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Abstract

In this paper, we show that Ricardian equivalence does not hold in a representative agent framework if one considers goods whose current consumption affect future marginal utilities. We find that, when the intertemporal elasticity of substitution changes over time, the timing of lump sum taxation has an asymmetric effect on current and future consumption. This in turn induces distinctive welfare consequences even if the government and individual budget constraints are unchanged in present value terms.

Keywords: Intertemporal preferences, Ricardian equivalence.
JEL classification: H2, H3.

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1 Introduction

Both in the academic and political arenas, there is an ongoing debate regarding the effects of the 2008 Troubled Asset Relief Program and the 2009 American Recovery and Reinvestment Act (more commonly known as "the stimulus package") on the U.S. economy. Besides the obvious question of whether and to which degree this fiscal policy will affect the GDP[1] (i.e. the magnitude of the fiscal multiplier), the debate has renewed the interest in another classical and important issue: what does all this mean in terms of current and future taxes? Recent estimates from the Congressional Budget Office projects a gloomy fiscal scenario for the U.S. economy over the next few decades as depicted in Figures 1 and 2.

A long standing tradition in macroeconomics argues that the timing of taxation does not matter. This so-called Ricardian equivalence has been recurrently the subject of scrutiny both at the empirical and theoretical levels since the seminal contribution of Barro (1974). Its reasoning goes as follows: in a frictionless environment with an infinitely lived agent, the scheme according to which the government finances its obligations does not change the household lifetime budget constraint. Since individuals are fully rational, which peculiar fiscal instrument is used by the government to finance its expenditures does not alter their lifetime disposable income. Hence their individually optimal consumption choices are unaffected. Whether government spending is financed through taxes or debt is irrelevant for the optimal consumption path chosen by the households as long as their budget constraint in present discounted value terms is balanced.

The literature has found several instances under which the Ricardian equivalence does not hold. In particular, the detailed survey articles by Bernheim (1987) and Seater (1993) provide theoretical and empirical appraisal of which assumptions are needed in order for Ricardian equivalence to fail. These premises can be categorized as follows: 1) if the economy has a finite horizon and there is no altruism or families are childless; 2) if capital markets are imperfect, in particular when individuals are liquidity constrained or there are heterogeneous borrowing rates; 3) if individuals have bequest motives that are not based on altruism; 4) when there is uncertainty regarding future income; 5) if fiscal policies have distributional or distortionary effects; 6) when consumers are boundedly rational; 7) when a sizeable part of the debt is foreign held.

This paper provides yet another important instance as to why the Ricardian equivalence may fail: intertemporally non-separable preferences. Although theoretical in nature, this motive is crucial for public policy purposes. There are several commodities whose past consumption levels affect current purchase decisions and current marginal utility. Examples include addictive goods (e.g. Becker and Murphy, 1988), storable goods (e.g. Dudine, Hendel and Lizzeri, 2006), goods with switching costs of consumption (e.g. Klemperer, 1995), and, most notably, durable goods (Bulow, 1982, and Stokey, 1981).[2] The intuition behind our result is simple: when marginal utilities are time varying, the timing of taxation does matter. With intertemporally non-separable preferences, the issue is that an extra

[1]For very recent studies on this topic see Alesina and Ardagna (2009), Mountford and Uhlig (2008), and Romer and Romer (2009).

[2]According to our own computations using Table 1.1.5 of the NIPA tables from Bureau of Economic Analaysis, durable goods in the U.S. constituted, on average, 13.73% of total consumption expenditures for the period of 1947:1-2009:3. Over the same period, on average, they made up 8.86% of GDP.

dollar of disposable income today yields a different marginal utility of consumption than what an extra dollar of disposable income yields tomorrow. Therefore, one should focus on the per period budget constraints rather than the intertemporal budget contrain when evaluating the welfare incidence of a marginal increase in upfront taxation versus issuing new debt.

Interestingly, in overlapping generation models, the presence of intertemporally non-separable preferences is a reason why Ricardian equivalence may actually *hold*. If the economy is populated by overlapping generations with finite lives, it is easy to create financing schemes that have income effects and alter the optimal consumption paths even in the presence of lump sum taxes by translating the burden of the taxes across generations. Robert Barro (1974) showed that it is possible to circumvent this instance if individuals have their descendants' utility function embedded in their own and want to bequeath them in order to offset the future consequences of current fiscal policies. On the contrary, the main result in our paper suggests that by modeling intertemporally non-separable preferences within the representative agent framework, the timing of taxation becomes crucial. The analysis we carry out in this study is disciplined: in the economy proposed, we carefully avoid all of the aforementioned reasons for Ricardian equivalence to fail.

The rest of the paper is organized as follows: in section 2, we provide an example of a simple two period endowment economy and show that a debt policy is preferred to a balanced budget policy under certain parameter values. In section 3, we lay out a model where labor is the only factor of production and show that with intertemporally non-separable preferences Ricardian equivalence does not hold. In section 4, we set up the standard non-stochastic neoclassical model of capital accumulation and we show that, if the government provides a public good and preferences are intertemporally non-separable, Ricardian equivalence in this framework does not hold either. Section 5 concludes.

2 A Motivating Example

To gain some intuition, we start by studying an example that slightly modifies the standard model presented in chapter 10 of Ljungqvist and Sargent (2004). The framework consists of an endowment economy where the representative consumer derives utility from consumption only and preferences are given by:

$$\sum_{t=0}^{T} \beta^t u(c_t^*) \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and T indicates the final period. The function $u(\cdot)$ is at least twice continuously differentiable, concave, and the Inada conditions hold such that $c_t^* > 0$. In our modified example, c_t^* represents effective consumption, which is defined as follows:

$$c_t^* = \alpha c_t + (1-\alpha)c_{t-1}; \tag{2}$$

c_t represents consumption at time t and $\alpha > 0$ is a parameter that defines the nature of the good. For instance, when $\alpha \in (0,1)$, then the good under consideration can be thought of as a durable good. On the other hand, when $\alpha > 1$ then it represents a good that is habit forming or addictive.

For simplicity, we consider two periods only ($T = 2$).[3] Thus, the consumer per period budget constraint is given by:

$$c_t = e_t - \tau_t + b_t - \frac{b_{t+1}}{R} \tag{3}$$

where R is the constant one period gross rate of return, e_t is the endowment at period t, b_t is the risk free loan to the government, and τ_t is the lump sum tax. To simplify exposition, we assume the endowment profile to be $e_1 > 1$ and $e_2 >1$ given the initial conditions, c_0, e_0 and b_0.

The government in this environment finances a stream of expenditures denoted by g_t. Financing is possible through the issuance of government bonds and/or lump sum taxation. Thus the government budget constraint is given by:

$$B_t + g_t = \tau_t + \frac{B_{t+1}}{R} \tag{4}$$

where B_t is the one period government debt due at time t. To simplify matters, we assume a particular stream of expenditures such that g_1=1 and g_2=0. The results derived in this section are robust to alternative expenditure schemes. Given that there is only one asset in this economy, in equilibrium we must have $B_t = b_t$. Furthermore, since there are only two periods, $b_t = 0$ for $t \geq 2$.

Now we turn to the issue of the timing of taxation. In order to do so, we evaluate two alternative fiscal policies. Fiscal policy $\mathcal{X}$ consists of an up front payment so that there is a balanced budget in the first period: τ_1=1, τ_2=0, and $B_1 = 0$. Fiscal policy $\mathcal{Y}$, instead, postpones the payment to the second period so that there is government debt in the first period, i.e. τ_1=0, τ_2= R, $B_1 = 1$. Note that both of these policies do not alter the intertemporal budget constraint of the household, nor that of the government. This is important because this fact is typically the reason why Ricardian equivalence holds. With non-distortionary taxes, if consumers can choose the same consumption set before and after different fiscal policies are implemented, and if the government budget constraint is not affected in present value terms, then it is argued that the overall welfare is unchanged. This line of reasoning breaks down once marginal utilities are allowed to vary over time. The timing of taxation matters even if intertemporal budget constraints remain the same in presen value terms. What really matters are the per period budget constraints, as an extra dollar of disposable income today yields a different marginal utility of consumption than what one extra dollar of disposable income yields tomorrow. To illustrate this point, let us evaluate the welfare under policy $\mathcal{X}$:

$$U(\mathcal{X}) \equiv u(\alpha(e_1 - 1) + (1-\alpha)c_0) + \beta\, u(\alpha e_2 + (1-\alpha)(e_1 - 1)). \tag{5}$$

Alternatively, welfare under policy $\mathcal{Y}$ would be:

$$U(\mathcal{Y}) \equiv u(\alpha e_1 + (1-\alpha)c_0) + \beta\, u(\alpha(e_2 - R) + (1-\alpha)e_1). \tag{6}$$

After some algebra, and exploiting the monotonicity properties of the intra period utility, it is possible to show that policy $\mathcal{Y}$ Pareto dominates policy $\mathcal{X}$, $U(\mathcal{Y}) > U(\mathcal{X})$, when this

[3] Our conclusion holds in the infinite case as well, as long as we impose a transversality condition.

sufficient condition holds $\alpha < \frac{1}{1+R}$. Two observations are in order. First, whenever we have positive rates of interest and the good under consideration is a durable good, then policy $\mathcal{Y}$ is preferred to policy $\mathcal{X}$. The reason why individuals would prefer to defer tax payment in the presence of durable goods is because they would have more disposable income to purchase those kinds of goods up front. Since durable goods provide a flow of consumption which accumulates over time (consumption of these goods in the first period improves utility in the subsequent period), then having a debt policy rather than a balanced budget in the first period yields higher welfare. The second observation is that when preferences are intertemporally separable ($\alpha = 1$), then $U(\mathcal{Y}) = U(\mathcal{X})$ and Ricardian equivalence holds.

As this simple example demonstrates, the nature of the good under consideration (durable or habit forming) can induce different welfare consequences. In the rest of the paper, we focus on a representative agent model with production and we show that the timing of fiscal policies has asymmetric effects on optimal consumption paths in the presence of intertemporally non-separable preferences.

3 A Model with Labor

In this section, we propose a richer model which allows us to study the dynamics of the government's financing decisions under intertemporally non-separable preferences in consumption and a linear production technology that uses labor. Let us consider an infinite horizon economy with discrete time where the representative consumer has the following lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t^*, l_t); \tag{7}$$

where $\beta \in (0,1)$ is the discount factor, l_t denotes leisure, and c_t^* represents effective consumption as previously defined.

Each period, the consumer is endowed with one unit of time. The problem of the representative consumer is to decide on an optimal sequence of consumption, labor, and bond holdings that maximizes lifetime utility taking wages and interest rates as given. Formally, the problem is to maximize (**??**) subject to the following budget constraint:

$$c_t = w_t(1 - l_t) - \tau_t - s_{t+1} + (1+r)s_t \tag{8}$$

where w_t denotes the wage rate, τ_t is the lump-sum tax, s_{t+1} is the quantity of bonds purchased in period t with maturity at period $t+1$, and r is the interest rate. In this environment, we will further assume the following:

$$\lim_{n\to\infty} \frac{s_n}{\prod_{i=1}^{n-1}(1+r)^i} = 0 \tag{9}$$

which states that the quantity of debt, discounted at t=0, must be equal to zero in the limit. This condition rules out infinite borrowing or *Ponzi schemes* and implies that we can write the sequence of budget constraints as a single intertemporal budget constraint. Repeated substitution yields the following lifetime budget constraint:

$$c_0 + \sum_{t=1}^{\infty} \frac{c_t}{\prod_{i=1}^{t}(1+r)^i} = w_0(1-l_0) - \tau_0 + \frac{w_t(1-l_t) - \tau_t}{\prod_{i=1}^{t}(1+r)^i}. \tag{10}$$

Now, maximizing utility subject to (??), we obtain the following optimality conditions:

$$\beta^t \alpha u_1(c_t^*, l_t) + \beta^{t+1}(1-\alpha)u_1(c_{t+1}^*, l_{t+1}) - \frac{\lambda}{\prod_{i=1}^t (1+r)^i} = 0 \tag{11}$$

$$\beta^t u_2(c_t^*, l_t) - \frac{\lambda w_t}{\prod_{i=1}^t (1+r)^i} = 0. \tag{12}$$

These optimality conditions imply a marginal rate of substitution that is changing over time. In particular, for an interior solution we have:

$$\frac{\alpha u_1(c_t^*, l_t) + \beta(1-\alpha)u_1(c_{t+1}^*, l_{t+1})}{u_2(c_t^*, l_t)} = \frac{1}{w_t} \tag{13}$$

$$\frac{\beta u_2(c_{t+1}^*, l_{t+1})}{u_2(c_t^*, l_t)} = \frac{w_{t+1}}{w_t(1+r)}. \tag{14}$$

Note that when α=1 we recover the standard result that the marginal rate of substitution of leisure for consumption in any given period equals the wage rate and that the intertemporal marginal rate of substitution of consumption equals the inverse of one plus the interest rate. These properties do not hold anymore once $\alpha \neq 1$.

In our environment, there is a representative firm producing output according to a linear production function given by:

$$y_t = z n_t \tag{15}$$

where z denotes productivity and n_t represents labor supply. The representative firm behaves competitively so that factor prices equal marginal products, which in our case implies $w_t = z$.

The government purchases g_t units of consumption goods in period t and these purchases are financed through lump sum taxation and by issuing one period government bonds. The government budget constraint is then given by:

$$g_t + (1+r)b_t = \tau_t + b_{t+1}; \tag{16}$$

where b_t is the number of one-period bonds issued by the government in period $t-1$. A bond issued in period t is a claim to $1+r$ units of consumption in period $t+1$. We further assume that b_0=0.

3.1 Equilibrium

A competitive equilibrium consists of quantities, $\{c_t, l_t, n_t, s_{t+1}, b_{t+1}\}_{t=o}^{\infty}$ and prices $\{w_{t+1}, r\}_{t=o}^{\infty}$ satisfying the first order conditions of the representative agent's and firm's problems, and a set of fiscal instruments $\{g_t, b_{t+1}, \tau_t\}_{t=o}^{\infty}$ that satisfy the government budget constraint. Finally, markets for goods, labor, and bonds clear such that:

$$s_{t+1} = b_{t+1} \quad \text{for} \quad \forall t \tag{17}$$

and

$$1 - l_t = n_t \quad \text{for} \quad \forall t. \tag{18}$$

Through repeated substitution, equations (??), (??), and (??) imply a single intertemporal government budget constraint:

$$g_0 + \sum_{t=1}^{\infty} \frac{g_t}{\prod_{i=1}^{t}(1+r)^i} = \tau_0 + \sum_{t=1}^{\infty} \frac{\tau_t}{\prod_{i=1}^{t}(1+r)^i}. \tag{19}$$

which states the present discounted value of government purchases equals the present discounted value of tax revenues. Since the government budget constraint must hold in equilibrium, we also have:

$$c_0 + \sum_{t=1}^{\infty} \frac{c_t}{\prod_{i=1}^{t}(1+r)^i} = w_0(1-l_0) - g_0 + \sum_{t=1}^{\infty} \frac{w_t(1-l_t) - g_t}{\prod_{i=1}^{t}(1+r)^i}. \tag{20}$$

The *Ricardian Equivalence Theorem* establishes that the present discounted value of government purchases and individual budget constraints are the relevant factors determining consumption and leisure while the timing of taxation is irrelevant. This result does not hold when we consider an intertemporal elasticity of substitution (IES) that is changing over time.

After substituting the optimal labor the decision into the optimal consumption decision of the agent, the resulting equilibrium takes the form:

$$\mathcal{F}(c_{t+1}, c_t, c_{t-1}) = g_t \tag{21}$$

where $\mathcal{F}(\cdot)$ is generally a non-linear function. More precisely equation (??) is a non-linear discrete dynamical system of order two with a non-autonomous term g_t. There is no general solution to this type of dynamical system and typically the solution can only be obtained numerically through successive linear approximations of $\mathcal{F}(\cdot)$. Even when $\mathcal{F}(\cdot)$ is linear, the general solution critically depends on g_t and the following property will typically hold:[4]

$$\frac{\partial c_{t+1}}{\partial g_t} \neq \frac{\partial c_t}{\partial g_t}$$

which emphasizes the importance of the timing of taxation on the optimal consumption paths. In the next section, we restrict the set of preferences so that an analytical solution can be obtained.

3.2 An Analytical Example: the Quadratic Case

The quadratic utility assumption allows for an analytical solution, which in turn allows us to derive how the timing of taxation changes the dynamics of consumption as a function of lump sum taxes or government expenditures. Suppose that the instantaneous utility function of the representative consumer is given by:

$$u(c_t^*, l_t) = d\left(c_t^* - \frac{(c_t^*)^2}{2}\right) + l_t - \frac{l_t^2}{2}$$

[4]For more on this class of dynamical systems we refer to Gil (2007).

where d is a positive constant that denotes the relative utility weight between effective consumption and leisure.

From the first order conditions, we have an equilibrium characterized by:

$$g_t - z^2 d(\alpha + \beta(1-\alpha)) + z^2 d\alpha(1-\alpha)c_{t-1} + z^2 d\left(\alpha^2 + \frac{\beta\alpha(1-\alpha)}{z^2 d}\right)c_t + z^2 d\beta\alpha(1-\alpha)c_{t+1} = 0.$$

Note that the previous dynamical system can be rewritten as follows:

$$\eta\, c_{t+1} + \rho\, c_t + \epsilon\, c_{t-1} = \phi - g_t; \tag{22}$$

where η, ρ and ϵ are functions of the primitives of our economy which are given by:

$$\eta = z^2 d\beta\alpha(1-\alpha) \qquad \rho = z^2 d\left(\alpha^2 + \frac{\beta\alpha(1-\alpha)}{z^2 d}\right)$$

$$\epsilon = z^2 d\alpha(1-\alpha) \qquad \phi = z^2 d(\alpha + \beta(1-\alpha)).$$

The general solution to the equilibrium for this economy is of the form:

$$c_t = f^h(t) + f^p(g_t)$$

where $f^h(t)$ and $f^p(t)$ correspond to the homogenous and particular solutions of equation (**??**), respectively. The homogeneous solution is of the form:

$$f^h(t) = \mathcal{S}q_1^t + \mathcal{Q}q_2^t$$

where $\mathcal{S}$ and $\mathcal{Q}$ depend on the initial conditions of the economy, and q_1 and q_2 are given by:

$$q_1, q_2 = \frac{-\rho \pm \sqrt{\rho^2 - 4\epsilon\eta}}{2\eta}.$$

The exact form of the particular solution, on the other hand, depends on the underlying process dictating government expenditures. For expositional purposes, let us assume that government expenditures are of the form $g_t = \phi + e^{-mt}$; where m is a positive constant. Then, the particular solution takes the form:

$$f^p(g_t) = -\mathcal{G}e^{-mt}$$

where $\mathcal{G} = \frac{1}{\eta e^{-m} + \rho + \epsilon e^m}$ which is a constant. Thus, the optimal consumption path can be written as:

$$c_t = \mathcal{S}q_1^t + \mathcal{Q}q_2^t - \frac{e^{-mt}}{\eta e^{-m} + \rho + \epsilon e^m}.$$

It is now easy to show that when α=1 the effect of changes in government spending through time, keeping aggregate resources the same, have no asymmetric effects on current and future consumption such that he Ricardian equivalence would hold. In particular,

$$\frac{\partial c_t}{\partial g_t} = -\frac{1}{z^2 d + 1}\forall t. \tag{23}$$

On the other hand, when $\alpha \neq 1$, we have that marginal effect of changes in government spending on current and future consumption are, respectively, given by:

$$\frac{\partial c_t}{\partial g_t} = -\frac{1}{\eta e^{-m} + \rho + \epsilon e^m} \tag{24}$$

and

$$\frac{\partial c_{t+1}}{\partial g_t} = -\frac{e^{-m}}{\eta e^{-m} + \rho + \epsilon e^m} \tag{25}$$

which are clearly not the same. Hence, the Ricardian equivalence does not hold. To summarize, under intertemporally non-separable preferences, the timing of taxation affects the agent's optimal allocation of resources. In particular, based on (**??**) and (**??**), it is possible to show that a marginal increase in current lump sum taxes has a different effect on future consumption than a marginal increase in government debt would have:

$$\frac{\partial c_{t+1}}{\partial b_{t+1}} > \frac{\partial c_{t+1}}{\partial \tau_t}. \tag{26}$$

The expression above implies that it is better to postpone taxation so that distortions on the margin do not grow over time. In the next section, we further explore this mechanism in a setting with capital accumulation.

4 A Model with Capital and Public Goods

Here, we consider capital instead of labor as the sole input of production. More precisely, our setting is the standard, non-stochastic neoclassical model of capital accumulation, extended to include a government providing a valued public good, and where individuals display intertemporally non-separable preferences over private consumption. We carry out our analysis with an AK model which in turn implies that, given the depreciation rate and the total factor productivity, the equilibrium interest rate is constant. We show that even in this framework Ricardian equivalence does not hold.

There is a continuum of homogeneous households with measure one.[5] Each household chooses consumption and savings in order to maximize lifetime utility, subject to a budget constraint and initial endowments of physical capital and public debt:

$$\max_{\{c_t,\ k_{t+1},\ b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t^*, g_t) \tag{27}$$

$$c_t + k_{t+1} + b_{t+1} = F(k_t) + (1-\delta)k_t + R_t b_t - \tau_t \tag{28}$$

$$\textit{given } k_0 > 0 \textit{ and } b_0$$

The function $u(\cdot)$ is the instantaneous utility, which depends on the effective consumption of the private good, c_t, as defined in (**??**) and the consumption of a public good, g_t. $u(\cdot)$ is

[5]This feature implies that, in equilibrium, individual variables are equal to aggregate variables. Therefore, with some abuse of notation, we do not differentiate between individual and aggregate variables in the presention below.

assumed to be continuously differentiable, increasing, and concave. The discount factor is $\beta \in (0,1)$. Individuals' total income is given by $F(k_t)$. Household's asset holdings are made up of physical capital, k_t, rented to firms at the rate r_t, as well as government bonds, b_t, which bear a gross interest denoted by $R_t = 1 + r_t$.

In this model with no uncertainty, one asset is enough to clear the financial market. Hence, in equilibrium, the rate of return on capital must be equal to the rate of return on bonds. Thus, the return on these two assets are denoted by the same symbol. Physical capital depreciates at a rate denoted by $\delta \in (0,1)$ and households are taxed with a lump sum τ_t.

Firms are competitive and produce an aggregate good according to the AK technology as in Romer (1986) and Lucas (1988). Formally, total production is given by:

$$y_t = F(k_t) = Ak_t \ \forall t \tag{29}$$

where A is total factor productivity. Profit maximization implies the following rental rate:

$$r_t = F_k(k_t) = A \ \forall t. \tag{30}$$

The government budget constraint is given by:

$$g_t + (1 + r_t)b_t = b_{t+1} + \tau_t \tag{31}$$

where the right-hand side of the equation represents government revenues generated by new debt issued, b_{t+1}, together with the revenues from taxation. The left hand side reflects government total expenditures, including the provision of the public good as well as the repaying of outstanding public debt and financial expenses.

4.1 Analysis

A competitive equilibrium consists of quantities, $\{c_t, c_{t+1}, k_{t+1}, b_{t+1}\}_{t=o}^{\infty}$ and prices $\{r_t\}_{t=o}^{\infty}$ satisfying the first order conditions of the representative agent's and firm's problems, and a set of fiscal instruments $\{g_t, b_{t+1}, \tau_t\}_{t=o}^{\infty}$ that satisfy the government budget constraint. Finally, the goods and the financial markets clear so that:

$$k_{t+1} = b_{t+1} \ \text{ for } \ \forall t \tag{32}$$

Solving the household's problem, we obtain the following optimality condition:

$$\frac{u_g(c_t^*, g_t)}{u_g(c_{t+1}^*, g_{t+1})} = \frac{\alpha u_c(c_t^*, g_t) + \beta(1-\alpha)u_c(c_{t+1}^*, g_{t+1})}{\alpha u_c(c_{t+1}^*, g_{t+1}) + \beta(1-\alpha)u_c(c_{t+2}^*, g_{t+2})} \tag{33}$$

The first ratio on the left hand side is the intertemporal marginal rate of substitution for the public good. The second ratio on the right hand side is the intertemporal marginal rate of substitution of consumption. The latter clearly implies that the intertemporal elasticity of substitution is time variant, a key element for the failure of Ricardian equivalence. The resulting equilibrium for an economy with capital and public goods is of the form:

$$\mathcal{H}(c_{t+1}, c_t, c_{t-1}) = h(g_{t+2}, g_{t+1}, g_t) \tag{34}$$

where $\mathcal{H}(\cdot)$ and $h(\cdot)$ depend on the underlying preferences and technology and are generally non-linear functions.

The equilibrium is described by a non-linear discrete dynamical system of order two with a non-autonomous term $h(g_{t+2}, g_{t+1}, g_t)$. As mentioned above, there is no general solution to this type of system and typically the solution can only be obtained numerically. Even when $\mathcal{H}(\cdot)$ is linear, the general solution critically depends on $h(g_{t+2}, g_{t+1}, g_t)$ and the following property will typically hold:

$$\frac{\partial c_{t+1}}{\partial g_t} \neq \frac{\partial c_t}{\partial g_t}.$$

This property emphasizes the importance of the timing of taxation on the optimal consumption and investment decisions.

4.2 An Analytical Example: the Logarithmic Case

In an effort to obtain sharper predictions, we need to impose further structure on the preferences. In particular, we set $u(\cdot) = log(\cdot)$ and assume separability between effective consumption and the public good. In this case, (**??**) can be rewritten as follows:

$$\max_{\{c_t,\ k_{t+1},\ b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t [log(c_t^*) + log(g_t)]. \tag{35}$$

When preferences are intertemporally separable in consumption (i.e. $\alpha = 1$), private consumption and the public good become indistinguishable from the perspective of the individual household. One extra unit of public good yields the same marginal utility as one extra unit of private consumption does. Condition (**??**) becomes:

$$\frac{g_{t+1}}{g_t} = \frac{c_{t+1}}{c_t} \tag{36}$$

Clearly, what matters is the sequence of the levels of public goods, but not how those are financed. With intertemporally non-separable preferences in consumption, however, things are different as the ratio of the marginal utilities is time varying. Hence, the method of financing public good is crucial.

With logarithmic utility and $\alpha \neq 1$, (**??**) becomes:

$$\frac{g_{t+1}}{g_t} = \frac{\frac{\alpha}{\alpha c_t + (1-\alpha)c_{t-1}} + \beta \frac{1-\alpha}{\alpha c_{t+1} + (1-\alpha)c_t}}{\frac{\alpha}{\alpha c_{t+1} + (1-\alpha)c_t} + \beta \frac{1-\alpha}{\alpha c_{t+2} + (1-\alpha)c_{t+1}}} \equiv MRS_c. \tag{37}$$

It is now possible to analytically compare the effects of two alternative fiscal policies for sustaining an exogenous increase in public good provision. First, we rewrite the left hand side of (**??**) using (**??**):

$$\frac{b_{t+2} - R_{t+1}b_{t+1} + \tau_{t+1}}{b_{t+1} - R_t b_t + \tau_t} = MRS_c. \tag{38}$$

Next, we consider the effect of a fiscal policy that increases current taxes only while leaving future taxes and public debt unchanged. This would imply:

$$\frac{\partial MRS_c}{\partial \tau_t} = -\frac{b_{t+2} - R_{t+1}b_{t+1} + \tau_{t+1}}{(b_{t+1} - R_t b_t + \tau_t)^2}. \tag{39}$$

On the other hand, a fiscal policy that changes only the future debt and does not change current or future taxes would yield:

$$\frac{\partial MRS_c}{\partial b_{t+1}} = -\frac{R_{t+1}(b_{t+1} - R_t b_t + \tau_t) + (b_{t+2} - R_{t+1}b_{t+1} + \tau_{t+1})}{(b_{t+1} - R_t b_t + \tau_t)^2} = \frac{\partial MRS_c}{\partial \tau_t} - \frac{R_{t+1}}{g_t}. \quad (40)$$

A comparison of (**??**) and (**??**) lead to the conclusion that there are asymmetric effects on consumption depending on the timing of taxation. Formally:

$$\frac{\partial MRS_c}{\partial b_{t+1}} < \frac{\partial MRS_c}{\partial \tau_t} \quad (41)$$

and thus we can conclude that the Ricardian equivalence does not hold.

The intuition is pretty clear: a policy of balanced budget induces up front taxation and a change in current period consumption. However, this has strong repercussions on the pattern of future consumption because the level of current consumption affects future effective consumption as well. In the case of a habit forming good, a lower current consumption lowers future consumption, and vice versa in the case of a durable good. A debt policy does not alter current consumption as much as up front taxation does. Therefore, there are less compounding distortionary effects on future effective consumption.

5 Conclusions

In this paper, we revisit the classical issue of Ricardian equivalence and the old age question of how the government should finance a given stream of expenditures. We show that the timing of taxation does matter when preferences are intertemporally non-separable. Keeping the usual assumptions that do not violate Ricardian equivalence (e.g. lump sum taxes, constant interest rates, no borrowing constraints, and the representative agent framework), we first examine a simple model with endowments, and then analyze a model where the only factor of production is labor. Finally, we also investigate the standard non-stochastic neoclassical model of capital accumulation. The only modification we introduce in these settings are intertemporally non-separable preferences. In all three instances, the fiscal instruments used by the government (debt or taxes) to finance public spending have asymmetric effects on individual consumption patterns across time.

When past consumption levels affect both current and future marginal utilities, the timing of the taxation is crucial even if fiscal instruments leave intertemporal budget constraints (of the government and of the individual) unaltered from a present discounted value point of view. In this case, what really matters is the per period budget constraint as an extra dollar of disposable income today yields a different marginal utility of consumption than one extra dollar of disposable income yields tomorrow. Our findings are important from a public policy perspective since preferences over many goods have been characterized using intertemporally non-separable utilities. Such goods come in many forms (e.g. addictive, durable, and storable) and constitute a significant fraction of GDP.

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