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DEBT, POLICY UNCERTAINTY AND EXPECTATIONS STABILIZATION

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# Debt, Policy Uncertainty and Expectations Stabilization*

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**Abstract**

This paper develops a model of policy regime uncertainty and its consequences for stabilizing expectations. Because of learning dynamics, uncertainty about monetary and fiscal policy is shown to restrict, relative to a rational expectations analysis, the set of policies consistent with macroeconomic stability. Anchoring expectations by communicating about monetary and fiscal policy enlarges the set of policies consistent with stability. However, absent anchored fiscal expectations, the advantages from anchoring monetary expectations are smaller the larger is the average level of indebtedness. Finally, even when expectations are stabilized in the long run, the higher are average debt levels the more persistent will be the effects of disturbances out of rational expectations equilibrium.

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## 1 Introduction

Following Taylor (1993) a large literature has developed arguing that a simple linear relationship between nominal interest rates, inflation and some measure of real activity, can account for the behavior of the Federal Reserve and central banks in a number of developed countries. Subsequent theoretical and applied work on monetary policy has introduced such rules as behavioral equations for policy makers in general equilibrium models. Simple rules have the desirable property of stabilizing expectations when policy is sufficiently active in response to developments in the macroeconomy. This property is often referred to as the Taylor principle. It assumes that fiscal policy is 'passive' and the resulting equilibrium Ricardian, implying that inflation and real activity are independent of fiscal variables, and that agents have complete knowledge of the economic environment; in particular, the monetary and fiscal regime.[1]

The appropriateness of this view rests on policy being of a particular kind and on the absence of regime change. Yet the recent U.S. financial crisis and recession demonstrates episodes of unconventional policy occasionally punctuate conventional policy. And in such times there exists profound uncertainty about the scale, scope and duration of the stance of stabilization policy — witness the extensive discussions of 'exit strategies' for monetary and fiscal policy. More generally, there are clearly historical episodes indicating on-going shifts in the configuration of monetary and fiscal policy in the U.S. post-war era. They suggest that policy might better be described by evolving combinations of active and passive policy rules, for which monetary policy may or may not satisfy the Taylor principle, and equilibrium may or may not be Ricardian.[2] Given these observations, it seems reasonable both to consider configurations of policy that differ to the standard account and to assume that in the initial phase of a given policy regime market participants lack full information about policy and its effects on the macroeconomy.

This paper evaluates the consequences of uncertainty about the prevailing policy regime for the efficacy of stabilization policy. We consider a model of near-rational expectations where market participants and policy makers have incomplete knowledge about the structure of the economy. Private agents are optimizing, have a completely specified belief system, but do not know the equilibrium mapping between observed state variables and market clearing prices.

[1] The term 'passive' follows the language of Leeper (1991). The descriptor 'Ricardian' follows Woodford (1996): for all sequences of prices, the fiscal accounts of the government are intertemporally solvent. Conversely, passive monetary and active fiscal policy lead to non-Ricardian equilibribria described later.

[2] The bond price support regime in the U.S. in the late 1940s discussed by Woodford (2001), and recent empirical evidence of shifting policy rules by Davig and Leeper (2006), are two examples.

By extrapolating from historical patterns in observed data they approximate this mapping to forecast exogenous variables relevant to their decision problems, such as prices and policy variables. Unless the monetary and fiscal authorities credibly announce the policy regime in place, agents are assumed to lack knowledge of the policy rules. Because agents must learn from historical data, beliefs need not be consistent with the objective probabilities implied by the economic model. Expectations need not be consistent with implemented monetary and fiscal policy — in contrast to a rational expectations analysis of the model.[3] This permits a meaningful notion of 'anchored expectations'.

A policy regime is characterized by a monetary policy rule that specifies nominal interest rates as a function of expected inflation and a tax rule that describes how the structural surplus is adjusted in response to outstanding public debt. The central bank has imperfect knowledge about the current state: it has to forecast the current inflation rate to implement policy. It, like households and firms, must learn from historical data. The central bank therefore reacts with a delay to changing economic conditions: argued to be characteristic of actual policy-making — see McCallum (1999). Stabilization policy is harder because it is more difficult to predict business cycle fluctuations.

Policy regime changes are not explicitly modelled. Instead, a stationary model environment is studied: policy rules are constant for all time. In contrast to rational expectations, we assume that initial expectations are not consistent with the policy regime in place. The environment constitutes a best-case scenario. If agents are unable to learn the policy reaction functions describing monetary and fiscal policy in a stationary environment, then learning such objects when there are changes in policy regime can only occur under more stringent conditions. As such, the analysis likely understates the severity of inference problems that agents face.

The analysis commences by identifying a class of policies that ensures determinacy of rational expectations equilibrium in our model. The requirements for determinacy are called the Leeper conditions — after Leeper (1991) — which define the set of policies under consideration. Within this class, policy rules are considered desirable if they have the additional property of stabilizing expectations under imperfect information, in the sense that expectations under learning dynamics converge to the rational expectations equilibrium associated

[3] A further implication of imperfect knowledge is agents respond with a delay to changes in policy. Given a change in policy regime, agents have few initial data points to infer the nature of the new regime and its implications for equilibrium outcomes. This accords with Friedman (1968), which emphasizes the existence of lags in monetary policy.

with a given policy regime. This is adjudged by the property of expectational stability developed by Marcet and Sargent (1989) and Evans and Honkapohja (2001). Good policy should be robust to both central bank and private agents' imperfect knowledge.

This robustness property is assessed in three scenarios which successively resolve uncertainty about the policy regime: i) agents have no knowledge of the monetary and fiscal policy regime; ii) agents understand the monetary policy strategy of the central bank. This implies all details of the central bank's monetary policy rule are correctly understood so that agents make policy-consistent forecasts. Monetary expectations are said to be anchored; and iii) agents further understand that fiscal policy is conducted to ensure the intertemporal solvency of the government budget. In which case, fiscal expectations are consistent with long-run policy and said to be anchored. Within each scenario two regimes are considered: one with active monetary and passive fiscal policy and one with passive monetary and active fiscal policy.

Four results are of note. First, under regime uncertainty, stabilization policy with simple rules is demonstrated to be more difficult than in a rational expectations analysis of the model: the menu of policies consistent with expectations stabilization is narrowed considerably relative to the Leeper conditions. Instability arises due to a failure of traditional aggregate demand management. It is shown that when both monetary and fiscal policy are not well understood, uncertainty about monetary policy is the main source of instability. As real interest rates are not accurately projected, anticipated future changes in monetary policy are less effective in managing current aggregate demand.

Second, resolving uncertainty about monetary policy and thereby anchoring monetary expectations improves the stabilization properties of simple rules, in the sense that a larger set of policies are consistent with stabilizing expectations. Independently of the policy regime in place, the improvement in macroeconomic stability stems from effective demand management, as the evolution of real interest rates becomes more predictable. However, the extent of advantage afforded by anchored monetary expectations depends on the economy's debt-to-output ratio. The more heavily indebted an economy, the smaller the menu of policies consistent with stability. Only in a zero debt economy are the full set of policies given by the Leeper conditions consistent with expectational stability. That average indebtedness mitigates the efficacy of stabilization policy stems from departures from Ricardian equivalence under learning — compare Barro (1974). These wealth effects on aggregate demand have magnitude proportional to the average debt-to-output ratio of the economy and can be destabilizing: tighter monetary policy to restrain inflation expectations can lead to positive valuation effects on holdings of the public debt, which stimulates demand. These findings resonate with practical policy-making, which frequently cites concern about the size of the public debt for stabilization policy.

Third, in addition to anchoring monetary expectations, anchoring fiscal expectations by communicating details of the long-run conduct of fiscal policy restores the full menu of policies described by the Leeper conditions. An economy with anchored fiscal expectations is shown to be isomorphic to a zero debt economy. This suggests that communication about fiscal policy may be as important as communication about monetary policy in high debt economies. The constraints imposed on monetary policy by indebtedness only matter to the extent that agents are unsure about the long-term consequences of fiscal policy. Fiscal uncertainty compromises monetary policy in economies with non-trivial public debt.

Fourth, because of departures from Ricardian equivalence, if agents are uncertain about the intertemporal solvency of the government accounts, the stock of debt can be a source of macroeconomic instability even when expectational stability is guaranteed. We analyze the dynamic response of the economy to a small shock to inflation expectations (equivalent to a change in the perceived inflation target) in a zero and high debt economy. Relative to zero debt economies, a shock to inflation expectations in high debt economies leads to persistent fluctuations in inflation and output before convergence to rational expectations equilibrium. Indebtedness fundamentally changes an economy's response to shocks and undermines the efficacy of simple rules for stabilization policy.

**Related Literature:** The analysis owes much to Leeper (1991) and the subsequent literature on the fiscal theory of the price level — see, in particular, Sims (1994), Woodford (1996) and Cochrane (1998). It also contributes to a growing literature on policy design under learning dynamics — see, inter alia, Howitt (1992), Bullard and Mitra (2002, 2006), Bullard and Eusepi (2008), Eusepi (2007), Evans and Honkapohja (2003, 2005, 2006), Preston (2005, 2006, 2008) — but is most directly related to Evans and Honkapohja (2007) and Eusepi and Preston (2008a). Evans and Honkapohja (2007) considers the interaction of monetary and fiscal policy in the context of Leeper's model under learning dynamics rather than rational expectations. The analysis here advances their findings by considering a model in which agents are optimizing conditional on their beliefs. Eusepi and Preston (2008a) analyzes the role of communication in stabilizing expectations. The presence or absence of knowledge about the policy regime is adapted from the notions of *full communication* and *no communication* developed in that paper. The results here differ in non-trivial ways as a broader class of fiscal policy is considered. Rather than assuming a zero-debt passive fiscal policy, which is understood by households, the analysis here considers a class of passive and active fiscal policies determined by the dual specification of a tax rule, which is unknown to agents, and choice of debt-to-output ratio. This engenders significantly richer model predictions regarding policy interactions and expectations stabilization, because agents must forecast future taxes to make current spending decisions and because holdings of the public debt are treated as net wealth. Wherefore this more general framework permits evaluating the advantages of communication about fiscal policy, an under studied topic.

# 2 A Simple Model

The following section details a model similar in spirit to Clarida, Gali, and Gertler (1999) and Woodford (2003). The major difference is the incorporation of near-rational beliefs delivering an anticipated utility model as described by Kreps (1998) and Sargent (1999). The analysis follows Marcet and Sargent (1989a) and Preston (2005), solving for optimal decisions conditional on current beliefs.

## 2.1 Microfoundations

**Households**: The economy is populated by a continuum of households which seeks to maximize future expected discounted utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[ \ln \left( C_T^i + g \right) - h_T^i \right] \tag{1}$$

where utility depends on a consumption index, $C_T^i$, the amount of labor supplied for the production of each good $j$, $h_T^i$, and the quantity of government expenditures $g > 0$.[4] The consumption index, $C_t^i$, is the Dixit-Stiglitz constant-elasticity-of-substitution aggregator of the economy's available goods and has associated price index written, respectively, as

$$C_t^i \equiv \left[ \int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad P_t \equiv \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \tag{2}$$

where $\theta > 1$ is the elasticity of substitution between any two goods and $c_t^i(j)$ and $p_t(j)$ denote household $i$'s consumption and the price of good $j$. The discount factor is assumed to satisfy $0 < \beta < 1$.

[4] The adopted functional form facilitates analytical results.

$\hat{E}_t^i$ denotes the beliefs at time $t$ held by each household $i$, which satisfy standard probability laws. Section 3 describes the precise form of these beliefs and the information set available to agents when forming expectations. Households and firms observe only their own objectives, constraints and realizations of aggregate variables that are exogenous to their decision problems and beyond their control. They have no knowledge of the beliefs, constraints and objectives of other agents in the economy: in consequence agents are heterogeneous in their information sets in the sense that even though their decision problems are identical, they do not know this to be true.

Asset markets are assumed to be incomplete. The only asset in non-zero net supply is government debt to be discussed below. The household's flow budget constraint is

$$B_{t+1}^i \leq R_t \left(B_t^i + W_t h_t^i + P_t \Pi_t - T_t - P_t C_t^i\right) \tag{3}$$

where $B_t^i$ is household $i$'s holdings of the public debt, $R_t$ the gross nominal interest rate, $W_t$ the nominal wage and $T_t$ lump-sum taxes. $\Pi_t$ denotes profits from holding shares in an equal part of each firm and $P_t$ is the aggregate price level defined below. Period nominal income is determined as

$$P_t Y_t^i = W_t h_t^i + \int_0^1 \Pi_t(j)\, dj$$

for each household $i$. Finally, there is a No-Ponzi constraint

$$\lim_{T\to\infty} \hat{E}_t^i R_{t,T} B_T^i \geq 0$$

where $R_{t,T} = \prod_{s=t}^{T-1} R_s^{-1}$ for $T \geq 1$ and $R_{t,t} = 1$.[5]

A log-linear approximation to the first-order conditions of the household problem provides the Euler equation

$$\hat{C}_t^i = \hat{E}_t^i \hat{C}_{t+1}^i - \left(\hat{\imath}_t - \hat{E}_t^i \hat{\pi}_{t+1}\right)$$

and intertemporal budget constraint

$$s_C \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \frac{\bar{b}}{\bar{Y}} \hat{b}_t^i + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^i - \frac{\bar{\tau}}{\bar{Y}} \hat{\tau}_T + \frac{\bar{b}}{\bar{Y}} \left(\beta \hat{\imath}_T - \hat{\pi}_T\right)\right] \tag{4}$$

[5] In general, No Ponzi does not ensure satisfaction of the intertemporal budget constraint under incomplete markets. Given the assumption of identical preferences and beliefs and aggregate shocks, a symmetric equilibrium will have the property that all households have non-negative wealth. A natural debt limit of the kind introduced by Aiyagari (1994) would never bind.

where

$$\hat{Y}_t \equiv \ln(Y_t/\bar{Y});\ \hat{C}_t^i \equiv \ln(C_t^i/\bar{C});\ \hat{\imath}_t \equiv \ln(R_t/\bar{R});\ \hat{\pi}_t = \ln\left(P_t/P_{t-1}\right);$$
$$\hat{\tau}_t \equiv \ln(\tau_t/\bar{\tau});\ \tau_t = T_t/P_t;\ \hat{b}_t^i = \ln\left(\tilde{B}_t^i/\bar{B}\right) \text{ and } \tilde{B}_t^i = B_t^i/P_{t-1}$$

and $\bar{z}$ denotes the steady-state value of any variable $z_t$.

Solving the Euler equation recursively backwards, taking expectations at time $t$ and substituting into the intertemporal budget constraint gives

$$\begin{aligned}\hat{C}_t^i &= s_C^{-1}\delta\left(\hat{b}_t^i - \hat{\pi}_t\right) + \\ &\quad s_C^{-1}\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)\left(\hat{Y}_T - \delta\hat{s}_T\right) - (1-\delta)\,\beta\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right]\end{aligned}$$

where

$$\hat{s}_t = \bar{\tau}\hat{\tau}_t/\bar{s};\ \ s_C = \bar{C}/\bar{Y} \ \text{ and } \ \delta = \bar{s}/\bar{Y} = (1-\beta)\,\bar{b}/\bar{Y}$$

are the structural surplus (defined below), the steady-state consumption-to-income ratio and the steady-state structural surplus-to-income ratio.[6] Optimal consumption decisions depend on current wealth and on the expected future path of after-tax income and the real interest rate.[7] The optimal allocation rule is analogous to permanent income theory, with differences emerging from allowing variations in the real rate of interest, which can occur due to variations in either the nominal interest rate or inflation. As households become more patient, current consumption demand is more sensitive to expectations about future macroeconomic conditions.

The steady-state structural surplus-to-income ratio, $\delta$, affects consumption decisions in three ways: i) it determines after-tax income; ii) it reduces the elasticity of consumption spending with respect to real interest rates; and iii) it indexes wealth effects on consumption spending that result from variations in the real value of government debt holdings. To interpret these effects further it is useful to consider aggregate consumption demand. Aggregating over the continuum and rearranging provides

$$\begin{aligned}\hat{C}_t &= s_C^{-1}\delta\left(\left(\hat{b}_t - \hat{\pi}_t\right) - \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)\,\hat{s}_T - \beta\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right]\right) \\ &\quad + s_C^{-1}\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)\,\hat{Y}_T - \beta\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right] \qquad (5)\end{aligned}$$

[6] Calculations are in an on-line appendix.

[7] Using the fact that total household income is the sum of dividend and wage income, combined with the first-order conditions for labor supply and consumption, delivers a decision rule for consumption that depends only on forecasts of prices: that is, goods prices, nominal interest rates, wages and dividends. However, we make the simplifying assumption that households forecast total income, the sum of dividend payments and wages received.

where

$$\int_0^1 \hat{C}_t^i di = \hat{C}_t; \quad \int_0^1 \hat{b}_t^i di = \hat{b}_t; \quad \text{and} \quad \int_0^1 \hat{E}_t^i di = \hat{E}_t$$

give aggregate consumption demand; total outstanding public debt; and average expectations. The second line gives the usual terms that arise from permanent income theory. The term premultiplied by $s_C^{-1}\delta$ in the first line is the intertemporal budget constraint of the government. In a rational expectations analysis of the model, this is an equilibrium restriction known to be equal to zero. However, agents might face uncertainty about the current fiscal regime.[8] And under arbitrary subjective expectations, households will in general incorrectly forecast future tax obligations and real interest rates, leading to holdings of the public debt being perceived as net wealth: Ricardian equivalence need not hold out of rational expectations equilibrium. The failure of Ricardian equivalence leads to wealth effects on consumption demand, and the magnitude of these effects is indexed by the structural surplus-to-output ratio, or equivalently the debt-to-output ratio as these steady-state quantities are proportional.[9] On average, the more indebted an economy the larger are the effects on demand. This is shown to be important in the design of stabilization policy.

Finally, note that if either the debt-to-output ratio is zero or the intertemporal budget constraint is for some reason known to hold by households, then consumption demand is determined by the second term only, delivering the model analyzed by Preston (2005, 2006).[10] Those papers consider the case of a zero-debt fiscal policy, understood to hold in all future periods so that households need not forecast taxes. This paper extends that analysis to a considerably broader class of fiscal policies that agents must learn about — with non-trivial consequence.

**Firms.** There is a continuum of monopolistically competitive firms. Each differentiated consumption good is produced according to the linear production function $y_t(j) = A_t h_t(j)$ where $A_t > 0$ denotes an aggregate technology shock. Each firm faces a demand curve $Y_t(j) = (P_t(j)/P_t)^{-\theta} Y_t$, where $Y_t$ denotes aggregate output, and solves a Calvo-style price-setting

[8] The tax rule is such that each household faces the same tax profile. However, agents are not aware of this: in forecasting future tax obligations they consider the possibility that their individual tax profile might have changed.

[9] Leith and von Thadden (2006) in Blanchard-Yaari model with rational expectations show that holdings of the public debt are treated as net wealth which has implications for determinacy of rational expectations equilibrium. However, the model structures are quite different. In their case, the probability of death gets built into the overall discount factor which in turn permits deviations from Ricardian equivalence. This is distinct to the structure inherent in (5).

[10] In general, assuming knowledge of the intertemporal budget constraint is questionable as it is just one of the many equilibrium restrictions that households are attempting to learn.

problem where prices can be optimally chosen in any period with probability $0 < 1 - \alpha < 1$. A price $p$ is chosen to maximize the expected discounted value of profits

$$\hat{E}_t^j \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_T^j (p) \text{ and } \Pi_T^j (p) = p^{1-\theta} P_T^{\theta} Y_T - p^{-\theta} P_T^{\theta} Y_T W_T / A_T$$

denotes period $T$ profits. Given the incomplete markets assumption it is assumed that firms value future profits according to the marginal rate of substitution evaluated at aggregate income $Q_{t,T} = \beta^{T-t} P_t Y_T / (P_T Y_t)$ for $T \geq t$.[11]

Denote the optimal price $p_t^*$. Since all firms changing prices in period $t$ face identical decision problems, the aggregate price index evolves according to

$$P_t = \left[ \alpha P_{t-1}^{1-\theta} + (1-\alpha) p_t^{*1-\theta} \right]^{\frac{1}{1-\theta}}.$$

Log-linearizing the first-order condition for the optimal price gives

$$\hat{p}_t = \hat{E}_t^i \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[ (1-\alpha\beta) \hat{\chi}_T + \alpha\beta \pi_{T+1} \right]$$

where $\hat{p}_t = \log(p_t^*/P_t)$ and $\hat{\chi}_t \equiv \ln(\chi_t/\bar{\chi})$ is average marginal costs defined below. Each firm's current price depends on the expected future path of real marginal costs and inflation. The higher the degree of nominal rigidity, the greater the weight on future inflation in determining current prices. The average real marginal cost function is $\chi_t = W_t/(P_t A_t) = Y_t/A_t$, where the second equality comes from the household's labor supply decision. Log-linearizing provides $\hat{\chi}_t = \hat{Y}_t - a_t$, where $a_t = \ln(A_t)$ so that current prices depend on expected future demand, inflation and technology.

## 2.2 Monetary and Fiscal Authorities

**Monetary Policy:** The central bank is assumed to implement monetary policy according to a one-parameter family of interest-rate rules $R_t = \bar{R} \left( E_{t-1}^{cb} \pi_t \right)^{\phi_\pi}$ where $E_{t-1}^{cb} \pi_t$ is a measure of current inflation and $\phi_\pi \geq 0$. The central bank does not observe inflation in real time and, like private agents, has an incomplete model of the economy. For simplicity, it is assumed the central bank has the same forecasting model for inflation as private agents. This is easily generalized. The nominal interest rate rule satisfies the approximation

$$\hat{\imath}_t = \phi_\pi E_{t-1}^{cb} \hat{\pi}_t. \tag{6}$$

[11] The precise details of this assumption are not important to the ensuing analysis so long as in the log-linear approximation future profits are discounted at the rate $\beta^{T-t}$.

This class of rule has had considerable popularity in the recent literature on monetary policy. It ensures determinacy of rational expectations equilibrium if the Taylor principle is satisfied. More importantly, the central bank is here appropriately modelled as an agent that must learn. Central banks face uncertainty about the current state, and particularly inflation. For example, in the U.S., in any given quarter only an estimate of the current CPI inflation rate is available from the BLS. Furthermore, even if uncertainty about a given inflation measure is small, there remains considerable uncertainty about to which measure of inflationary pressure ought the central bank respond. Aside from measurement issues, the informational assumption is congruous with identification strategies adopted in vector autoregression studies on the effects of monetary policy shocks — see, for example, Christiano, Eichenbaum, and Evans (1999). It also resonates with evidence adduced by Rotemberg and Woodford (1997) on the response of spending and pricing decisions to monetary policy shocks. While the present study assumes households and firms make decisions based on time $t$ information, rather than time $t-1$ information, Eusepi and Preston (2008a) makes clear that such timing would tend to exacerbate instability from learning since agents possess less information about the determination of prices.

Similar results would obtain in a model in which monetary policy is conditioned on expectations of next-period inflation given time $t$ information. Preference is given to (6) because of the above mentioned measurement issues and because it implies identical determinacy conditions to a policy in which the central bank perfectly observes current inflation. Regardless, what is to be emphasized is the central bank is realistically described as learning about the current state.

**Fiscal Policy:** The fiscal authority finances government purchases of $g$ per period by issuing public debt and levying lump-sum taxes. Denoting $B_t$ as the outstanding government debt at the beginning of any period $t$, and assuming for simplicity that the public debt is comprised entirely of one-period riskless nominal Treasury bills, government liabilities evolve according to

$$B_{t+1} = (1 + i_t) [B_t + gP_t - T_t] .$$

It is convenient to rewrite this constraint as

$$b_{t+1} = (1 + i_t) \left( b_t \pi_t^{-1} - s_t \right)$$

where $s_t = T_t/P_t - g$ denotes the primary surplus and $b_t = B_t/P_{t-1}$ a measure of the real value of the public debt. Observe that $b_t$ is a predetermined variable since $W_t$ is determined a period in advance.[12] The government's flow budget constraint satisfies the log-linear approximation

$$\hat{b}_{t+1} = \beta^{-1}\left(\hat{b}_t - \hat{\pi}_t - (1-\beta)\,\hat{s}_t\right) + \hat{\imath}_t. \tag{7}$$

The model is closed with an assumption on the path of primary surpluses $\{s_t\}$.[13] Analogous to the monetary authority, it is assumed that the fiscal authority adjusts the primary surplus according to the one-parameter family of rules

$$s_t = \bar{s}\left(\frac{b_t}{\bar{b}}\right)^{\phi_\tau}$$

where $\bar{s}, \bar{b} > 0$ are constants coinciding with the steady-state level of the primary surplus and the public debt respectively. $\phi_\tau \geq 0$ is a policy parameter. The fiscal authority faces no uncertainty about outstanding liabilities as they are determined a period in advance. The tax rule satisfies the log-linear approximation

$$\hat{s}_t = \phi_\tau \hat{b}_t. \tag{8}$$

### 2.3 Market clearing and aggregate dynamics

General equilibrium requires goods market clearing,

$$\int_0^1 C_t^i di + g = C_t + g = Y_t. \tag{9}$$

This relation satisfies the log-linear approximation

$$s_C \int_0^1 \hat{C}_t^i di = s_C \hat{C}_t = \hat{Y}_t.$$

It is useful to characterize the natural rate of output — the level of output that would prevail absent nominal rigidities under rational expectations. Under these assumptions, optimal price setting implies the log-linear approximation $\hat{Y}_t^n = a_t$. Movements in the natural rate of output are determined by variations in aggregate technology shocks. Using this definition, aggregate dynamics of the economy can be characterized in terms of deviations from the flexible price equilibrium. Finally, asset market clearing requires

$$\int_0^1 B_t^i di = B_t \text{ and } \int_0^1 \hat{b}_t^i di = \hat{b}_t,$$

[12] See Eusepi and Preston (2007) for a more general analysis with multiple-maturity debt.

[13] This is without loss of generality. It would be straightforward to specify separate policies for the revenues and expenditures of the government accounts without altering the substantive implications of the model.

implying the sum of individual holdings of the public debt equals the supply of one-period bonds.

Aggregating household and firm decisions provides

$$\begin{aligned}\hat{x}_t &= \delta\beta^{-1}\left(\hat{b}_t - \hat{\pi}_t\right) - \beta^{-1}\delta\hat{s}_t + \\ &\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)\left(\hat{x}_{T+1} - \delta\hat{s}_{T+1}\right) - (1-\delta)\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) + r_T^n\right] \end{aligned} \quad (10)$$

assuming for analytical convenience, and without loss of generality, $g = 0$, so that $s_C = 1$, and

$$\hat{\pi}_t = \kappa\hat{x}_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left[\kappa\alpha\beta\hat{x}_{T+1} + (1-\alpha)\beta\hat{\pi}_{T+1}\right] \quad (11)$$

where $\int_0^1 \hat{E}_t^i di = \hat{E}_t$ gives average expectations; $x_t = \hat{Y}_t - \hat{Y}_t^n$ denotes the log-deviation of output from its natural rate; $r_t^n = \hat{Y}_{t+1}^n - \hat{Y}_t^n$ the corresponding natural rate of interest — assumed to be an identically independently distributed process; and $\kappa = (1-\alpha)(1-\alpha\beta)\alpha^{-1} > 0$.

The average expectations operator does not satisfy the law of iterated expectations due to the assumption of completely imperfect common knowledge on the part of all households and firms. Because agents do not know the beliefs, objectives and constraints of other households and firms in the economy, they cannot infer aggregate probability laws. This is the property of the irreducibility of long-horizon forecasts noted by Preston (2005).

To summarize, the model comprises the structural relations (6), (7), (8), (10) and (11). The model is closed with the specification of beliefs, described next.

## 3 Learning: Belief Formation and the Policy Regime

**Beliefs.** Optimal decisions of households and firms require forecasting the evolution of future real interest rates, income, taxes and inflation. The central bank has only to forecast the current inflation rate. For inflation and income (or output gap), agents are assumed to use a linear econometric model, relating inflation and income to the evolution of real government debt. That is

$$\hat{x}_t = \omega_{0,t}^x + \omega_{1,t}^x \hat{b}_t + e_t^x \quad (12)$$

$$\hat{\pi}_t = \omega_{0,t}^\pi + \omega_{1,t}^\pi \hat{b}_t + e_t^\pi \quad (13)$$

where $e_t^x$ and $e_t^\pi$ are i.i.d. disturbances. The model contains the same variables that appear in the minimum-state-variable rational expectations solutions to the model that result under the various policy configurations described in the next section.[14] And while the rational expectations solution does not contain a constant, it has a natural interpretation under learning of capturing uncertainty about the steady state.

Concerning the nominal interest rate, the fiscal surplus and debt dynamics, agents' forecasts depend on their knowledge about the monetary and fiscal regimes in place. Consider first the monetary policy regime. As in Eusepi and Preston (2008a), uncertainty about the monetary policy regime is captured by assuming that agents do not know the monetary policy rule (6). In this case agents use the model

$$\hat{\imath}_t = \omega^i_{0,t} + \omega^i_{1,t}\hat{b}_t + e^i_t \tag{14}$$

which is consistent with the minimum-state-variable rational expectations solutions under the various monetary and fiscal regimes described in the next section. If agents know the current monetary policy regime, then, given their beliefs about future inflation, they use the rule (6) to compute policy consistent forecasts of the future path of the nominal interest rate.[15] In this case, monetary policy expectations are said to be anchored.

Throughout most of the paper we assume that market participants face uncertainty about the fiscal regime. Agents need to forecast the future evolution of the fiscal surplus and the future evolution of debt (which is also needed to predict the evolution of output and inflation). Their model is

$$\hat{s}_t = \omega^s_{0,t} + \omega^s_{1,t}\hat{b}_t + e^s_t \tag{15}$$

and

$$\hat{b}_{t+1} = \omega^b_{0,t} + \omega^b_{1,t}\hat{b}_t + e^b_t, \tag{16}$$

which, again, is consistent with the different monetary and fiscal regimes described in the next section. Changes in beliefs resulting from knowledge of the fiscal regime are noted as they arise. When agents possess such knowledge, fiscal expectations are said to be anchored.

**Beliefs updating and forecasting**. Each period, as additional data become available, agents update the coefficients of their parametric model given by (12)-(16) using a recursive least-squares estimator. Letting $\omega' = (\omega_0, \omega_1)$ be the vector of coefficients to estimate, $z_t =$

[14] For example, in a rational expectations equilibrium under a Ricardian regime: $\omega^x_0 = 0$, $\omega^x_1 = 0$ and $e^x_t = \phi_0 r^n_t$.

[15] Eusepi and Preston (2008a) consider the intermediate case where agents know the policy rule but have to estimate the rule's coefficients and show that this does not alter the stability properties of the equilibrium.

$\left(\hat{x}_t, \hat{\pi}_t, \hat{\imath}_t, \hat{s}_t, \hat{b}_{t+1}\right)$ and $q_{t-1} = \left(1, \hat{b}_t\right)$, the algorithm can be written in recursive terms as

$$\hat{\omega}_t = \hat{\omega}_{t-1} + g_t^{-1} R_t^{-1} q_{t-1} \left(z_t - \hat{\omega}_{t-1}' q_{t-1}\right)' \tag{17}$$

$$R_t = R_{t-1} + g_t^{-1} \left(q_{t-1} q_{t-1}' - R_{t-1}\right) \tag{18}$$

where $g_t$ is a decreasing sequence and where $\hat{\omega}_t$ denotes the current-period's coefficient estimate.[16] Agents update their estimates at the end of the period, after making consumption, labor supply and pricing decisions. This avoids simultaneous determination of the parameters defining agents' forecast functions and current prices and quantities.[17]

**True Data Generating Process**. Using (12)-(16) to substitute for expectations in (5) and solving delivers the actual data generating process

$$z_t = \Gamma_1 \left(\hat{\omega}_{t-1}\right) q_{t-1} + \Gamma_2 \left(\hat{\omega}_{t-1}\right) r_t^n \tag{19}$$

$$\hat{\omega}_t = \hat{\omega}_{t-1} + g R_t^{-1} q_{t-1} \left(\left[\left(\Gamma_1 \left(\hat{\omega}_{t-1}\right) - \hat{\omega}_{t-1}'\right) q_{t-1} + \Gamma_2 \left(\hat{\omega}_{t-1}\right) r_t^n\right]\right)' \tag{20}$$

$$R_t = R_{t-1} + g \left(q_{t-1} q_{t-1}' - R_{t-1}\right) \tag{21}$$

where $\Gamma_1(\hat{\omega})$ and $\Gamma_2(\hat{\omega})$ are nonlinear functions of the previous-period's estimates of beliefs. The actual evolution of $z_t$ is determined by a time-varying coefficient equation in the state variables $\hat{b}_t$ and $r_t^n$, where the coefficients evolve according to (20) and (21). The evolution of $z_t$ depends on $\hat{\omega}_{t-1}$, while at the same time $\hat{\omega}_t$ depends on $z_t$. Learning induces self-referential behavior. The dependence of $\hat{\omega}_t$ on $z_t$ is related to the fact that outside the rational expectations equilibrium $\Gamma_1(\hat{\omega}_{t-1}) \neq \hat{\omega}_{t-1}'$ and similarly for $\Gamma_2$. This self-referential behavior emerges because each market participant ignores the effects of their learning process on prices and income, and this is the source of possible divergent behavior in agents' expectations.

**Expectations Stability**. The data generating process implicitly defines the mapping between agents' beliefs, $\omega$, and the actual coefficients describing observed dynamics, $\Gamma_1(\omega)$. A rational expectations equilibrium is a fixed point of this mapping. For such rational expectations equilibria we are interested in asking under what conditions does an economy

[16] For example, $\omega_0 = (\omega_0^x, \omega_0^\pi, \omega_0^i, \omega_0^s, \omega_0^b)$. It is assumed that $\sum_{t=1}^{\infty} g_t = \infty$, $\sum_{t=1}^{\infty} g_t^2 < \infty$ — see Evans and Honkapohja (2001).

[17] To compare the model under learning with the predictions under rational expectations, we assume that agents' expectations are determined simultaneously with consumption, labor supply and pricing decisions, so that agents observe all variables that are determined at time $t$, including $\hat{b}_{t+1}$. For example, the one-period-ahead forecast for $\hat{\pi}_t$ is $\hat{E}_t \hat{\pi}_{t+1} = \hat{\omega}_{0,t-1}^\pi + \hat{\omega}_{1t-1}^\pi \hat{b}_{t+1}$ where $\hat{\omega}_{0,t-1}^\pi$ and $\hat{\omega}_{1t-1}^\pi$ are the previous-period's estimates of belief parameters that define the period-$t$ forecast function. They observe the same variables as a 'rational' agent. The only difference is that they are attempting to learn the 'correct' coefficients that characterize optimal forecasts. Finally, the central bank interest rate decision is predetermined since it is based on $t-1$ information (including the estimates of belief parameters).

with learning dynamics converge to each equilibrium. Using stochastic approximation methods, Marcet and Sargent (1989b) and Evans and Honkapohja (2001) show that conditions for convergence are characterized by the local stability properties of the associated ordinary differential equation

$$\frac{d(\omega_0, \omega_1)}{d\tau} = \Gamma_1(\omega) - \omega, \tag{22}$$

where $\tau$ denotes notional time.[18] The rational expectations equilibrium is said to be expectationally stable, or E-Stable, when agents use recursive least squares if and only if this differential equation is locally stable in the neighborhood of the rational expectations equilibrium.[19]

## 4 Rational Expectations: Leeper Revisited

The following characterizes the set of unique equilibria under the rational expectations assumption. The analysis is analogous to Leeper (1991), though in the context of the model of section 2. All proofs are collected in the on-line appendix unless otherwise noted.

**Proposition 1** *There exist unique bounded rational expectations equilibria of the indicated form if and only if the following conditions are satisfied: either*

1. *Monetary policy is active and fiscal policy is passive such that*

$$1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ and } \phi_\pi > 1$$

*with inflation dynamics determined as*

$$\hat{\pi}_t = \phi_0 r_t^n; \quad or$$

2. *Monetary policy is passive and fiscal policy is active such that*

$$0 \leq \phi_\pi < 1 \text{ and } 0 \leq \phi_\tau < 1 \text{ or } \phi_\tau > \frac{1+\beta}{1-\beta}$$

*with inflation dynamics determined as*

$$\hat{\pi}_t = \phi_1 \hat{b}_t + \phi_2 r_t^n.$$

*The coefficients $\{\phi_0, \phi_1, \phi_2\}$ are reported in the on-line appendix.*

[18] If $\Gamma_1(\omega) = \omega$, it follows from subsequent results in section 4 that $\Gamma_1(\omega) = 0$ and $\Gamma_2(\omega) = \phi_0$ in the case of a Ricardian regime and $\Gamma_1(\omega) = \phi_1$ and $\Gamma_2(\omega) = \phi_2$ in the case of a non-Ricardian regime.

[19] Standard results for ordinary differential equations imply that a fixed point is locally asymptotically stable if all eigenvalues of the Jacobian matrix $D[\Gamma(\omega_0, \omega_1) - (\omega_0, \omega_1)]$ have negative real parts (where $D$ denotes the differentiation operator and the Jacobian is understood to be evaluated at the relevant rational expectations equilibrium).

These requirements, called the Leeper conditions, define the set of policies about which subsequent analysis is focused. When $1 < \phi_\tau < (1+\beta)/(1-\beta)$ the eigenvalue of the difference equation (7) is inside the unit circle, and, for all bounded sequences $\{\pi_t,\ i_t\}$, real debt converges to its steady state value. Because taxes are adjusted to ensure intertemporal solvency of the government accounts for all possible paths of the price level, this configuration of policy is termed locally Ricardian, where locally refers to the use of a log-linear approximation. In contrast, if either $0 \leq \phi_\tau < 1$ or $\phi_\tau > (1+\beta)/(1-\beta)$, then the eigenvalue is outside the unit circle and real debt dynamics are inherently explosive. It is this property that requires a specific path of the price level to ensure solvency of the intertemporal accounts. Hence, locally non-Ricardian.[20]

# 5 Monetary and Fiscal Regime Uncertainty

Having laid out preparatory foundations, the analysis turns to the consequences of regime uncertainty for stabilization policy. One final assumption is required to facilitate analytical results: the economy is assumed to have only a small degree of nominal rigidity. Formally, the conditions for expectational stability are studied in the neighborhood of the limit, $\alpha \to 0$. This is not equivalent to analyzing a flexible price economy. For an arbitrary degree of nominal friction, $0 < \alpha < 1$, analytical results are unavailable except in two special cases.[21] Section 8 provides some more general numerical examples.

Under regime uncertainty, the following results obtain.

**Proposition 2** *Stabilization policy ensures expectational stability if and only if*

1. *Monetary policy is active and fiscal policy is passive such that*

$$1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ and } \phi_\pi > \frac{1}{1-\beta}; \text{ or}$$

2. *Monetary policy is passive and fiscal policy is active such that* $0 \leq \phi_\pi < 1$, *and either*

[20] Two other classes of equilibria are possible. One is the case of passive fiscal policy combined with a passive monetary policy satisfying $0 < \phi_\pi < 1$ for which there is indeterminacy of rational expectations equilibrium for all parameter values. None of these equilibria are stable under learning — see Preston (2005). The second is the case of active fiscal policy with active monetary policy. Under rational expectations it can be shown that there exist a class of unbounded equilibria that have explosive debt and inflation dynamics. Evans and Honkapohja (2007) show in a model where only one-period-ahead expectations matter that such equilibria are learnable if the agents' regression model is appropriately transformed to handle the implied non-stationarity. While not denying its obvious interest, we eschew such an analysis in preference of working with a consistent belief structure.

[21] These cases are discussed on page 17.

(a)

$$0 \leq \phi_\tau < \min\left(\phi_\tau^*, 1\right) \ \textit{where}\ \phi_\tau^* = \frac{2}{[(1-\beta\phi_\pi)^{-1} + (1-\beta)]};\ \textit{or}$$

(b)

$$\phi_\tau > \frac{1+\beta}{1-\beta}.$$

The proof is sketched in appendix A.1. This proposition extends results found in Preston (2006) and Eusepi and Preston (2008a) in two dimensions: by examining economies with non-zero steady-state debt and by examining configurations of policy that deliver non-Ricardian equilibria. The mechanism giving rise to instability is the same in both Ricardian and non-Ricardian equilibria. Because monetary policy expectations are unanchored, agents fail to accurately predict real interest rates. This leads to a failure in traditional aggregate demand management through interest rate policy — see Eusepi and Preston (2008a, 2008b) for a detailed discussion and examples. When both monetary and fiscal policies are not well understood, uncertainty about the monetary policy rule is the dominant source of instability. As a consequence, stability is independent of average indebtedness. Determinacy of rational expectations equilibrium is similarly independent of this object. The sequel demonstrates that under non-rational expectations this is not generally true.

Regime uncertainty constrains the menu of policies consistent with expectations stabilization relative to the class of policies given by the Leeper conditions. If fiscal policy is passive then monetary policy must be highly aggressive to prevent self-fulfilling expectations. For many monetary policies satisfying the Taylor principle there is no choice of fiscal policy that can guarantee stability. The restriction on the choice of monetary policy depends on the households' discount factor, $\beta$, since this parameter regulates the impact of revisions to expectations about future macroeconomic conditions on current spending and pricing decisions. The more patient are households the larger will be the impact on current macroeconomic conditions.

If fiscal policy is active, policy choices are again restricted relative to a rational expectations analysis of the model. Under rational expectations, conditional on monetary policy being passive, any choice of active fiscal policy delivers a unique bounded rational expectations equilibrium. Under regime uncertainty this is no longer true. The precise choice of monetary policy constrains the set of fiscal policies consistent with macroeconomic stability. However, for a given choice of monetary policy there always exists a choice of fiscal policy that prevents expectations-driven instability. Part 2(b) of the proposition shows that a fiscal policy, characterized by either an exogenous surplus or an extremely aggressive fiscal rule, is conducive to macroeconomic stability for all parameter configurations.[22] Thus, perhaps surprisingly, non-Ricardian regimes appear to be more robust to learning dynamics.

The learning behavior of *both* private agents and the central bank engenders the instability result. If the central bank could perfectly observe current inflation, then the stability conditions under learning are the Leeper conditions: the same restrictions implied by local determinacy.[23] This result is special to the model at hand. Permitting multiple maturity debt is sufficient to undermine stability even when inflation is accurately observed — see Eusepi and Preston (2007). This is because bond prices themselves depend on expectations about future policy, which tends to make the equilibria more susceptible to instability.

# 6 Resolving Uncertainty about Monetary Policy

To isolate the role of uncertainty about the fiscal regime, follow Eusepi and Preston (2008a), and consider the benefits of credibly communicating the monetary policy rule to firms and households. The precise details of the monetary policy rule are announced, including the policy coefficients and conditioning variables. Knowledge of this rule serves to simplify firms' and households' forecasting problems. Agents need only forecast inflation: policy-consistent forecasts of future nominal interest rates can then be determined directly from the announced policy rule. It follows that credible announcements have the property that expectations about future macroeconomic conditions are consistent with the policy strategy of the monetary authority. In this sense, monetary policy expectations are anchored.

Under communication of the policy regime the aggregate demand equation becomes

$$\begin{aligned} \hat{x}_t &= \delta\beta^{-1}\left(\hat{b}_t - \hat{\pi}_t\right) - \beta^{-1}\delta\hat{s}_t - (1-\delta)\,\phi_\pi \hat{E}_{t-1}\hat{\pi}_t \\ &\quad + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)\left(\hat{x}_{T+1} - \delta\hat{s}_{T+1}\right) - (1-\delta)\left(\phi_\pi\beta - 1\right)\hat{\pi}_{T+1} + r_T^n\right] \end{aligned} \quad (23)$$

determined by direct substitution of the monetary policy rule into equation (10). The remaining model equations are unchanged with the exception of beliefs. As nominal interest rates need not be forecast, an agent's vector autoregression model is estimated on the restricted state vector $z_t = \left(\hat{x}_t, \hat{\pi}_t, \hat{s}_t, \hat{b}_{t+1}\right)$. Knowledge of the monetary policy regime does not

[22] With an interest peg and an exogenous surplus, E-stability holds for an arbitrary degee of nominal rigidities. A proof is available in the on-line appendix. Proposition 2 also implies that for $\beta \to 1$, $\phi_\pi < 0.5$ guarantees stability independently of $\phi_\tau$. For higher values of $\phi_\pi$ stability depends on the fiscal rule. Furthermore, a fiscal rule with $\phi_\tau > \phi_\tau^*$ can be shown to weaken the rational expectations equilibrium relation between real debt and inflation, making inflation expectations less responsive to the level of real debt.

[23] A proof for arbitrary degree of nominal rigidity is available in the on-line appendix.

eliminate uncertainty about the statistical laws determining state variables, as future output, inflation, taxes and real debt must still be forecasted to make spending and pricing decisions. And, in particular, the details of the fiscal policy regime remain uncertain.

**Proposition 3** *Under anchored monetary policy expectations, stabilization policy ensures expectational stability if the following conditions are satisfied: either*

1. *Monetary policy is active and fiscal policy is passive such that*

$$1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ and } \phi_\pi > \frac{1}{1-\beta\delta} \text{ ; or}$$

2. *Monetary policy is passive,* $0 \leq \phi_\pi < 1$*, and fiscal policy is active such that*

   *(a)*

$$0 \leq \phi_\tau < 1 \quad \text{and} \quad \delta < \min\left[\frac{\left(1-\beta+\beta^2\phi_\pi\right)\left(1-\phi_\pi\right)}{\phi_\pi\beta\left(1-\beta\phi_\pi\right)}, 1\right] \quad \text{or}$$

   *(b)*

$$\phi_\tau > \frac{1+\beta}{1-\beta}.$$

**Remark 4** *The conditions in 1. and 2.(b) are also necessary conditions.*

Appendix A.2 provides a sketch of the proof. Regardless of the regime, guarding against expectations-driven instability for a given choice of tax rule, $\phi_\tau$, requires a choice of monetary policy rule that depends on two model parameters: the household's discount factor, $\beta$, and the steady-state ratio of the primary surplus to output, $\delta$ (or equivalently the steady-state debt-to-output ratio since $\bar{s} = (1-\beta)\bar{b}$). The choice of fiscal regime, reflected in the implied average debt-to-output ratio, imposes constraints on stabilization policy. Less fiscally responsible governments have access to a smaller set of monetary policies to ensure learnability of rational expectations equilibrium. In the case of passive fiscal policies, the higher is the average debt-to-output ratio, the more aggressive must monetary policy be to protect the economy from self-fulfilling expectations.

Similarly, under active fiscal policies, the choice of monetary policy is again constrained by the average level of indebtedness of the economy. The higher are average debt levels the more passive must be the adopted monetary policy rule. Regardless of the policy regime, for $0 < \delta < 1$, the menu of policies consistent with stabilizing expectations is larger than when agents are uncertain about monetary policy strategy — compare proposition 2. This discussion is summarized in the following proposition which presents two special cases of the above results.

**Proposition 5** *Anchored monetary policy expectations unambiguously improves stabilization policy under learning dynamics. For $0 < \delta < 1$, a larger menu of fiscal and monetary policies is consistent with expectations stabilization under knowledge of the monetary policy regime than under monetary policy regime uncertainty. When $\delta \to 1$, the regions of stability in the communication and no communication cases coincide. When $\delta = 0$, the Leeper conditions are restored.*

That the stability of expectations depends on a steady-state quantity through $\delta$ is surprising when compared to a rational expectations analysis: determinacy conditions are independent of this quantity. What then is the source of this dependence?

Proposition 3 makes clear that the choice of monetary policy, $\phi_\pi$, and the steady-state structural surplus-to-output ratio, $\delta$, play a crucial role in determining stability, in both Ricardian and non-Ricardian policy equilibria. The main source of instability is a class of wealth effects arising from violations of Ricardian equivalence: agents perceive real bonds to be net wealth out of rational expectations equilibrium.

**Active Monetary Policy and Passive Fiscal Policy**. To provide intuition, consider a regime with active monetary policy and passive fiscal policy. Suppose that inflation expectations increase. Because of communication agents correctly predict a steeper path of the nominal interest rate (as forecasts satisfy the Taylor principle), which restrains aggregate demand, leading to lower actual inflation. In an economy with zero net debt, expectations of future inflation decline, driving the economy back to equilibrium. But with holdings of the public debt treated as net wealth, lower inflation generates a positive wealth effect, stimulating aggregate demand and increasing inflationary pressures. The increase in real debt is larger if the monetary authority does not observe current prices because the nominal interest rate does not immediately decrease with inflation. On the one hand, active policy restrains demand as agents expect higher future real interest rates. On the other hand, larger real debt and higher expected nominal interest rates generate wealth effects with inflationary consequences. If the monetary policy rule is not sufficiently active and the stock of government debt is large the latter prevail, leading to instability.

Further insight into this result can be obtained by considering a more general form of utility function with constant consumption intertemporal elasticity of substitution, $\sigma > 0$. Aggregate demand becomes

$$\begin{aligned}\hat{x}_t &= \delta\beta^{-1}\left(\hat{b}_t - \hat{\pi}_t\right) - \beta^{-1}\delta\hat{s}_t - (1-\delta)\,\phi_\pi\hat{E}_{t-1}\hat{\pi}_t \\ &\quad + \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)\left(\hat{x}_{T+1} - \delta\hat{s}_{T+1}\right) - (\sigma-\delta)\left(\phi_\pi\beta - 1\right)\hat{\pi}_{T+1} + r_T^n\right].\end{aligned}$$

A lower $\sigma$ mitigates the negative output response to an expected increase in the real rate.[24]

**Proposition 6** *Assume $\sigma > \delta$. In a passive fiscal regime and knowledge of the active monetary policy regime, expectational stability obtains if*

$$\phi_\pi > \frac{1}{1 - \beta\frac{\delta}{\sigma}}.$$

A smaller intertemporal elasticity of substitution reduces the stabilizing effects of anticipated shifts in the expected path of the nominal interest rate while increasing the relative importance of destabilizing wealth effects. As $\sigma \to \delta$, the stability condition is the same as in the case of uncertainty about the monetary policy regime.[25] However, the underlying mechanism that generates instability is quite different. In one case, instability arises from departures from Ricardian equivalence; in the other from a failure of interest rate policy to restrain demand because interest rate forecasts do not satisfy the Taylor principle.

**Active Fiscal Policy and Passive Monetary Policy**. A similar logic operates with passive monetary policy and active fiscal policy. Following an increase in inflation expectations, output and inflation increase, stimulated by a decline in real interest rates. The positive relation between real debt and inflation — engendered from the structure of beliefs local to rational expectations equilibrium — drives the economy back to equilibrium.[26] But higher inflation can also have a destabilizing effect because it leads to a higher expected path for the nominal interest rate, increasing the real value of interest payments on outstanding government debt. This positive wealth effect increases aggregate demand and inflation. If the latter effect is sufficiently strong the combination of monetary and fiscal policy can be destabilizing.[27] That is, if monetary policy is sufficiently aggressive and the steady-state level of real debt is sufficiently high, then inflationary effects dominate, leading to instability. Fiscal policy, as reflected in average indebtedness, may compromise monetary policy.

# 7 Resolving Uncertainty about Fiscal Policy

Emphasis has so far been given to resolving uncertainty about monetary policy. This apparent asymmetry resonates with actual policy making. Much has been made of the purported

[24] In this case, the Phillips curve coeffcient $\kappa$ is substituted by $\tilde{\kappa} = \kappa\sigma^{-1}$.

[25] Notice that the conditions for determinacy are not affected by $\sigma$.

[26] Recall part 2 of proposition 1 which demonstrates inflation and debt to be positively related in non-Ricardian equilibria. See also section 5.2 of Eusepi and Preston (2008c) for a simple example and further intuition.

[27] It can be shown that the higher $\phi_\tau$, the smaller the parameter set for which we have stability. In fact the larger is $\phi_\tau$ the weaker the relation between real debt and inflation, and the more important the wealth effects from higher nominal rates.

advantages of clear communication strategies in both the theory and practice of monetary policy. However, there have been limited parallel developments in the literature on fiscal policy. While a detailed discussion of these issues is not possible here, the model does provide some indication that developments of the kind proposed by Leeper (2009) may be desirable. One model-based interpretation of the benefits of communicating about fiscal policy can be given by asking what are the consequences of agents understanding the fiscal accounts to be intertemporally solvent. Alternatively stated, what if agents correctly understand that, conditional on future monetary policy and inflation, outstanding debt is fully backed by future taxation — in which case we say agents' fiscal expectations are anchored. Aside from this implied restriction, agents' beliefs are specified as in section 6.

**Proposition 7** *Suppose agents have anchored fiscal expectations, so that households and firms understand that the restriction*

$$\left(\hat{b}_t - \hat{\pi}_t\right) = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\, \hat{s}_T - \beta \left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right] \tag{24}$$

*holds at all times. Anchored fiscal expectations restore the Leeper conditions as necessary and sufficient for stability.*

The proof follows directly from proposition 3. Proposition 7 suggests that communication about fiscal policy may be as important to stabilization objectives as communication about monetary policy. If both monetary and fiscal expectations are anchored in the sense that i) forecasts of inflation and nominal interest rates satisfy the monetary policy rule (6); and ii) forecasts of inflation, debt, nominal interest rates and taxes satisfy the intertemporal solvency condition (24), then the Leeper conditions are necessary and sufficient for stabilizing expectations. Contrasting propositions 3 and 7 reveals a deeper insight: absent anchored fiscal expectations, anchoring monetary policy expectations yields fewer stabilization benefits — a smaller set of monetary policies are consistent with stability — the larger is the debt-to-output ratio. In the limit $\delta \to 1$, failure to communicate about fiscal policy completely undermines any stabilization benefits from anchoring monetary policy expectations. This raises serious questions about the efficacy of monetary policy in situations of considerable macroeconomic uncertainty in which there are dramatic expansions in the scale and scope of fiscal activities, as witnessed in many economies in response to the financial crisis of 2008.

The special case $\delta = 0$ is isomorphic in terms of stability of expectations to the more general case of anchored fiscal expectations. This underscores the importance of debt dynamics to expectations stabilization: from an expectational stability viewpoint, indebtedness only matters to the extent agents are uncertain about the long-run consequences of fiscal policy. Absent anchored monetary expectations, anchored fiscal expectations provide no advantages from an expectational stability perspective — the conditions of proposition 3 are operative. Implications for the sequencing of institutional reform suggests themselves.

Finally, note that for the question of expectational stability, no specific assumption need be made about how the government actually intends to achieve intertemporal solvency. All that is required is agents believe government promises that the fiscal regime is consistent with fiscal solvency at any point in time. Given observed current debt and inflation, and forecasts of future interest rates and inflation, agents can infer the expected present discounted value of taxes that satisfies (24). This does not mean fiscal variables are irrelevant — the projected evolution of debt is still used in forecasts. And the precise details of how taxes are actually adjusted will matter for dynamics out-of-rational-expectations equilibrium.

# 8 The Public Debt and Macroeconomic Dynamics

The preceding analytical results have focused on the implications of regime uncertainty on the ability of monetary and fiscal policy makers to stabilize expectations in the long run. A no less interesting and pressing question in the light of the Global Financial Crisis concerns the consequences of policy uncertainty for macroeconomic dynamics, even when expectational stability is guaranteed asymptotically. Specifically, do anchored fiscal expectations improve stabilization policy out of rational expectations equilibrium? And do high debt levels impair macroeconomic control by monetary and fiscal policy makers? The following examines model-implied impulse response functions to an inflation shock to answer these questions.

## 8.1 Generating Impulse Response Functions

The impulse response functions to a shock to inflation expectations are generated as follows. The model is simulated 5000 times assuming shocks to the natural rate, monetary policy and tax policy have standard deviations: $\sigma_r = 1$, $\sigma_i = 0.1$ and $\sigma_\tau = 0.1$.[28] A quarterly model is assumed giving a discount factor $\beta = 0.99$. In contrast to the analytical results, more general assumptions are made about the degree of nominal rigidities and the elasticity of intertemporal substitution. The Calvo parameter is fixed at $\alpha = 0.6$, consistent with Blinder, Canetti, Lebow, and Rudd (1998) and Bils and Klenow (2004). A utility function with intertemporal

[28] Shocks to the policy rules are added to prevent agents from learning the policy coefficients after few data points. However, their inclusion does not affect the stability results.

elasticity of substitution of consumption equal to 0.3 is considered, consistent with broad findings in the macroeconomics literature. Monetary policy is specified as $\phi_\pi = 1.5$ and tax policy as $\phi_\tau = 4$. Monetary expectations are anchored; fiscal expectations are not.

Two levels of average indebtedness are considered: a low debt economy, which has a debt-to-output ratio of zero on average, $\delta = 0$; and a high debt economy with a debt-to-output ratio of $4\bar{b}/\bar{Y} = 2.3$ (in annual terms). While the latter is arguably large, it is chosen to emphasize the dynamics that operate in a high debt economy. It is also the only asset that can be held in this economy — there is no capital. Note also that these two economies are sufficient to answer the two questions posed above. High average indebtedness is only relevant to dynamics when fiscal expectations are not anchored — recall section 7.

The impulse response functions are computed by perturbing each simulated path by an expectational shock. The difference between these perturbed paths and the original paths provides the impulse response functions. They are non-linear because of the learning dynamics and the plotted paths correspond to the median impulse response over 5000 simulations. The perturbation is done by increasing the initial beliefs about the constant in the inflation equation, $a_{\pi,0}$, from zero (the parameter's rational expectations equilibrium value) to 0.01. This represents an increase in inflation expectations at all forecast horizons. It can be interpreted as a small shift in the perceived inflation target, or in the long-run inflation average. All other coefficients are initially set to their rational expectations values.

A decreasing gain is employed so that $g_t = g_{t-1} + 1$ where $g_0$ is chosen to be large enough to ensure that beliefs remain in the basin of attraction — recall the theoretical results are local characterizations of dynamics. Hence, with sufficient data the analytical results of the paper guarantee beliefs will converge to the rational expectations equilibrium of the model, given appropriate choice of policy. For the described calibration, we numerically verify that both economies satisfy local expectational stability conditions. A high choice of $g_0$ is equivalent to having a tight prior on the initial beliefs (in our experiments we chose $g_0 = 50$). A consequence, relevant to interpreting the impulse response functions, is the slow convergence to rational expectations equilibrium. There is no attempt here for empirical realism. Rather we seek to draw out general lessons about the mechanisms underlying model dynamics.[29]

[29] However, Eusepi and Preston (2008b) demonstrate that learning dynamics represent a promising approach to fitting observed business cycles.

## 8.2 The Role of Indebtedness: fiscal effects in a Ricardian regime

Figure 1 plots the impulse responses for output, inflation and nominal interest rates. The impulse responses for the high debt economy are distinguished by smaller impact effects and much greater persistence. Substantial indebtedness fundamentally changes how the economy responds to shocks. Concomitantly, unanchored fiscal expectations impairs control of inflation and output.

To understand the nature of these differences it is useful to decompose aggregate demand into the following terms

$$\begin{aligned}\hat{x}_t &= \delta\left(\beta^{-1}\left(\hat{b}_t-\hat{\pi}_t\right)-\beta^{-1}\hat{s}_t+\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[\left(\hat{\imath}_T-\hat{\pi}_{T+1}\right)-\left(1-\beta\right)\hat{s}_{T+1}\right]\right)\\ &\quad+\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[\left(1-\beta\right)\hat{x}_{T+1}-\sigma\left(\hat{\imath}_T-\hat{\pi}_{T+1}\right)+r_T^n\right]\\ &= \Psi_{\delta,t}+\Psi_{R,t} \end{aligned} \tag{25}$$

where

$$\Psi_{\delta,t}=\delta\left(\beta^{-1}\left(\hat{b}_t-\hat{\pi}_t\right)-\beta^{-1}\hat{s}_t+\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[\left(\hat{\imath}_T-\hat{\pi}_{T+1}\right)-\left(1-\beta\right)\hat{s}_{T+1}\right]\right)$$

and $\Psi_{R,t}$ captures remaining terms. The variable $\Psi_{R,t}$ isolates terms that would obtain in a zero-debt economy, or equivalently, one in which fiscal expectations are anchored. $\Psi_{\delta,t}$ captures departures from this benchmark, representing deviations from Ricardian equivalence because holdings of the public debt are treated as net wealth. It is the real value of holdings of the public debt once future tax and interest obligations are accounted for.

Figure 2 plots these two terms. It is immediate that $\Psi_{\delta,t}$ generates destabilizing demand effects in a high debt economy. In a regime with zero steady-state debt, active monetary policy increases the expected future path of real rates reducing demand, and, in turn, curbing inflation until the economy returns to rational expectations equilibrium — see Figure 2 which also plots the real long rate defined as

$$\rho_t=\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{\imath}_T-\hat{\pi}_{T+1}\right).$$

In an economy with high steady-state debt this channel is still present. However, deviations from Ricardian equivalence drive aggregate demand in the opposite direction. The term $\Psi_{\delta,t}$ initially rises because: i) taxes are predetermined at the time of the shock and only rise over time; ii) agents anticipate higher future real interest rates, which deliver a positive income effect from holding the public debt; and iii) there is a valuation effect from the initial fall in inflation. For these reasons, the impact effects of inflation shock on output and inflation are smaller in the high debt economy.

However, the high debt economy displays a more sluggish response to the shock. Two components of the non-Ricardian term $\Psi_{\delta,t}$ delay the adjustment of output and inflation. First, as shown in Figure 1 the value of real debt outstanding rises over time increasing expected surpluses and thus future fiscal tightening.[30] Second, with active monetary policy, inflation below steady state induces lower expected real rates, generating income effects for debt holders. The decrease in $\Psi_{\delta,t}$ mutes the stimulative effects of lower expected real rates on aggregate demand, which are captured by the term $\Psi_{R,t}$.

Over time long rates and taxes adjust to reduce outstanding public debt and stabilize inflation, inducing convergence. In this experiment, there is a tight link between monetary and fiscal policy. Active monetary policy might not be sufficient to stabilize expectations if market participants face uncertainty about the fiscal regime and the government issues a sufficiently large quantity of debt on average.

## 9 Conclusions

This paper develops a model of policy regime uncertainty and its consequences for stabilizing expectations. Uncertainty about monetary and fiscal policy is shown to restrict, relative to a rational expectations analysis, the set of policies consistent with macroeconomic stability. Anchoring expectations about monetary and fiscal policy enlarges the set of policies consistent with stability. However, absent anchored fiscal expectations, the advantages from anchoring monetary expectations are smaller the larger is the average level of indebtedness. Finally, even when expectations are stabilized in the long run, the higher are average debt levels the more persistent will be the effects of disturbances out of rational expectations equilibrium.

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[30] The rise in real debt occurs because the central bank over predicts inflation and thus only gradually reduces the nominal interest rate in response to lower inflation. As a result, actual inflation is below the nominal interest rate (which is a function of expected inflation).

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# A Appendix

## A.1 Proof of Proposition 2

**Ricardian regime** The results reported in this proof can be reproduced using the Matlab file: *fiscal_ delay_ benchmark.m.* First, it can be shown that the beliefs $(\omega_0, \omega_1)$ evolve according to two separate sub-systems

$$\dot{\omega}_0 = (A - I_5)\,\omega_0 \text{ and } \dot{\omega}_1 = (B - I_5)\,\omega_1.$$

where $A$ and $B$ represent components of the Jacobian of $\bar{T}\,(\omega_0, \omega_1)$, evaluated at the rational expectations equilibrium $\omega_0^*, \omega_1^*$, defined in the previous proposition.

Consider the evolution of the intercept $\omega_0$. We take three steps in order to reduce the matrix $A$ from a five dimensional to three dimensional object. First, evaluating the matrix $A$ reveals that

$$\dot{\omega}_0^s = -\omega_0^s.$$

Hence, the intercept in the fiscal rule equation converges for all parameter values, independently of the other elements of the beliefs vector. This reduces dimensionality by one. Second, using the restriction

$$A_{5,j} = -\beta^{-1} A_{2,j} + A_{3,j} \quad \text{for } j = 1...5$$

delivers the three dimension system (see the Matlab file for the details of the variables' transformation):

$$\begin{bmatrix} \dot{\omega}_0^x \\ \dot{\omega}_0^\pi \\ \dot{\omega}_0^i \end{bmatrix} = \tilde{A} \begin{bmatrix} \omega_0^x \\ \omega_0^\pi \\ \omega_0^i \end{bmatrix}.$$

For the real parts of the three eigenvalues to be negative requires

$$Tr(\tilde{A}) < 0, \quad \det(\tilde{A}) < 0 \text{ and } M_{\tilde{A}} = -Sm(\tilde{A}) \cdot Tr(\tilde{A}) + \det(\tilde{A}) > 0$$

where $Sm(\tilde{A})$ denotes the sum of all principle minors of $\tilde{A}$. We are interested in the limit case where $\alpha \to 0$. In this case, the trace, determinant and $M_{\tilde{A}}$ become arbitrarily large. Consider the trace first. We can calculate the limit $\lim_{\alpha\to 0^+}\alpha\cdot Tr(\tilde{A}) = -\left(\phi_\pi - 1 - \phi_\pi\right)\beta\left(1-\beta\right)^{-1}$ which is negative if and only if

$$\phi_\pi > \frac{1}{1-\beta}. \tag{26}$$

Likewise, the determinant $\lim_{\alpha\to 0^+}\alpha\det(\tilde{A}) = -\left(\phi_\pi - 1\right)\left(1-\beta\right)^{-1}$ which is negative if and only if $\phi_\pi > 1$. For $M_{\tilde{A}}$ we have

$$\lim_{\alpha\to 0^+}\alpha^2\cdot M_{\tilde{A}} = \frac{\left(2-2\phi_\pi+\phi_\pi\beta\right)\left(1-\phi_\pi+\phi_\pi\beta\right)}{\left(1-\beta\right)^2}$$

which is positive provided (26).

Consider now the coefficients on real debt. An identical process reduces the dimensionality of the matrix $B$ to a three dimensional matrix $\tilde{B}$. Considering the trace we get

$$\lim_{\alpha\to 0^+}\alpha\cdot Tr(\tilde{B}) = \frac{1-\phi_\tau\left(1-\beta\right)\left(\beta\phi_\pi+1\right)}{\phi_\tau\beta(1-\beta)}$$

which is decreasing in $\phi_\tau$. In a Ricardian regime, $\phi_\tau > 1$. Evaluating the expression at $\phi_\tau = 1$, if (26) then the trace of the $\tilde{B}$ matrix is negative. Evaluating the determinant we get

$$\lim_{\alpha\to 0^+}\alpha\cdot\det(\tilde{B}) = \frac{1-\beta\phi_\pi-\beta^{-1}\phi_\tau\beta(1-\beta)}{\phi_\tau\beta(1-\beta)}$$

which is decreasing in $\phi_\tau$. Again, imposing $\phi_\tau = 1$ gives

$$\lim_{\alpha\to 0^+}\alpha\cdot\det(\tilde{B}) = -\frac{\phi_\pi-1}{\phi_\tau(1-\beta)} < 0.$$

Finally,

$$\lim_{\alpha\to 0^+}\alpha^2\cdot M_{\tilde{B}} = \frac{\left[\phi_\tau\left(\beta-1\right)\left(\beta\phi_\pi+2\right)-\beta\phi_\pi+2\right]\left[\phi_\tau\left(\beta-1\right)\left(\beta\phi_\pi+1\right)+1\right]}{\beta^2\phi_\tau^2(1-\beta)^2}.$$

which is, again, decreasing in $\phi_\tau$.Imposing $\phi_\tau = 1$ yields

$$\lim_{\alpha\to 0^+}\alpha^2\cdot M_{\tilde{B}} = \frac{\left(2-2\phi_\pi+\beta\phi_\pi\right)\left(1-\phi_\pi+\beta\phi_\pi\right)}{\beta\phi_\tau^2(1-\beta)^2}$$

which is positive if (26) is satisfied.

**Non-Ricardian regime** The matrices $A$ and $B$ corresponding to the non-Ricardian regime can be reduced to three dimensional matrices by following the same steps as above. To further simplify the problem, we use two Lemmas.

**Lemma 8** *Consider the model where $\alpha\to 0$. Then $\lambda_2\to\phi_\pi$.*

**Proof.** Recall that

$$\lambda_2 = \frac{1}{2\beta}\left[1+\beta+\kappa(\alpha)-\sqrt{(1+\beta+\kappa(\alpha))^2-4\beta(1+\kappa(\alpha)\phi_\pi)}\right].$$

We can then evaluate

$$\lim_{\alpha\to 0}\lambda_2 =$$

$$\frac{1}{2\beta}\lim_{\alpha\to 0}\frac{\left\{\begin{array}{c}\left[1+\beta+\kappa(\alpha)+\sqrt{(1+\beta+\kappa(\alpha))^2-4\beta(1+\kappa(\alpha)\phi_\pi)}\right]\times\\ \left[1+\beta+\kappa(\alpha)-\sqrt{(1+\beta+\kappa(\alpha))^2-4\beta(1+\kappa(\alpha)\phi_\pi)}\right]\end{array}\right\}}{\left[1+\beta+\kappa(\alpha)+\sqrt{(1+\beta+\kappa(\alpha))^2-4\beta(1+\kappa(\alpha)\phi_\pi)}\right]} =$$

$$\frac{1}{2\beta}\lim_{\alpha\to 0}\frac{4\beta(1+\kappa(\alpha)\phi_\pi)}{\left[1+\beta+\kappa(\alpha)+\sqrt{(1+\beta+\kappa(\alpha))^2-4\beta(1+\kappa(\alpha)\phi_\pi)}\right]}.$$

Using L'Hôpital

$$\frac{1}{2\beta}\lim_{\alpha\to 0}\left[4\beta\kappa'(\alpha)\phi_\pi/\left(\kappa'(\alpha)+\frac{(1+\beta+\kappa(\alpha))\kappa'(\alpha)-2\beta\kappa'(\alpha)\phi_\pi}{\sqrt{(1+\beta+\kappa(\alpha))^2-4\beta(1+\kappa(\alpha)\phi_\pi)}}\right)\right] =$$

$$\frac{1}{2\beta}\lim_{\alpha\to 0}\left[4\beta\phi_\pi/\left(1+\frac{(1+\beta+\kappa(\alpha))-2\beta\phi_\pi}{\sqrt{(1+\beta+\kappa(\alpha))^2-4\beta(1+\kappa(\alpha)\phi_\pi)}}\right)\right] =$$

$$\frac{1}{2\beta}\lim_{\alpha\to 0}2\beta\phi_\pi = \phi_\pi.$$

■

We then conjecture that as $\alpha\to 0$, one eigenvalue of $\tilde{A}$ and $\tilde{B}$ tends to $-1$. The conjecture is verified in the following Lemma.

**Lemma 9** *Consider the model where $\alpha\to 0$. Then one eigenvalue $\psi$ of $\tilde{A}$ and $\tilde{B}$ converges to $-1$.*

**Proof.** The characteristic equations of $\tilde{A}$ and $\tilde{B}$ are

$$\Delta^{\tilde{A}}(\psi) = \psi^3 - tr\left(\tilde{A}\right)\psi^2 + Sm(\tilde{A})\psi - \det(\tilde{A})$$

and

$$\Delta^{\tilde{B}}(\psi) = \psi^3 - tr\left(\tilde{B}\right)\psi^2 + Sm(\tilde{B})\psi - \det(\tilde{B}).$$

It can be shown that[31]

$$\lim_{\alpha\to 0}\Delta^{\tilde{A}}(-1) = -1 - tr\left(\tilde{A}\right) - Sm(\tilde{A}) - \det(\tilde{A}) = 0$$

[31] The limit is computed in the matlab file fiscal_delay_benchmark.m.

and

$$\lim_{\alpha\to 0}\Delta^{\tilde{B}}(-1) = -1 - tr\left(\tilde{B}\right) - Sm(\tilde{B}) - \det(\tilde{B}) = 0.$$

■

Let us consider the local stability of the intercept coefficients. The remaining two eigenvalues $z_1$ and $z_2$ of $\tilde{A}$ are negative if

$$tr(\tilde{A}) + 1 = z_1 + z_2 < 0 \text{ and } \det(\tilde{A}) = -z_1 z_2 < 0.$$

The trace is

$$tr\left(\tilde{A}\right) = -\left[1 + \frac{1 - \left[1 - \beta\phi_\pi(1-\beta)\right]\phi_\tau - \beta\phi_\pi}{1 - (1-\beta)\phi_\tau - \beta\phi_\pi}\right]. \tag{27}$$

**(a)** Consider the case of $0 \leq \phi_\tau < 1$: the trace can be re-arranged to deliver the following relationship between $\phi_\tau$ and $\phi_\pi$ at $tr\left(\tilde{A}\right) = 0$,

$$\phi_\tau = \frac{2}{\left[(1-\beta\phi_\pi)^{-1} + (1-\beta)\right]}$$

in the text. Then, $0 \leq \phi_\tau < \min\left(\phi_\tau^*\left(\phi_\pi\right), 1\right)$, where

$$\phi_\tau^*\left(\phi_\pi\right) = \frac{2}{\left[(1-\beta\phi_\pi)^{-1} + (1-\beta)\right]},$$

and where we use $\partial tr\left(A\right)/\partial\phi_\tau > 0$ for $\phi_\tau \in [0,1)$. The determinant is

$$-\det\left(\tilde{A}\right) = \frac{\left(1-\phi_\tau\right)\left(\beta\phi_\pi - 1\right)}{\left(1-\beta\right)\phi_\tau + \beta\phi_\pi - 1} > 0. \tag{28}$$

Finally, consider the $B$ matrix. Proceeding in the same way as for the $\tilde{A}$ matrix, the trace can be shown to be

$$tr(\tilde{B}) = -2 - \frac{(1-\beta)\beta^2\phi_\pi^2\phi_\tau}{(-(1-\beta)\phi_\tau - \beta\phi_\pi + 1)(\beta\phi_\pi - 1)}. \tag{29}$$

which gives the following expression for

$$\phi_\tau^{**}\left(\phi_\pi\right) = \frac{2}{\left(\beta^2\phi_\pi^2 + 2\left(1-\beta\phi_\pi\right)\right)\frac{(1-\beta)}{(1-\beta\phi_\pi)^2}}$$

which solves $tr(\tilde{B}) = 0$ (also shown to have positive derivative with respect to $\phi_\tau$). It can be shown that $\phi_\tau^{**}\left(\phi_\pi\right) > \phi_\tau^*\left(\phi_\pi\right)$.[32] The determinant of the $\tilde{B}$ matrix is equal to $-1$ for every parameter value.

**(b)** Straightforward algebraic manipulations of (27)-(29) show that the stability condition holds for all parameter values with $\phi_\tau > \left(1+\beta\right)/\left(1-\beta\right)$.

[32] It can be shown that the difference betwen the denominator of $\tau^*$ and the denominator in $\tau^{**}$ is equal to

$$\left(\beta\phi_\pi - 1\right)^{-2}\left(1-\phi_\pi\right)\beta > 0.$$

## A.2 Proposition 3

The proof follows the same steps as in Proposition 2.

**Ricardian Regime.** The matrices $A$ and $B$ are three dimensional, given that agents do not have to forecast the nominal interest rate and the surplus. The expressions below are calculated using the file *fiscal_ analytical_ trsp.m*. Let us consider first the matrix $A$. We find

$$\lim_{\alpha \to 0^+} \alpha \cdot tr(A) = \frac{1 + (\beta\delta - 1)\phi_\pi}{1 - \beta} \tag{30}$$

which gives the stability condition in the main text. Thus, for $\delta = 0$, we the Taylor principle obtains. Using $\delta = (1 - \beta)\frac{\bar{b}}{\bar{y}}$ we can re-write the stability condition as

$$\phi_\pi(1 - \beta(1 - \beta)\frac{\bar{b}}{\bar{y}}) - 1 > 0$$

so that for high levels of debt-to-output ratio and for intermediate values of the discount factor instability is likely to arise. As $\alpha \to 0$, the determinant is

$$\lim_{\alpha \to 0^+} \alpha \cdot \det(A) = -\frac{\phi_\pi - 1}{1 - \beta}$$

and negative provided $(\phi_\pi - 1) > 1$. Finally,

$$\lim_{\alpha \to 0^+} \alpha^2 \left[-Sm(A) \cdot Tr(A) + \det(A)\right] = \frac{\left(\phi_\pi \left(2 - \beta\delta\right) - 2\right)\left(\phi_\pi \left(1 - \beta\delta\right) - 1\right)}{\left(1 - \beta\right)^2},$$

which is positive provided (30) is satisfied.

Consider now the matrix $B$. The trace is satisfies

$$\lim_{\alpha \to 0^+} \alpha \cdot tr(B) = \frac{(-1 + \beta + \beta^2\phi_\pi\delta - \beta\phi_\pi\delta)\phi_\tau - \left(1 - \delta\right)\beta\phi_\pi + 1}{\left(1 - \beta\right)\beta\phi_\tau}$$

and is negative provided the trace of the matrix for the constants is negative ($\phi_\tau > 1$ in the Ricardian fiscal regime). As $\alpha \to 0$, the determinant is always negative, that is

$$\lim_{\alpha \to 0^+} \alpha \cdot \det\left(B\right) = -\beta^{-1} - \frac{\left(\beta\phi_\pi - 1\right)}{\left(1 - \beta\right)\beta\phi_\tau} < 0,$$

if (30) is satisfied.

Finally, letting $\alpha \to 0$, the sum of all principle minors becomes

$$\lim_{\alpha \to 0^+} \alpha^2 \left[-Sm(B) * Tr(B) + \det(B)\right] =$$

$$\frac{\left[(-2\beta - \beta^2\phi_\pi\delta + \beta\phi_\pi\delta + 2)\phi_\tau - 2 - \beta\phi_\pi\delta + 2\beta\phi_\pi\right]\left[(1 + \beta\phi_\pi\delta - \beta - \beta^2\phi_\pi\delta)\phi_\tau - 1 + \beta\phi_\pi - \beta\phi_\pi\delta\right]}{\phi_\tau^2\beta^2(1 - \beta)^2},$$

which is positive provided $\phi_\tau > 1$ (Ricardian fiscal Regime) and (30) is satisfied.

**Non-Ricardian Regime.** As in proposition 2, it can be shown that one eigenvalue of both matrices $A$ and $B$ is equal to $-1$. We define the trace of the constant coefficients as

$$\lim_{\alpha \to 0^+} tr(A) + 1 = \Phi^A(\phi_\tau, \phi_\pi, \delta).$$

First notice that

$$\Phi^A_\delta(\phi_\tau, \phi_\pi, \delta) = \frac{(1-\beta\phi_\pi)\beta^2\phi_\pi}{(1-\beta)\left(1-(1-\beta)\,\phi_\tau - \beta\phi_\pi\right)} > 0$$

if $0 \leq \phi_\tau < 1$ and

$$\Phi^A_{\phi_\tau}(\phi_\tau, \phi_\pi, \delta) = \frac{(1-\beta\phi_\pi)(\beta\phi_\pi\delta - \phi_\pi + 1)\beta}{(-1+\phi_\tau - \phi_\tau\beta + \beta\phi_\pi)^2} > 0$$

for all admissible values of $\delta$, $\phi_\pi$, and $\phi_\tau$, where $\Phi^A_x$ denotes the derivative of $\Phi^A$ with respect to the argument $x$. Second we show that for values of $\delta < \delta^{TA}$ the trace is negative. Consider $\phi_\tau < 1$. Using the inequality above, we can solve for $\delta^{TA}$ as

$$\Phi^A(1, \phi_\pi, \delta^{TA}) = -\frac{(\beta^2\phi_\pi - \beta^2\phi_\pi^2 + \beta^2\phi_\pi^2\delta^{TA} + \beta\phi_\pi - \beta\phi_\pi\delta^{TA} - \beta - \phi_\pi + 1)}{(1-\beta)(1-\phi_\pi)} = 0$$

where

$$\delta^{TA} = \frac{\left((1-\beta+\phi_\pi\beta^2\right)(1-\phi_\pi)}{\phi_\pi\beta(1-\beta\phi_\pi)} > 0.$$

If $\phi_\tau > (1+\beta)/(1-\beta)$ then $\Phi^A_\delta(\phi_\tau, \phi_\pi, \delta) < 0$. Evaluating $\Phi^A$ at $\delta = 0$ gives

$$\Phi^A(\phi_\tau, \phi_\pi, 0) \quad = \quad \frac{(C_{\phi_\tau}\phi_\tau - \beta^3\phi_\pi^2 + 3\beta^2\phi_\pi - 2\beta\phi_\pi - 2\beta + 2)}{(1-\beta)((1-\beta)\phi_\tau - 1 + \beta\phi_\pi)}$$

where

$$C_{\phi_\tau} = (\beta^3\phi_\pi - 2\beta^2\phi_\pi - \beta^2 + \beta\phi_\pi + 3\beta - 2).$$

The denominator is positive for $\phi_\tau > (1+\beta)/(1-\beta)$. For the numerator, substituting $\phi_\tau = ((1+\beta)/(1-\beta))$ gives

$$(C_{\phi_\tau}\phi_\tau - \beta^3\phi_\pi^2 + 3\beta^2\phi_\pi - 2\beta\phi_\pi - 2\beta + 2) = \left(\beta^2 - \beta\right) + \left(\beta^2\phi_\pi - \beta\phi_\pi\right) + (2\beta^2\phi_\pi - 2\beta) - \beta^3\phi_\pi - \beta^3\phi_\pi^2 < 0$$

Last, the coefficient $C_{\phi_\tau}$ is positive since

$$\left(\beta^3\phi_\pi - \beta^2\phi_\pi\right) + R(\phi_\pi, \beta) < 0$$

where

$$R(\phi_\pi, \beta) = -\beta^2\phi_\pi - \beta^2 + \beta\phi_\pi + 3\beta - 2$$

$$R(0,\beta) = -(\beta-1)^2 - 1 + \beta < 0, \qquad R(1,\beta) = -2(\beta-1)^2 < 0$$

and

$$R_{\phi_\pi}(\phi_\pi,\beta) = -\beta^2 + \beta > 0.$$

Hence, for $\phi_\tau > (1+\beta)/(1-\beta)$ the trace is negative. Finally, the determinant of the Jacobian is

$$\lim_{\alpha\to 0^+} [-\det(A)] = \frac{(1-\phi_\tau)(1-\beta\phi_\pi)}{1-(1-\beta)\,\phi_\tau - \beta\phi_\pi} > 0.$$

Following the same steps for matrix $B$ :

$$\lim_{\alpha\to 0^+} tr(B) + 1 = \Phi^B(\phi_\tau,\phi_\pi,\delta).$$

First notice that

$$\Phi^B_\delta(\phi_\tau,\phi_\pi,\delta) = \frac{\beta^2\phi_\pi^2}{(1-\beta)\,(1-(1-\beta)\,\phi_\tau - \beta\phi_\pi)} > 0$$

for $0 < \phi_\tau < 1$ and

$$\Phi^B_{\phi_\tau}(\phi_\tau,\phi_\pi,\delta) = \frac{(1-\beta)\delta\beta^2\phi_\pi^2}{(-1+\phi_\tau-\phi_\tau\beta+\beta\phi_\pi)^2} > 0.$$

Solving for $\delta^{TB}$ from

$$\Phi^B(1,\phi_\pi,\delta^{TB}) = \frac{(\beta^2\phi_\pi^3 - \beta^2\phi_\pi^3\delta^{TB} - \beta^2\phi_\pi^2 - 2\beta\phi_\pi^2 + \beta\phi_\pi^2\delta^{TB} + 2\beta\phi_\pi + 2\phi_\pi - 2)}{(1-\beta\phi_\pi)(1-\phi_\pi)} = 0$$

provides

$$\delta^{TB} = \frac{\big((1-\beta\phi_\pi) + \phi_\pi\beta^2 + (1-\beta\phi_\pi) - \phi_\pi\beta^2(1-\phi_\pi)\big)\,(1-\phi_\pi)}{\beta\phi_\pi^2\,(1-\beta\phi_\pi)} > \delta^{TA}.$$

Moreover, for $\phi_\tau > (1+\beta)/(1-\beta)$

$$tr(B) = \Phi^B_\delta(\phi_\tau,\phi_\pi,\delta) < 0, \quad \text{and} \quad \Phi^B(\phi_\tau,\phi_\pi,0) = -(\beta^2\phi_\pi^2 - 2\beta\phi_\pi + 2)/(1-\beta\phi_\pi) < 0.$$

Finally, the determinant is equal to one for all parameter values.

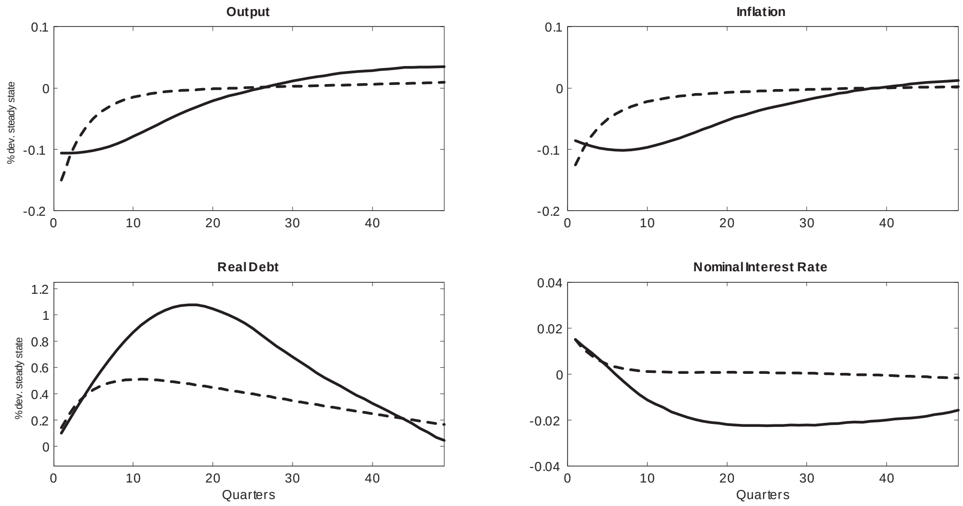


Figure 1: Impulse response functions to a shock to inflation expectations. Solid line corresponds to the high debt economy; dashed line the zero debt economy.

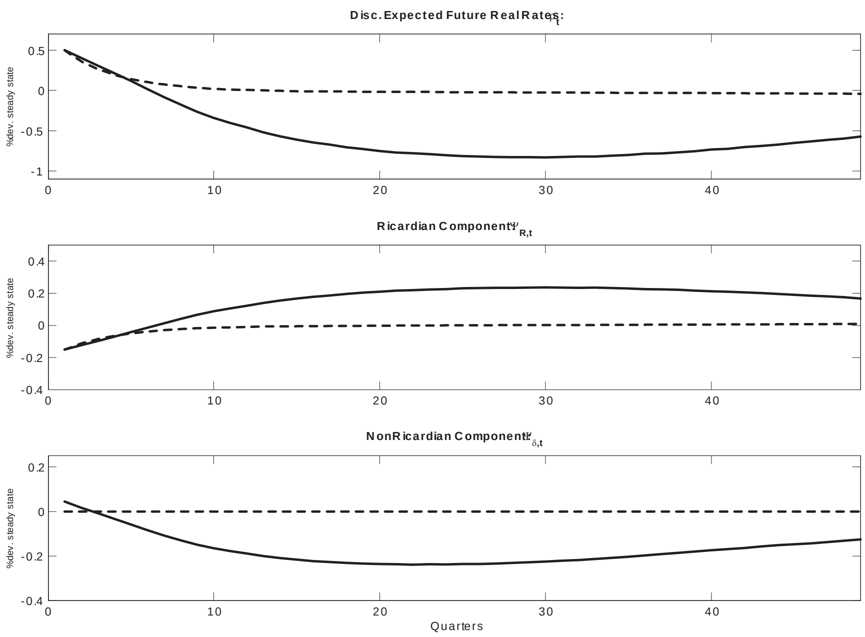


Figure 2: Impulse response functions to a shock to inflation expectations. Solid line corresponds to the high debt economy; dashed line the zero debt economy.