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Single Source of Error State Space Approach to the Beveridge Nelson Decomposition

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Abstract

A well known property of the Beveridge Nelson decomposition is that the innovations in the permanent and transitory components are perfectly correlated. We use a single source of error state space model to exploit this property and perform a Beveridge Nelson decomposition. The single source of error state space approach to the decomposition is computationally simple and it incorporates the direct estimation of the long-run multiplier.

Keywords: Beveridge Nelson decomposition; Long-run multiplier; Single source of error; State-space models.

JEL classification: C22, C51, E32

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1. Introduction

Two stylized facts associated with most macroeconomic time series are long run growth and recurrent fluctuations around the growth path. This has often led to exercises which decompose macroeconomic series into permanent and transitory components, where the permanent component represents long run growth or trend in the economy, and the transitory component represents the business cycle. Canova (1998) outlines several different methods for trend/cycle decomposition, focussing on differences between "economic" and "statistical" procedures.

One decomposition that has attracted considerable attention in the applied macroeconomics literature is that first proposed by Beveridge and Nelson (BN) (1981). They defined the permanent component of an integrated time series as the level of the long run forecast (minus the deterministic trend, if any), and the transitory component as the difference between the present level and the permanent component. By-products of this definition are that the same innovation drives the BN permanent and transitory components, so that innovations in these components are perfectly correlated.

Another well known decomposition is the unobserved components (UC) decomposition advocated by Harvey (1989), in which innovations in the trend and cyclical components have zero correlation by assumption. This decomposition is popular in forecasting circles, but it is also used in applied macroeconomic analysis. An alternative UC forecasting approach advocated by Ord, Koehler and Snyder (1997) is the class of state space models with a single source of disturbance. In these latter models, innovations in the unobserved state components are perfectly correlated because they are driven by the same disturbance. It is this similarity with the BN property that motivates our use of a single source of error (SSOE) state space forecasting approach to undertake a BN decomposition.

Our suggested SSOE approach to the BN decomposition focusses on the well known fact that BN permanent and transitory components correspond to those from a UC trend and cycle decomposition with perfectly correlated disturbances. Previous approaches, including those by Miller (1988), Newbold (1990) and Morley (2002) have focussed on overcoming difficulties associated with truncating infinite sums in the BN permanent component. We avoid truncation issues by working with an (equivalent) BN decomposition outlined in Stock and Watson (1988). Our SSOE approach also delivers a direct estimate of the long-run multiplier, which is often used as a measure of "persistence" in applied macroeconomic analysis.

2. Beveridge Nelson Decomposition

Assume that y_t is a I(1) variable with a Wold representation given by

$$\Delta y_t = \mu + \gamma\left(L\right)\varepsilon_t, \tag{2.1}$$

where μ is the drift, $\gamma(L)$ is a polynomial in the lag operator L with $\gamma(0) = 1$ and $\Sigma_{i=0}^{\infty} |\gamma_i| < \infty$, and ε_t is an iid $\left(0, \sigma^2\right)$ one-step-ahead forecast error of y_t. Using the well known identity that $\gamma(L) = \gamma(1) + (1 - L)\gamma^*(L)$, we can rewrite (2.1) as

$$\Delta y_t = \mu + \gamma\left(1\right)\varepsilon_t + (1 - L)\gamma^*(L)\varepsilon_t, \tag{2.2}$$

or equivalently as

$$y_t = \frac{\mu}{(1-L)} + \gamma\left(1\right)\frac{\varepsilon_t}{(1-L)} + \gamma^*(L)\varepsilon_t = \tau_t + c_t. \tag{2.3}$$

The BN permanent component is given by $\tau_t = \frac{\mu}{(1-L)} + \gamma\left(1\right)\frac{\varepsilon_t}{(1-L)}$, while the transitory component is $c_t = \gamma^*(L)\varepsilon_t$, showing the perfect correlation between innovations to τ_t and c_t. The expression for τ_t can be rewritten as

$$\tau_t = \mu + \tau_{t-1} + \gamma\left(1\right)\varepsilon_t, \tag{2.4}$$

and if $\gamma(L)$ is an ARMA(p, q) process with $\gamma(L) = \frac{\theta(L)}{\phi(L)} = \frac{1+\theta_1 L+...+\theta_q L^q}{1+\phi_1 L+...\phi_p L^p}$ then

$$c_t = \phi_p^*(L)c_t + \psi_n^*(L)\varepsilon_t + (1 - \gamma(1))\varepsilon_t, \tag{2.5}$$

where $\phi_p^*(0) = \psi_n^*(0) = 0$ and the orders of $\phi_p^*(L)$ and $\psi_n^*(L)$ are p and n with $n \leq \max(p - 1,\, q - 1)$.

Economists are often interested in the long-run multiplier $\gamma\left(1\right)$, which measures the long-run effect of a shock ε_t on y_t. Here, we note from (2.4) and (2.5) that $\gamma\left(1\right)$ determines the relative size of (contemporaneous) innovations to each component.

3. Single Source of Error State Space Models

The single source of error state space model proposed by Snyder (1985) is

$$y_t = \beta' x_{t-1} + e_t \tag{3.1}$$

with

$$x_t = F x_{t-1} + \alpha e_t, \tag{3.2}$$

where (3.1) and (3.2) are measurement and system equations. The k-vector x_t represents the unobserved state of the underlying process at time t, e_t is an i.i.d. $\left(0, \sigma^2\right)$ innovation, and α, β and F are respectively $(k \times 1)$, $(k \times 1)$ and $(k \times k)$ matrices. The key feature of this specification is that all equations are driven by the same innovation. Details regarding the stability and estimation of this model can be found in Snyder (1985) and Ord, Koehler, and Snyder (1997).

4. Single Source of Error State Space Approach to BN Decomposition

A comparison of the BN decomposition in Section 2 with the framework in Section 3 reveals that the former fits into the SSOE framework if one substitutes (2.4) and (2.5) into $y_t = \tau_t + c_t$ to obtain a measurement equation given by

$$y_t = \mu + \tau_{t-1} + \phi_p^*(L) c_t + \psi_n^*(L) \varepsilon_t + \varepsilon_t, \tag{4.1}$$

and then uses (2.4) and (2.5) as system equations (so that α in (3.2) becomes $\alpha = (\gamma(1), (1 - \gamma(1))))$. These system equations are similar to those in standard UC decompositions, but they identify the unobserved components by assuming that the trend and cycle innovations have perfect correlation, rather than zero correlation.

Our SSOE approach uses the perfect correlation between the contemporaneous innovations in (2.4) and (2.5) to parameterize the BN decomposition, and this allows direct estimation of $\gamma(1)$. In contrast, Morley's (2002) state space approach is based on a parameterization of Δy_t that facilitates the direct estimation of each ARMA coefficient.

Following the convention of calling the two BN components “the trend” and “the cycle”, the parameter of interest in empirical studies of (the logarithms) of output is typically $\gamma(1)$, which measures the long run percentage increase in GDP resulting from a 1% shock in GDP in one quarter. If $\gamma(1) < 1$ then the trend and cycle will have perfect positive correlation and both components will share in the variation of the data. However, if $\gamma(1) > 1$, then innovations in the trend and cycle will have perfect negative correlation, and τ_t will be more variable than y_t. Proietti (2002) and others have questioned whether one should call τ_t a "trend" when it is more volatile than output itself, but Morley et al (2003) (who found that $\gamma(1) > 1$ for real US GDP) observed that a shock to output can shift the trend so that output is behind trend until it catches up. Thus it is quite reasonable for "trend innovations" to be negatively correlated with "cycle innovations" and for trend innovations to be more variable than output innovations.

5. Applications

We illustrate the use of the SSOE approach to performing BN decompositions for ARIMA(2,1,2) models of the logarithms of real output for the United States, the United Kingdom and Australia. The US model coincides with one used by Stock and Watson (1988) when studying the contribution of the trend component to real US GNP, and we examine corresponding models for the UK and Australia to demonstrate the relative contribution of trends in other countries. The permanent and transitory components of the ARIMA(2,1,2) model are

$$\tau_t = \mu + \tau_{t-1} + \gamma(1)\varepsilon_t \quad \text{and}$$

$$c_t = -\phi_1 c_{t-1} - \phi_2 c_{t-2} + \theta_1 \varepsilon_{t-1} + (1 - \gamma(1))\varepsilon_t,$$

which lead to a SSOE state space form given by

$$y_t = \mu + \begin{bmatrix} 1 & -\phi_1 & -\phi_2 & \theta_1 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \\ \varepsilon_{t-1} \end{bmatrix} + \varepsilon_t \quad \text{and}$$

$$\begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\phi_1 & -\phi_2 & \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \gamma(1) \\ 1-\gamma(1) \\ 0 \\ 1 \end{bmatrix} \varepsilon_t.$$

We use quarterly data for USA GNP (1947:1 to 2003:1), for UK GDP (1960:1 to 2003:1) and for Australian GDP (1979:3 to 2003:3). Our estimates based on the state space representations correspond with direct maximum likelihood estimation of the ARIMA model, and they satisfy the relevant stability conditions. We remove θ_1 in the UK model, after finding that it is statistically insignificant.

Measures of trend are reported in Table 1, while the implied transitory components are illustrated in Figure 1 together with reference recessions published by the NBER and the ECRI. While there are often pronounced declines in the transitory components around NBER/ECRI peak to trough episodes, there are also clear differences between BN output cycles and conventional business cycles. This is unsurprising, given that each type of cycle has been constructed to serve different purposes, and has been based on different information sets.

Our primary measure of trend is $\gamma(1)$ ($\frac{1+\theta_1+\theta_2}{1+\phi_1+\phi_2}$ in terms of the ARMA coefficients for Δy_t), which is Campbell and Mankiw's (1987) persistence measure that predicts the long run increase in output resulting from a 1% shock in output in one quarter. We find that $\gamma(1) > 1$ for all countries, implying strong persistence. This result also implies negative correlation between innovations to "trend" and "cycle". To provide an alternative perspective on trend, Table 1 also reports Stock and Watson's (1988) R^2 measure of the fraction of the variance in the quarterly change in real output that can be attributed to changes in trend. These R^2 measures indicate that trend makes relatively lower contributions in the USA and Australian cases than it does in the UK.

6. Conclusion

This paper demonstrates the use of a single source of error state space approach to compute the permanent and transitory components of the BN decomposition. This approach exploits the BN property that the two components are perfectly correlated, and it allows direct inference on the long-run multiplier $\gamma(1)$ as opposed to indirect inference based on the ARIMA coefficients.

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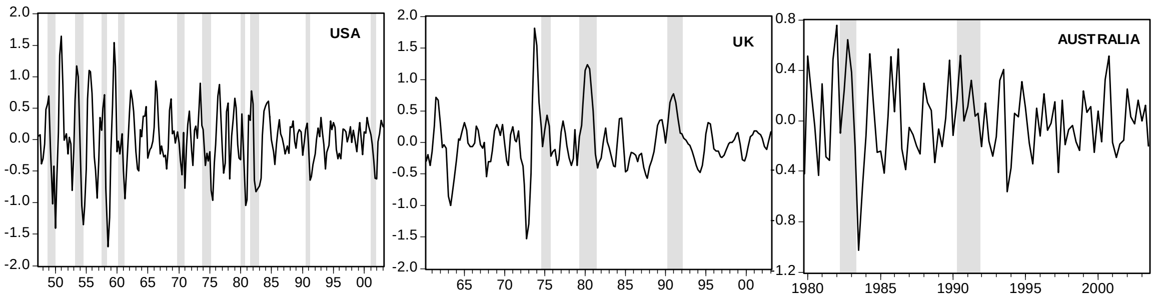
Table 1: Measures of trend in log real GNP/GDP

Country	Long-run multipliers ($\gamma(1)$) and standard errors	R^2
USA	1.2653 (0.1459)	0.8458
UK	1.2267 (0.1587)	0.9686
Australia	1.3733 (0.0460)	0.8822

R^2 statistics are obtained by regressing the quarterly change in GNP against the change in BN trend

Figure 1: Transitory components for ARIMA(2,1,2) models of output for USA, UK and AUSTRALIA



Shaded areas on the graphs indicate peak to trough episodes (recessions) recorded by the NBER (USA) and the ECRI (UK and Australia)