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Fiscal Rigidity in a Monetary Union: The Calvo Timing and Beyond[1]

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Abstract

The paper analyzes the interactions between monetary and fiscal policies. Its emphasis is on a monetary union; one in which (some of) the governments are excessively ambitious. In contrast to conventional games, our novel game theoretic framework allows for *stochastic timing* of policy actions. The fact that moves occur with some ex-ante probability distribution (rather than certainty every period) enables us to model various *degrees of fiscal rigidity* and indiscipline that are *heterogeneous* across the member countries. We examine a number of specifications in discrete and continuous time, such as the widely-used Calvo (1983) timing, as well as a fully general probability distribution of the timing of policy actions. We derive the necessary and sufficient *degree of monetary commitment* that eliminates socially inferior (subgame perfect Nash) equilibria. This degree is shown to be increasing in (i) the degree of fiscal rigidity of each member country, (ii) their relative economic size, (iii) the structure of the economy (that determines eg inflation and output variability costs), and (iv) the degree of the central banker's impatience. Interestingly, such a strong monetary commitment - interpretable as a sufficiently *explicit* numerical inflation target - does not only ensure high credibility of the central bank, but it also indirectly *'disciplines'* the fiscal policymaker(s). As such, it leads to an improvement in monetary-fiscal policy cooperation and outcomes of *both* policies. We conclude by calibrating the model with European Monetary Union data. This exercise aims at providing some quantitative predictions regarding the required explicitness of the European Central Bank's commitment to an inflation target.

Keywords: fiscal-monetary policy interaction, commitment, rigidity, Calvo timing, inflation targeting, dynamic games, asynchronous moves, stochastic timing, coordination games, Battle of Sexes.

JEL classification: E42, E61, C70, C72

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1. Introduction

Fiscal and monetary policies are strongly inter-related, as the actions of one policy affect the outcomes of the other policy. This is true even if the central bank is formally and legally independent. Such inter-dependence implies the following two questions that have concerned central bankers in many countries (including the European Union and United States): 1) Can the observed excessive spending of fiscal policy compromise the anti-inflation credibility of monetary policy?, and 2) Can the central bank's institutional design influence the government's behaviour, and the long-run stance of fiscal policy?

To contribute to this debate - in both a *single country* and a *monetary union* setting - we propose a game theoretic framework that allows us to model the fiscal-monetary interactions in a more general context. Our main contribution relates to the *timing* of the policy actions.[4]

The existing literature has, explicitly or implicitly, studied the policy interaction as a standard repeated game.[5] In such a setting all policy moves are: (i) deterministic, ie they occur with certainty at a pre-specified time, (ii) repeated every period, and (iii) simultaneous, ie unobservable by the opponent(s) in real time. Our framework relaxes these three assumptions that are arguably unrealistic in the macroeconomic policy context. It allows for the timing of the policies' moves to be *stochastic*, ie only occur with some probability, and only in some periods. This also means that the policies often move sequentially in an asynchronous fashion.

By doing so the framework extends the existing game theoretic literature on *asynchronous* move games, that has primarily examined the (deterministic) case of alternating moves.[6] Nevertheless, to be able to compare the results with those in the existing literature, we only deviate in one aspect - the rest of our assumptions are conventional. Specifically, as in the standard repeated game: (i) all players move, with certainty, simultaneously every r periods (including the initial move), and (ii) all past period moves can be observed (perfect monitoring).

Our generalization lies in the fact that one or more players *may* also make *additional moves* in between these standard simultaneous moves. We consider a number of specifications (probability distributions) of these extra moves, both in discrete and continuous time. In addition to a fully general probability distribution, we also examine uniform, normal, and binomial distributions, the latter following the increasingly utilized timing of Calvo (1983).[7]

We use such dynamic timing to postulate the concepts of *monetary commitment* and *fiscal rigidity*, where both refer to the respective policymaker's *inability to move*. Unlike the conventional game theoretic concept of commitment that involves Stackelberg leadership, our commitment concept allows for:

(i) *concurrent* commitment (both policies may be committed at the same time);

(ii) *partial* commitment (policies may only be committed to a certain *degree*).

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[4]It should be noted that our framework is applicable to all types and classes of games, ie its contribution extends well beyond the macroeconomic application examined in this paper.

[5]See for example Hughes Hallett and Libich (2007), Persson et al. (2006), Eggertsson and Woodford (2004), Dixit and Lambertini (2003), Barnett (2001), Bhattacharya and Haslag (1999), Persson and Tabellini (1994), Leeper (1991), Sargent and Wallace (1981), Mundell (1962), or Tinbergen (1954).

[6]See Cho and Matsui (2005), Wen (2002), Lagunoff and Matsui (1997), or Maskin and Tirole (1988). These papers provide a strong justification and motivation for our general approach; for example, Cho and Matsui (2005) argue that: *'[a]lthough the alternating move games capture the essence of asynchronous decision making, we need to investigate a more general form of such processes'*. Let us stress that our framework with stochastic timing of moves is very different from the so-called stochastic games, in which the random element is some 'state' (see eg Neyman and Sorin (2003) or Shapley (1953)).

[7]It is worth mentioning that despite the increasingly frequent use of the Calvo (1983) timing in macroeconomic models, this is limited to price/wage setting behaviour. The policymakers are still assumed (either explicitly through the repeated game or implicitly throught the rational expectations solution) to be able to alter their policy instruments every period. This is true under both *discretion* and (timeless perspective) *commitment*.

In answering question 1) posed in our opening paragraph, it is demonstrated that the macroeconomic outcomes of the policy interaction, as well as monetary policy credibility, crucially depend on the degree of monetary commitment *relative* to the degree of fiscal rigidity of the union members (where naturally larger countries carry a greater weight). If this degree is above a certain necessary and sufficient threshold we derive, then monetary policy credibility and outcomes will *not* be threatened by excessively ambitious fiscal policymakers. If however monetary commitment is insufficient (below this threshold), monetary policy is likely to lack credibility and miss its inflation objective. This is due to the spillover effects from fiscal policies, and it holds even if the central bank is independent, conservative, and targets the natural rate of output. The fact that the threshold is a function of the structure of the economy, and of the society's preferences offers valuable insights.

We interpret the degree of monetary commitment as the degree of *explicitness* (transparency), with which a numerical inflation target is grounded in the central banking legislation/statutes. This is because a legislated target can arguably be less frequently/likely altered by the central bank than an implicit target - due to reputational and accountability considerations. Interestingly, the analysis thus implies that central banks such as the European Central Bank and the Federal Reserve can better safeguard their credibility in the presence of fiscal pressures by more explicitly committing to their long-run inflation targets.[8]

In relation to our question 2), we show that by doing so central banks are also able to '*discipline*' their fiscal counterparts. The reason for such a 'disciplining effect' of monetary commitment on fiscal policy is two-fold. First, to achieve its explicit target a more strongly committed central bank will counter-act the expansionary effects of excessive fiscal spending more vigorously. Second, such behavior reduces, or fully eliminates, the short-term political benefits of excessive spending to the government, and hence provides stronger incentives for a reform towards fiscal sustainability.

This 'disciplining' result seems robust as it holds in all considered scenarios, and has been derived in Libich, Hughes Hallett, and Stehlík (2007) in a different (deterministic) setting. That paper includes a case study in relation to this finding written by Dr Don Brash, the Governor of the Reserve Bank of New Zealand during 1988-2002, in which he argues that *'New Zealand provides an interesting case study illustrating the arguments in the article'.* He describes the policy developments in New Zealand shortly after strengthening monetary commitment by the adoption of explicit inflation targeting. When the government brought down an excessively expansionary budget in the pre-election period of mid-1990, he was forced to tighten monetary conditions in order to offset the budget's effect, and honour the bank's commitment to the newly legislated inflation target. He documents that these events had a *'profound effect on thinking about fiscal policy in both major parties in Parliament.'* Among other, he recalls that:

> *'Some days later, an editorial in the "New Zealand Herald", New Zealand's largest daily newspaper, noted that New Zealand political parties could no longer buy elections because, when they tried to do so, the newly instrument-independent central bank would be forced to send voters the bill in the form of higher mortgage rates'.*

The rest of the paper proceeds as follows. Section 2 presents the fiscal-monetary interaction as a *coordination game* (specifically the Battle of Sexes). Section 3 postulates the game theoretic framework with stochastic timing of moves. Section 4 reports a general result on the outcomes of the interaction that encompasses any arbitrary probability distribution of the timing of moves.

[8]What is required is a more explicit commitment in terms of long-run/average inflation. This means that the central bank does *not* need to become more conservative, and compromize the flexibility in stabilizing the real economy in response to shocks.

		$\mathcal{F}$	
		L	H
$\mathcal{M}$	L	a, w	b, x
	H	c, y	d, z

Figure 1. Fiscal-monetary interaction: general payoff matrix

Sections 5-7 then demonstrate the intuition using specific probability distributions and report several additional insights. Section 8 calibrates the most appropriate setup to the case of the European Monetary Union. Sections 9-10 examine two extensions of the analysis. The former shows how the framework can elegantly deal with any *combinations* of probability distributions by using their mean value. The latter section incorporates discounting. Section 11 then summarizes and concludes.

2. Fiscal-Monetary Interaction as the Battle of Sexes

2.1. **Policies.** There exist a monetary policymaker, $\mathcal{M}$ (male, he), and N (finite or infinite) number of fiscal policymakers, denoted by $\mathcal{F}_n$ (females, she), where $n \in \{1, 2, ..., N\}$.[9] Our companion paper Libich, Hughes Hallett, and Stehlík (2007) presents a macroeconomic model of the $\mathcal{F}$-$\mathcal{M}$ policy interaction, in which the policies are formally independent, but their decisions are inter-dependent through spillovers of their actions onto the other policy, and onto macroeconomic outcomes. To maintain a high degree of generality, we do not postulate a specific macroeconomic model here. Instead, our game theoretic representation reported in Figure 1 can nest any such model (ad hoc as well as micro-founded), in which monetary and fiscal policies interact strategically.

2.2. **Instruments and Actions.** Each policy has one (independent) instrument, and chooses between two different action levels, L (low) and H (high), see Figure 1. The instrument of $\mathcal{M}$ policy can be thought of as choosing a level of the interest rate/inflation, whereas the $\mathcal{F}$ policy instrument as choosing the size of the budget deficit/debt. As we assume the public (society) to prefer the L outcomes to the H outcomes for both policies, L and H can also be interpreted as policy *discipline* and *indiscipline* respectively.

It is however important to note that our interest lies in the $\mathcal{F}$-$\mathcal{M}$ interaction of a *long-run* rather than short-run nature. Therefore, the players' actions should be interpreted as setting some *average* levels of their instruments, ie determining the steady state outcomes in terms of $\mathcal{M}$ and $\mathcal{F}$ discipline. Our focus is *not* on short-run deviations from trend due to shocks and business cycle fluctuations.

2.3. **Payoffs.** The payoffs $\{a, b, c, d, w, x, y, z\}$ in Figure 1 are functions of the structural parameters of the underlying macroeconomic model after the action sets have been truncated to $\{L, H\}$ (for examples of such truncation see Cho and Matsui (2005)).[10] Libich, Hughes Hallett, and Stehlík (2007) focus on the case studied in the literature, in which the central bank's objectives are perfectly aligned with those of the society, ie $\mathcal{M}$ is conservative, patient and *responsible* (targets the natural rate of output). In contrast, the government's objectives are overly *ambitious*

[9]To simplify the notation we will use $\mathcal{F}$ and $\mathcal{M}$ to denote the respective policymakers as well as their policies.

[10]They argue that it is natural to set the L levels to the socially optimal, but time-inconsistent values, whereas the H levels to the time-consistent but socially inferior ones.

		$\mathcal{F}$	
		L	H
$\mathcal{M}$	L	0, -1	-2, -2
	H	-2, -2	-1, 0

FIGURE 2. Fiscal-monetary interaction as the Battle of Sexes: a numerical example

($\mathcal{F}$ policy's output target is above the natural rate). This may be due to either political economy reasons (lobby groups, myopia, unionization, welfare schemes, naïve voters) or structural features (aging population, pay-as-you-go health and pension systems, high outstanding debt etc).

The paper shows that despite the central bank's best efforts, $\mathcal{M}$ policy outcomes can be, under a range of reasonable parameter values, socially inferior (unless the central bank is a strict inflation targeter - an 'inflation nutter' - and fully ignores output volatility). Specifically, due to the spillover effects from $\mathcal{F}$ policy, $\mathcal{M}$ policy may lack credibility and overshoot its inflation target L on a permanent basis in an attempt to also stabilize the real economy.[11] In game theoretic terms, it is shown that the socially inferior outcome (H, H) may result, either as a unique Nash or as one of several Nash equilibria.

Due to its game theoretic 'richness' - presence of multiple equilibria - this paper examines one such feasible scenario that has the structure of the *Battle of Sexes* game - see Figure 2 for an example.[12] In such case the payoffs in Figure 1 satisfy the following general conditions

$$a = 0 > d > b, d > c \quad \text{and} \quad z = 0 > w > x, w > y, \tag{1}$$

where we have normalized a and z to zero without loss of generality. We will refer to $-d$ and $-w$ as an *inflation cost* since it is associated with $\mathcal{M}H$, whereas b, c, x, y can be thought of as the *output volatility cost* since output deviates from the natural rate.

2.4. **Nash Equilibria.** The stage game has multiple Nash equilibria: two in pure strategies, (L, L) and (H, H), and one in mixed strategies. Two issues are worth noting. First, each player *prefers* (ie has the highest payoff from) a different pure Nash. $\mathcal{M}$ prefers the socially optimal (L, L) outcome featuring discipline of both policies, whereas $\mathcal{F}$ prefers the socially inferior (H, H) outcome featuring indiscipline.[13]

Second and therefore, the mixed Nash equilibrium is a likely outcome. The fact that it delivers outcomes inferior to either pure Nash, for both policymakers, is a reason for concern. It further

[11] It was shown that the standard quadratic welfare function that underlies such inflation/output stabilization effort can be derived from micro-fundations, see Woodford (2003). It was further documented that real world central banks, even those with a legal 'unitary' mandate for inflation stabilization, are flexible inflation targeters and attempt to stabilize output in practice, see eg Cecchetti and Ehrmann (1999) or Kuttner (2004).

[12] Under other parameter values other classes of games can occur. It will become apparent that results for some of them (including all other types of coordination games) can be directly obtained from our results.

[13] In this respect the game is equivalent to the Game of Chicken - which has been frequently used to study the policy interaction, see eg Barnett (2001) or Bhattacharya and Haslag (1999). Libich, Hughes Hallett, and Stehlík (2007) however show that the Game of Chicken is, contrary to the Battle of Sexes, unlikely to arise from a realistic macroeconomic model of policy interaction featuring a *responsible* central bank. This is because such bank will never have an incentive to respond to $\mathcal{F}L$ by $\mathcal{M}H$.

points towards two seemingly contradictory aspects of the game - the *policy conflict* (the first issue) and the *policy coordination problem* (the second issue).

2.5. **Outcomes Under Standard Commitment.** To get sharper predictions this type of interaction has often been studied allowing for commitment - the *Stackelberg leadership* of one player. Under this standard game theoretic notion of commitment, the following is true: (i) the game has a unique outcome whereby the leader's preferred pure Nash equilibrium obtains, and (ii) this is regardless of the players' discount factors and the exact payoffs. Specifically, under the central bank's leadership ($\mathcal{M}$-dominance) the (L, L) outcome obtains, whereas under the government's leadership ($\mathcal{F}$-dominance) the (H, H) outcome obtains.

In the rest of the paper we examine the outcomes of the interaction allowing for a more general timing of moves. It will become apparent that the conventional conclusions (i) and (ii) are refined and partly qualified, even if the assumption of a simultaneous initial move is preserved.

3. The Game Theoretic Setup With Stochastic Moves

3.1. **Definitions and Assumptions.** Time is denoted by $t \in \mathbb{R}$. For comparability with the results of the standard repeated game, all our assumptions follow this conventional approach. First, the timing of all players' moves is common knowledge. Second, all past periods' moves can be observed (ie perfect monitoring). Third, all players are rational, have common knowledge of rationality, and for expositional clarity they have complete information about the structure of the game and the opponent's payoffs. Fourth, all players are assumed to move, with certainty, simultaneously every $r \in \mathbb{N}$ periods - starting in period $t = 0$. Note that all of these assumptions are non-essential for our analysis, and all can be easily relaxed.

Definition 1. ***Standard moves*** *refer to moves made with certainty every $r \in \mathbb{N}$ periods, and* ***non-standard moves*** *refer to those (potentially) made in between standard moves. A player that can, from an ex-ante perspective,* ***only*** *make standard moves will be called the* ***standard player****, with r expressing his/her degree of* ***commitment*** *or* ***rigidity****.*

Throughout the paper we assume at least one of the $N + 1$ players in the game to be the standard player. This is to provide a benchmark for the other players' moves, and examine the timing differences in relative terms (which simplifies the analysis a great deal). As our focus is on a monetary union with a common central bank but multiple $\mathcal{F}$ policymakers, $\mathcal{M}$ will have the role of the standard player. Nevertheless, in a single country setting we will also examine the opposite situation of $\mathcal{F}$ being the standard player.

Definition 2. *The* ***reaction function***

$$F(t) : [0, r] \to [0, 1], \text{ where } F(0) = 0, \quad (2)$$

is an arbitrary non-decreasing function summarizing the timing of non-standard moves of the standard player's ***opponent(s)****.*

Several specific examples of $F(t)$ are examined below and graphically depicted in Figures 3-5 and 8. Let us note four issues regarding the interpretation of $F(t)$. First, while we define $F(t)$ on a closed interval $[0, r]$ for ease of exposition (eg to be able to include the natural assumption $F(0) = 0$), the function relates to the opponent's non-standard moves only. The standard moves occurring every r periods are not included - to indicate this $F(t)$ is called the *reaction* function in Definition 2.

Second, $F(t)$ does *not* fully describe the exact timing all moves (potentially) made by the standard player's opponents. It only describes their *first* non-standard move following each standard move, ie their first available response. The reader can therefore think of $F(t)$ as the 'first-reaction' function. Such description is however without loss of generality, since in all subsequent moves on the interval $(0, r)$, the same action would be selected. This is because (i) the choice is made under identical circumstances as the first available response on that interval, (ii) the opponents' action can be altered with certainty at the next standard move (at $t = r$), and (iii) the standard player cannot move before its next standard move (at $t = r$).

Third, $F(t)$ can be interpreted in a number of ways, depending on the particular scenario. Under stochastic non-standard moves, $F(t)$ expresses their *cumulative distribution function* (CDF) in the $N = 1$ case, and hence a weighted sum of (heterogeneous) CDFs in the $N > 1$ case. Put differently, in the former case $F(t)$ expresses the *probability* that the opponent has had the opportunity to respond on the interval $[0, t]$, and in the latter it is the *proportion* of opponents that have had such an opportunity.

Fourth, $\int_0^r F(t)\mathrm{d}t$ describes the (overall) 'reaction speed' of the standard player's opponent(s). The integral $\int_0^r (1 - F(t))\,\mathrm{d}t \in (0, r]$ can then be interpreted as the (overall) degree of rigidity or commitment of the standard player's opponent(s).[14] In statistical terms, it is a (weighted average of) *complementary* CDF(s). Therefore, we will refer to

$$\frac{r}{\int_0^r (1 - F(t))\,\mathrm{d}t} \in [1, \infty) \tag{3}$$

as the degree of the standard player's *relative commitment.* Note that such specification nests the conventional repeated game, in which (i) $t \in \mathbb{N}$, (ii) all players are standard, and (iii) all have $r = 1$.[15]

3.2. **Equilibrium and (Non)-Repetition.** We will use a standard equilibrium refinement, subgame perfection, that eliminates non-credible threats. *Subgame perfect Nash equilibrium* (SPNE) is a strategy vector (one strategy for each player) that forms a Nash equilibrium after any history.

While the r period stage game may be repeated, we can restrict our analysis to the stage game (ie one move of the standard player). This is possible since we will throughout be interested in deriving conditions for the game to have a *unique SPNE* - one that is *efficient.* Due to these two properties, if the conditions are satisfied then repeating the game, and allowing for the standard player's reputation building of some form, would not affect the reported equilibrium. Put differently, in the further repetitions the same outcomes obtain - for all histories.[16] The uniqueness also implies that we can only focus on pure strategies without loss of generality.

Definition 3. *The standard player will be called to* ***win the Battle*** *of Sexes, and the opponent(s) to* ***lose the Battle****, if the game has a unique SPNE, and that SPNE has the standard player's* ***preferred*** *(highest payoff) outcome uniquely on its equilibrium path.*

Since $\mathcal{M}$'s preferred outcome (L, L) is the socially optimal one, we will throughout be deriving the circumstances under which $\mathcal{M}$ wins (and $\mathcal{F}$s lose) the Battle, ie under which L obtains on the equilibrium path of the unique SPNE for *both* policies, and in *all* moves.

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[14]The *overall* label applies to the cases $N > 1$. Further, while a game theorist will think in terms of commitment (since his interest lies in the *effect* on the outcomes of the game), a macroeconomist may find it natural to interpret this as either commitment or rigidity (based on the *source* of the inability to move). We will throughtout talk about $\mathcal{M}$ commitment, but $\mathcal{F}$ rigidity.

[15]In Section 9 we use the fact that, under $F(r) = 1$, we have $\int_0^r (1 - F(t))\,\mathrm{d}t = \mu$, where μ is the mean of the underlying probability distribution. This will allow us to easily combine different probability distributions.

[16]In this sense we can think of our analysis as the worst case scenario, in which reputation cannot help the players cooperate in coordination games.

4. General Result: Arbitrary Moves

In order to make the analysis more illustrative we will in Sections 4-8 abstract from the standard player's discounting the future (and only incorporate it in Section 10, where it is shown not to change any of the qualitative predictions of the analysis).

Specifically, Section 4 will first report a general result that holds for both discrete and continuous time, as well as for any probability distribution. The subsequent Sections 5-8 will then demonstrate the intuition by examining several specific scenarios. This will be complemented by Section 8 which will report a calibrated example, the case of the European Monetary Union.

Theorem 1. *Consider the Battle of Sexes policy interaction described by (1) and an arbitrary reaction function $F(t)$. The necessary and sufficient condition for the standard player $\mathcal{M}$ to discipline the $\mathcal{F}$ policymaker(s) and win the Battle (ensuring socially optimal outcomes) is*

$$\frac{r}{\int_0^r (1 - F(t))\,\mathrm{d}t} > \frac{b}{d}. \tag{4}$$

It therefore follows that, for any $F(t)$, the degree of $\mathcal{M}$ commitment required for $\mathcal{M}$ to win the Battle is increasing in: (i) his output volatility cost relative to his inflation cost; and (ii) the (overall) degree of $\mathcal{F}$ rigidity.[17]

Proof. For $\mathcal{M}$ to win the Battle, it suffices to show that $\mathcal{M}$ finds it optimal to play L in all his nodes, and regardless of the opponents' past or simultaneous moves. This is because then the $\mathcal{F}$ opponent(s) will play their unique best response L in every node on the equilibrium path. Specifically, we need to show that L is, in his every move, $\mathcal{M}$'s unique best response to both (currently obtaining) L and H played by $\mathcal{F}$. As the former is trivially satisfied (due to $a > c$), let us derive the conditions for the latter. For reasons explained in Section 3.2 we can focus on the stage game lasting r periods without loss of generality.

Using backwards induction, we know that when/if a particular $\mathcal{F}$ policymaker gets a chance to *respond* anytime on the interval $[0, r]$ to $\mathcal{M}$'s standard move at $t = 0$, she will play the level played by $\mathcal{M}$ in that move. This is because she will be able to alter her move with certainty at the next standard move (at $t = r$), whereas $\mathcal{M}$ will *not* be able to do so. Put differently, she will, regardless of her discount factor, play the best response to the currently occurring move of $\mathcal{M}$. Knowing this, $\mathcal{M}$ would in his initial move play $\mathcal{M}L$ against $\mathcal{F}H$ if and only if

$$b\int_0^r (1 - F(t))\,\mathrm{d}t + a\int_0^r F(t)\mathrm{d}t > dr. \tag{5}$$

The left-hand side (LHS) and the right-hand side (RHS) of this *necessary and sufficient* condition report $\mathcal{M}$'s payoffs, under $\mathcal{F}H$, from playing L and H respectively. Specifically, the LHS of (5) states that if $\mathcal{M}$ plays L he will get the off-diagonal payoff b for interactions with $\mathcal{F}$s that have not been able to move yet, and a with those who have (and have therefore switched to $\mathcal{F}L$). The two elements on the LHS can therefore be thought of as $\mathcal{M}$'s initial 'credibility investment' and a subsequent 'credibility reward'.[18] The RHS states that from playing H, $\mathcal{M}$ will get the payoff

[17] Analogously, $\mathcal{F}$ in the role of the standard player wins the Battle if and only if $\frac{r}{\int_0^r (1-F(t))\mathrm{d}t} > \frac{x}{w}$.

[18] We use the term *credibility* in the intuitive sense of the literature (see eg the quantification of Faust and Svensson (2001)): the further inflation expectations are from the target the lower the credibility. While we do not model expectation formation explicitly in this paper, in a long-run analysis average expectations will, under most specifications (including rational expectations), equal the average equilibrium inflation level played by the central bank, ie either L or H. Therefore, if the game has a unique SPNE with only $\mathcal{M}L$ ($\mathcal{M}H$) on the equilibrium path, $\mathcal{M}$ policy *never* (*always*) lacks credibility. If there exists SPNE with both $\mathcal{M}L$ and $\mathcal{M}H$ on the equilibrium path, then $\mathcal{M}$ policy *may* lack credibility.

d throughout the stage game. Collecting the terms in (5), using $a = 0$, and dividing by $-b > 0$ we obtain (4). □

The theorem highlights the importance of *relative commitment* - what matters is how frequently/likely a player can move relative to the opponent(s). Further, the relative cost/benefit of commitment (credibility investment vs reward) also plays a crucial role. As the payoffs b and d are functions of the structural parameters of the underlying macroeconomic model, and of the policy preferences, one can consider how changes in various features of the economy affect the required relative commitment.

In order to further develop the intuition and provide additional insights, we will examine several cases of interest. These will feature various timing specifications (probability distributions, ie types of $\mathcal{F}$ rigidity), and are summarized in the following table.

Case	Moves	Time
1	deterministic	discrete
2	uniformly distributed	continuous
3	binomially distributed (Calvo)	discrete
Ext 1	combinations (including normally distributed)	continuous

In each case we first focus on a *monetary union* scenario with a single $\mathcal{M}$ and any number $N \geq 1$ of $\mathcal{F}$ policymakers. These can be heterogenous not only in terms of their degree of $\mathcal{F}$ rigidity, but also in terms of their economic size (influence). To do so we denote the relative weights of the union members by $w_1, w_2, \ldots, w_N$, such that $\sum_{n=1}^{N} w_n = 1$. Such monetary union setting will nest a *single country* setting ($N = 1$), which will be reported separately in each considered case.

The deterministic Case 1 provides a benchmark. Note that it is still a generalization of the standard repeated game (as well as alternating move games of Maskin and Tirole (1988)), as it allows for the *frequency of moves* to differ across players. Such property is in line with Tobin (1982), who observed that '*Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer. It would be desirable in principle to allow for differences among variables in frequencies of change...*'.

The Calvo (1983) timing of Case 3 has become increasingly used in the macroeconomic literature when modelling the moves of the price/wage-setters. We will therefore use it for calibration in Section 8.

5. Case 1: Deterministic Moves

5.1. Monetary Union. In this benchmark case each $\mathcal{F}$ policymaker n moves with a *constant frequency.* Specifically she does so every $t = jr_n^{\mathcal{F}}$, where $j \in \mathbb{N}, r_n^{\mathcal{F}} \in \mathbb{N}$, and $\left\lfloor \frac{r}{r_n^{\mathcal{F}}} \right\rfloor = \frac{r}{r_n^{\mathcal{F}}}, \forall n$ (which implies $r_n^{\mathcal{F}} \leq r$, as well as synchronization of the standard moves). Then the reaction function (2) has the following specific form (see Figure 3 for a graphical depiction)

$$F(t) = \sum_{n: r_n^{\mathcal{F}} \leq t}^{N} w_n. \tag{6}$$

Note that this case's $F(t)$ nests the conventional repeated game, which obtains under $r_n^{\mathcal{F}} = r, \forall n$.

Proposition 1. *Consider Case 1 of the Battle of Sexes policy interaction described by (1) and (6). The greater the* ***economic size*** *of the member country w_n, the more her $\mathcal{F}$ rigidity increases the necessary and sufficient degree of $\mathcal{M}$ commitment ensuring $\mathcal{M}$'s win.*

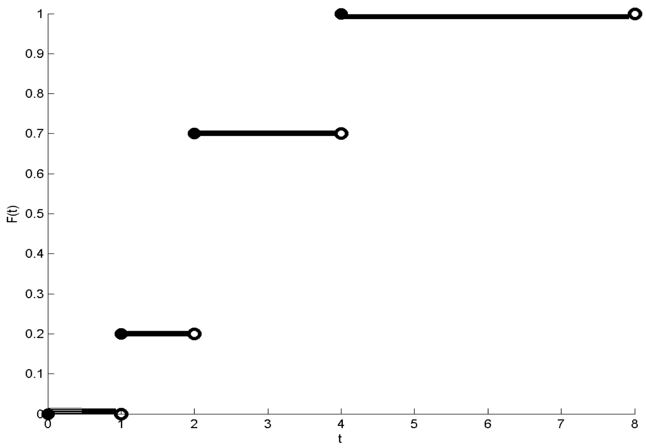

FIGURE 3. $F(t)$ of Case 1 featuring $r = 8$ and three opponents with weigths $w_1 = 0.2, w_2 = 0.5, w_3 = 0.3$ and rigidities $r_1^{\mathcal{F}} = 1, r_2^{\mathcal{F}} = 2, r_3^{\mathcal{F}} = 4$.

Proof. Integrating $F(t)$ from (6) over $[0, r]$ we obtain

$$\int_0^r F(t)\mathrm{d}t = w_1(r - r_1^{\mathcal{F}}) + w_2(r - r_2^{\mathcal{F}}) + ... + w_N(r - r_N^{\mathcal{F}}) = r - \sum_{n=1}^{N} w_n r_n^{\mathcal{F}}.$$

Substituting this integral into (4) we get the necessary and sufficient condition

$$\frac{r}{\sum_{n=1}^{N} w_n r_n^{\mathcal{F}}} > \frac{b}{d}. \tag{7}$$

The fact that the LHS is decreasing in w_n and $r_n^{\mathcal{F}}$ for all n proves the claim. □

Intuitively, the greater a union member's economic influence, and the more fiscally rigid the member is, the more she determines (increases) the required degree of $\mathcal{M}$ commitment that will discipline her, and other member countries.

5.2. **Single Country.** Under $N = 1$ the reaction function (6) and its integral become

$$F(t) = \begin{cases} 0 & \text{if} \quad t < r^{\mathcal{F}}, \\ 1 & \text{if} \quad t \geq r^{\mathcal{F}}, \end{cases} \quad \text{and} \quad \int_0^r F(t)\mathrm{d}t = r - r^{\mathcal{F}}. \tag{8}$$

Then the necessary and sufficient condition for $\mathcal{M}$ to win, (7), simplifies into

$$\frac{r}{r^{\mathcal{F}}} > \frac{b}{d}, \tag{9}$$

where $\frac{r}{r^{\mathcal{F}}}$ expresses $\mathcal{M}$'s relative commitment (the specific form of (3) for Case 1). Note that since $\frac{b}{d} > 1$ form (1), $r = r^{\mathcal{F}}$ is never sufficient to deliver $\mathcal{M}$'s win. In such case $\mathcal{M}$ commitment is not strong enough relative to $\mathcal{F}$ rigidity to eliminate the undesirable Nash equilibria arising from $\mathcal{F}$'s excessive output ambition.

Analogously, if the roles are reversed and $\mathcal{F}$ is the standard player, for her to win the necessary and sufficient condition is

$$\frac{r}{r^{\mathcal{M}}} > \frac{x}{w}, \tag{10}$$

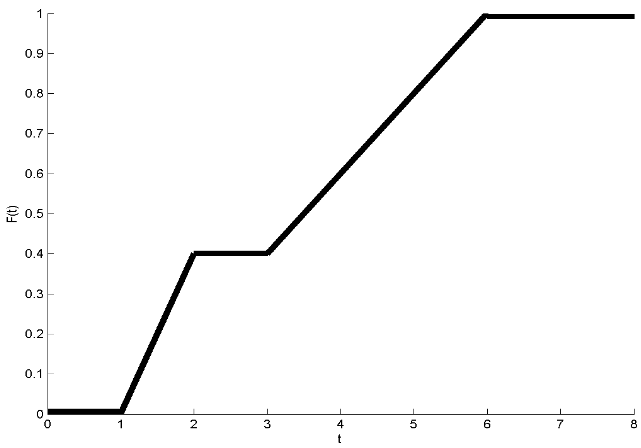

FIGURE 4. $F(t)$ of Case 2 featuring $r = 8$ and two opponents with $w_1 = 0.4, w_2 = 0.6, g_1 = 1, h_1 = 2, g_2 = 3, h_2 = 6$, ie uniformly distributed probability of moves on intervals $[1, 2]$ and $[3, 6]$.

where $r^{\mathcal{M}}$ is the analog of $r^{\mathcal{F}}$ defined above. Obviously, if the payoffs are symmetric then the conditions in (9) and (10) are equivalent.

6. Case 2: Uniformly Distributed Moves

6.1. **Monetary Union.** Consider some g_n and h_n, such that $0 \leq g_n < h_n \leq r, \forall n$, as the minimum and maximum $\mathcal{F}$ rigidity of the nth country respectively. Further assume that each $\mathcal{F}$ policymaker n can first respond with uniformly distributed probability on the interval $[g_n, h_n] \subseteq [0, r]$.[19] Then the reaction function (2) has the following specific form (see Figure 4 for a plot)

$$F(t) = \sum_{n=1}^{N} w_n F_n(t), \tag{11}$$

where $F_n(t)$ are the individual members' reaction functions

$$F_n(t) = \begin{cases} 0 & \text{if} \quad t \in [0, g_n), \\ \frac{t-g_n}{h_n-g_n} & \text{if} \quad t \in [g_n, h_n), \\ 1 & \text{if} \quad t \in [h_n, r]. \end{cases} \tag{12}$$

Proposition 2. *Consider Case 2 of the Battle of Sexes policy interaction described by (1) and (11). The necessary and sufficient degree of $\mathcal{M}$ commitment ensuring $\mathcal{M}$'s win is, among other, increasing in the minimum and maximum $\mathcal{F}$ rigidity, g_n and $h_n, \forall n$.*

Proof. Integrating the composite $F(t)$ from (11) over $[0, r]$ we get

$$\int_0^r F(t)\mathrm{d}t = \sum_{n=1}^{N} w_n[r - \frac{1}{2}(g_n + h_n)] = r - \frac{1}{2}\sum_{n=1}^{N} w_n(g_n + h_n) \tag{13}$$

[19]Let us note that if $g_n = h_n$, for all n, we get Case 1.

Substituting this integral into (4) the necessary and sufficient condition becomes

$$\frac{r}{\frac{1}{2}\sum_{n=1}^{N} w_n(g_n+h_n)} > \frac{b}{d} \tag{14}$$

The fact that the LHS is decreasing in g_n and h_n for all n completes the proof. □

Note that the proposition is a complement to Theorem 1, suggesting that *relative* commitment is what matters. This is because, for all n, a higher g_n and h_n reduce $\mathcal{M}$'s relative commitment.

6.2. **Single Country.** Under $N = 1$ the reaction function (11) and its integral become

$$F(t) = \begin{cases} 0 & \text{if} \quad t \in [0, g), \\ \frac{t-g}{h-g} & \text{if} \quad t \in [g, h), \\ 1 & \text{if} \quad t \in [h, r], \end{cases} \quad \text{and} \quad \int_0^r F(t)\mathrm{d}t = r - \frac{1}{2}(g+h). \tag{15}$$

Then the necessary and sufficient condition for $\mathcal{M}$ to win, (14), simplifies into

$$\frac{r}{\frac{1}{2}(g+h)} > \frac{b}{d}. \tag{16}$$

If the roles are reversed and $\mathcal{F}$ is the standard player, for her to win the necessary and sufficient condition becomes

$$\frac{r}{\frac{1}{2}(g+h)} > \frac{x}{w}, \tag{17}$$

where g and h now refer to the $\mathcal{M}$ policymaker's moves.

7. Case 3: Binomially Distributed (Calvo) Moves

7.1. **Monetary Union.** Assume that each $\mathcal{F}$ policymaker n moves every uniformly distributed discrete period t (eg $t \in \mathbb{N}$), but only with probability $(1-\theta_n)$ - that is independent across time and players.[20] Then θ_n can be interpreted as the probability that $\mathcal{F}$ is *unable*, after observing the central bank's determination to fight inflation regardless of the associated costs ($\mathcal{M}L$ as a response to $\mathcal{F}H$), to discipline her actions and start producing a balanced budget on average, $\mathcal{F}L$, despite this being her optimal play.[21]

Arguably, there exist a number of obstacles for a government to reform its $\mathcal{F}$ actions and put them on a sustainable path, even if it wishes to do so. These are primarily political economy influences (lobby groups, unionization, welfare schemes, naïve voters), as well as structural features (eg aging population, pay-as-you-go health and pension systems, high outstanding debt etc). Naturally, the greater the extent of these factors in a country n, the larger θ_n is, which will be postulated quantitatively in Section 8.

The composite reaction function of the $\mathcal{F}$ players can then be written as (see Figure 5 for a graphical depiction)

$$F(t) = \sum_{i=0}^{\lfloor t\rfloor -1} \sum_{n=1}^{N} w_n (1-\theta_n)\,\theta_n^i = 1 - \sum_{n=1}^{N} w_n \theta_n^{\lfloor t\rfloor}. \tag{18}$$

[20]This specification à la Calvo (1983) nests some of the above scenarios. In particular, if $\theta_n = 1, \forall n$, then we get $F(t) = 0$, which corresponds to Case 1 under $r_n^{\mathcal{F}} = r, \forall n$, and hence the conventional repeated game. If $\theta_n = 0$ we get Case 1 with $r_n^{\mathcal{F}} = 1$.

[21]Obviously, it may be the case in the real world that for some individual governments the best response to $\mathcal{M}L$ is $\mathcal{F}H$, not $\mathcal{F}L$ as assumed in (1). This may be because such 'selfish' member countries do not fully internalize the cost (negative externality) of their indisciplined actions on the union as a whole. Including this aspect would only alter our findings quantitatively - the required degree of $\mathcal{M}$ commitment would be increasing in the degree of member countries' 'selfishness'. As explicit modeling of such a scenario is straightforward (through modifying the payoffs), and does not add any game theoretic insights, we do not consider it here.

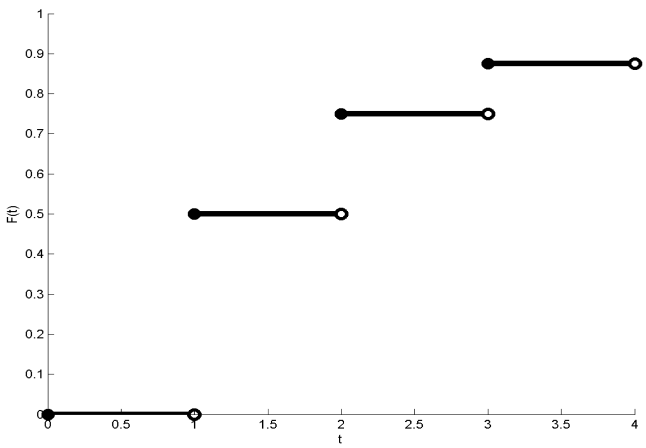

FIGURE 5. $F(t)$ of Case 3 featuring $r = 4$ and one opponent with $\theta = 0.5$.

Proposition 3. *Consider Case 3 of the Battle of Sexes policy interaction described by (1) and (18). The necessary and sufficient degree of $\mathcal{M}$ commitment ensuring $\mathcal{M}$'s win is increasing in the probabilities θ_n that the $\mathcal{F}$ policymaker(s) are unable to run sustainable budgets (on average).*

Proof. Integrating $F(t)$ from (18) over $[0, r]$ we obtain

$$\int_0^r F(t)\mathrm{d}t = r - \sum_{i=0}^{r-1}\sum_{n=1}^{N} w_n \theta_n^i = r - \sum_{n=1}^{N} w_n \frac{1-\theta_n^r}{1-\theta_n}. \tag{19}$$

Using our general necessary and sufficient condition in (4) we get, after rearranging

$$\frac{r}{\sum_{n=1}^{N} w_n \frac{1-\theta_n^r}{1-\theta_n}} > \frac{b}{d}. \tag{20}$$

The fact that the LHS is decreasing in θ_n for all n completes the proof. □

Note that the proposition again highlights the role of *relative $\mathcal{M}$ commitment*, since θ_n is the only determinant of the degree of $\mathcal{F}$ rigidity of the member countries.

7.2. **Single Country.** Under $N = 1$ the reaction function (18) and its integral become

$$F(t) = \sum_{i=0}^{\lfloor t \rfloor - 1} \theta^i (1-\theta) = 1 - \theta^{\lfloor t \rfloor} \quad \text{and} \quad \int_0^r F(t)\mathrm{d}t = \sum_{i=0}^{r-1} 1 - \theta^i. \tag{21}$$

Thus, the necessary and sufficient condition for $\mathcal{M}$ to win is

$$\frac{r}{(1 + \theta + \theta^2 + \ldots + \theta^{r-1})} > \frac{b}{d}, \tag{22}$$

If the roles are reversed and $\mathcal{F}$ is the standard player, for her to win the necessary and sufficient condition becomes

$$\frac{r}{(1 + \theta + \theta^2 + \ldots + \theta^{r-1})} > \frac{x}{w}, \tag{23}$$

where θ now refers to the $\mathcal{M}$ policymaker's probability of not being able to move each period.

8. Calibration of Case 3: European Monetary Union

Let us apply the above theory to a real world situation - the world's largest monetary union: the European Monetary Union (EMU). As of the writing of this paper, there are fifteen member countries that have adopted the common currency Euro (the so-called Eurozone), namely Austria, Belgium, Cyprus, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, the Netherlands, Portugal, Slovenia, and Spain. Therefore, we set $N = 15$.

While $\mathcal{M}$ policy is conducted by a common $\mathcal{M}$ authority, the European Central Bank (ECB), each country has an independent $\mathcal{F}$ policy. In order to consider multiple $\mathcal{F}$ policymakers the ECB will be the standard player that moves every r periods. In terms of the $\mathcal{F}$ policymakers, two types of heterogeneity are arguably the most important ones in regards to the EMU. First, it is the economic size that differs greatly across the member countries. Second and more crucially, it is the degree of $\mathcal{F}$ rigidity and indiscipline. As both types of $\mathcal{F}$ heterogeneity are present in Case 3, and the Calvo probabilistic timing seems appropriate in this context, we will utilize it here.

To calibrate θ_n in the necessary and sufficient condition (20), we propose the following matrix for assigning a value to the EMU members, based on their $\mathcal{F}$ performance in the (recent) past

$$\theta_n = \begin{cases} \frac{\alpha S_n}{\alpha S_n - 1} & \text{if} \quad S_n \leq 0, \\ 0 & \text{if} \quad S_n > 0, \end{cases} \tag{24}$$

where α is some positive constant (that determines the exact slope of θ_n), and S_n is the arithmetic *mean* of country n's fiscal surplus as a percentage of the gross domestic product (GDP) over the period 2001-2006 (inclusive) using Eurostat data (see Appendix A). This implies that $S_n > 0, S_n < 0$, and $S_n = 0$ indicate an average surplus, deficit and balanced budget respectively. We start the sample in 2001 rather than in 1999 (the year in which the Euro was officially adopted) in order to exclude the idiosyncratic effects of the Maastricht criteria on $\mathcal{F}$ policy outcomes around the time of the Euro's adoption. Nevertheless, this time frame is arguably sufficient to show the medium/long-run stance of $\mathcal{F}$ policy in these countries, and eliminate short-run (business cycle) fluctuations.[22]

The choice of the most realistic α depends on the interpretation of the length of each period, t, and the frequency of the central bank's moves, r. It was stressed in Section 2 that we examine the long-run (ie trend) outcomes of the $\mathcal{F}$-$\mathcal{M}$ interaction. Therefore, we interpret t as one year, which is the frequency of the government proposing and implementing the budget (and hence getting a chance to be fiscally sound).

In terms of $\mathcal{M}$ policy, the 'instrument' that affects long-run outcomes, ie average inflation, is the level of the *inflation target*. Under such interpretation, the frequency with which the target can be altered, r, is arguably an increasing function of the *explicitness* with which the target is stated in the central banking legislation or statutes. This is because the more explicitly the target is grounded, the less frequently it can be changed.[23]

As a baseline we set $\alpha = 1$ in (24) - see Figure 6 for a plot that also shows the EMU countries. Such a specification implies that a country with $S_n = -1$ (such as Austria) has a 50% probability

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[22]Using cyclically adjusted deficits and/or including the size of each country's debt as a percentage of GDP into the specification of S_n would not change the quantitative nature of the results.

[23]For example, the 1989 Reserve Bank of New Zealand Act states that the inflation target may only be changed in a Policy Target Agreement (PTA) between the Minister of Finance and the Governor, and this can only be done on *pre-specified regular* occasions (eg when a new Governor is appointed). Since late 1990 the PTA was 'renegotiated' (but not necessarily altered) five times, ie roughly every three years, which would imply $r = 3$. For more discussion see Libich (2008).

Obviously, the reader can think of $\mathcal{M}$ commitment r as the *expected* length of time that $\mathcal{M}$ is unable to change it target (whereby $r = \frac{1}{1-\theta_{\mathcal{M}}}$, with $\theta_{\mathcal{M}}$ being the probability in any one period).

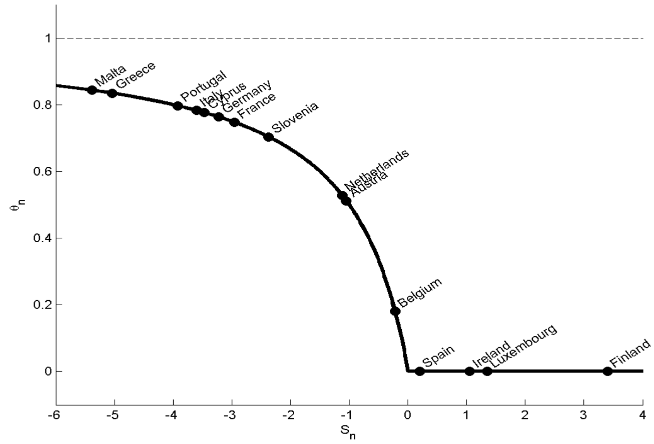

FIGURE 6. Dependence of θ_n on S_n, see (24), depicting the EMU countries.

($\theta_n = \frac{1}{2}$) of making $\mathcal{F}$ finances sustainable in any one budget, whereas a country with $S_n = -3$ (such as Germany or France) only has a 25% probability of doing so each year ($\theta_n = \frac{3}{4}$). For obvious reasons, θ_n in (24) is truncated by zero from below for countries with a surplus on average, $S_n > 0$. This means that the four such countries in the sample - Finland, Ireland, Luxemburg, and Spain - are able to play $\mathcal{F}$ discipline immediately.[24]

In terms of the weights w_n, we use the country's real GDP share of the EMU's total. Specifically, using Eurostat data, we calculate the average annual GDP for each EMU member country over 2001-2006, and divide it by the EMU's average over that period (see Appendix A).

There are two remaining parameters, b and d. The fraction $\frac{b}{d}$ depends not only on the $\mathcal{M}$ policymaker's (and society's) costs of inflation variability relative to output variability (and hence the structural parameters of the economy), but also on the relative weights assigned to these two standard objectives in the policy loss function. Therefore, we do not calibrate these parameters. Instead, we report in Figure 7 the necessary and sufficient condition (20) in terms of a threshold $\bar{r}$, as a function of $\frac{b}{d}$, above which $\mathcal{M}$ wins the Battle as (20) is satisfied. The reader can then choose values of $\frac{b}{d}$ as they see fit, and read off the necessary and sufficient degree of $\mathcal{M}$ commitment (explicitness of the target) $\bar{r}$ on the y axis. This degree does not only ensure the ECB's credibility, but it also disciplines fiscal policies of the member countries.[25]

The calibration implies a tentative conclusion: the necessary explicitness of the ECB's inflation target may be substantial. Specifically, the target should be explicit enough for 'all' to believe that it will not be abandoned or altered for at least 4-6 years.

[24]If the reader finds the values implied by $\alpha = 1$ too much on the optimistic side in terms of the reform opportunities (ie if $\mathcal{F}$ indiscipline is more persistent), s/he may want to select some $\alpha > 1$, which will increase the value of θ_n. Let us note that there also exists an alternative, short-run interpretation of t and r. The $\mathcal{M}$ policy instrument that affects short-run outcomes is the short-term interest rate. Therefore, the length r can be interpreted as (roughly) one month - the frequency of the ECB's Governing Council meetings at which the interest rate is decided. Then time period t can be thought of as one week or one day, and α selected accordingly to yield realistic values of θ_n.

[25]Recall that under $\frac{b}{d} < 1$ any r value uniquely ensures the L outcomes for both policies, since (L, L) is the unique Nash equilibrium of the standard one-shot game. Also note that while under $r < \bar{r}$ the $\mathcal{M}$ policymaker does not win the Battle, it does *not* imply that he loses it. There still exist multiple SPNE including the one with the L levels throughout, ie socially optimal outcomes *may* still obtain.

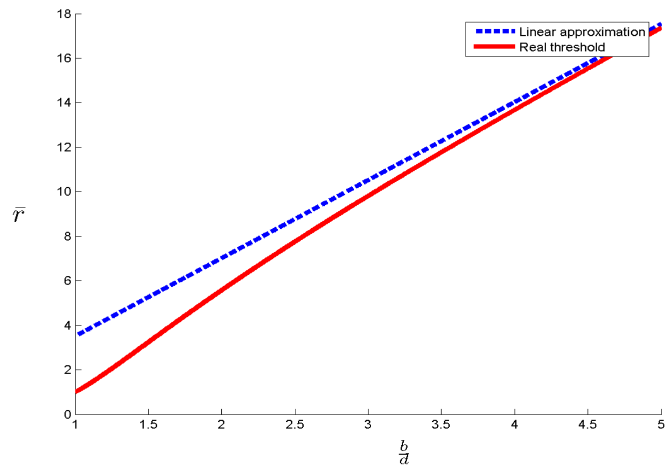

Figure 7. Dependence of $\bar{r}$ on $\frac{b}{d}$, see (20). For large r, the term θ_n^r in (20) vanishes and the relationship becomes linear (the linear approximation is indicated by the dashed line).

There seem to be two specific arrangements regarding the ECB's objectives that could help strengthen this belief and the ECB's commitment to the inflation target. Both (i) abandoning the monetary pillar and (ii) specifying the target as a point value (say 2%), not as an interval (under 2%) would arguably clarify the Bank's goals and allow the Bank better communicate its commitment to (long-term) price stability. This would equip the Bank with more ammunition when interacting with ambitious governments.

9. Extension 1: Combinations of Probability Distributions

This section reports a result which simplifies the above analysis, and importantly, allows us to easily examine any *combinations* of different probability distributions. Therefore, in contrast to the above Cases 1-3 that allowed for the union members to differ in the *degree* of $\mathcal{F}$ rigidity, this section also allows for their *type* of $\mathcal{F}$ rigidity to differ.

Lemma 1. *Consider $F(t)$ from Definition 2 such that*

$$F(r) = 1. \tag{25}$$

Then

$$\int_0^r (1 - F(t))\,\mathrm{d}t = \mu, \tag{26}$$

where μ is the mean value of the underlying probability distribution.

Proof. This is a known result in statistics, see eg Lemma 2.4 in Kallenberg (2002). □

Let us mention the interpretation of (25): it ensures that (all) the opponent(s) have the opportunity to make at least one non-standard move between their standard moves. Lemma 1 implies that, if (25) holds, even probability distributions expressing very complicated timing of moves can be 'summarized' without loss of generality by their first moments. Put differently, if (25) is satisfied then μ fully describes the overall degree of rigidity of the standard player's opponents, and $\frac{r}{\mu}$ expresses the standard player's relative commitment. The necessary and sufficient condition (4) of Theorem 1 can then be rewritten as

$$\frac{r}{\mu} > \frac{b}{d}. \tag{27}$$

The following result uses Lemma 1 to extend Theorem 1 for any combinations of probability distributions. Note however that it is not its full-fledged generalization since (25) is required to hold - for every n (which was not the case for validity of Theorem 1).

Theorem 2. *Consider the Battle of Sexes policy interaction described by (1), whereby each member n's $\mathcal{F}$ rigidity is described by an arbitrary probability distribution with a mean value of μ_n. Under $F_n(r) = 1, \forall n$, the necessary and sufficient condition for the standard player $\mathcal{M}$ to discipline the $\mathcal{F}$ policymaker(s) and win the Battle is*

$$\frac{r}{\sum_{n=1}^{N} w_n \mu_n} > \frac{b}{d}. \tag{28}$$

Proof. The proof is analogous to the proof of Theorem 1. Using backwards induction the necessary and sufficient condition for $\mathcal{M}L$ to be the best response to $\mathcal{F}H$ is

$$\sum_{n=1}^{N} w_n \left(b \int_0^r (1 - F_n(t))\, \mathrm{d}t + a \int_0^r F_n(t) \mathrm{d}t \right) > dr. \tag{29}$$

The RHS, ie $\mathcal{M}$'s payoff from playing $\mathcal{M}L$ under $\mathcal{F}H$, is now a weighted average of $\mathcal{M}$'s interactions with each individual $\mathcal{F}$. Using $a = 0$ and rearranging one obtains

$$\frac{r}{\sum_{n=1}^{N} w_n \int_0^r (1 - F_n(t))\, \mathrm{d}t} > \frac{b}{d}, \tag{30}$$

which, using Lemma 1, yields (28). □

It is worth noting that the second moments of the probability distributions do not play any role (in the absence of discounting). To demonstrate the usefulness of this 'shortcut', let us report an example that combines the above Case 2 with normally distributed moves.

Example 1. *Consider a monetary union consisting of two equally sized member countries, whose $\mathcal{F}$ policymakers' timing of moves has the following form:*

- *Country 1: uniformly distributed moves of Case 2, namely $F(t)$ from (15),*
- *Country 2: normally distributed moves, such that*

$$F_2(t) = \frac{\Phi_{\mu_2,\sigma^2}(t)}{\Phi_{\mu_2,\sigma^2}(r) - \Phi_{\mu_2,\sigma^2}(0)} \tag{31}$$

where

$$\Phi_{\mu_2,\sigma^2}(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{(x-\mu_2)^2}{2\sigma^2}} \mathrm{d}x,$$

is the CDF of a normal distribution (truncated on the interval $[0, r]$), and where μ_2 and σ are its mean and standard deviation (and $x \in \mathbb{R}$). Then the necessary and sufficient degree of $\mathcal{M}$ commitment for $\mathcal{M}$ to win the Battle of Sexes is

$$\frac{r}{\mu_1 + \mu_2} > \frac{b}{d}, \tag{32}$$

where $\mu_1 = \frac{g+h}{2}$ is the mean value of the probability distribution in (15).

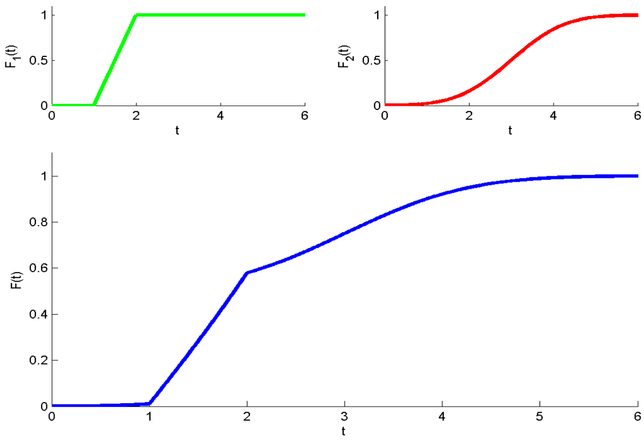

Figure 8. $F(t)$ of Example 1 featuring $r = 6$ and two equally weighted opponents - one with uniformly distributed probability on $[1, 2]$, and the other with (truncated) normally distributed moves with $\mu = 3$ and $\sigma^2 = 1$.

For a graphical depiction of the composite $F(t)$ see Figure 8. Let us note two things. First, the condition (25) is implicitly satisfied for both countries. Second, σ does not determine the threshold value of r.

10. Extension 2: Discounting

As was apparent in the proof of Theorem 1, discounting by the *opponent* of the standard player has neither qualitative nor quantitative effect on the outcomes of the game. This section shows that while discounting by the standard player himself does have an effect, it is only a quantitative one. Specifically, the standard player's impatience works in the predicted direction of making it harder for the player to win.

Let us assume that the standard player's payoff is discounted by $e^{-\rho t}$, where $\rho \in [0, \infty)$.[26]

Theorem 3. *Consider the Battle of Sexes policy interaction described by (1), in which the standard player's discounts the future by* $e^{-\rho t}$*. The necessary and sufficient degree of the standard player's commitment to win the Battle is increasing in the degree of his impatience (discounting),* ρ*.*

Proof. See Appendix B. □

Importantly, the necessary and sufficient degree of $\mathcal{M}$ commitment is still increasing in (overall) $\mathcal{F}$ rigidity, as well as in $\frac{b}{d}$. It is further straightforward to show that there exists a sufficient threshold $\overline{\rho}$ such that, for all $\rho < \overline{\rho}$, we can find a finite r that delivers $\mathcal{M}$'s win (ie $\bar{r}$ exists). These facts imply that the above findings are robust to discounting.

Nevertheless, in contrast to the outcome under standard commitment, in which Stackelberg leadership delivers a win to $\mathcal{M}$ regardless of his discount factor, even infinitely strong $\mathcal{M}$ commitment may be insufficient in our framework. Put differently, there exists a threshold $\hat{\rho}$ such

[26]It should also be noted that the analysis of the standard player's discounting can be made more parsimonious by incorporating it into the function $F(t)$. We however do not do so in order to keep the intuition of $F(t)$ as a reaction function of the opponent(s).

that, for all $\rho > \hat{\rho}$, there is no r that satisfies the necessary and sufficient condition for $\mathcal{M}$'s win (ie $\bar{r}$ does not exist).

11. Summary and Conclusions

The paper models the interaction between fiscal ($\mathcal{F}$) and monetary ($\mathcal{M}$) policy - in a monetary union as well as in a single country setting. The aim is to consider two policy relevant questions: 1) Under what circumstances, if any, can excessive $\mathcal{F}$ policies undermine the credibility of monetary policy, and cause deviations from the central bank's inflation target?, and 2) Is there anything the central bank can do to indirectly induce a change in the undesirable fiscal stance?

The paper's main contribution lies in examining the interaction of $\mathcal{M}$ and (any number of) $\mathcal{F}$ policies in a novel game theoretic setting, in which the *timing* of the policies' actions is no longer deterministic but *stochastic*. Our framework is general enough to allow for an *arbitrary probability distribution* of the policymakers' moves, as well as arbitrary *combinations* of probability distributions. For illustration we complement the results for a fully general setting by depicting several realistic scenarios, namely uniform, binomial, and normal distributions.

All settings show that if the central bank is *sufficiently strongly* committed, it can resist $\mathcal{F}$ pressure and ensure the credibility of its target. Furthermore, such strong $\mathcal{M}$ commitment $\bar{r}$ - interpretable as a sufficiently *explicit numerical inflation target* - has the potential of *disciplining* fiscal policy, and hence improving outcomes of both policies. We show that $\bar{r}$ is an increasing function of: (i) $\mathcal{F}$ rigidities of the member countries, (ii) their relative economic size, (iii) the relative cost of output vs inflation volatility, and (iv) the central banker's impatience.

The latter implies that a *less* patient central bank needs to commit *more* strongly (explicitly) to ensure its credibility. Interpreting patience as an increasing function of the degree of central bank independence, this offers an explanation for the fact that inflation targets were more explicitly grounded in countries originally lacking central bank independence (such as New Zealand, UK, Canada, and Australia) than in those with a rather independent central bank (such as the US, Germany, and Switzerland).

12. References

Barnett, R. 2001. Inflation, Taxes, and the Coordination of Monetary and Fiscal Policy by Use of a Game of Chicken. Canadian Journal of Economics 34, 82-99.

Bhattacharya, J. and Haslag, J.H. 1999. Monetary policy arithmetic: some recent contributions. Economic and Financial Policy Review Federal Reserve Bank of Dallas, Q III, 26-36.

Cecchetti, S.G. and Ehrmann, M. 1999. Does Inflation Targeting Increase Output Volatility? An International Comparison of Policymakers' Preferences and Outcomes. NBER Working Papers: 7426.

Cho, I. and Matsui, A. 2005. Time Consistency in Alternating Move Policy Games. Japanese Economic Review 56 (3) 273–294. .

Dixit, A. and Lambertini, L. 2003. Interactions of Commitment and Discretion in Monetary and Fiscal Policies. American Economic Review 93(5), 1522-1542.

Eggertsson, G.B. and Woodford, M. 2004. Optimal Monetary and Fiscal Policy in a Liquidity Trap. National Bureau of Economic Research Inc NBER Working Papers: 10840.

Faust, J. and Svensson, L.E.O. 2001. Transparency and Credibility: Monetary Policy with Unobservable Goals. International Economic Review 42, 369-397.

Hughes Hallett, A. and Libich, J. 2007. Fiscal-Monetary Interactions: The Effect of Fiscal Restraint and Public Monitoring on Central Bank Credibility. Open Economies Review 18 (5), 559-576.

Kallenberg, O. 2002. Foundations of Modern Probability Series: Probability and its Applications. 2nd ed.

Kuttner, K.N. 2004. A snapshot of inflation targeting in its adolescence. In C. Kent and S. Guttman (eds.), The Future of Inflation Targeting, pp. pp.6-42. Sydney: The Reserve Bank of Australia.

Lagunoff, R. and Matsui, A. 1997. Asynchronous Choice in Repeated Coordination Games. Econometrica 65, 1467-1477.

Leeper, E.M. 1991. Equilibria under 'active' and 'passive' monetary and fiscal policies. Journal of Monetary Economics 27 (1), 129-147.

Libich, J. 2008. An explicit inflation target as a commitment device. Journal of Macroeconomics 30 (1), 43-68.

Libich, J., Hughes Hallett, A. and Stehlik, P. 2007. Monetary and Fiscal Policy Interaction with Various Degrees and Types of Commitment. CEPR DP 6586.

Maskin, E. and Tirole, J. 1988. A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs. Econometrica 56, 549-569.

Mundell, R. 1962. Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability. IMF Staff Papers, 9, 70-79.

Neyman, A. and Sorin, S. 2003. Stochastic Games and Applications. Kluwer Academic Press.

Persson, M., Persson, T. and Svensson, L.E.O. 2006. Time Consistency of Fiscal and Monetary Policy: A Solution. Econometrica 74(1), 193-212.

Persson, T. and Tabellini, G. (eds.) 1994. Monetary and fiscal policy. Volume 1. Credibility. Cambridge and London: MIT Press.

Sargent, T.J. and Wallace, N. 1981. Some unpleasant monetarist arithmetic. Federal Reserve Bank of Minneapolis Quarterly Review 5, 1-17.

Shapley, L.S. 1953. Stochastic games. Proc. Nat. Acad. Science: 39:1095-1100.

Tinbergen, J. 1954. Centralization and Decentralization in Economic Policy. Amsterdam.

Wen, Q. 2002. Repeated Games with Asynchronous Moves. Department of Economics Vanderbilt University Working Papers: 0204.

Woodford, M. 2003. Interest and prices. Princeton and Oxford: Princeton University Press.

Appendix A. EMU Data

We use the data from Eurostat to create our variables S, θ, and w for each member country n, reported in the following Table. The way these are created is described in the main text.[27]

[27] The only exception is Greece, for which the 2001 budget balance is not stated in the Eurostat database, and hence the 2002-2006 period is averaged over to create S. Further, while the individual country values the table are rounded to two or three decimal places, the Eurozone averages as well as the calculations in the main text have been done with nine decimal places.

Country n	Weight w_n	Surplus S_n	$\mathcal{F}$ Rigidity θ_n
Austria	0.033	−1.05	0.51
Belgium	0.039	−0.22	0.18
Cyprus	0.001	−3.47	0.78
Finland	0.021	3.4	0
France	0.218	−2.95	0.75
Germany	0.327	−3.22	0.76
Greece	0.019	−5.04	0.83
Ireland	0.015	1.05	0
Italy	0.147	−3.6	0.78
Luxembourg	0.004	1.35	0
Malta	0.001	−5.38	0.84
The Netherlands	0.061	−1.12	0.53
Portugal	0.016	−3.92	0.8
Slovenia	0.003	−2.37	0.7
Spain	0.094	0.2	0
Average (non-weighted)	0.067	−1.75	0.5
Average (weighted)		−2.4	0.61

Appendix B. Proof of Theorem 3

Proof. The proof is again analogous to the proof of Theorem 1 in all its aspects. If $\mathcal{M}$ is the standard player, the necessary and sufficient condition (5) becomes, under discounting

$$b\int_0^r e^{-\rho t}\left(1-F(t)\right)\mathrm{d}t + a\int_0^r e^{-\rho t}F(t)\mathrm{d}t > d\int_0^r e^{-\rho t}\mathrm{d}t, \tag{33}$$

It suffices to show that if (33) holds for some $\rho > 0$, then the inequality holds for $\rho = 0$ - the case without discounting in (5). The proof that this is true for any pair of ρ_1, ρ_2 such that $\rho_1 > \rho_2$ is equivalent.

The inequality in (33) can be rewritten into

$$\int_0^r e^{-\rho t}\left[\frac{b}{d}\left(1-F(t)\right)-1\right]\mathrm{d}t < 0. \tag{34}$$

Let us define $\varpi \in [0, r]$ as the point in which

$$\left[\frac{b}{d}\left(1-F(\varpi)\right)-1\right] = 0.$$

The integral in (34) can thus be split into the positive and the negative part, ie

$$\int_0^{\varpi} e^{-\rho t}\left[\frac{b}{d}\left(1-F(t)\right)-1\right]\mathrm{d}t + \int_{\sigma}^{r} e^{-\rho t}\left[\frac{b}{d}\left(1-F(t)\right)-1\right]\mathrm{d}t < 0.$$

Consequently, the mean value theorem and non-increasing $e^{-\rho t}$ imply that there exist positive numbers ρ_1, ρ_2 (with $1 > \rho_1 > \rho_2 > 0$), such that

$$\rho_1\int_0^{\varpi}\left[\frac{b}{d}\left(1-F(t)\right)-1\right]\mathrm{d}t + \rho_2\int_{\sigma}^{r}\left[\frac{b}{d}\left(1-F(t)\right)-1\right]\mathrm{d}t < 0.$$

The LHS can be rewritten into

$$(\rho_1 - \rho_2) \int\limits_0^{\varpi} \left[\frac{b}{d} (1 - F(t)) - 1 \right] \mathrm{d}t + \rho_2 \int\limits_0^{r} \left[\frac{b}{d} (1 - F(t)) - 1 \right] \mathrm{d}t < 0,$$

which yields, after rearranging

$$\int\limits_0^{r} \left[\frac{b}{d} (1 - F(t)) - 1 \right] \mathrm{d}t < \frac{\rho_1 - \rho_2}{-\rho_2} \int\limits_0^{\varpi} \left[\frac{b}{d} (1 - F(t)) - 1 \right] \mathrm{d}t < 0.$$

And thus the desired inequality (5) holds. □