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THE FISCAL MULTIPLIER AND SPILLOVER IN A GLOBAL LIQUIDITY TRAP

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The Fiscal Multiplier and Spillover in a Global Liquidity Trap*

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Abstract

We consider the fiscal multiplier and spillover in an environment in which two countries are caught simultaneously in a liquidity trap. Using a standard New Open Economy Macroeconomics (NOEM) model, an optimizing two-country sticky price model, we show that the fiscal multiplier and spillover are contrary to those predicted in textbook economics. For the country with government expenditure, the fiscal multiplier exceeds one, the currency depreciates, and the terms of trade worsen. The fiscal spillover is negative if the intertemporal elasticity of substitution in consumption is less than one and positive if the parameter is greater than one. Incomplete stabilization of marginal costs due to the existence of the zero lower bound is a crucial factor in understanding the effects of fiscal policy in open economies.

JEL Classification: E52, E62, E63, F41

Keywords: Zero lower bound; two-country model; fiscal policy; beggar-thy-neighbor

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1 Introduction

The liquidity trap has become an issue of global concern. The economic downturn following the financial turmoil that first began to emerge in 2007 has resulted in monetary policy with virtually a zero lower bound on nominal interest rates simultaneously in a number of countries, including Japan, the United Kingdom, and the United States. As a result, many countries are attempting to stimulate aggregate demand and production via fiscal expansion.

In this paper, we theoretically investigate the effects of fiscal policy in which two countries are caught simultaneously in a liquidity trap, and compare the results with that under normal circumstances. Using the standard New Open Economy Macroeconomics (NOEM) model, which is a two-country sticky price model, we analyze the fiscal multiplier—the extent to which one country's government expenditure increases production in that country—and the fiscal spillover—the extent to which one country's government expenditure boosts production in the other country. We examine whether fiscal expansion yields a beggar-thy-neighbor situation.

Fiscal expansion is considered to yield differing outcomes, depending on interest rate environments. According to textbook economics, fiscal expansion under flexible exchange rates is thought to be not effective (Dornbusch, Fisher, and Startz, 2008). In the Mundell-Fleming model (Mundell, 1967), where the economy is characterized by flexible exchange rates, fixed domestic prices, and perfect capital mobility, fiscal expansion builds up upward pressure on interest rates in the home country. Subsequently, the exchange rate appreciates. That offsets the increase in demand for home-produced goods by crowding out exports.[1]

Recently, the abovementioned NOEM model launched by Svensson and Van Wijnbergen (1989) and Obstfeld and Rogoff (1995) has become the workhorse model for the monetary policy analysis in an open economy. In the NOEM model with producer currency pricing,

[1] Whether this crowing out effect completely offsets the effects on the demand depends on the assumptions about the model. For example, another standard text book by Krugman and Obstfeld (2006) shows that not permanent but temporal fiscal expansion can have a positive short-run effect on output. Even with permanent fiscal expansion, if the economy is not initially at the long-run equilibrium, the fiscal multiplier can be positive.

Clarida, Gali, and Gertler (2002) and Benigno and Benigno (2003, 2006) show that the optimal monetary policy is aiming at price stability. When the central banks follow optimal monetary policy, as will be explained in detail later, domestic interest rates are raised after an increase in the domestically produced goods price stemming from a positive government shock. The nominal exchange rate appreciates and the terms of trade improve for the domestic country. This induces the production switching from domestic to foreign country. Consequently, the fiscal multiplier becomes less than one.

However, such conventional wisdom obtained from the Mundell-Fleming model or the NOEM model with optimal monetary policy for the ineffectiveness of fiscal policy may be overturned in a liquidity trap. This is because in the liquidity trap, nominal interest rates are kept low despite fiscal expansion. Low interest rates prevent the exchange rate from appreciating. The economic activity in the home country is thus stimulated rather than dampened. This mechanism in a liquidity trap is somewhat similar to the case when monetary policy does not completely stabilize but rather accommodates the distortions from a government spending shock, as examined by Obstfeld and Rogoff (1995). There, monetary policy, namely money supply, is assumed to be constant.[2] Relatively lowered domestic consumption due to a positive government shock yields a downward pressure on the home money demand.[3] To keep money supply constant, a nominal interest rate falls, resulting in a depreciation of the nominal exchange rate and worsening in the terms of trade for the domestic country. Consequently, the multiplier becomes larger than unity. In both cases, the key factor is incomplete stabilization of marginal costs.[4]

Using a standard two-country sticky price, namely the NOEM model, we demonstrate

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[2]In early NOEM models represented by Obstfeld and Rogoff (1995) or Corsetti and Pesenti (2001), money supply is the tool of monetary policy for nominal spending control. It is not trivial to express the problem regarding the zero lower bound on nominal interest rates with such NOEM models. Therefore, we employ the recently widely used NOEM model represented by Clarida, Gali and Gertler (2002) or Benigno and Benigno (2003), where short-term nominal interest rates are the monetary policy tool and nominal rigidity is introduced through the Calvo friction rather than one-period-ahead price setting.

[3]Relative domestic consumption decreases, because of the assumption of an incomplete market. In Obstfeld and Rogoff (1995), households in two countries have access to not state-contingent assets but an integrated world real bond.

[4]Note that this is true irrespective of whether we consider the closed or open economy.

that the size of multipliers and the sign (positive or negative) associated with the spillover in a global liquidity trap are contrary to those predicted in the Mundell-Fleming model or in the NOEM model with optimal monetary policy. In a global liquidity trap, the fiscal multiplier exceeds one. The fiscal spillover is negative if the intertemporal elasticity of substitution in consumption is less than one and positive if the parameter is greater than one.

Incomplete stabilization of marginal costs due to the existence of the zero lower bound is a crucial factor in understanding the effects of fiscal policy in open economies. Thanks to this, government spending in the home country raises the marginal costs of home-produced goods, which increases expected inflation rates and decreases real interest rates. Intertemporal optimization causes consumption to increase, so that the fiscal multiplier exceeds one. This effect functions to increase demand for foreign-produced goods, as well, but the fiscal spillover may be negative due to the following mechanism. While government spending continues, the price of home-produced goods increases more than that of foreign-produced goods. Expecting that two countries are at symmetric equilibrium when government spending ends, the home currency depreciates and the home terms of trade worsen on impact when government spending begins. That shifts demand for goods from foreign-produced goods to home-produced ones. The fiscal spillover thus may become negative depending on the intertemporal elasticity of substitution in consumption.

Reflecting Japan's experience around 2000 as well as the global financial crisis that began in the summer of 2007, several studies analyze the role of fiscal policy in a liquidity trap. Christiano (2004) demonstrates that the fiscal multiplier exceeds one in the presence of the zero lower bound.[5] A number of papers have examined his results using richer

[5] Regarding monetary policy, Fujiwara et al. (2009) investigate the optimal monetary policy when two countries are simultaneously caught in a liquidity trap in the form of a commitment policy. For a closed economy model, see also Reifschneider and Williams (2000), Kato and Nishiyama (2005), Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006, 2007), and Nakov (2008). For an extension to an open economy model, see Svensson (2001, 2003), Coenen and Wieland (2003), and Nakajima (2008).

In relation to the fiscal theory of the price level, see Benhabib, Schmitt-Grohe, and Uribe (2002) and Iwamura, Kudo, and Watanabe (2005).

frameworks with nonlinearity, different monetary policy, and/or various shocks and policy measures.[6] Our model does not deal with some of their developments, but instead discuss the fiscal multiplier and spillover in a global liquidity trap, which have not been studied in the previous literature.

The closest research to ours is Cook and Devereux (2011).[7] Cook and Devereux (2011) also analyze the effects of fiscal policy in a global liquidity trap using a sticky price model, reporting almost the same findings. Yet, there exist several differences. First, two papers introduce a different assumption regarding the duration of government expenditure. In Cook and Devereux (2011), the duration of government expenditure is the same as that of the adverse shock that brings about the liquidity trap. In our model, the duration of government expenditure is up to two periods, which is shorter than the duration of the adverse shock. By using this assumption, we simplify our analysis and clarify the underlying mechanism that produces the multiplier greater than one and the spillover which can be either positive or negative. Second, as our original contributions, we highlight the importance of the size of the intertemporal elasticity of substitution in influencing the sign of spillover. We also study the effects of incomplete markets and local currency pricing, instead of complete markets and producer currency pricing, which are common both in Cook and Devereux (2011) and ours. Especially, this paper is the first study to show that fiscal spillover in a global liquidity trap becomes positive with local currency pricing even when the intertemporal elasticity of substitution is less than one. In addition, we consider a less strict inflation targeting rule. Third, as the unique contribution of Cook and Devereux (2011), their paper incorporates home bias and analyzes optimal policy.

Other related studies include Corsetti, Meier, and Muller (2010) and Freedman et al. (2009), which evaluate the fiscal multiplier and spillover in a similar context. Their analyses suggest that a low interest rate environment is the key to yielding a greater fiscal multiplier and changing the sign associated with the fiscal spillover. Their models are, however, not strictly a model of a zero lower bound on nominal interest rates, and the

[6] See Bodenstein et al. (2009), Braun (2009), Christiano et al. (2009), Eggertsson (2009), and Woodford (2011).

[7] We found this paper after we released our discussion paper in March 2010.

method of maintaining low interest rates seems controversial. For example, Corsetti, Meier, and Muller (2010) establish a low interest rate environment by proposing a fiscal expansion policy that is followed by a reduction in spending over time. Our setup of considering a low interest rate environment by means of the zero lower bound is natural, and the bound is now realized. Moreover, while their analysis is numerical, we can show analytically the degree of the fiscal multiplier and the sign of the spillover. It helps us understand the underlying mechanism and determine the sensitivity of our results to the intertemporal elasticity of substitution in consumption.[8]

The structure of the paper is as follows. Section 2 introduces a standard two-country sticky price model (the NOEM model) used in this paper. In Section 3, we analyze the fiscal multiplier and spillover without the presence of the zero lower bound. In Section 4, we analyze the fiscal multiplier and spillover in the presence of the zero lower bound. Section 5 extends the model by incorporating incomplete financial markets, less strict interest rate rules against inflation and local currency pricing and Section 6 concludes.

2 Model

2.1 Model Setup

Our two-country sticky price (NOEM) model is a conventional one, similar to that used in Clarida, Gali, and Gertler (2002), Benigno and Benigno (2003), and Fujiwara et al. (2009). The economy consists of a home country H and a foreign country F. Labor is not mobile and it is used to produce a continuum of differentiated goods on the unit intervals $[0, 1]$ in both countries.

Household A representative household in the home country maximizes

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \prod_{\tau=1}^{t} \beta_\tau \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{h_t^{1+\omega}}{1+\omega} \right),$$

[8] A number of empirical studies also exist regarding the international as well as the domestic transmission of fiscal shocks (e.g., Perotti [2007], Corsetti and Muller [2008], Kim and Roubini [2008], and Feyrer and Shambaugh [2009]). In the studies on the international transmission of fiscal shocks, questions are related to whether we observe twin deficits: budget deficits and current account deficits, and their empirical results are mixed.

subject to the budget constraint:

$$P_tC_t + \mathrm{E}_t\left(Q_{t,t+1}D_{t+1}\right) + B_{t+1} = W_th_t + D_t + i_{t-1}B_t + \Pi_t - T.$$

The consumption bundle *per capita* C_t is defined by

$$C_t = \left(\frac{C_{H,t}}{n}\right)^n \left(\frac{C_{F,t}}{1-n}\right)^{1-n}. \tag{1}$$

As a result, the utility based aggregate price index (CPI) P_t, is described as

$$P_t = P_{H,t}^n P_{F,t}^{1-n}. \tag{2}$$

The consumption index represents bundles of differentiated home-produced goods $C_{H,t}$ and foreign-produced goods $C_{F,t}$, both spent in the home country *per capita*. The aggregate price index composes of a domestic good price $P_{H,t}$ and a foreign good price $P_{F,t}$. We denote the weight for the bundle of home-produced goods by n $(0 < n < 1)$ and for foreign-produced goods by $1-n$. We can use n to interpret as the relative size of the home country. The intratemporal elasticity of substitution between home and foreign goods is one. We denote hours worked by h_t, nominal wage by W_t, lump-sum profits by Π_t, taxes by T, bond holdings by B_t and the gross nominal interest rate by i_t. The object E_tD_{t+1} is an Arrow security, standing for the quantity of home currency to be delivered in period $t+1$ if state s_{t+1} is realized at $t+1$, conditional on history up to t. The associated price is $Q_{t,t+1}$. Household preferences are governed by factors common to the two countries: discount factor $0 < \beta_t < 1$, the inverse of the intertemporal elasticity of substitution in consumption σ, the utility weight on hours worked χ, and the Frisch elasticity of labor supply ω.

Following Eggertsson and Woodford (2003), Christiano (2004) and Christiano, Eichenbaum, and Rebelo (2009), we assume that the discount factor is stochastic and is the origin of the liquidity trap. We call this natural rate shock. In the zero-inflation steady state, the natural rate is positive, so the zero bound does not bind. However, if a large-scale shock decreases the natural rate to negative, the zero bound starts to bind. The real interest rate remains too high, contracting the real economic activity and rendering the economy sub-optimal.

The preferences are set as

$$C_{H,t}+G_t=\left[\int_0^1 C_{H,t}(j)^{1-\frac{1}{\varepsilon}}\,\mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where G_t represents government expenditure in the home country. Similarly, we define

$$C^*_{H,t}=\left[\int_0^1 C^*_{H,t}(j)^{1-\frac{1}{\varepsilon}}\,\mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

for home-produced goods $C^*_{H,t}$ that are spent in the foreign country, where superscript * denotes foreign variables.

Firms Regarding firms, Y_t and Y^*_t represent aggregate home and foreign productions respectively. An intermediate good is produced by a continuous number of monopolists using the production technology:

$$Y_t(j)=h_t(j).$$

Each monopolist maximizes its profits subject to its demand curve and the Calvo-type price friction where θ is the probability that the monopolist cannot re-optimize its price. Under the setting, each monopolist maximizes the discounted sum of the profits:

$$\begin{aligned}&\sum_{k=0}^{\infty}\theta^k \mathrm{E}_t\prod_0^k \beta_{t+k-1}\frac{C_{t+k}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+k}}\left[\bar{P}_{H,t}C_{H,t+k}(j)-W_{t+k}C_{H,t+k}(j)\right]\\ =\;&\sum_{k=0}^{\infty}\theta^k \mathrm{E}_t\prod_0^k \beta_{t+k-1}\frac{C_{t+k}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+k}}\left(C_{H,t+k}+G_{t+k}\right)\left[\bar{P}_{H,t}\left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}-W_{t+k}\left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}\right],\end{aligned}$$

where $\bar{P}_{H,t}$ is the reset price.

We assume producer currency pricing and therefore the law of one price. Let the aggregate price (inflation rate) of domestic goods and foreign goods be P_H (π_H) and P^*_F (π^*_F), respectively. Also, let a nominal exchange rate as the number of home currency units per unit of foreign currency be S_t. Then, the price of domestic goods sold in the foreign country is described simply as

$$P^*_{H,t}=\frac{P_{H,t}}{S_t}.$$

Fiscal and Monetary Policy Regarding fiscal policy, as is illustrated in equation (3), we assume that the government buys only domestic goods for G_t. This assumption yields the deviation of the terms of trade from its steady-state level. Fully coordinated fiscal spending in the two countries would make our results isomorphic to those in a closed economy framework.[9]

As for monetary policy, following Christiano (2004), we assume that domestic and foreign central banks conduct an optimal discretionary policy to stabilize inflation.

Market Clearing Finally, the resource constraint is given by

$$nY_t(j) = nC_{H,t}(j) + (1-n)\, C^*_{H,t}(j).$$

Output of the homogeneous home good is related to aggregate employment by

$$Y_t = \Delta_{H,t} h_t,$$

where the relative price dispersion term $\Delta_{H,t}$ is given by

$$\Delta_{H,t} = \int_0^1 \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon} \mathrm{d}j.$$

Using this, the resource constraint is rearranged as

$$nY_t = n\Delta_{H,t}\left(C_{H,t} + G_t\right) + (1-n)\, \Delta^*_{H,t} C^*_{H,t}. \tag{4}$$

Under the Calvo price frictions, the dynamics of $\Delta_{H,t}$ are defined by

$$\Delta_{H,t} = (1-\theta)\left[\frac{1-\theta\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{1-\varepsilon}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} + \theta\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\varepsilon}_{H,t-1} \Delta_{H,t-1},$$

where the gross inflation rate of $P_{H,t}$ is defined as

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}.$$

[9] For simplicity, we neglect government expenditure in the foreign country. Given symmetry, it is straightforward to analyze this, except for the size of home country n. The effect of government expenditure in both countries can be analyzed easily, too, by summing the effect of government expenditure in the home country and that in the foreign country, as long as the first-order approximation is accurate.

2.2 Linearized System of Equations

Using the above setup, we can derive the log-linearlized system of equations with respect to the four variables of $\hat{Y}$, $\hat{Y}^*$, $\hat{\pi}_H$, and $\hat{\pi}_F^*$, as follows:

$$\frac{1+n\left(\sigma-1\right)}{1-g\_y}(\Delta\hat{Y}_{t+1}-\Delta\hat{G}_{t+1})+\left(1-n\right)\left(\sigma-1\right)\Delta\hat{Y}_{t+1}^{*}=\hat{\imath}_t-\hat{\pi}_{H,t+1}+\hat{\beta}_t, \tag{5}$$

$$\left[1+\left(1-n\right)\left(\sigma-1\right)\right]\Delta\hat{Y}_{t+1}^{*}+\frac{n\left(\sigma-1\right)}{1-g\_y}(\Delta\hat{Y}_{t+1}-\Delta\hat{G}_{t+1})=\hat{\imath}_t^{*}-\hat{\pi}_{F,t+1}^{*}+\hat{\beta}_t, \tag{6}$$

$$\begin{aligned}\hat{\pi}_{H,t} &= \beta\hat{\pi}_{H,t+1}\\ &+\frac{\left(1-\theta\right)\left(1-\theta\beta\right)}{\theta}\left\{\left[\omega+\frac{1+n\left(\sigma-1\right)}{1-g\_y}\right]\hat{Y}_t+\left(1-n\right)\left(\sigma-1\right)\hat{Y}_t^{*}-\frac{1+n\left(\sigma-1\right)}{1-g\_y}\hat{G}_t\right\}\end{aligned} \tag{7}$$

and

$$\begin{aligned}\hat{\pi}_{F,t}^{*} &= \beta\hat{\pi}_{F,t+1}^{*}\\ &+\frac{\left(1-\theta\right)\left(1-\theta\beta\right)}{\theta}\left\{\left[\omega+1+\left(1-n\right)\left(\sigma-1\right)\right]\hat{Y}_t^{*}+\frac{n(\sigma-1)}{1-g\_y}\left(\hat{Y}_t-\hat{G}_t\right)\right\}.\end{aligned} \tag{8}$$

The circumflex (ˆ) indicates the log-linearized deviation of the variable from its steady state, with the exception of $\hat{G}_t=(G_t-G)/Y$. A parameter g_y represents the steady state ratio of government expenditure to output. The first two equations represent IS curves with respect to domestic goods and foreign goods. Deducting $\hat{G}_t$ from $\hat{Y}_t$, the IS curves indicate the Euler equations with respect to consumption. The last two equations represent the New Keynesian Phillips curves with respect to inflation in the home country and in the foreign country. Government spending influences the marginal costs of domestic and foreign goods via the resource constraints and the labor market equilibrium conditions.

3 Fiscal Multiplier and Spillover under Normal Circumstances

Before calculating the fiscal multiplier and spillover step by step, we summarize findings in this paper in Table 1. Numbers show the actual multiplier or spillover. Signs inside brackets represent a comparison of the fiscal multiplier and spillover with those of a closed economy: +, 0, and – imply that the multiplier or spillover is larger than, equal to, and smaller than in the closed economy, respectively. As we will see, incomplete stabilization

Table 1: Fiscal Multiplier and Spillover

		Multiplier		Spillover	
		$0 < \sigma < 1$	$\sigma > 1$	$0 < \sigma < 1$	$\sigma > 1$
Normal	$\hat{G}_t$	+, <1 (+)	+, <1 (−)	−	+
ZLB	$\hat{G}_t$	1 (0)	1 (0)	0	0
ZLB	$\hat{G}_{t+1}$	+ (−)	+ (+)	+	−

of marginal costs due to the existence of the zero lower bound is a crucial factor for these results.

The first row represents the multiplier or spillover under normal circumstances, provided contemporaneous government spending. The table demonstrates that the multiplier is below one, irrespective of the inverse of the intertemporal elasticity of substitution in consumption σ. Spillover can be positive or negative, depending on σ.

The second row represents the multiplier or spillover in a global liquidity trap where two countries are constrained by the zero lower bound, provided contemporaneous government spending. The table demonstrates that the multiplier is one, which is greater than that under normal circumstances. Spillover is zero. Government spending has no effect on foreign production.

The third row represents the fiscal multiplier and spillover in a global liquidity trap at time t, when government spending is anticipated for the next period $t+1$. The table demonstrates that the multiplier is positive. In other words, when the next period government spending is anticipated, home production increases on impact before its materialization. If contemporaneous government spending is also present, the multiplier becomes the sum of the second and third rows, hence exceeding one. Spillover is positive or negative, depending on σ. The sign flips over from that under normal circumstances.

3.1 Fiscal multiplier

To discuss Table 1 in detail, we first consider a case in which neither central bank in two countries is constrained by the zero lower bound of nominal interest rates, using the above

NOEM model summarized by equations (5) to (8). With producer currency pricing, there are only two kinds of aggregate inflation. Therefore, irrespective of the inflation index to be targeted, two policies in two countries are enough to stabilize inflation in two countries.[10]

Further, as shown by the NOEM models of Clarida, Gali, and Gertler (2002) and Benigno and Benigno (2003), since price stability of producer prices is the optimal discretion and the commitment monetary policy, central banks achieve zero inflation in both countries.[11] Under this optimal monetary policy, marginal costs expressed in the first term of the right-hand side of the Phillips curves in equations (7) and (8) become zero. Thus, analysis in this section can be considered as one to use the simple international real business cycle model without capital, namely under the flexible price equilibrium.

As a result, the fiscal multiplier associated with government spending in the home country is given by

$$\frac{d\hat{Y}_t}{d\hat{G}_t} = 1 - \frac{\omega(1-g\_y)\left[1+\omega+(\sigma-1)(1-n)\right]}{\left[\omega(1-g\_y)+1-n+\sigma n\right]\left[1+\omega+(\sigma-1)(1-n)\right]+(\sigma-1)^2 n(1-n)}.$$

Since the second term is positive under standard calibration of parameters, the fiscal multiplier is lower than one. Compared to the fiscal multiplier in a closed economy model in Christiano (2004)

$$\frac{d\hat{Y}_t}{d\hat{G}_t}|_{\text{closed}} = \frac{\sigma}{\omega(1-g\_y)+\sigma},$$

we can see

$$\begin{aligned} &\operatorname{sign}\left\{\frac{d\hat{Y}_t}{d\hat{G}_t} - \frac{d\hat{Y}_t}{d\hat{G}_t}|_{\text{closed}}\right\} \\ = \ & -(\sigma-1)(1-n)\left[1+\omega+(\sigma-1)(1-2n)\right]. \end{aligned}$$

The two multipliers naturally become equal when $n = 1$, that is, when the relative size of the home country is infinite. The inverse of the intertemporal elasticity of substitution

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[10] Note that CPI inflation rates, namely inflation rates based on the welfare based price index, are the same in both countries. Under the complete international financial market and the settings of preferences in this paper, there exists only one real interest rate. According to the Fisher equation, CPI inflation expectations are also the same in both countries.

[11] The optimal targeting rule of the home central bank is static and does not contain any foreign variables. This is because we do not include any distortionary shock, such as the cost-push shock examined in Clarida, Gali, and Gertler (2002). Therefore, there is no trade-off in stabilizing between price and output.

in consumption σ influences the relative size of the fiscal multiplier to that in a closed economy. When $\sigma > 1$, the fiscal multiplier is smaller than it is in a closed economy. When $0 < \sigma < 1$, the fiscal multiplier becomes larger than that in a closed economy. When $\sigma = 1$, as shown by Clarida, Gali, and Gertler (2002), the two countries become insular. Hence, the fiscal multiplier equals that in a closed economy, irrespective of the size of n.

3.2 Fiscal spillover

The fiscal spillover of the home country's government spending in the foreign country is given by

$$\frac{d\hat{Y}_t^*}{d\hat{G}_t} = \frac{\omega n(\sigma - 1)}{\left[\omega(1 - g\_y) + 1 + n\left(\sigma - 1\right)\right]\left[1 + \omega + \left(\sigma - 1\right)\left(1 - n\right)\right] + \left(\sigma - 1\right)^2 n\left(1 - n\right)}.$$

The fiscal spillover is positive if $\sigma > 1$ and negative if $0 < \sigma < 1$. If $\sigma = 1$, the two countries become insular and there is no spillover. Naturally, there is no spillover in a closed economy model.

3.3 Mechanism

The intuition behind these results and the differences from the case under the closed economy can be understood as follows. The key is the effects from the terms of trade ToT_t. The terms of trade are denoted by

$$ToT_t = \frac{S_t P_{F,t}^*}{P_{H,t}} = \frac{Y_t - G_t}{Y_t^*}, \tag{9}$$

which is log-linearized as

$$\widehat{ToT}_t = \frac{\hat{Y}_t - \hat{G}_t}{1 - g\_y} - \hat{Y}_t^*. \tag{10}$$

Government spending directly increases production and employment in the home country. That yields an upward pressure on home prices, and to prevent this, monetary policy is tightened and real interest rates rise. Monetary tightening generates home exchange rate appreciation and improves the terms of trade as shown in equation (9). As Clarida, Gali, and Gertler (2002) and Fujiwara et al. (2009) point out, the terms of trade have two opposing effects on real marginal costs MC_t. This is clear from the equation below, which is derived by combining the resource constraint and the labor market equilibrium condition:

$$\begin{aligned} MC_t &= h_t^{\omega} C_t^{\sigma} ToT_t^{1-n} \\ &= \Delta_{H,t}^{-\sigma} h_t^{\sigma+\omega} ToT^{(1-\sigma)(1-n)}. \end{aligned}$$

This equation implies that, first, by affecting output prices, the improvement in the terms of trade directly decreases real marginal costs in the home country.[12] On the other hand, it increases real marginal costs in the foreign country. To completely stabilize inflation rates so that real marginal costs become constant, home employment needs to increase while foreign employment needs to decrease. Second, the improvement in the terms of trade induces production switching from domestic to foreign goods. Given domestic output, that requires a rise in output in the foreign country. Total consumption rises in both countries, and due to risk sharing, both domestic and foreign consumption rise by the same amount. The rise in domestic consumption raises real marginal costs in the home country, and to offset the rise in prices, home employment needs to decrease.[13] Its size grows with σ.

Consequently, if $\sigma > 1$, the first channel is dominated by the second channel. Home employment decreases, which makes the fiscal multiplier smaller than that in a closed economy. Foreign employment increases, which yields positive fiscal spillover. If $0 < \sigma < 1$, the first channel dominates the second channel. Home employment increases, which makes the fiscal multiplier larger than that in a closed economy. Foreign employment decreases, which yields negative fiscal spillover. If $\sigma = 1$, the two channels offset each other. The fiscal multiplier equals that in a closed economy and the fiscal spillover becomes zero, since the two countries are insular.[14]

It is worth noting that the implications for welfare are different. Higher production means higher employment. That has the effect of decreasing welfare, while because of

[12] This is the *terms of trade effects* according to Clarida, Gali, and Gertler (2002).

[13] This is the *risk sharing effects* according to Clarida, Gali, and Gertler (2002).

[14] The sensitivity to σ depends on the form of utility. For example, Greenwood, Hercowitz, and Huffman (1988) use the following utility function:

$$\frac{1}{1-\sigma}\left(C_t - \chi \frac{h_t^{1+\omega}}{1+\omega}\right)^{1-\sigma}.$$

That function, not additively separable, abstracts the income effect on labor. Real marginal costs are independent of σ.

perfect risk sharing, consumption in the two countries is the same.[15] Recall that, if $\sigma > 1$, the fiscal multiplier is lower and the fiscal spillover is higher compared with the case of a closed economy. Therefore, if $\sigma > 1$, government spending produces superior social outcomes for the country than would be seen in the case of a closed economy, because households in the foreign country work more to produce more goods. That leads to *the beggar-thy-neighbor* problem. Also note that, as Tille (2001) and Benigno and Benigno (2003) point out, welfare implications are highly dependent on other parameters, such as the intratemporal elasticity of substitution between home and foreign goods, which in this paper is assumed to be 1. Consequently, whether σ is larger or smaller than unity is crucial in the analyses of this paper.

4 Fiscal Multiplier and Spillover in a Global Liquidity Trap

We next consider a case in which the zero lower bound constrains central banks. Central bank cannot achieve complete price stability any more. We follow Christiano (2004) and Christiano, Eichenbaum, and Rebelo (2009). The natural rate shock $\hat{\beta}_t$ occurs at periods $t = 1, \cdots, T-1$, and returns to zero at period T. Central banks pursue optimal discretionary policy to stabilize inflation. For simplicity, we further assume that government spending in the home country G_t is implemented only for two periods, at $t = 1$ and $t = 2 < T$.[16] The government spending is not large enough to push the economy out of the liquidity trap.

Using lag-operator L, we arrange equations (5) to (8). Since we assume that government spending in the home country G_t is implemented for only two periods, at $t = 1$ and $t = 2$, first-order approximation up to L^{-1} is sufficient. The fiscal multiplier is approximated up

[15] Note that we do not assume any utility gain from government expenditure. For an analysis on the optimal level of government expenditure with a small utility gain from it, see Christiano, Eichenbaum, and Rebelo (2009).

[16] Extending the periods of government spending does not change our main results. Also for simplicity, we focus on the fiscal multiplier and spillover only on the initial date. We can compute those on the future dates following Christiano (2004), but the analysis becomes less intuitive.

to the first order as

$$\begin{aligned}\hat{Y}_t &= C(L^{-1})(\hat{\imath}_t + \hat{\beta}_t) + C^*(L^{-1})(\hat{\imath}_t^* + \hat{\beta}_t) \\ &\quad + \left[1 + \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{n+\sigma(1-n)}{\sigma}\omega(1-g\_y)L^{-1}\right]\hat{G}_t, \qquad (11)\end{aligned}$$

where $C(L^{-1})$ and $C^*(L^{-1})$ are the first-order functions of L^{-1}, which are of little importance for the fiscal multiplier and spillover in a global liquidity trap.

4.1 Fiscal multiplier

The coefficients on $\hat{G}_t$ and $\hat{G}_{t+1}$ represent the fiscal multiplier. Equation (11) suggests that completely temporary government spending $\hat{G}_t$ has a multiplier of one. It already exceeds the multiplier in the previous section. Government spending in the next period $\hat{G}_{t+1}$ has a multiplier of

$$\frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{n+\sigma(1-n)}{\sigma}\omega(1-g\_y),$$

in period t. Multi-period government spending increases the fiscal multiplier. As σ falls, the fiscal multiplier becomes larger. Government spending in the past period, t-1, has no effect on output at time t, because the model is purely forward-looking.

According to Christiano (2004), the fiscal multiplier in a closed economy is 1 for $\hat{G}_t$. A one-time government expenditure has a multiplier of the same amplitude even in a global liqudity trap. The fiscal multiplier in a closed economy for $\hat{G}_{t+1}$ is given by

$$\frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{\omega}{\sigma}(1-g\_y).$$

If $\sigma > 1$, the fiscal multiplier in an open economy is larger than it is in a closed economy. If $0 < \sigma < 1$, the fiscal multiplier is smaller than that in a closed economy. If $\sigma = 1$, the fiscal multiplier equals that in a closed economy, since the two countries become insular. Interestingly, the results here are quite contrary to those obtained in the previous section.

4.2 Fiscal spillover

The fiscal spillover is given by

$$\begin{aligned}\hat{Y}_t^* &= D^*(L^{-1})(\hat{\imath}_t^* + \hat{\beta}_t) + D(L^{-1})(\hat{\imath}_t + \hat{\beta}_t) \\ &\quad - \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{(\sigma-1)n\omega}{\sigma}L^{-1}\hat{G}_t,\end{aligned}$$

where $D(L^{-1})$ and $D^{*}(L^{-1})$ are the first-order functions of L^{-1}. The coefficients on $\hat{G}_t$ and $\hat{G}_{t+1}$ are zero and

$$-\frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{(\sigma-1)\,n\omega}{\sigma},$$

respectively. Completely temporary government spending $\hat{G}_t$ has no spillover while government spending for the next period $\hat{G}_{t+1}$ has negative (positive) spillover if $\sigma > (<)1$. If $\sigma = 1$, there is no spillover. As n rises, the absolute size of the spillover tends to increase. Note that the signs of the spillover computed here are the opposite of those obtained under normal circumstances.

4.3 Mechanism

For the fiscal multiplier, we discuss the differences from the cases without a liquidity trap in the open economy as well as with a liquidity trap in a closed economy. For the fiscal spillover, since there is no spillover effect in a closed economy, we only compare the results without a liquidity trap to these with a liquidity trap, using the same NOEM model. Incomplete stabilization of marginal costs due to the existence of the zero lower bound is a crucial factor in determining the size of the fiscal multiplier and spillover.

4.3.1 Fiscal Multiplier

The mechanism for the higher fiscal multiplier in a global liquidity trap compared to the case without zero lower bound is basically the same as those obtained in Christiano (2004), Christiano, Eichenbaum, and Rebelo (2009), and Woodford (2011).[17] Government spending directly increases production and employment in the home country. That yields an upward pressure on home prices. Because of the zero lower bound, inflation rates and marginal costs are not stabilized.[18] They increase, and real interest rates drop. Intertemporal

[17] As written in Introduction, this mechanism is somewhat similar to the situation supposed in Obstfeld and Rogoff (1995) that monetary policy does not completely stabilize the inefficient fluctuations stemming from a government spending shock.

[18] The importance of the fluctuations in marginal costs on the fiscal multiplier is also pointed out by Braun (2009) and implicitly by Christiano, Eichenbaum, and Rebelo (2009). If monetary policy is conducted following the Taylor type rule and induces incomplete stabilization, multipliers become larger in this case than under complete price stability.

optimization causes consumption to increase. That yields a greater fiscal multiplier than that in the previous section. Thus, the increase in consumption is larger as σ is lower.

There exist open economy effects via the changes in the terms of trade through σ, that yield differences from the case in a closed economy, as is explained in the previous section. The sign of the fiscal multiplier does, however, not change by σ as shown in Table 1.

These altogether imply that the main factor for the higher multiplier in a global liquidity trap compared to the case without zero lower bound is the inability of complete stabilization of inefficient fluctuations of marginal costs.

4.3.2 Fiscal Spillover

Table 1 shows the quite contrary results for the fiscal spillover depending on whether economies are caught in a global liquidity trap or not. Again, the difference is stemming from the incomplete stabilization of marginal costs in a global liquidity trap, but we here explain the detailed mechanism behind this result.

Government spending has two opposing effects. First, it shifts demand for goods from foreign-produced goods to home-produced goods. When government spending ends, the terms of trade are at equilibrium: the price of home-produced goods is as expensive as that of foreign-produced goods. While government spending continues, the price of home-produced goods increases more than that of foreign-produced goods. In other words, reflecting the higher inflation of home-produced goods in the future, nominal exchange rates jump toward deprecation immediately after the government spending shock. Therefore, when government spending begins, the terms of trade worsen: home-produced goods are relatively cheaper than foreign-produced goods. It increases demand for home-produced goods, and decreases demand for foreign-produced goods. Second, the increase in expected inflation rates lowers real interest rates and increases demand for foreign-produced goods. The second effect becomes weaker, as the inverse of the intertemporal elasticity of substitution in consumption, σ, becomes greater. If σ is greater (less) than one, the first effect dominates (is dominated by) the second channel. Fiscal spillover becomes negative (positive).

More precise mechanism runs as follows. For completely temporary government spending, domestic production rises by the same amount while home consumption does not change. Real marginal costs rise, which increases prices of home-produced goods on impact. In the foreign economy, production and consumption do not change, so prices of foreign-produced goods measured in the foreign currency do not change. The terms of trade do not change because the currency of the home country depreciates on impact.

The fact that the nominal exchange rates change may seem contradictory with the uncovered interest parity condition, given that nominal interest rates in both countries are bound at zero. Yet, it is not contradictory at all: the expected change in nominal exchange rates remains unchanged. Also note that because government spending is completely temporary, expected inflation rates in two countries do not change. That makes real interest rates and consumption unchanged in both countries.

Then, consider a case in which there is government spending at $t = 2$ as well as at $t = 1$. Backward induction addresses the underlying mechanism. The economy at $t = 2$ is close to the economy when there is completely temporary government spending at $t = 1$. The terms of trade are at equilibrium at $t = 2$. Prices of home-produced goods rise by $\hat{\pi}_{H,2}$ while prices of foreign-produced goods do not change. The only difference is in the nominal exchange rates. From $t = 1$ to $t = 2$, nominal exchange rates do not move due to the uncovered interest parity condition. Expected appreciation (depreciation) is zero reflecting the zero nominal interest rates in both countries. It suggests that the terms of trade at $t = 1$ are positive by $\hat{\pi}_{H,2}$, meaning that the terms of trade worsen or nominal exchange rates depreciate at $t = 1$ for the home country.[19] From equation (10), we have

$$\hat{\pi}_{H,2} = \frac{\hat{Y}_1 - \hat{G}_1}{1 - g\_y} - \hat{Y}_1^*. \tag{12}$$

Worsening of the terms of trade increases home-produced goods but decreases foreign-produced goods. On the other hand, because of a rise in the home-produced good at $t = 2$, the aggregate inflation rate in the home country rises by $n\hat{\pi}_{H,2}$, which lowers the real interest rate by $n\hat{\pi}_{H,2}$ at $t = 1$. Due to intertemporal optimization, that increases aggregate consumption in the two countries as

$$\frac{n\hat{\pi}_{H,2}}{\sigma} = n\frac{\hat{Y}_1 - \hat{G}_1}{1 - g\_y} + (1 - n)\hat{Y}_1^*. \tag{13}$$

[19] Numerical exercise in Figure 1 shown later illustrates these dynamics.

A decrease in the real interest rate increases both home-produced goods and foreign-produced goods. Clearly, equations (12) and (13) altogether suggest that, for $0 < \sigma < 1$, $\hat{Y}_1^*$ increases, while for $\sigma > 1$, $\hat{Y}_1^*$ decreases.[20] As σ is larger, the effect of the real interest rate on $\hat{Y}_1^*$ becomes smaller. The increase in $\hat{Y}_1^*$ is dominated by the decrease in $\hat{Y}_1^*$ stemming from the worsening (improvement) of the terms of trade for the home (foreign) country. The fiscal spillover thus becomes negative for a large σ.

Regarding welfare, a lower fiscal multiplier or higher spillover is better for the country where the government spending takes place. Therefore, if $\sigma > 1$, government spending is socially worse for the country with government spending than in a closed economy, since households in the foreign country work less to produce less goods. It does not yield the *beggar-thy-neighbor* problem, but ends up with *beggar-thy-self.*

5 Extension

In this section, we extend the model by incorporating incomplete financial markets, a less strict interest rate rule against inflation, or local currency pricing.

5.1 Incomplete Financial Markets

In the main part of this paper, we assumed complete financial markets. The household has an access to the Arrow security, leading to perfect international risk sharing:

$$C_t^{*-\sigma} = C_t^{-\sigma} \frac{S_t P_t^*}{P_t} = C_t^{-\sigma} e_t.$$

Since our model does not assume a home bias in demand for goods and the representative households of the two countries are equally wealthy in the initial period, the real exchange rate e_t remains unity. Home and foreign consumption is perfectly aligned:

$$\hat{C}_t = \hat{C}_t^*.$$

When markets are incomplete, such perfect international risk sharing is not always attainable. As a particular and rather extreme type of incompleteness, let us consider

[20] Regarding exchange rates, the validity of the uncovered interest parity condition has often been called into question. Our results are, however, robust to the uncovered interest parity condition because there are no interest rate differences between two countries.

financial autarky. Net nominal export must be zero, and the above risk sharing equation is replaced by

$$(1-n)P_{H,t}C^*_{H,t} - nP_{F,t}C_{F,t} = 0.$$

Using demand equations and log-linearizing the above equation, we obtain

$$\hat{p}_{H,t} + (-\hat{p}_{H,t} + \hat{C}^*_t) = \hat{p}_{F,t} + (-\hat{p}_{F,t} + \hat{C}_t).$$

Therefore, the same international risk sharing equation holds:

$$\hat{C}_t = \hat{C}^*_t.$$

Such a result appears surprising, but actually is consistent with the finding discovered by Cole and Obstfeld (1991). They reveal that fluctuations in the terms of trade generate a similar outcome to international risk sharing, because a country's terms of trade are negatively correlated with growth in its export sector. Speaking of our model, in the above equation of net nominal export, the log-deviation of the terms of trade $\hat{p}_{F,t} - \hat{p}_{H,t}$ has two opposing effects. The improvement of the terms of trade raises the unit price of net export, while it decreases the quantity of net export by shifting demand from home-produced goods to import goods. Those two effects cancel each other completely, when the intratemporal elasticity of substitution between home and foreign goods is one. This terms of trade effect provides perfect insurance.

Consequently, our previous results are unchanged under incomplete financial markets. When the intratemporal elasticity of substitution between home and foreign goods deviates from one, our previous results will change. But its degree is minor under plausible parameter values, according to Cole and Obstfeld (1991) and Woodford (2010).

5.2 Interest Rate Rule

To examine the effects of monetary policy on the fiscal multiplier and spillover, we simulate the model using four variants of the following interest rate rule:

$$\hat{\imath}_t = \rho\hat{\imath}_{t-1} + \alpha\hat{\pi}_{H,t}.$$

The first is our benchmark, which is equivalent to $\rho = 0$ and $\alpha \to \infty$. Second, we consider the rule of $\rho = 0$ and $\alpha = 1.5$. Third and fourth, we consider the rules with smoothing, captured by $\rho = 0.5$ and $\alpha = 1.5$ and $\rho = 0.9$ and $\alpha = 1.5$, respectively. Since our interest is in the fiscal multiplier and spillover in the global liquidity trap, we focus only on the case in which the two countries are faced with the zero lower bound. In the simulation, a sequence of large negative shocks to the natural rate of interest ($\hat{\beta} = 0.05$) is given for ten quarters. We assume fiscal spending with the size of 1 % of GDP only in the domestic country. The duration of such fiscal spending is set to be two quarters. Parameters are calibrated to the conventional values as in Table 2.

Table 2: Parameters

Parameters	Values	Explanation
σ	5	Relative risk aversion
β	0.99	Subjective discount factor
ω	1	Frisch elasticity
θ	0.75	Calvo parameter
n	0.5	Home country size
g_y	0.1	Share of fiscal expenditure

Figure 1 demonstrates the time-series paths of eight key variables: outputs, interest rates, changes in producer prices in the two countries, the terms of trade, and changes in the nominal exchange rate. All lines illustrate their percentage deviations from those without fiscal spending. Simulation results are consistent with our prediction made in the previous section. As equation (11) suggests, completely temporary government spending $\hat{G}_t$ has a multiplier of one in period 2. In period 1, government spending yields the multiplier larger than 1. Since we assume $\sigma > 1$ in this simulation, the fiscal spillover is negative. The terms of trade and the nominal exchange rate both increase, suggesting a worsening of the terms of trade and a currency depreciation.

We cannot find any difference in both multiplier and spillover among examined policy rules. As is obvious from the two figures in the second row, in order to compute the multiplier and spillover in the global liquidity trap, we deliberately create a situation where nominal interest rates are constrained at zero irrespective of fiscal spending. It is

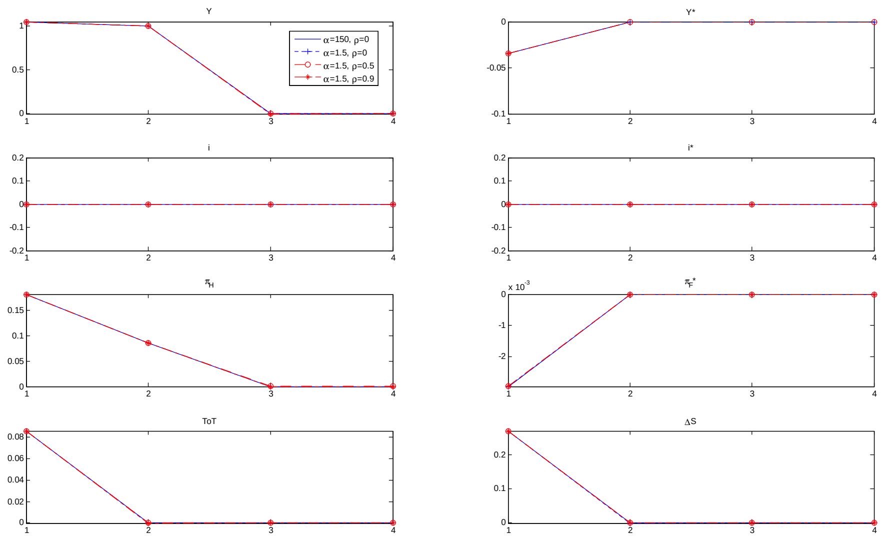


Figure 1: Multiplier and Spillover (PCP)

true that a difference among interest rate rules can alter the optimal paths of variables after the termination of the zero bound period, which also induces a jump in the nominal exchange rate in period one. This can lead to some difference in the multiplier and spillover, if negative shocks to the natural rate of interest rate are small and the duration of such shocks is short.

5.3 Local Currency Pricing

Under local currency pricing, the law of one price no longer holds. Instead, home firms optimize the price of goods sold in the foreign country, $P^*_{H,t}$, by maximizing the following profits:

$$\sum_{k=0}^{\infty}\theta^k \mathrm{E}_t \prod_{0}^{k}\beta_{t+k-1}\frac{C_{t+k}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+k}}\left[\bar{P}^*_{H,t}C^*_{H,t+k}(j)-\frac{W_{t+k}}{S_{t+k}}C^*_{H,t+k}(j)\right].$$

We here assume the same price stickiness θ for $P^*_{H,t}$ as that for $P_{H,t}$. Similarly, $P_{F,t}$ can be calculated for foreign goods sold in the home country.

Under those setup, nine equations are obtained for nine variables $\hat{Y}$, $\hat{Y}^*$, $\hat{e}$, p_H, p^*_F, $\hat{\pi}$,

$\hat{\pi}^*$, $\hat{\pi}_H$, and $\hat{\pi}_F^*$:

$$\begin{aligned}\hat{Y}_t =& -\frac{1-g\_y}{\sigma}\left[\hat{\imath}_t - \hat{\pi}_{t+1} + \hat{\beta}_t\right] + \hat{Y}_{t+1} - \Delta\hat{G}_{t+1} \\ &+n(1-g\_y)\Delta\hat{p}_{H,t+1} - \frac{(1-n)^2}{n}(1-g\_y)\Delta\hat{p}^*_{F,t+1} \\ &+\frac{(1-n)(1-g\_y)}{\sigma}\Delta\hat{e}_{t+1},\end{aligned}$$

$$\begin{aligned}\hat{Y}_t^* =& -\frac{1}{\sigma}\left[\hat{\imath}_t^* - \hat{\pi}^*_{t+1} + \hat{\beta}_t\right] + \hat{Y}^*_{t+1} \\ &+(1-n)\Delta\hat{p}^*_{F,t+1} - \frac{n^2}{1-n}\Delta\hat{p}_{H,t+1} \\ &-\frac{n}{\sigma}\Delta\hat{e}_{t+1},\end{aligned}$$

$$\frac{1}{1-g\_y}\hat{Y}_t - \frac{1}{1-g\_y}\hat{G}_t + \frac{n}{1-n}\hat{p}_{H,t} - \frac{1-n}{n}\hat{p}^*_{F,t} = \hat{Y}_t^*, \tag{14}$$

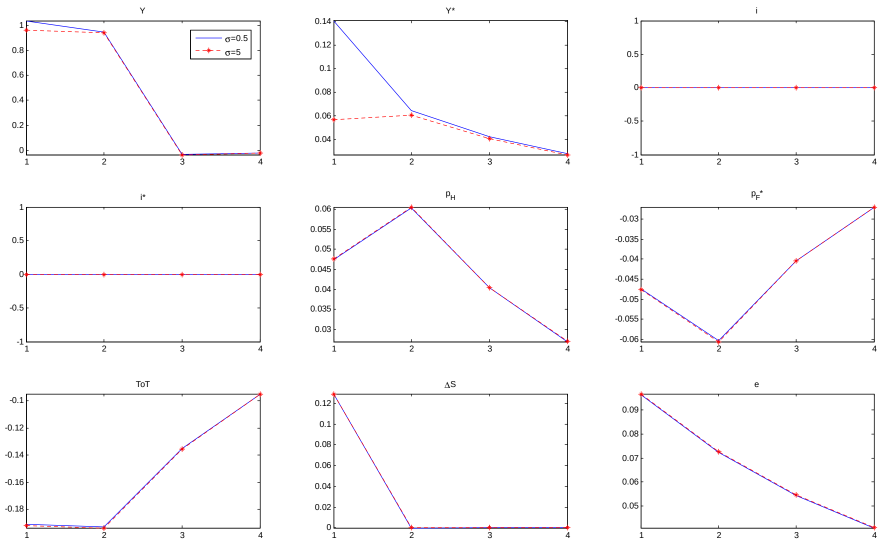
$$\begin{aligned}&\hat{\pi}_{H,t} \\ =& \frac{(1-\theta)(1-\theta\beta)}{\theta}\left\{\begin{array}{c}\left[\omega + \frac{\sigma}{1-g\_y}\right]\hat{Y}_t - \frac{\sigma}{1-g\_y}\hat{G}_t \\ +(\sigma n - 1)\hat{p}_{H,t} - \frac{\sigma(1-n)^2}{n}\hat{p}^*_{F,t} + (1-n)\hat{e}_t\end{array}\right\} \\ &+\beta\hat{\pi}_{H,t+1},\end{aligned}$$

$$\begin{aligned}&\hat{\pi}^*_{F,t} \\ =& \frac{(1-\theta)(1-\theta\beta)}{\theta}\left\{\begin{array}{c}(\omega+\sigma)\hat{Y}_t^* \\ +\left[\sigma(1-n)-1\right]\hat{p}^*_{F,t} - \frac{\sigma n^2}{1-n}\hat{p}_{H,t} - n\hat{e}_t\end{array}\right\} \\ &+\beta\hat{\pi}^*_{F,t+1},\end{aligned}$$

$$\begin{aligned}&\hat{\pi}_t^* - \frac{1-n}{n}\Delta\hat{p}^*_{F,t} \\ =& \frac{(1-\theta)(1-\theta\beta)}{\theta}\left\{\begin{array}{c}\left[\omega + \frac{\sigma}{1-g\_y}\right]\hat{Y}_t - \frac{\sigma}{1-g\_y}\hat{G}_t \\ +\sigma n\hat{p}_{H,t} + \frac{(1-n)\{1-\sigma(1-n)\}}{n}\hat{p}^*_{F,t} - n\hat{e}_t\end{array}\right\} \\ &+\beta\left(\hat{\pi}^*_{t+1} - \frac{1-n}{n}\Delta\hat{p}^*_{F,t+1}\right),\end{aligned}$$

$$\begin{aligned}&\hat{\pi}_t - \frac{n}{1-n}\Delta\hat{p}_{H,t} \\ =& \frac{(1-\theta)(1-\theta\beta)}{\theta}\left[\begin{array}{c}(\omega+\sigma)\hat{Y}_t^* \\ +\sigma(1-n)\hat{p}^*_{F,t} + \frac{n(1-\sigma n)}{1-n}\hat{p}_{H,t} + (1-n)\hat{e}_t\end{array}\right] \\ &+\beta\left(\hat{\pi}_{t+1} - \frac{n}{1-n}\Delta\hat{p}_{H,t+1}\right),\end{aligned}$$

Figure 2: Multiplier and Spillover (LCP)



$$\hat{\pi}_{H,t} = \hat{\pi}_t + \Delta \hat{p}_{H,t},$$

and

$$\hat{\pi}^*_{F,t} = \hat{\pi}^*_t + \Delta \hat{p}^*_{F,t}.$$

Figure 2 illustrates the time-series paths of nine key variables: outputs, interest rates, producer prices in the two countries, the terms of trade, changes in the nominal exchange rate, and the real exchange rate.[21] In this simulation, we assume an interest rate rule where $\alpha = \infty$ and $\rho = 0$. Solid blue lines represent a case when $\sigma = 0.5$, while dotted red lines represent a case when $\sigma = 5$.

Under local currency pricing, we observe positive fiscal spillover irrespective of whether σ is larger (smaller) than unity and the economy is in period 2 as well as period 1. The mechanism of positive spillover under local currency pricing is understood via the global resource constraint in equation (14). The global resource constraint in equation (14) is obtained first by log-linearizing equations (1), (2) and (4) for each country and then com-

[21] Under producer currency pricing, the real exchange rate remains unity, so was not depicted in Figure 1.

bining them. Since the terms of trade are defined by

$$ToT_t = \frac{P_{F,t}}{S_t P^*_{H,t}}$$

under local currency pricing, equation (14) is transformed into

$$\frac{\hat{Y}_t - \hat{G}_t}{1 - g\_y} - e_t - \widehat{ToT}_t = \hat{Y}^*_t, \tag{15}$$

when $n = 0.5$. Basically, this equation tells when the terms of trade improve for the domestic country or when the domestic real exchange rate appreciates, the level of domestic production decreases relatively more to the level of the foreign production. Figure 2 illustrates that irrespective of σ, the terms of trade decrease, indicating the improvement for the domestic country. This is because a domestic currency depreciation improves the terms of trade for the country under local currency pricing. In equation (15), the effect of the worsening of the terms of trade dominates that of the depreciation of the real exchange rate. This is due to the currency misalignments that are formally defined by Engel (2011). Under local currency pricing, changes in the nominal exchange rate do not feed into the export price. As a result, the fiscal spillover becomes positive under local currency pricing, when the zero bound of nominal interest rates is binding.

On the other hand, under producer currency pricing, the terms of trade are defined by equation (9), and the global resource constraint is given by equation (10). As shown in Figure 1, a relative increase in the inflation rate in the domestic country induces an initial jump in the nominal exchange rate toward depreciation. These developments worsen the terms of trade. This makes home-produced goods relatively cheaper than foreign-produced goods and increases demand for home-produced goods, and decreases demand for foreign-produced goods. Consequently, fiscal expenditure in the domestic country has negative spillover effects on the foreign output when $\sigma > 1$.

6 Conclusion

In this paper, we find that the fiscal multiplier exceeds one in a global liquidity trap. On the other hand, the fiscal spillover is negative if the intertemporal elasticity of substitution in consumption is less than one and positive if the parameter is greater than one. The size of the multiplier and the sign (positive or negative) associated with the spillover are quite contrary to those in the undergraduate textbook, those without the zero lower bound, and those under the flexible price equilibrium or the sticky price equilibrium with optimal monetary policy. Incomplete stabilization of marginal costs due to the existence of the zero lower bound is crucial in understanding the effects of fiscal policy in open economies. In addition, we also show that the fiscal spillover in a global liquidity trap becomes positive under local currency pricing, even when the intertemporal elasticity of substitution is less than unity. This result stems from the fact that a currency deprecation reflecting fiscal expansion in the domestic country improves the terms of trade under local currency pricing while it deteriorates the terms of trade under producer currency pricing.

We have so far considered only symmetric cases. A further question could be, in a situation where only the foreign country is constrained by the zero lower bound, how does government expenditure in the home country influence the foreign country's economy. Another issue is how the government spending is used. A part of the government spending may contribute directly to a household's utility. Government spending may be used to purchase not only home goods but also foreign goods. Local currency pricing instead of producer currency pricing may alter our results. The validity of uncovered interest parity becomes crucial when we consider asymmetric cases between two countries. These issues will be addressed in future research.

References

[1] Adam, Klaus, and Roberto M. Billi (2006). "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates." Journal of Money, Credit and Banking, 38(7), 1877–1905.

[2] Adam, Klaus, and Roberto M. Billi (2007). "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates." Journal of Monetary Economics, 54(3), 728–752.

[3] Benhabib, Jess, Stephanie Schmitt-Grohe, and Martin Uribe (2002). "Avoiding Liquidity Traps." Journal of Political Economy, 110(3), 535–563.

[4] Benigno, Gianluca, and Pierpaolo Benigno (2003). "Price Stability in Open Economies." Review of Economic Studies, 70(4), 743–764.

[5] Benigno, Gianluca, and Pierpaolo Benigno (2006). "Designing Targeting Rules for International Monetary Policy Cooperation." Journal of Monetary Economics, 53(3), 473–506.

[6] Bodenstien, Martin, Christopher J. Erceg, and Luca Guerrieri (2009). "The Effects of Foreign Shocks when U.S. Interest Rates are at Zero." Mimeo.

[7] Braun, Richard A. (2009). "Fiscal and Monetary Policy Interactions in a Low Interest Rate Environment." Mimeo.

[8] Christiano, Lawrence J. (2004). "The Zero-Bound, Zero-Infltion Targetting, and Output Collapse." Mimeo, Northwestern University.

[9] Christiano, Lawrence J., Martin Eichenbaum, and Sergio Rebelo (2009). "When is the Government Spending Multiplier Large?." NBER Working Paper 15394, National Bureau of Economic Research.

[10] Clarida, Richard, Jordi Gali, and Mark Gertler (2002). "A Simple Framework for International Monetary Policy Analysis." Journal of Monetary Economics, 49(5), 879–904.

[11] Coenen, Gunter, and Volker Wieland (2003). "The Zero-Interest-Rate Bound and the Role of the Exchange Rate for Monetary Policy in Japan." Journal of Monetary Economics, 50(5), 1071–1101.

[12] Cook, David and Michael B. Devereux (2011). "Optimal Fiscal Policy in a World Liquidity Trap." European Economic Review, 55(4), 443-462.

[13] Corsetti, Giancarlo, and Paolo Pesenti (2000). "Cross-border Spillovers from Fiscal Stimulus." International Journal of Central Banking, 6(1), 5–37.

[14] Corsetti, Giancarlo, and Gernot J. Muller (2008). "Twin Deficits, Openness, and the Business Cycle." Journal of the European Economic Association, 6(2-3), 404–413.

[15] Corsetti, Giancarlo, Andre Meier, and Gernot J. Muller (2010). "Cross-border Spillovers from Fiscal Stimulus." International Journal of Central Banking, 6(1), 5–37.

[16] Dornbusch, Rudiger, Stanley Fischer, and Richard Startz (2008). Macroeconomics. New York: McGraw-Hill, 10th edition.

[17] Eggertsson, Gauti B. (2009). "What Fiscal Policy is Effective at Zero Interest Rates?." Federal Reserve Bank of New York Staff Report 402.

[18] Eggertsson, Gauti B., and Michael Woodford (2003). "The Zero Bound on Interest Rates and Optimal Monetary Policy." Brookings Papers on Economic Activity, 34(2003-1), 139–235.

[19] Engel, Charles, (2011). "Currency Misalignments and Optimal Monetary Policy: A Reexamination." American Economic Review, 101(6), 2796-2822.

[20] Feyrer, James, and Jay C. Shambaugh (2009). "Global Savings and Global Investment: The Transmission of Identified Fiscal Shocks." NBER Working Paper 15113, National Bureau of Economic Research.

[21] Freedman, Charles, Michael Kumhof, Douglas Laxton, and Jaewoo Lee (2009). "The Case for Global Fiscal Stimulus." IMF Staff Position Note, International Monetary Fund.

[22] Fujiwara, Ippei, Tomoyuki Nakajima, Nao Sudo, and Yuki Teranishi (2009). "Global Liquidity Trap." Mimeo, Institute for Monetary and Economic Studies, Bank of Japan.

[23] Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman (1988). "Investment, Capacity Utilization, and the Real Business Cycle." American Economic Review, 78(3), 402–417.

[24] Iwamura, Mitsuru, Takeshi Kudo, and Tsutomu Watanabe (2005). "Monetary and Fiscal Policy in a Liquidity Trap: The Japanese Experience 1999-2004." NBER Working Paper 11151, National Bureau of Economic Research.

[25] Jung, Taehun, Yuki Teranishi, and Tsutomu Watanabe (2005). "Optimal Monetary Policy at the Zero-Interest-Rate Bound." Journal of Money, Credit and Banking, 37(5), 813–835.

[26] Kato, Ryo, and Shin-Ichi Nishiyama (2005). "Optimal Monetary Policy when Interest Rates are Bounded at Zero." Journal of Economic Dynamics and Control, 29(1-2), 97–133.

[27] Kim, Soyoung, and Nouriel Roubini (2008). "Twin Deficiit or Twin Divergence? Fiscal Policy, Current Account, and Real Exchange Rate in the U.S.." Journal of International Economics, 74(2), 362–383.

[28] Krugman, Paul, and Maurice Obstfeld, International Economics. Boston: Pearson, 7th edition.

[29] Mundell, Robert (1967). International Economics. New York: Macmillan.

[30] Nakajima, Tomoyuki (2008). "Liquidity Trap and Optimal Monetary Policy in Open Economies." Journal of the Japanese and International Economies, 22(1), 1–33.

[31] Nakov, Anton (2008). "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate." International Journal of Central Banking, 4(2), 73–127.

[32] Cole, Harold L. and Maurice Obstfeld (1991). "Commodity Trade and International Risk Sharing." Journal of Monetary Economics, 28, 3–24.

[33] Obstefld, Maurice and Kenneth Rogoff (1995). "Exchange Rate Dynamics Redux." Journal of Political Economy, 103(3), 624-660.

[34] Perotti, Roberto (2007). "In Search of the Transmission Mechanism of Fiscal Policy." NBER Macroeconomics Annual, 22, 169–226.

[35] Reifschneider, David, and John C. Williams (2000). "Three Lessons for Monetary Policy in a Low-Infltion Era." Journal of Money, Credit and Banking, 936–978.

[36] Svensson, Lars E. O. and Sweder van Wijnbergen (1989). "Excess Capacity, Monopolistic Competition, and International Transmission of Monetary Disturbances." Economic Journal, 99, 785–805.

[37] Svensson, Lars E. O. (2001). "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap." Jonetary and Economic Studies, 19(S1), 277–312.

[38] Svensson, Lars E. O. (2003). "Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others." Journal of Economic Perspectives, 17(4), 145–166.

[39] Tille, Cedric (2001). "The Role of Consumption Substitutability in the International Transmission of Monetary Shocks." Journal of International Economics, 53(2), 421–444.

[40] Woodford, Michael (2010). "Globalization and Monetary Control." in International Dimensions of Monetary Policy ed. by Jordi Gali and Mark Gertler, 13–77.

[41] Woodford, Michael (2011). "Simple Analytics of the Government Expenditure Multiplier." American Economic Journal: Macroeconomics, 3(1), 1–35.