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Ramsey Fiscal and Monetary Policy under Sticky Prices and Liquid Bonds*

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Abstract

We construct a monetary model where government bonds also provide liquidity service. Liquid government bonds affect equilibrium allocations, inflation and create an endogenous interest-rate spread. How this new feature alters optimal fiscal-monetary policy in a stochastic sticky-price environment is considered. The trade-off confronting a planner, shown in recent literature, between using inflation surprise and labor-income tax is eradicated by the existence of the liquid bond. We find that the more sticky prices become, the more the planner stabilizes prices, but the planner also creates less distortionary and less volatile income taxes by resorting to taxing the liquidity service of bonds.

Keywords: Optimal fiscal and monetary policy; sticky prices; liquid bonds

JEL classification: E42; E52; E63

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1. Introduction

There has been recent renewed interest amongst macroeconomists on the issue of optimal fiscal and monetary policy. The benchmark framework approaches this issue from the point of view of a Ramsey planner. In earlier literature on optimal fiscal and monetary policy, the analyses were often carried out using competitive flexible-price monetary models without capital, for example, Lucas and Stokey (1983), Calvo and Guidotti (1993), and Chari et. al. (1991). The general conclusion was that optimal fiscal-monetary policy entails volatile and serially uncorrelated inflation rate while labor income taxes are smooth. This is because the planner uses surprise inflation as a lump-sum tax on household financial wealth, while minimizing the distortionary effect of labor income tax. However, in such economies, the dynamics of government bonds play no instrumental role in the optimal fiscal-monetary policy plan.

Government debt dynamics also take a back seat in terms of optimal fiscal-monetary policy in typical Ricardian economies with sticky prices and costly inflation. In the seminal works of Schmitt-Grohé and Uribe (2004b) and Siu (2004), the authors provide a variation on the results found in the optimal fiscal-monetary policy literature. In such economies, inflation is costly in terms of real resources such that the planner has to trade-off between minimizing tax distortions and minimizing costly inflation volatility. On one hand, in order to minimize tax distortions on private work incentives, the planner would like to use unexpected variations in the price level as a means for taxing household wealth, which leads to greater inflation volatility. This is the same effect found in the earlier class of flexible price competitive economies. On the other, the existence of price-adjustment cost affects household welfare via their feasibility constraint. This discourages the planner from trading off unexpected inflation with labor income tax variations, resulting in lower inflation volatility. Schmitt-Grohé and Uribe (2004b) find that the second effect dominates. In other words, for modest degrees of price stickiness, the tension is resolved in the direction in favor of price stability or low inflation volatility. Furthermore, the tax rate on labor is still reasonably smooth or "near random walk", but this tends to be less so, when there is imperfect competition; or even less when there exist sticky prices. Siu (2004) also has very similar conclusions. Siu (2004) specifically reports that under an optimal Ramsey policy, the volatility of inflation decreases while that of the labor tax rate increases as the degree of price stickiness in the economy rises. He also finds that the tax distortion can be smoothed over time.[1]

In this paper, we introduce a not unrealistic feature of government bonds that provide some liquidity service.[2] This feature affords us two key outcomes that contrast with the current optimal

[1]The result in Siu (2004) and Schmitt-Grohé and Uribe (2004b), in terms of a near-unit-root feature of optimal income tax, echoes the outcome in Aiyagari et. al. (2002). In Aiyagari et. al. (2002), the model is perfectly competitive but features incomplete markets where there is only real non-state-contingent government debt. In our model, we do not assume away complete asset markets as Aiyagari et. al. (2002) do. On the contrary, there exists a hypothetical bond that replicates a complete asset market used for intertemporal consumption and risk-sharing. At the same time, private agents also want to hold assets in the form of government debt in exchange for their liquidity service although they pay a lower return than the hypothetical bond.

[2]Canzoneri and Diba (2005) provide a factual argument or example that, "... U.S. Treasury bills clearly facilitate transactions in a number of ways: they serve as collateral in many financial markets, banks hold them to manage the liquidity of their portfolios, and individuals hold them in money-market accounts that offer checking services."

fiscal-monetary policy literature that builds on models with costly inflation, in particular Schmitt-Grohé and Uribe (2004b) and Siu (2004). First, our environment differs from theirs. We show that government bonds affect the intertemporal allocations of resources simply because government bonds are valued by the private sector in terms of their transactions service. This provides an avenue for fiscal policy, in terms of government debt, to affect the path of inflation and real intertemporal allocations.[3]

Second, the existence of liquid, interest-bearing government bonds creates a spread between the returns on illiquid private bonds and liquid government bonds that acts as an additional tax instrument.[4] This is important for a planner to achieve the second best in an economy with monopolistic distortions and price rigidity. We find that the more sticky prices become, the more the optimal Ramsey plan favors price stability *but* the planner can also afford a less distortionary and *less volatile* income tax scheme. The latter result is opposite to that of Schmitt-Grohé and Uribe (2004b) and Siu (2004). To achieve the latter, the planner allows for more "interest-spread surprises" or volatility in the return on liquid government bonds relative to market bond return, and drives the price of liquidity of government bonds to be equal to that of money (a Friedman rule with respect to liquid bonds). It is possible for the optimal fiscal-monetary policy plan to do so in successively more price-sticky economies because the implicit tax on liquid government bonds merely distorts the distribution of liquidity holdings between real money balances and government bonds as a first-order effect. In contrast, inflation-surprise and labor income taxes have additional intertemporal distorting effects on household allocations of consumption and leisure. In short, the planner exploits the role of government bonds as shock absorber more than the usual suspects of surprise inflation and labor income tax.

The new addition in our model has a close counterpart in Canzoneri and Diba (2005). However, they were concerned with the issue of price level determinacy in a deterministic, partial-equilibrium and flexible-price model with simple monetary- and fiscal-policy rules. In their economy, fiscal policy can provide a nominal anchor, even when monetary policy does not. Their result arises because government bonds can provide liquidity services and this allows bonds to affect the equilibrium process for inflation. They allow for bonds to enter a cash-in-advanced (CIA) constraint and to act as imperfect substitutes for money. We generalize their assumption to a general equilibrium production economy with costly price adjustment. Furthermore, we consider optimal policy from the point of view of the benchmark Ramsey planner.

The remainder of the paper is as follows. We outline the model primitives and assumptions in Section 2. We show how a recursive decentralized equilibrium in the model can be solved as a Ramsey planning problem in Section 3. We calibrate the model and perform some numerical experiments to study the behavior of the Ramsey equilibria in Section 4. Finally we conclude in Section 5.

[3] In typical Ricardian models used to study optimal fiscal and monetary policy, often there is a single short-term interest rate. Furthermore, in this class of dynamic general equilibrium monetary models, the determination of equilibrium paths of real variables and inflation are independent of government debt dynamics (see e.g. Canzoneri and Diba (2005) and Walsh (2003), chapter 4).

[4] One can envision that the private sector can also issue liquid assets or bonds (e.g. credit cards, commercial paper and etc.). However, for the sake of clarity and exposition, we assume that there only exist a nominally risk-free private bond that is illiquid and the liquid government bond.

2. The Model

Consider an economy populated by a large number of infinitely lived identical households. Each household derives utility from consumption, c, and leisure, $1-h$ where time endowment is unity and h is the fraction of time spent working. Households are also monopolistic firms producing a differentiated intermediate good. Fiscal and monetary policy will be determined jointly by a Ramsey planner. We begin by specifying the exogenous stochastic processes in the model.

2.1. Exogenous stochastic processes

There are two exogenous forcing processes in the model. These can be interpreted as demand and supply shocks. On the demand side, government spending is a Markov process, where

$$\ln g_t = \left(1-\rho_g\right)\ln \overline{g} + \rho_g \ln g_{t-1} + u_{g,t};\;\; \rho_g \in [0,1), u_{g,t} \sim \text{i.i.d.}\left(0,\sigma_g^2\right). \tag{1}$$

where $\overline{g}$ is steady state government consumption. On the supply side, economy-wide shocks to production technology is given by the Markov process

$$\ln z_t = \rho_z \ln z_{t-1} + u_{z,t};\;\; \rho_z \in [0,1), u_{z,t} \sim \text{i.i.d.}\left(0,\sigma_z^2\right). \tag{2}$$

It is assumed that $(g_t, z_t)' \in \mathcal{S}$ where $\mathcal{S} \subset \mathbb{R}_+^2$ is compact.

2.2. Household-firm problem

Households are monopolistic firms producing a differentiated intermediate good, thus the demand for this monopolist's good is $d\left(\widetilde{P}_t/P_t\right)Y_t$, where $d'\left(\widetilde{P}_t/P_t\right) < 0$, $d(1) = 1$, and $d'(1) < -1$. The household-firm employs labor, $\widetilde{h}_t$, with a competitive nominal wage $w_t P_t$, and produce using a technology

$$d\left(\frac{\widetilde{P}_t}{P_t}\right)Y_t = z_t \widetilde{h}_t \tag{3}$$

Because each household-firm is monopolistic, they can set $\widetilde{P}_t$ and following Rotemberg (1982), we assume they face a real convex cost of price adjustment

$$C\left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}}\right) = \frac{\theta}{2}\left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}} - \overline{\Pi}\right)^2. \tag{4}$$

where θ will be a parameter governing the degree of price-stickiness and $\overline{\Pi} \geq 1$ is steady-state inflation.

Let $m = M/P$, $b = B/P$, $\Pi_t = P_t/P_{t-1}$ and $p_t = \widetilde{P}_t/P_t$ respectively denote real money balances, real government bond holdings, the inflation factor, and a firm-specific price relative to the average price level. The sequence of household budget constraints is given by

$$c_t + m_t + b_t + b_t^* \leq \frac{m_{t-1}}{\Pi_t} + R_{t-1}\frac{b_{t-1}}{\Pi_t} + R_{t-1}^*\frac{b_{t-1}^*}{\Pi_t} + \left[p_t Y_t d\left(p_t\right) - w_t \widetilde{h}_t - \frac{\theta}{2}\left(\frac{p_t}{p_{t-1}}\Pi_t - \overline{\Pi}\right)^2\right] + \left(1-\tau_t\right) w_t h_t. \tag{5}$$

for $t = 0, 1, 2, ...$, where $b_t^* \in \mathcal{B}^* \subset \mathbb{R}$ is a private bond that pays a nominally risk-free return of R_t^* in period $t+1$. The household's time-0 payoff is measured as the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{6}$$

where E_0 is the mathematical expectations operator, taken over the sequence of functions $U(c_t, h_t)$ measurable with respect to the information set generated by $\{z_t, g_t, b_t^*, b_t\}$ at time 0.[5] $U(\cdot)$ satisfies the Inada conditions: $\lim_{x \searrow 0} U'(x) = +\infty$ for $x = c$ or $x = 1 - h$. The household maximizes (6) subject to (5) and a cash-in-advance (CIA) constraint:

$$m_t + k(b_t) \geq c_t. \tag{7}$$

The transactions service of bonds is reflected in the function $k(b_t)$ which satisfies the following properties.

Assumption 1 *The function $k(b_t)$ satisfies:*

A1 $k(b_t) = 0$ *for* $b_t \leq 0$*;*

A2 $k'(b_t) > 0$ *and* $k''(b_t) < 0$ *for* $b_t > 0$*;*

A3 $\lim_{b \searrow 0} k'(b_t) < 1$, $\lim_{b \nearrow +\infty} k'(b_t) = 0$ *and* $\lim_{b \nearrow +\infty} k(b_t) < c_t$.

Assumption A1 ensures that negative bond holdings do not provide any transactions value so that $b_t \in \mathcal{B} \subset \mathbb{R}_+$, and A2 ensures that positive government bond holdings provide increasing transactions service, but the marginal transactions service is decreasing. Lastly, A3 ensures that these bonds are never sufficient to fund all consumption purchases.[6] That is, there will still be positive holdings of money.[7]

Let the Lagrange multiplier on the constraints (7) and (5) respectively μ_t be λ_t, and the multiplier on the technology constraint (3), when inserted into (5) be $mc_t\lambda_t$. The first-order conditions are

$$U_c(c_t, h_t) = \lambda_t + \mu_t \tag{9}$$

$$\lambda_t = \beta R_t^* E_t \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \tag{10}$$

[5] Specifically at time zero, the information set or sigma algebra is $\mathcal{F}_0 = \mathcal{B}_0 \times \mathcal{B}_0^* \times \mathcal{S}$, where $\mathcal{F}_0 \subset \mathcal{F}_1 \cdots \subset \mathcal{F}_t$.

[6] In terms of practical implementation, to ensure the CIA binds at all times and still satisfies positive money holdings, we will assume shocks with small bounded supports, and admit only the parameter $\lim_{b \nearrow +\infty} k(b_t) = \phi$ such that for sufficiently large steady-state consumption, $\bar{c} > \phi$, consumption c_t will almost surely be bounded above $k(b_t)$ for all t and all histories leading up to and including date t.

[7] Alternatively we could have modeled the CIA constraint as

$$m_t + k(b_t) c_t \geq c_t. \tag{8}$$

where k still satisfies Assumption 1. This would be closer to the CIA constraint in the endowment economy of Canzoneri and Diba (2005), where $c_t = y := 1$. In this case, m_t will be strictly positive since c_t is nonnegative under the Inada conditions, and $k(b_t) \in (0, 1)$. However, this assumption creates additional nonlinearities in the optimality conditions with respect to liquid bonds for households and the planner, without affording much difference in the qualitative implications of the model.

$$\lambda_t = R_t \beta E_t \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) + \mu_t k' (b_t) \tag{11}$$

$$\lambda_t = \beta E_t \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) + \mu_t \tag{12}$$

$$U_h (c_t, h_t) = -\lambda_t (1 - \tau_t) w_t \tag{13}$$

$$\frac{w_t}{z_t} = mc_t \tag{14}$$

$$\begin{aligned}\lambda_t \left[Y_t d (p_t) + p_t Y_t d' (p_t) - \theta \left(\frac{\Pi_t p_t}{p_{t-1}} - \overline{\Pi} \right) \frac{\Pi_t}{p_{t-1}} - mc_t Y_t d' (p_t) \right] \\ + \beta E_t \left[\lambda_{t+1} \theta \left(\frac{\Pi_{t+1} p_{t+1}}{p_t} - \overline{\Pi} \right) \frac{\Pi_{t+1} p_{t+1}}{p_t^2} \right] = 0\end{aligned} \tag{15}$$

The last two conditions (14) and (15), respectively, characterize the optimal labor demand by the household-firm and the optimal price-setting condition which depends on expected future prices. These first-order conditions are quite standard, apart from (11). Combining (9)-(12), we can express the optimal demand for government bonds as

$$k' (b_t) = \frac{R_t^* - R_t}{R_t^* - 1}. \tag{16}$$

At the optimum, the household will demand government bonds up to the point where the marginal transactions value of such bonds are equal to the marginal opportunity cost of holding government bonds, relative to the hypothetical bond which pays a return of R_t^*. Notice that as long as $b_t > 0$ it must be that, $R_t^* - R_t > 0$ since $k' (b_t) > 0$. Thus, as long as the government issues bonds with transactions value for private agents, there will exist an interest-rate spread in the model.[8]

Another important feature in our model that is different from standard monetary models is that real money demand is now affected by the process of government bonds, b_t, directly. This can be seen by combining the CIA constraint (7), when it binds, with (9) to yield real money demand as $m_t = U_c^{-1} (\lambda_t + \mu_t) - k (b_t)$ and λ_t and μ_t are pinned down by (10)-(12) which explicitly involve the demand for government bonds $k' (b_t)$. Similarly, government bonds affect optimal inflation dynamics (15) through the real marginal cost of production, mc_t, and this comes directly from its immediate effect on the marginal value of wealth λ_t in (11) and hence optimal labor supply and demand, (13) and (14).

[8] There are many empirical studies, notably Weil (1983), Giovannini and Labadie (1991), Bansal and Coleman (1996), and Canzoneri et. al. (2002), that find a sizeable equity premium, or a large spread between the average return on equity and the return on treasury bills. In our model, care has to be taken to interpret the interest-rate spread literally as an "equity premium". As Canzoneri and Diba (2005) suggest, one might attempt to measure our return on the illiquid hypothetical bond, R^*, using consumption and price data on our household's Euler equation. Further, one can take the return on liquid government bonds, R, as that for a three-month T-bill. In that instance, our notion of an interest-rate spread, $R^* - R$, should have a magnitude that is close to what is observed as the equity premium.

2.3. *Symmetric pricing equilibrium*

For identical households, in equilibrium, the state-contingent real asset $b_t^* = 0$ since identical households have no desire to borrow or lend to each other. Also, in a symmetric equilibrium, all household-firms charge the same price, so that $p_t = 1$. That is, all households will charge the same price as the average price, or $\widetilde{P}_t = P_t$, for all t. Given the same production technology and competitive wage rate, it must be that the amount of labor supplied by each household equals its demand in its production such that $h_t = \widetilde{h}_t$. The demand for each monopolist's good is $d(p_t) Y_t$ so that the elasticity of demand for each good is $\epsilon(p_t) = d'(p_t) p_t Y_t / d(p_t) Y_t$.

In a symmetric equilibrium, $p_t = 1$ so that under our assumption that $d(1) = 1$, we get the elasticity of demand faced by each household-firm is constant, $\eta \equiv d'(1) < -1$. Also, the marginal revenue for each monopolist is $[1 + \epsilon(p_t)] d(p_t) Y_t$. In the symmetric equilibrium, marginal revenue for all monopolists is $(1 + \eta) Y_t$. The optimal pricing condition (15), together with the fact that in a symmetric equilibrium, $Y_t = z_t h_t$ and also using (14), can be expressed as

$$\left(\Pi_t - \overline{\Pi}\right) \Pi_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1}\right] + \frac{\eta z_t h_t}{\theta} \left[\frac{1+\eta}{\eta} - \frac{w_t}{z_t}\right]. \tag{17}$$

This is an expectations-augmented Phillips curve, which says that time-t inflation depends on the contemporaneous gap real marginal cost and steady-state real marginal cost, $\eta^{-1}(1 + \eta)$, and expected discounted next-period inflation. Also, the greater is the cost of prices adjustment, $\theta \to \infty$, the closer is expected discounted next-period inflation to current inflation. That is, prices are expected not to change very much the more costly is price adjustment. The greater is the elasticity of demand, $\eta \to -\infty$, the more positive and sensitive is the response of current inflation to real marginal cost (limiting case of perfect competition).

2.4. *Resource constraint*

The resource constraint is given by

$$z_t h_t = c_t + g_t + \frac{\theta}{2} \left(\Pi_t - \overline{\Pi}\right)^2 \tag{18}$$

which is the market clearing condition for consumption goods, private and government, where some of that produced resources is dissipated in terms of a price-adjustment cost.

2.5. *Government budget constraint*

The sequence of government budget constraint is

$$M_t + B_t + \tau_t P_t w_t h_t = M_{t-1} + R_{t-1} B_{t-1} + P_t g_t. \tag{19}$$

This says that government spending and the payment of public debt with interest, is financed with either the issue of new money, new debt or income tax receipts. We can re-write this in real terms as

$$m_t + b_t + \tau_t w_t h_t = \frac{m_{t-1}}{\Pi_t} + \frac{R_{t-1} b_{t-1}}{\Pi_t} + g_t \tag{20}$$

for $t = 0, 1, 2, ...$ Notice that with higher inflation, the government can relax the one-period government budget constraint by lowering the real liability of money holding m_{t-1}/Π_t. This also makes the real return on government bonds, R_{t-1}/Π_t, state-contingent.

2.6. Recursive decentralized equilibrium

The following defines the recursive decentralized equilibrium (RDE) for a given feasible policy rule.

Definition 1 *A recursive decentralized equilibrium is the sequence of bounded prices* $\{R_t^*\}_{t=0}^{\infty}$, *allocations* $\{c_t, h_t, m_t, w_t, \Pi_t, mc_t\}_{t=0}^{\infty}$, *and a given policy rule* $\{\tau_t, R_t, b_t\}_{t=0}^{\infty}$ *respecting the optimality conditions (9)-(14) and (17), satisfying the feasibility constraints (18) and (19) and the transversality condition*

$$\lim_{s\to\infty} E_t \left(\prod_{i=0}^{s} R_{t+i}^{-1} \right) (R_{t+s} B_{t+s} + M_{t+s}) = 0, \tag{21}$$

for given stochastic processes (1)-(2).

3. Ramsey Problem

Instead of solving for a decentralized equilibrium, we re-cast the problem in terms of a Ramsey planning problem which also implements the recursive decentralized equilibrium. We take the primal approach to optimal policy as in Atkinson and Stiglitz (1980), Lucas and Stokey (1983), Chari et. al. (1991), which characterizes the equilibrium in terms of allocations (and the inflation rate) as far as possible. The problem can be characterized in the following way: a planner chooses private allocations that would maximize private welfare and then finds the policies that would support such an equilibrium. It is assumed that the Ramsey planner commits to a once-and-for-all plan at time 0.

The existence of costly price adjustment implies that the Ramsey plan underlying the primal form of the decentralized equilibrium can no longer be described by a single present-value implementability constraint as is usually done in flexible-price economies. The intuition from Schmitt-Grohé and Uribe (2004b) is that the sequence of prices is uniquely determined when real allocations are obtained in the primal form of a flexible price equilibrium. These prices then imply a sequence of real discount factors that ensure the transversality condition in the competitive equilibrium is respected in all dates and states. However, when a Phillips curve exists under sticky prices, it poses additional constraint on the path of prices. So in order for the resulting Ramsey plan to deliver a sequence of prices that is consistent with that in a RDE, the plan has to satisfy both the RDE's transversality condition and the Phillips curve, and some version of an implementability constraint, where intertemporal government budget solvency and private optimality conditions hold. Furthermore, in our model with an interest-rate differential, we can show that government bonds and the return on bonds (or it's parallel in terms of the market return) impose additional constraints on the implementability constraint.

The following proposition shows that the equilibrium plan under such a Ramsey planner also satisfies the condition of a RDE in Definition 1.

Proposition 1 *The plans* $\{c_t, h_t, \Pi_t, mc_t, b_t, R_t^*\}_{t=0}^{\infty}$ *respecting the resource constraint (18), the sequence of government budget constraints:*

$$c_t - k(b_t) + b_t + \left(mc_t z_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t) / \left(2 - R_t^{*-1}\right)} \right) h_t = \frac{c_{t-1} - k(b_{t-1})}{\Pi_t} + \frac{\left[R_{t-1}^* - \left(R_{t-1}^* - 1\right) k'(b_{t-1})\right] b_{t-1}}{\Pi_t} + g_t \quad (22a)$$

for $t \geq 1$ *and*

$$c_0 - k(b_0) + b_0 + \left(mc_0 z_0 + \frac{U_h(c_0, h_0)}{U_c(c_0, h_0) / \left(2 - R_0^{*-1}\right)} \right) h_0 = \frac{M_{t-1} + R_{-1} B_{-1}}{P_{-1} \Pi_0} + g_0 \quad (22b)$$

the expectational Phillips curve

$$\left(\Pi_t - \overline{\Pi}\right) \Pi_t = \beta E_t \left[\frac{U_c(c_{t+1}, h_{t+1}) / \left(2 - R_{t+1}^{*-1}\right)}{U_c(c_t, h_t) / \left(2 - R_t^{*-1}\right)} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1} \right] + \frac{\eta z_t h_t}{\theta} \left[\frac{1+\eta}{\eta} - mc_t \right] \quad (23)$$

and the present-value implementability constraint,

$$E_t \sum_{s=0}^{\infty} \frac{\beta^s}{\Delta_{t,t+s}} \frac{U_c(c_{t+s}, h_{t+s})}{\left(2 - R_{t+1}^{*-1}\right)} \left\{ \left[1 + \frac{\left(R_{t+s}^* - 1\right)\left(1 - k'(b_{t+s})\right)}{R_{t+s}^* - \left(R_{t+s}^* - 1\right) k'(b_{t+s})} \right] c_{t+s} - \frac{\left(R_{t+s}^* - 1\right)\left(1 - k'(b_{t+s})\right)}{R_{t+s}^* - \left(R_{t+s}^* - 1\right) k'(b_{t+s})} k(b_{t+s}) + (mc_{t+s} - 1) z_{t+s} h_{t+s} + \frac{U_h(c_{t+s}, h_{t+s}) h_{t+s}}{U_c(c_{t+s}, h_{t+s}) / \left(2 - R_{t+1}^{*-1}\right)} \right\} = \frac{U_c(c_0, h_0)}{\left(2 - R_0^{*-1}\right)} \left[\frac{R_{t-1}^* - \left(R_{t-1}^* - 1\right) k'(b_{t-1}) b_{t-1} + c_{t-1} - k(b_{t-1})}{\Pi_t} \right] \quad (24)$$

where $\Delta_{t,t+s} = \prod_{i=1}^{s} \left[1 - \left(1 - R_{t+i-1}^{*-1}\right) k'(b_{t+i-1})\right]$, *for all states and* $t = 0, 1, 2, ...,$ *and given initial conditions* $\left(R_{-1} B_{-1} + M_{-1}\right) / P_{-1}$ *also satisfy the recursive decentralized equilibrium in Definition 1.*

Proof. See Appendix A. ■

Remark 1 *Note that* b_t *appears in the implementability constraint (24). This is a manifestation of the notion that "bonds do matter" in the equilibrium dynamics, in this case, of the Ramsey equilibrium. This is not the case in typical one-interest-rate Ricardian models.*

We show how to formulate and practically implement the Ramsey planner's problem in Appendix B.

4. Dynamics of Ramsey Equilibrium

In this section, we present numerical solutions and examples of the Ramsey equilibrium. First, we consider how the optimal Ramsey program behaves as price stickiness – the measure that governs the cost of inflation – changes. The benchmark sticky-price economy is calibrated using post-war US data. Second, we examine how the new feature of the model, captured by the substitutability between

Parameter	**Value**	**Description**
β	0.956	Subjective discount factor
s_g	0.2	Share of government consumption in GDP
$\overline{\Pi}$	1.042	Gross inflation rate
$\overline{z}$	1	Steady-state level of technology
δ	3.017	Labor supply parameter
ϕ	0.149	Bond substitutability parameter
b/zh	0.44	Share of government debt in GDP
θ	17.5/4	Degree of price stickiness
η	-6	Elasticity of demand
ρ_z	0.82	Autocorrelation of technology
σ_z	0.0229	Std. deviation of technology shock
ρ_g	0.9	Autocorrelation of government spending
σ_g	0.0302	Std. deviation of government spending shock

Table 1: Calibration

money and bonds, ϕ, affects the optimal fiscal-monetary policy plan. This also serves as a robustness check on our main result.

In order to implement the model numerically, we impose functional forms on the model's primitives. We assume the period utility of the representative household to be $U(c, h) = \ln c + \delta \ln(1 - h)$. A bonds transactions-service function, which satisfies Assumption 1, is $k(b) = \phi\left(1 - e^{-\frac{b}{\overline{c}}}\right)$ where $\phi \leq \overline{c}$ and $\overline{c}$ is steady-state consumption.

4.1. Calibration

The calibration is summarized in Table 1. The calibration of β, given steady-state inflation $\overline{\Pi}$, ensures that steady-state nominal return on the hypothetical bond is $R^* = 1.09$. Given the share of government debt in GDP of about 44 percent, we can calibrate of ϕ to ensure that the interest rate spread, $R^* - R$ in steady state is about 5 percent, following the findings of Bansal and Coleman (1996). The parameter δ is solved endogenously using the government budget constraint at steady state, and is consistent with a fraction of hours worked, $h = 0.2$. The details of calibrating ϕ and δ can be found in Appendix C. The rest of the parameters follow the calibration of Schmitt-Grohé and Uribe (2004b). We employ a second-order accurate perturbation method by Schmitt-Grohé and Uribe (2004a) to solve for the optimal state transition and policy functions around the non-stochastic steady-state. See Appendix D for a short summary.

4.2. Ramsey policy and price stickiness

In this section we explore the behavior of the Ramsey equilibrium in the face of changes in price stickiness θ. This parameter also determines the cost on real resources of changing prices. We consider the volatility, serial correlation and unconditional mean for the key variables implied in the model.[9]

[9]The reported percentage standard deviations and serial correlations are obtained from averaged statistics of Monte Carlo simulated time series of length $T = 100$, and $H = 500$ simulation draws of sample paths.

These variables are the return on market assets (R^*), the return on liquid government bonds (R), inflation (Π), real marginal cost (mc), labor income tax rate (τ), labor effort (h), consumption (c), real money balances (m) and the holding of liquid government bonds (b).

4.2.1. Volatility and persistence

Figure 1 plots the standard deviation of the key variables as a function of the degree of price stickiness, θ, for the Monte Carlo simulations. Of particular note is that in the face of shocks to government spending and technology, optimal policy is geared towards greater price stability. It can be seen that as θ rises, the volatility of Π decreases. However, we also see a rise in the volatility of R relative to R^* and also in the volatility of b. In order to achieve lower inflation volatility since inflation is more costly as price stickiness rises, the planner creates more volatility in the return on government debt and the government debt itself. The greater volatility in the return on government debt and the debt itself means that the planner can use debt as a shock absorber whilst minimizing the shock absorbing role of inflation or labor income tax when financing government spending.

It can also be seen that labor income tax becomes less volatile as θ increases. We reproduce this in a larger diagram in Figure 2. This result stands in stark contrast to that of Siu (2004) and Schmitt-Grohé and Uribe (2004b). Specifically, Siu (2004) showed that as price-stickiness increases, the volatility of labor income tax rate rises, because the planner in his case forgoes minimizing labor tax distortions in terms of volatility, in favor of a lower inflation volatility. Our result is different because government bonds are held by households partly to provide liquidity. Thus, instead of distorting labor supply and hence output by increasing the volatility of labor tax rate, the planner in our model chooses to distort the distribution of liquidity between government bonds and money. Thus we see a greater volatility on R and b, while a lower volatility on τ as θ rises.

Part of the optimal tax program also takes into account the effect on private expectations of future policy paths. Figure 3 plots the distributions of first-order autocorrelations. This manifests itself as increasingly persistent labor income tax rate, τ, and bond rates, R^* and R, which make up the implicit tax on liquid bond holdings. The former has a first-order serial correlation of about 0.8, while R is increasingly near random walk as θ rises. The converse is true for inflation. In order to minimize the costly effect of inflation when price stickiness increases, the optimal program would make inflation less and less autocorrelated, in an attempt to mimic price flexibility.

Figure 4 plots the averaged contemporaneous correlations with output from the Monte Carlo experiments. Of note are the correlations of R and τ, with output, y. On average, the former correlation is negative while the latter is positive. A negative correlation between R and y suggests that in good times the planner would like to partially reduce its debt burden by lowering the return on government debt. This is equivalent to increasing the tax rate on bond liquidity. Similarly, in good times, when y is high, the planner would like to tax labor, τ, at a higher rate. Both these outcomes are consistent with a planner that aims to smooth out tax distortions over time.

In summary, we find that the more sticky prices become, the optimal Ramsey plan favors more price stability but the planner can also afford a less distortionary income tax. That is as price stickiness increases, the less volatile and persistent is inflation and the less volatile is labor income tax, but the more volatile and persistent is the interest rate on liquid bonds and the quantity of government bonds. Also, the interest-rate spread is increasing with the degree of price stickiness, reflecting the increasing tax on bond liquidity.

4.2.2. Unconditional means and government revenues

In Figure 5 we plot the results of the asymptotic unconditional mean of the key variables over different values for the degree of price stickiness, θ. It can be seen that as θ increases, the interest-rate spread rises. The planner achieves this increasing spread $R^* - R$, as a function of θ, by lowering the return on government debt R while causing the market return R^* to rise with θ. Consumption and inflation increases, while government bond holdings and income tax rate τ fall with θ. As prices become more sticky, the planner is more concerned about price stabilization, since the real cost from inflation rises. Thus it has to give up even more of its ability to front-load its budget via creating inflation surprises and taxing money holdings. The planner resorts more to taxing government bonds holdings by increasing the spread between market return and the return on government debt. But in equilibrium, this means that households will hold less of the government debt due to the fall in R. Average inflation rises as a means of taxing monopoly profits accruing to the household firms. The planner does so by lowering the average price markup, or equivalently in the model, raising the average real marginal cost.

It would also be interesting to calculate the average revenues, both explicit and implicit, that are due to the government under different price-stickiness environments. There are three sources, in real terms, from which the government budget can be relaxed. Figure 6 plots these revenues as θ increases. First we have money seigniorage, calculated as Πm. As θ rises, money seigniorage increases. As prices become more sticky, households substitute liquid bonds with more real money balances, since the planner engineers less inflation volatility, while creating more volatility on the return on liquid bonds. Recall that the average opportunity cost of holding liquid bonds, $R^* - R$, rises as well.

This takes us to the second source of "revenue". There is an implicit tax revenue from the taxation of the liquidity services of bonds, calculated as $(R^* - R)\, b/\Pi$. Of particular note is the size of the implicit bond seigniorage revenue. It first rises slightly then declines as price stickiness increase. This can be explained as a Laffer curve effect. With greater price-stickiness and hence real cost of inflation, the planner increases the average tax rate, $R^* - R$, on bond liquidity as part of an optimal program of using liquid bonds as shock absorber, in place of surprise inflation. As the tax rate $R^* - R$ increases sharply the demand for bonds falls accordingly. The net effect is that the implicit bond seigniorage revenue first rises with the increase in $R^* - R$ (or θ) but eventually falls with a sharp decrease in demand for liquid government bonds.

Lastly, the real income tax revenue is $\tau mczh$. Labor income tax revenue is increasing with θ. This is driven by a higher level of labor effort and real marginal cost (or lower average price markup) even though the tax rate on average is falling.

4.3. Robustness and the effect of money-liquid-bond substitutability

It is important that we investigate the effect of the new feature of this model, the degree of substitutability between money and bonds, ϕ, on the optimal policy plan. First, we break the analysis of the effect of ϕ down to individual shocks to technology and government spending. This is shown by impulse response analysis. Second, we repeat the exercise of analyzing the optimal policy under different price-stickiness environments, across different values of ϕ. This exercise allows us to see how ϕ affects the optimal policy plan when both technology and government-spending shocks are present, and also serves as a check on the sensitivity of our main result in Section 4.2.

4.3.1. Impulse response dynamics and liquidity

In Figure 7 we consider a one-standard-deviation ($\sigma_g = 0.023$) positive shock to government spending. In Figure 8 we consider a one-standard-deviation ($\sigma_z = 0.03$) shock to technology. We keep the parameterization of the model as in the benchmark case in Table 1 but vary ϕ. For example, under the positive government spending shock, the optimal policy plan generates a persistent decline in the interest spread, $R^* - R$, in order to encourage more government bond holdings. Labor taxes are also raised but kept on a persistently positive deviation path, while consumption and real money holdings fall. With higher bond liquidity effect, ϕ, The path of inflation and labor tax are kept remarkably similar to the case with near zero bond liquidity, whilst the optimal plan allows the interest spread to adjust by larger amplitudes and thus using government bond holdings more as the shock absorber. A similar effect can be seen in Figure 8 for the case of the technology shock.

As these impulse responses show, the effect of government bond liquidity, ϕ, merely serves to provide an interest-rate-surprise tax avenue in the optimal tax program, while the optimal responses of inflation and labor income tax are remarkably stable or unchanged across degrees of money-bond substitutability, ϕ.

4.3.2. Volatilities, stickiness and liquidity

Figure 9 plots the volatility of the key variables as functions of a set of economies indexed by (ϕ, θ), where each economy is made to share the same set of histories of stochastic technology and government spending shocks. Thus we can consider the effect on the optimal volatility of our variables of interest as we vary the degree of price stickiness θ, for different cases of ϕ.

Two observations stand out in this experiment. First, given a particular degree of bond-money substitutability, ϕ, and the same histories of stochastic government spending and technology shocks, there is a rise in the volatility of government bond return, R, relative to the market-bond return, R^*, but a fall in the volatility of inflation and labor tax, as an economy become more price sticky. In other words, the government can use debt as a shock absorber in order to lower two kinds of social costs – inflation cost which increases with price stickiness and labor distortion cost which increases with the volatility of income tax. This robustifies, with respect to the new parameter ϕ, our result from Section 4.2.

Second, for each given price stickiness level, θ, the greater is ϕ the more the planner can afford to reduce the uncertainty of inflation and labor tax rates while increasing the volatility of the interest spread between market and bond returns. Intuitively, in an economy with greater liquidity effect of government bonds (higher ϕ), the "cost" of using bond tax is lower relative to the cost of using inflation tax and labor tax. This is because for equal opportunity cost of holding liquid bonds $(R^* - R)$, a higher money-bond substitutability results in a larger demand for government bonds which means a larger tax base in terms of bond tax, since $k'(b;\theta) > k'\left(b;\widetilde{\theta}\right)$ for all $\widetilde{\theta} > \theta$. This argument is shown graphically in Figure 10. This effect is further enhanced by the planner allowing for a lower spread on average, $(R^* - R)$, as shown in Figure 11, as ϕ increases. Thus, with relatively greater holdings of liquid government bonds as ϕ rises, the planner allows for more volatility on the bond rate – a surprise interest-rate tax, given inflation tax is too costly – for a given degree of price stickiness.

5. Conclusion

We constructed a model where government bonds provide liquidity service, an idea that goes back to the work of Tobin (1965) and Patinkin (1965) and which is also supported by facts such as US Treasury bills having a role in facilitating transactions. This assumption, allows bonds to matter and leads to new and contrasting results for economies with sticky prices and inflation cost. Specifically, we found that the more sticky prices become, the optimal Ramsey plan favors more price stability but the planner can also afford less distortionary income taxes by resorting to taxing the liquidity service of bonds.

We found that as price stickiness increases, the less volatile and persistent is inflation and the less volatile is labor income tax, but the more volatile and persistent is the quantity of government debt and its return to the debt holder. Further, the labor income tax rate remains very persistent, reflecting a tax-smoothing outcome. Also, the interest-rate spread is increasing with the degree of price stickiness, reflecting the increasing tax on bond liquidity. Thus, with increasing price-stickiness the Ramsey optimal monetary policy is to stabilize inflation, foregoing the shock-absorbing role of inflation, and the corresponding optimal fiscal policy is to minimize labor income tax distortions, over time (tax smoothing) and across states (lower volatility). In return for the gain in low inflation volatility and low intertemporal income tax distortions, the optimal policy uses liquid government bonds as a means of shock absorption. We show that this result is robust to different degrees of substitutability between liquidity holdings in terms of real money balances and real government bonds.

This paper had considered the effect of bond liquidity and the resulting premium on market interest rates and its role in the design of optimal fiscal and monetary policy. While one may wish to consider deeper microfoundations for assets with different liquidities and for rationalizing a liquidity premium, that is not the main message here. There is, nevertheless, interesting work done on "deepening" the microfoundations of money, asset prices and liquidity, for instance Lagos (2005), using search theory.

It would be interesting to extend such an approach to general equilibrium environments with optimal policy.

Appendix

A Proof of Proposition 1

First show that the plans $\{c_t, h_t, \Pi_t, mc_t, b_t, R_t^*\}_{t=0}^{\infty}$ satisfying Definition 1 also satisfy (18), (22a)-(24). Use (7) to eliminate m_t, (16) to eliminate R_t, and (13)-(14) to eliminate τ_t, from the real government budget constraint (20). This yields (22a)-(22b) for $t \geq 0$. Using (11), (10) and (12) we can construct $\lambda_t = U_c(c_t, h_t) / \left(2 - R_t^{*-1}\right)$ for all t and all states, and use this to eliminate λ_t and λ_{t+1} from (17) to yield (23). To show that the RDE satisfies the time-t implementability constraint, for $t, s \geq 0$, (19) can be written as

$$M_{t+s} + B_{t+s} + P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s} = R_{t+s-1}B_{t+s-1} + M_{t+s-1} + P_{t+s}g_{t+s}. \tag{25}$$

Let $D_{t+s} := \prod_{i=0}^{s} R_{t+i-1}^{-1}$ and $W_{t+s} := R_{t+s-1}B_{t+s-1} + M_{t+s-1}$.

Thus we can write $B_{t+s} = \left(W_{t+s+1} - M_{t+s}\right) R_{t+s}^{-1}$. Substituting these definitions into (25), and multiplying (25) with D_{t+s} we obtain

$$D_{t+s}M_{t+s}\left(1 - R_{t+s}^{-1}\right) + D_{t+s}R_{t+s}^{-1}W_{t+s+1} - D_{t+s}W_{t+s} = D_{t+s}\left(P_{t+s}g_{t+s} - P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s}\right).$$

Summing this from $s = 0$ to $S > 0$, and taking expectations conditional on information at time t:

$$E_t \sum_{t=0}^{S} \left[D_{t+s}M_{t+s}\left(1 - R_{t+s}^{-1}\right) - D_{t+s}\left(P_{t+s}g_{t+s} - P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s}\right)\right]$$
$$= E_t D_{t+S+1}W_{t+S+1} + D_t W_t.$$

Let $S \to \infty$ and invoking (21), we have $\lim_{S\to\infty} E_t D_{t+S+1}W_{t+S+1} = 0$ and thus,

$$E_t \sum_{t=0}^{\infty} \left(\prod_{i=1}^{s} R_{t+i-1}^{-1}\right)\left[M_{t+s}\left(1 - R_{t+s}^{-1}\right) - \left(P_{t+s}g_{t+s} - P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s}\right)\right] = W_t. \tag{26}$$

Making use of (10) to find $R_t^* R_{t+1}^* \cdots R_{t+s-1}^*$, we can derive

$$E_t\left[\beta^s\left(\frac{\lambda_{t+s}P_t}{\lambda_t P_{t+s}}\right)\prod_{i=1}^{s} R_{t+i-1}^*\right] = 1.$$

Multiply both sides of (26) with this to obtain

$$E_t \sum_{t=0}^{\infty}\left(\prod_{i=1}^{s} R_{t+i-1}^{-1}R_{t+i-1}^*\right)\frac{\beta^s\lambda_{t+s}}{P_{t+s}}\left[M_{t+s}\left(1 - R_{t+s}^{-1}\right)\right.$$
$$\left. - \left(P_{t+s}g_{t+s} - P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s}\right)\right] = \frac{\lambda_t W_t}{P_t}.$$

and using (16), (7), (13)-(14) and $\lambda_t = U_c(c_t, h_t) / \left(2 - R_t^{*-1}\right)$, to eliminate R_{t+s}, λ_t, λ_{t+s}, M_{t+s}/P_{t+s}, and using (18) to eliminate g_{t+s} we can obtain (24).

Going backwards. Now show that $\{c_t, h_t, \Pi_t, mc_t, b_t, R_t^*\}_{t=0}^{\infty}$ satisfying (18), (22a)-(24) can implement the RDE in Definition 1. Suppose that the economy is determined by the Ramsey plan satisfying (18), (22a)-(24). The planner can construct λ_t that satisfies (11), (10), (12), and (13)-(14) and (7). From these and (22a) we can recover $\{\tau_t, m_t, g_t\}$ that satisfy (19). Given λ_t and λ_{t+1} we can recover (17) from (23). Further $\{R_t\}$ can be recovered from (16) for given $\{b_t, R_t^*\}$. It remains to show that the RDE's transversality condition will not be violated. Since (19) can be recovered, re-write this at $t+s$ in time-t value as

$$E_t \sum_{t=0}^{S} \left[\frac{D_{t+s} M_{t+s}}{P_t D_t} \left(1 - R_{t+s}^{-1}\right) - \frac{D_{t+s}}{P_t D_t} \left(P_{t+s} g_{t+s} - P_{t+s} \tau_{t+s} mc_{t+s} z_{t+s} h_{t+s}\right) \right] = E_t \frac{D_{t+S+1}}{P_t D_t} W_{t+S+1} + \frac{W_t}{P_t}. \quad (27)$$

Since the time-t implementability constraint is satisfied in the Ramsey plan, the limit of the LHS of (27) necessarily exists when $S \to \infty$, and this limit is W_t/P_t such that the present value of the government budget equals exactly the initial condition on government liabilities. This implies $\lim_{S\to\infty} E_t D_{t+S+1} W_{t+S+1} = 0$. And re-writing for the definition of D_{t+S+1} and W_{t+S+1}, we have

$$\lim_{s\to\infty} E_t \left(\prod_{i=0}^{s} R_{t+i}^{-1} \right) \left(R_{t+s} B_{t+s} + M_{t+s}\right) = 0$$

which is (21). $\square$

B The Ramsey Problem

The Lagrangian for the Ramsey problem is

$$\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U(c_t, h_t) + \lambda_t^c \left[U_c(c_t, h_t) - \lambda_t \left(2 - \frac{1}{R_t^*}\right) \right] + \lambda_t^b \left[\lambda_t - \beta R_t^* E_t \frac{\lambda_{t+1}}{\Pi_{t+1}} \right] \\
& + \lambda_t^s \Bigg[c_t - k(b_t) + b_t + \left(mc_t z_t + \frac{U_h(c_t, h_t)}{\lambda_t} \right) h_t - \frac{c_{t-1} - k'(b_{t-1})}{\Pi_t} \\
& - \frac{\left(R_{t-1}^* - \left(R_{t-1}^* - 1\right) k(b_{t-1})\right) b_{t-1}}{\Pi_t} - g_t \Bigg] \\
& + \lambda_t^r \left[z_t h_t - c_t - g_t - \frac{\theta}{2} \left(\Pi_t - \overline{\Pi}\right)^2 \right] + \lambda_t^p \Bigg[\beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1} \right) \\
& + \frac{\eta}{\theta} z_t h_t \left(\frac{1+\eta}{\eta} - mc_t \right) - \left(\Pi_t - \overline{\Pi}\right) \Pi_t \Bigg] \Bigg\}
\end{aligned}$$

with the first-order conditions for $t \geq 1$,

$$U_c(c_t, h_t) + \lambda_t^c U_{cc}(c_t, h_t) + \lambda_t^s - \beta E_t \frac{\lambda_{t+1}^s}{\Pi_{t+1}} - \lambda_t^r = 0$$

$$U_h(c_t,h_t) + \lambda_t^s \left(mc_t z_t + \frac{U_{hh}(c_t,h_t)\,h_t + U_h(c_t,h_t)}{\lambda_t} \right) + \lambda_t^r z_t + \lambda_t^p \frac{\eta}{\theta} z_t \left(\frac{1+\eta}{\eta} - mc_t \right) = 0$$

$$-\lambda_t^c \left(2 - \frac{1}{R_t^*} \right) + \lambda_t^b - \lambda_{t-1}^b \frac{R_{t-1}^*}{\Pi_t} - \frac{\lambda_t^s}{\lambda_t^2} U_h(c_t,h_t)\,h_t - \lambda_t^p \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t^2} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1} \right) + \frac{\lambda_{t-1}^p}{\lambda_{t-1}} \left(\Pi_t - \overline{\Pi}\right) \Pi_t = 0$$

$$-\frac{\lambda_t^c \lambda_t}{(R_t^*)^2} - \lambda_t^b \beta E_t \frac{\lambda_{t+1}}{\Pi_{t+1}} + \beta E_t \frac{\lambda_{t+1}^s}{\Pi_{t+1}} \left(1 - k'(b_t)\right) b_t = 0$$

$$\lambda_t^s \left[1 - k'(b_t)\right] - \beta E_t E_t \frac{\lambda_{t+1}^s}{\Pi_{t+1}} \left\{ \left[R_t^* - (R_t^* - 1)\left(k'(b_t) + b_t k''(b_t)\right) \right] + k'(b_t) \right\} = 0$$

$$\lambda_{t-1}^b R_{t-1}^* \frac{\lambda_t}{\Pi_t^2} + \frac{\lambda_t^s}{\Pi_t^2} \left[R_{t-1}^* - \left(R_{t-1}^* - 1\right) k'(b_{t-1}) \right] b_{t-1} + \frac{\lambda_t^s}{\Pi_t^2} \left[c_{t-1} - k(b_{t-1}) \right] - \theta \lambda_t^r \left(\Pi_t - \overline{\Pi}\right) + \left(\frac{\lambda_{t-1}^p \lambda_t}{\lambda_{t-1}} - \lambda_t^p \right) \left(2\Pi_t - \overline{\Pi}\right) = 0$$

$$\lambda_t^s = \frac{\eta}{\theta} \lambda_t^p$$

$$U_c(c_t,h_t) = \lambda_t \left(2 - \frac{1}{R_t^*} \right)$$

$$\lambda_t = \beta R_t^* E_t \frac{\lambda_{t+1}}{\Pi_{t+1}}$$

$$c_t - k(b_t) + b_t + \left(mc_t z_t + \frac{U_h(c_t,h_t)}{\lambda_t} \right) h_t = \frac{c_{t-1} - k(b_{t-1})}{\Pi_t} + \frac{\left(R_{t-1}^* - \left(R_{t-1}^* - 1\right) k'(b_{t-1}) \right) b_{t-1}}{\Pi_t} + g_t$$

$$z_t h_t = c_t + g_t + \frac{\theta}{2} \left(\Pi_t - \overline{\Pi}\right)^2$$

$$\left(\Pi_t - \overline{\Pi}\right) \Pi_t = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1} \right) + \frac{\eta}{\theta} z_t h_t \left(\frac{1+\eta}{\eta} - mc_t \right).$$

and the first-order conditions for $t = 0$,

$$U_c(c_0,h_0) + \lambda_0^c U_{cc}(c_0,h_0) + \lambda_0^s - \beta E_0 \frac{\lambda_1^s}{\Pi_1} - \lambda_0^r = 0$$

$$U_h(c_0,h_0)+\lambda_0^s\left(mc_0z_0+\frac{U_{hh}(c_0,h_0)h_0+U_h(c_0,h_0)}{\lambda_0}\right)$$
$$+\lambda_0^r z_0+\lambda_0^p\frac{\eta}{\theta}z_0\left(\frac{1+\eta}{\eta}-mc_0\right)=0$$

$$-\lambda_0^c\left(2-\frac{1}{R_0^*}\right)+\lambda_0^b-\frac{\lambda_0^s}{\lambda_0^2}U_h(c_0,h_0)h_0-\lambda_0^p\beta E_0\left(\frac{\lambda_1}{\lambda_0^2}\left(\Pi_1-\overline{\Pi}\right)\Pi_1\right)=0$$

$$-\frac{\lambda_0^c\lambda_0}{(R_0^*)^2}-\lambda_0^b\beta E_0\frac{\lambda_1}{\Pi_1}+\beta E_0\frac{\lambda_1^s}{\Pi_1}\left(1-k'(b_0)\right)b_0=0$$

$$\lambda_0^s\left[1-k'(b_0)\right]-\beta E_0E_0\frac{\lambda_1^s}{\Pi_1}\left\{\left[R_0^*-(R_0^*-1)\left(k'(b_0)+b_0k''(b_0)\right)\right]+k'(b_0)\right\}=0$$

$$\frac{\lambda_0^s}{\Pi_0^2}\left[R_{-1}^*-\left(R_{-1}^*-1\right)k'(b_{-1})\right]b_{-1}$$
$$+\frac{\lambda_0^s}{\Pi_0^2}\left[c_{-1}-k(b_{-1})\right]-\theta\lambda_0^r\left(\Pi_0-\overline{\Pi}\right)-\lambda_0^p\left(2\Pi_0-\overline{\Pi}\right)=0$$

$$\lambda_0^s=\frac{\eta}{\theta}\lambda_0^p$$

$$U_c(c_0,h_0)=\lambda_0\left(2-\frac{1}{R_0^*}\right)$$

$$\lambda_0=\beta R_0^*E_0\frac{\lambda_1}{\Pi_1}$$

$$c_0-k(b_0)+b_0+\left(mc_0z_0+\frac{U_h(c_0,h_0)}{\lambda_0}\right)h_0=\frac{c_{-1}-k(b_{-1})}{\Pi_0}$$
$$+\frac{\left(R_{-1}^*-\left(R_{-1}^*-1\right)k'(b_{-1})\right)b_{-1}}{\Pi_0}+g_0$$

$$z_0h_0=c_0+g_0+\frac{\theta}{2}\left(\Pi_0-\overline{\Pi}\right)^2$$

$$\left(\Pi_0-\overline{\Pi}\right)\Pi_0=\beta E_0\left(\frac{\lambda_1}{\lambda_0}\left(\Pi_1-\overline{\Pi}\right)\Pi_1\right)+\frac{\eta}{\theta}z_0h_0\left(\frac{1+\eta}{\eta}-mc_0\right).$$

where $\lambda_{-1}^c=\lambda_{-1}^b=\lambda_{-1}^s=\lambda_{-1}^r=\lambda_{-1}^p=0$.

C Calibrating ϕ and δ

From the Ramsey planner's version of the government budget constraint, we have at steady state

$$\left[\overline{c}-k\left(\overline{b}\right)\right]\left(1-\overline{\Pi}^{-1}\right)+\overline{b}\left(1-\frac{\left(\overline{R}^*-\left(\overline{R}^*-1\right)k'\left(\overline{b}\right)\right)}{\overline{\Pi}}\right)+\left(\overline{mcz}+\frac{U_h\left(\overline{c},\overline{h}\right)}{\overline{\lambda}}\right)\overline{h}-s_g\overline{z}\overline{h}=0 \quad (28)$$

and given our assumption on functional forms, we have

$$U_h\left(\overline{c},\overline{h}\right)=-\delta/\left(1-\overline{h}\right),\ k\left(\overline{b}\right)=\phi\left(1-e^{-\frac{\overline{b}}{\overline{c}}}\right),\ k'\left(\overline{b}\right)=\frac{\phi}{\overline{c}}e^{-\frac{\overline{b}}{\overline{c}}}.$$

Given $\overline{h}$ and s_g, we can solve for $\overline{c}$ from the resource constraint (18) at steady state. And $\overline{\Pi},\overline{b},\overline{R}^*$ are known values, while $\overline{\lambda}$ can be solved from the first-order condition $U_c\left(\overline{c},\overline{h}\right)=1/\overline{c}=\overline{\lambda}\left(2-1/\overline{R}^*\right)$. Using the optimality condition (16) at steady state, we can calibrate ϕ from

$$k'\left(\overline{b}\right)=\frac{\phi}{\overline{c}}e^{-\frac{\overline{b}}{\overline{c}}}=\frac{\overline{R}^*-\overline{R}}{\overline{R}^*-1}$$

given an estimate of $\overline{R}$. Once all the required values are known, one can solve for δ from (28).

D Second-order approximate solution

In this paper we utilize a second-order approximation of the state transition and policy function in solving the Ramsey planner's problem. This method of solution is due to Sims (2000) and Schmitt-Grohé and Uribe (2004a). Since the first-order conditions of the Ramsey problem in Appendix B are at most up to second order in the state and control variables, this solution method will provide an accurate approximation of the true solution. Very briefly, the system of optimality conditions in Appendix B conforms to the general nonlinear system of expectational difference equations

$$E_tF\left(x_{t+1},x_t,y_{t+1},y_t\right)=0 \tag{29}$$

where x is of dimension $n_x\times 1$ and y is $n_y\times 1$. The state vector can be partitioned as $x=\left[x^1;x^2\right]$ where x^1 are endogenous predetermined state variables and x^2 are exogenous state variables. The exogenous stochastic processes follow the law of motion

$$x_t^2=\Lambda x_{t-1}^2+\sigma\widetilde{\eta}\varepsilon_t$$

where $\sigma\geq 0$ is a scalar scaling the size of the exogenous shocks, $\widetilde{\eta}$ is a known matrix, and $\varepsilon\sim$i.i.d.$(0,I)$. In our application,

$$\Lambda=\begin{bmatrix}\rho_z & 0\\ 0 & \rho_g\end{bmatrix},\sigma=1,\widetilde{\eta}=\begin{bmatrix}\sigma_z & 0\\ 0 & \sigma_g\end{bmatrix}.$$

To solve the model, we guess a solution of the form

$$x_{t+1}=h\left(x_t,\sigma\right)+\sigma\eta\varepsilon_{t+1} \tag{30}$$

$$y_t=g\left(x_t,\sigma\right) \tag{31}$$

where $\eta'=[0,\widetilde{\eta}]$, by approximating the functions $g:\mathbb{R}^{n_x}\times\mathbb{R}_+\to\mathbb{R}^{n_y}$ and $h:\mathbb{R}^{n_x}\times\mathbb{R}_+\to\mathbb{R}^{n_x}$ around the non-stochastic steady state $\overline{x}=0$ and $\sigma=0$. We can find the non-stochastic steady state values by solving $F(\overline{x}, \overline{x}, \overline{y}, \overline{y}) = 0$.Schmitt-Grohé and Uribe (2004a) proved that at least up to a first-order approximation, which is the equivalent of linear perturbation solution methods like Klein (2000), the constant term in the optimal linear policy and state-transition functions are independent of the size of variance of exogenous shocks, σ^2. However, when a second-order approximation is used, they prove that the only difference between a second-order approximation to the solution of the stochastic model (29) and its non-stochastic counterpart are constant terms, in the optimal state-transition and policy functions, which are functions of σ^2. Furthermore, the linear terms are independent of σ^2. Thus, we have the following theorem for our numerical solution method from Schmitt-Grohé and Uribe (2004a):

Theorem 1 *The second-order approximation around* $(x, \sigma) = (\overline{x}, 0)$ *of the solution to the model (29) given as (30) and (31) have the properties that* $g_\sigma(\overline{x}, 0) = 0$, $h_\sigma(\overline{x}, 0) = 0$, $g_{x\sigma}(\overline{x}, 0) = 0$, *and* $h_{x\sigma}(\overline{x}, 0) = 0$.

Practically, given the first-order approximate solution, finding the second-order coefficients is just a matter of solving a linear system of equations. Specifically, the coefficients on the i-th term, where $i = 1, 2, ..., n_x + n_y$, of the j-th order of approximation are determined by the coefficients of i-th term of the i-th order approximation, for $j > 1$ and $i < j$.[10] We use the `MATLAB` codes provided by Schmitt-Grohé and Uribe (2004a) to solve the model. The solution also requires the `MATLAB` Symbolic Math Toolbox in finding symbolic expressions of the first-order and second-order term derivatives of the function F.

References

Aiyagari, R., Marcet, A., Sargent, T.J., Seppälä, J., 2002. Optimal taxation without state-contingent debt. Journal of Political Economy 110, 1220-1254.

Aruoba, S.B., Fernandez-Villaverde, J., Rubio-Ramirez, J.F., 2004. Comparing solution methods for dynamic equilibrium economies. Manuscript. University of Pennsylvania.

Atkinson, A.B., Stiglitz, J.E., 1980. Lectures on Public Economics. McGraw-Hill, New York.

Bansal, R., Coleman, W.J., 1996. A Monetary explanation of the equity premium, term premium and risk-free rate puzzles. Journal of Political Economy 104, 1135-1171.

Calvo, G.A., Guidotti, P.E., 1993. On the flexibility of monetary policy: the case of the optimal inflation tax. Review of Economic Studies, 60, 667-687.

Canzoneri, M.B., Cumby, R.E., Diba, B.T., 2002. Euler equations and money market interest rates: a challenge for monetary policy models. Manuscript. Georgetown University, Washington D.C.

Canzoneri, M.B., Diba, B.T., 2005. Interest rate rules and price determinacy: the role of transactions services of bonds. Journal of Monetary Economics, 52, 329-343.

[10] An extension of this solution algorithm to higher-order approximations has been implemented by Aruoba et. al. (2004) in the `Mathematica` environment.

Chari, V.V., Christiano, L. J., Kehoe, P.J., 1991. Optimal fiscal and monetary policy: some recent results. Journal of Money, Credit and Banking, 23, 519-539.

Chari, V.V., Christiano, L. J., Kehoe, P.J., 1995. Policy analysis in business cycle models, in Cooley, T.F. (ed), Frontiers of Business Cycle Research, Princeton University Press, New Jersey.

Giovannini, A., Labadie, P., 1991. Asset prices and interest rates in cash-in advance models. Journal of Political Economy, 99, 1215-1251.

Klein, P., 2000. Using the generalized schur form to solve a multivariate linear Rational expectations model. Journal of Economic Dynamics and Control, 24, 1405-1423.

Lagos, R., 2005. Asset prices and liquidity in an exchange economy. Manuscript, October 2005.

Lucas Jr, R.E., Stokey, N., 1983. Optimal fiscal and monetary policy in an economy without capital. Journal of Monetary Economics, 12, 55-93.

Patinkin, D., 1965, Money, Interest and Prices: an integration of monetary and value theory. Second edition: MIT Press, 1989, Cambridge, MA.

Rotemberg, J. J., 1982. Sticky prices in the united states. Journal of Political Economy, 90, 1187-1211.

Schmitt-Grohé, S., Uribe, M., 2004a. Solving dynamic general equilibrium models using a second-order approximation to the policy function. Journal of Economic Dynamics and Control, 28, 755-775.

Schmitt-Grohé, S., Uribe, M., 2004b. Optimal fiscal and monetary policy under sticky prices. Journal of Economic Theory, 114, 198-230.

Sims, C., 2000. Second order accurate solution of discrete time dynamic equilibrium models. Manuscript. Princeton University, Princeton.

Siu, H., 2004. Optimal fiscal and monetary policy with sticky prices. Journal of Monetary Economics, 51, 575-607.

Tobin, J., 1965. The interest-elasticity of transactions demand for cash. Review of Economics and Statistics, 38, 241-247.

Walsh, C.E., 2003. Monetary Theory and Policy. Second edition: MIT Press, Cambridge, MA.

Weil, P., 1989. The equity premium puzzle and the riskfree rate puzzle. Journal of Monetary Economics, 24, 401-421.

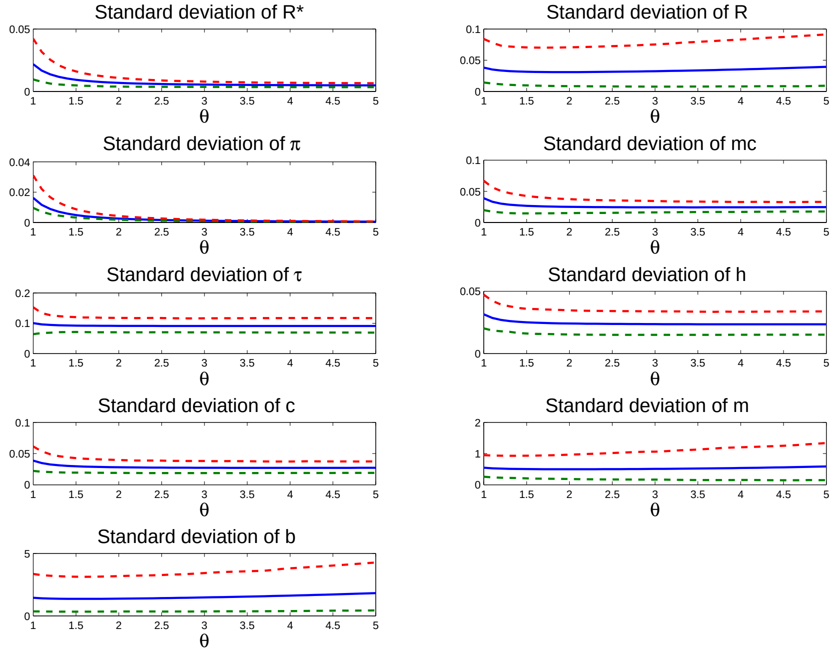

Figure 1: Average for Monte Carlo simulation $T = 100, H = 500$ for percentage standard deviations, under different price stickiness environments. (- - 90% confidence interval.)

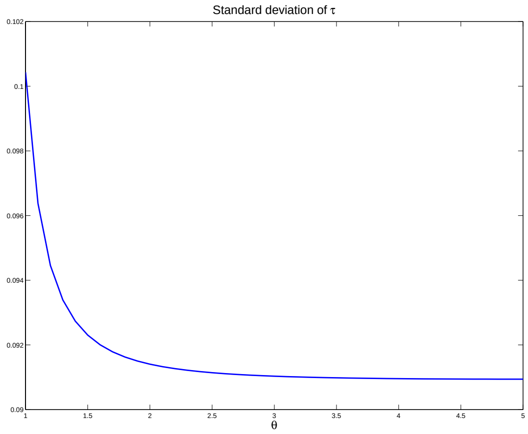

Figure 2: Average for Monte Carlo simulation $T = 100, H = 500$ for percentage standard deviation of labor tax rate, τ, under different price stickiness environments.

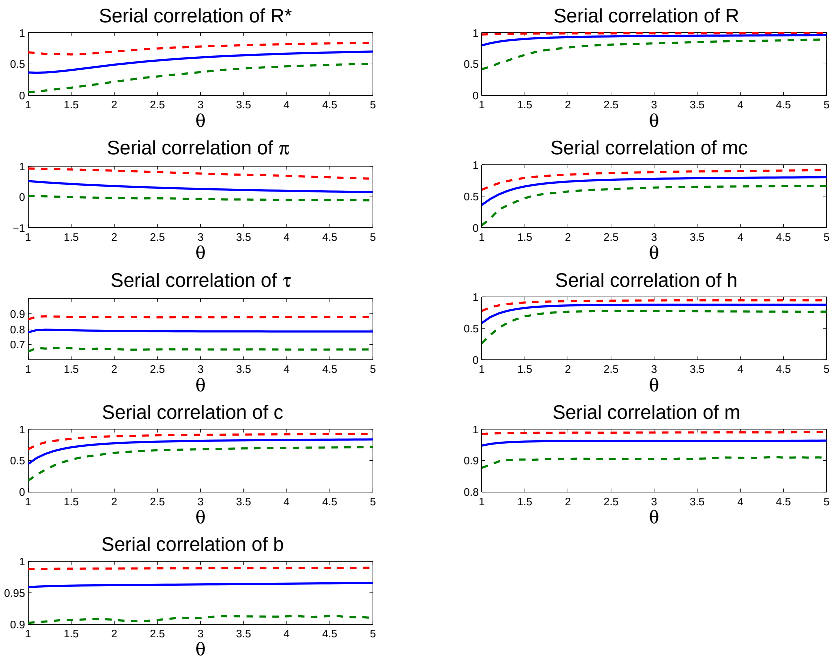

Figure 3: Average for Monte Carlo simulation $T = 100, H = 500$ for serial correlations, under different price stickiness environments. (- - 90% confidence interval.)

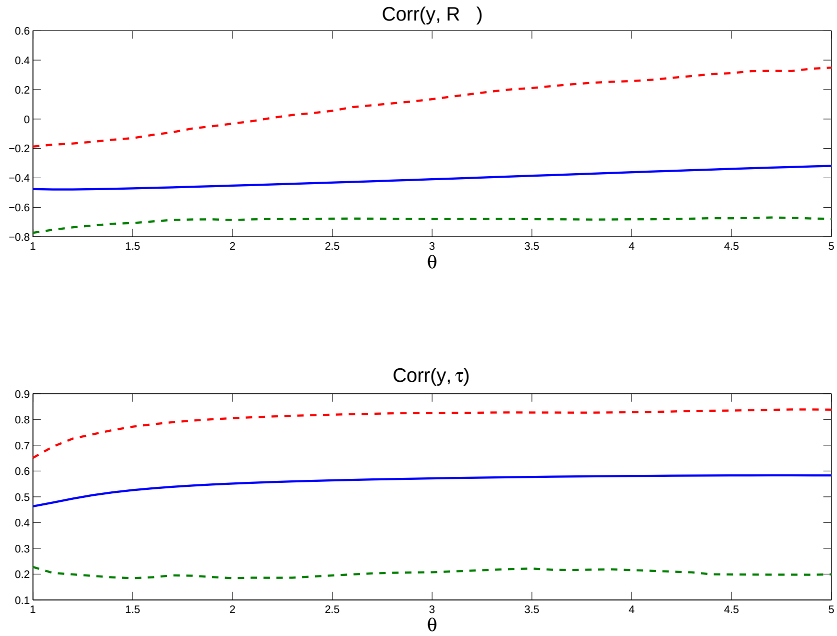

Figure 4: Contemporaneous correlation of GDP with the return on liquid bonds and labor income tax rate.

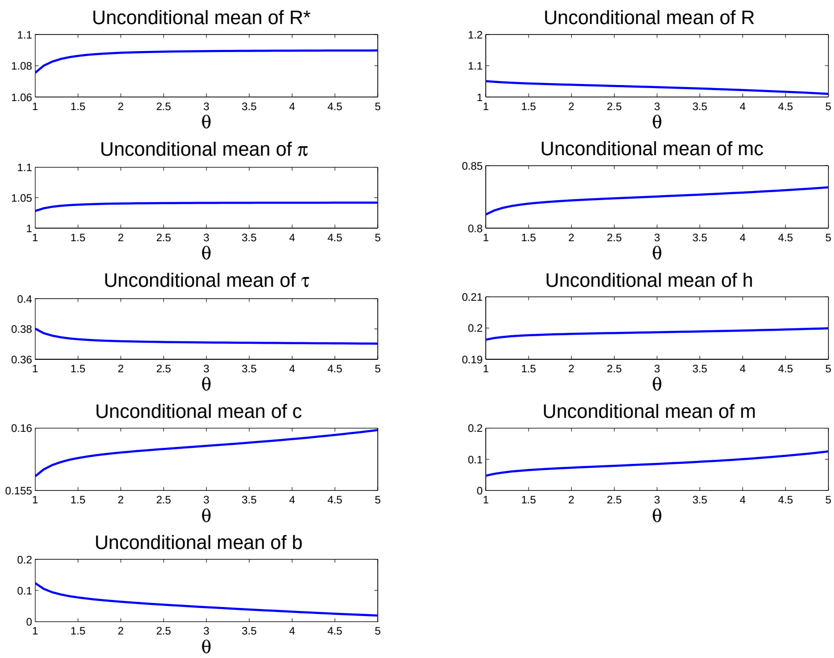

Figure 5: Asymptotic unconditional means (in levels) under different price stickiness environments.

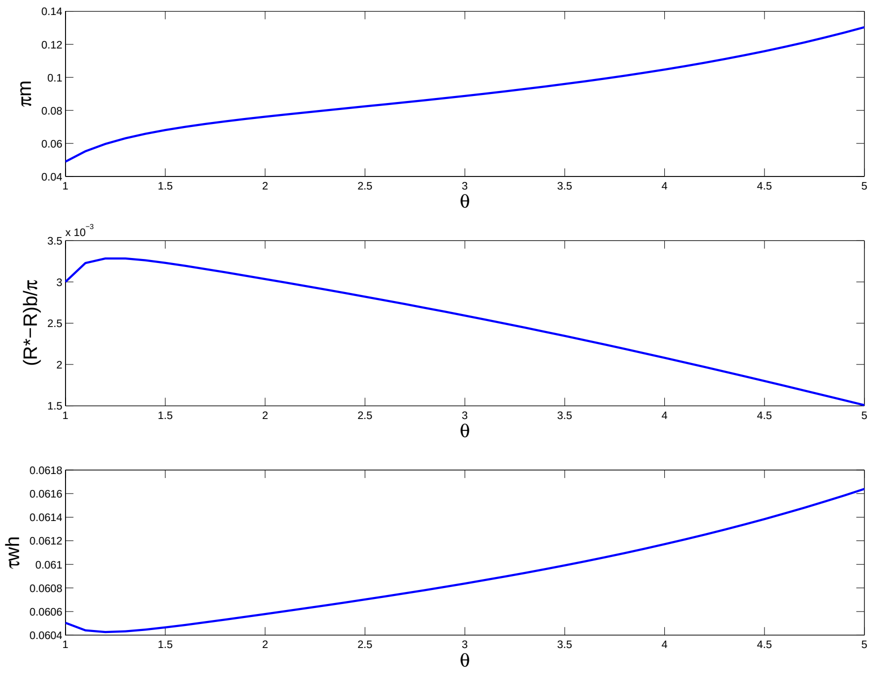

Figure 6: Asymptotic means (in levels) of money seigniorage, implicit tax revenue on bonds, and labor income tax revenue, under different price stickiness environments.

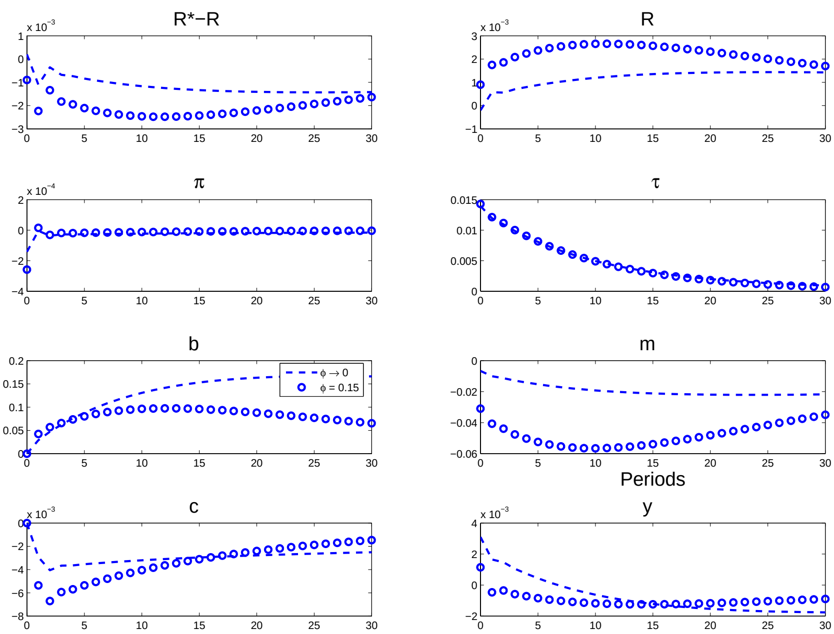

Figure 7: One standard deviation ($\sigma_g = 0.03$) i.i.d. government spending shock in two economies with different ϕ's.

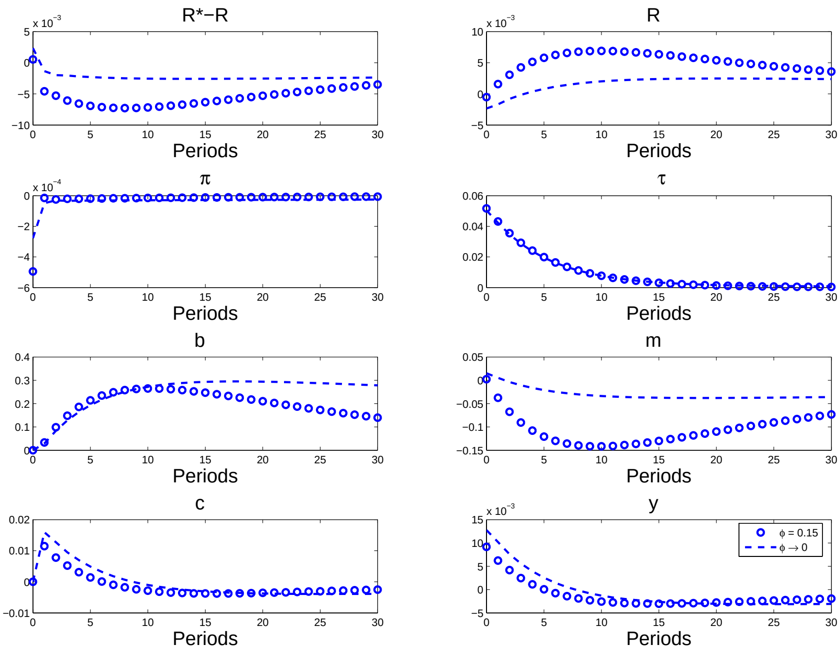

Figure 8: One standard deviation ($\sigma_z = 0.023$) i.i.d. technology shock in two economies with different ϕ's.

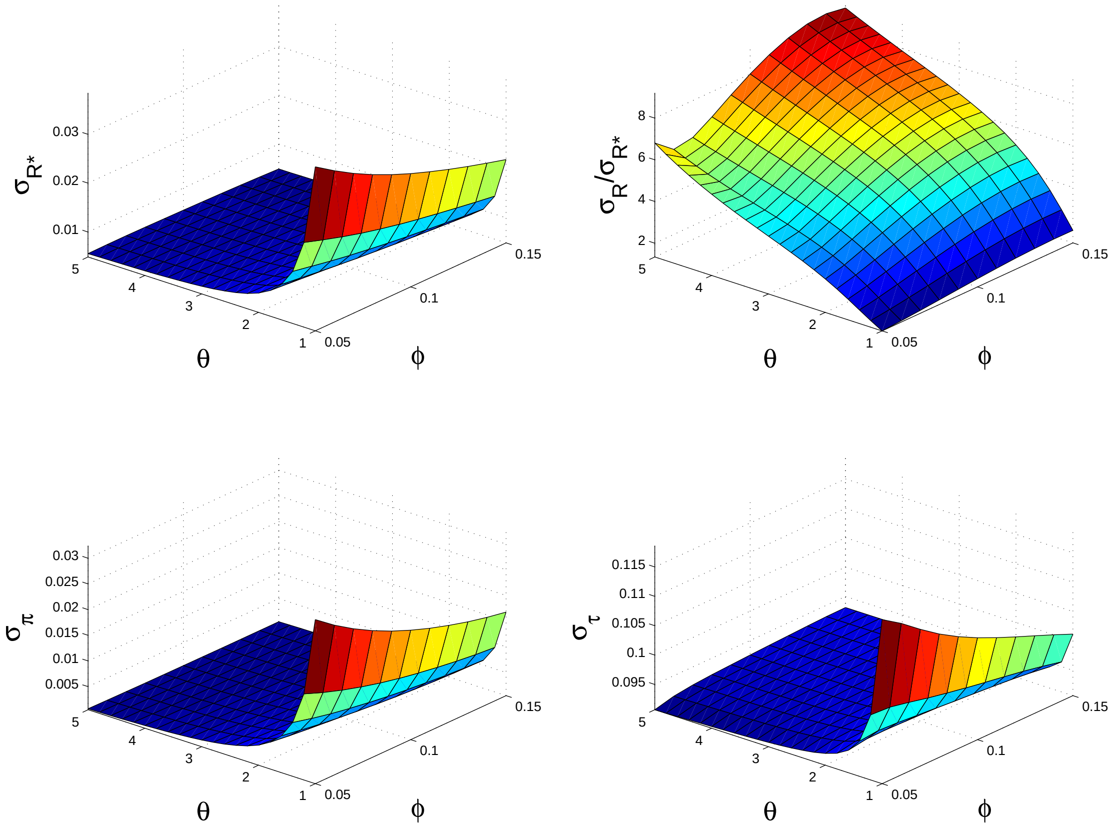

Figure 9: Tax-instrument volatilities as functions of economies indexed by (ϕ, θ). Each point on the surfaces are generated by the same set of histories of exogenous shocks. Averages of statistics for Monte Carlo simulation $T = 100, H = 500$.

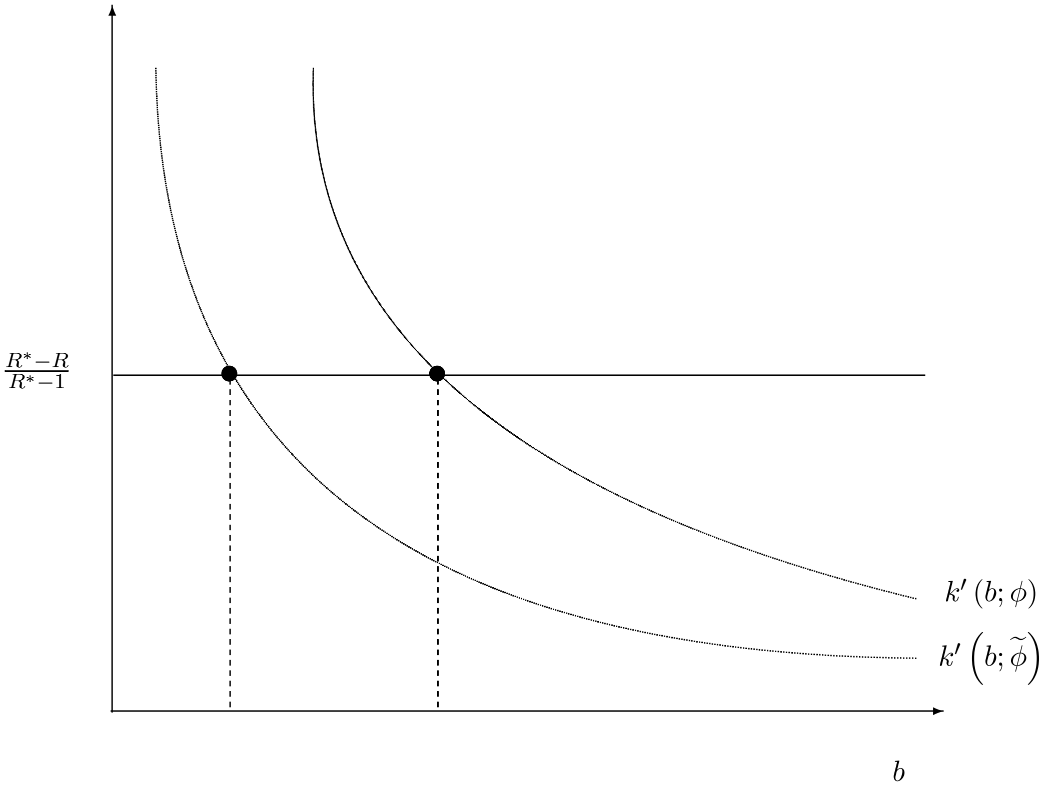

Figure 10: Example with $\phi > \widetilde{\phi}$. For equal opportunity cost of holding government bonds, a higher ϕ, results in higher bond holdings.

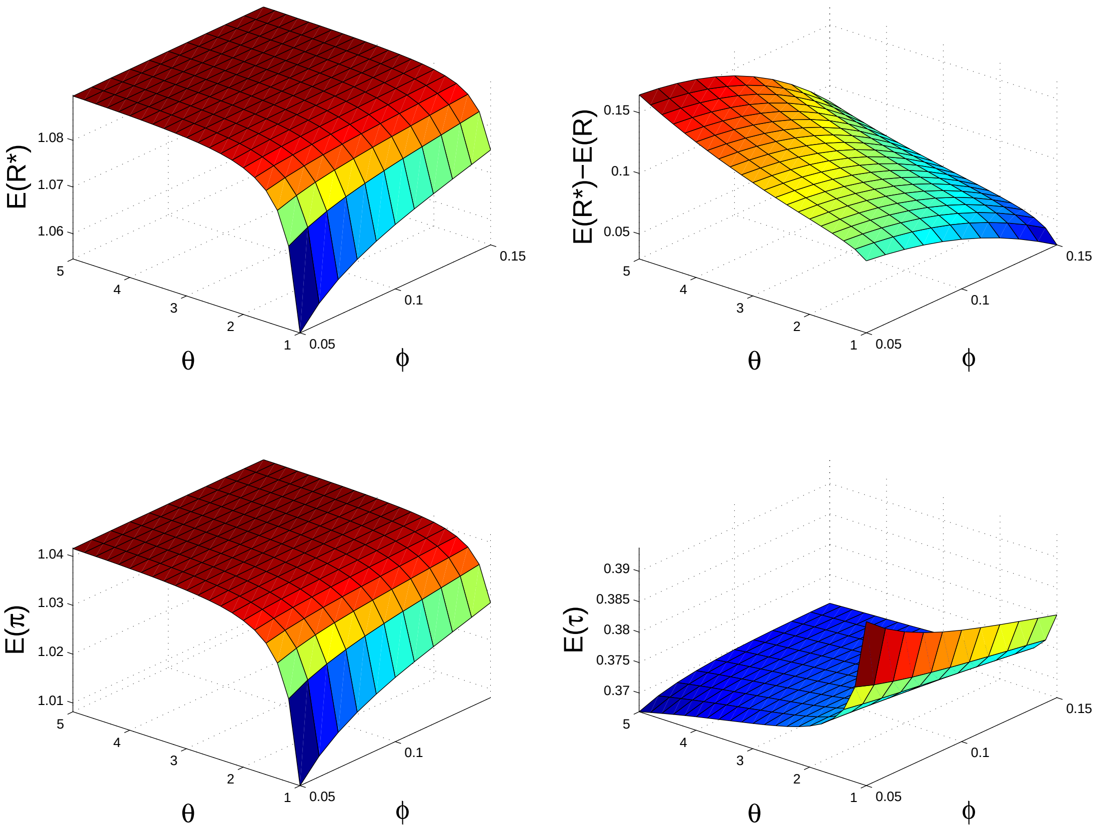

Figure 11: Tax-instrument unconditional means as functions of economies indexed by (ϕ, θ). Each point on the surfaces are generated by the same set of histories of exogenous shocks. Averages of statistics for Monte Carlo simulation $T = 100, H = 500$.