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A Note on the Anchoring Effect of Explicit Inflation Targets

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Abstract

Empirical literature provided convincing evidence that explicit (ie legislated) inflation targets anchor expectations. We propose a novel game theoretic framework with generalized timing that allows us to formally capture this beneficial *anchoring effect*. Using the framework we identify several factors that influence whether and how strongly expectations are anchored, namely: (i) the public's cost of decision-making, (ii) the public's inflation aversion, (iii) the slope of the Phillips curve, (iv) the magnitude of supply shocks, (v) the degree of central bank conservatism, and under many (but not all) circumstances, (vi) the explicitness of the inflation target.

Keywords: anchoring effect, updating expectations, rational inattention, endogenous timing, decision-making cost, explicit inflation targets.
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1. Introduction

Private expectations of the future play a central role in the optimal setting of monetary policy, as well as its outcomes. The extent to which the policy's design can affect expectation formation has been a matter of debate. Several recent empirical papers contributed to this debate by showing that in countries with an explicit (ie legislated) inflation target expectations are better anchored, ie less responsive to various pieces of news and policy announcements.[2]

This paper offers a formal model of how explicit inflation targets anchor expectations, ie why they may make private agents inattentive. Anchoring is of interest to central bankers since anchored expectations give them more leverage over the real interest rate, and hence make their stabilization efforts more effective. For this reason Kohn (2008) argued that: '*This anchoring is critical.*' - see also Bernanke (2007) or Mishkin (2007).

To show the anchoring effect, we postulate a novel game theoretic framework that generalizes the timing of the players' actions. Specifically, the policymaker and the public will no longer necessarily move every period, and/or in a simultaneous fashion, which has been the case in most macroeconomic as well as game theoretic settings.[3] Instead, the players will be moving with a certain *constant frequency*.[4]

Our framework allows the frequency of actions to differ across the players, and be *endogenous*. Under our general timing setup, for expectations to be called 'anchored' they have to be: 1) infrequently updated, and 2) on average equal to the long-run inflation goal of the central bank (not necessarily an explicit one).

Such specification implies two features about the agents' responses to new data, both of which are consistent with Bernanke's (2007) interpretation of 'anchored expectations' as meaning '*relatively insensitive to incoming data*'. First, in many periods private agents do not react to shocks at all. Second, even in periods in which agents do act, they respond less aggressively (ie incorporate a smaller portion of the observed transitory shock), since they know that their decision will not be reconsidered for several periods over which the shock will be fading away.

Let us now sketch how expectations get anchored in the model. The public is 'economically rational', and hence it optimally chooses the frequency with which it updates expectations - based on a cost/benefit calculation. This frequency affects the public's utility in three respects, which we refer to as: (i) the *decision-cost motive*, (ii) the *accuracy motive*, and (iii) the *monitoring motive.*

In terms of (i), we assume that the public faces some cost of decision-making, eg gathering and processing information, changing its previous actions etc. Naturally, this cost

[2] See for example Beechey et al. (2008), Gürkaynak et al. (2006) and (2005), Levin et al. (2004), or Kuttner and Posen (1999).

[3] At least since Barro and Gordon (1983), the interaction between the central bank and private agents has been commonly studied as a standard repeated game. In such setting - under both discretion and pre-commitment (timeless perspective) - the players' instruments are adjusted *simultaneously* at *each* period. The same is implicitly assumed in conventional rational expectations models.

[4] Such timing is motivated by Tobin (1982), who in his Nobel lecture argued that: '*Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer except when extraordinary events compel revisions. It would be desirable in principle to allow for differences among variables in frequencies of change and even to make these frequencies endogenous. But at present, models of such realism seem beyond the power of our analytical tools.*'

is increasing in the frequency of updating expectations. As such, it constitutes a reason for the public to be *rationally inattentive* (ie optimally choose to update expectations *less frequently* - see eg Reis (2006)), and for expectations to be anchored.

In contrast, the two motives (ii) and (iii) will go in the opposite direction, and provide the public with reasons for updating expectations *more frequently.* The (short-run) accuracy motive refers to the public's attempt to correctly respond to shocks in real time. Frequent updating of expectations ensures that current shocks are incorporated into the public's decision, which minimizes the expectation errors. We identify several factors that influence the degree of expectation anchorness, but show that the explicitness of the target is *not* one of them. Therefore, the accuracy motive alone is unable to explain why explicit targets have an anchoring effect.

In contrast, the (long-run) monitoring motive provides an explanation. It refers to the public's attempt to discourage the policymaker from deviating from the optimal long-run inflation level. By frequent updating, the public can 'punish' the policymaker and reduce his output gain from small surprises - eliminating incentives to carry them out. Since an explicit inflation target can be 'reconsidered' less frequently/likely by the monetary policymaker than an implicit one, the public's punishment lasts for longer, and can thus be less frequent. Therefore, the optimal frequency of expectation updating is a decreasing function of the explicitness of the target - ie the anchoring effect occurs.

2. Macroeconomic Model

2.1. **Economy.** For illustration, we use a familiar New Keynesian model - a simplified version of Clarida et al. (1999). Two equations - a Phillips curve and an IS curve - describe the economy, but we will only need the former for our purposes

$$\pi_t = \lambda x_t + e_t + u_t, \tag{1}$$

where $\lambda > 0$, π denotes inflation, e denotes inflation expectations, x expresses the output gap, and t denotes time. For algebraic convenience we assume the periods to be discrete steps of an arbitrarily small length. The timing of e will be endogenous and described below, but in any case the public is, like the policymaker, *forward looking* and acts rationally. Both players also have common knowledge of rationality and complete information about the economy and the structure of the game. The variable u is an inflation shock with a zero mean and variance σ_u^2. The shock is observable in real time by the players that can move in that period.[5]

2.2. **Players.** The preferences of the players are as follows (we will assume out discounting for simplicity). The policymaker has the standard quadratic one-period utility function, namely

$$U_t^g = -\alpha(x_t - x^T)^2 - (\pi_t - \pi^O)^2, \tag{2}$$

where π^O is the socially optimal low inflation level. The positive parameter α expresses the degree of the central bank's conservatism, and we will restrict our attention to the

[5]It will become apparent that the nature of our results is largely independent of the details of the macroeconomic model. For example, the intuition obtains for various forms of the shock process (including AR1) that have a zero mean.

realistic cases $\alpha \in (0,1)$.[6] The output *gap* target is $x^T \in \mathbb{R}$, ie the output target itself may be above, below, or equal to potential output. We will first consider the case $x^T = 0$, and then examine $x^T \neq 0$ (the possible reasons for which will be discussed in Section 5).

The public's one-period utility function is the following

$$U_t^p = -(\pi_t - e_t)^2 - C_\pi - C_e, \tag{3}$$

where the three components will underlie the three motives discussed in the introduction. The first element is a common representation of rational expectations, whereby the public suffers disutility from incorrectly predicting the inflation rate (see Backus and Driffill (1985)). We will refer to it as the *inaccuracy cost.* The C_π element is an *inflation cost*, and the C_e variable is a *decision-making cost* (both will be postulated below).

2.3. **Solution.** In a conventional one-shot (one-period) game our model yields outcomes analogous to Clarida et al. (1999). To demonstrate this, set up the Lagrangian and derive the familiar targeting rule under discretion

$$\pi_t - \pi^O = -\frac{\alpha}{\lambda}(x_t - x^T). \tag{4}$$

Substituting (4) into the Phillips curve and imposing rational expectations then implies

$$\pi_t^* = \pi^O + \frac{\alpha}{\lambda}x^T + \frac{\alpha}{\alpha+\lambda^2}u_t \quad \text{and} \quad x_t^* = -\frac{\lambda}{\alpha+\lambda^2}u_t, \tag{5}$$

which are the standard values of inflation and the output gap in equilibrium (all equilibrium values will be denoted by asterisk throughout).

2.4. **Two Policy Instruments.** In modern macroeconomic models the central bank's instrument is the *short-term interest rate, i*, which determines the level of inflation in each period, π. Likewise in our model, but we suppress the demand side for parsimony.

In addition, there is another policy 'instrument'. The policymaker chooses the level of its *long-run inflation target*, π^T. Long-run expresses the fact that it is the policymaker's preferred *average level* of inflation, $\bar{\pi}$ (all averages will be denoted by bar). Therefore, in the presence of shocks it does not need to be achieved each and every period, it only needs to be delivered *on average* over the medium-long-run (business cycle).

Specifically, in a certain period the central bank may, in order to stabilize output, optimally select an inflation rate that deviates from its inflation target, $\pi_t^* \neq \pi^T$. Therefore, when stating that the long-run inflation *target is achieved* or *deviated from*, our meaning is always in an *average* (long-run) sense.[7] Such mutual consistency of the short-run and long-run instruments, generally present in most macroeconomic models, can be seen in (5), where the supply shock does not affect the average (long-run) values.

3. Rational Inattention and the Anchoring Effect

In our game theoretic model, the public and the central bank will not necessarily 'move' every period. Instead, they will be *able* to adjust their instruments with a certain frequency - in line with Tobin's (1982) call quoted above.

[6] Research shows that the central bank's relative weight on output in industrial countries has been fairly low, see for example Clarida et al. (1998).

[7] Most explicit inflation targets in industrial countries are specified as a long-run objective, and interpreted in such a flexible fashion that allows for output stabilization.

3.1. **Timing of Moves.** To generalize the timing of the standard repeated game but still keep the framework as comparable as possible, we will assume that the frequency of moves is *constant.* In terms of the central bank, we assume that it can adjust i - and hence π - every period, whereas it can only adjust its inflation target π^T every r^b periods. In interpreting r^b we will assume that since a more explicit target is more visible by the public, it can be less frequently reconsidered and altered.[8] One can think of institutional (legislative) constraints or reputational consequences following frequent changes of the inflation target. Naturally, these can be stronger if the target is explicit than if it is implicit. Therefore, we have the following definition:

Definition 1. *The variable r^b expresses the degree of* ***explicitness of the inflation target****.*

In our companion paper Libich (2008a) we model r^b as an endogenous variable. Nevertheless, in order to keep the focus of the presented paper on the behavior of expectations we will treat r^b as exogenous here.

In terms of the public, it will update its expectations every r_p periods, whereby r_p will be endogenously determined (optimally selected by the public). Let us define some terminology that will be used throughout.

Definition 2. *The level r^{p*} optimally selected by the public expresses the* ***degree of rational inattention****. The public is* ***rationally inattentive*** *if r^{p*} is strictly positive (does not approach zero). Inflation expectations are* ***anchored*** *if (i) the public is rationally inattentive, and (ii) expectations are on average equal to the optimal inflation target, $\bar{e} = \pi^O$. In such case the variable r^{p*} also expresses the degree of* ***anchorness of expectations****.*

The definition implies that while anchored expectations in our model always imply some degree of rational inattention, the reverse is not true. In both cases we have a strictly positive (and potentially large) r^{p*}, but in the latter case expectations may or *may not* equal the optimal inflation level on average.

Definition 3. *An explicit inflation target will be called to have an* ***anchoring effect*** *if (i) expectations are anchored, and (ii) the equilibrium degree of expectation anchorness, r^{p*}, is a non-decreasing function of the target's explicitness, r^b.*

To study the anchoring effect of explicit inflation targets effectively let us assume that

$$\frac{r^b}{r^p} = \left\lfloor \frac{r^b}{r^p} \right\rfloor > 1, \tag{6}$$

where $\lfloor . \rfloor$ denotes the integer value (the so-called floor function). This purely technical restriction will ensure that the game is closer to the standard repeated game setup, as it features both *synchronized* (ie simultaneous) and *asynchronized* moves.[9] It implies

[8]For example, the 1989 Reserve Bank of New Zealand Act states that the explicit inflation target may only be changed in a Policy Target Agreement between the Minister of Finance and the Governor, and that this can only be done on *pre-specified regular* occasions (eg when a new Governor is appointed). Since late 1990 the PTA was 'renegotiated' (but not necessarily altered) five times, ie roughly every three years.

[9]It is important to note, however, that this special case is representative of the more asynchronous cases with $\frac{r^b}{r^p} \neq \left\lfloor \frac{r^b}{r^p} \right\rfloor$, see Libich (2008b).

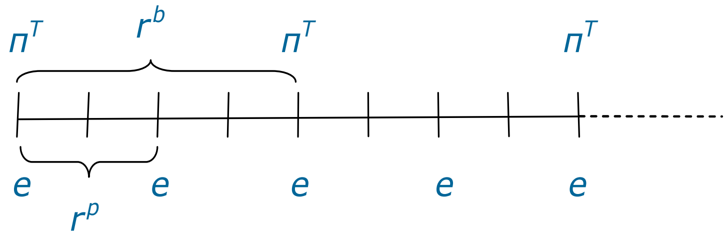

FIGURE 1. An example of the timing with $r^b = 4, r^p = 2$ showing two repetitions of the dynamic stage game (the short–run policy instrument $\pi(i)$ adjustable every period has been left out for clarity).

that: (i) every r^b the public updates expectations and the central bank reconsiders the inflation target simultaneously (ie have synchronized moves), and that (ii) expectations can also get adjusted in between these synchronized moves since $r^b > r^p$.[10] Combining these points implies that (iii) the *dynamic stage game* is r^b periods long, and that it gets regularly repeated. We will denote the *horizon* of the game, ie the total number of periods by T.

Let us summarize the timing of moves - an example of a time line is in Figure 1.

(1) At the beginning of the game, in $t = 0$, the public chooses r^p - observing r^b.
(2) Still in $t = 0$ and observing r^p, r^b, and the current shock, the policymaker and the public make a synchronized first move of all their instruments $\left\{\pi(i), \pi^T, e\right\}$.
(3) The policymaker then reconsiders the interest rate (and hence inflation) every period, and the long-run inflation target every r^b periods. The public updates expectations every r^p periods. All these moves are made observing all past and current shocks, as well as all past moves of the opponent.
(4) The payoffs are accrued every period until period T after which the game finishes.

3.2. **Three Motives in Terms of r^{p*}.** To understand the public's decision about the optimal frequency of updating expectations we need to examine the three motives discussed in the introduction. The public's incentive to update expectations *less frequently*, which provides reasons for rational inattention and anchored expectations, is due to the associated cost of doing so, C_e. For example, Mankiw and Reis (2002) discuss the existence of costs related to *'changing wage contracts and information-gathering, decision making, negotiation and communication'*. We interpret our C_e in such broad sense.

The related body of literature assumes, either implicitly or explicitly, that this cost is a per-period fee increasing in the number of updating/processing. The same will be assumed in our model, $\frac{\partial C_e}{\partial r^p} < 0, \forall r^p$. To obtain a clear-cut and illustrative analytical solution we will use the following functional form

$$C_e = \frac{c_e}{r^p}, \tag{7}$$

where $c_e > 0$. This implies that a higher inattention r^p leads to a lower cost C_e.

[10]Our results will imply that in the $r^p \geq r^b$ case there will never be an anchoring effect.

In contrast, the public's incentive to update expectations *more frequently* si due to the two remaining elements in its objective function. The first - short-run - reason is the *accuracy motive*, which works for any $x^T \in \mathbb{R}$ of the policymaker. In a period in which the public does not update expectations, it does not optimally react to the current shock (if any). Expectations will therefore be set incorrectly and deviate from actual inflation, which is costly to the public. A lower r^p will decrease the proportion of periods $\frac{r^p}{T}$ with such inaccuracy cost, and hence increase the public's utility.

We will show that the accuracy motive can identify a number of variables that determine the degree to which expectations are anchored. Nevertheless, the explicitness of the inflation target is *not* among them, ie this motive alone cannot explain the anchoring effect of explicit targets.

Therefore, we also examine another - long-run - reason for updating expectations more frequently, the *monitoring motive*. It is determined by two factors. First, the public is averse to deviations of long-run inflation from the optimal long-run level, which we called the inflation cost C_π. Let us postulate it as the following fixed per-period cost

$$C_\pi = \begin{cases} c_\pi > 0 & \text{if} \quad \bar{\pi} \neq \pi^O, \\ 0 & \text{if} \quad \bar{\pi} = \pi^O. \end{cases} \tag{8}$$

The second driver of the monitoring motive is the fact that the policymaker's output target differs from potential, $x^T \neq 0$. He then has an incentive to carry out inflation/deflation surprises to achieve its output objective. As these are costly to the public (increase C_π), the public may find it optimal to keep the policymaker in check and eliminate this incentive. It has a way of doing so: since the policymaker can adjust its long-run inflation only every $r^b > r^p$ periods, the public can punish the policymaker for such behavior.[11]

The fact that the size (length) of the punishment is increasing in $\frac{r^b}{r^p}$ implies that under a more explicit target (higher r^b), less frequent expectation updating (higher r^p) is required to deliver a punishment of the same magnitude, and eliminate the policymaker's temptation (ie minimize C_π). The fact that the public also wants to economize on its C_e cost then implies that r^{p*} is an increasing function of r^b.

In order to improve the exposition, we will examine the accuracy and the monitoring motive separately in the next two sections.

4. The Short-run Accuracy Motive

This section will deal with the public's tradeoff between infrequent updating (and minimizing the decision-making cost C_e), and frequent updating (and minimizing the inaccuracy cost $(\pi_t - e_t)^2$).

Proposition 1. *Assume* ***no monitoring motive,*** $x^T = 0$. *The public is rationally inattentive and its expectations are anchored, whereby the* ***equilibrium anchorness*** *is decreasing in (i) the variance of shocks* σ_u^2*, and increasing in: (ii) the cost of decision-making* c_e*, (iii) the output sensitivity of inflation* λ*, (iv) the time horizon* T*, and (v) the policy conservatism* $\frac{1}{\alpha}$*. An explicit inflation target has however* ***no anchoring effect.***

[11] Importantly, note that this punishment is the public's optimal choice, not an arbitrary rule (trigger strategy) of the Barro-Gordon variety.

Proof. Under $x^T = 0$ there is no temptation to surprise inflate/deflate, and hence (5) shows that *average* inflation and expectations are at the O level throughout, $\bar{e}^* = \bar{\pi}^* = \pi^O$. It then follows from (8) that $C_\pi = 0$.

The inaccuracy cost differs in periods in which expectations get updated (whose proportion over all periods is $\frac{T-r^p}{T}$), and those in which this does not happen (whose proportion is $\frac{r^p}{T}$). The public's *expected* one period utility (denoted by EU_t^p) is therefore a weighted average of utilities from these two types of periods

$$EU_t^p = -\left(\frac{T-r^p}{T}0 + \frac{r^p}{T}\left(\frac{\alpha}{\alpha+\lambda^2}\sigma_u\right)^2\right) - 0 - \frac{c_e}{r^p}. \tag{9}$$

This summarizes the implications of (1) and (3) that in the updating periods the inaccuracy cost is zero, since expectations are set accurately, $e_t^* = \pi_t^*$, and that in the non-updating periods the cost $(\pi_t^* - e_t^*)^2$ is of the expected size $\left(\frac{\alpha}{\alpha+\lambda^2}\sigma_u\right)^2$. The latter is because the policymaker can adjust its short-term interest rate instrument every period, choosing the optimal level π_t^* from (5), whereas expectations will be pre-set at the long-run component of π_t^* from (5), π^O, since the shock cannot be predicted.

Differentiating (9) with respect to r^p, setting equal to zero, and rearranging yields[12]

$$\hat{r}^p = \frac{\alpha+\lambda^2}{\alpha}\sqrt{\frac{2Tc_e}{\sigma_u^2}}. \tag{10}$$

The fact that $\hat{r}^p$ in (10) is a function of the five variables in Proposition 1 with the desired signs, but not a function of r^b, completes the proof. □

Note that all the five determinants of the degree of anchorness work in the expected direction. For example, if shocks are larger the inaccuracy cost rises and the public therefore chooses to update expectations more frequently. The fact that explicit targets play no anchoring role is also intuitive. In the absence of the policymaker's temptation to deviate from potential output, the long-run inflation target is always 'credible', and hence the public has no incentive to monitor - it simply economizes on the decision-making cost vis-à-vis the inaccuracy cost. The next section therefore looks for the anchoring effect elsewhere, and examines the monitoring motive of the public.

5. The Long-run Monitoring Motive

The public only has an incentive to monitor the policymaker under $x^T \neq 0$.[13] The literature has identified several possible reasons for $x^T \neq 0$, such as (i) mismeasurement of potential output (eg Orphanides (2001)), (ii) market imperfections (eg Barro and Gordon (1983)), (iii) political economy reasons (eg Faust and Svensson (2001)), or (iv) a shortcut for an asymmetry in the policy preferences (eg Cukierman and Gerlach (2003)).

[12] Note that due to our asynchronicity restriction (6), $\hat{r}^p$ should be rounded to the nearest real value such that $\frac{r^b}{r^p}$ is an integer. Nevertheless, since (6) is a purely technical assumption, in what follows we will use the more illustrative original condition in (10).

[13] In many settings in which the public is *uncertain* about the value of x^T, the monitoring motive is likely to exist even under $x^T = 0$.

The public monitors to minimize its inflation cost C_π, ie reduce deviations of average inflation from the optimal long-run inflation level. The monitoring motive is therefore about the *average level*, at which expectations are anchored. It was shown in (5) and discussed in Section 2.4 that the average level of inflation and expectations is unaffected by zero-mean shocks. This implies that we can, without loss of generality, separate the monitoring motive from the accuracy motive, and examine the former by abstracting from shocks and short run deviations. Put differently, the setting of the policymaker's short-run instrument $\pi(i)$ becomes superfluous in this section, since we know that on average it will be set to be consistent with the selected long-run inflation target, π^T. This allows us to only focus on the π^T and e actions, whereby the latter should also be interpreted as choosing some *average* level, $\bar{e}$.

In order to better communicate the intuition of the anchoring effect, we will streamline the exposition of the rest of our analysis by making several assumptions. First, we normalize $\lambda = 1$ and $\pi^O = 0$ throughout. Second, in presenting the normal form of the game we truncate the long-run action sets of the players to two *average* levels (the short-run levels will however remain *unrestricted*). Specifically, we follow the standard truncation (see eg Cho and Matsui (2005)), and choose the two levels of interest: one is the socially optimal level, O, and the other is the equilibrium (but potentially time inconsistent) long-run level from (5), which we denote by S as suboptimal

$$\pi^T \in \{\pi^O, \pi^S = \pi^O + \frac{\alpha}{\lambda} x^T\} \ni \bar{e}. \tag{11}$$

Note that under the considered $x^T \neq 0$ the O and S levels are different for all α.

5.1. **Equilibrium of the (Standard) Static Stage Game.** Let us first examine the outcomes of the standard *static* stage game (lasting one period), which are unaffected by r^b and r^p. The payoff matrix can be obtained using the macroeconomic outcomes, (1)-(3), with the truncation (11). Denoting the payoffs by $\{\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d},\mathbf{w},\mathbf{x},\mathbf{y},\mathbf{z}\}$ the payoff matrix is then as follows.[14]

		Public	
		$\bar{e}^O$	$\bar{e}^S$
Central	π^O	$\mathbf{a}= -\alpha x^{T^2}$; $\mathbf{w}= -C_e$	$\mathbf{b}= -\alpha(\alpha+1)^2 x^{T^2}$; $\mathbf{x}= -\alpha^2 x^{T^2} - C_e$
bank	π^S	$\mathbf{c}= \left(-\alpha^3+\alpha^2-\alpha\right) x^{T^2}$; $\mathbf{y}= -\alpha^2 x^{T^2} - c_\pi - C_e$	$\mathbf{d}= -\alpha(\alpha+1) x^{T^2}$; $\mathbf{z}= -c_\pi - C_e$

Note that the payoffs of the public satisfy, for all parameter values

$$\mathbf{w} > \mathbf{x} \ \text{ and } \ \mathbf{z} > \mathbf{y}. \tag{12}$$

Equation (3) then implies that the public's static best response is always to choose the action level equivalent to the central bank's, ie set expectations in line with inflation. Further note that for all considered α the policymaker's payoffs satisfy

$$\mathbf{c} > \mathbf{a} \ \text{ and } \ \mathbf{d} > \mathbf{b}. \tag{13}$$

[14] Let us point out that our game theoretic representation is quite general - it can nest any macroeconomic model, whereby the payoffs $\{\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d},\mathbf{w},\mathbf{x},\mathbf{y},\mathbf{z}\}$ are simply functions of the deep parameters of the selected model.

The relationships in (12)-(13) imply that the standard static game has a unique Nash equilibrium, $\left(\pi^S, \bar{e}^S\right)$, which is Pareto-inefficient - inferior to $\left(\pi^O, \bar{e}^O\right)$. This is because π^S is the policymaker's strictly dominant strategy - due to $x^T \neq 0$. It will become apparent below that allowing for the players' actions to be infrequent, ie considering the *dynamic* stage game, may alter these outcomes.

5.2. **Equilibrium of the Dynamic Stage Game.** As explained in the timing Section 3.1, the full game consists of a dynamic stage game r^b periods long that gets repeated ($\frac{T}{r^b}$ times). The way we solve the game is determined by the specific results we are interested in. It is beyond the scope of this paper to fully describe all the equilibrium outcomes of the game under all circumstances. Instead, in search for the anchoring effect our interest lies in equilibrium *uniqueness* and *efficiency.* In doing so we use subgame perfection - a conventional equilibrium refinement.

Definition 4. *Any subgame perfect Nash equilibrium (SPNE) that has, on its equilibrium path, all players playing in all long-run moves: (i) the socially optimal O levels will be called **Ramsey**; (ii) the inferior S levels will be called **anti-Ramsey**.*

Obviously, in addition to these two *types of SPNE* that are symmetric, there may exist a number of other non-Ramsey SPNE with both the O and S levels.

In this paper, we will focus on deriving conditions under which the *dynamic stage game* has (i) a *unique* SPNE, that is (ii) of the Ramsey type, and hence *Pareto-efficient.* Under these conditions repeating the game will not affect the set of SPNE - for a proof see Libich and Stehlík (2008).[15] The uniqueness condition also means that we can focus on pure strategies without loss of generality.

5.3. **Results.** The following result relates to the public's monitoring motive.

Proposition 2. *Updating inflation expectations **more frequently** reduces the incentives of the central bank to carry out inflation/deflation surprises, if any. If the public updates expectations **sufficiently frequently**, $r^p < \tilde{r}^p$, then the long-run inflation target is not deviated from (on average) even under $x^T \neq 0$.*[16]

Proof. We can, without loss of generality, restrict our attention to the r^b period dynamic stage game for reasons explained in Section 5.2. Solving backwards, ie taking both r^p and r^b as given, let us analyze the players' π^T and $\bar{e}$ actions. We know that the public will find it optimal to play the same action in all its asynchronized moves - they are all made under the same circumstances. The public's rationality and (3) imply that the optimal action selected in all these moves will be the static best response to the policymaker's initial (now observable) move, ie $\bar{e}^*_{t\in(0,r^b)} = \pi^T_0$.

Moving backwards, we now need to determine the policymaker's optimal play in $t = 0$. For the optimal inflation target to be *time consistent* (and for a Ramsey SPNE to exist),

[15]For the fact that *the Folk theorem* may not apply in some asynchronous games, which is the case here, see eg Takahashi and Wen (2003).

[16]Note that the proposition does *not* state that inflation never deviates from the inflation target. It is only in the long-run (average) sense, see the discussion of Section 2.4.

it is required that π_0^O be the best response to $\bar{e}_0^O$. This is guaranteed by the following condition

$$\mathbf{a}r^b \geq \mathbf{c}r^p + \mathbf{d}(r^b - r^p). \tag{14}$$

Both the left-hand side (LHS) and the right-hand side (RHS) are derived assuming that the public plays $\bar{e}_0^O$. The LHS expresses the fact that if the policymaker chooses π_0^O, he will achieve the payoff **a** in all r^b periods. In contrast, the RHS describes the scenario of the policymaker playing π_0^S and initially getting output closer to his preferred level x^T through an inflation/deflation surprise, and the **c** payoff. This however only lasts for r^p periods, after which the public switches to $\bar{e}^S$, and punishes the policymaker (with a **d** payoff) for the rest of the stage game, $(r^b - r^p)$ periods. Substituting in the respective values $\{\mathbf{a}, \mathbf{c}, \mathbf{d}\}$ from the payoff matrix in Section 5.1 and rearranging yields

$$r^p \leq \frac{r^b}{2 - \alpha}. \tag{15}$$

While (15) ensures the *existence* of a Ramsey SPNE, we are interested in deriving conditions under which there is a *unique* Ramsey SPNE. This is to make sure S level expectations never occur on the equilibrium path. For this to be the case π_0^O must be a strictly dominant strategy, thus in addition to (15) (with strict inequality) it is required that π_0^O is the unique best response also to $\bar{e}_0^S$. We can think of this as the willingness of the central bank to carry out a disinflation even if it knows that the disinflation will lack credibility, and will therefore be costly.

The following condition, derived in the same way as (14), but assuming that the public plays $\bar{e}_0^S$, ensures this

$$\mathbf{b}r^p + \mathbf{a}(r^b - r^p) > \mathbf{d}r^b. \tag{16}$$

It states that the policymaker prefers to select π^O even if he knows that $\bar{e}^S$ will be played, and hence he will suffer some temporary output cost **b**. He does so knowing that he will be 'rewarded' by the public's switching to $\bar{e}^O$ after r^p periods, and thus gaining the **a** payoff for $(r^b - r^p)$ periods. Rearranging (16) and using the payoff matrix yields

$$r^p < \tilde{r}^p = \frac{r^b}{2 + \alpha}. \tag{17}$$

Comparing the two conditions implies that (17) is stronger than (15) for all considered α. It is therefore the necessary and sufficient condition for *existence* and *uniqueness* of Ramsey SPNE.[17] If this condition holds then we obtain, for all considered α and r^b, the long-run inflation (target) at the socially optimal O level in all periods. □

Let us summarize the outcomes of the game as a function of r^p and r^b. The proof implies that:

(1) under $\frac{r^p}{r^b} < \frac{1}{2+\alpha}$ we have a unique SPNE, and it is of the Ramsey type;
(2) under $\frac{r^p}{r^b} > \frac{1}{2-\alpha}$ there is a unique SPNE, and it is of the anti-Ramsey type;

[17] The discussion of footnote 12 applies to equation (10) as well (except that $\tilde{r}^p$ has to be rounded *down* since (17) is an inequality).

(3) under $\frac{r^p}{r^b} \in \left[\frac{1}{2+\alpha}, \frac{1}{2-\alpha}\right]$ there exist *multiple* SPNE, one of which is Ramsey, one of which is anti-Ramsey, and the rest of which are other non-Ramsey types with both O and S levels on the equilibrium path, for one or both players.

With respect to the latter region of multiple equilibria, standard game theoretic concepts do not offer a good way to determine which of the SPNE ends up being played. In order to simplify the presentation of the results, we will throughout assume that only the symmetric equilibria (Ramsey or anti-Ramsey) would be selected. Specifically, let us assume that: (i) under the threshold value, $r^p = \tilde{r}^p = \frac{r^b}{2+\alpha}$, the Ramsey SPNE would obtain (for which a weak-dominance argument can be used), and (ii) under all the remaining values of the interval in (3) the anti-Ramsey SPNE would obtain.[18]

The question that remains to be answered is whether the public will indeed choose some $r^p \leq \tilde{r}^p$ to guarantee itself optimal inflation on average (minimizing the cost C_π from (8)), or whether it is too costly for the public to do so (in terms of paying a higher updating cost C_e in (7)). The following proposition addresses this decision.

Proposition 3. *If the public's inflation cost is sufficiently large, $c_\pi \geq \tilde{c}_\pi$, then the public always chooses to update expectations sufficiently frequently, $r^{p*} = \min\{\hat{r}^p, \tilde{r}^p\}$, to uniquely ensure a Ramsey SPNE. Then an explicit inflation target has an* ***anchoring effect.***

Proof. In its r^{p*} decision, the public solves backwards and takes into account the equilibrium outcomes in the later periods of the dynamic stage game derived in Section 5.3. The public therefore compares its utility from choosing some $r^p > \tilde{r}^p$ (and hence getting the $\mathbf{z}$ payoff with $C_\pi = c_\pi$ from the anti-Ramsey SPNE), and from choosing some $r^{p*} \leq \tilde{r}^p$ (and getting the $\mathbf{w}$ payoff with $C_\pi = 0$ from the Ramsey SPNE).

Let us first consider the case in which $\hat{r}^p$ from (10) falls into this interval, ie $\hat{r}^p \leq \tilde{r}^p$. Setting up the inequality, this is true iff

$$r^b \geq \tilde{r}^b = \frac{(2+\alpha)\left(\alpha + \lambda^2\right)}{\alpha}\sqrt{\frac{2Tc_e}{\sigma_u^2}}. \tag{18}$$

In such case we know that $r^{p*} = \hat{r}^p$, since this is the maximum utility level based on the accuracy motive, and the constraint $r^{p*} \leq \tilde{r}^p$ coming from the monitoring motive and ensuring $C_\pi = 0$ is automatically satisfied.

In the opposite case, $\hat{r}^p > \tilde{r}^p$, the monitoring motive and minimization of C_π however requires a more frequent (and hence more costly) updating than the optimal frequency implied the accuracy motive alone. Therefore, for the public to update sufficiently frequently in such case, $r^{p*} \leq \tilde{r}^p$, an extra condition must be satisfied (as C_e is decreasing in r^p the public will only rationally consider the highest r^p value in this interval, ie $\tilde{r}^p$). Specifically, for $U_t^p(r^p = \tilde{r}^p) \geq U_t^p(r^p = \hat{r}^p)$ in the presence of shocks it is required that

$$-\frac{\tilde{r}^p}{T}\left(\frac{\alpha}{\alpha+\lambda^2}\sigma_u\right)^2 - \frac{c_e}{\tilde{r}^p} \geq -\frac{\hat{r}^p}{T}\left(\frac{\alpha}{\alpha+\lambda^2}\sigma_u\right)^2 - \frac{c_e}{\hat{r}^p} - c_\pi.$$

[18] This assumption is only made to be able to compare the players' utility under various r^p and r^b values, and hence may affect our results below quantitatively, but not qualitatively.

Intuitively, the c_π cost has to be sufficiently high *relative* to the c_e cost to justify more frequent updating. Substituting in the $\tilde{r}^p$ and $\hat{r}^p$ values yields

(19)

$$c_\pi \geq \tilde{c}_\pi = \frac{\left(\alpha r^b - (2+\alpha)\left(\alpha+\lambda^2\right)\sqrt{\frac{2Tc_e}{\sigma_u^2}}\right)\left(\alpha r^b \sigma_u^2\left(\alpha+\lambda^2\right)\sqrt{\frac{2Tc_e}{\sigma_u^2}} - \left(\alpha+\lambda^2\right)^2(2+\alpha)\,Tc_e\right)}{(2+\alpha)\left(\alpha+\lambda^2\right)^3 Tr^b\sqrt{\frac{2Tc_e}{\sigma_u^2}}}.$$

If this condition is satisfied we can summarize the equilibrium degree of expectation anchorness as follows

$$r^{p*} = \begin{cases} \hat{r}^p = \sqrt{\frac{2Tc_e}{\sigma_u^2}\left(\frac{\alpha+\lambda^2}{\alpha}\right)^2} & \text{if} \quad r^b \geq \tilde{r}^b, \\ \tilde{r}^p = \frac{r^b}{2+\alpha} & \text{if} \quad r^b \leq \tilde{r}^b. \end{cases} \tag{20}$$

Noting that r^{p*} is a non-decreasing function of r^b completes the proof. □

It is interesting to note that the target's explicitness only increases expectation anchorness up to a point, $\tilde{r}^b$, after which further enhancements in explicitness do not strengthen the degree of anchoring. The same is true for the target's credibility.

Remark 1. *If the public's monitoring motive exists but is insufficiently strong, ie $x^T \neq 0$ but $c_\pi < \tilde{c}_\pi$, then expectations may be inattentive, but not anchored at the optimal inflation level.*

The previous proof implies that if neither (18) nor (19) are satisfied, ie if the inflation target is insufficiently explicit and the inflation cost is insufficient high, $r^b < \tilde{r}^b$ and $c_\pi < \tilde{c}_\pi$, then $r^{p*} = \hat{r}^p$, but the anti-Ramsey SPNE with S level expectations obtains.

6. Summary and Conclusions

Academics and central bankers alike have pointed out that well-anchored expectations of private agents are crucial for monetary policy effectiveness. We offer a formal way of modeling one avenue that may lead to anchored expectations - the effect of an explicit (ie legislated) inflation target. This is motivated by a recent stream of empirical literature reporting that inflation expectations are better anchored (less sensitive to new data) in countries with an explicit inflation target.

The paper proposes a game theoretic framework with endogenous timing, that allows for rational inattention. In particular, the public can optimally choose to update expectations infrequently to reduce its decision-making cost. We derive the circumstances under which this happens, and expectations are anchored at the inflation target level (agents look-through shocks). We further identify several variables that determine how much expectations are anchored, one of which is the explicitness of the target.

This analysis is complemented by Libich (2008a), who formally shows how, and under what circumstances, anchored expectations translate into an improvement in macroeconomic stabilization. Both papers imply two important caveats to the above conclusions.

First, the beneficial effects of explicit inflation targets occur under many - but not all - circumstances. For example, the inflation target must be specified as a *long-run* objective, ie one that only needs to be achieved on average, not every point in time (which would unduly reduce the central bank's flexibility to stabilize the real economy).

Second, real world explicit inflation targets are, in delivering the anchoring effect, likely to work in conjunction with the conventional channels of improved monetary policy such as reputation, central bank independence, and effective communication. Put differently, explicit targets are *not* a *sufficient* condition for anchored expectations.

Let us also stress that the paper has not given an overall evaluation of explicit inflation targets - it focused specifically on the empirically relevant anchoring effect. More research is required to provide a complete welfare assessment of this institutional arrangement, especially in light of the current financial turbulences.

7. References

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