

CENTRE FOR APPLIED MACROECONOMIC ANALYSIS

The Australian National University



CAMA Working Paper Series

August, 2005

SOME ISSUES IN USING VARS FOR MACROECONOMETRIC RESEARCH

Renee Fry

Centre for Applied Macroeconomic Analysis
The Australian National University

Cambridge Endowment for Research in Finance
University of Cambridge

Adrian Pagan

Centre for Applied Macroeconomic Analysis
The Australian National University

Queensland University of Technology

Some Issues in Using VARs for Macroeconometric Research*

Renée Fry[†] and Adrian Pagan[‡]

August 9, 2005

Contents

1	Introduction	2
2	Information Needed for Deriving Impulse Responses	4
3	Estimating the VAR (D_j)	5
4	Estimating Impact Impulses (B_0^{-1})	8
4.1	Long-Run Restrictions	8
4.1.1	Instrumental Variable Approach	9
4.1.2	Direct Approach from the VAR	10
4.2	Two Examples of Long-Run Restrictions	12
4.2.1	Blanchard and Quah	12
4.2.2	Gali	13
4.3	Sign Restrictions	14
4.3.1	Blanchard and Quah	17
4.3.2	Peersman	19
4.3.3	Can we recover true impulses?	20
4.4	Bayesian Methods	23

*We would like to thank Jim Nason and Jon Faust for comments on this paper.

[†]Centre for Applied Macroeconomic Analysis, Australian National University, Canberra, A.C.T. Australia 0200, and CERF, University of Cambridge.

[‡]Centre for Applied Macroeconomic Analysis, Australian National University, and Queensland University of Technology. Pagan's research was supported by ESRC Grant No. 000 23-0244.

5	Measuring Uncertainty	24
5.1	Long-Run Restrictions	24
5.1.1	Blanchard and Quah	26
5.1.2	Gali	27
5.2	Sign Restrictions	30
5.2.1	Blanchard-Quah	31
5.2.2	Peersman	31
6	What do Variance Decompositions Tell us about the Cycle?	33
7	Conclusion	35
8	References	36

1 Introduction

VARs have become one of the major ways of extracting information about the macro economy. One might cite three major uses of them in macroeconomic research.

1. Quantifying impulse responses to macroeconomic shocks.
2. The measurement of the degree of uncertainty about the impulse responses or other quantities formed from them.
3. The contribution of different shocks to business cycles and forecast errors through variance decompositions.

In this paper we look at some issues that have arisen in the literature about the ability of VARs to provide the requisite information.

The issues that arise in connection with the first of the trio of uses above relate to how one is to identify the shocks. Essentially this is a question of how to convert a VAR to an SVAR and how to identify the parameters of such a system. Initially the basic approach was to assume that the macroeconomic system could be represented as a set of simultaneous equations that recursively determined economic variables. Later some non-recursivity was allowed and often the requisite identifying information became inertial restrictions e.g. monetary policy had no impact on real variables for two quarters. In more recent times there has been a move away from the imposition of short-run restrictions, as in recursive and inertial systems, to either long-run restrictions or the incorporation of qualitative and quantitative information. The latter generally involve the use of either sign restrictions or

other prior information. To date little has been written that compares these different methods and asks what difficulties arise in their use.

All of the methods mentioned above have a feature in common viz. that they use as part of their identifying information the assumption that the shocks to the macroeconomic system are uncorrelated. This is a strong assumption. As with all specification errors, if it is wrong it can lead to very poor estimates of impulse responses- Giordani (2004) and Cooley and Dwyer (1995) provide examples. In Giordani the errors in the VAR equations involve a common omitted variable - the level of potential output - since the VAR is specified in the levels of output rather than the output gap. In Cooley and Dwyer there is a single shock in the model so that the error terms in the VAR must be constructed from this single shock and so must be perfectly correlated. Assuming that they are uncorrelated will produce specification errors that bias impulse response estimators.

There are also other possible sources of specification errors in VARs. One of these is that the VAR is assumed to be of finite order. There are quite a few models in which this would not be true. Indeed, in some cases it may not be possible to represent the data as a VAR at all - see Lippi and Reichlin (1994), the example in Cooley and Dwyer (1995) and the conditions for the existence of a stationary VAR in Fernandez-Villaverde et al. (2004). In other instances the VAR may need to be of infinite order. Mostly, the latter case arises when the system follows a finite order VAR in n variables but the investigator works with a VAR in $m < n$ variables, so that the reduced system is actually a VARMA rather than a VAR process. The latter problem has been remarked upon by a number of authors e.g. Cooley and Dwyer (1995), Giannone et al (2004), Kapetanios et al. (2005), Canova and Pina (2002) etc. In the next section of this paper we will study the question of when a number of variables in a finite-order VAR will result in a VARMA process in the reduced number of variables. We establish some situations in which the VAR will remain finite and show what this would imply for an RBC model and the first example in Cooley and Dwyer. Although in principle it would be better to proceed to modelling under our recommended procedure there may be some practical difficulties involving data. Nevertheless it is useful to understand the fact that whether a reduction in the number of variables results in an infinite or finite VAR depends a great deal upon which variables are retained in the VAR and which are deleted.

In this paper we look at these issues from both a theoretical and empirical perspective. In our empirical work we use three examples- the original Blanchard-Quah two variable system featuring demand and supply side shocks, a four variable IS-LM system used by Gali (1994) and a four variable system used by Peersman (2005) which analysed the causes of the early-

millenium slow down.

2 Information Needed for Deriving Impulse Responses

For simplicity we will consider a structural VAR, SVAR(1), of first order,

$$B_0 z_t = B_1 z_{t-1} + \varepsilon_t$$

with underlying VAR

$$z_t = A_1 z_{t-1} + v_t.$$

We immediately have that $v_t = B_0^{-1} \varepsilon_t$. The solution to the VAR(1) is the MA form

$$z_t = D(L)v_t$$

where $D(L) = I + D_1 L + D_2 L^2 + \dots$, with the D_j being the impulse responses of z_{t+j} to a unit change in v_t . It follows that the MA form for the SVAR is

$$z_t = C(L)\varepsilon_t$$

where $C(L) = C_0 + C_1 L + \dots$, with the impulse responses to ε_t being $C_j = D_j B_0^{-1}$.

The VAR is easily estimated by OLS regardless of the nature of the SVAR. Hence D_j can always be found once the lag length in the VAR is specified. It is possible that there is no finite order VAR for the data however, in which case the estimation of a finite order one would produce inconsistent estimators of D_j . In turn this would result in inconsistent estimators of C_j , even when B_0 is known. For this reason we begin with the question of when it is that the system we are interested in estimating can be represented as a finite order VAR, and then subsequently move on to the question of how to estimate B_0 .

To get the impulse responses with respect to ε_t however, requires that one estimate B_0^{-1} . Traditionally this entails first estimating B_0 . However, if one only wants to estimate impulse responses for a sub-set of the shocks, one can get these without knowing all of the B_0 . To illustrate this, suppose that we want to find the impulse response for the first shock, and that the elements of B_0 are $\{\beta_{ij}^0\}$, while those of B_0^{-1} are $\{\beta_0^{ij}\}$. Then, if we can estimate the parameters β_{1j}^0 , we can recover the residuals of the first equation $\hat{\varepsilon}_{1t}$ as our estimate of the first shock.

Now if $\delta' = [\beta_0^{11} \quad \dots \quad \beta_0^{n1}]$ we see that the impulse functions for ε_{1t} , C_j^1 , are related to those for v_t through $C_j^1 = D_j\delta$, so to form C_j^1 we will need to estimate δ . But

$$v_t = \varepsilon_{1t}\delta + \eta_t,$$

where η_t are linear combinations of the remaining ε_{jt} . Since ε_{1t} is uncorrelated with η_t we can regress the VAR residuals \hat{v}_t against the $\hat{\varepsilon}_{it}$ and consistently estimate δ . This can then be used to construct the impulse responses for the shock ε_{1t} . Notice that the basic requirement is that we be able to estimate the parameters of the equations defining the shocks of interest.

3 Estimating the VAR (D_j)

An old literature, due to Zellner and Palm (1974) and Wallis (1977), has noted that when a system is a VAR(p) in n variables and it is reduced to a smaller system with $m < n$ variables, the smaller system will generally be a VARMA process. Since many applications involve such compression of variables it might be expected that most models should therefore be estimated as VARMA processes, and that the use of a VAR can lead to specification issues. It might be expected that a very high-order VAR could compensate for this mis-specification but the order may in fact be far too high for the data sets one is normally faced with. For this reason it seems important to look at the issue in more detail than is normal in the literature.

Since many of the cases that have arisen in which there are approximation problems with VARs come from DSGE models, it is useful to look at the compression issue in the context of such models. Hence we think of the system in n variables as having the form

$$z_t = \bar{A}z_{t-1} + \bar{B}E_t(z_{t+1}) + u_t \tag{1}$$

where $\dim(z_t) \geq \dim(u_t)$. We will assume that the system has been reduced as much as possible to get this form. Now, following Pesaran and Binder (1995), we have the VAR(1) solution (assuming that u_t is *i.i.d.*)

$$z_t = Pz_{t-1} + Gu_t.$$

Now consider what happens if we model only a subset of the variables. We will call the modelled sub-set z_{1t} and the omitted variables z_{2t} . We can decompose the VAR above as

$$z_{1t} = P_{11}z_{1t-1} + P_{12}z_{2t-1} + G_1u_t \tag{2}$$

and we will assume that the following relation holds between z_{1t} and z_{2t}

$$z_{2t} = \bar{D}_0 z_{1t} + \bar{D}_1 z_{2t-1} + \bar{D}_2 u_t.$$

Substituting this in we get

$$z_{1t} = (P_{11} + P_{12} \bar{D}_0) z_{1t-1} + P_{12} \bar{D}_1 z_{2t-2} + G_1 u_t + P_{12} \bar{D}_2 u_{t-1}$$

so that the sufficient conditions for there to be a finite order VAR in z_{1t} will be that either

1. $P_{12} = 0$ i.e. z_{2t} does not Granger cause z_{1t} - see Lütkepohl (1993 p 55) and Quenouille (1957, p. 43-44).
2. $\bar{D}_1 = 0, \bar{D}_2 = 0$ i.e. the variables to be eliminated must be connected to the retained variables through an identity and there can be no “own lag” in the omitted variables in the relation connecting z_{1t} and z_{2t} . This observation looks trivial but, in fact, it happens that many of the problems that have arisen where a finite order VAR does not obtain come from the fact that the variables which are omitted are connected with the retained variables through an identity, but one that contains an “own lag”.

Let us illustrate this fact with the first example in Cooley and Dwyer (1995). Using the notation in that paper we have the system of equations to be solved as

$$\begin{aligned} \Delta Y_t &= \varepsilon_{Dt} - \Delta P_t + a \varepsilon_{St} \\ \Delta Y_t &= N_t - N_{t-1} + \varepsilon_{St} \\ \Delta P_t &= \Delta W_t - \varepsilon_{St} \\ \Delta W_t &= -U_t - N_t + N_{t-1} \\ U_t &= N_t^* - N_t \end{aligned}$$

where Y_t is output, P_t the price level, N_t is employment, W_t is the wage rate, U_t is unemployment and N_t^* is the labour force. The system is clearly a VAR in $\Delta Y_t, \Delta P_t, \Delta W_t, U_t$ and N_t . Cooley and Dwyer follow Blanchard and Quah and retain only two of these variables, ΔY_t and U_t , in their condensed system. Using our principle it is clear that this is a bad choice since N_{t-1} appears in the system and therefore N_t potentially needs to be one of the retained variables. There is an identity that connects U_t and N_t so that we could have used their system if it had incorporated N_t^* as well. Since N_t^* is an exogenous variable this would mean that the system would need to be a

VARX one in which N_{t-1}^* appeared in each of the equations for U_t and Y_t . If the variables retained had been ΔY_t and N_t then the system would be a finite order VAR(1). Thus if one had analysed the latter system one would have perfectly recovered the demand and supply shocks. If one uses the $\Delta Y_t, U_t$ choice the assumption of a finite order VAR will be in error unless N_t^* was constant and an intercept appeared in each equation. Failure to use a VARX system here will cause biases in the estimated impulse responses.

A second example would be the standard RBC model. Using the simple model in Uhlig (1999) after log linearization around the steady state we would get

$$l_t = y_t - c_t \quad (3)$$

$$C^*c_t + K^*k_t = Y^*y_t + (1 - \delta)K^*k_{t-1} \quad (4)$$

$$c_t = E_t(c_{t+1} - \alpha\gamma(y_{t+1} - k_{t+1}))$$

$$y_t = a_t + \alpha k_{t-1} + (1 - \alpha)l_t,$$

where c_t is consumption, a_t is the technology shock, k_t is capital stock, l_t is the labour force, and y_t is output. An asterisk denotes steady state values and γ is the steady state share of capital in output. The solution to this system can be made a VAR(1) in c_t, l_t, y_t and k_t . It's clear that we could eliminate any of c_t, l_t or y_t since these do not appear as a lagged variable in the system. Equally clearly k_t cannot be eliminated unless we can find an identity relating it to other variables that does not involve k_{t-1} but the identity (4) shows that this is not possible. If a preference shock appeared in the model so that (3) was no longer an identity the same circumstances would hold. The solved system would be a VAR(1) in c_t, k_t, l_t and y_t . Thereupon the two identities can be used to end up with a two variable VAR(1), provided k_t is one of the two variables. Most of the literature that seeks to establish that a SVAR cannot approximate a DSGE model - Chari et al (2004), Erceg et al (2003), Cooley and Dwyer (1995) - substitute out k_t , and so end up with a non-finite order VAR. This suggests that one should always try to formulate VARs in the capital stock as it is unlikely that one will get a finite order VAR if it does not appear in the variable set. It should be realized that the problems of a VARMA versus a VAR are due to the variables we select to appear in the VAR and do not come from the fact that we are using a model like an RBC one. It doesn't seem quite accurate to say, as Cooley and Dwyer (p. 77) do, that "...in general the SVAR approach rules out compatibility with large classes of economic models.."

It may be a problem to construct data on the capital stock, but in those models that are using the VAR approximation to a DSGE model the capital

stock can be constructed from the parameters of the model and the investment data being used as part of the estimation strategy e.g. del Negro et al (2004) (although this model is more complex than the basic RBC model and the initial system solved has lagged prices and wages in it and these are probably impossible to remove as there are no suitable identities). As a general principle it would seem that one would want to incorporate a variable that represents stocks into a VAR since these are very slow to adjust. It is an odd feature that, almost invariably, VARs simply incorporate flow variables.

4 Estimating Impact Impulses (B_0^{-1})

There are a number of ways proposed to do this in the literature

4.1 Long-Run Restrictions

Let us assume that the VAR does correctly describe the system and consider how one would quantify the impulse responses. We need some identifying information. A model would produce this but there has always been a search for restrictions that are compatible with many models. Blanchard and Quah (1989) originated the idea of using long-run restrictions as identifying information. They have a bivariate SVAR system

$$z_{1t} = \beta_{11}^1 z_{1t-1} + \beta_{12}^1 z_{2t-1} + \beta_{12}^0 z_{2t} + \epsilon_{1t} \quad (5)$$

$$z_{2t} = \beta_{21}^1 z_{1t-1} + \beta_{22}^1 z_{2t-1} + \beta_{21}^0 z_{1t} + \epsilon_{2t} \quad (6)$$

with z_{1t} being the change in the log of output, z_{2t} being unemployment, ϵ_{1t} being a supply shock and ϵ_{2t} a demand shock. For simplicity we work with an SVAR(1) as the essential issues become clearer if one does this.

Blanchard and Quah argued that a demand shock should have a zero long-run effect on the log level of output while a supply shock will not. Let z_{1t} be the change in output and ϵ_{2t} be the demand shock. Now the system can be written as

$$B(L)z_t = \epsilon_t$$

where $B(L) = B_0 - B_1L$ and L is the lag operator.

$$B(L) = \begin{bmatrix} 1 - \beta_{11}^1 L & -\beta_{12}^0 - \beta_{12}^1 L \\ -\beta_{21}^0 - \beta_{21}^1 L & 1 - \beta_{22}^1 L \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 1 & -\beta_{12}^0 \\ -\beta_{21}^0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} \beta_{11}^1 & \beta_{12}^1 \\ \beta_{21}^1 & \beta_{22}^1 \end{bmatrix}$$

Consequently

$$z_t = B(L)^{-1}\epsilon_t = C(L)\epsilon_t = (C_0 + C_1L + \dots)\epsilon_t \quad (7)$$

where the elements C_j are the j period ahead impulse responses of z_t to a transitory unit rise in the shocks ϵ_t i.e. $C_j = \frac{\partial z_{t+j}}{\partial \epsilon_t}$. The sum of these C_j are the long-run responses of z_{1t} (output) and z_{2t} to the shocks. Now $C(1) = \sum C_j$ and, in the two-variable case, $C(1)$ has the form

$$C(1) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix}.$$

where the zero in the second column comes from the fact that this shock is transitory.

Imposing the uncorrelatedness assumption on the shocks that

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \sim i.i.d. \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \right)$$

and using the triangular structure on $C(1)$ enables the consistent estimation of the parameters in B_0 and B_1 .

There are two ways to see how the information just described is used to estimate the unknown parameters in B_j - an indirect method that uses instrumental variable methods and a direct method that works with quantities that can be extracted from the VAR. Each has its uses so we proceed to describe them both.

4.1.1 Instrumental Variable Approach

Because $C(L) = B(L)^{-1}$ it follows that $C(L)B(L) = I_2$ and $C(1)B(1) = I_2$. Hence

$$\begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} 1 - \beta_{11}^1 & -\beta_{12}^0 - \beta_{12}^1 \\ -\beta_{21}^0 - \beta_{21}^1 & 1 - \beta_{22}^1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

from which $c_{11}(-\beta_{12}^0 - \beta_{12}^1) = 0$, giving $(-\beta_{12}^0 - \beta_{12}^1) = 0$, as $c_{11} = 0$ would mean that $C(1)$ was singular. This gives a restriction on the parameters of the first equation, $\beta_{12}^0 = -\beta_{12}^1$, so it can be written as

$$z_{1t} = \beta_{11}^1 z_{1t-1} + \beta_{12}^0 \Delta z_{2t} + \epsilon_{1t}. \quad (8)$$

Now consider the estimation of (8) and (6). (8) can be estimated by using z_{1t-1}, z_{2t-1} as instruments for z_{1t-1} and Δz_{2t} . Having estimated these coefficients we can get the residuals from this equation $\hat{\varepsilon}_{1t}$ and then use $\hat{\varepsilon}_{1t}, z_{1t-1}$ and z_{2t-1} as instruments for z_{1t-1}, z_{2t-1} and z_{1t} in (6). Note that unlike the recursive case we can't use z_{1t} as an instrument.

4.1.2 Direct Approach from the VAR

The VAR(1) underlying the SVAR(1) above is

$$z_t = A_1 z_{t-1} + v_t$$

where $A_1 = B_0^{-1} B_1$ and $v_t = B_0^{-1} \varepsilon_t$. Hence $A(L) = I - A_1 L$ and $C(L) = A(L)^{-1} B_0^{-1}$ i.e. $C(1)B_0 = A(1)^{-1}$. Therefore

$$\begin{aligned} c_{11} &= A^{11}(1) \\ -c_{11}\beta_{12}^0 &= A^{12}(1) \end{aligned}$$

and hence

$$\hat{\beta}_{12}^0 = -\frac{\hat{A}^{12}(1)}{\hat{A}^{11}(1)} = \frac{\hat{A}_{12}(1)}{\hat{A}_{22}(1)}. \quad (9)$$

Note that

$$\Delta z_{2t} = A_{22}(1)z_{2t-1} + A_{21}z_{1t-1} + v_{2t}.$$

Since the instrument z_{1t-1} already appears in the Δz_{2t} equation, the only useful instrument is z_{2t-1} , and the correlation between it and Δz_{2t} will be determined by $A_{22}(1)$. This shows the connection between the two approaches.

Now we can also estimate β_{21}^0 since the errors in the VAR, v_t , have covariance matrix V and we have

$$B_0 V B_0' = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

Hence

$$\begin{aligned} \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} &= \begin{bmatrix} 1 & -\beta_{12}^0 \\ -\beta_{21}^0 & 1 \end{bmatrix} \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{21}^0 \\ -\beta_{12}^0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} V_{11} - \beta_{12}^0 V_{21} & V_{12} - \beta_{12}^0 V_{22} \\ -\beta_{21}^0 V_{11} + V_{21} & -\beta_{21}^0 V_{12} + V_{22} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{21}^0 \\ -\beta_{12}^0 & 1 \end{bmatrix} \end{aligned}$$

and this yields the restriction

$$-\beta_{21}^0(V_{11} - \beta_{12}^0 V_{21}) + V_{12} - \beta_{12}^0 V_{22} = 0$$

so that

$$\hat{\beta}_{21}^0 = \frac{\hat{V}_{12} - \hat{\beta}_{12}^0 \hat{V}_{22}}{\hat{V}_{11} - \hat{\beta}_{12}^0 \hat{V}_{21}} \quad (10)$$

Once $\hat{\beta}_{21}^0$ and $\hat{\beta}_{12}^0$ are found one can form regressors like $z_{1t} - \hat{\beta}_{12}^0 z_{2t}$ and regress these against z_{t-1} to estimate the remaining parameters in B_1 . An alternative approach to estimating β_{21}^0 is to use the residuals from (8) in (6) as an instrument for z_{1t} .

More generally suppose that the long run response of output to a demand shock is not zero but μ i.e.

$$C(1) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} c_{11} & \mu \\ c_{21} & c_{22} \end{bmatrix}.$$

Then we would have

$$\begin{aligned} c_{11} - \mu\beta_{21}^0 &= A^{11}(1) \\ -c_{11}\beta_{12}^0 + \mu &= A^{12}(1) \end{aligned}$$

so that

$$\begin{aligned} \beta_{12}^0 c_{11} - \mu\beta_{12}^0\beta_{21}^0 &= \beta_{12}^0 A^{11}(1) \\ -c_{11}\beta_{12}^0 + \mu &= A^{12}(1) \end{aligned}$$

and

$$\mu(1 - \beta_{12}^0\beta_{21}^0) = \beta_{12}^0 A^{11}(1) + A^{12}(1)$$

The other restriction from the covariance matrix does not depend on the long run restriction so we have

$$\mu(\beta_{12}^0\beta_{21}^0 - 1) + \beta_{12}^0 A^{11}(1) + A^{12}(1) = 0 \quad (11)$$

$$-\beta_{21}^0(V_{11} - \beta_{12}^0 V_{21}) + V_{12} - \beta_{12}^0 V_{22} = 0 \quad (12)$$

and this produces two equations to solve for β_{12}^0 and β_{21}^0 . Notice that there is no simple relation between the coefficients any longer. It is possible to study how sensitive the estimates of β_{12}^0 and β_{21}^0 and the impulses are to values of μ different from zero and one might work out an asymptotic theory for them being local to zero.

The example given above generalizes. To see this suppose that there are two shocks that have permanent effects. The first affects the first variable while the second shock can have a permanent effect on both the first and

second variables. Suppose that there are four variables. This means that $C(1)$ has the structure

$$\begin{bmatrix} c_{11} & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and thus we would have

$$\begin{bmatrix} c_{11} & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & \beta_{12}^0 & \beta_{13}^0 & \beta_{14}^0 \\ \beta_{21}^0 & 1 & \beta_{23}^0 & \beta_{24}^0 \\ \beta_{31}^0 & \beta_{32}^0 & 1 & \beta_{34}^0 \\ \beta_{41}^0 & \beta_{42}^0 & \beta_{43}^0 & 1 \end{bmatrix} = \begin{bmatrix} A^{11}(1) & A^{12}(1) & A^{13}(1) & A^{14}(1) \\ A^{21}(1) & A^{22}(1) & A^{23}(1) & A^{24}(1) \\ A^{31}(1) & A^{32}(1) & A^{33}(1) & A^{34}(1) \\ A^{41}(1) & A^{42}(1) & A^{43}(1) & A^{44}(1) \end{bmatrix}$$

It is clear that we have

$$\begin{aligned} c_{11} &= A^{11}(1) \\ \beta_{1j}^0 &= A^{1j}(1)/A^{11}(1) \end{aligned}$$

so that the first row of B_0 can be found in this way. To determine the second row we note that there will be five unknowns $c_{12}, c_{22}, \beta_{21}^0, \beta_{23}^0$ and β_{24}^0 but only four values $A^{2j}(1)$ that these relate to. Thus we need another equation to solve for the five unknowns and this comes from the assumption that the first and second shocks are uncorrelated. Alternatively, one can use the residuals (shocks) from the first equation as an instrument to estimate the second equation. Once we can estimate the two shocks we can determine their impulse responses as was discussed earlier.

There are variants of this situation in the literature. Altig et al (2005) have two shocks- but they actually constrain $c_{12} = 0$ as well. Therefore they do not need the zero correlation assumption and so they can isolate the two permanent shocks as above i.e. in the second case $c_{22} = A^{22}(1)$ and $\beta_{2j}^0 = A^{2j}(1)/A_{11}(1)$. Notice that their VAR is over-identified and so one could test the assumption that $c_{12} = 0$. There are other cases where one does not know that there is only one permanent shock e.g. in Peersman (2005) the oil price shock may or may not have a permanent effect.

4.2 Two Examples of Long-Run Restrictions

4.2.1 Blanchard and Quah

Blanchard and Quah (1989) estimate a bivariate SVAR of quarterly real GNP growth (Δy_t) and the unemployment rate (U_t) to disentangle demand ($\varepsilon_{D,t}$) and supply ($\varepsilon_{S,t}$) shocks for the US. Long-run restrictions are used to identify

the shocks. Following Blanchard and Quah some adjustments are made to the data - unemployment is detrended using a linear time trend and the GNP growth series is split into two sections between 1973:4 and 1974:1, with the sample mean of the respective sub-samples removed prior to estimation of the VAR. The order of the VAR is 8.

The two equation SVAR system looks like

$$\begin{aligned}\Delta y_t &= \beta_{12}^0 U_t + lags + \varepsilon_{S,t} \\ U_t &= \beta_{21}^0 \Delta y_t + lags + \varepsilon_{D,t}\end{aligned}\tag{13}$$

(13) can be re-parameterized as

$$\Delta y_t = \beta_{12}^0 \Delta_8 U_t + lags \text{ in } (\Delta_8 U_t, \Delta y_t) + \phi U_{t-8} + \varepsilon_{S,t}$$

and the long-run restriction relating to demand shock effects means that $\phi = 0$. Hence it is possible to use U_{t-8} as an instrument for $\Delta_8 U_t$ in order to estimate β_{12}^0 . After that the residuals from this equation are used as an instrument for Δy_t in the U_t equation to estimate β_{21}^0 . The latter enforces the assumption that the demand and supply shocks are uncorrelated. The direct approach can also be used to estimate the parameters.

Thus we see that MLE estimates of β_{12}^0 and β_{21}^0 come from imposing linear restrictions using the VAR estimates of the $A_1 \dots A_8$ and the covariance matrix V . Once $\hat{\beta}_{ij}^0$ are found the initial impulse responses can be computed from $\hat{C}_0 = \hat{B}_0^{-1}$. Since $\hat{C}_j = \hat{D}_j \hat{C}_0$ and \hat{D}_j are just least squares estimators it is clear that the distribution of \hat{C}_j will be heavily influenced by that of \hat{C}_0 and so we focus upon it in the examples of this paper.

4.2.2 Gali

Gali (1992) has a four variable SVAR(4) with variables $\Delta y_t, \Delta i_t, i_t - \Delta p_t$ and $\Delta m_t - \Delta p_t$, where y_t is the log level of GDP, i_t is the nominal interest rate, p_t is the log of the price level and m_t is the log of the money supply. He maintains the assumptions that the nominal interest rate and inflation are both $I(1)$ processes, while the real interest rate and other growth variables are $I(1)$. The equations form a structural system with four structural shocks that are identified as aggregate supply ε_{1t} , money supply and demand (ε_{2t} and ε_{3t}) and aggregate demand (ε_{4t}). Essentially it is an IS/LM model augmented by a Phillips curve. The contemporaneous part is highlighted below.

$$\begin{aligned}\Delta y_t &= \beta_{12}^0 \Delta i_t + \beta_{13}^0 (i_t - \Delta p_t) + \beta_{14}^0 (\Delta m_t - \Delta p_t) + lags + \varepsilon_{1t} \\ \Delta i_t &= \beta_{21}^0 \Delta y_t + \beta_{23}^0 (i_t - \Delta p_t) + \beta_{24}^0 (\Delta m_t - \Delta p_t) + lags + \varepsilon_{2t} \\ i_t - \Delta p_t &= \beta_{31}^0 \Delta y_t + \beta_{32}^0 \Delta i_t + \beta_{34}^0 (\Delta m_t - \Delta p_t) + lags + \varepsilon_{3t} \\ \Delta m_t - \Delta p_t &= \beta_{41}^0 \Delta y_t + \beta_{42}^0 \Delta i_t + \beta_{43}^0 (i_t - \Delta p_t) + lags + \varepsilon_{4t}.\end{aligned}$$

The system is then identified by making a number of assumptions.

1. The shocks are uncorrelated between themselves.
2. Demand and monetary shocks have no long run effect on output.
3. Money shocks have no contemporaneous effect on output i.e. they only affect y_{t+1} and not y_t .
4. In setting the money supply the monetary authority does not contemporaneously react to the inflation rate.

The impact of these restrictions upon the system is discussed in Pagan and Robertson (1998). The long run restriction enables Δi_{t-4} , $(i_{t-4} - \Delta p_{t-4})$, $(\Delta m_{t-4} - \Delta p_{t-4})$ to be used as instruments for $\Delta_4 \Delta i_t$, $\Delta_4(i_t - \Delta p_t)$ and $\Delta_4(\Delta m_t - \Delta p_t)$ respectively in the first equation. The third restriction means that the error in the VAR equation for $\Delta y_t, v_{1t}$, is uncorrelated with ε_{2t} and ε_{4t} so it can be used as an instrument in those equations. Finally, the absence of contemporaneous inflation in the money supply equation means that $\beta_{24}^0 = -\beta_{23}^0$. To estimate the parameters of the second equation one uses $\hat{\varepsilon}_{1t}$ and \hat{v}_{1t} as instruments for the two endogenous variables Δy_t and $i_t - \Delta m_t$. We have used a longer data sample than Gali did but find that the parameter estimates of the system are not too far away from those reported in Pagan and Robertson(1998).

4.3 Sign Restrictions

A strategy that has become increasingly popular for estimating SVARs is that of sign restrictions. Suppose that we have ordered the variables in some recursive way and then computed estimates of B_0 . This will mean that the VAR residuals \hat{v} are related to the structural residuals as $\hat{v}_t = \hat{B}_0^{-1} \hat{\varepsilon}_t$. If we call S^{-1} the matrix that has the estimated standard deviations of the ε on the diagonal and zeros elsewhere we could write $\hat{v}_t = \hat{B}_0^{-1} S^{-1} S \hat{\varepsilon}_t = T \hat{\eta}_t$ where η_t has unit variances.

Now suppose we could find a square matrix Q such that $Q'Q = QQ' = I$. Then

$$\begin{aligned} \hat{v}_t &= TQ'Q\hat{\eta}_t \\ &= T^*\hat{\eta}_t^* \end{aligned}$$

and we have a new set of estimated shocks $\hat{\eta}_t^*$ that also have the property that their covariance matrix is I since it will be $QE(\hat{\eta}_t \hat{\eta}_t')Q' = I$. Thus we

have found a combination of the shocks η_t^* that have the same covariance matrix as η_t but which will have a different impact upon v_t and hence the variables z_t .

One example of such a Q is simply a re-ordering of the variables. This produces a new set of shocks and impulses but the shocks will still be orthogonal. One often sees the comment that other orderings were tried with the same result in terms of impulse responses. It's also often the case that people try different orderings but then choose between them based on the estimated impulse responses as one can't choose between them from the data as they have identical VARs. But this raises the question of whether all of the different orderings on the variables exhausts the ways of combining together the shocks while keeping them orthogonal to one another i.e. retaining the identity matrix as covariance matrix. The answer is no and it is this fact that the sign restriction literature uses.

Consider a 4×4 matrix Q_{23} of the form

$$Q_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i.e. the matrix is the identity matrix in which the (2,3) and (3,2) elements have been replaced by the cosine and sine terms and θ lies between 0 and π . Q_{23} is called a Givens rotation. Then

$$\begin{aligned} Q'_{23}Q_{23} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 \theta + \sin^2 \theta & -\cos \theta \sin \theta + \sin \theta \cos \theta & 0 \\ 0 & -\cos \theta \sin \theta + \sin \theta \cos \theta & \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= I_4 = Q_{23}Q'_{23} \end{aligned}$$

So it's clear that we could use $\hat{\eta}^* = Q_{23}\hat{\eta}$ as possible shocks as these are orthogonal. Their impact upon the z_t will then be $T^* = TQ'_{23}$. Obviously there are many such combinations. For a four variable system one could choose other Givens rotations $Q_{12}, Q_{13}, Q_{14}, Q_{23}, Q_{24}, Q_{34}$ as potential combining matrices. In practice most users of the approach have used the multiple of the

basic set of Givens matrices e.g. in the four variable case

$$Q = Q_{12}(\theta_1) \times Q_{13}(\theta_2) \times Q_{14}(\theta_3) \times Q_{23}(\theta_4) \\ \times Q_{24}(\theta_5) \times Q_{34}(\theta_6)$$

Now, the matrix Q above depends upon six values for θ and, as we change θ_j , we will get different values for the Q matrix. Canova and de Nicolo (2002) suggested that one make a grid of M values for each of the values of θ_j between 0 and π , and then compute the 12^6 possible values of Q . Of course all of these are indistinguishable in terms of fitting the first two moments of z_t . They therefore propose that one uses sign information about the impulses to decide which of these is the better combination e.g. maybe one says that a positive interest shock should have a negative effect upon output and inflation from 2-6 lags out. Then only those combinations that produced a shock that had such a feature would be retained for further analysis.

In recent times a quasi-Bayesian approach has become popular instead of the grid method. The θ_j are taken to be uniformly distributed over $(0, \pi)$ and then (in the four variable case) realizations are made from the product of six independent $U(0, \pi)$ densities. This is really just a useful scheme for generating values of θ_j that can be used to construct candidate Q matrices rather than a Bayesian analysis per se. Uhlig (2005) and Peersman (2005) use this approach, although a Bayesian treatment is also given of the VAR coefficients which produce the base set of impulse responses that are combined together with Q . For any realization of the VAR coefficients and error variance one can perform the Givens rotation analysis and tabulate those that satisfy the sign restrictions.

There are some issues which arise due to the fact that there is a lack of uniqueness in the impulse responses that one gets with this approach. There may be many impulse responses that satisfy the sign restrictions. What does one do about this? There have been two strategies for dealing with the issue. One is to compute all the impulse responses $C_j^{(k)}$, where k indexes the impulses that satisfy sign restrictions, and to then report some summary measure of these such as a median over k of $C_j^{(k)}$. We discuss the soundness of this strategy when looking at issues of uncertainty later. The second approach, as in Faust (1998) is to specify a criterion function that can be used to discriminate between the various values of θ e.g. one might choose the value of θ that maximize the contribution of monetary policy shocks to the explanation of the variance of output. We have little to say about this strategy as it clearly revolves around whether the criterion function is a reasonable one.

4.3.1 Blanchard and Quah

Rather than estimate the Blanchard and Quah model using long-run restrictions one might seek to use sign restrictions to identify the shocks. As mentioned earlier this requires the selection of some base set of orthogonal shocks using estimates of A_1 and V . These are then re-combined (rotated) with the Givens matrices. Since there are only two variables in the Blanchard Quah system there will be a single Givens matrix Q_{12} , and so all the impulse responses generated are indexed by a single parameter θ . As θ is drawn from $(0, \pi)$ we get a new set of impulse responses which are compared to the sign restrictions. If these are satisfied then the set of impulses is kept. Draws of θ continue until we have a set of 1000 impulse responses that obey all the restrictions. Uhlig (2005) and Peersman (2005) produce more impulse responses by drawing new values of A_1 and V from a posterior distribution for VAR estimators. In our paper we will simply use the OLS estimates of A_1 and V i.e. we adopt a classical rather than a Bayesian approach so the range of impulses we look at come simply from the fact that many impulse responses may satisfy the restrictions for given values of A_1 and V rather than because new realizations of A_1 and V are being generated.

We need to decide on what sign restrictions are to be employed to identify the shocks. These were loosely based on the empirical results of Blanchard and Quah. Blanchard and Quah force supply shocks to have permanent effects on GNP, while demand shocks do not. In their results a positive demand shock reduces unemployment, but it is unclear how unemployment would be contemporaneously affected by a supply shock. However, it is assumed that unemployment will fall in the medium term, and will then return to its original value. To formally implement these responses, the signs of the impulse response functions were constrained as follows.

First, the impulse response function for the effect of a positive demand shock on real GNP (y_t) is constrained to be positive for four quarters following the initial shock, where

$$C_{y,j}^{\epsilon_D} \geq 0, \quad j = 0, 1, 2, 3, 4,$$

where $C_{y,j}^{\epsilon_D}$ was the j 'th impulse response of output to a demand shock. Second, the corresponding impulse response function for the effect of a positive demand shock on unemployment is constrained to be negative for four quarters:

$$C_{U,j}^{\epsilon_D} \leq 0, \quad j = 0, 1, 2, 3, 4.$$

Thirdly, the supply shock is assumed to have a long run effect on real GNP. To implement this criterion, it is assumed that real GNP will be posi-

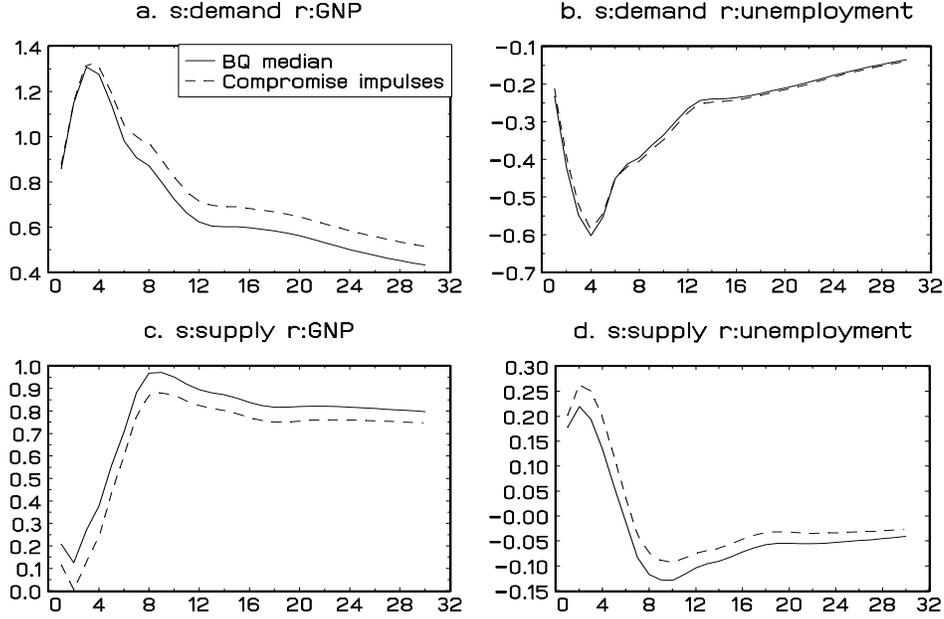


Figure 1: Blanchard and Quah (1989) impulse response functions from sign restriction estimation. BQ median $med(C_j^{(k)})$ and the optimal median $\min(\phi^{(k)'}\phi^{(k)})$.

tive for 3 years following the initial shock

$$C_{y,j}^{\epsilon S} \geq 0, t = 0, 1, \dots, 12.$$

Finally, while the effect of a supply shock on unemployment is ambiguous in the short term, it was assumed to be positive after a certain number of periods. To accommodate this restriction without being too stringent in selecting the appropriate impulses, no sign restriction is imposed on the impact impulse response ($j = 0$) but they begin after three quarters has elapsed following the shock i.e.

$$C_{U_j}^{\epsilon S} \geq 0, j = 3, 4.$$

Figure 1 presents the median of the impulse responses as a solid line i.e. it presents $med(C_j^{(k)})$. The other line on the graphs is an alternative measure that we discuss later.

4.3.2 Peersman

Peersman (2005) estimates a four-variate VAR for the Euro region and the US. The variables are the first difference of the log of oil prices (Δoil_t), output growth (Δy_t), consumer price inflation (Δp_t), and the short term nominal interest rate (s_t). In this section we replicate the model only for the US data, which involves a quarterly VAR(3) estimated over the period 1980Q1 to 2002Q2, with both a constant and a time trend included. Peersman identifies four shocks using the sign restriction methodology. These are a demand shock (ε_t^D), a monetary policy shock (ε_t^M), and two supply shocks: the first being an oil price shock (ε_t^O), and the second is labelled a supply shock (ε_t^S). The method of generating candidate impulse responses is the same as described earlier.

Sign restrictions are only applied to the contemporaneous effects of oil prices and interest rate shocks but, for the other two shocks, the sign of impulses over four periods is used. Specifically, a positive demand shock is expected to generate a positive response in oil prices (oil_t), output (y_t), prices (p_t), and the interest rate as follows.

$$\begin{aligned} C_{oil,j}^{\varepsilon^D} &\geq 0, \quad j = 0, \\ C_{y,j}^{\varepsilon^D} &\geq 0, \quad j = 0, 1, 2, 3, \\ C_{p,j}^{\varepsilon^D} &\geq 0, \quad j = 0, 1, 2, 3, \\ C_{s,j}^{\varepsilon^D} &\geq 0, \quad j = 0. \end{aligned}$$

A positive monetary policy shock is expected to generate a negative response in oil prices, output and the price level, and a positive response in interest rates. This leads to the restrictions:

$$\begin{aligned} C_{oil,j}^{\varepsilon^M} &\leq 0, \quad j = 0, \\ C_{y,j}^{\varepsilon^M} &\leq 0, \quad j = 0, 1, 2, 3, \\ C_{p,j}^{\varepsilon^M} &\leq 0, \quad j = 0, 1, 2, 3, \\ C_{s,j}^{\varepsilon^M} &\geq 0, \quad j = 0. \end{aligned}$$

A favourable supply shock is expected to lead to an increase in output and a decline in prices and the interest rate. A positive oil price shock has a positive impact on oil prices, while the effect of a supply shock on oil is ambiguous. To distinguish between the oil shock and the supply shock, it is assumed that the impact of an oil price shock on oil prices is larger than a

supply shock on oil prices. Thus a positive supply shock has these effects:

$$\begin{aligned} C_{y,j}^{\varepsilon S} &\geq 0, \quad j = 0, 1, 2, 3, \\ C_{p,j}^{\varepsilon S} &\leq 0, \quad j = 0, 1, 2, 3, \\ C_{s,j}^{\varepsilon S} &\geq 0, \quad j = 0, \end{aligned}$$

Finally, a positive oil price shock involves

$$\begin{aligned} C_{oil,j}^{\varepsilon O} &\geq 0, \quad j = 0, \\ C_{y,j}^{\varepsilon O} &\leq 0, \quad j = 0, 1, 2, 3, \\ C_{p,j}^{\varepsilon O} &\geq 0, \quad j = 0, 1, 2, 3, \\ C_{s,j}^{\varepsilon O} &\geq 0, \quad j = 0 \end{aligned}$$

and

$$C_{oil,j}^{\varepsilon O} \geq C_{oil,j}^{\varepsilon S}, \quad j = 0.$$

As in the Blanchard and Quah replication above, the Bayesian methodology is not adopted to estimate the parameters of the VAR in the current application. Rather, the VAR is estimated using OLS. On average, 92 values of θ have to be drawn to generate a single set of impulses which satisfy all restrictions. The results are presented in Figure 2 which shows the same information as for the Blanchard and Quah model.

4.3.3 Can we recover true impulses?

Can sign restrictions recover a set of shocks from a model, in the sense of producing the correct quantitative information? To look at this consider the simplest possible case of a demand and a supply function with associated shocks but abstracting from any dynamics i.e. the system is

$$\begin{aligned} q_t &= -\beta p_t + \varepsilon_{Dt} \\ q_t &= \gamma p_t + \varepsilon_{St}, \end{aligned}$$

where $\beta > 0, \gamma > 0$ and the shocks are distinguished as demand (D) and supply (S). We have

$$\begin{aligned} -\beta p_t + \varepsilon_{Dt} &= \gamma p_t + \varepsilon_{St} \\ \therefore p_t &= \frac{\varepsilon_{Dt}}{\beta + \gamma} - \frac{\varepsilon_{St}}{\beta + \gamma}, \end{aligned}$$

and so a demand shock raises prices and a supply shock reduces them. This is the sign restriction information and we now ask whether it is possible to recover the correct shocks.

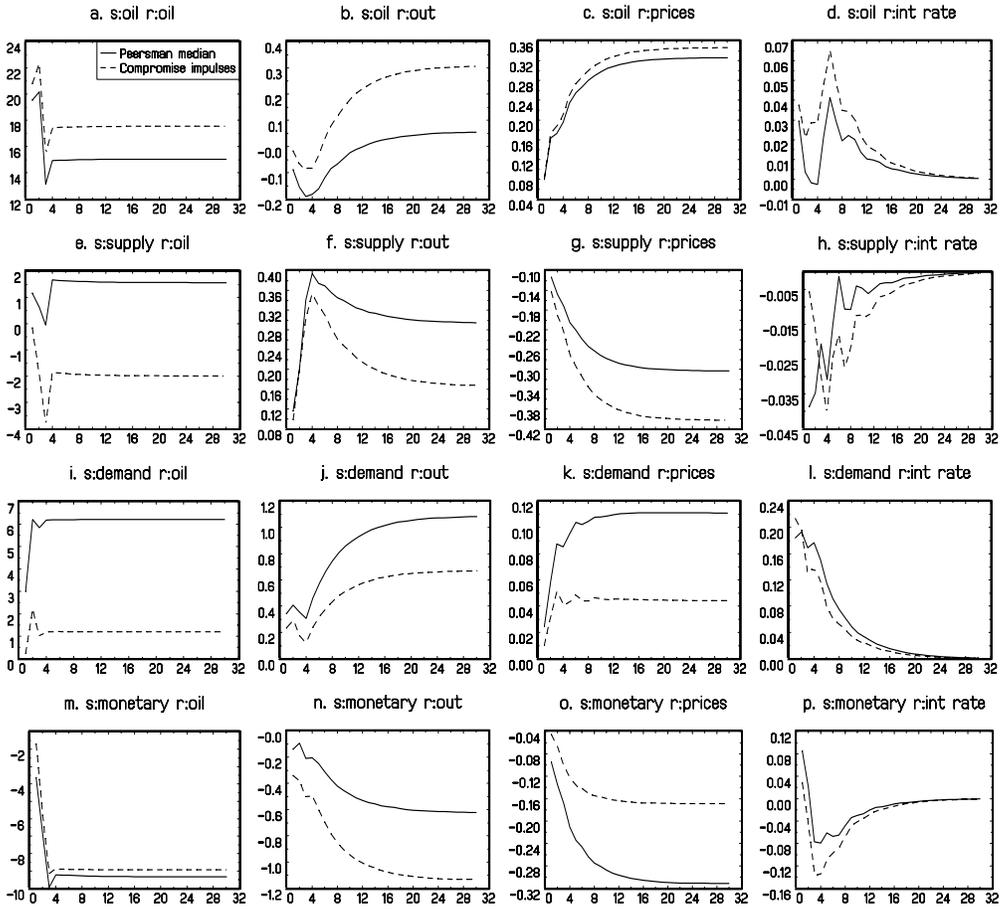


Figure 2: Peersman (2005) impulse response functions from sign restriction estimation. Peersman median $med(C_j^{(k)})$ and the optimal median $\min(\phi^{(k)'} \phi^{(k)})$.

Since we need to begin with a base set of orthogonal shocks a recursive system is assumed with q_t being ordered before p_t i.e. the base system is

$$\begin{aligned} q_t &= \eta_{1t} \\ q_t + \pi p_t &= \eta_{2t}. \end{aligned}$$

Now the first question we might ask is whether these two orthogonal shocks in the recursive system can be identified as the true supply and demand ones. To investigate this we have the assumed solution for p_t :

$$\pi p_t = \eta_{2t} - \eta_{1t},$$

which gives the impact of the shocks η_{jt} on p_t as equal and opposite with absolute magnitude of $\frac{1}{|\pi|}$. This is the same qualitative characteristic as the true shocks, and so η_{1t} and η_{2t} would satisfy the sign constraints. But would this identification produce impulses of the correct magnitude? To assess this we need to look at the plim of $\hat{\pi}$ found by regressing q_t on $-p_t$ since this is the appropriate estimator given the recursive assumption. The probability limit is found as

$$\begin{aligned} p \lim \hat{\pi} &= -(E(p_t^2))^{-1} E(p_t q_t) \\ &= (E(p_t^2))^{-1} (\beta E(p_t^2) - E(p_t \varepsilon_{Dt})) \\ &= \beta - \left[\frac{\sigma_D^2 + \sigma_S^2}{(\beta + \gamma)^2} \right]^{-1} \frac{\sigma_D^2}{\beta + \gamma} \\ &= \beta - \left[\frac{\sigma_D^2 + \sigma_S^2}{\sigma_D^2 (\beta + \gamma)} \right]^{-1} \\ &\neq \frac{1}{\beta + \gamma} \end{aligned}$$

Thus the shocks identified from a recursive system give the right sign but the wrong magnitude for the impulse responses.

Now suppose we use the orthogonal shocks η_{1t} and η_{2t} as the basis for our Givens rotation and ask whether the rotated shocks could be the true demand and supply ones. In this instance there is only one Givens matrix Q_{12} so that we will have

$$\begin{bmatrix} \eta_{1t}^* \\ \eta_{2t}^* \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}$$

and so

$$\begin{aligned}
\begin{bmatrix} \eta_{1t}^* \\ \eta_{2t}^* \end{bmatrix} &= \begin{bmatrix} \eta_{1t} \cos \theta - \eta_{2t} \sin \theta \\ \eta_{1t} \sin \theta + \eta_{2t} \cos \theta \end{bmatrix} \\
&= \begin{bmatrix} q_t \cos \theta - (q_t + \pi p_t) \sin \theta \\ q_t \sin \theta + (q_t + \pi p_t) \cos \theta \end{bmatrix} \\
&= \begin{bmatrix} q_t(\cos \theta - \sin \theta) + \pi p_t \sin \theta \\ q_t(\cos \theta + \sin \theta) + \pi p_t \cos \theta \end{bmatrix}
\end{aligned}$$

This will mean that to recover the shocks perfectly we will need

$$\begin{aligned}
q_t(\cos \theta - \sin \theta) + \pi p_t \sin \theta &= q_t + \beta p_t \\
q_t(\cos \theta + \sin \theta) + \pi p_t \cos \theta &= q_t - \gamma p_t
\end{aligned}$$

i.e.

$$\begin{aligned}
\cos \theta - \sin \theta &= 1 \\
\sin \theta + \cos \theta &= 1 \\
\pi \sin \theta &= \beta \\
\pi \cos \theta &= -\gamma
\end{aligned}$$

Its clear that the first two conditions imply that $2\cos \theta = 2$ and so $\theta = 0$ is needed. From the second set of restrictions we would then need that $\pi = -\gamma$ and that is only true if $\beta = 0, \sigma_{11} = \sigma_{22} = 0$. Hence it will be impossible to recover the shocks using sign restrictions when there are arbitrary values for $\beta, \gamma, \sigma_{11}$ and σ_{22} . Thus, despite the fact that there are innumerable responses that satisfy the sign restrictions, none of these would ever produce the correct ones. This should not be surprising since sign restrictions are very weak information. However, it is unsettling that there seems little chance of it producing the correct quantitative information about impulse responses. Indeed, it is interesting to observe that one of the persistent themes in sign restriction work has been that monetary policy seems incredibly strong compared with what one would obtain from SVARs identified in other ways -see for example Peersman (2005). This suggests that one has to be very careful when using sign restrictions, checking that the implied responses to shocks produce plausible estimates i.e. although qualitative results may accord with sign priors this may be achieved by distorting the magnitudes.

4.4 Bayesian Methods

Bayesian methods essentially estimate a VAR subject to some prior restrictions. These will induce some prior restrictions upon SVAR parameters.

When the SVAR comes from a recursive simultaneous system these have been well studied but there seems no comparable analysis for long-run restrictions. The key to this analysis is (9) and (10). Given some realizations from the posteriors for the estimated VAR coefficients i.e. A_1 and V , we can use these equations to derive realizations from the posteriors for $\hat{\beta}_{12}^0$ and $\hat{\beta}_{21}^0$. The fundamental issue is what priors on A_1 and V are to be used. For Blanchard-Quah and Gali type models using the Minnesota prior wouldn't make sense, since z_{1t} is the growth in output, and one would not want to assume that it was a unit root process. Such a specification would imply a prior mean on A_{22} that equals zero, which would represent a very poor strategy, since A_{22} appears on the denominator and one wants to keep it well away from zero.

An extension of this if we think that the long-run restriction is not zero is to think about putting a prior on μ . We would then draw from the priors on the A_1 and V and also from a prior on μ . Once we have realizations of these we can compute realizations of the β_{ij} .

5 Measuring Uncertainty

5.1 Long-Run Restrictions

Can long-run restrictions produce estimators of the quantities of interest that are reliable? Reference is often made to Faust and Leeper (1997) as having demonstrated that it is not e.g. Erceg et al (2004, p1) say "..as emphasized by Faust and Leeper (1997), structural VARs (SVARs) that achieve identification through long-run restrictions may perform poorly when estimated over the sample periods typically utilized" (p1). Such statements are a little misleading. As Faust and Leeper (p 345) say " We show that conventional inferences regarding impulse responses will be badly biased in *all sample sizes*" (italics added). Here any small sample effect is simply a consequence of the fact that the bias remains in large samples. It is important to understand what the source of this large sample bias is. Basically it stems from assuming that there is an unknown process with true impulses C_j and then asking whether we can estimate $C(1)$, in (7), unbiasedly with a VAR whose order increases with the sample size. As they observe, the answer is in the negative, unless one can restrict C_j . Summability of these is not sufficient since it is possible to construct C_j that produce a spike in the spectral density, and this will not be captured with a function that effectively assumes smoothness. If the true process generating the C_j is a stationary finite order VAR, or a stationary and invertible VARMA process, then one can estimate the $C(1)$

without bias (asymptotically) by estimating the appropriate process. So, if we are working with data simulated from such systems, as Erceg et al do, the Faust and Leeper point would not be immediately relevant in small samples¹ However, it is possible that, if the VAR has a unit root in it, then the fact that such a process has power at the origin in the "spectral density" might be regarded as behaving like a spike in its effects, and so one might project the existence of some small-sample inference problems..

The source is that the estimators of B_0 involve instrumental variables and it is possible that one may have weak instruments, which can cause densities for the estimated coefficients in B_0 to depart substantially from a normal density centered on the true values of parameters in a finite sample. An alternative way of viewing this is that the estimated parameters in B_0 are functions of the $A_{ij}(1)$ and generally involve ratios of the latter - see (9). Whenever one has ratios of random variables it is unlikely that the density function will be normal in a finite sample. In fact the situation can be worse as we want to estimate impulse responses i.e. we ultimately want \hat{B}^{-1} . Thus the impact impulse response of the first variable to the second shock in the Blanchard-Quah model is

$$-\frac{\beta_{12}^0}{1 - \beta_{12}^0 \beta_{21}^0},$$

which is also a ratio. Consequently, even if $\hat{\beta}_{12}^0$ and $\hat{\beta}_{21}^0$ have normal distributions, the impulse responses may not.

Might we expect weak instruments? In the two variable set up of the Blanchard-Quah model the answer to such a question revolves around how correlated Δz_{2t} is with z_{2t-1} i.e. how persistent is the z_{2t} process? If $z_{2t} = \rho z_{2t-1} + v_t$ then

$$\Delta z_{2t} = (\rho - 1)z_{2t-1} + v_t$$

Stock et al (2002, p5) and argue that the F statistic ratio which tests if $\rho - 1$ is zero should exceed 10 if instruments are not to be regarded as weak. Assuming that $\rho \neq 1$, the variance of $\hat{\rho} - 1$ used in forming the F statistic is

$$\frac{\sigma_v^2}{TE(y_{t-1}^2)} = \frac{1}{T \frac{1}{1-\rho^2}},$$

and so the F statistic criterion translates into $\frac{(1-\rho)T}{(1+\rho)} > 10$. Roughly this means that for values of ρ over .82 there will be a weak instrument (when $T = 100$). Since many macroeconomic and financial time series have a value

¹Of course, as Erceg et al show and measure, there is a specification error if the process is VARMA but one estimates with a finite order VAR.

of ρ of over .8 it is clear that there is a very strong possibility of weak instrument problems surfacing in empirical work with long-run restrictions. Thus this makes a connection with the Faust and Leeper argument since instruments are weak when there is a unit root and we have noted that one might think of their analysis as pertaining to that case. However, it needs to be recognized that one can have other sources of weak instruments in VARs, and we will see this later in connection with the model in Gali (1992). Moreover, the weak instrument point is one that relates specifically to finite samples i.e. the inference problems are not there in large samples, whereas the Faust and Leeper argument involves positing the existence of a situation where there are large sample inference problems and then arguing that it should apply in small samples.

To translate this into a correlation between Δz_{2t} and z_{2t-1} we note that

$$\begin{aligned} E(\Delta z_{2t} z_{2t-1}) &= (\rho - 1) E z_{2t-1}^2 \\ E(z_{2t-1}^2) &= \frac{\sigma_v^2}{1 - \rho^2} \\ E(\Delta z_{2t}^2) &= (\rho - 1)^2 E(z_{2t-1}^2) + \sigma_v^2, \end{aligned}$$

so that the correlation is

$$\frac{\rho - 1}{\sqrt{2(1 - \rho)}}$$

and so values of the correlation that are smaller than .3 in absolute value would produce weak instruments (for $T = 100$).

We first look at this in the context of Blanchard and Quah and Gali's model that was described earlier.

5.1.1 Blanchard and Quah

As a preliminary we compute the correlation between U_{t-1} and ΔU_t and find that it is -.06 which strongly signals that we are likely to have weak instruments and so the distributions of \hat{B}_0 and \hat{B}_0^{-1} are unlikely to be normal. Figure 3 presents histograms and distributions of these quantities. The histograms rather than the kernel estimators of the densities are presented for $\hat{\beta}_{ij}^0$, since huge outliers are generated, and these distort any kernel estimators. It is clear that $\hat{\beta}_{12}^0$ and $\hat{\beta}_{21}^0$ deviate significantly from normality, while the impulse response distributions (the last four graphs) bear little resemblance to a normal random variable. Indeed there may be near-unit root issues here. Thus the standard methods of evaluating confidence intervals

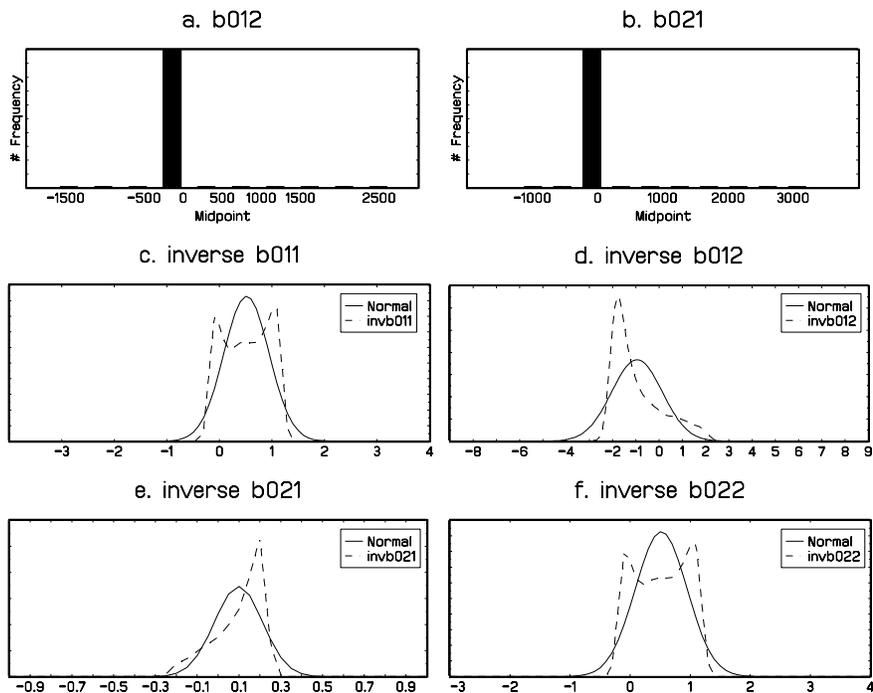


Figure 3: Blanchard and Quah (1989). Panels a. to b. histogram of selected elements of \widehat{B}_0 . Panels c. to f. empirical distributions of the elements of \widehat{B}_0^{-1} compared to normal.

with Blanchard-Quah type models that effectively assume normality will be misleading in indicating the uncertainty surrounding the estimated impulse responses. It may be possible to utilize the weak instrument literature to produce correct confidence intervals since we know the mapping between the impulses and the β_{ij}^0 (using the direct method).

The origin of the non-normality is quite clear from (9). If $\hat{A}_{22}(1)$ was centered on zero we will have a very non-normal density for $\hat{\beta}_{12}$. This scenario eventuates, as seen in Figure 4 which shows the histogram of $\hat{A}_{22}(1)$.

5.1.2 Gali

In this model there are possible potential problems with instrument quality as Δi_{t-1} , $i_{t-1} - \Delta p_{t-1}$ and $\Delta m_{t-1} - \Delta p_{t-1}$ are being used as instruments for $\Delta^2 i_t$, $\Delta i_t - \Delta^2 p_t$ and $\Delta^2 m_t - \Delta^2 p_t$ in the Δy_t equation. It is possible that these instruments are reasonable e.g. Δi_{t-1} may be correlated with $\Delta^2 i_t$ since it is likely that i_t is close to a pure random walk and so Δi_t will be white

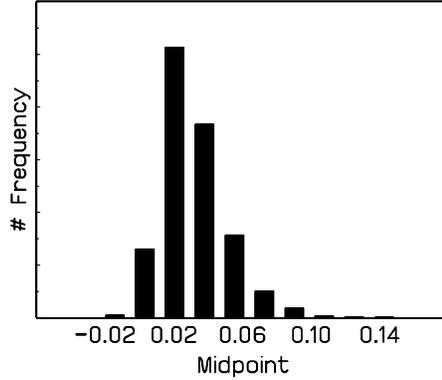


Figure 4: Blanchard and Quah (1989). Histogram of $\widehat{A}_{22}(1)$.

noise. Indeed in this case the correlation would be $\frac{1}{\sqrt{2}}$. As outlined earlier Gali also uses VAR residuals as instruments, and so there are a number of potential sources of weak instrument problems - the transition from the densities for $\widehat{\beta}_{01j}$ to $\widehat{\beta}_{02j}$ suggests that it is the latter which is the source of difficulties; this agrees with the analysis in Pagan and Robertson (1998) of the weak instrument problems in Gali's model. Since all we have done here is to update the data set there is little point in going over that analysis in any detail again, except to note that Figure 5 shows that the problems still persist with the longer sample.

The figures shows the distribution of the estimators of the 12 unknown coefficients in B_0 where we have derived these by assuming that the point estimates obtained from the data represent the true values and then asking what would happen if one tried to estimate such a SVAR using the Gali restrictions. The problems of weak instruments stand out, with few of the distributions looking as if they can be represented by a normal density. As one moves down these graphs we have once again replaced kernel estimators of the densities of $\widehat{\beta}_{ij}^0$ with histograms, because of the huge outliers. It should be clear from this diagram that treating the distribution of $\widehat{\beta}_{ij}^0$ as normally distributed, as is typically done when drawing standard errors around impulse responses, is seriously inaccurate. One needs to generate confidence intervals taking account of the weak instrument problem, but how to do this when there are multiple endogenous variables instruments is still a research area in its infancy.

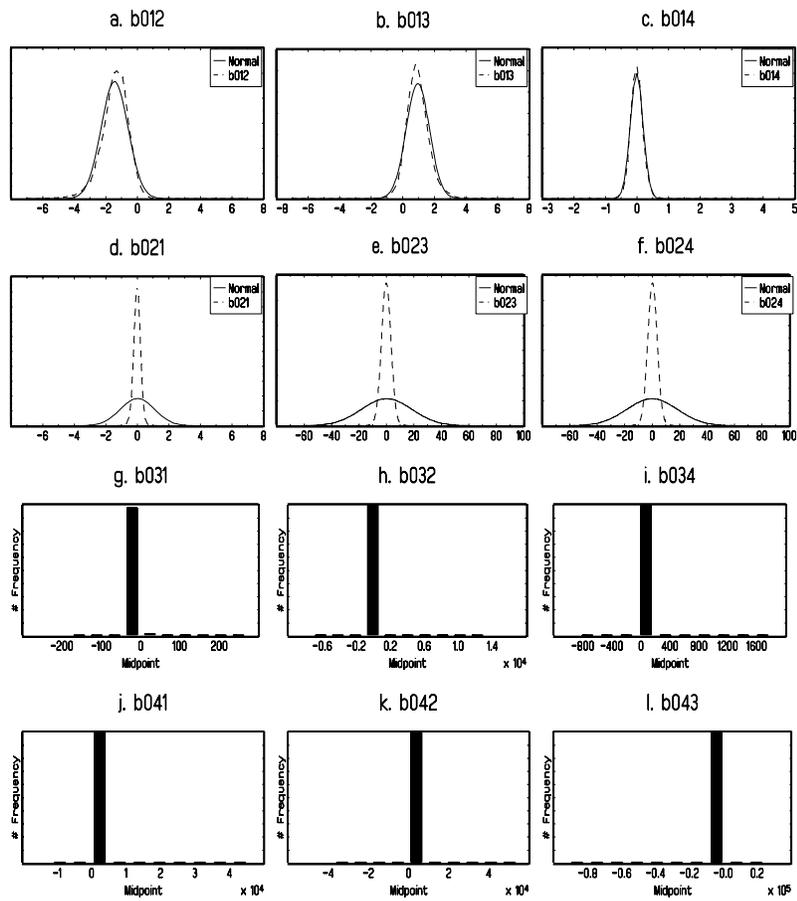


Figure 5: Gali (1992). Panels a. to f. empirical distributions of selected elements of \widehat{B}_0 compared to normal. Panels g. to l. histograms of selected elements of \widehat{B}_0 .

5.2 Sign Restrictions

How much uncertainty is there about impulse responses when we utilize sign restrictions. This is a complex question. For any given estimator of A and V one will probably be able to produce a set of impulse responses that satisfy the sign restrictions. Since \hat{A} and \hat{V} are random variables this will induce a distribution on the impulse responses. But what complicates the picture is that there is no unique set of impulses that satisfy the sign restrictions for any given \hat{A}, \hat{V} . In existing presentations both of these sources of uncertainty are combined together, but it seems important to analyse them separately, as some issues arise from the non-uniqueness do not seem to have been adequately addressed.

Let the matrix T^* indicate the impact impulse responses to the shocks. We know that the impulse responses for any lag j are T^*D_j . Fixing the VAR coefficients at their estimates \hat{A}, \hat{V} from the data would mean that \hat{D}_j wouldn't change, so we will study the distribution of \hat{T}^* alone. Now consider a two-variable SVAR where $T^* = \{\tau_{ij}^*\}$ is a 2×2 matrix. In this instance there is only one Givens rotation matrix Q_{12} and hence this depends on a scalar θ . Suppose we look at the effect of the first shock on the first variable i.e. $\hat{\tau}_{11}^*$. Because of the lack of uniqueness we may have quite a lot of different values for τ_{11}^* , each corresponding to the different θ which govern the value of Q_{12} . Let us therefore write $\tau_{11}^{*(k)}$ to indicate the k 'th value of τ_{11}^* , where $T^{*(k)}$ would be T^* evaluated at $\theta^{(k)}$. In the work reported on earlier we had 1000 values of θ and so $k = 1, \dots, 1000$.

How do we deal with this non-uniqueness? Uhlig (2005) and Peersman (2005) suggest that one report the median of $\tau_{11}^{*(k)}$. To investigate the utility of this we need to recognize that there is more than one impulse response to be computed. Suppose we look at the impact on the second variable of the second shock, τ_{22}^* . It would appear that the suggested way of summarizing the multiple estimates would be to use $med(\tau_{22}^{*(k)})$. Now here we may have a problem. The value of $med(\tau_{22}^{*(k)})$ was found by inserting some value of θ in the Givens matrix. Let us assume that this is the second value θ . It will also be the case that a value of θ produced $med(\tau_{11}^{*(k)})$, and this will be designated as $\theta^{(1)}$. Now there is nothing that makes these two values of θ the same. Since the *shocks* are identified by using a single T^* , and this depends upon the value of θ , we see that there may no longer be a single set of shocks that generates the recorded (median) impulse responses. Or, put another way, if we constructed shocks using the separate median impulse responses, we could no longer be sure that they are uncorrelated, as that requires the use of a single value of θ .

5.2.1 Blanchard-Quah

Let us look at this issue in the context of the Blanchard-Quah model. There are four impulse responses in B_0^{-1} and, when we compute the medians over the 1000 realizations, we find that the values of θ associated with these medians would be 0.180909, 0.180909, 0.180462 and 0.180909. Since these are close to one another it wouldn't seem as if one would be very concerned.

If the values of θ had been more disparate we would need to have another way of selecting a single value for θ that can be used to produce impulse responses and shocks. One way of doing this is to choose that value of $\theta^{(k)}$ that produces impulses that are as close to the median responses as possible. To devise a criterion to do this we need to recognize that the impulses are not unit free, so that we first standardize them by subtracting off their median, and then divide that quantity by their standard deviation over the 1000 realizations. These standardized impulses are then grouped into a vector $\phi^{(k)}$ (in the Blanchard and Quah case ϕ is 4×1 as there are four impulses) for each value $\theta^{(k)}$. Subsequently we choose the k that minimizes $\phi^{(k)'}\phi^{(k)}$, and then use that $\theta^{(k)}$ to calculate impulses. Whether this strategy produces a unique k is an empirical question, although in our applications it turns out to do so. The dashed lines in figure 1 showing the impulse responses for Blanchard-Quah from sign restrictions are the impulses found by this method. It is interesting that the value of θ found by this method is $\theta = 1.64545$ which emphasizes how insensitive the impulse responses are the values of θ are in this case. and that the lack of uniqueness of impulse responses in the Blanchard-Quah model is a minor issue. Consequently, more attention would naturally be paid to variation that comes from the fact that A and V need to be estimated.

An alternative we thought about was to use $med(\theta^{(k)})$ as the representative value of θ . The problem with this can be seen in figure 6 which reports the histogram of the 1000 θ 's that satisfy the sign restrictions in the Blanchard Quah model. It is also clear from this diagram how one managed to get a value of $\theta = 1.65$ as the solution to our synthetic optimization approach.

5.2.2 Peersman

Now as the number of variables expand the situation becomes much more complex. For example, in the four variable and four shock case of Peersman's model there will be six Givens matrices which are combined into a single one with the form

$$Q = Q_{12}(\theta_1) \times Q_{13}(\theta_2) \times Q_{14}(\theta_3) \times Q_{23}(\theta_4) \\ \times Q_{24}(\theta_5) \times Q_{34}(\theta_6).$$

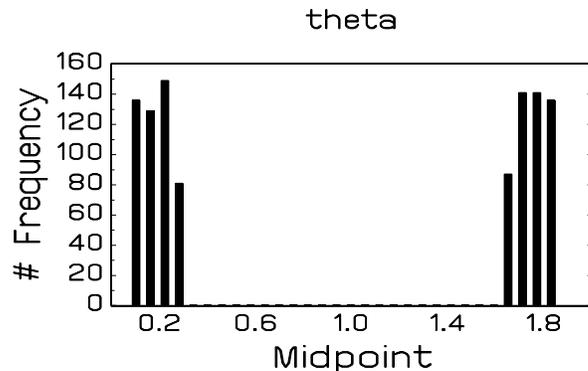


Figure 6: Histogram of $\phi^{(k)}$ that satisfy the Blanchard and Quah (1989) sign restrictions.

If we look at the values of θ that produce the median effect of output and price responses to an oil price shock we get the two implied vectors for θ of

$$[.627 \quad 1.820 \quad 1.830 \quad .250 \quad .418 \quad 1.150]$$

and

$$[.693 \quad 1.750 \quad 1.830 \quad 2.930 \quad 1.310 \quad .645]$$

respectively. Unlike the Blanchard-Quah case these are very different vectors, which may produce very different shocks, and so it becomes very unclear what the system actually is that the median of the impulse responses represent.

We applied our alternative method to this model and again the dashed lines in Figure 2 shows what we would get if we choose a single θ to represent the impulses with the constraint that we are trying to get as close to the medians as possible. Now ϕ is a 16×1 vector and it is clear that there are sometimes very different quantitative values assigned to the impulse responses. This emphasizes a point made earlier that it may not be possible to produce accurate quantitative values for impulses if we simply use sign restrictions.

The point of the exercises conducted above therefore has been to emphasize that the non-uniqueness of the impulses found using sign restrictions raises a question mark over how one is to summarize the information from the searches and that this is a question that has not been canvassed enough in the literature.

6 What do Variance Decompositions Tell us about the Cycle?

Suppose that a model has been fitted which enables one to write down a decomposition of y_t into a sum of past shocks:

$$y_t = \sum_{i=1}^K \sum_{j=0}^{\infty} C_{ij} \varepsilon_{it-j},$$

where ε_{it} are uncorrelated shocks and the impulse responses of y_t to a unit rise in ε_{it-j} are C_{ij} . We will think of y_t as being the log of a variable. The forecast variance $y_{t+L} - y_t$ can be written in terms of the sums of C_{ij}^2 and the fraction of the variance explained by each of the shocks can be determined. Often L is set to the “business cycle horizon” and the dominant contributor to the variance of $y_{t+L} - y_t$ is regarded as the “cause” of the business cycle.

What has this variance decomposition got to do with the business cycle? Although oft-repeated, the connection seems to be more one of assertion than coming from any analysis. We will illustrate this by looking at an updated version of the system set out in Gali (1992) whose estimation was described earlier. The estimated model is simulated to find the implied level of GDP (a deterministic trend is added back on to the cumulated values of Δy_t) and this simulated data was then used to investigate the nature of the business cycle implied by the model.² Thereafter we omit some shocks in order to see what their role is in the business cycle.

Table 1 US Business Cycle Characteristics,
Data and Gali’s SVAR Model: 1980-2002

	All shks	$\varepsilon_{jt} = 0(j = 2, 3, 4)$	$\varepsilon_{jt} = 0(j = 2, 3)$	Data
Dur Con	4.4	2.9	4.2	2.9
Dur Expan	18.3	34.9	19.8	20.2
Amp Con	-2.9	-1.1	-2.5	-2.1
Amp Expan	21.8	32.4	22.8	22.0

Table 1 shows that, if supply side shocks were the only factor present in the economy, the business cycle would be quite long, and that demand shocks play a major role in reducing the cycle to a length comparable to

²The turning points in y_t and the characteristics of the business cycle were found with the BBQ algorithm described in Harding and Pagan (2002).

that actually observed. Money demand and supply shocks play a minor role in the cycle since it stays the same length even when they are deleted. Thus an implication of the analysis above would be that, if demand shock volatility can be reduced, then one might expect expansions to be of the order of nine years or so. It may well be that this is an explanation of the improved performance in the past few decades. It is known that the volatility of GDP growth has declined and, if this was a consequence of a reduction in the volatility of demand shocks, we would expect a lengthening of the cycle. An interesting concomitant of the conclusion just reached is that it is contrary to Gali's conclusion which came from studying variance decompositions. He noted that using the latter showed that, when predicting variables 10 quarters out, 92% of the variance of the forecast error is explained by supply side shocks and only 3% for demand. This led him to comment (p722) that, "The results here seem less akin to a traditional Keynesian view of economic fluctuations".

The reason why variance decompositions at long horizons tells us little about business cycle determinants is simply that the turning points in the series y_t are found from the DGP of Δy_t , as the latter is the indicator of whether y_t is still rising (falling) or has started to fall (rise), and not that of a long difference $\Delta_L y_t$. To crystallize this, suppose we think of a recession as involving two periods of negative growth. Then we are looking at the probability of getting such an event and it will depend on the type of DGP that one has for Δy_t . Suppose Δy_t was an AR(1) of the form

$$\Delta y_t = \mu + \rho \Delta y_{t-1} + \sigma e_t$$

where $e_t \sim n.i.d.(0, 1)$. Then the probability of a turning point will depend upon the $var(\Delta y_t)$, since that will be a major factor in determining whether a negative value of Δy_t can be realized i.e. it is the variance of Δy_t which is important and *not* the $var(\Delta_L y_t)$, where L is the "business cycle frequency". The discrepancy between the predictions of impulse response analysis and the nature of the business cycle is apparent in many papers that plot the predictions of fitted SVAR (or DSGE) models. Indeed, in Gali's paper the plots of the tracking performance of the SVAR he fits shows that technology shocks did not have as dominant a role in the cycle as one would expect from his variance decompositions. The problem with this literature arises from a failure to carefully define what is meant by a business cycle. Ask the wrong question, and you get the wrong answer.

7 Conclusion

Our paper has shown that many of the applications of VAR methods to macroeconomic data need to be treated with extreme caution. One difficulty that was analysed is the fact that the order of the VAR will change as the set of variables included in it changes. We argued that existing VARs fail to capture the stock dimensions of macroeconomic outcomes and that more attention should be paid to retaining stock variables in the analysis rather than substituting them out, as has been the common practice.

We then turned to an analysis of how one was to estimate impulse responses to shocks of interest. Recursive system methods have been widely used, and these can produce strong results, albeit at the cost of imposing restrictions that are often incredible when viewed from the perspective of a macro-economic system. It is not surprising then that a literature has arisen that seeks to use much weaker information. It emerges from our analysis that the two most popular methods for doing this - long-run and sign restrictions - may be incapable of producing precise quantitative estimates of impulse responses. In some ways this should not be surprising. It is a powerful adage that weak information produces weak results, and both long-run and sign restrictions are weak information. It is extraordinary how we constantly think that this adage only applies to past methods and that, somehow, the new method will be exempt from it. Based on the frequency of use, and some of the claims made about these methods, one would be easily led to think that this is true for long-run and sign restriction methods of analysing data. We hope to have shown that this is far from the truth. This does not mean that the methods cannot be useful but it is clear that we need to develop techniques that enable us to precisely assess how much information they actually do provide. At the moment the information provided gives an impression of spurious accuracy. At least in the case of long-run restrictions there are developments that suggest ways that we may make an allowance for the weak instrument problems that plague this method. For sign restrictions we have made a suggestion about a procedure that tries to summarize the information pertaining to the non-uniqueness of the estimates of impulses from this technique, but no doubt there may be better methods of doing this.

Even when we do get good estimates of impulse responses we wish to use them to answer some question. An important one has always been what shocks drive the business cycle. Here the standard approach in the VAR literature has been to utilize a variance decomposition to provide an answer. We show that this procedure does not give a satisfactory answer, simply because it does not work with any well-defined notion of the business cycle. When one does, it becomes clear why variance decompositions will often give

the wrong answer.

8 References

Altig, D., L. Christiano, M. Eichenbaum and J. Linde (2005), "An Estimated Model of the US Business Cycle", paper presented to the *Macroeconomics and Reality: 25 Years Later* Conference, Pompeu-Fabra University, Barcelona.

Blanchard, O.J. and D. Quah (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances", *The American Economic Review*, 79, 655-673.

Binder, M and M.H. Pesaran (1995), "Multivariate Rational Expectations Models and Macroeconomic Modelling: A Review and Some New Results" in M.H. Pesaran and M. Wickens (eds) *Handbook of Applied Econometrics: Macroeconomics*, Basil Blackwell, Oxford.

Canova, F. and G. de Nicolò (2002), "Monetary Disturbances Matter for Business Fluctuations in the G7", *Journal of Monetary Economics*, 49, 1131-1159.

Canova, F. and Pina, J.P. "Monetary Policy Misspecification in VAR models", mimeo, Universitat Pompeu Fabra.

Chari, V.V., P.J. Kehoe and E. R. McGrattan (2004), "A Critique of Structural VARs Using Real Business Cycle Theory", *Working Paper no 631 Federal Reserve Bank of Minneapolis*.

Cooley, T.F. and M. Dwyer (1995), "Business Cycles Without Much Theory: A Look at Structural VARs", *Journal of Econometrics*, 83, 57-88.

del Negro, M., F. Schforeide, F. Smets and R. Wouters (2004), "On the Fit and Forecasting Performance of New-Keynesian Models", mimeo, Federal Reserve Bank of Atlanta.

Erceg, C.J., L. Guerrieri and C. Gust (2004), "Can Long-Run Restrictions Identify Technology Shocks?", *International Finance Discussion Paper 792*, Board of Governors of the Federal Reserve System.

Faust, J. (1998), "The Robustness of Identified VAR Conclusions about Money", *Carnegie-Rochester Series on Public Policy*, 49, 207-244.

Faust, J. and E.M. Leeper, (1997), "When do Long-Run Identifying Restrictions Give Reliable Results", *Journal of Business and Economic Statistics*, 15, 345-353.

Fernandez-Villaverde, J., J. Rubio-Ramirez and T.J. Sargent (2004), "The A,B,C (and D's) of VARs", mimeo, New York University.

Gali, J. (1992), "How Well does the ISLM Model Fit Postwar U.S. Data?", *Quarterly Journal of Economics*, 107, 709-735.

Giannone, D., L. Reichlin and L. Sala (2005), "VARs, Common Factors and the Empirical Validation of Equilibrium Business Cycle Models", *Journal of Econometrics* (forthcoming).

Giordani, P. (2004), "An Alternative Explanation of the Price Puzzle" , *Journal of Monetary Economics*, 51, 1271-1296.

Harding, D. and A.R. Pagan, (2003), "A Comparison of Two Business Cycle Dating Methods", *Journal of Economic Dynamics and Control*, 27, 1681-1690.

Kapetanios, G., A.R. Pagan and A. Scott (2005), "Making a Match: Combining Theory and Evidence in Policy-oriented Macroeconomic Modeling", CAMA Discussion Paper 2005/1, Australian National University.

Lippi, M. and L. Reichlin (1994), "VAR Analysis, Non Fundamental Representations, Blaschke Matrices", *Journal of Econometrics*, 63, 307-325.

Lütkepohl, H. (1993), *An Introduction to Multiple Time Series* (Springer-Verlag, Berlin)

Pagan, A.R. and Robertson, J.C. (1998), "Structural Models of the Liquidity Effect", *Review of Economics and Statistics*, 202-217.

Peersman, G. (2005), "What Caused the Early Millenium Slowdown? Evidence Based on Autoregressions", *Journal of Applied Econometrics*, 20, 185-207.

Quenouille, M.H. (1957), *The Analysis of Multiple Time Series*, Griffin's Statistical Monographs and Course No 1 (Griffin, London)

Stock, J.H., J.H. Wright and M. Yogo (2002), "A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments", *Journal of Business and Economic Statistics*, 20, 518-529.

Uhlig, H. (1999), "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily", in R. Marimon and A. Scott (eds) *Computational Methods for the Study of Dynamic Economies*, Oxford University Press, 30-61.

Uhlig H. (2005), "What are the Effects of Monetary Policy: Results from an Agnostic Identification Approach", *Journal of Monetary Economics*, 52, 381-419.

Wallis, K.F. (1977), "Multiple Time Series and the Final Form of Econometric Models", *Econometrica*, 45, 1481-1497.

Zellner, A. and F. Palm (1974), "Time Series Analysis and Simultaneous Equation Econometric Models", *Journal of Econometrics*, 2, 17-54.