/\partial h'(\overline{\pi})=-\frac{\partial F}{\partial f^{-1}}\cdot\frac{\partial f^{-1}}{\partial L}\cdot\frac{\partial L}{\partial h'}<0$$

since $\frac{\partial F}{\partial f^{-1}}>0\ \forall f^{-1}$ being F the cdf of a normal distribution; $\frac{\partial f^{-1}}{\partial L}<0$ being f^{-1} the inverse function of a normal pdf that takes positive values and, by the

Inverse Function Theorem, $\left(f^{-1}\right)' = 1/f'$; and, finally, being from equation (45):

$$\frac{\partial L}{\partial h'} = \frac{-\varphi}{\{[\alpha + \varphi h'(\overline{\pi})]\}^2} < 0, \ \forall h'$$

Then:

$$\frac{dh'(\overline{\pi})}{d\overline{\pi}} = -\frac{1}{-\frac{\partial F}{\partial f^{-1}} \cdot \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'}} > 0$$

Applying the Chain rule to equation (43) we find how $\overline{\phi}$ changes with $\overline{\pi}$:

$$\begin{aligned}
\frac{\partial \overline{\phi}}{\partial \overline{\pi}} &= \alpha \cdot \frac{\partial F}{\partial f^{-1}} \cdot \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \overline{\pi}} \\
&\quad + \varphi \cdot \frac{\partial h}{\partial F} \cdot \frac{\partial F}{\partial f^{-1}} \cdot \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \overline{\pi}} - \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \overline{\pi}} \\
&= \left[\underbrace{\frac{\partial F}{\partial f^{-1}} \left(\alpha + \varphi \frac{\partial h}{\partial F} \right) - 1}_{>0} \right] \cdot \underbrace{\frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \overline{\pi}}}_{>0} > 0 \\
&\quad \text{for } \alpha \text{ and } \varphi \text{ large enough}
\end{aligned}$$

■

Lower bound $\underline{\phi}$ Recall that we are assuming $\underline{\pi} < \pi_0$, although this is not essential to the derivation of what follows.

The tangency conditions between the function $C_{\underline{\phi}} = F[\alpha \underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}]$ and the 45° line are:

$$\underline{\pi} = F[\alpha \underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}] \tag{46}$$

$$1 = f[\alpha \underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}] \cdot [\alpha + \varphi h'(\underline{\pi})] \tag{47}$$

From equation (47) we have (recall that $f^{-1}(\cdot)$ is defined as the inverse function of $f(\cdot)$ that takes positive values, and thus we need to change the sign of $\alpha \underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}$, being this negative since $\underline{\pi} < \pi_0$):

$$-\left[\alpha \underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}\right] = f^{-1}\left[\frac{1}{\alpha + \varphi h'(\underline{\pi})}\right] \tag{48}$$

Substituting out $\underline{\pi}$ and rearranging we obtain:

$$\begin{aligned}
\underline{\phi} &= \alpha F\left\{-f^{-1}\left[\frac{1}{\alpha + \varphi h'(\underline{\pi})}\right]\right\} + \varphi h\left(F\left\{-f^{-1}\left[\frac{1}{\alpha + \varphi h'(\underline{\pi})}\right]\right\}\right) \\
&\quad + f^{-1}\left[\frac{1}{\alpha + \varphi h'(\underline{\pi})}\right]
\end{aligned} \tag{49}$$

Proposition 8 *If α and φ are large enough, the lower bound of the range of fundamentals for which there are multiple equilibria increases with the probability of devaluation.*

Proof. In order to prove this proposition we need to know the sign of $h'(\underline{\pi})$ and of $h''(\underline{\pi})$. By Proposition 3 $h'(\underline{\pi}) > 0 \ \forall \underline{\pi}$. From equations (46) and (48) we have:

$$\underline{\pi} = F\left(-f^{-1}\left\{L\left[h'(\underline{\pi})\right]\right\}\right) \tag{50}$$

where:

$$L\left[h'(\underline{\pi})\right] = \frac{1}{\alpha + \varphi h'(\underline{\pi})} \tag{51}$$

Again, we can sign $h''(\underline{\pi})$ applying the IFT to equation (50). Define:

$$G\left[\underline{\pi}, h'(\underline{\pi})\right] \equiv \underline{\pi} - F\left(-f^{-1}\left\{L\left[h'(\underline{\pi})\right]\right\}\right)$$

Then:

$$\frac{dh'(\underline{\pi})}{d\underline{\pi}} = -\frac{\partial G/\partial \underline{\pi}}{\partial G/\partial h'(\underline{\pi})}$$

where:

$$\partial G/\partial \underline{\pi} = 1$$

and

$$\partial G/\partial h'(\underline{\pi}) = -\frac{\partial F}{\partial\left(-f^{-1}\right)} \cdot \frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} > 0$$

since $\frac{\partial F}{\partial(-f^{-1})} > 0 \ \forall \left(-f^{-1}\right)$; $\frac{\partial(-f^{-1})}{\partial L} > 0$, and from equation (51):

$$\frac{\partial L}{\partial h'} = \frac{-\varphi}{\left\{\left[\alpha + \varphi h'(\underline{\pi})\right]\right\}^2} < 0, \ \forall h'$$

Then:

$$\frac{dh'(\underline{\pi})}{d\underline{\pi}} = -\frac{1}{-\frac{\partial F}{\partial(-f^{-1})} \cdot \frac{\partial(-f^{-1})}{\partial L} \cdot \frac{\partial L}{\partial h'}} < 0$$

From (49) we have:

$$\begin{aligned}
\frac{\partial \underline{\phi}}{\partial \underline{\pi}} &= \alpha \cdot \frac{\partial F}{\partial\left(-f^{-1}\right)} \cdot \frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \underline{\pi}} \\
&\quad + \varphi \cdot \frac{\partial h}{\partial F} \cdot \frac{\partial F}{\partial\left(-f^{-1}\right)} \cdot \frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \underline{\pi}} - \frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \underline{\pi}} \\
&= \left[\underbrace{\frac{\partial F}{\partial\left(-f^{-1}\right)}\left(\alpha + \varphi \frac{\partial h}{\partial F}\right) - 1}_{>0}\right] \cdot \underbrace{\frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \underline{\pi}}}_{>0} > 0 \\
&\quad \text{for } \alpha \text{ and } \varphi \text{ large enough}
\end{aligned}$$

■

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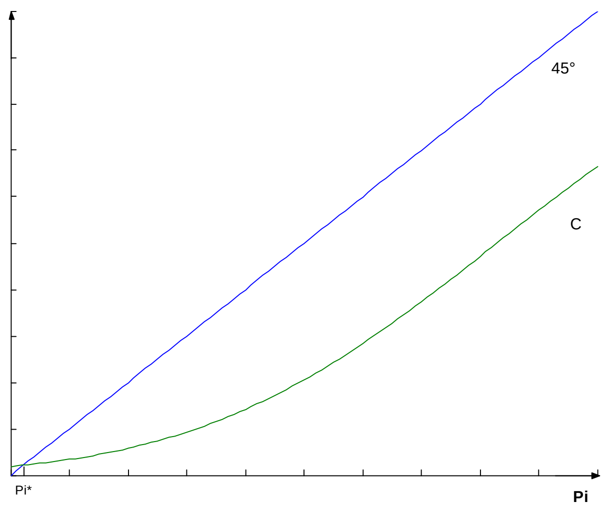

Figure 1. Unique equilibrium.

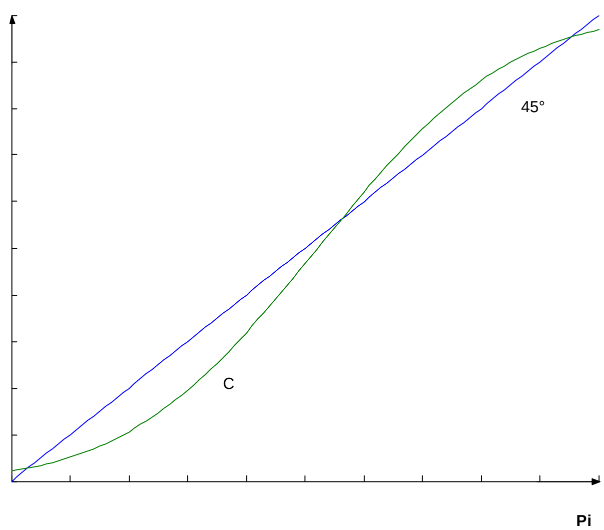

Figure 2. Multiple equilibria.

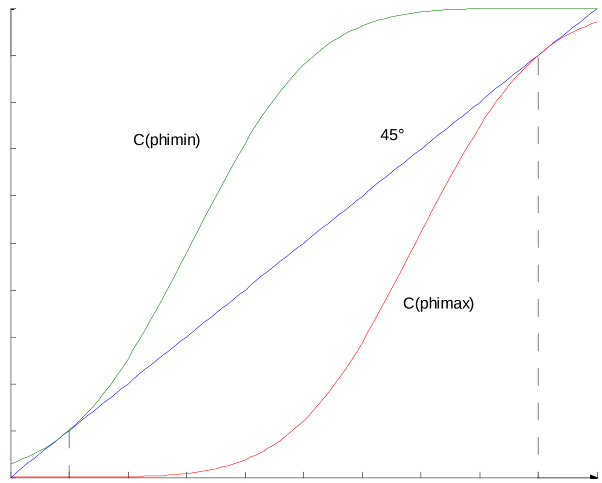

Figure 3. Critical values of fundamentals.