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1 Introduction

A persistent question arising in the development of models for the analysis of macroeconomic policy has been the relative role of economic theory and evidence (data) in their construction. Although this question is present in academic work as well, it is particularly pressing in a policy environment where there is a natural tendency to emphasize the ability of the selected model to match the data. At the same time, after five decades of such modelling it has become clear that this should not be done at the expense of failing to have a clear conceptual foundation for any decisions that are informed by the model. For this reason a strategy has generally emerged that identifies three different stages to the construction of a policy-relevant model. At each stage one has a model type that aims to perform a certain task and later model types build on the earlier ones.

The three model types are:

1. The *conceptual model (CM).* In this a framework is set out to explicate the preferred conceptual view of the economy that policy makers or advisers think is appropriate. This model needs to be much larger than

the miniature models that often guide academic research e.g. models such as IS-LM, New Keynesian models, Real Business Cycle (RBC) models etc. Such miniature models are rarely able to capture the complexities of any given situation e.g. referring to aggregate demand, as in an IS curve, rather than the components, is unlikely to produce a very convincing analysis for any policy discussion. Moreover, policy makers have increasingly been required to be precise about the arguments in support of a particular policy action (and sometimes the information that is an input into it, such as forecasts), and this points to the need to expand the size of the model while retaining the clarity that a strong theoretical perspective brings to analysis.

2. The *Data Adjusted Model (DAM).* The conceptual model is unlikely to be able to match an existing data set without becoming impossibly complex and thereby losing its main attraction of having a readily comprehendible structure. Hence it will generally need to be adjusted in order to match the evidence. We choose the word “ match” very carefully here. Replicating the data is rarely the objective. There are many reasons for this. Data is often subject to substantial revisions for long periods of time, so one may be replicating a chimera; it can also be influenced by many special factors that are hard to specify; and, finally, it is rarely the case that there is a precise mapping between the variables in the conceptual model and their measurable counterparts.

3. The *operational model (OM).* Policy discussion generally involves forecasts. Given that the opinions of decisions makers and any relevant current information must be incorporated into such forecasts it may be necessary to adjust the DAM during a forecasting "round" i.e. rather than just achieving a match to past data as in the DAM, it may be desirable to further adjust the model to reflect perceptions about future data patterns. In many instances it can be difficult to incorporate information about the near-term future into a conceptual model e.g. often surveys of the opinions of stock analysts about expected earnings growth are a better guide to actual equity price outcomes than the predictions from a DAM. It is also true that there is a great deal of useful information about current and future shocks to the economy in data on the future anticipations of households and firms, but this data is voluminous, and generally needs to be summarized in some way, possibly

via a preliminary factor analysis. Thus much information may need to be used in addition to that present in the DAM in order to convert it to the model that will actually be used to produce the forecasts that will be the foundation of the policy discussion.

The information used to transform the DAM to the OM is very context specific so that there is little one can generally say about the transformation. One possibility is to conceive of all of the different sources of information as being represented by a small number of factors which might then be added to the DAM in a fairly mechanical way to produce the operational model that is the source of the forecast. Methods for such compression have been recounted in a voluminous literature in the past decade, but often these factor models have been viewed as supplemental to the forecasts from the primary CM or DAM used by a policy making institution, rather than as providing additional information to be incorporated into these models. We will not review this factor literature. Moreover, because of the specificity of the conversion process of a DAM to an OM we will concentrate upon the first two stages in this paper.

The following two sections discusses issues relating to conceptual models. What needs to be put into them, what structure might be expected from them, what one wishes to achieve with them and how they might be modified for particular tasks. We distinguish between the economic and econometric structure of such models. In order to discuss the economic structure it proves useful to build a conceptual model that incorporates many of the approaches used in existing work. The process of construction resembles that increasingly used in central bank policy modeling, and so it might be regarded as representative of the ways that policy models are constructed so as to reflect evidence. The CM we use is smaller than those used in practice, out of a desire to succinctly describe it within a journal article. But it is much larger than academic models. We will refer to it as the C-model.

Section 4 then asks whether the C-model does produce a good match to the data. Although one does not expect a perfect match the model incorporates many of the features that academic quantitative research has found to be important to describing macro-economic outcomes. Despite this, there are still notable deficiencies in the match. To improve the match one needs to perform some adjustments along the line of the DAM. One possibility is to allow quite general patterns of serial correlation in the shocks that drive the model. A more traditional one is to add "tracking shocks" to the model.

We perform such adjustments in section 5 so as to formalize the degree to which the C-model fails to capture the data. In doing this analysis we wish to highlight the tools that might be used in exploring the match of the CM to data and some general methods for producing a match. Thus our C-model is a vehicle for exposition and is not necessarily meant to be a model that would be used in an actual policy environment.

Finally, section 6 turns to considering why the strategy of working with a CM has become so popular. We address this question by examining an alternative modelling strategy that dominates in some academic work. In this one starts with a very general model that is designed to fit the data closely and then an attempt is made to impose some theoretical structure upon it. In practice this has meant beginning with a Vector Autoregressive (VAR) process and then imposing structure upon it to produce a structural VAR (SVAR). We investigate how successful this strategy would be if the economy for which data is available was our C-model. Even though we proceed by working with an SVAR that correctly identifies the shocks we find that the SVAR is not capable of extracting the impulse responses for some of the C-model shocks, even if presented with 30000 observations. The reason is simple: the C-model economy involves many more variables than the typical VAR does and so the reduction to a smaller dimensional system means that an extremely high order VAR is needed to reproduce the impulses. Indeed, this is of such an order that it would be much larger than that used in many empirical papers which utilize statistical selection methods such as AIC or BIC. The problem can be ameliorated with a judicious choice of variables to appear in the VAR system but the example suggests that, since one will never be sure that the choice of variables is appropriate, it is better to start with a CM and work towards making it match the data than attempting the converse. One might say that this is a point that has always been recognized by those performing macroeconometric modelling for policy purposes. Thus, in the Cowles Commission approach, the emphasis was always upon deriving the restrictions that a structure placed upon a reduced form, rather than beginning with a reduced form and then deriving a structure. Section 7 then concludes.

2 The Economic Structure of Conceptual Models

2.1 Evolution of CMs

Initially models like IS-LM and AS-AD were the miniature models that formed the basis of CMs, but only in a very loose way. Moreover, there was generally no distinction between the CM and the OM. Inevitably, the model selected was effectively data- rather than theory-driven. Increasingly however we have seen CMs that are much more tightly linked with theory and whose structure is less driven by the idea that they should be capable of producing a tight fit to the data e.g. the builders of the influential Quarterly Projection Model (QPM) of the Bank of Canada commented on this thus:

> "There had been a systematic tendency towards over-fitting equations and too little attention to capturing the underlying economics. It was concluded that the model should focus on capturing the fundamental economics necessary to describe how the macro economy functions and, in particular, how policy works, and that it be calibrated to reflect staff judgement of appropriate properties rather than estimated by econometric techniques" (Coletti et al (1996,p14)).

Today many models are either present or are being built in central banks that reflect this philosophy. Since these models must represent the data to some degree it is natural that their features reflect those that have proved helpful in making miniature models produce a closer match to the data. In recent years a large number of these features have been experimented with and have become a standard part of CMs. Some limit has to be placed on the number and nature of these, otherwise one could lose the economic story that is being told in the CM. The objective is to try to focus the discussion in the policy context around some stories rather than degenerating into a discussion of many special events, as can easily happen with data driven models.

2.2 Overview of the C-Model

In this section we present a model that is representative of a typical CM. The key objective in constructing a CM is the provision of an acceptable theoretical shell. Although this shell might be added to in particular contexts, it primarily aims to set out a picture of the economy that has a sound theoretical base and is in accord with the conceptions of policy makers and advisers about how that specific economy functions. Some of the latter views are given expression through the choice of values to be assigned to model parameters. The model we construct reflects what might be regarded as a consensus view of the structure of CMs. Of course there may be a number of ways of incorporating these effects and we have had to choose a particular strategy, but it can be argued that most of the elements built into our CM e.g. habit persistence in consumption, inertia in price levels and inflation, are common to a large number of the CMs that appear in policy institutions and in the academic literature. It needs to be said that the CM used for policy work is very unlikely to be at the frontiers of academic work. The latter models are often highly experimental and many turn out to be ephemeral. Some however survive and the ideas expressed in them become widely accepted. Consequently, it is almost inevitable that the CMs used by a policy institution will incorporate only those ideas that have survived scrutiny, and have proven their worth over a longer period of time, rather than being focussed upon a set of novel ideas that are still untested.

The C-model described here centers upon the interactions of key agents – households, firms, and policy-makers – in an open economy. The equations for the behavior of households and firms arise from utility- and profit-maximising specifications, respectively, while policy is described by simple assumptions about targets and instruments. We assume that households attempt to maximize utility, defined over the consumption of domestically-produced and imported nondurable goods, holdings of real money balances, and time spent in leisure. They receive wage income from their labour, rental income from capital, and supernormal profits from goods markets. They own the domestic capital stock, and can reach a higher or lower desired asset position by saving in one-period consumption bonds that are issued by the government and foreigners. Firms are assumed to maximize profits by the sale of goods in domestic and foreign markets, for which they utilize a Cobb-Douglas technology and pay rents on labour and capital. They price at a mark-up over real marginal cost on each unit of output. The government raises taxes and debt and purchases output and services existing debt. A monetary authority anchors the nominal side via a simple inflation-targeting rule. The model has full stock-flow consistency and a well defined steady-state. Standard no-arbitrage conditions are assumed to hold, although in the uncovered interest parity condition there is a twist – a penalty on deviations away from an exogenous level of net foreign assets, rationalized as a time-varying sovereign risk premium – that is used to render the consumption and financial asset paths stationary in the face of transitory shocks.[1,2]

The C-model has many of the key elements of the current macroeconomic paradigm. Specifically, goods and labour markets are assumed to be monopolistically competitive, in this case as per Blanchard and Kiyotaki (1987). This provides a rationale for the introduction of nominal rigidities (that is, the suppliers have market power). In the case of the goods market, a continuum of intermediate goods producers provides differentiated goods to a final goods supplier (a "retailer"), which is assumed to be perfectly competitive. Intermediate goods firms are able to each extract a rent from the sale of their good to domestic and foreign markets. We assume that firms face intangible nominal price adjustment costs, as per Rotemberg (1982). This captures the idea that individual firms dislike changing prices because they fear losing market share. In the flexible price equilibrium, the mark-up over real marginal costs would be a constant, but, with price adjustment costs, it is time varying.

In the case of the labour market, a continuum of households provides differentiated labour services to a final labour supplier (an "employment agency"), which is assumed to be perfectly competitive. Households are therefore able to extract a rent from the sale of their labour services, and this will be based on a mark-up over the real marginal disutility of working. This implies that the labour market equilibrium is non-competitive. Such a mechanism allows us to introduce nominal wage rigidity in the same way as with firms. For simplicity, we make the assumption that there is a mapping between fluctuations in hours worked and unemployment (see Gali, 1996).

It is these rigidities that provide a link between nominal conditions and real activity. On the real side, we also impose capital and labour adjustment

[1] An implication of the representative agent paradigm in the open economy with free flow of capital is that consumption and asset accumulation are a random walk (see, for example, Barro and Sala-i-Martin, 1995, chapter 3). In a typical open-economy RBC model, the domestic economy produces the same good as the rest of the world and net trade is equivalent to exchanges of consumption bonds. The real exchange rate is then constant at unity. In our case, we want a model in which the current account and the real exchange rate are able to respond to shocks, but are stationary if the shocks are transitory. There are several methods for rendering consumption stationary; see Schmidt-Grohe and Uribe (2003) for a comparison.

[2] Note that, because the model is perfect foresight and simulated under an assumption of certainty equivalence, the term "risk premium" is a loose one.

costs. These are tangible quadratic costs that come out of the cashflow streams of firms and affect marginal returns, but are assumed to be zero in steady state.[3] Together, these nominal and real rigidities imply that monetary policy has an effect on real activity in the short run, and that the adjustment to the steady state is not instantaneous.

We add some more features designed to introduce richer propagation of shocks than would be standard. Inflation inertia is introduced by an indexation assumption, with a proportion of firms raising prices according to a rule of thumb that is based on lagged and steady-state inflation.[4] Persistence in consumption is introduced by the assumption of so-called external habits in utility.

The model is quite stark in its assumptions. There are no market frictions and no locational specificity. For example, there is no banking sector and no specific role for money and credit in the monetary transmission mechanism. There are no market frictions and distortions, no fixed costs or discontinuities. The model assumes a representative household and a symmetric equilibrium for firms. Above all, markets are assumed to clear at all times, as all agents have complete knowledge of the economy and complete understanding of shocks when they hit. In sum, the model contains assumptions that are almost guaranteed to be violated by the data, especially in the short run.

2.3 Decision Rules of Agents

There is a continuum of households that are differentiated by the labour they can provide. The maximization problem for an individual $j$can be expressed as follows:

$$\max \sum_{i=0}^{\infty} \beta^i \left\{ \begin{array}{c} \frac{1}{1-\frac{1}{\sigma^c}} \left(c_{t+i}\left(j\right) - \rho \cdot \bar{c}_{t+i}\right)^{1-\frac{1}{\sigma^c}} + \frac{\theta^M}{1-\frac{1}{\sigma^M}} \left(\frac{M_{t+i}(j)}{P^c_{t+i}}\right)^{1-\frac{1}{\sigma^M}} \\ +\frac{\theta^h}{1-\frac{1}{\sigma^h}} \left(1 - h_{t+i}\left(j\right)\right)^{1-\frac{1}{\sigma^h}} - \frac{\chi^W}{2} \left(\frac{W_{t+i}(j)/W_{t+i-1}(j)}{1+wdot_{t-1}} - 1\right)^2 h_{t+i} \end{array} \right\},$$

where c denotes real consumption, $\bar{c}$ is a reference habit level of consumption, M is nominal money, P^c is the consumption basket price level, h is hours worked, W is nominal wages, and nominal wage inflation is $wdot_t \equiv \frac{W_t}{W_{t-1}} -$

[3]This has the implication that there is a difference between gross output, $F\left(K, L\right)$, and net output $y = F\left(K, L\right) - \left(\tilde{\chi}^K + \tilde{\chi}^L\right)$, where $\tilde{\chi}$ represents the costs of capital or labour adjustment.

[4]See, for example, Smets and Wouters (2003).

1. β represents the household discount rate; σ^c, σ^M and σ^h represent the elasticities of inter-temporal substitution for consumption, money and hours, respectively; θ^M and θ^h are preference weights on money balances and hours, respectively; ρ is the weight on the habit level; and χ^W is the weight on costs of adjusting nominal wages.

This maximand is subject to a labour market clearing condition,

$$h_{t+i}(j) = \left[\frac{W_{t+i}(j)}{W_{t+i}}\right]^{-\eta^W} h_{t+i},$$

and a (real) period-by-period budget constraint,[5]

$$\begin{aligned}
& \frac{b^f_{t+i}(j)}{q_{t+i}} + b^g_{t+i}(j) + k_{t+i}(j) + p^c_{t+i}\left(\frac{M_{t+i}(j)}{P^c_{t+i}}\right) \\
= \quad & \left(1 + r^f + \frac{\phi}{2}\left(2b^{fTAR}_{t+i-1} - b^f_{t+i-1}(j)\right)\right)\frac{b^f_{t+i-1}(j)}{q_{t+i}} \\
& + \frac{1 + R_{t+i-1}}{1 + pdot_{t+1}} b^g_{t+i-1}(j) \\
& + \left(1 + r^k_{t+i-1} - \delta - \frac{(1 + rf)(rf + \delta)}{1 + \omega}\left((u_{t+i-1})^{1+\omega} - 1\right)\right) k_{t+i-1}(j) \\
& + \frac{p^c_{t+i-1}}{1 + pdot_{t+i}}\left(\frac{M_{t+i-1}(j)}{P^c_{t+i-1}}\right) \\
& + \frac{W_{t+i}(j)}{P_{t+i}} h_{t+i}(j) - p^c_{t+i} c_{t+i}(j) + \pi_{t+i} - \tau_{t+i}.
\end{aligned}$$

where b^f and b^g are the levels, respectively, of (net) foreign bonds and government bonds held by an individual; k is the capital stock; p^c is the relative price of the consumption bundle (to the domestic good); r^f is the real foreign interest rate; b^{fTAR} is an exogenous "target" level of foreign debt (assumed to be set by world capital markets); R is the nominal one-period interest rate; *pdot* is the inflation rate in the numeraire price of domestic goods: $pdot_t \equiv \frac{P_t}{P_{t-1}} - 1$; u is a rate of capital utilization; π is a lump-sum transfer of

[5]There is a cost to being away from a target level of foreign debt and this produces a linear term for a risk premium in the UIP condition. One wants a risk premium that is asymmetric in the deviations of the debt target from the level of debt and this is the simplest penalty function one could use.

supernormal profits; and τ is a lump-sum tax payment.[6] The real exchange rate is denoted by q; the convention here is that a rise in q is an appreciation.

The maximization problem describes the motivations of an individual agent who wants to maximize the benefits from consuming (non-durable) goods, holding real money balances, spending time at leisure, while minimizing the disutility from changing nominal wages. The individual has income from capital and labour rents, as well as a profit transfer, and pays for consumption and taxes. The individual can use foreign and domestic bonds to smooth consumption over time. The choice variables for individual householders are therefore consumption, $c(j)$; labour supply, $h(j)$; real money balances, $\frac{M_{t+i}(j)}{P^c_{t+i}}$; and portfolio demands for capital, $k(j)$, government bonds, $b^g(j)$, and foreign bonds, $b^f(j)$.

There is a continuum of firms that are differentiated by the goods they can provide. An individual firm j maximizes $U^K_t(j) = \sum_{i=0}^{\infty} \beta^i \varphi^K_{t+i} \xi_{t+i}$ where

$$\xi_{t+i} = \left\{ \begin{array}{c} P_{t+i}(j) \left[\frac{P_{t+i}(j)}{P_{t+i}}\right]^{-\eta} y^h_t + P^x_{t+i}(j) \left[\frac{P^x_{t+i}(j)}{P^x_{t+i}}\right]^{-\eta^x} x_{t+i} \\ -W_{t+i} \cdot h_{t+i}(j) \\ -\left(\begin{array}{c} \frac{1+R_{t+i-1}}{1+pdot_{t+i}} - 1 + \delta \\ +\frac{(1+rf)(rf+\delta)}{1+\omega}\left((u_{t+i-1}(j))^{1+\omega} - 1\right) \end{array} \right) \cdot P_{t+i} \cdot k_{t+i-1}(j) \\ -LM_{t+i}(j) \left[\begin{array}{c} \left[\frac{P_{t+i}(j)}{P_{t+i}}\right]^{-\eta} y^h_{t+i} + \left[\frac{P^x_{t+i}(j)}{P^x_{t+i}}\right]^{-\eta^x} x_{t+i} \\ -z^{1-\alpha}_{t+i}\left(u_{t+i}(j)\, k_{t+i-1}(j)\right)^{\alpha} h^{1-\alpha}_{t+i}(j) \\ +\frac{\chi^k}{2}\left(\frac{k_{t+i}(j)}{k_{t+i-1}(j)} - 1\right)^2 k_{t+i} \\ +\frac{\chi^h}{2}\left(\frac{h_{t+i}(j)}{h_{t+i-1}(j)} - 1\right)^2 h_{t+i} \end{array} \right] \\ -\frac{\chi^p}{2}\left(\frac{P_{t+i}(j)/P_{t+i-1}(j)}{\rho^P \cdot (1+pdot_{t+i-1}) + (1-\rho^P)\cdot(1+pdottar_{t+i-1})} - 1\right)^2 \cdot P_{t+i} \cdot y^h_{t+i} \\ -\frac{\chi^{px}}{2}\left(\frac{P^x_t(j)/P^x_{t-1}(j)}{\rho^P \cdot (1+pdot_{t+i-1}) + (1-\rho^P)\cdot(1+pdottar_{t+i-1})} - 1\right)^2 \cdot P^x_{t+i} \cdot x_{t+i} \end{array} \right\}.$$

Denoting domestic demand for output by $y^h_t = c^h_t + i_t + g_t$, where c^h denotes

[6] Note that the value of money-in-utility is in terms of its consumption-bundle purchasing power, so $mon_t = \frac{M_t}{P^c_t}$. In terms of a real-valued budget constraint denominated in units of the *domestic* good (i.e. where "real" money would be $\frac{M_t}{P_t}$), we need to reflate by the relative price to get $\frac{P^c_t}{P_t} \cdot \frac{M_t}{P^c_t}$. When detrending for inflation, lagged money in terms of domestic goods is correspondingly lagged notional stock in current nominal prices: $\frac{M_{t-1}}{P^c_t} = \frac{M_{t-1}}{P^c_{t-1}} \cdot \frac{P^c_{t-1}}{P_t} = \frac{M_{t-1}}{P^c_{t-1}} \cdot \frac{P_{t-1}}{P^c_t} \cdot \frac{P^c_{t-1}}{P_{t-1}} = \frac{M_{t-1}}{P^c_{t-1}} \cdot \frac{1}{1+pdot_t} \cdot p^c_{t-1}$.

home-produced consumption, i is capital investment, and g is government spending, the first set of terms in the maximand is a cashflow expression that expresses the profit of the firm as the rent for its good sold in domestic and foreign markets over factor payments represented by nominal wage payments and capital costs. This is subject to a market-clearing condition that demand meets final output, with a Lagrange multiplier denoted by LM. Here η and η^x are the elasticities of demand for domestically-sold and exported goods, respectively; δ is the depreciation rate on capital; ω is the order of the capacity adjustment costs; z is a factor productivity scalar; α is the exponent on capital intensity in production; χ^k, χ^h, $\chi^P$and χ^{Px} are weights on the adjustment costs for capital, hours, domestic prices and export prices, respectively; and ρ and $\rho^P$are the geometric weights on the lagged inflation rate and inflation target in the price indexation scheme. Each firm therefore makes choices about capital, $k(j)$, the capital utilization rate, $u(j)$, labour input, $h(j)$, its price in domestic markets, $P(j)$, and its price in foreign markets, $P^x(j)$.

Policy is assumed to follow simple instrument rules, given targets. There is a consolidated fiscal agency that redistributes resources. Government spending and debt are simply assumed to be maintained as targets to real output:

$$b_t^g = b_t^{gTAR} \cdot y_t$$
$$g_t = (1-\rho^g) \cdot \left(g_t^{TAR} \cdot y_t\right) + \rho^g \cdot g_{t-1},$$

(with the equation for g_t allowing for the possibility that government spending is persistent). Government expenditures include spending and servicing debt. Revenue includes new debt flotation, seigniorage, and taxation. Given money demand and debt targeting, the government budget constraint can be used to define the lump-sum household tax as the fiscal instrument:

$$\tau_t = g_t - b_t^g + \frac{1+R_{t-1}}{1+pdot_t} b_{t-1}^g - p_t^c \cdot mon_t + \frac{p_{t-1}^c}{1+pdot_t} \cdot mon_{t-1}.$$

The role of the monetary authority is to anchor the nominal side of the economy. We specify a simple monetary rule in which the nominal interest rate is moved away from its steady-state level in response to deviations of inflation from its target, with some smoothing on it:

$$R_t = \left(1-\rho^R\right) \cdot \left[R_t^{SS} + \mu_1 \cdot \left(pcdot_t - pcdot_t^{TAR}\right)\right] + \rho^R \cdot R_{t-1}.$$

It has been pointed out that the assumption of smoothing is not well founded in theory but it is widely used in policy-oriented CMs and we therefore retain it here. The inflation targeting rule might be replaced by a more conventional "Taylor rule", but then we would need to decide on exactly what variable should enter the relation - marginal costs, "output gap" etc. Although the model features a production function, and so can yield an output gap, in practice this is unlikely to be what policy makers would use. Bearing these issues in mind, and the emphasis upon inflation targeting in the U.K., it seemed simpler to specify a rule based on an observable.

The maximization problems for households and firms and the assumptions about policy formation, when supplemented with assumptions for export demand and price pass-through, lead to a core set of 26 "behavioral" relations, although identities mean that there are more than 26 variables in our C-model. The aggregate behavior of households is described by an aggregate consumption Euler equation, equations that disaggregate the consumption choice into demands for imported- and home-produced consumption, a demand for money equation, a labour supply equation, and the household budget constraint (which can be thought of as an equation for the demand for foreign bonds). The aggregate behavior of firms is described by a production function, a labour demand curve, a desired capital equation, a first order condition for capacity utilization, the mark-up equation, and a profit definition. Policy is summarized by the monetary reaction function, government spending path, government debt supply, and the government budget constraint (which can be thought of as a fiscal reaction function). Financial markets are effectively reduced to a UIP equation and an expression for the competitive rental rate on capital. Equations for export demand, relative prices, and a market clearing condition for domestic production complete the system.

2.4 Parameterization

The model is parameterized by imposing parameters to observe theoretical constraints and to loosely match some desired features of the data, mainly first and second moment conditions. The data are from the UK national accounts, at current prices. Each period is assumed to be a quarter, which is reflected in the parameterization of rates.

First, to achieve a steady-state equilibrium, the household discount rate $(0 < \beta < 1)$ has to be restricted to 0.99 to be the inverse of the gross foreign real rate, $1 + r^f = 1.01$. The elasticity of inter-temporal substitution for consumption ($0 \leq \sigma^c < \infty$) is imposed at 0.5, which is a "reasonable" value for macro models (and creates a role for opposing income and substitution effects). The elasticity of intertemporal substitution for hours ($0 \leq \sigma^h < \infty$), however, is imposed at 1 in order to be neutral to the real interest rate (see Burnside and Eichenbaum, 1996). The elasticity of intertemporal substitution for real money balances ($0 \leq \sigma^M < \infty$) is then imposed at 0.5, so that the income elasticity of demand for real money balances relative to consumption is the same. The preference weight on money ($0 \leq \theta^M < \infty$) is chosen at 0.0001 to achieve a ratio of real money balances to output close to the observed recent average of approximately 0.005. This implies a definition of money that is very narrow. Since the interest rate rule is the nominal anchor this choice is not an issue. The preference weight on leisure ($0 \leq \theta^h < \infty$) is set to 55 to achieve a value for hours worked close to the observed recent average of 0.24. The weight on imports in the Cobb-Douglas consumption aggregator ($0 < \psi < 1$) is imposed at 0.1 to achieve a good compromise for the ratio of aggregate imports to output.[7] Finally, the habit weight ($0 \leq \rho < 1$) is imposed at a value of 0.90, in line with recent empirical evidence.

We parameterize the exponent on capital in production ($0 \leq \alpha \leq 1$) to match observed averages for the return to capital from the national income accounts, and to calibrate a steady-state capital-output ratio of just over 2 (for an annual flow). The depreciation rate on capital ($0 < \delta \leq 1$) is chosen at a reasonable value of 0.025, which will also affect the capital:output ratio via the rental rate of capital. The order of utilizations adjustment costs ($0 \leq \omega < \infty$) is imposed at 0.4. This parameter has no effect on the steady state, but is a key parameter in determining the balance between output and inflation responses to demand shocks. Lowering ω will allow demand fluctuations to be met more by output than by changing prices.

Similarly, adjustment cost parameters ($0 < \chi^k, \chi^h, \chi^P, \chi^{Px}, \chi^W < \infty$) have no effect on the steady state but are chosen judgementally to match priors about the time taken to re-equilibrate following permanent shocks. Values of 2.5 are chosen for both capital and employment adjustment costs.

[7] Note that the theoretical structure of this model is not rich enough to account properly for observed quantities of imports – we should be adding structure for the demand for imports by firms and government, as well as a demand for imported intermediates. The parameterisation chosen is too much for the ratio c^m to c, but chosen so that net trade and current account flows are more realistic.

The nominal adjustment parameters χ^P,χ^{Px} and χ^W are used to parameterize the inflation responses to a demand shock and have values of, respectively, 10, 10 and 50. As in Christiano, Eichenbaum and Evans (2003), we find that nominal wage rigidity is important for matching priors about the responses to demand shocks).

Demand elasticities have important effects for both the steady state and dynamic properties. As the elasticities go to infinity, we approach perfect competition and zero returns to entrepreneurship. As they approach 1 (the minimum value) the mark-up increases. The elasticity of demand for domestic goods sold in domestic markets, η, is imposed at 5, which implies a 20 per cent profit per unit sold in domestic markets. Demand conditions for domestic goods sold in export markets are assumed to be more competitive, such that η^x is imposed at 20. The labour demand parameter, η^W, is imposed at 50 to achieve a value close to the observed recent average for the labour share of income of around 0.67.

The exponent in the export demand condition, ε, is chosen to be 0.9 in order to ensure that the Marshall-Lerner conditions hold. The weight on the "UIP premium" $(0 < \phi < \infty)$ is kept at quite a low value of 0.25 in order to keep the real exchange rate path reasonably close to the "pure" UIP path. This parameter also works in conjunction with ε to determine the volatility of current account flows, which also influenced the choice of the value. The value of the debt target, b^{fTAR}, is set at 0.01, which, given the import penetration values for domestic consumption, investment and government expenditures, and relative prices of exports and imports, helps us to achieve a small trade deficit in steady state, in line with recent observations.[8]

Fiscal targets are chosen to match typical ratios of (annual) debt and spending to output, at 0.4 and 0.2 respectively. The inflation target is 2.5 annually (though the model is superneutral, so this is not an important choice). The parameter governing smoothness in the government spending rule $(0 \leq \rho^g < 1)$ is arbitrarily set at 0.75. The equivalent parameter describing smoothness of the monetary reaction function $(0 \leq \rho^R < 1)$ is set at 0.65, which is consistent with estimates of instrument rules for the UK using recent data. Based on the same evidence, the weight on inflation $(0 < \mu^{pdot} < \infty)$ is set at 1.8.

[8]However, we pay a price in terms of the steady-state value for the consumption-income ratio. In short, there is just not enough saving in the economy to support a higher equilibrium level of consumption, and so the outcome is considerably lower than observed values.

The resulting steady state can be summarized (in annual units) as follows:

Table 1: Steady State Ratios for the C-Model	
aggregate consumption:output ratio	0.43
domestic good consumption:output ratio	0.53
imported good consumption:output ratio	0.06
investment:output ratio	0.20
government expenditure:output ratio	0.20
export:output ratio	0.07
trade balance:output ratio	-0.00012
capital:output ratio	2.00
government debt:output ratio	0.40
foreign bond:output ratio	0.01
labour share of income	0.66
hours	0.21
real interest rate	0.04
rental rate of capital	0.15
real money balances	0.0043
inflation rate	0.025
relative price of output	0.98
relative price of aggregate consumption	1.38
relative price of imported consumption	1.00
relative price of exports	0.84

3 The Econometric Structure of Conceptual Models

3.1 Dynamic Structure

Conceptual models derive very heavily from the Dynamic Stochastic General Equilibrium (DSGE) approach to modelling, albeit there are some differences that will be elaborated on later. For this reason the discussion of dynamics will be couched as if we were analyzing a relatively small DSGE model.

After linearization such models can be thought of as a set of equations with the stylized structure

$$y_t = BE_t(y_{t+1}) + Ay_{t-1} + Fu_t, \tag{1}$$

where u_t are the exogenous stochastic variables (shocks) that drive the system.[9] Some of these equations may be identities but the key ones are the Euler equations representing optimal choices in the face of uncertainty. All variables in y_t above should be thought of as being deviations from some *deterministic* steady state values - in the case of variables such as output these are mostly log deviations from a steady state path while, for variables such as interest rates and inflation, they are levels deviations from a constant steady state rate. The model is quantified by setting B, F to some values that have emerged as part of the exercise producing the model. Exactly how this is done varies a great deal from institution to institution, but the parameterization of the C-model in the previous section provides the flavour of the process.

The solution to this model has the general form - see Binder and Pesaran (1995)-

$$y_t = Py_{t-1} + D\sum_{j=0}^{\infty} S^j E_t u_{t+j} \tag{2}$$

where P satisfies $P - BP^2 - A = 0, D = (I - BP)^{-1}F$, and $S = (I - BP)^{-1}B$. It is clear then that the ultimate dynamic structure for y_t will come from two sources. One of these derives from the theoretical structure - that is the Euler equations and constraints - and is represented by P. The second source of dynamics stems from the nature of the u_t. To analyze the latter, we adopt the assumption, common to many DSGE models, that

$$u_t = \Phi u_{t-1} + \eta_t, \tag{3}$$

and so the solution for y_t will be

$$\begin{aligned} y_t &= Py_{t-1} + Gu_t \\ &= Py_{t-1} + G\Phi u_{t-1} + G\eta_t, \end{aligned}$$

where $G = D\sum_{j=0}^{\infty} S^j \Phi^j$.

Now, if the rank of G equals $\dim(u_t) \leq \dim(y_t)$ then $G^+ = (G'G)^{-1}G'$ is the generalized inverse of G, and so $u_t = G^+(y_t - Py_{t-1})$ and this relation can be used to get

$$\begin{aligned} y_t &= Py_{t-1} + G\Phi G^+(y_{t-1} - Py_{t-2}) + G\eta_t \\ &= (P + G\Phi G^+)y_{t-1} - G\Phi G^+ P y_{t-2} + G\eta_t. \end{aligned} \tag{4}$$

[9] In practice systems will have longer backward and, possibly, forward lags.

This expression makes clear that the *intrinsic dynamics* described by the theoretical model (represented by P), is augmented by *extrinsic dynamics* captured by Φ, with the consequence that the evolutionary process for y_t changes from a VAR(1) to a VAR(2). This raises the possibility that, by choosing a general enough serial correlation structure for u_t, it may be possible to reproduce the dynamic structure for y_t i.e. it might appear that there is a good match of model to data but it is due to the exogenous specification of the shock processes u_t rather than the economics, as represented by P. Consequently, it should always be the case that evidence is presented on which component is responsible for the match, so that any user of such a model fully understands how the match to evidence is being made. To date few DSGE models provide this information although an early analysis along these lines is Cogley and Nason (1993).

It is worth examining the solution above in more detail. By definition P must satisfy

$$P = (I - BP)^{-1}A$$

so that

$$rank(P) = \min\{rank(I - BP), rank(A)\}$$

Thus the $rank(A)$ will be a key determinant of the possibility of rank reduction in P. To be more precise, if a lagged value of one of the y_t does not appear in any of the equations, then A (and hence P) will have reduced rank. This seems a likely occurrence. For example, in standard open economy models there will be no lagged value of the exchange rate in the UIP condition. Therefore, unless there is delayed pass through of the exchange rate to inflation, or some lagged effects of exchange rates in the IS curve, it will not appear anywhere in the system. Thus, in that case, the rank of P must be one less than the number of variables in y_t. This may provide a simple explanation for the reduced rank in P noted by Nason and Rodgers (2005) for small open economy models. As we will see later, it is a characteristic of our C-model.

The reduced dynamic rank just discussed is different to that which has been commented upon when the number of shocks is less than $\dim(y_t)$, which we might call "covariance reduction". Such rank reduction occurs in (say) the basic RBC model, where there is a single technology shock- see Giannone et al.(2003). The rank reduction that we have just identified is closer to that seem in the common-trend/common-cycle representation of Vahid and Engle (1993) and the common features work of Engle and Kozicki (1993). If there is rank reduction in P then it can be written as $P = \gamma\delta'$, and there will exist $\gamma_{\perp}$ such that $\gamma'_{\perp}\gamma = 0$. Consequently, the combination $\gamma'_{\perp}y_t = \gamma'_{\perp}u_t$ will be white noise if u_t is white noise i.e. there would be a serial correlation common feature in that there exists a combination of the y_t which will be white noise. The contrast between dynamic rank reduction and covariance reduction is very clear if one had a basic RBC model in which there were at least as many shocks as variables in y_t. Then we would see rank reduction in P simply because such models feature only one lagged value, capital stock, (as the basic model abstracts from items such as habit persistence, time to build, adjustment costs etc.), but there would be no rank reduction coming from the number of shocks.

3.2 The "Long-run" Econometric Structure of CM Models

A special case of interest is when some of the eigenvalues of Φ are unity i.e. some of the shocks are permanent. It is possible to find the moving average representation for y_t from a model such as (1):

$$y_t = D(L)\eta_t, \tag{5}$$

where the elements D_j in the polynomial $D(L) = D_0 + D_1L + ...$ are the j period impulse responses. In CMs the D_j can generally be found analytically, while simulations of the larger models provide numerical solutions. There will be different values for the D_j depending on the value of Φ. We will describe the shocks as purely permanent or purely transitory when $\Phi = I$ and $\Phi = 0$ respectively.

When there are permanent shocks the value of D_∞ will indicate the long run responses. Any variable whose associated column in D_∞ has non-zero elements will be an $I(1)$ variable and the rank of D_∞ will indicate how many co-integrating vectors there are among the $I(1)$ variables. Rather than examining D_∞ it is sometimes simpler to work with the representation

$$\Delta y_t = C(L)\eta_t, \tag{6}$$

where

$$C(L) = C_0L + C_1L + ...,$$

since this is closer to that employed in the co-integration literature. It is then easily seen that $C(1) = D_\infty$ and the co-integrating vectors are the β

that set $\beta' C(1) = 0$. Of course the β are not unique and some identifying assumptions need to be placed upon them.

Now, let y_t be partitioned as $\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}$, where y_{1t} and y_{2t} are $I(1)$ and $I(0)$ variables respectively. Then, once the β have been decided upon, it is possible to construct the co-integrating errors $\psi_t = \beta' y_{1t}$ and to re-write the system as a Vector Error Correction Model (VECM) (assuming that it is reasonable to propose that y_t follows a VAR(1))[10]

$$\begin{bmatrix} \Delta y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \alpha' \\ \gamma' \end{bmatrix} \psi_{t-1} + M y_{2t-1} + v_t. \tag{7}$$

As is well known the common permanent components of y_{1t} may be constructed as $y_{1t}^P = J\tau_t$ where

$$\begin{aligned} J &= \beta_\perp (\alpha'_\perp \beta_\perp)^{-1}, & (8) \\ \tau_t &= \alpha_\perp \sum_{t=1}^{T} v_{t-j}, & (9) \end{aligned}$$

and $\alpha'_\perp \alpha = 0, \beta'_\perp \beta = 0$. Studying the ability of y_{1t}^P to track the data on y_{1t} will be important since the VECM forces any forecasts made with the CM to revert to y_{1t}^P, and so forecast errors are inevitable if this permanent component does not track the data properly -see Clements and Hendry (1999). Basically this is an issue of whether there is co-integration in the sample period and in the forecast period. Indeed it may well be that the operational model may need to change the parameters embodied in a CM, given views about what will happen over a forecast horizon.

There are a number of implications of the above point. First, changing the model parameters so as to produce values for α and β such that one gets co-integration (if it exists in the data), is an important part of any exercise, and it will not be sufficient to tie model parameters down by the matching of impulse responses, as has often been done e.g. by Christiano et al. (2003) and Murchison et al. (2004). An extra difficulty with this

[10] This two-step strategy is advocated by Garratt et al. (2003). A small CM is specified and some parts of the co-integrating vectors follow from the nature of that model, while the remaining elements are found by standard estimation methods with I(1) processes. Our C-model is a larger version of their CM but, as outlined earlier, it was quantified by a variety of calibration methods rather then purely econometric approaches.

latter modeling approach is that one needs to be careful that the SVAR used to generate the impulse responses that are to be matched does not use identification conditions that are incompatible with the CM. Thus SVARs identified by recursive (ordering) assumptions will rarely be appropriate as they would be incompatible with the simultaneity existing between exchange rates and interest rates. Secondly, it will generally not be a good idea to filter any data so as to remove the permanent component as this will not ensure that the re-constituted model will satisfactorily track the levels of the data, and it is these that will be important in the forecast. Consequently, the error correction term will be of an incorrect magnitude and this will lead to incorrect forecasts of Δy_t. We will illustrate this point later.

3.3 Differences between DSGE and CM

As is standard in this area, the equations of policy-oriented CMs are solved under the assumption of perfect foresight and simulated under the assumption of certainty equivalence. Examples of policy related CMs that adopt this approach are the QPS model at the Bank of Canada - Coletti et al (1996)- the FPS model at the Reserve Bank of New Zealand-Black et al (1997) - and the BEQM model at the Bank of England - Harrison et al. (2005). Cast in terms of the DSGE model we can see two cases that correspond to perfect foresight viz. when $\Phi = 0$ or I. In the first case agents know the current value of the shock and set it to zero in future periods. In the second, the value used for the shock remains the same in all periods. Because these types of shocks are very common in policy-oriented CMs, Pagan (2003) refers to the latter as incomplete DSGE (IDSGE) models. It is rare to see the familiar approach in DSGE modelling where time-series processes are assumed for the shocks and projections are effectively made of future values based on such assumptions e.g. Φ is set to an intermediate value between 0 and I, as in Smets and Wouters (2003).

There are both advantages and disadvantages to working with IDSGE models. The disadvantage is that one gives up some generality. The main advantage arises in a forecasting context where often paths for shocks are to be specified based on the priors of the policy makers, and these rarely fit a simple structure like (3). Moreover not having to specify a path for the shocks simplifies computation of optimal solutions a good deal and produces a neat separation between what the theory can provide in the way of dynamics and what is being imposed as an auxiliary assumption, something we will illustrate later.

3.4 Can a CM be represented as an SVAR?

If the series y_t follows a VECM (or VAR) then, if the CM is to match the data, the error terms in it, v_t, must have the form $C_0\eta_t$. The consequences of this equivalence are that $cov(v_t) = C_0 cov(\eta_t) C_0'$ and, once the CM model has been quantified by some method, C_0 is fixed and, therefore, it may not be possible to find a diagonal form for $cov(\eta_t)$ that replicates the $cov(v_t)$ that can be estimated from the residuals of a VECM (or VAR) fitted to data. If one wishes to make the $cov(\eta_t)$ diagonal it will be necessary to make some of the elements in C_0 free parameters. This contrasts with standard just-identified SVAR studies where the parameters of such models are estimated by *imposing* such a restriction. For most CM models such a restriction is not needed as it is the cross equation restrictions coming from the dynamic structure which should provide the requisite identifying information. If the shocks in the SVAR were in fact not uncorrelated then this mis-specification would bias the SVAR coefficient estimators -an example of such an effect being Giordani (2004). There is not much work done on this issue although Cho and Moreno (2004) look at the covariance matrix of shocks in a New Keynesian model and report that there is a correlation. Thus most DSGE models seem to assume it even though the restriction is not actually needed in estimation and can be tested. This issue represents a major problem in matching CM models to the data and one where there is no clear answer. *A priori* there seems no reason to think that shocks should be made uncorrelated, although it certainly helps in interpretations if they are.

A further problem in making a match is when the number of shocks in the CM exceeds the number of variables in y_t. If this situation arises one has to distinguish between the cases where there are observable and unobservable shocks in u_t. The former could be things like terms of trade, foreign output, tax rates etc., which are observable quantities, and so the shocks into them can be isolated, in contrast to unobservable items like demand shocks. One can absorb the variables associated with observable shocks into y_t and then the issue becomes whether the number of unobservable shocks exceeds the number of pre-augmentation variables in y_t. If it does then the final representation for y_t will not be a VAR but will most likely be a VARMA process (an exception would be when each of the unobservable shocks follows the same univariate autoregressive process). Thus fitting a VAR to data generated from such a CM would produce mis-specified estimates of impulse responses.

If the number of unobserved shocks is less than $\dim(y_t)$ then the implied VAR would have a singular covariance matrix for its errors, but this does not produce any mis-specification. In practice however it has sometimes been the case that a selection is made of y_{1t} from y_t such that $\dim(y_{1t}) = \dim(u_t)$ and then the VAR is fitted to y_{1t} and not y_t. Then, even if y_t followed a VAR, y_{1t} will not. The classic analyses are in Zellner and Palm (1974) and Wallis (1977), who showed that a VAR in y_t would generally become a VARMA process in y_{1t}. An exception to this is when the extra variables in y_t are related to those in y_{1t} via identities, as that would leave the order of the VAR unchanged. Some of the debate over the utility of SVARs in recent years e.g. Giovanni et al, (2002), Chari et al. (2004) are simply instances of the effects of variable reduction. In section 6 we look at this issue in the context of our C-model.[11]

3.5 Moving towards the DAM

Assuming that the CM cannot match the data as closely as desired one has to ask how it might be augmented so as to produce a better match. In many DSGE models a standard approach has been to relate the values of the CM model variables y_t^* to the data, y_t, with "errors in variables" or "tracking shocks" ε_t i.e. $y_t = y_t^* + \varepsilon_t$ - see Altug (1989) and Ireland (2004). Assuming that we can represent the model as a VAR (1), and there is no serial correlation in u_t, we can represent the situation as

$$y_t = y_t^* + \varepsilon_t \tag{10}$$

$$y_t^* = Py_{t-1}^* + Gu_t. \tag{11}$$

Then, as Ireland (2004) has noted, when it is assumed that $cov(\varepsilon_t, u_t) = 0$ this is a state space system and one can estimate the parameters underlying it using the innovations representation of the log likelihood due to Schweppe (1965). This representation exploits the Kalman filter. In Altug's version ε_t were white noise while Ireland allows them to follow a VAR process.

Suppose then that one makes the ε_t a VAR(1)

$$\varepsilon_t = \Psi\varepsilon_{t-1} + e_t, \tag{12}$$

[11] The biases that come from variable reduction were explored by one of the authors and John McDermott originally for the FPS model of the New Zealand economy. These results were reported in Pagan(2002).

where e_t is white noise. Then, it is easily seen that the two equations (10) and (12) imply

$$\Delta y_t = \Delta y_t^* + (I - \Psi)(y_{t-1}^* - y_{t-1}) + e_t. \tag{13}$$

If some "levels" adjustments needed to be made to the CM i.e. $y_t = \Gamma y_t^* + \varepsilon_t$, (13) would become

$$\Delta y_t = \Gamma \Delta y_t^* + (I - \Psi)(\Gamma y_{t-1}^* - y_{t-1}) + e_t, \tag{14}$$

which is a VECM linking y_t and y_t^*. Thus this mechanism provides an "average" reconciliation of the data and the CM outcomes y_t^*. How good the tracking performance is will depend upon the magnitude and nature of e_t. Of course y_t^* is unobservable but, since the equations constitute a state space form, it is possible to form estimates of y_t^* using the Kalman filter once model parameters and Ψ have been specified or estimated.

The analysis above suggests that, if y_t^* can be estimated by specifying some history for the model shocks, one might actually treat (14) as a way of moving from the CM to the DAM. Indeed, since (14) is a VECM, one might consider adding other non-model variables to it in order to reduce the size of e_t. That is effectively what was done in the Bank of England's core/non-core strategy for building and using the BEQM model described in Harrison et al. (2005). The data and the core model output (y_t^*) were essentially connected by an ECM, augmented with variables that had been difficult to incorporate into the core model but which were felt to be important to accurate tracking of the data. A necessary restriction in such an approach is that these variables needed to be added in such a way that, in steady state, they had a zero effect, so that $y_t = y_t^*$. Of course this latter restriction implies the necessity of checking that the core model is tracking the level of the data accurately, a theme we return to in the next section.

4 Some Analysis of the C-Model

It is important that one know the extent to which a CM can replicate data. There are two dimensions to this. One is to ask how well it tracks the longer-run movements in the data while the other relates to how it accounts for dynamic adjustments around this path. It is particularly important that the CM track the longer -run movements since the way in which these models have been used generally means that this is the path towards which the economy is seen to be adjusting. The implied long-run path can be determined relatively easily from the output of impulse responses to selected shocks. We give an example of this strategy in the next sub-section using our C-model as a representative of CMs. In contrast, succinct expressions for the dynamic adjustments around the path implied by the CM, and the extent to which outcomes are influenced by considerations about the future, are much harder to derive. For this reason we describe a method for generating data from the CM - termed the *pseudo data*- which can then be used to fit a variety of statistical and economic models, so as to provide information on such questions. Using these tools we provide an analysis of our C-Model. Our comparisons are with a U.K. data set since the C-model was calibrated to reflect beliefs about the U.K. economy.

4.1 The Match of the Long-run Path of the C-Model

The C-model can output many variables. In looking at the long-run path we need to focus upon the variables that have permanent components i.e. are $I(1)$. This decision is determined by the nature of the data. In this section we look at four variables - the log of domestic output (y_t), the log of foreign output (y_t^f), the log of investment (i_t) and the log of the real exchange rate (e_t). All variables were detrended using a first order polynomial in time. The resulting series seem to be clearly $I(1)$ when constructed from U.K. data over 1977/2-2003/2. Some care had to be taken in constructing the data so as to match model quantities. The most difficult correspondence to effect related to what we have called domestic output, since the CM we are using has been simplified a good deal from what it would be like in a policy context In particular, it does not include inventories, housing investment, other investment, and imputed rents. These are therefore removed from the UK data on GDP.

Using data on the four variables above there is mixed evidence on the number of co-integrating vectors. With a VAR(1) there is strong evidence of two co-integrating vectors but, with a VAR(2) (and higher), Johansen's max test suggests there is a single vector, while the trace test indicates either two or (less clearly) three. To illustrate the methods it was decided to assume that there are two such vectors. Once this decision was made, the two permanent shocks implied by the system needed to be named, and we took these to be foreign demand and TFP. Thus one of the shocks is potentially observable but the other isn't. The two-step procedure just described needs to be applied to all CMs. In the first step the number of permanent shocks needs to be determined from some data analysis. The second step involves a choice of the types of permanent shocks and this decision should reflect what policy makers and their advisers feel have been the main factors with a lasting impact over the historical data period.

To derive the cointegrating vectors we need the matrix $C(1)$ in (6) (D_0 in (5)). As mentioned above two permanent shocks were imposed on the C-model. These were a permanent 1% rise in foreign demand and the level of TFP respectively. Table 2 gives the values of $C(1)$ coming from these two shocks.

Table 2 $C(1)$ for Various Variables for the Permanent TFP and Foreign Demand Shocks

	TFP	For Demand
Dom Out (y)	.8406	-.0198
Log Real ER (r)	-.9243	1.3167
For Demand (y^f)	0	1
Investment(i)	.8443	-.0205

The two permanent components among the four variables imply that there must be two co-integrating vectors. To find the 4×2 vector β such that $\beta' C(1) = 0$ we need identifying information. We decided to exclude investment from the first vector, and foreign demand from the second, so that the co-integrating relations were found to be (to two decimal places)

$$y = -.91r + y^f \tag{15}$$
$$i = y \tag{16}$$

It is interesting to look into whether the co-integrating vectors found for the C-model agree with those found by applying an estimator to data over 1977/1-2003/2. Using the same identification scheme and Johansen's estimator one would get[12]

$$y = -.03r + 1.53y^f \tag{17}$$
$$i = 4.28y + .51r \tag{18}$$

[12]The calculations were done with Microfit 5.

Clearly there are substantial differences between the two sets of vectors. Qualitatively those of the C-model seem more reasonable. Indeed, one feels that most analysts might well be happier with those from the C-model. This emphasizes the fact that, simply relying upon data to re-produce long-run relations, rather than adopting a theoretical perspective, may not yield outcomes that are attractive. Nevertheless, such comparisons are often very useful as they do direct attention to possible difficulties in the long-run match to data of a calibrated CM. This may well be the case for the C-model, where it would seem that the exchange rate effects on output implied by it might be too strong. Consequently, the parameters of the C-model might be manipulated to improve on this aspect.

Another view of the correspondence of the C-model with the "levels" of the data is found by constructing the implied permanent components. Given the values for β in (15) and (16), ECM terms were formed and used in a VECM(2) fitted to the data on the four variables. These provide estimates of the matrix α in (7). β and α are then used to construct J and $\alpha_{\perp}$ in (8) and the permanent components of each of the series can be extracted using (9). Figure 1 then plots the permanent components of domestic output implied by the C-model. Also present are the same quantities formed from the Johansen parameter estimates in (17) and (18), along with the data. It is clear that the ability of the C-Model to track the levels of the data suffered a good deal after the beginning of 1997. This is a period in which a very strong appreciation in the real exchange rate occurred. The C-Model would have predicted a strong decline in the level of output but this did not occur. Because the Johansen estimates have a very weak exchange rate effect it does not have the same feature. However, it tends to err in the other direction, over-estimating the level of output. It is also noticeable that the Johansen estimates produced much poorer tracking ability in the 1980s. In general this experiment suggests that the impact of the exchange rate needs to be closely investigated as it is likely that this can make the level of forecasts go significantly off track.

4.2 Generating the Pseudo Data for CMs

We have observed a number of times that it will be important to be able to generate pseudo-data from CMs. In doing this we have to recognize some constraints that govern the method to be used. First, for most purposes it is

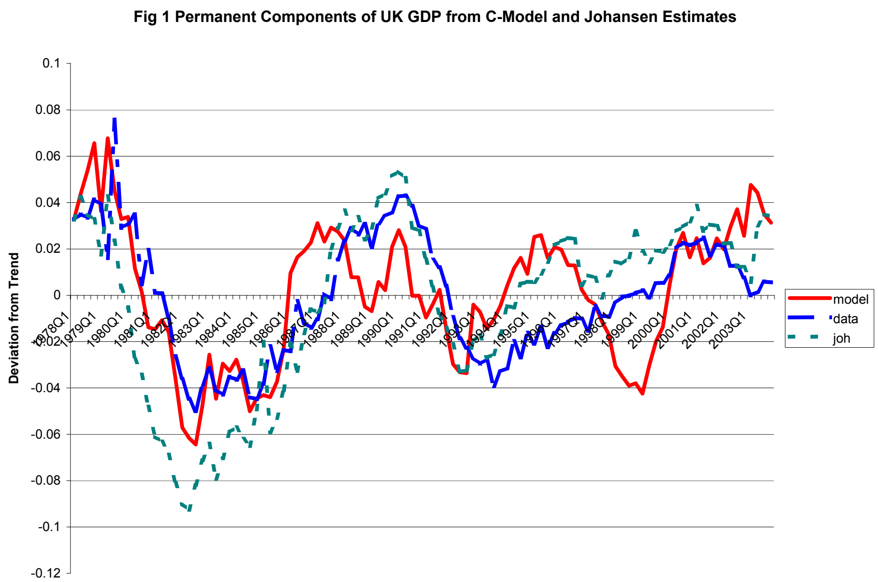

Figure 1:

not necessary to find the responses of all variables to a given set of shocks. It is likely that a relatively small set of variables will be those of most interest. Second, most CMs are known from the properties of their impulse responses rather than from any statistical model such as a VAR i.e. the D_j are what is most easily obtained and analyzed. For this reason we find pseudo-data not by generating data from a VAR, as most of the DSGE literature does, but from the impulse responses themselves. The method seems to work quite well and is essentially a version of that in Masson (1988), where the exogenous variables are the shocks being applied.

The method proceeds by first extracting the impulse responses corresponding to the variables of interest, y_t (a much smaller set of variables than are in the CM), and then generating data on y_t using

$$y_t = D(L)\eta_t, \tag{19}$$

where η_t are $i.i.d.(0, V)$. It is necessary to determine a suitable V. The central problem here is that, if one is interested in whether the CM can generate the correct dynamics, one doesn't want to use information that is based on dynamics when determining a value for V. Thus, if one chose V to replicate the variance of y_t (or Δy_t if y_t was $I(1)$), then there would be an implicit comparison of the model value of $D(L)$ with that of the data, since the latter shows up in the sample variance of y_t. For this reason it seems best to vary the method of selection of V depending on what we are trying to discover about the CM. Thus, suppose we wanted to see if a VAR fitted to the data matched the VAR implied by the CM. Let the estimated covariance matrix from the VAR errors (when fitted to data on y_t) be $\hat{\Omega}$. Then we could choose V as the solution to

$$\hat{\Omega} = D_0 V D_0',$$

where D_0 is a known quantity from the CM. This means that we are treating the CM as having an SVAR model representation whose implied VAR shocks have a covariance matrix equal to that from the data. This separates out the issue of how closely the dynamics can be replicated from the identification of the shocks. We use this same technique in a different context in section 6. It is important to note that there is no guarantee that V will be diagonal and so the shocks may not be uncorrelated.

4.3 The Match of the Univariate Properties of Variables in the C-Model

We use the method just described to investigate a number of issues that arise in policy-oriented macroeconomic modeling. Our first experiment involves simulating data on four variables from the $C-$model - domestic output, a real exchange rate, inflation and a real interest rate .[13] There are five shocks corresponding to these variables- to foreign demand, to total domestic factor productivity (TFP), to government expenditure, to the inflation target and to the sovereign risk premium. The foreign debt constraint was raised by .001 to produce a risk premium shock, the inflation target was increased to 3.5% p.a. and all other items were raised by 1%. In the first experiment all shocks are pure impulse. In the second, TFP and foreign demand shocks are both permanent.

In order to complete the experiment we need to select the covariance matrix of the shocks. In the case of purely transitory shocks we fitted a

[13] There are actually five variables but because the foreign demand process is exogenous it is simulated separately from the C-model.

$VAR(2)$ to data on the five variables and then found a value of V for the five shocks that matched the covariance matrix of the $VAR(2)$ errors. V was far from diagonal, as seen from the fact that the correlations of TFP shocks with government expenditure (-.95), the inflation target (.73) and the risk premium (.76). There were also some significant correlations with the foreign shock, although none above .5, and this would imply an equivalent rank reduction in V. Some evidence of a rank reduction in Ω was available as the minimum eigenvalue of the correlation matrix formed from Ω was .05. Once V was determined, pseudo-data was generated from the C-model with it. Finally, to encapsulate the differences between the data and the C-model we fit $AR(2)$ models to both actual and pseudo data for the four variables of interest. Table 3 shows these AR(2) coefficients; they are designated as γ_1 and γ_2.

Table 3 AR(2) models fitted to Data
and Pseudo-Data with Transitory Shocks

	Data		C-Model	
	γ_1	γ_2	γ_1	γ_2
Dom Out (ly)	.58	.36	.77	-.35
Log Real ER (lrer)	1.13	-.24	.71	-.25
Inflation (inf)	.40	.36	.86	.05
Real Int Rate (rr)	.29	.23	.38	-.30

It is clear that, while the C-model generates substantial persistence in output and real exchange rate movements, it falls short of the near-I(1) processes that are a feature of the data on these variables. In terms of inflation and the real interest rate it shows a pattern that is very different from the data. Tests that the two sets of parameters are the same lead to strong rejections, with $\chi^2(2)$ tests of over 25.

To generate more persistence we assume that there are two permanent shocks. Now a VECM(2) rather than a VAR(2) is fitted to the data. Since there are three $I(1)$ variables and two permanent shocks the first of the co-integrating variables given in (15) produces the requisite error correction term. The covariance matrix for the shocks V is then derived from the VECM errors in the same way as before. Once again these are correlated, although more weakly than before.

Table 4 presents the same information as in Table 3, except that, because there is now a unit root in output and the real exchange rate, the AR(2) is fitted to the first differences of those series. It is clear that the model still fails to reproduce the univariate data characteristics, although the test statistics indicating a formal rejection of the coefficients being the same are much lower than they were when shocks were purely transitory. In summary, it is clear that, to produce a match to the data, it will be necessary to allow the shocks into the C-model to be more than just purely transitory or purely permanent.

Table 4 AR(2) models fitted to Data
and Pseudo-Data with Permanent Shocks

	Data		C-Model	
	γ_1	γ_2	γ_1	γ_2
Log Dom Out (ly)	-.37	.07	.09	.03
Log Real ER (lrer)	.22	-.10	-.16	-.08
Inflation (inf)	.49	.39	.45	-.08
Real Int Rate (rr)	.31	.24	.78	-.01

4.4 Some Other Features and Uses of the C-Model

4.4.1 Dynamic Rank Reduction

We noted earlier that one might expect some dynamic rank reduction in CMs. We investigated this in the context of the C-model in the following way. Throughout this analysis we are concentrating upon the case where there are just transitory shocks

If there is dynamic rank reduction then a VAR(p) in n variables can be represented as a reduced rank VAR (RVAR) model of the form, see Velu et al. (1986)

$$y_t = \gamma[\sum_{i=1}^{p} \delta_i' y_{t-i}] + v_t \tag{20}$$

$$= \gamma\delta' z_t + v_t, \tag{21}$$

where γ and δ_i, $i = 1, ..., p$, are (n, r^*) matrices, $r^* \leq n$, $\delta = \begin{pmatrix} \delta_1' & . & . & \delta_p' \end{pmatrix}'$ and $z_t = \begin{pmatrix} y_{t-1}' & . & . & y_{t-p}' \end{pmatrix}'$, $t = 1, ..., T$. In (21) it is assumed that the true rank of the matrices γ and δ are identical and equal to r^*, which is thus referred to as the rank of the system (21). However, note that the ranks of δ_i,

$i = 1, ..., p$, need not equal r^*; in particular, $rank(\delta_i) \leq r^*$, $i = 1, ..., p$. Note that the number of free parameters in this model is equal to $r^*(n + np - r)$ as opposed to n^2p for a standard VAR model.

Given the system rank r^*, Velu et al. suggested an estimation method for the parameters γ and δ which may be shown to be quasi-maximum likelihood. Denote the sample second moment matrices by $S_{YY} = T^{-1}Y'Y$, $S_{YZ} = T^{-1}YZ'$, $S'_{ZY} = S_{YZ}$ and $S_{ZZ} = T^{-1}ZZ'$, where Y and Z are matrices containing $\{y_t\}$ and $\{z_t\}$. Hence, the covariance matrix of the unrestricted OLS residuals, $S_{YY.Z} = S_{YY} - S_{YZ}S_{ZZ}^{-1}S_{ZY}$, is the unrestricted quasi-ML estimator of the error process variance matrix $\Sigma_{\mathbf{vv}}$. Additionally, let $\{\hat{\lambda}_i^2\}_{i=1}^n$, $\hat{\lambda}_1^2 \geq ... \geq \hat{\lambda}_n^2 \geq 0$, denote the ordered squared eigenvalues of the $n \times n$ matrix $S_{YY.Z}^{-1/2}S_{YZ}S_{ZZ}^{-1}S_{ZY}S_{YY.Z}^{-1/2}$ with associated eigenvectors $\{\hat{\boldsymbol{\nu}}_i\}_{i=1}^n$ subject to the normalization $\hat{\nu}_i'\hat{\nu}_j = 1$if $i = j$ and 0 otherwise. Therefore, the quasi-ML estimators for γ and δ in (21), with system rank r^* minimize

$$tr\{S_{YY.Z}^{-1/2}(Y - \gamma\delta'Z)(Y - \gamma\delta'X)'S_{YY.Z}^{-1/2}\},$$

and are given by

$$\widehat{\gamma} = S_{YY.Z}^{1/2}\widehat{V}, \qquad \hat{\delta} = S_{ZZ}^{-1}S_{ZY}S_{YY.Z}^{-1/2}\widehat{V}$$

where $\widehat{V} = (\hat{\nu}_1, ..., \hat{\nu}_{r^*})$. Here $\hat{\gamma}$ and $\hat{\delta} = (\hat{\delta}_1', ..., \hat{\delta}_{r^*}')'$ satisfy the induced normalization $\hat{\gamma}'S_{YY.Z}^{-1}\hat{\gamma} = I_{r^*}$ and $\hat{\delta}_i'S_{ZZ}^{-1}\hat{\delta}_j = \hat{\lambda}_i^2$ if $i = j$, and 0 otherwise - see Velu et al. (1986) for more details.

In order to determine the rank of the system we use a standard test of rank. The test of rank of a matrix suggested by Bartlett (1947) which examines whether the $n - r^*$ smallest canonical correlations ρ_i, $i = r^* + 1, \ldots, n$, between $\{y_t\}$ and $\{z_t\}$ are zero *i.e.* it looks at the matrix $\Upsilon = S_{YZ}S_{ZZ}^{-1}$. Under $H_{r^*} : rk(\Upsilon) = r^*$, $\rho_{r^*+1} = \ldots = \rho_n = 0$. The test statistic of Bartlett(1947) is given by $T\sum_{i=r^*+1}^n \ln(1+\hat{\lambda}_i^2)$, where $\hat{\lambda}_i = \hat{\rho}_i/(1-\hat{\rho}_i^2)^{1/2}$, and $1 \geq \hat{\rho}_1^2 \geq ... \geq \hat{\rho}_n^2 \geq 0$ are the ordered squared sample canonical correlations between $\{y_t\}$ and $\{z_t\}$, which is identical to the (log-) likelihood ratio (LR) test for $H_{r^*} : rank(\Upsilon) = r^*$ against $H_{r^*}' : rank(\Upsilon) > r^*$; see Reinsel and Velu (1998, Section 2.6). Under H_{r^*}, $T\sum_{i=r^*+1}^n \ln(1+\hat{\lambda}_i^2)$ has a limiting chi-square distribution with $(n - r^*)(np - r^*)$degrees of freedom. Note that the test is obtained *via* a singular value decomposition of the unrestricted or full rank LS (quasi-ML) estimator $\widehat{\Upsilon} = S_{YZ}S_{ZZ}^{-1}$.

Applying these tests to a VAR fitted to the pseudo-data from the C-model with purely transitory shocks and a VAR order chosen by AIC - roughly a VAR(7) - one finds that there is evidence of rank reduction to an RVAR(3). Thus it seems very likely that many CM models will feature dynamic rank reduction. Note that the rank reduction here is not that due to co-integration as all shocks are transitory and so the variables are not I(1). When there is dynamic rank reduction one can think of y_t as being driven by factors $f_i = \delta_i' y_{t-i}$, although these factors would be correlated and so are different to standard factor models. It has often been found that the co-integrating restrictions can improve forecasts over a longer forecast horizon and the reduced dynamic rank indicated here may also have the same effect but at shorter horizons. Indeed, Vahid and Issler (2002) found that there were significant gains in forecast accuracy when the rank restrictions were imposed, particularly over short-term horizons.

4.4.2 Forward Looking Aspects of the C-Model

The pseudo-data can be used for many purposes e.g. to fit a DSGE model of some sort that is of much smaller dimension than a policy-oriented CM is. Sometimes this is useful for interpreting the CM in terms of results that come from miniature models that have been extensively studied. As an illustration we fitted a New Keynesian Phillips curve of the form

$$\Delta\pi_t = \phi\pi_{t+1}^e + (1-\phi)\pi_{t-1} + a_1 y_t + a_2 y_{t-1}$$

to the pseudo-data from the C-model obtained in the permanent shock case earlier, where π is inflation, π_{t+1}^e is expected inflation and y_t is an output gap. We used as instruments for π_{t+1} and y_t, the variables π_{t-1}, y_t^f, y_{t-1} and y_{t-2}. Two interesting facts emerged from this exercise. One was that ϕ was estimated to be .52 i.e. the C-model seems balanced between the present and future in terms of influences on inflation, and the second was the very strong evidence that $a_2 = -a_1$ i.e. inflation was affected by the growth in output rather than the output gap per se. The value of $a_1$was .17.

5 Towards the DAM: Matching via Shocks

To progress towards the DAM requires that one determine what information is available to the modeler. If it is possible to solve the CM then it makes sense to proceed in the way done by the Bank of England in its core/non-core distinction which was discussed earlier. But this does require that one can measure the shocks that enter the CM and it seems likely that not all of these will be observable. For this reason we describe an approach that estimates these shocks and then defines tracking errors that need to be added on to the CM to replicate the data. The key to our analysis is the recognition that there are inevitably more variables in the CM than there are shocks to it.

Let the CM have the solved form

$$y_t = G(L)x_t + H(L)e_t \tag{22}$$

where now we have separated out the observable (x_t) from unobservable (e_t) shocks. An example of an observable shock might be foreign demand. We get $G(L)$and $H(L)$ by computing impulse responses to shocks.

Observable shocks have the property that the process driving them can be determined from the data so that these cannot be manipulated to produce a better match of the base model with the data. However, the unobservable model shocks might be. If we write the observations upon the variables y_t as z_t, and let $H(L) = H_0 + H_1L + ...$, one might then reverse-engineer these shocks by estimating them as

$$\hat{e}_t = H_0^+[z_t - G(L)x_t - (H_1L + ...)\hat{e}_t]$$

where $H_0^+ = (H_0'H_0)^{-1}H_0$ and we are recognizing here that $\dim(e_t)$ will generally be much smaller than $\dim(y_t)$. This is a recursive relation giving

$$\begin{aligned}
\hat{e}_1 &= H_0^+[z_1 - G_0x_1] \\
\hat{e}_2 &= H_0^+[z_2 - G_0x_2 - G_1x_1 - H_1\hat{e}_1].
\end{aligned}$$

etc.[14] If $\dim(e_t) = \dim(y_t)$ this could enable the CM to produce a perfect match with the data, although the nature of the estimated shock processes might be unacceptable to policy makers e.g. these might have very complex serial correlation patterns that would be hard to motivate. When $\dim(e_t) < \dim(y_t)$ we will not be able to reproduce y_t exactly as there are not enough shocks. To achieve an exact re-production we define *tracking* residuals as

$$\hat{\varepsilon}_t = z_t - B(L)x_t - C(L)\hat{e}_t.$$

[14] Note that we need to assign a value to e_0. This is the same probelm as arises with the use of the Kalman filter to extract estimates of the shocks in DSGE models. Normally the Kalman filter initiates the recursion with the expected value of the shock, and that will be zero in this context.

Once these have been determined both the model shocks and the tracking residuals can be examined e.g. by looking for serial correlation in them, relating them to observed variables etc. We note that, by construction, the rank of the matrix (E, Ξ), where $E = (e_1, e_2, ..., e_T)'$ and $\Xi = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_T)'$ is n, where n is the dimension of y_t. As E is of full column rank, Ξ has reduced column rank. Hence the number of independent tracking shocks is $m = n - \dim(e_t)$.

We now present some results of our analysis of the nature of the shocks - both model and tracking - that would be needed for the C-model to reproduce the data. Data on four variables - log of output, log of the real exchange rate, inflation and the real interest rate - constituted z_t. For illustrative purposes the model shocks were all assumed unobservable i.e. the foreign demand shock was treated as unknown. We then assumed that the model was only driven by two shocks - the foreign one and TFP. The recursion above then determines what the history of these shocks would be, after which four tracking shocks were added on to replicate the four variables (of course there are only two *independent* tracking shocks since $rank(\Xi) = 2$).

Table 5 AR(2) models fitted to Implied Model and Tracking Shocks

	γ_1	γ_2
TFP	-.26	.78
Foreign Demand	-.54	.46
1st Tracking Shock	-.18	.80
2nd Tracking Shock	-.17	.66

There is a reasonable degree of persistence in all shocks but well away from a unit root. However both the TFP and foreign demand shocks seem to exhibit somewhat peculiar behavior and have much less persistence than one might expect. Effectively, this is extra evidence of the inadequate performance of the C-model.

6 An Alternative Modelling Strategy?

A question that naturally arises is why the CMs used in most policy related work are much larger than those typically used in academic research. One possible explanation is that one needs models of such a size in order to adequately capture the responses found in actual economies. That this may be an issue has already been alluded to. Academic modelers tend to work with a relatively small number of variables whereas most economies have a much larger number. A standard academic approach is to choose a relatively small number of variables, z_t, to estimate a VAR, and then make a choice about the order of the VAR using statistical criteria such as AIC and BIC. Resistance to a large number of variables in the VAR comes from the fact that, if there are n variables in z_t, there are $p \times n$ lagged values on the RHS of each equation of a VAR(1). But the problem with this way of thinking is in keeping p fixed as one increases n. For a given degree of approximation to the true impulse responses, it may pay to work with a relatively large number of variables in z_t, as one can thereby keep the VAR order relatively small. Choice of variables in VARs has been given little attention compared to choice of lag length, but is at least as important. The consequence of this variable reduction will generally be to make it very hard to capture the impulse responses to shocks with smaller models, even when very flexible dynamics are assumed.

To illustrate this point *we take the C-Model as the DGP* and simulate data from it[15]. We then ask if we can recover the impulse responses to its shocks if one proceeded in the standard ways used in academic modelling. Since the shocks we apply to the C-model are "structural" it is necessary to convert a VAR into an SVAR using identifying information. In practice this is not an easy task, and it may well be that the restrictions imposed to do this are incompatible with a CM. But we wish to abstract from this difficulty in order to concentrate upon the dynamic approximation problem. Consequently we assume that the investigator knows the impact impulse response matrix D_0 and so can recover the true shocks η_t and, thereby, the implied impulse responses of y_t to η_t from an estimated VAR.[16] Let the SVAR impulse polynomial be $D^{\eta,p}(L)$ for a fitted p'th order VAR. Then it is of interest to ask about the relation between $D^{\eta,p}(L)$ and the true impulse responses constructed from the $D(L)$ of the $C-$model as p is varied. Since $D_j^{v,p} = D_0^{v,p} \cdot f(A_1, ..., A_p)$, where A_j are the VAR coefficients, once one knows D_0 any incorrect impulses implied by the SVAR are due to the inability to estimate the A_j accurately, and this will simply reflect the mis-specification

[15] The fact that the C-model does not produce a good match to U.K. data is therefore irrelevant to the comparisons of this section.

[16] An alternative which we also look at involved estimating D_0 by regressing the VAR shocks against the known C-model shocks. The estimates of D_0 found in this way were quite close to the true value and so the impulse responses were much the same.

due to the reduction of the C-model in N variables to a VAR in n variables.

We start with the question of what order VAR would be chosen using standard statistical model selection criteria if we had 200 observations, which seems an upper limit to the amount of information available with quarterly macroeconomic data. To get these 200 pseudo-observations we generate 500 samples of 10200 observations and then drop the first 10000 in order to remove the effect of initial conditions, which were set to zero. We consider a set of five variables in the VAR: domestic output, the real exchange rate, inflation, the real interest rate and foreign demand, while the shocks are the five transitory shocks mentioned earlier. Information criteria are used to select lag orders for the VAR model. Results are as expected, in that AIC chooses a relatively high average lag order of 7 for the VAR, while BIC chooses a much lower order of around 4. Since the C-model features dynamic rank reduction it is instructive to note Vahid and Issler's (2002) conclusion from Monte Carlo experiments that, in such instances, there is a downward bias in the order selected by AIC.[17] We therefore set p to either 4 or 7.

Now we could fit the VAR to the 200 pseudo-data points and compare the impulse responses to the true values. We do such an exercise for $p = 4, 7$ calling these the VAR4/200 and VAR7/200 cases. But this confounds possible approximation errors of the VAR with a small sample size, and it is useful to separate out the two effects. Accordingly, we also generate 500 samples of 40000 observations, drop the first 10000 in order to remove the effect of initial conditions, and then fit the VAR for the same values of p as before. These results then would just show the approximation error and are labelled $VAR4/30K$ and $VAR7/30K$.

For productivity, monetary policy and foreign demand shocks, there is a good correspondence between the VAR implied impulse responses and the true ones from the C-model. This is not true however of the risk premium and the aggregate demand (fiscal) shocks. In those cases a very high order VAR may be needed to reproduce the impulse responses of the C-model. Figures 2 and 3 show the comparison of the impulse responses for the fiscal policy shock from the C-model. It is clear that the VAR selected with AIC ($p = 4$) and just 200 data points can be a long way from the true values. This is true even with 30000 observations. A VAR(7) does better, but is still inaccurate, even

[17] We have also looked at the ability of VARMA models to reproduce the impulses but these did not seem any more successful than a VAR and so the results are not reported here.

with 30000 observations. It should be said that, using a VAR(50) with 30000 observations does produce estimated impulse responses that are essentially indistinguishable from the true values, so that there are no "fundamentals" problems of the sort identified by Lippi and Reichlin (1994). It should be stressed once again that the close fit for the early lags is simply an artifact of the fact that C_0 is assumed to be known in all cases, and the difficult problem of how one is to identify the shocks has been sidestepped in order to focus on approximation issues. In practice, C_0 would have to be estimated as well, and it is not easy to estimate this response given that these models will rarely be recursive. Imposing long-run restrictions generally results in many weak instrument problems and there can be substantial biases in the estimation of the elements in C_0 - see Pagan and Robertson (1998), *inter alia*.

There seem to be three lessons one can learn from the experiment. First, the order of a VAR needed to reproduce correct impulse responses for actual economies is likely to be far higher than has ever been suggested in practice and quite infeasible given the sample sizes in macroeconomics. Second, using a VAR with order chosen by some statistical criterion is unlikely to produce good estimates of the items of ultimate interest viz. the impulse responses. Finally, it would seem better to begin with a CM and then attempt to modify it rather than to try to recover responses by using very little prior information. Even a relatively simple CM produces a complex set of inter-relationships in an economy and these are hard to capture when little prior information is used.

7 Conclusion

We have discussed the construction of modern policy-oriented macro-economic models and their relation to evidence. These have a strong theoretic base and so the issue that arises is one of how to match them up with evidence. Our paper has looked at some ways in which econometric methods have been, and are being, used for that task. We illustrate these methods by constructing a theoretical model that is representative of the type of model increasingly being used in policy analysis. This model was also used to determine if standard academic approaches to recovering information on shocks via SVARs could be an effective substitute. We find that it is very hard to recover such impulses using the types of VARs that have been popular for some time.

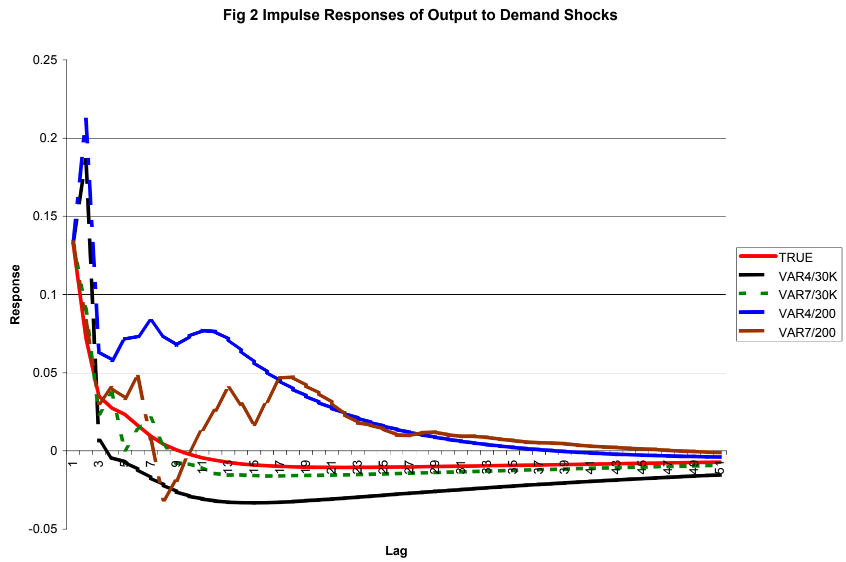

Figure 2:

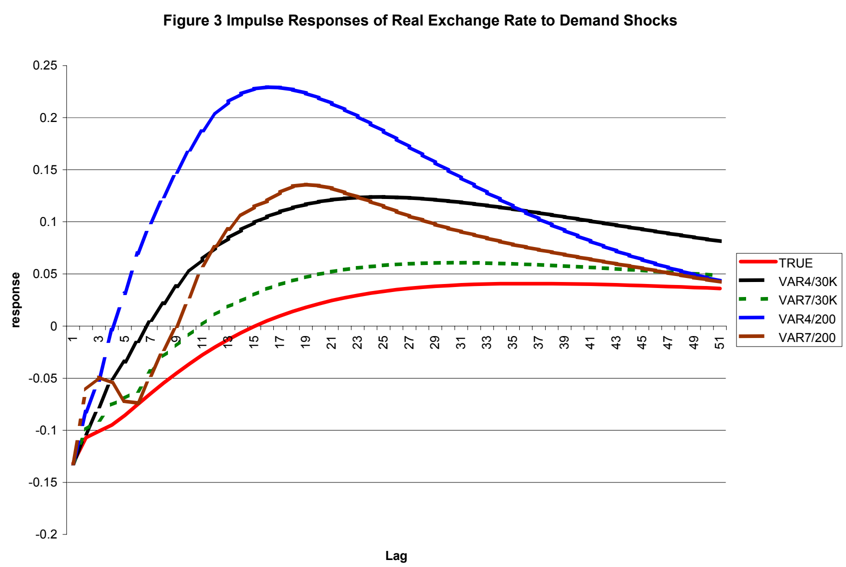

Figure 3:

This does not deny a role for miniature models in applied macroeconomics. Indeed, we have argued that one can learn about some of the characteristics of larger models by examining data produced by it though the lens of these smaller models.

8 References

Altug, S. (1989), "Time to Build and Aggregate Fluctuations: Some New Evidence" , *International Economic Review*, 30, 889-920.

Binder,M and M.H.. Pesaran (1995), "Multivariate Rational Expectations Models and Macroeconomic Modelling: A Review and Some New Result" in M.H. Pesaran and M Wickens (eds) *Handbook of Applied Econometrics: Macroeconomics,* Basil Blackwell, Oxford.

Barro, R. J. and X. Sala-i-Martin (1995), *Economic Growth,* New York: McGraw-Hill Inc.

Bartlett, M. S. (1947), "Multivariate Analysis" , *Journal of the Royal Statistical Society, Series B*, Vol. 9, pp. 176-197.

Black, R., V. Cassino, A. Drew, E. Hansen, B. Hunt, D. Rose and A. Scott (1997), "The Forecasting and Policy System: The Core Model" *Reserve Bank of New Zealand Research Paper 43*.

Blanchard, O. J. and N. Kiyotaki (1987), "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review* 77(4), 647-66.

Burnside, C. and M. Eichenbaum (1996), "Factor Hoarding and the Propagation of Business Cycle Shocks." *American Economic Review*, 86(5), 1154-1178.

Chari, V.V., P.J. Kehoe and E. R. McGrattan (2004), "A Critique of Structural VARs Using Real Business Cycle Theory", *Working Paper no 631 Federal Reserve Bank of Minneapolis*

Cho, S. and A. Moreno (2004), " A Small-Sample Study of the New Keynesian Macro Model", mimeo, Departamento de Economia, Universidad de Navarra, Spain

Christiano, L.J., M. Eichenbaum, and C.L. Evans (2003), "Nominal rigidities and the dynamic effects of a shock to monetary policy" mimeo, Northwestern University

Cogley, T. and J.M. Nason (1993), "Impulse Dynamics and Propagation Mechanisms in a Real Business Cycle Model", *Economics Letters*, 43, 77-81.

Coletti, D., B. Hunt, D. Rose and R. Tetlow (1996), "The Bank of Canada's New Quarterly Projection Model, Part 3: The dynamic model: QPM ", *Bank of Canada, Technical Report 75.*

Clements, M. and D.F. Hendry (1999) *Forecasting Non-stationary Economic Time Series*, M.I.T. Press, Cambridge Mass.

Engle, R.F. and S. Kozicki, (1993) "Testing for Common Features," *Journal of Business and Economic Statistics*, 11, p. 369-380.

Forni, M., M.Lippi and L. Reichlin (2003), "Opening the Black Box: Structural Factor Models versus Structural VARs", ECARES, Universite Libre de Bruxelles, mimeo.

Gali, J (1996), "Unemployment in dynamic general equilibrium economies." *European Economic Review*, 40, 839-45.

Garratt, A. K.Lee, H.M. Pesaran and Y. Shin (2003), "A Long-run Structural Macroeconometric Model for the U.K.", *Economic Journal*, 113, 412-455.

Giannone, D., L. Reichlin and L. Sala (2002), "VARs, Common Factors and the Empirical Validation of Equilibrium Business Cycle Models ", *Journal of Econometrics* (forthcoming)

Giordani, P. (2004), "An Alternative Explanation of the Price Puzzle" , *Journal of Monetary Economics,* 51, 1271-1296.

Harrison, R., K. Nikolov, M. Quinn, G.Ramsay, A. Scott and R. Thomas (2005), *The Bank of England Quarterly Model*, Bank of England.

Ireland, P. (2004), "A Method for Taking Models to the Data" *Journal of Economic Dynamics and Control,* 28, 1205-1226.

Lippi, M. and L. Reichlin (1994), "VAR Analysis, Non Fundamental Representations, Blaschke Matrices", *Journal of Econometrics*, 63, 307-325

Masson, P. (1988), "Deriving Small Models from Large Models, in R. Bryant (ed) *Empirical Macroeconomics for Interdependent Economies*, Brookings Institution.

Murchison, S., A. Rennison and Z. Zhu (2004), "A Structural Small Open-Economy Model for Canada", *Bank of Canada Working Paper*, 2004-4.

Nason, J.M. and J.H. Rodgers (2005) "The Sparkle in Exchange Rates: Restricting its Present-Value Model with Common Trends and Common Cycles", mimeo, Federal Reserve Bank of Atlanta

Pagan, A.R. and J. Robertson, "Structural Models of the Liquidity Effect", *Review of Economics and Statistics*, 80, 202-217.

Pagan, A. R.(2002) "What is a Good Macroeconomic Model for a Central Bank to Use?", Federal Reserve Bank of San Francisco *Conference on Macro-*

economic Models for Monetary Policy, (available at http://www.frbsf.org.economics/conferences/0203/index.html)

Pagan, A.R. (2003) " Report on Modelling and Forecasting at the Bank of England" . *Bank of England Quarterly Bulletin*, Spring, 1-29

Reinsel, G.C. and R.P. Velu (1998), *Multivariate Reduced Rank Regression* (Springer-Verlag).

Rotemberg, J. J. (1982), "Sticky prices in the United States.", *Journal of Political Economy*, 90(6), 1187-1211.

Schmitt-Grohe, S„ and M. Uribe (2003), "Closing small open economy models, *Journal of International Economics* 61, 163-185.

Schweppe, F.C. (1965), "Evaluation of Likelihood for Gaussian Signals", *I.E.E.E. Transactions on Information Theory*, 11, 61-70.

Smets, F. and R. Wouters (2003),"An Estimated Dynamic Stochastic General Equilibirum Model of the Euro Area", *Journal of the European Economic Association*, 1, 1123-1175.

Vahid, F. and J.V. Issler (2002), "The Importance of Common Cyclical Features in VAR Analysis: A Monte Carlo Study", *Journal of Econometrics,* 109, 341-363.

Velu, R. P., G. C. Reinsel and D. W. Wichern (1986), "Reduced Rank Models for Multiple Time Series" , *Biometrika*, 73, pp. 105-118.

Wallis, K.F. (1977), "Multiple Time Series and the Final Form of Econometric Models" , *Econometrica*, 45, 1481-1497.

Zellner, A. and F. Palm (1974), "Time Series Analysis and Simultaneous Equation Econometric Models" , *Journal of Econometrics*, 2, 17-54.