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Abstract: Contests often involve players vying for the same prize year after year. This paper characterizes equilibrium effort, both individual and aggregate, in a general parameterization of such repeated contests.

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1. Introduction

Contests over a fixed prize are often repeated, such as firms competing each year for recurring government contracts, or agents vying to represent new and established talent. While the importance of dynamic contests is widely acknowledged,[1] insight has been mixed regarding how dynamic interactions affect effort expended. Tullock (1988) conjectured that dynamic contests might induce more wasteful effort than their static counterparts, but did not formally analyze that issue. More recent dynamic models suggest the opposite (e.g., Cairns, 1989; Wirl, 1994; Shaffer and Shogren, 2008).[2]

This paper expands our understanding of rent-seeking behavior in dynamic contests by analyzing equilibrium effort levels in a general parameterization of a repeated logit contest.[3] Our goal is to explore which patterns of conduct are robust to generic dynamic equilibrium concepts, to complement prior studies that restricted attention to one or two special cases. We show that the share of the prize dissipated by effort is independent of the size of the prize for any dynamic contest, while individual and aggregate effort respond in opposite directions to either the number of players or returns to effort.

Importantly, both individual and aggregate effort decline with the degree of cooperation. Since repeated interaction affords an opportunity for dynamic enforcement mechanisms, we might generically expect more cooperative behavior than in static one-shot contests. Our finding then suggests that the recent literature contrasting with Tullock's (1988) conjecture may hold quite generally.

2. Analysis

Consider a repeated contest between two players, 1 and 2, who expend effort, x_1 and x_2, for an exogenous prize of common value G. Following Tullock (1980) and others, assume a logit payoff function, which can be interpreted as a probability of winning an indivisible prize or as a share of a divisible prize. We adopt the latter terminology below, though our analysis is valid for either interpretation. Denote player 1's share of the prize in the stage game as:

$$p(x_1; x_2, \alpha, \gamma) = \alpha x_1^{\gamma} / (\alpha x_1^{\gamma} + x_2^{\gamma}), \tag{1}$$

where α is an ability parameter reflecting the relative strength of the players, and γ parameterizes the productivity of effort. Without loss of generality, assume player 1 is the contest favorite — his share of the prize exceeds one-half at the static Nash equilibrium ($\alpha > 1$) — and player 2 is the underdog (Dixit, 1987). If instead $\alpha = 1$, the players are identical and the payoffs symmetric.

The net payoffs to player 1 and 2 are $\pi_1 = Gp(x_1;x_2,\alpha,\gamma) - x_1$ and $\pi_2 = G[1 - p(x_2;x_1,\alpha,\gamma)] - x_2$. If the players behave according to the static Nash equilibrium in the stage game, standard calculations give the following symmetric effort levels (Shogren and Baik, 1992):

$$x_1 = x_2 = [\alpha\gamma/(1 + \alpha)^2]G. \tag{2}$$

Player 1's (2's) net payoff is $\alpha G(1+\alpha-\gamma)/(1+\alpha)^2$ (respectively, $G(1+\alpha-\alpha\gamma)/(1+\alpha)^2$). Equation (2) is an increasing function of γ, reflecting the players' equilibrium response to the higher marginal payoff to rent-seeking associated with larger values of γ.[4] Shaffer and Shogren (2008) establish restrictions on the parameters implied by the second-order conditions.

A repeated contest (with either a finite or infinite horizon) will in general induce different choices of effort than the static Nash choice. Because there are many possible dynamic equilibrium concepts, a general characterization of repeated contests requires a model that parameterizes the conduct of the players. Many studies have shown that the conjectural variation, though often interpreted as a static conduct parameter, can explicitly represent conduct in a wide variety of dynamic games, even with incomplete information or bounded rationality (Worthington, 1990; Dockner, 1992; Cabral, 1995; Pfaffermayr, 1999; Itaya and Shimomura, 2001; Friedman and Mezzetti, 2002; Itaya and Okamura, 2003; Figuieres et al., 2004; Saracho, 2005). Indeed, though the conjectural variation cannot provide a theoretical basis for selecting among alternative equilibrium concepts, its only limitation in representing diverse equilibria lies in its implicit assumption that players have continuous reaction functions, which Friedman and Samuelson (1990, 1994) have shown is not restrictive.

The conjectural variation λ_i is defined as $\sum_j \partial x_{j\neq i}/\partial x_i$ for arbitrary numbers of players i and j, where larger positive values of λ_i correspond to greater degrees of cooperation.[5] In the symmetric case ($\alpha = 1$ and $\lambda_i = \lambda$) with n players, the contest payoff for player i is $x_i^\gamma G/\sum_j x_j^\gamma - x_i$ and the first-order condition for equilibrium effort is:

$$0 = \gamma x_i^{\gamma-1}G/\textstyle\sum_j x_j^\gamma - 1 - x_i^\gamma G[\gamma x_i^{\gamma-1} + \sum_{j\neq i} \lambda\gamma x_j^{\gamma-1}]/(\sum_j x_j^\gamma)^2 \qquad (3)$$

which by symmetry reduces to:

$$0 = \gamma x_i^{\gamma-1}G/nx_i^\gamma - 1 - \gamma x_i^{2\gamma-1}G[1 + (n-1)\lambda]/n^2x_i^{2\gamma} \qquad (4)$$

implying an equilibrium effort in each period of:

$$x_i = \gamma G(n-1)(1 - \lambda)/n^2. \qquad (5)$$

The share of the prize dissipated by each player's effort, x_i/G, is independent of the size of the prize in any such equilibrium. The share of the prize dissipated in aggregate, nx_i/G, is also independent of G but increasing in n for all $\lambda < 1$ and $\gamma > 0$. Since $\partial x_i/\partial\lambda = -\gamma G(n-1)/n^2 < 0$, greater cooperation (i.e., larger λ) reduces effort for a given number of players, returns to effort (γ), and size of prize. Aggregate effort is decreasing in λ since $\partial nx_i/\partial\lambda = -\gamma G(1 - 1/n) < 0$.

More players reduces individual effort for all $n > 2$ and $\lambda_i < 1$, since $\partial x_i/\partial n = \gamma G(1 - \lambda)(2 - n)/n^2$, but increases aggregate effort for all $\lambda < 1$ since $\partial nx_i/\partial n = \gamma G(1 - \lambda)/n^2$. For unbounded numbers of players, aggregate effort asymptotically approaches $\gamma G(1 - \lambda)$. Greater returns to effort increases both individual and aggregate effort for all $n > 1$ and $\lambda < 1$, since $\partial x_i/\partial\gamma = G(n - 1)(1 - \lambda)/n^2$ and $\partial nx_i/\partial\gamma = G(n - 1)(1 - \lambda)/n$. These results are summarized as:

Proposition 1. *The general, imperfectly cooperative, symmetric dynamic contest parameterized by conjectural variations exhibits the following properties:*

(a) individual effort declines with the number of players and the degree of cooperation, but increases with returns to effort;
(b) aggregate effort increases with the number of players and returns to effort, but declines with the degree of cooperation;
(c) the share of the prize dissipated by effort is independent of the size of the prize.

In the two-player contest parameterized by conjectural variations where player 1 is the favorite ($\alpha > 1$) and the payoff corresponds to equation (1), equilibrium effort for player 1 is:

$$x_1 = \alpha\gamma G(1 - \lambda)/(1 + \alpha)^2. \qquad (6)$$

A change in ability affects equilibrium effort according to $\partial x_1/\partial\alpha = \gamma G(1 - \lambda)(1 - \alpha)/(1 + \alpha)^3$. If player 1 is the favorite ($\alpha > 1$), then $\partial x_1/\partial\alpha < 0$ for all $\lambda < 1$, implying that the favorite's effort is a decreasing function of the size of her advantage over the underdog. That is, the favorite reduces effort as the contest becomes more unevenly matched (see also Baik and Shogren, 1992). Also,

$\partial x_1/\partial\lambda = -\alpha\gamma G/(1 + \alpha)^2 < 0$ as in the symmetric game, while $\partial x_1/\partial\gamma = \alpha G(1 - \lambda)/(1 + \alpha)^2 > 0$ for all $\lambda < 1$. Proposition 2 summarizes these results:

Proposition 2. *In the asymmetric, imperfectly cooperative, dynamic contest parameterized by conjectural variations, the favorite's effort responds as follows:*

(a) decreases with the size of her advantage over the underdog;
(b) responds to the degree of cooperation as in the symmetric game analyzed above;
(c) responds to the returns to effort as in the symmetric game analyzed above.

3. Concluding Comments

This note explores properties of repeated rent-seeking games in a general parametric framework. We characterize how unequal ability, additional players, and more productive effort affect individual and aggregate effort levels. When the behavior of contestants is parameterized by conjectural variations, and if everyone has equal ability, additional players and smaller returns to effort lead to less individual effort. More players, however, will increase aggregate effort. With unequal ability, the favorite's effort is inversely related to the size of her advantage over the underdog — the more lopsided the contest, the less effort expended. More cooperative behavior, such as sustained by dynamic enforcement mechanisms, leads to lower individual and aggregate effort, a finding that generalizes previous analysis of dynamic contests and contrasts with a conjecture by Tullock (1988).

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Footnotes

1. See for example Brooks and Heijdra (1989).

2. Other studies of dynamic contests include Leininger and Yang (1994), Monahan and Sobel (1994), Schnytzer (1994), and Hausken (1995).

3. See Tullock (1980), Dixit (1987), Baik and Shogren (1992) for the standard model.

4. The marginal payoff to x_1 changes with γ as $\partial^2 p/\partial\gamma\partial x_1 = \alpha/[x_1(1 + \alpha)^2] > 0$ at $x_1 = x_2$, while the marginal payoff to x_2 changes with γ as $\partial^2 p/\partial\gamma\partial\, x_2 = 1/[\, x_2(1 + \alpha)^2] > 0$ at $x_1 = x_2$.

5. A few of the specific types of dynamic games analyzed in the conjectural variations literature include open-loop, closed-loop, k-period punishment phase, and optimal punishment games.