# CENTRE FOR APPLIED MACROECONOMIC ANALYSIS

The Australian National University

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**CAMA Working Paper Series** **May, 2011**

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## AN APPLICATION OF MODELS OF SPECULATIVE BEHAVIOUR TO OIL PRICES

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CAMA Working Paper 11/2011
http://cama.anu.edu.au

# An Application of Models of Speculative Behaviour to Oil Prices*

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**Abstract**

We estimate three different models of speculative behaviour using oil price data. There are two major results: (i) The three-regime model of Brooks and Katsaris (2005) and a three-regime variant of van Norden and Schaller (2002) fit the oil price data reasonably well; and (ii) Both models show that the probabilities of being in a bubble collapsing state and a bubble expansion state spike in late-2008/early-2009. This provides some support for the claim by Phillips and Yu (2010) and Gilbert (2010) that a bubble in oil prices existed for short period in 2008.
JEL Classification: C5, Q4
Keywords: Oil price, bubble, speculative behavior, three-regime, estimation

*We would like to thank Pedro Gomis-Porqueras for encouraging us to take up this topic.
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# 1 Introduction

In this paper we evaluate the claim of Phillips and Yu (2010) and Gilbert (2010) that a bubble existed in the oil price during 2008. We estimate three different models of bubbles: the three-regime model of Brooks and Katsaris (2005, BK hereafter), the two-regime model of van Norden and Schaller (2002), and a three-regime variant of van Norden and Schaller (2002, VNS hereafter). We first gauge how well each of these models fits the oil price data. We then calculate the probability that the oil price was in either an explosive or a collapsing bubble state during a particular time period. We believe these tests make two contributions. First, they are a robustness check on the findings of Phillips and Yu (2010) and Gilbert (2010). Second, to our knowledge no previous paper has extended these speculative behaviour models to commodity prices. Thus our estimations are a novel application of each.

The benchmark model we use is the three-regime model of Brooks and Katsaris (2005). This model follows the rational asset pricing literature by separating the current price of an asset into fundamental and bubble components. It assumes that the bubble component is limited to one of three regimes (with positive probability): deterministic, surviving, and collapsing. This assumption, together with the associated probabilities of being in a particular regime, lead to a three-equation model which can be estimated to assess the probability of being in any of the three regimes at any given point in time. The two-regime model of van Norden and Schaller (2002) has only the bubble surviving and collapsing regimes. Its three-regime extension adds the deterministic regime of BK, but uses different explanatory variables.

The data used for estimation differ from those in BK and VNS. Because their focus is on the stock market, fundamental values in those papers are dividends. Here, the fundamental value of the oil price is taken to be the current and expected discounted convenience yield that accrues from holding inventories. This convenience yield is the value of any benefits that holding inventories of oil might provide (Pindyck, 1993). The current convenience yield is obtained using a non-arbitrage condition between oil spot and futures prices, along with the appropriate discount rate. The fundamental value of oil at any given point in time is then calculated by using these convenience yield values in conjunction with the method of Campbell and Shiller (1987).

We find that both the BK and three-regime VNS models fit the oil price data reasonably well. In both cases, all of the necessary restrictions on coefficient signs are met. A likelihood test further shows that these two models are statistically indistinguishable. In both models the probability of being in a bubble collapsing state spikes in late-2008/early-2009, shortly after oil prices collapsed. The probability of being in a bubble expansion regime rises just before the probability of collapse. This is in-line with the findings of Phillips and Yu (2010) and Gilbert (2010). The estimations support the claim that a bubble existed during this time and was short-lived.

# 2 The Models

This section provides a brief introduction to the models used for estimation, beginning with that of BK. Following Lucas (1978), the price of an asset at any time ($P_t$), assuming a constant discount rate ($r_f$), can be separated into a dividend component (where $\delta_t$ is the dividend at $t$) and a bubble component (where $B_t$ is the bubble component at $t$):

$$P_t = \sum_{k=0}^{\infty} (1 + r_f)^{-(k+1)} E_t(\delta_{t+k}) + B_t. \quad (1)$$

In the presence of bubbles, the transversality condition fails and the bubble component is shown to be a submartingale process:

$$E_t(B_{t+1}) = (1 + r_f) B_t. \quad (2)$$

It is assumed that the bubble component can be in one of three states (or regimes): a deterministic ($D$) regime, collapsing ($C$) regime, or a surviving ($S$) regime. If the bubble is in the deterministic regime, its expected value is given by (where $s_{t+1}$ is the regime next period):

$$E_t(B_{t+1}|s_{t+1} = D) = (1 + r_f) B_t. \quad (3)$$

In the collapsing regime its expected value is given by:[1]

$$E_t(B_{t+1}|s_{t+1} = C) = g(b_t) P_t, \quad (4)$$

where $b_t$ is the relative size of the bubble in period $t$ (i.e. $b_t = B_t/P_t$).

Assume the probability of being in a deterministic regime at period $t+1$ is $n_t$, and the probability of being in a survival regime is $(1 - n_t)q_t$. Then the probability of being in a collapsing regime is $(1 - n_t)(1 - q_t)$. Using these probabilities, along with equations (2), (3) and (4) the expected value in a surviving regime is:

$$E_t(B_{t+1}|s_{t+1} = S) = \frac{(1 + r_f)}{q_t} - \frac{(1 - q_t)}{q_t} g(b_t) P_t. \quad (5)$$

After some manipulation, these expected values give the expected gross returns ($R_{t+1}$):

$$\begin{aligned} E_t(R_{t+1}|s_{t+1} = D) &= M, & (6)\\ E_t(R_{t+1}|s_{t+1} = B) &= M + \frac{1 - q_t}{q_t} [M b_t - g(b_t)], & (7)\\ E_t(R_{t+1}|s_{t+1} = C) &= M(1 - b_t) + g(b_t), & (8) \end{aligned}$$

where $M = (1 + r_f)$. The probability of being in a deterministic regime is assumed to be related to the absolute bubble size $|b_t|$ and the return spread $S_t^{fa}$:[2]

$$n_t = \Omega\left(\beta_{n0} + \beta_{nb}|b_t| + \beta_{ns} S_t^{fa}\right), \quad (9)$$

where $\Omega$ is the standard normal cumulative density function and the $\beta$'s are coefficients. The probability $q_t$ is assumed to be a function of the absolute bubble size and abnormal trading volume:

[1] Here, $g(B_t)$ is a continuous and everywhere differentiable function such that, $g(0) = 0$ and $0 \leq \partial g(b_t) \partial b_t \leq 1 + i$.

[2] This is measured as the absolute value of the average 12-month actual returns minus the absolute value of the average 12-month returns of the estimated fundamental values.

$$q_t = \Omega\left(\beta_{q0} + \beta_{qb}\left|b_t\right| + \beta_{qv}V_t\right), \tag{10}$$

where $V_t$ is the percentage deviation of last month's volume from the 12 month moving average. Equations (6)-(10) constitute the three-regime BK model.

The two-regime VNS model consists of the bubble survival regime and the bubble collapsing regime only. It also does not utilize information from the return spread and the abnormal trading volume. It is formalized by equations (7) and (8) with $q_t = \Omega\left(\beta_{q0} + \beta_{qb}\left|b_t\right|\right)$. A three-regime extension of the VNS model consists of equations (6) - (10), but does not have terms related to abnormal trading volume or the return spread.

# 3 Model Estimation

This section outlines estimation of the BK model. The two variants of the VNS models are estimated in a similar manner and use the same data. Before it can be used, the BK model requires some simplification. This is accomplished by linearizing equations (6)-(8) using first-order Taylor series approximations around an arbitrary $b_0$ and $V_0$:

$$\begin{aligned}
R_{t+1}^D &= \beta_{D,0} + u_{t+1}^D, \quad u_{t+1}^D \sim N\left(0, \sigma_D^2\right) && (11)\\
R_{t+1}^S &= \beta_{S,0} + \beta_{S,b}b_t + \beta_{S,V}V_t + u_{t+1}^S, \quad u_{t+1}^S \sim N\left(0, \sigma_S^2\right) && (12)\\
R_{t+1}^C &= \beta_{C,0} + \beta_{C,b}b_t + u_{t+1}^C, \quad u_{t+1}^C \sim N\left(0, \sigma_C^2\right). && (13)
\end{aligned}$$

The residuals of each equation are assumed to be normally distributed with zero mean and a given variance. The log likelihood function of this system is:

$$l\left(R_1, R_2, \cdots, R_T; \Psi\right) = \sum_{t=1}^{T-1} \ln\left[n_t \frac{\phi_D}{\sigma_D} + \left(1 - n_t\right) q_t \frac{\phi_S}{\sigma_S} + \left(1 - n_t\right)\left(1 - q_t\right) \frac{\phi_C}{\sigma_C}\right],$$

where $\Psi$ contains all of the unknown parameters, the $\sigma$'s are standard deviations, and $\phi_D$, $\phi_S$ and $\phi_C$ are the probability density functions of $N\left(R_{t+1}^D - \beta_{D,0}, \sigma_D^2\right)$, $N\left(R_{t+1}^S - \beta_{S,0} - \beta_{S,b}b_t - \beta_{S,V}V_t, \sigma_S^2\right)$ and $N\left(R_{t+1}^C - \beta_{C,0} - \beta_{C,b}b_t, \sigma_C^2\right)$ respectively.

This likelihood function is unbounded.[3] To avoid this problem we use the Quasi-Bayesian approach of Hamilton (1991), where one adjusts the log likelihood function to be:

$$l^*\left(R_1, \cdots, R_T; \Psi\right) = l\left(R_1, R_2, \cdots, R_T; \Psi\right) - \sum_{k \in \{D,S,C\}} \frac{a_k}{2} \log \sigma_k^2 - \sum_{k \in \{D,S,C\}} \frac{b_k}{2\sigma_k^2},$$

where the ratio $(b_k/a_k)$ corresponds to our prior for $\sigma_k^2$ and $a_k$ characterizes the weight of the prior. The model is estimated using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm with 100 sets of randomly generated start-up values and we take estimates associated with the largest likelihood value.

[3] The unbounded likelihood function problem associated with the mixture normal model has been well documented in the literature. See Fruhwirth-Schnatter (2006).

The data required for the estimation include values of gross returns, bubble size, abnormal trading volume and return spread for oil. To obtain the bubble size, we follow the present value model of commodity pricing as in Pindyck (1993). The fundamental price of oil at any point in time is defined as the sum of the current and expected convenience yield. Current monthly convenience yield ($Y_{t,1}$) can be calculated using a non-arbitrage condition between the spot ($P_t$) and futures price ($F_{t+1}$) of oil:

$$Y_{t,1} = (1 + r_t) P_t - F_{t+1}. \tag{14}$$

Here, we take $P_t$ as the price of the nearest-month West Texas Intermediate (WTI) futures contract, $F_{t+1}$ is the price of the next-to-nearest month WTI contract, and $r_t$ is the 3-month Treasury Bill rate. The oil price data is taken at a monthly frequency from the U.S. Energy Information Administration (EIA). The real Treasury Bill rates are taken at a monthly frequency from the Federal Reserve Bank of St. Louis's Federal Reserve Economic Data (FRED). We deflate the price of the nearest-month WTI futures contract and the calculated convenience yield by the U.S. Consumer Price Index (CPI), which comes from the U.S. Bureau of Labour Statistics (BLS).

Once the convenience yield is obtained, the fundamental price and bubble size are calculated using the method of Campbell and Shiller (1987). The only modification made to the procedure is in using the convenience yield in lieu of dividends. Finally, futures trading volume data is based on nearest-month futures contracts of WTI, and comes from DataStream International.

# 4 Results

Our data is sampled from January 1985 to December 2010. Likelihood ratio tests (bottom panel of Table 1) indicate that the three-regime VNS model fits the oil price data better than its two-regime counterpart over the sample period. The test statistics also show that the three-regime VNS and BK models are not statistically different. The implication of this is that, for the oil price, coefficients relating to abnormal trading volume and return spread in the BK model are jointly insignificant. The top panel of Table 1 shows that the BK and three-regime VNS model estimates have similar magnitudes. Figures 1 and 2 further show that the probabilities in either model of being in an expanding or collapsing state seem to move in similar patterns. Given this, the remainder of the discussion will focus on the results from the BK model.

The top panel of Table 1 also shows that the BK model meets all of the necessary restrictions on the sign of its coefficients.[4] The associated Wald statistics and P-values for the restrictions are in the middle panel of Table 1. The only concern is that we fail to reject the null hypotheses that $\beta_{q,b} = 0$ and $\beta_{q,V} = 0$ at the 10% significance level, although both coefficients have the expected sign. As expected, the bubble surviving regime has the highest expected return and the bubble collapsing regime has the lowest expected return. The conditional volatility in the bubble collapsing regime (i.e. $\sigma_C$) is higher than those in the other two regimes.

Figures 1 and 2 show that the probability of being in a bubble collapsing regime spikes when the oil price hits a trough in late-2008/early-2009. This is consistent in both three-regime models and indicates that there may have been some deviation of the price from fundamental values.

[4]The restrictions are: (i) $\beta_{S,b} > \beta_{C,b}$; (ii) $\beta_{S,V} > 0$; (iii) $\beta_{C,b} < 0$; (iv) $\beta_{n,b} < 0$; (v) $\beta_{n,S} < 0$; (vi) $\beta_{q,b} < 0$; and (vii) $\beta_{q,V} < 0$. One can refer to Brooks and Katsaris (2005) for interpretations of these restrictions.

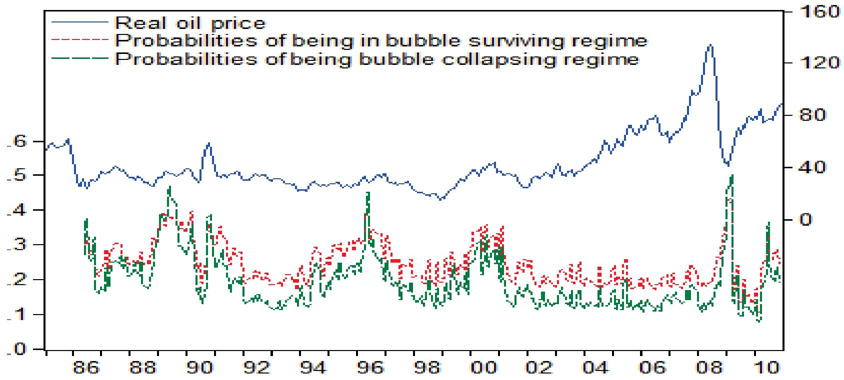


Figure 1: BK probabilities of being in a surviving or collapsing regime.

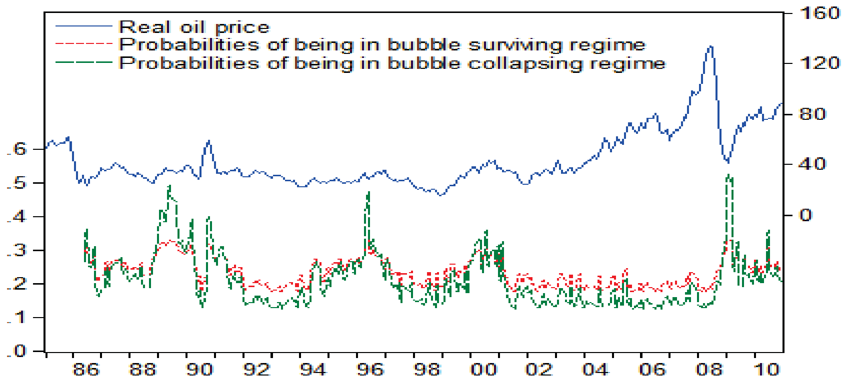


Figure 2: Three-Regime VNS probabilities of being in a surviving or collapsing regime.

Interestingly, this spike follows a period where the probability of being in a bubble expansion does not build up gradually. In fact, the figures show that in both models the probability of being in an expanding regime spikes just before, or during the same time as the probability of being in a collapsing regime. This suggests that any bubble expansion and collapse was relatively short-lived.

## 5 Conclusions

We assess the claim of a recent bubble in oil prices by estimating three different models of speculative behaviour. Our results indicate that the three-regime models of Brooks and Katsaris (2005) and van Norden and Schaller (2002) fit recent oil price data reasonably well. Results from each show that the probability of being in a bubble collapsing regime jumps in late-2008/early-2009, just after oil prices collapsed. The estimations also show a rise in the probability of being in a bubble expansion regime slightly before or at the same time as being in a collapsing regime.

One implication of this is that it provides some support for findings of Phillips and Yu (2010) and Gilbert (2010). The rises in probabilities of expansion/collapse are in-line with the timing in either of those papers. Additionally, the estimated model probabilities in both the expansion and collapsing regimes spike for a short period and then quickly fall. As in Phillips and Yu (2010) and Gilbert (2010), this indicates that any bubble which did expand/collapse was short-lived. Another implication is that the speculative behaviour models of Brooks and Katsaris (2005) and van Norden and Schaller (2002) can be extended to include commodity prices in addition to their original use with the stock market.

## References


**Brooks, Chris and Apostolos Katsaris**, "A Three-Regime Model of Speculative Behaviuor: Modelling the Evolution of the S&P 500 Composite Index," *The Economic Journal*, 2005, *115*, 767–797.

**Campbell, John Y. and Robert J. Shiller**, "Cointegration and Tests of Present Value Models," *The Journal of Political Economy*, 1987, *95*, 1062–1088.

**Fruhwirth-Schnatter, Sylvia**, *Finite Mixture and Markov Switching Models*, Springer, 2006.

**Gilbert, Christopher L.**, "Speculative Influences on Commodity Futures Prices 2006-2008," Discussion Paper 197, United Nations Conference on Trade and Development 2010.

**Hamilton, James D.**, "A quasi-Bayesian approach to estimating parameters for mixture of normal distribution," *Journal of Bussiness and Economic Statistics*, 1991, *9*, 27–39.

**Lucas, Robert E. Jr.**, "Asset Prices in an Exchange Economy," *Econometrica*, 1978, *46* (6), 1429–1445.

**Phillips, Peter C.B. and Jun Yu**, "Dating the Timeline of Financial Bubbles during the Subprime Crisis," Cowles Foundation Discussion Paper 1770, Cowles Foundation for Research in Economics 2010.

**Pindyck, Robert S.**, "The Present Value Model of Rational Commodity Pricing," *The Economic Journal*, 1993, *103*, 511–530.

**van Norden, Simon and Huntley Schaller**, "Fads or Bubbles?," *Empirical Economics*, 2002, *27* (2).

Table 1: Estimation Results

| | BK Model | | Extension of VNS Model | | VNS Model | |
|---|---|---|---|---|---|---|
| | Coeff. | P-value | Coeff. | P-value | Coeff. | P-value |
| $\beta_{D,0}$ | 1.017 | 0.0000 | 1.020 | 0.0000 | - | - |
| $\beta_{S,0}$ | 1.134 | 0.0000 | 1.137 | 0.0000 | 1.034 | 0.0000 |
| $\beta_{S,b}$ | 0.049 | 0.0000 | 0.050 | 0.0000 | 0.169 | 0.0000 |
| $\beta_{S,V}$ | 0.054 | 0.0147 | - | - | - | - |
| $\beta_{C,0}$ | 1.009 | 0.0000 | 1.004 | 0.0000 | 1.034 | 0.0000 |
| $\beta_{C,b}$ | -0.350 | 0.0000 | -0.334 | 0.0000 | -0.187 | 0.0000 |
| $\beta_{n,0}$ | 0.551 | 0.0008 | 0.528 | 0.0011 | - | - |
| $\beta_{n,b}$ | -1.808 | 0.0001 | -1.635 | 0.0003 | - | - |
| $\beta_{n,S}$ | -0.947 | 0.1747 | - | - | - | - |
| $\beta_{q,0}$ | 0.287 | 0.2632 | 0.216 | 0.3971 | -0.230 | 0.1532 |
| $\beta_{q,b}$ | -0.411 | 0.4753 | -0.533 | 0.3434 | -1.214 | 0.0104 |
| $\beta_{q,V}$ | -0.239 | 0.7757 | - | - | - | - |
| $\sigma_D$ | 0.031 | 0.0000 | 0.030 | 0.0000 | - | - |
| $\sigma_S$ | 0.015 | 0.0000 | 0.014 | 0.0000 | 0.018 | 0.0000 |
| $\sigma_C$ | 0.091 | 0.0000 | 0.091 | 0.0000 | 0.070 | 0.0000 |
| lld | 1254.9711 | | 1252.4986 | | 1166.8571 | |
| Restriction $H_1$ | Statistic | P-value | Statistic | P-value | Statistic | P-value |
| $\beta_{D,0} \neq \beta_{S,0} \neq \beta_{C,0}$ | 776.865 | 0.0000 | 864.338 | 0.0000 | 0.001 | 0.4997 |
| $\beta_{C,b} < 0$ | 57.648 | 0.0000 | 51.519 | 0.0000 | 120.762 | 0.0000 |
| $\beta_{S,b} > \beta_{C,b}$ | 73.541 | 0.0000 | 998.773 | 0.0000 | 311.716 | 0.0000 |
| $\beta_{S,V} > 0$ | 6.058 | 0.0069 | - | - | - | - |
| $\beta_{q,V} < 0$ | 0.082 | 0.3876 | - | - | - | - |
| $\beta_{q,b} < 0$ | 0.511 | 0.2374 | 0.901 | 0.1712 | 6.648 | 0.0050 |
| $\beta_{n,S} < 0$ | 1.850 | 0.0869 | - | - | - | - |
| $\beta_{n,b} < 0$ | 14.921 | 0.0001 | 13.232 | 0.0001 | - | - |
| LR Test | 4.945[a] | 0.1759 | 171.283[b] | 0.0000 | - | - |

For the BK model and the extension of VNS model, the prior coefficients are: $a_k = 0.01$ for all $k \in \{D, S, C\}$ and $b_D = 0.00005, b_S = 0.0001, b_C = 0.001$. For the VNS model, the prior coefficients are: $a_k = 0.01$ for all $k \in \{S, C\}$ and $b_S = 0.0001, b_C = 0.001$.

*a* Testing the extension of VNS model against the BK model.

*b* Testing the VNS model against the extension of the VNS model.