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Keywords

Staggered loan interest rate, economic fluctuation, optimal monetary policy

JEL Classification

E32, E44, E52, G21

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Abstract

We investigate a new source of economic stickiness: namely, staggered loan interest rate contracts under monopolistic competition. The paper introduces this mechanism into a standard New Keynesian model. Simulations show that a response to a financial shock is greatly amplified by the staggered loan contracts though a response to a productivity, cost-push or monetary policy shock is not much affected. We derive an approximated loss function and analyse optimal monetary policy. Unlike other models, the function includes a quadratic loss of the first-order difference in loan rates. Thus, central banks have an incentive to smooth the policy rate.

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Imperfections in financial markets affect the design of monetary policy because it is implemented through financial markets. Here, financial market imperfection is understood as the presence of a wedge between the optimal and actual levels of loan interest rates. A very influential study in this regard is Bernanke, Gertler and Gilchrist (1999, henceforth BGG), stressing that credit market imperfections have a significant influence on business cycle dynamics. In the BGG model, this financial market wedge is determined by time-varying leverage in that endogenous mechanisms in credit markets work to amplify and propagate shocks to the economy, the so-called financial accelerator mechanism. Consequently, a considerable number of existing theoretical studies on monetary policy and financial market imperfections follow the BGG model.[1] However, unlike the present analysis, these studies assume that loan rates can change costlessly in each period, and therefore do not focus on a realistic imperfection observed in many credit markets: namely, the sticky adjustment of loan rates.

Many studies report the stickiness in loan rates for non-financial corporations in several economies.[2] More specifically, in US work, Berger and Udell (1992) employ micro-level data to show that it takes at least two quarters and perhaps more for private banks to adjust the loan rates for newly contracted loans against the market rate proxying for the policy rate. Gambacorta (2008) conducts a similar analysis for Italy and finds the presence of a sticky adjustment of about two quarters in the response of loan rates to a policy rate change for newly contracted short-term lending. Hülsewig, Mayery and Wollmershäuserz (2007) and Gerali *et al.* (2010) also assume staggered loan contracts in New Keynesian models and find empirically that frictions in loan markets play an important role in the propagation of shocks in the euro area on account of incomplete pass-through from policy rates to loan rates. Finally, in Japan, the Bank of Japan (2007, Chart 50) reports that major city banks require five quarters and local banks seven quarters on average to adjust their loan rates in

[1]For example, see Christiano, Trabandt and Walentin (2007) and Christiano, Motto and Rostagno (2007).

[2]Graham and Wright (2007) show interest rate stickiness in consumer loans.

response to a change in the policy rate for all loans. The Bank of Japan (2008, Chart 1-29) similarly reports loan rate stickiness in local banks. Together, these studies suggest that regardless of whether loans are newly contracted or outstanding, or short or long term, loan rates are sufficiently sticky in response to changes in policy rates.[3] Thus, we should consider staggered loan rates, along with other types of financial market imperfections, as an important factor in the conduct of monetary policy.

The first main contribution of this study is to introduce in a tractable manner a staggered nominal loan rate contract with microfoundations derived from the optimizing behaviour of agents into a simple New Keynesian model. We assume staggered loan contracts between a bank and a firm under *monopolistic competition*.[4] In related work, Sander and Kleimeier (2004), Gropp, Sørensen and Lichtenberger (2007) and van Leuvensteijn *et al.* (2008) show that the wedge between the loan rate and the policy rate is a result of the imperfect competition between banks, which induces the staggered loan rates. This staggered contract mechanism highlights a new source of economic rigidity that adds to pre-existing sources of stickiness in the literature. Moreover, by introducing a new equation for the financial market in our model, *i.e.*, the loan rate curve, we can incorporate financial market shocks into our analysis. In the impulse response analysis, we reveal the following property: the cost channel and its stickiness alter economic fluctuations. In particular, a response to a financial shock is greatly amplified by the presence of staggered loan contracts though a response to a productivity, cost-push or monetary policy shock is not much affected.

The second main contribution is that we derive a new objective function for the central bank given a loan rate friction in the financial market, and therefore more

[3] In terms of outstanding loans, loan rate stickiness is natural because private banks and firms do not modify all loan contracts in every period. This is also generally true for long-term loans because most of these have fixed loan rates.

[4] In Hülsewig, Mayery and Wollmershäuserz (2007), an ad hoc loan demand function is induced from the distorted aggregation of loans. Alternatively, in Gerali *et al.* (2010), the assumption is that the loan adjustment cost derives the sticky loan rate dynamics. Both of these mechanisms differ from that assumed in our model. Moreover, neither study focuses on optimal monetary policy.

fully investigate the characteristics of optimal monetary policy. We show that this approximated utility-based welfare criterion holds a specific property not shared by other welfare functions in previous work, including Rotemberg and Woodford (1997), Giannoni (2000), Erceg, Henderson and Levin (2000), Aoki (2001), Steinsson (2003), Benigno (2004) and Ravenna and Walsh (2006).[5] In contrast to these studies, using a model with staggered loan rate contracts, we show that the approximated welfare function includes the first-order difference in loan interest rates, and this in turn induces a reduction in the magnitude of the policy rate changes. As a result, optimal monetary policy has the characteristic of policy rate smoothing. Also, with a modest degree of loan stickiness, we can quantitatively rationalize the smoothing parameter of policy rate estimated in the literature. These outcomes explain the fact that a central bank changes its policy rate through a series of small adjustments in the same direction, as discussed in previous studies such as Goodfriend (1991) and Woodford (2003b). It is the staggered property of financial markets that induces the central bank to optimally smooth the policy rate. We also show the response of optimal monetary policy to a variety of shocks, whereby the directions of the responses of optimal monetary policy and monetary policy conducted under a Taylor-type rule differ for both a loan rate shock and a marginal cost shock.

The remainder of the paper is organized as follows. Section 1 constructs the baseline model. In Section 2, we provide the impulse responses of the model under a

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[5] Giannoni (2000) derives a second-order approximation to the consumer utility function in a model with monetary transaction costs. Erceg, Henderson and Levin (2000) derive an approximated welfare function in a model with staggered wage contracts. Aoki (2001) derives the approximated welfare criteria for a central bank in a model with heterogeneous price-setting sectors, comprising a flexible-price sector and a sticky-price sector. Steinsson (2003) provides an approximated welfare function for a model in which one agent behaves by following Calvo-type price setting and the other sets prices according to a rule of thumb, which induces a hybrid Phillips curve, including both forward- and backward-looking terms. Benigno (2004) extends the discussion of welfare criteria to an international macro framework. Ravenna and Walsh (2006) derive the welfare criteria and investigate optimal monetary policy under a flexible cost channel, *i.e.*, under flexible loan contracts between firms and private banks.

Taylor-type rule. In Section 3, we derive a second-order approximation to the consumer utility function and derive an optimal monetary policy rule. Section 4 details the properties of the optimal monetary policy. In Section 5, we conclude the paper.

1 Model

The model comprises four agents: a consumer, a firm, a central bank and a private bank. The representative consumer plays four roles in our model. First, the consumer consumes differentiated goods determined through a cost minimization problem given an aggregate consumption level. Second, the consumer chooses the optimal amount of aggregate consumption, bank deposits and investment in risky assets given the deposit rate set by the central bank. Third, the consumer provides differentiated labour services and, because it holds monopolistic power, decides the wage of each differentiated type of labour. Lastly, the consumer owns both the bank and the firm, and so receives dividends in each period.

The representative firm consists of three layers: a president, a continuum of project groups populated on the $[0,1]$ interval under the president and a continuum of business units populated on the $[0,1]$ interval in each project group. Here, we assume that the business unit h in each project group is characterized by a differentiated type of labour h. The firm thus plays two roles in our model. First, the president decides how many differentiated workers to hire, which is determined through a cost minimization problem in which a fraction of the labour cost must be financed through an external loan from a private bank, the so-called *cost channel*. The cost of the differentiated type of labour is financed by a differentiated loan. Second, in a monopolistic environment (an individual demand curve on differentiated consumption goods offered by the consumer), each project manager sets a differentiated goods price and produces one good using the external loan assigned by the president to finance some of the labour costs in order to maximize profit. We assume staggered price setting for goods following the Calvo (1983)–Yun (1992) framework.[6]

The representative private bank consists of two layers: a president and a continuum of working groups populated over $[0, 1]$ under the president. The private bank plays two roles in our model. First, the president receives a deposit from the consumer and divides the deposit among the working groups.[7] Second, under monopolistic competition, each working group lends to the firm by setting differentiated nominal loan rates according to the loan's demand curve. As explained below, we assume that each working group can set the differentiated nominal loan rate according to the business unit property which is characterized by the differentiated labour type. In the baseline model, we replicate the staggered property of loan rates through the Calvo (1983)–Yun (1992) framework in which the private bank perfectly fixes loan rates for a certain period. Finally, the central bank sets the deposit rate.[8]

1.1 *Cost Minimization*

In this model, we have two cost minimization problems. The first determines the optimal allocation of differentiated goods for the consumer. The second determines the optimal allocation of labour services, given the loan rates and wages, for the firm's president.

For the consumer, we assume that the consumer derives utility from the con-

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[6] In terms of the first firm role, we can alternatively assume a representative labour aggregator, *i.e.*, an employment co-ordinator, as in Erceg, Henderson and Levin (2000), instead of a firm president. In this case, it is natural to assume the coexistence of many independent firms producing different goods using a differentiated labour service instead of a single firm.

[7] BGG make the same assumption. We can also assume the existence of many different private banks providing loans to different business units in a firm or to different firms instead of a single private bank. In this case, each private bank receives a deposit from the consumer and lends the entire deposit to a particular firm. Thus, the total amount of deposit for each private bank should equal the total deposits of the consumer.

[8] Appendix A provides details of the optimization problem, the derivation of the first-order conditions and the log-linearizations. Appendix is available on my website (http://www.geocities.jp/yuki_teranishi/).

sumption index, which is defined as a Dixit–Stiglitz (1977) aggregator, of bundles of differentiated goods $f \in [0,1]$ produced by a firm's project groups as follows:

$$C_t \equiv \left[\int_0^1 c_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where C_t is aggregate consumption, $c_t(f)$ is a particular differentiated good along a continuum produced by the firm's project group f and $\theta > 1$ is the elasticity of substitution across goods produced by project groups. For the consumption aggregator, the appropriate consumption-based price index is given by

$$P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}}, \tag{2}$$

where P_t is the aggregate price and $p_t(f)$ is the price of a particular differentiated good $c_t(f)$. As in other applications of the Dixit–Stiglitz aggregator, the consumer's allocation across differentiated goods at each time must solve a cost minimization problem. This means that the relative expenditure on a particular good is decided according to the following:

$$c_t(f) = C_t \left[\frac{p_t(f)}{P_t} \right]^{-\theta}. \tag{3}$$

An advantage of this consumption distribution rule is that the consumer's total expenditure on consumption goods is given by $P_t C_t$. We use this demand function for differentiated goods in the firm sector.

On the firm side, the president optimally allocates labour services from the consumer to each project group according to another cost minimization problem. The labour index L_t is given by

$$L_t \equiv \left[\int_0^1 l_t(h)^{\frac{\epsilon-1}{\epsilon}} dh \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $l_t(h)$ is the differentiated labour supply of type $h \in [0,1]$ that goes to firm business unit h within each project group. $\epsilon > 1$ is the elasticity of substitution across differentiated labors. Thus, the differentiated labour types are not perfectly substitutable. Because of the simplified homogeneous project group assumption, each project group eventually uses all types of workers in the same proportions.[9] Here, the firm uses a differentiated loan to hire a differentiated worker. Furthermore, we assume that aggregate labour L_t is used for production. Given the model set-up in which the firm must finance a fraction γ of the labour cost of the business unit h, $\gamma w_t(h) l_t(h)$ (where $0 \leq \gamma \leq 1$), through a loan from working group h in the private bank, the cost minimization problem of the president is given by

$$\min_{l_t(h)} \int_0^1 (1 + \gamma r_t(h)) w_t(h) l_t(h) dh, \tag{5}$$

subject to Eq. (4), where $r_t(h)$ is the nominal loan rate during time t to the business unit h, which is set by the working group h in the private bank, and $w_t(h)$ is the nominal wage for labour supply to h business unit of all project groups, which is set by the consumer. Note that we use the same notation h for the differentiated nominal loan rate, the business unit, the working group, the differentiated nominal wage and the differentiated labour supply, which correspond to each other. Here, working groups in the private bank can set different loan rates for different business units in the firm, each characterized by the type of labour, under monopolistic power. Importantly, the private bank interprets differences in the type of labour as differences in the risk of business units. In the real economy, a firm borrows different loans according to when, why, how much and for how long it requires external finance.[10] Given the aggregate labour L_t determined by the demand for goods, the relative demand for each differentiated type of labour, which is decided by the firm's president, is given by

$$l_t(h) = L_t \left[\frac{(1 + \gamma r_t(h)) w_t(h)}{\Omega_t} \right]^{-\epsilon}, \tag{6}$$

[9] Erceg, Henderson and Levin (2000) assume the same situation for employment. In other words, all project groups solve the cost minimization problems under the same situation, especially under the same labour index in this model.

[10] We can also justify this environment when considering project finance. For project finance, a firm uses different loans for different businesses.

$$\Omega_t \equiv \left\{ \int_0^1 \left[(1 + \gamma r_t(h)) w_t(h) \right]^{1-\epsilon} dh \right\}^{\frac{1}{1-\epsilon}}. \tag{7}$$

Then, we have

$$\int_0^1 (1 + \gamma r_t(h)) w_t(h) l_t(h) dh = \Omega_t L_t. \tag{8}$$

Using the assumption that a specific fraction of the firm's labour cost associated with labour type h is financed through a loan h, then the amount the firm needs to borrow per labour type is

$$q_t(h) = \gamma w_t(h) l_t(h). \tag{9}$$

Then, we also have

$$q_t(h) = \gamma L_t \left[\frac{(1 + \gamma r_t(h)) w_t(h)}{\Omega_t} \right]^{-\epsilon} w_t(h). \tag{10}$$

By defining $Q_t \equiv \int_0^1 q_t(h) dh$, we have

$$q_t(h) = \left[\frac{(1 + \gamma r_t(h))^{-\epsilon} (w_t(h))^{1-\epsilon}}{\Omega_t^*} \right] Q_t, \tag{11}$$

where $\Omega_t^* \equiv \int_0^1 (1 + \gamma r_t(h))^{-\epsilon} (w_t(h))^{1-\epsilon} dh$. This is the demand function for loans by business unit of type h in the firm. Here, the president of the firm changes the allocation of loans according to the business cost, *i.e.*, the labour and loan costs, for each business. When the business cost in h business unit increases, the proportion of business operations through h business unit used for production decreases. Note that because of the differentiated type of labour, the demand for loans is differentiated without assuming a distorted aggregator of loans (money). We use this demand function for private banks.

1.2 *Consumer*

We consider a representative consumer that derives utility from consumption and disutility from the supply of labour. The consumer maximizes the following welfare function:

$$U_t = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T, \nu_T) - \int_0^1 V(l_T(h), \nu_T) dh \right] \right\}, \tag{12}$$

where E_t is an expectation conditional on the state of nature at period t. $U(\cdot)$ is an increasing and concave function in the consumption index C_t and $V(\cdot)$ is an increasing and convex function in labour supply $l_t(\cdot)$. ν_t is an exogenous disturbance of preference, where the steady state value of ν_t is given by $\overline{\nu} = 1$ (hereafter, we denote the steady state value of k_t as $\overline{k}$ for any variable except the output gap and the inflation rate). The budget constraint of the consumer is given by

$$\begin{aligned} P_t C_t + \mathrm{E}_t \left[X_{t,t+1} B_{t+1} \right] + D_t \;\; \leq \;\; & B_t + (1 + i_{t-1}) D_{t-1} + (1 + \tau_w) \int_0^1 w_t(h) l_t(h) dh \\ & + \int_0^1 \Pi_t^B(h) dh + \int_0^1 \Pi_t^F(f) df + T_t, \end{aligned} \tag{13}$$

where B_t is a set of risky assets, D_t is the amount of bank deposits, i_t is the nominal deposit rate (policy rate) set by the central bank from t to $t+1$, τ_w is the subsidy for income, $w_t(h)$ is the nominal wage for labour supply $l_t(h)$ to the firm's business unit of type h, $\int_0^1 \Pi_t^B(h)dh$ is the nominal dividend from owning the bank, $\int_0^1 \Pi_t^F(f)df$ is the nominal dividend from owning the firm, T_t is a subsidy and $X_{t,t+1}$ is the stochastic discount factor between t and $t+1$. We assume a complete financial market for risky assets. Thus, we have a unique discount factor and can characterize the relationship between the deposit rate and the stochastic discount factor as follows:

$$\frac{1}{1 + i_t} = \mathrm{E}_t \left[X_{t,t+1} \right]. \tag{14}$$

Given the optimal allocation of consumption expenditure across the differentiated goods, the consumer must choose the total amount of consumption, the optimal amount of risky assets to hold and an optimal amount to deposit in each period to maximize the welfare function. The necessary and sufficient conditions are given by

$$U_C(C_t, \nu_t) = \beta(1 + i_t) \mathrm{E}_t \left[U_C(C_{t+1}, \nu_{t+1}) \frac{P_t}{P_{t+1}} \right]. \tag{15}$$

In this model, the consumer provides differentiated types of labour to the firm and so holds the power to decide the wage of each type of labour, as in Erceg, Henderson and Levin (2000). We assume that each project group hires all types of workers in the same proportions. The consumer sets each wage $w_t(h)$ for any h in every period to maximize its utility subject to the budget constraint given by Eq. (13) and the demand function of labour given by Eq. (6).[11] Then we have the following relation:

$$(1+\tau_w)\,\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon-1}\frac{V_l(l_t(h),\nu_t)}{U_C(C_t)}. \tag{16}$$

In this paper, we assume that the consumer supplies labour only for the firm, not the private bank. We use the relation given by Eq. (16) for the firm. To eliminate the distortion from monopolistic labour supply, we set $1+\tau_w = \frac{\epsilon}{\epsilon-1}$.

1.3 *Firm*

As explained above, we first assume that the president determines the allocation of hiring differentiated labour, given the aggregate labour determined by the demand for goods, using a cost minimization problem in which a fraction of the labour costs must be financed through external loans from a private bank. We also assume that in a monopolistically competitive goods market, each project manager employs all types of workers, borrows all types of external loans and produces a single good with resetting the firm's price at certain intervals.

Under the Calvo (1983)–Yun (1992) framework, the f project manager resets the firm's price with probability $1-\alpha$ and maximizes the firm's present discounted value of profit:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T}\left[(1+\tau_p)\,p_t(f) y_{t,T}(f) - G_T \Omega_T L_T(f)\right]. \tag{17}$$

G_t is the exogenous disturbance on the marginal cost as in Woodford (2003a), where $\overline{G} = 1$. We use the consumer's (shareholder's) marginal rate of substitution $X_{t,T}$

[11] We assume a flexible wage setting in the sense that the consumer can change her wage in every period.

between t and T for each firm's project group. τ_p is the subsidy for profit, where $1+\tau_p = \frac{\theta}{\theta-1}(1+\gamma\bar{r})$ eliminates two distortions arising from monopolistic competition in the goods market and the positive loan (policy) rate to make the marginal cost of production equal to one.

The first and second terms in the bracket for Eq. (17) denote sales profit $(1+\tau_p)\,p_t(f)y_{t,T}(f)$ and production cost $G_T\Omega_T L_T(f)$, respectively. Given that the production cost depends on the loan rate, in addition to the labour cost, the firm sets prices according to the aggregate loan rate, as shown below. The optimal price $p_t(f)$ in this Calvo environment is as follows:

$$\begin{aligned}
& \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \left[(1+\tau_p) \frac{\theta-1}{\theta} \frac{p_t(f)}{P_T} \right] \\
= \; & \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \\
& \times G_T \left[\int_0^1 \left((1+\gamma r_T(h)) \frac{V_l(l_T(h), \nu_T)}{U_Y(Y_T, \nu_T)} \frac{\partial L_T(f)}{\partial y_{t,T}(f)} \right)^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}}, \qquad (18)
\end{aligned}$$

where we use $y_{t,T}(f) \equiv Y_T \left[\frac{p_t(f)}{P_T}\right]^{-\theta}$ from Eq. (3) under the goods market clearing such that the supply of each differentiated good equals its demand, $c_t(f) = y_t(f)$ and $C_t = Y_t$ for any t, use Eq. (16) and assume that the firm's linear production function is given by $y_t(f) = A_t L_t(f)$, where A_t is an exogenous disturbance of productivity. Project groups that are allowed to reset their goods prices will set the same goods price, so the solution of $p_t(f)$ in Eq. (18) is expressed by p_t^*.

In the Calvo (1983)–Yun (1992) setting, the evolution of the aggregate price index is described by the following motion:

$$P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)\left(p_t^*\right)^{1-\theta}. \qquad (19)$$

1.4 *Private Bank*

Each working group manager can reset the group's loan rate with probability $1-\varphi$. To explain this stickiness, behind the model, we assume a situation where (1) the bank re-evaluates the risks associated with financing a firm's businesses in some interval periods citing limitations on informational transactions or costs associated with re-evaluation, or (2) the firm makes long-term fixed loan rate contracts with the private bank. We assume that each working group can set different loan rates that depend on the business units' labour type under monopolistic power. We can then define the maximization problem for working group h, where the objective is to maximize the present discounted value of profit:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \varphi^{T-t} X_{t,T} \left[M_T (1+\tau_r)(1+r_t(h)) - (1+i_T) \right] q_{t,T}(h), \tag{20}$$

where we define $q_{t,T}(h) = \left[\frac{(1+\gamma r_t(h))^{-\epsilon} (w_T(h))^{1-\epsilon}}{\Omega_T^*} \right] Q_T$ from Eq. (11), $1+\tau_r = \frac{\epsilon\gamma(1+\bar{r})}{\epsilon\gamma(1+\bar{r})-1-\gamma\bar{r}}$ is the profit subsidy to eliminate the distortion from monopolistic competition in the loan market as $1+\bar{i} = 1+\bar{r}$ and $r_t(h)$ is the nominal loan interest rate during time t set by working group h in the private bank.[12] M_t is the exogenous disturbance from the time-varying subsidy, as followed by Woodford (2003a), where $\overline{M} = 1$. We use the consumer's (shareholder's) marginal rate of substitution X for each working group of the bank. Furthermore, in equilibrium, we assume that the supply of deposits equals demand: $D_t = Q_t$. Thus, the president of the private bank implicitly allocates deposits to each working group. We can transform Eq. (20) as follows:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} q_{t,T}(h) \left[M_T (1+\tau_r)(1+r_t(h)) - (1+i_T) \right]. \tag{21}$$

Then, the optimal loan rate $r_t(h)$ in this Calvo setting solves the following equation:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} q_{t,T}(h) \left[(1+\tau_r) - \epsilon\gamma \frac{M_T (1+\tau_r)(1+r_t(h)) - (1+i_T)}{1+\gamma r_t(h)} \right] = 0. \tag{22}$$

[12] If we interpret $r_t(h)$ as the interest rate from t to $t+1$, the dividend from the private bank in the consumer's budget constraint is given by $\int_0^1 \Pi_{t-1}^B(h)dh$. However, even in this case, the model does not change.

Working groups that are allowed to reset their loan rates will set the same loan rate, so the solution of $r_t(h)$ in Eq. (22) is expressed by r_t^*. On the other hand, we have the following evolution of the aggregate loan rate index:

$$1 + R_t = \varphi(1 + R_{t-1}) + (1 - \varphi)(1 + r_t^*), \tag{23}$$

where we define $1 + R_t \equiv \int_0^1 \frac{q_t(h)}{Q_t}(1 + r_t(h))dh$.[13]

1.5 *Log-linearization and the Closed System of Economy*

We log-linearize the equation around the constant steady state that is efficient because of subsidies (hereafter, we denote the log-linearized value of k_t as $\widehat{k}_t$ for any variable except the output gap and the inflation rate). We define the output gap as that between output Y_t and the natural rate of output Y_t^n that is defined from Eq. (18) as the output obtained under a flexible price and constant loan rate as

$$\left\{ \int_0^1 \left[\frac{V_l(l_t^n(h), \nu_t)}{U_Y(Y_t^n, \nu_t)G_t} \right]^{1-\epsilon} dh \right\}^{\frac{1}{1-\epsilon}} = 1, \tag{24}$$

where we assume a flexible price setting $p_t^*(f) = P_t$, the constant loan rate $\widehat{r}_t(h) = \overline{r}$ and the subsidies for wage income, firm profit and bank profit, and $l_t^n(h)$ is the amount of labour type h employed under Y_t^n. Thus, the disturbances given by ν_t and A_t induce the disturbance in the natural rate of output.

Together with Eq. (14), the condition given by Eq. (15) expresses the intertemporal optimal allocation on aggregate consumption. Assuming the market clears, we finally obtain the standard New Keynesian IS curve by log-linearizing Eq. (15):

$$x_t = \mathrm{E}_t x_{t+1} - \sigma \left(\widehat{i}_t - \mathrm{E}_t \pi_{t+1} - \widehat{r}_t^n \right), \tag{25}$$

where we name $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$ the output gap, $\pi_t \equiv \ln P_t / P_{t-1}$ inflation, $\widehat{r}_t^n \equiv \sigma^{-1} \left(-\widehat{Y}_t^n + \mathrm{E}_t \widehat{Y}_{t+1}^n \right)$ the natural rate of interest and $\sigma \equiv -\frac{U_Y}{U_{YY}\overline{Y}}$ is the intertemporal

[13] Kobayashi (2008) derives a similar loan rate curve under a different model by assuming bank's monopoly for firm.

elasticity of substitution of aggregate expenditure. We refer to $\widehat{r}_t^n$ as a productivity shock.

By log-linearizing Eq. (18) and Eq. (19), we obtain the following augmented New Keynesian Phillips curve:

$$\pi_t = \kappa x_t + \xi \widehat{R}_t + \beta \mathrm{E}_t \pi_{t+1} + \frac{\kappa}{(\nu + \sigma^{-1})} \widehat{G}_t, \tag{26}$$

where $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)(\nu+\sigma^{-1})}{\alpha}$ and $\xi \equiv \frac{\kappa\gamma(1+\bar{r})}{(\nu+\sigma^{-1})(1+\gamma\bar{r})}$ are positive parameters, where $\nu \equiv \frac{\overline{L}V_{ll}}{V_l}$ is the elasticity of the desired real wage to the quantity of labour demanded. $\widehat{G}_t$ is a marginal cost shock. In contrast to the standard New Keynesian Phillips curve, this augmented form includes the loan rate because the production cost depends on the loan rate in addition to the labour cost.

By log-linearizing Eq. (22) and Eq. (23), we can characterize the relationship between the loan rate and the deposit rate as follows:

$$\widehat{R}_t = \lambda_1 \mathrm{E}_t \widehat{R}_{t+1} + \lambda_2 \widehat{R}_{t-1} + \lambda_3 \widehat{i}_t - \frac{\lambda_3}{1+\bar{i}} \widehat{M}_t, \tag{27}$$

where $\lambda_1 \equiv \frac{\varphi\beta}{1+\varphi^2\beta}$, $\lambda_2 \equiv \frac{\varphi}{1+\varphi^2\beta}$, and $\lambda_3 \equiv \frac{\epsilon(1-\varphi\beta)(1-\varphi)}{(\epsilon-1)(1+\varphi^2\beta)} \frac{1}{1+\tau_r}$. $\widehat{M}_t$ is interpreted as a loan rate shock. One of the key contributions in this paper is to incorporate shocks to the financial market, specifically the shock to the loan rate curve, into an otherwise standard New Keynesian model.

Finally, the closed system of economy consists of three equations, Eq. (25), Eq. (26) and Eq. (27), in addition to a monetary policy defined in the following sections for four endogenous variables: x_t, π_t, $\widehat{i}_t$ and $\widehat{R}_t$.

2 Properties of the Model

2.1 *Parameters and Monetary Policy Rule*

To show the impulse responses, we assume that the central bank sets a deposit rate, *i.e.*, policy rate, in every period using a Taylor-type rule as follows:

$$\widehat{i}_t = (1-\mu_i)(\mu_\pi \pi_t + \mu_x x_t) + \mu_i \widehat{i}_{t-1} + S_t, \tag{28}$$

where μ_π, μ_x, and μ_i are positive parameters. S_t is a monetary policy shock. In simulations, we set the policy parameters in the Taylor-type rule as $\mu_\pi = 1.72$, $\mu_x = 0.34$ and $\mu_i = 0.71$ following Clarida, Gali and Gertler (2000) to compare impulse responses under this instrumental rule with those under the optimal monetary policy in Section 5. Some empirical studies, such as Judd and Rudebusch (1998) and Orphanides (2003), also show that the policy rate smoothing parameter μ_i has been around 0.7 to 0.8 since the 1980s.

To set empirically supported parameters, we use parameters from empirical studies as much as possible in Table 1. We set σ as 0.2 from Chari, Kehoe and McGrattan (2002), α as 0.75 from Steinsson (2007) and φ as 0.58 from Fujiwara and Teranishi (2011). Values for the other parameters are from Woodford (2003a). We use the parameters in Table 1 as the base case. To show the effects of the cost channel and the staggered loan rate, we illustrate two additional cases by changing parameters $\varphi = 0$ (the flexible loan rate case) and $\gamma = 0.5$ (the weak cost channel case) alongside the other parameters given in Table 1.

2.2 *Impulse Responses under the Taylor-type Rule*

2.2.1 *Loan Rate Shock*

Figure 1 depicts the simulation outcomes under the Taylor-type rule. We assume an unexpected 1% positive shock on the loan rate in the loan rate curve, where the autocorrelation of the shock is 0.6. The figure shows the percentage deviations from the steady state.

The shock to the loan rate increases the inflation rate because of an increase in cost. The policy rate then increases. In turn, a high policy rate induces a negative output gap. To compare the base case and that for the flexible loan rate, the loan rate shock induces significantly larger and more persistent economic fluctuations through staggered loan contracts.[14] In addition, comparing the base case with that for the

[14]As the persistence of the shock or loan rate stickiness increases, the difference in the impulse response between the two cases becomes more marked.

weak cost channel, as the share of external finance to labour cost increases, economic fluctuations become larger. Thus, both the cost channel and the stickiness of the cost channel potentially play an important role in explaining economic fluctuations.

2.2.2 *Other Shocks*

Figure 2 provides the simulation outcomes under the Taylor-type rule to an unexpected 1% positive shock on the rate of inflation in the augmented New Keynesian Phillips curve, on the output gap in the IS curve, or on the policy rate in the Taylor rule, where the autocorrelation of the shock is 0.6. We only show impulse responses under the base case in Figure 2 because the impulses of the cases for the weak cost channel and the flexible loan rate do not significantly differ from ones of the base case.[15] The figure shows the percentage deviations from the steady state.

In response to the marginal cost shock, the rate of inflation increases, and this induces increases in the policy rate and the loan rate. In turn, these changes induce a negative output gap. The shock to productivity increases the output gap and the inflation rate. In turn, these raise the policy rate and the loan rate. As shown, the positive shock to monetary policy decreases the rate of inflation, even with an increase in the loan rate. The output gap decreases according to the real policy rate. Even assuming the weak cost channel case or the flexible loan rate case, impulse responses do not significantly change, in particular for the inflation rate and the output gap. Thus, the response to the productivity, cost-push or monetary policy shock is not much affected by the presence of staggered loan contracts.

3 Optimal Monetary Policy

First, we derive a second-order approximation to the welfare function.[16] Second, we derive an optimal monetary policy when the central bank is credibly committed to

[15] The figures of the cases for the weak cost channel and the flexible loan rate are shown in Appendix B.

[16] Appendix C provides details of these derivations and explanations.

a policy rule in the *timeless perspective*.

3.1 *Approximated Welfare Function*

Under the goods market clearing, a second-order approximation to the welfare function of Eq. (12) around the efficient steady state is finally given by

$$U_t \simeq -\Lambda \mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} J_T, \tag{29}$$

where

$$J_t = \lambda_\pi \pi_t^2 + \lambda_x x_t^2 + \lambda_R \left(\widehat{R}_t - \widehat{R}_{t-1} \right)^2, \tag{30}$$

and $\Lambda \equiv \frac{1}{2}\overline{Y}u_c$, $\lambda_\pi \equiv \frac{\alpha\theta}{(1-\alpha)(1-\alpha\beta)}$, $\lambda_x \equiv (\sigma^{-1}+\nu)$ and $\lambda_R \equiv \frac{\epsilon^2\varphi\left(\nu+\epsilon^{-1}\right)}{(1+\nu\epsilon)^2(1-\varphi)(1-\varphi\beta)} \left(\frac{\gamma(1+\bar{r})}{1+\gamma\bar{r}}\right)^2$. The loss function J_t includes a quadratic loss of the first-order difference in loan rates in addition to quadratic losses in the inflation rate and the output gap because the disaggregated loan rates disperse, which distorts production through distorted labour supplies, given infrequent price change.[17]

In the case of flexible loan contracts, the loss function only includes quadratic losses of the inflation rate and the output gap, *i.e.*, $\lambda_R = 0$.[18] This loss function is consistent with that in Ravenna and Walsh (2006) with a flexible loan contract and in Woodford (2003b) without a banking sector. Thus, in a model with staggered loan rates, the central bank should pay attention to loan rate fluctuations, particularly the first-order difference in loan rates.[19]

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[17] From the technical point of derivation, the dispersion of disaggregated variables can be approximated by the quadratic loss of the first-order difference of an aggregated variable. Thus, the quadratic loss of the first-order difference in loan rates is included as the quadratic loss of the first-order difference in price levels expressed by the inflation rate is included.

[18] Even in the case where there is no loan contract between the firm and the private bank, we have $\lambda_R = 0$.

[19] The relative values of λ_R to λ_π and λ_x increase as the staggeredness of loan rate contracts rises. Furthermore, the relative values of λ_R to λ_π and λ_x increase as the fraction of the firm's external finance increases.

3.2 *Derivation for Optimal Monetary Policy*

We consider an optimal monetary policy when the central bank is credibly committed to a policy rule in the *timeless perspective*.[20] Here, as shown in Woodford (2003a), the central bank conducts monetary policy in a forward-looking way by paying attention to future economic variables and by taking account of the effects of monetary policy on these future variables.

The objective of monetary policy is to minimize the expected value of the loss function given by Eq. (29) under the standard New Keynesian IS curve given by Eq. (25), the augmented Phillips curve given by Eq. (26) and the loan rate curve given by Eq. (27). The optimal monetary policy is expressed by the solution of the optimization problem, which is represented by the following Lagrangian problem:

$$\mathcal{L} = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \begin{array}{c} J_T + 2\Xi_{1,T} \left[x_{T+1} - \sigma \left(\widehat{i}_T - \pi_{T+1} \right) - x_T \right] \\ +2\Xi_{2,T} \left[\kappa x_T + \xi \widehat{R}_T + \beta \pi_{T+1} - \pi_T \right] \\ +2\Xi_{3,T} \left[\lambda_1 \widehat{R}_{T+1} + \lambda_2 \widehat{R}_{T-1} + \lambda_3 \widehat{i}_T - \widehat{R}_T \right] \end{array} \right\} \right\}, \tag{31}$$

where Ξ_1, Ξ_2, and Ξ_3 are the Lagrange multipliers associated with the constraints of the IS, Phillips and loan rate curves, respectively. We differentiate the Lagrangian with respect to π_t, x_t, $\widehat{R}_t$ and $\widehat{i}_t$ to obtain the following first-order conditions:

$$\lambda_\pi \pi_t + \beta^{-1} \sigma \Xi_{1,t-1} - \Xi_{2,t} + \Xi_{2,t-1} = 0, \tag{32}$$

$$\lambda_x x_t - \Xi_{1,t} + \beta^{-1} \Xi_{1,t-1} + \kappa \Xi_{2,t} = 0, \tag{33}$$

$$\lambda_R \left(\widehat{R}_t - \widehat{R}_{t-1} \right) - \beta \lambda_R \left(\mathrm{E}_t \widehat{R}_{t+1} - \widehat{R}_t \right) + \xi \Xi_{2,t} - \Xi_{3,t} + \beta^{-1} \lambda_1 \Xi_{3,t-1} + \beta \lambda_2 \mathrm{E}_t \Xi_{3,t+1} = 0, \tag{34}$$

$$\Xi_{3,t} - \lambda_3^{-1} \sigma \Xi_{1,t} = 0. \tag{35}$$

These four conditions, together with the IS curve, the Phillips curve and the loan rate curve equations, are the conditions governing the loss minimization for $t \geq 0$.

[20] Detailed explanations about the timeless perspective are in Woodford (2003a).

In other words, the sequence of policy rates determined by these conditions is the optimal interest rate path. Note that all variables are in the steady state before $t = 0$, so we have $\widehat{R}_{-1} = \Xi_{1,-1} = \Xi_{2,-1} = \Xi_{3,-1} = 0$.

For simplicity, we can better understand optimal policy by reducing the number of conditions to

$$\begin{aligned} & (1 - z_1 L)(1 - z_2 L) \left[\lambda_R \left(\triangle \widehat{R}_t - \beta \mathrm{E}_t \triangle \widehat{R}_{t+1}\right) - \kappa^{-1} \xi \lambda_x x_t\right] \\ = \;& \mathrm{E}_t \left[\frac{\beta \lambda_2 \sigma}{\lambda_3} z_3 (1 - z_3^{-1} F)(1 - z_4 L)(\kappa \lambda_\pi \pi_t + \lambda_x \triangle x_t)\right], \end{aligned} \tag{36}$$

where z_1, z_2, z_3 and z_4 are parameters, satisfying $z_1 + z_2 = 1 + \beta^{-1} + \kappa\sigma\beta^{-1}$, $z_1 z_2 = \beta^{-1}$ ($z_1 > 1$, $0 < z_2 < 1$), $z_3 + z_4 = -\frac{\lambda_3}{\beta\lambda_2\sigma}(\frac{\sigma}{\lambda_3} - \frac{\xi}{\kappa})$, and $z_3 z_4 = \frac{\lambda_3}{\beta\lambda_2\sigma}(\frac{\xi}{\beta\kappa} - \frac{\sigma\lambda_1}{\beta\lambda_3})$. L is the lag operator and F is the forward operator. Using this, we confirm that the central bank has an incentive to pay attention to the first-order difference in loan rates, as well as to the standard concerns of the output gap and the inflation rate. This property is induced by the staggered loan contracts. There are both forward-looking and backward-looking terms in the optimal policy. Thus, not only does the optimal rule imply history dependence, but it also has a pre-emptive property (precautionary property). This pre-emptive property arises from the inertia in the loan rate curve. In the case of flexible loan contracts, *i.e.*, $\varphi = 0$, λ_R is zero, and so the optimal monetary policy rule reduces to

$$-\kappa^{-1} \xi \lambda_x (1 - z_1 L)(1 - z_2 L) x_t = \left(\frac{\sigma}{\lambda_3} - \frac{\xi}{\kappa} - \frac{\xi}{\beta\kappa} L\right)(\kappa \lambda_\pi \pi_t + \lambda_x \triangle x_t). \tag{37}$$

Under flexible loan rate contracts, the central bank does not have an incentive to pay attention to either loan rates or forward-looking terms. In a model in which no part of the labour cost must be paid through a loan, *i.e.*, $\gamma = 0$, the optimal monetary policy rule reduces to that in a standard New Keynesian model of Woodford (Ch. 4, 2003a) as follows:

$$\kappa \lambda_\pi \pi_t + \lambda_x \triangle x_t = 0. \tag{38}$$

4 Analysis of Optimal Monetary Policy

4.1 *Policy Interest Rate Smoothing*

In reality, central banks often change their policy rates through a series of small adjustments in the same direction, as discussed in previous studies such as Goodfriend (1991) and Woodford (2003b). Woodford (2003b) suggests that optimal commitment policy can induce this gradualism, *i.e.*, the history-dependent property of monetary policy. However, in Woodford's (2003b) model, there is no term that measures the change in interest rates in its loss function. We show that in a staggered loan contract setting, the central bank does indeed have this additional term, which implies that the central bank has the incentive to smooth policy rates.

Giannoni (2000) and Woodford (2003a) theoretically introduced an interest rate term into the loss function by assuming monetary friction. Their loss function, however, includes the quadratic loss of the policy rate deviation from its steady state value rather than that of the change in the policy rates. As such, this loss function is inconsistent with the fact that central banks typically attempt to smooth policy rate changes. In discussing this difference, Woodford (2003b) refers to the *delegation problem*. He shows that a central bank can achieve exactly the same equilibrium as in the optimal commitment policy for the standard loss function consisting of quadratic losses in the inflation rate and the output gap in Woodford (Ch. 6, 2003a) when a central bank minimizes a loss function day by day:[21]

$$J_t = \lambda_\pi \pi_t^2 + \lambda_x x_t^2 + \lambda_i \widehat{i}_t^2 + \lambda_\triangle \left(\widehat{i}_t - \widehat{i}_{t-1} \right)^2, \tag{39}$$

where $\lambda_\triangle$ is a positive parameter.[22] Note that an additional term measuring the change in the policy rates is included. This desirable outcome, however, holds only in a specific environment. Thus, in the delegation problem, the loss function given

[21] The form of day-by-day minimization, known as discretionary policy, is defined in Woodford (2003b).

[22] We set $i^* = 0$ in Woodford (2003b).

by Eq. (39) cannot generally induce equilibrium responses achieved by the optimal commitment policy for the standard loss function.

In contrast to the discussion in Woodford (2003b), the model with staggered loan rates directly modifies the loss function in a way that induces it to smooth policy rates over time. Using the loan rate curve given by Eq. (27), we can transform $\left(\widehat{R}_t - \widehat{R}_{t-1}\right)^2$ in the loss function given by Eq. (30) as

$$\left(\widehat{R}_t - \widehat{R}_{t-1}\right)^2 = \left\{\left[\lambda_1^{-1} n_1^{-1}(1-n_2 L)^{-1}(1-n_1^{-1}F)^{-1}\right]\left[\lambda_3\left(\widehat{i}_t - \widehat{i}_{t-1}\right) - \frac{\lambda_3}{1+\bar{i}}\left(\widehat{M}_t - \widehat{M}_{t-1}\right)\right]\right\}^2, \tag{40}$$

where we have $n_1 + n_2 = \lambda_1^{-1}$ and $n_1 n_2 = -\lambda_1^{-1}\lambda_2$. If there is no shock in the loan rate curve, *i.e.*, $\widehat{M}_t = 0$ for any t, the loan rate stabilization directly implies policy rate stabilization. Thus, when faced with shocks to marginal cost and productivity, the central bank has the incentive to minimize any change in policy rates. The central bank then conducts monetary policy by generating realistic time paths of smoothed policy rates under staggered loan contracts. Another important finding is that the policy rates can be more volatile so as to offset the loan rate shock in the last bracket of the loss function. Loan rates, however, are still adequately stabilized.

We can check the property of the monetary policy smoothing through simulations. Following Steinsson (2007), Table 2 provides a median value of the resulting distribution of the autocorrelation of the policy rate. We assume three types of shock, a productivity shock, a marginal cost shock and a loan rate shock, and two types of monetary policy, an optimal monetary policy and an optimal policy with no loan rate smoothing. The optimal policy with no loan rate smoothing denotes the optimal monetary policy with $\lambda_R = 0$ to evaluate the role of the loan rate smoothing term. We set the autocorrelation of the shock to 0.6 for the base case and 0.4 for the shorter persistence case.

In the base case, for the productivity shock, the optimal monetary policy induces the autocorrelation of the policy rate to be 0.87. This is close to the empirically estimated values of 0.7 to 0.8. Compared with this case, the optimal monetary policy with no loan rate smoothing induces a lower autocorrelation of the policy rate, with a value of 0.66. The difference in persistence arises from the quadratic loss of the first-order difference of loan rates in the approximated welfare function. Thus, the central bank's incentive to smooth the loan rate is an important element in explaining policy rate smoothing in addition to the policy commitment. We can see a similar result for the marginal cost shock. Following the revealed property of the optimal monetary policy, for the loan rate shock, the autocorrelation of the policy rate is smaller at 0.27 under the optimal policy than under the optimal monetary policy with no loan rate smoothing, where it takes a value of 0.43.[23]

Even for the shorter persistence case, optimal monetary policy induces higher autocorrelation of the policy rate than the optimal monetary policy with no loan rate smoothing does for the productivity and marginal cost shocks. We observe the reverse for the loan rate shock.

4.2 *Impulse Responses under Optimal Monetary Policy*

In the following analyses, we use the parameters in Table 1.

4.2.1 *Loan Rate Shock*

Figure 3 depicts the simulation outcomes under optimal monetary policy. We assume an unexpected 1% positive shock on the loan rate in the loan rate curve, where the autocorrelation of the shock is 0.6. The figure shows the percentage deviations from the steady state.

For the loan rate shock, the optimal monetary policy lowers the policy rate to offset the shock for the first few periods, as shown in Eq. (40), although the Taylor-type rule raises the policy rate in response to increased inflation through the cost

[23] These properties do not change even when we assume $\gamma = 0.5$ alongside the other parameters given in Table 1. See details in Appendix D.

channel.[24] Thus, the impulse responses to the other variables differ between the two forms of monetary policy. In particular, the impulse response of the inflation rate is sufficiently mitigated under the optimal monetary policy to reduce the welfare loss.[25]

4.2.2 *Productivity Shock*

Figure 4 provides the simulation outcomes under the optimal monetary policy. We assume an unexpected 1% positive shock on the output gap in the IS curve, where the autocorrelation of the shock is 0.6. The figure shows the percentage deviations from the steady state.

For the productivity shock, the policy rate rises under both rules. The optimal monetary policy, however, increases the policy rate more than the Taylor-type rule, which thereby induces different impulse responses for the other variables. In particular, the impulse responses of the inflation rate and the output gap are sufficiently mitigated under the optimal monetary policy.

4.2.3 *Marginal Cost Shock*

Figure 5 details the simulation outcomes under the optimal monetary policy. We assume an unexpected 1% positive shock on the rate of inflation in the augmented New Keynesian Phillips curve, where the autocorrelation of the shock is 0.6. The figure shows the percentage deviations from the steady state.

For the marginal cost shock, the policy rate rises under both rules. The optimal monetary policy raises the policy rate much more than the Taylor-type rule, which thereby induces different impulse responses for the other variables. As a result, the inflation rate is stabilized more under the optimal monetary policy than under the Taylor-type rule.

[24] $\widehat{M_t}$ is negative to produce a positive loan rate shock.

[25] These outcomes do not change even when we assume $\gamma = 0.5$ alongside the other parameters given in Table 1. See details in Appendix D.

5 Concluding Remarks

In this paper, we introduce staggered nominal loan interest rate contracts between a private bank and a firm under monopolistic competition into the standard New Keynesian model in a tractable way. Simulation results for this model show that a response to a financial shock is greatly amplified by the presence of staggered loan contracts though a response to a productivity, cost-push or monetary policy shock is not much affected. In this sense, staggered loan rates effectively change economic fluctuations.

The normative question of what the central bank should seek to accomplish is a primary concern of our study. We show that a second-order approximation to the consumer's welfare function includes a quadratic loss of the first-order difference term in the loan rate. This is the novel contribution of this analysis. This property implies that the central bank desires to smooth the policy rate over time. In reality, the central bank adjusts the policy rate through a series of small adjustments in the same direction, and it is the staggered property of the loan rate contracts that implies that such small adjustments are theoretically optimal.

In terms of future work, it would be interesting to investigate indeterminacy in the model with staggered loan interest rate contracts. It is also important to introduce a sticky loan interest rate into dynamic stochastic general equilibrium models so as to implement quantitatively rich analysis in these models when explaining economic fluctuations.

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Table 1: *Parameter Values*

Parameters	Values	Explanation
β	0.99	Discount factor
σ	0.2	Intertemporal elasticity of substitution of aggregate expediture
α	0.75	Probability of price unchange
φ	0.58	Probability of loan rate unchange
θ	7.66	Substitutability of differentiated consumption goods
ϵ	7.66	Substitutability of differentiated labors
γ	1	Fraction of external finance
ν	0.11	Elasticity of the desired real wage to the quantity of labor demanded
μ_π	1.72	Coefficient of inflation in Taylor rule
μ_x	0.34	Coefficient of the output gap in Taylor rule
μ_i	0.71	Coefficient of the policy rate lag in Taylor rule

Table 2: *Autocorrelation of Policy Rate*

Shock\Policy	Optimal Monetary Policy	Monetary Policy with No Loan Rate Smoothing
AR(1) of Shock=0.6		
Productivity	0.87	0.66
Marginal Cost	0.91	0.8
Loan Rate	0.27	0.43
AR(1) of Shock=0.4		
Productivity	0.76	0.45
Marginal Cost	0.87	0.79
Loan Rate	0.14	0.44

Notes. The median value of the resulting distribution of AR(1) is calculated by simulating 1000 data series from each model, in which each data length is 100, following Steinsson (2007).

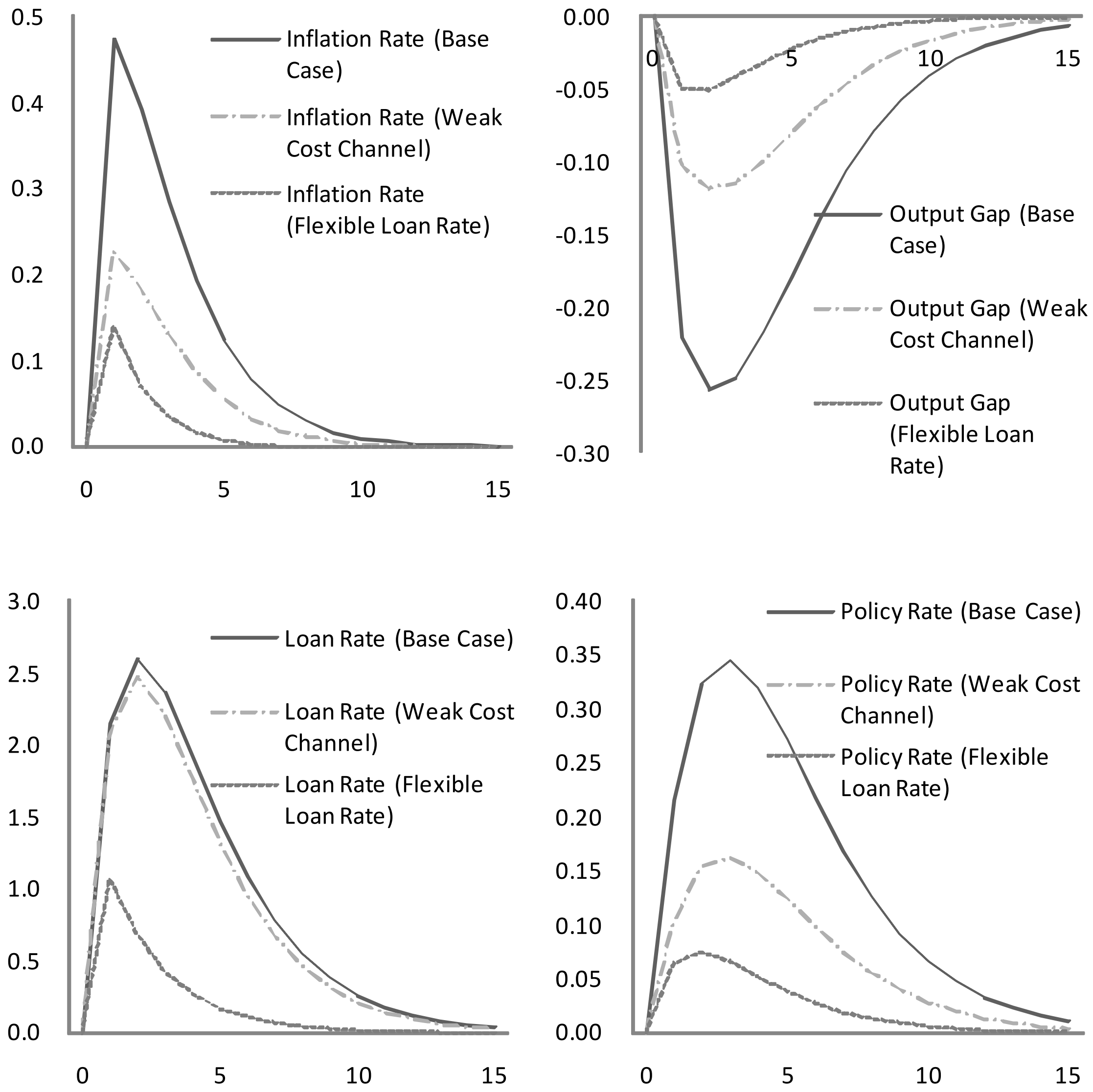

Fig. 1. Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Taylor-type Rule for Loan Rate Shock.

Notes. In the base case, the parameters in Table 1 are used. The model assumes the ratio of the external finance being 0.5 in the case of weak cost channel and assumes the probability of loan rate unchange being zero in the case of flexible loan rate alongside the other parameters given in Table 1.

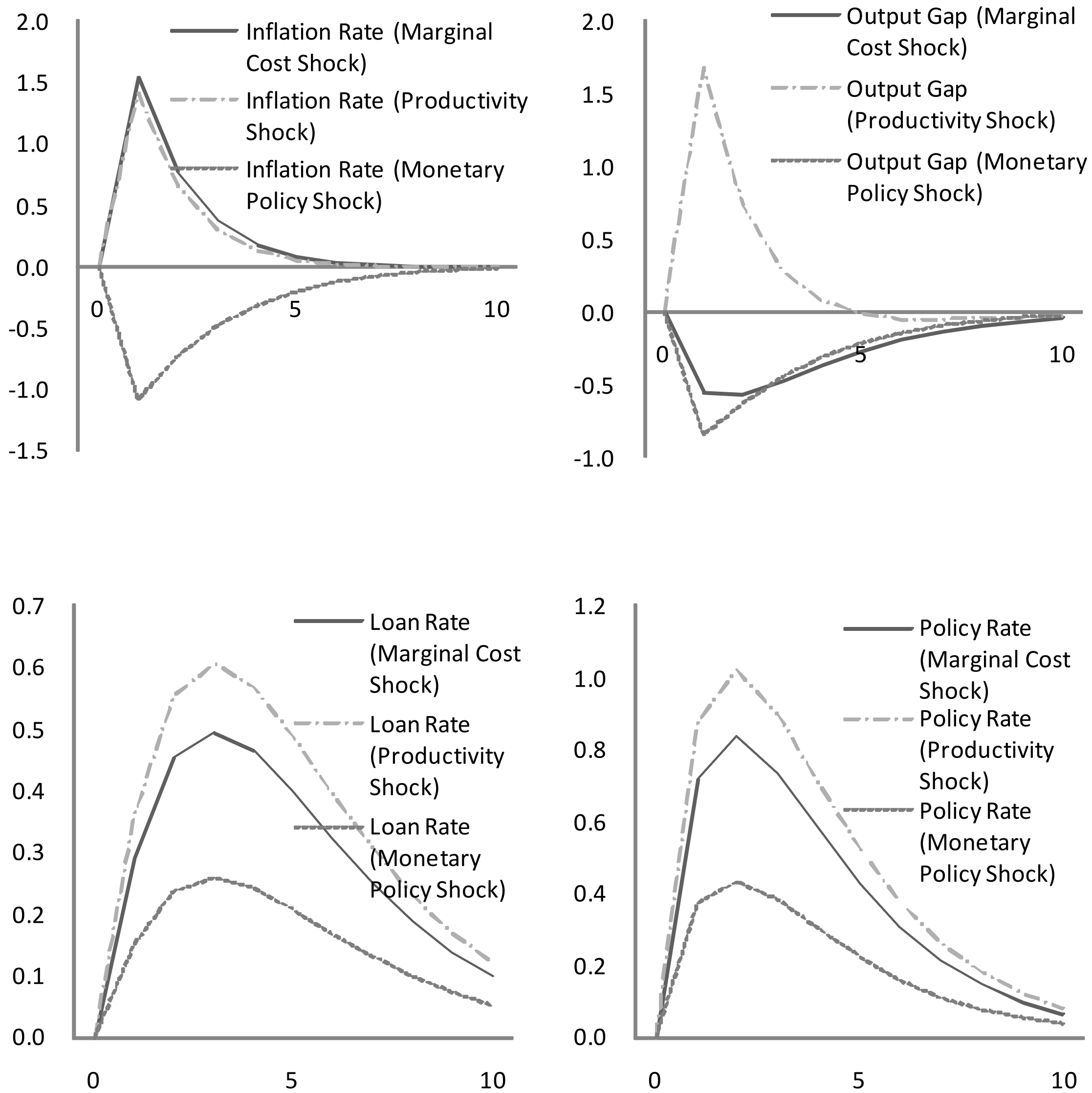

Fig. 2. Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) in Base Case under Taylor-type Rule for Marginal Cost Shock, Productivity Shock, and Monetary Policy Shock.

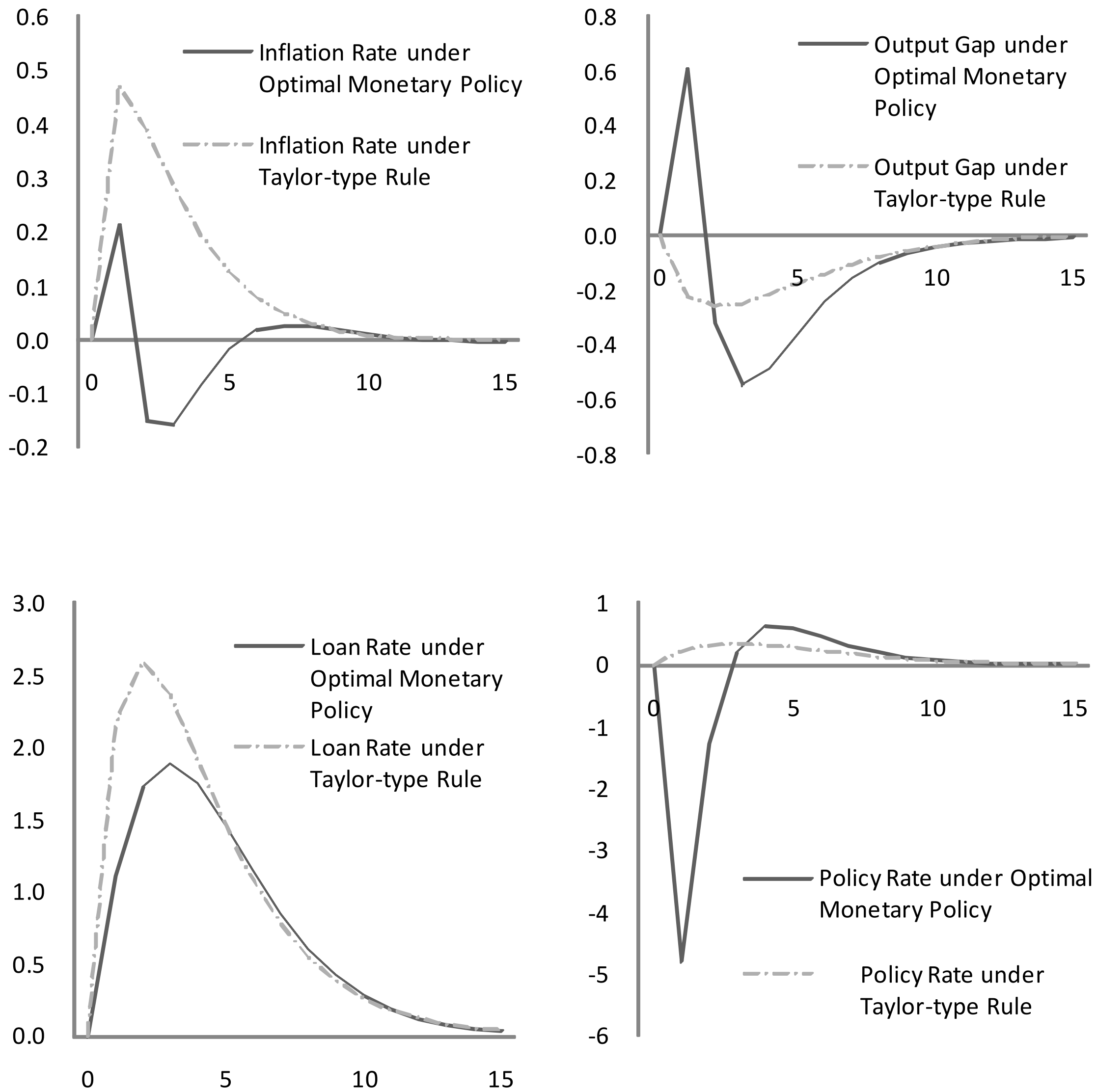

Fig. 3. Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Optimal Monetary Policy and Taylor-type Rule for Loan Rate Shock.

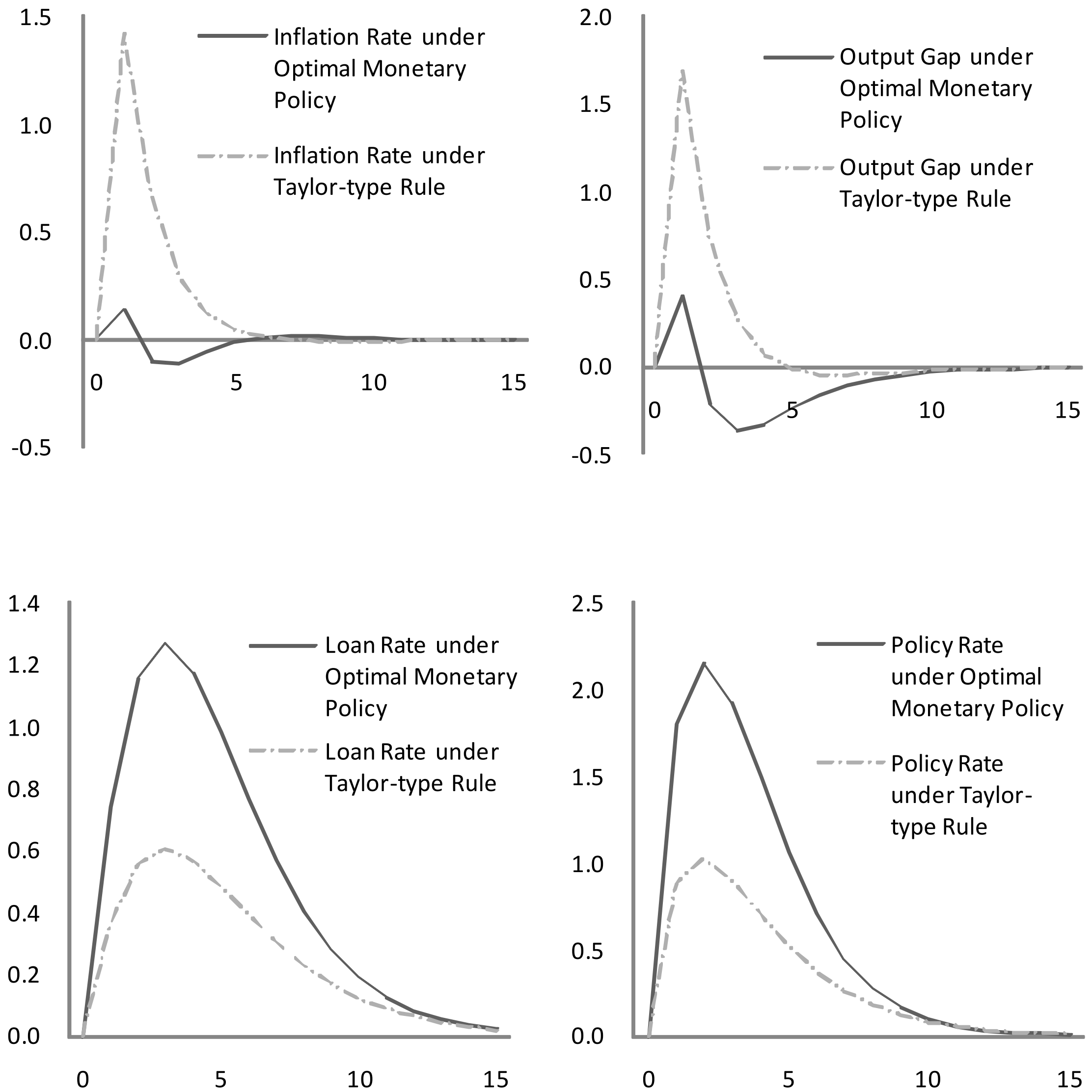

Fig. 4. Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Optimal Monetary Policy and Taylor-type Rule for Productivity Shock.

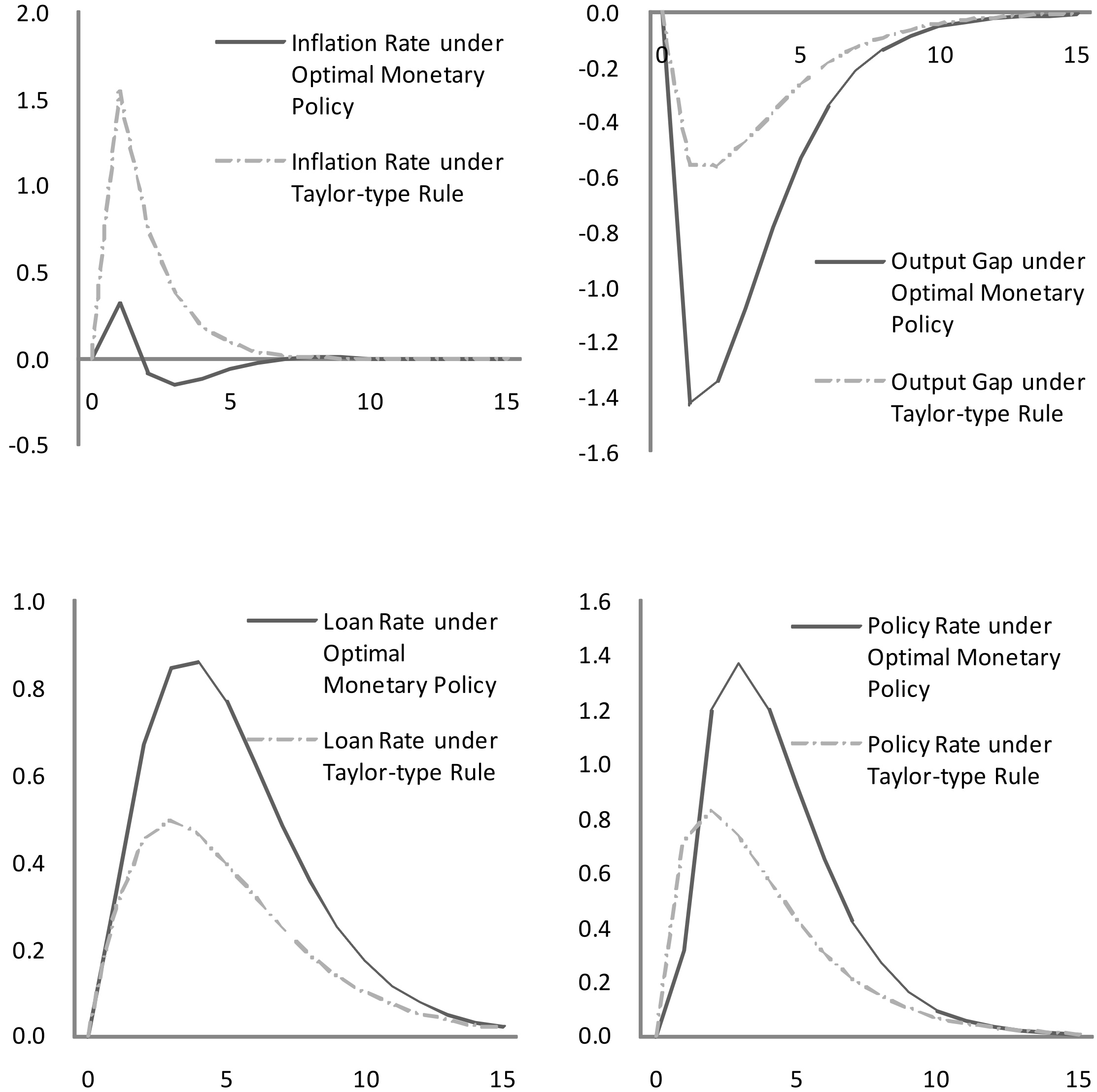

Fig. 5. Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Optimal Monetary Policy and Taylor-type Rule for Marginal Cost Shock.

Appendix (Not for Publication)

In this appendix, I explain the detailed derivation of the model and approximated welfare function.

A Baseline Model

Except x_t and π_t, log-linearized version of variable k_t is expressed by $\widehat{k}_t = \ln(k_t/\overline{k})$, where $\overline{k}$ is steady state value of k_t.

A.1 *Consumer*

A cost minimization problem of consumer on differentiated consumption bundle is given by

$$\min_{c_t(f)} \int_0^1 c_t(f)p_t(f)df,$$

subject to

$$C_t \equiv \left[\int_0^1 c_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}.$$

By defining a following consumption-based price index as

$$P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}},$$

we can derive a relative expenditure on (demand for) differentiated goods as follows:

$$c_t(f) = C_t \left[\frac{p_t(f)}{P_t}\right]^{-\theta}.$$

Then the consumer maximizes the objective function:

$$U_t = \mathrm{E}_t \left\{\sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T, \nu_T) - \int_0^1 V(l_T(h), \nu_T)dh\right]\right\},$$

subject to the budget constraint:

$$P_tC_t + \mathrm{E}_t\left[X_{t,t+1}B_{t+1}\right] + D_t \leq B_t + (1+i_{t-1})D_{t-1} + (1+\tau_w)\int_0^1 w_t(h)l_t(h)dh + \int_0^1 \Pi_t^B(h)dh + \int_0^1 \Pi_t^F(f)df.$$

The consumer chooses C_t, B_{t+1}, D_t, and $w_t(h)$ in every period under given optimal allocation of differentiated goods, then we have following relations:

$$\frac{U_C(C_t,\nu_t)}{U_C(C_{t+1},\nu_{t+1})} = \frac{\beta}{X_{t,t+1}}\frac{P_t}{P_{t+1}}, \tag{41}$$

$$U_C(C_t,\nu_t) = \beta(1+i_t)\mathrm{E}_t\left[U_C(C_{t+1},\nu_{t+1})\frac{P_t}{P_{t+1}}\right], \tag{42}$$

$$(1+\tau_w)\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon-1}\frac{V_l(l_t(h),\nu_t)}{U_C(C_t,\nu_t)},$$

where $1+\tau_w = \frac{\epsilon}{\epsilon-1}$. Under assumption of Eq. (14), we can find that the conditions given by Eq. (41) and the one given by Eq. (42) are same. Thus we use the relation given by Eq. (42). Before log-linearization, under equilibrium $C_t = Y_t$ for any t, we interpret Eq. (42) as

$$U_Y(Y_t,\nu_t) = \beta(1+i_t)\mathrm{E}_t\left[U_Y(Y_{t+1},\nu_{t+1})\frac{P_t}{P_{t+1}}\right]. \tag{43}$$

Under the definitions of $\pi_t \equiv \ln P_t/P_{t-1}$ and $\widehat{i}_t \equiv \ln(1+i_t)/\left(1+\bar{i}\right)$, we log-linearize Eq. (43) around the efficient steady state, then we have

$$x_t = \mathrm{E}_t x_{t+1} - \sigma\left(\widehat{i}_t - \mathrm{E}_t\pi_{t+1} - \widehat{r}_t^n\right),$$

where $\widehat{r}_t^n \equiv \sigma^{-1}\left[-\widehat{Y}_t^n + \mathrm{E}_t\widehat{Y}_{t+1}^n\right]$ and $\sigma \equiv -\frac{U_Y}{U_{YY}\overline{Y}} > 0$. The definition of the output gap is given by the following section.

A.2 *Firm*

As explained, the demand function of loans by a firm is given by

$$q_t(h) = \left[\frac{(1 + \gamma r_t(h))^{-\epsilon} (w_t(h))^{1-\epsilon}}{\Omega_t^*} \right] Q_t. \tag{44}$$

Under given optimal allocation of loans by a president, the h project manager uses loan to finance a part of wage, re-sets its price $p_t(h)$ with probability $1 - \alpha$ to maximize present discounted value of profit given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[(1 + \tau_p) \, p_t(f) y_{t,T}(f) - G_T \int_0^1 (1 + \gamma r_T(h)) w_T(h) l_T(h) dh \right],$$

$$\Longrightarrow \mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[(1 + \tau_p) \, p_t(f) \left[\frac{p_t(f)}{P_T} \right]^{-\theta} Y_T - G_T \Omega_T L_T(f) \right],$$

where we use the outcome from the cost minimization problem and use the demand function on differentiated goods $y_{t,T}(f) \equiv Y_T \left[\frac{p_t(f)}{P_T} \right]^{-\theta}$ from Eq. (3) under $c_t(f) = y_t(f)$ and use $C_t = Y_t$ for any t. G_t is the marginal cost shock as in Woodford (2003), where $\overline{G} = 1$. Here we use consumer's (shareholder's) marginal rate of substitution $X_{t,t+1}$ as given discount rate for each firm's project group. For specifying the derivation, we put f on $l_t(h)$ and L_t. In this case, in the relation of L_t, we may have

$$L_t = \int_0^1 L_t(f) df.$$

It notes that the price setting of firm's project group is independent from the loan rate setting of bank's working group. Then, we can transform the present discounted value of profit as

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} \left[(1 + \tau_p) \, p_t(f) \left[\frac{p_t(f)}{P_T} \right]^{-\theta} Y_T - G_T \Omega_T L_T(f) \right].$$

We can find the optimal price setting $p_t^*(f)$ in a following first-order condition:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{U_Y(C_T, \nu_T)}{P_T} \left[\begin{array}{c} (1 + \tau_p) \, (1 - \theta) y_{t,T}(f) \\ -G_T \left[\int_0^1 ((1 + \gamma r_T(h)) w_T(h))^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}} \frac{\partial L_T(f)}{\partial p_t(f)} \end{array} \right] = 0.$$

$$
\begin{aligned}
\implies & \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \left[(1+\tau_p) \frac{\theta-1}{\theta} \frac{p_t^*(f)}{P_T} \right] \\
= & \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \\
& \times G_T \left\{ \int_0^1 [1+\gamma r_T(h)]^{1-\epsilon} \left[\frac{V_l(l_T(h), \nu_T)}{U_Y(Y_T, \nu_T)} \frac{\partial L_T(f)}{\partial y_{t,T}(f)} \right]^{1-\epsilon} dh \right\}^{\frac{1}{1-\epsilon}}, \qquad (45)
\end{aligned}
$$

due to Eq. (16). Here we assume that the firm's linear production functions is given by $y_t(f) = A_t L_t(f)$. A_t is an exogenous disturbance of technology. Then we can transform Eq. (45) again as

$$
\begin{aligned}
& \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \left[(1+\tau_p) \frac{\theta-1}{\theta} \frac{p_t^*(f)}{P_t} \frac{P_t}{P_{t+1}} \frac{P_{t+1}}{P_{t+2}} \cdots \frac{P_{T-1}}{P_T} \right] \\
= & \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) G_T \left[\int_0^1 (1+\gamma r_T(h))^{1-\epsilon} mc_{t,T}^{1-\epsilon}(h,f) dh \right]^{\frac{1}{1-\epsilon}}, \qquad (46)
\end{aligned}
$$

where we define $mc_{t,T}(h,f) \equiv \frac{V_l(l_T(h),\nu_T)}{U_Y(Y_T,\nu_T)} \frac{\partial L_T(f)}{\partial y_{t,T}(f)}$. By log-linearizing Eq. (46) around the efficient steady state, we have a following equation:

$$
\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\widehat{\widetilde{p}}_t^*(f) - \sum_{\tau=t+1}^{T} \pi_\tau - \frac{\gamma(1+\bar{r})}{1+\gamma\bar{r}} \widehat{R}_T - \widehat{G}_T - \widehat{mc}_T + \omega_p \theta (\widehat{\widetilde{p}}_t^*(f) - \sum_{\tau=t+1}^{T} \pi_\tau) \right] = 0, \qquad (47)
$$

where we define $1 + R_t \equiv \int_0^1 \frac{q_t(h)}{Q_t}(1 + r_t(h))dh$, $\widehat{r}_t(h) \equiv \ln(1 + r_t(h))/(1 + \bar{r})$, and $\widehat{R}_t \equiv \ln(1 + R_t)/(1 + \bar{r})$, and so we have $\widehat{R}_t \equiv \int_0^1 \widehat{r}_t(h)dh$. Also, we define $\widehat{mc}_t(f) \equiv \int_0^1 \widehat{mc}_t(h,f)dh$, $\widehat{mc}_t(h,f) \equiv \ln(mc_t(h,f)/\overline{mc})$, $\widetilde{p}_t^*(f) \equiv \frac{p_t^*(f)}{Pt}$, and $\widehat{\widetilde{p}}_t^*(f) \equiv \ln(\widetilde{p}_t^*(f)/\overline{\widetilde{p}}_t^*)$. It notes that log-linearization for $mc_t(h) \equiv \frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)} \frac{\partial L_t}{\partial Y_t}$ is given by $\widehat{mc}_t \equiv \int_0^1 \widehat{mc}_t(h)dh$, and we make use of the relation of $\widehat{mc}_{t,T}(f) = \widehat{mc}_T - \omega_p \theta(\widehat{\widetilde{p}}_t(f) - \sum_{\tau=t+1}^{T} \pi_\tau)$, where $\omega_p \equiv \frac{f f_{LL}}{(f_L)^2}$. By transforming Eq. (47), we have

$$
\frac{1}{1-\alpha\beta} \widehat{\widetilde{p}}_t^*(f) = \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[(1+\omega_p\theta)^{-1} \left(\widehat{mc}_T + \frac{\gamma(1+\bar{r})}{1+\gamma\bar{r}} \widehat{R}_T + \widehat{G}_T \right) + \sum_{\tau=t+1}^{T} \pi_\tau \right]. \qquad (48)
$$

Thus, all project groups which change prices at time t set the same price. Then, by taking average of f, Eq. (48) can be transformed to

$$\frac{1}{1-\alpha\beta}\widehat{\widetilde{p}}_t^* = \mathrm{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[(1+\omega_p\theta)^{-1}\left(\widehat{mc}_T + \frac{\gamma(1+\overline{r})}{1+\gamma\overline{r}}\widehat{R}_T + \widehat{G}_T\right) + \sum_{\tau=t+1}^{T}\pi_\tau\right], \tag{49}$$

where $(p_t^*)^{1-\theta} \equiv \int_0^1 p_t^*(f)^{1-\theta}df$, and so $\widehat{\widetilde{p}}_t^* = \int_0^1 \widehat{\widetilde{p}}_t^*(f)df$. In the Calvo (1983) - Yun (1992) setting, the evolution of aggregate price index is described by the following motion:

$$\int_0^1 p_t(f)^{1-\theta}df = \alpha\int_0^1 p_{t-1}(f)^{1-\theta}df + (1-\alpha)\int_0^1 p_t^*(f)^{1-\theta}df,$$

$$\Longrightarrow P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(p_t^*)^{1-\theta}. \tag{50}$$

By log-linearizing Eq. (50), we have

$$\widehat{\widetilde{p}}_t^* = \frac{\alpha}{1-\alpha}\pi_t. \tag{51}$$

After substituting Eq. (51) into Eq. (49), we have a following relation:

$$\frac{\alpha}{1-\alpha}\pi_t = (1-\alpha\beta)\mathrm{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[(1+\omega_p\theta)^{-1}\left(\widehat{mc}_T + \frac{\gamma(1+\overline{r})}{1+\gamma\overline{r}}\widehat{R}_T + \widehat{G}_T\right) + \sum_{\tau=t+1}^{T}\pi_\tau)\right]. \tag{52}$$

Then, by considering of $\frac{\alpha}{1-\alpha}\pi_t - \alpha\beta\mathrm{E}_t\frac{\alpha}{1-\alpha}\pi_{t+1}$ in Eq. (52), we finally have the augmented Phillips curve:

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega_p\theta)}\left(\widehat{mc}_t + \frac{\gamma(1+\overline{r})}{1+\gamma\overline{r}}\widehat{R}_t\right) + \beta\mathrm{E}_t\pi_{t+1} + \frac{\kappa}{\omega+\sigma^{-1}}\widehat{G}_t.$$

On the other hand, according to the discussion in Woodford (2003a), we define the natural rate of output Y_t^n from Eq. (45) as

$$(1+\tau_p)\frac{\theta-1}{\theta} - (1+\gamma\overline{r})\left\{\int_0^1\left[\frac{G_tV_l(l_t^n(h),\nu_t)}{U_Y(Y_t^n,\nu_t)}\right]^{1-\epsilon}dh\right\}^{\frac{1}{1-\epsilon}} = 0, \tag{53}$$

$$\left\{\int_0^1\left[\frac{G_tV_l(l_t^n(h),\nu_t)}{U_Y(Y_t^n,\nu_t)}\right]^{1-\epsilon}dh\right\}^{\frac{1}{1-\epsilon}} = 1, \tag{54}$$

where we assume a flexible price setting $p_t^*(f) = P_t$ and assume constant loan rate $\widehat{r}_t(h) = \overline{r}$ as in Ravenna and Walsh (2006) under the natural rate of output, so hold $y_t(f) = Y_t^n$. Also, $l_t^n(h)$ is the amount of labor type h employed under Y_t^n. The disturbances by ν_t and A_t induces the disturbance of the natural rate of output. The definition of the natural rate of output is slightly different from one defined in Friedman(1968) and Woodford (2003a) in terms of treatment of loan rates in Eq. (53).[26] Here, $1 + \tau_p$ is given by $\frac{\theta}{\theta-1}(1 + \gamma\overline{r})$. Then, we have

$$\widehat{mc}_t = (\omega + \sigma^{-1})(\widehat{Y}_t - \widehat{Y}_t^n),$$

where $\widehat{Y}_t \equiv \ln(Y_t/\overline{Y})$, $\widehat{Y}_t^n \equiv \ln(Y_t^n/\overline{Y})$, and $\omega \equiv \omega_p + \omega_w$.[27] Here ω_w is the elasticity of marginal disutility of work with respect to output increase in $\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)}$, which is given by $\frac{\overline{L}V_{ll}}{V_l}$ in a case of a linear production function. Then, by defining $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$, we finally have

$$\pi_t = \kappa x_t + \xi \widehat{R}_t + \beta \mathrm{E}_t \pi_{t+1} + \frac{\kappa}{\omega + \sigma^{-1}} \widehat{G}_t,$$

where $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)(\omega+\sigma^{-1})}{\alpha(1+\omega_p\theta)}$ and $\xi \equiv \frac{(1-\alpha)(1-\alpha\beta)\gamma(1+\overline{r})}{\alpha(1+\omega_p\theta)(1+\gamma\overline{r})}$.

A.3 *Private Bank*

Then under given demand function of loan set by Eq. (44), each working group of private bank re-sets its loan rates, $r_t(h)$, with probability $1 - \varphi$ to maximize present discounted value of profit given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} \varphi^{T-t} X_{t,T} \left[M_T (1 + \tau_r)(1 + r_t(h)) - (1 + i_T) \right] q_{t,T}(h), \tag{55}$$

where we define $q_{t,T}(h) = \left[\frac{(1+\gamma r_t(h))^{-\epsilon}(w_T(h))^{1-\epsilon}}{\Omega_T^*} \right] Q_T$ from Eq. (11) and $Q_T \equiv \int_0^1 q_{t,T}(h)dh$, i_t is deposit rates which is set by a central bank and is same for all working groups. We assume $z_t(h)$ is zero, $z(h) = 0$ and $D_t = Q_t$ in equilibrium. Then, we can

[26] Friedman, M., 1968. The role of monetary policy. American Economic Review 58, 1–17.

[27] We can see more detailed derivation in Woodford (Ch. 3, 2003).

transform Eq. (55) as

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} q_{t,T}(h) \left[M_T (1+\tau_r)(1+r_t(h)) - (1+i_T) \right].$$

Then an optimal loan rate setting of $r_t(h)$ under the situation in which managers can re-set their loan rates with probability $1-\varphi$ is given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} q_{t,T}(h) \left[M_T (1+\tau_r) - \epsilon\gamma \frac{M_T (1+\tau_r)(1+r_t(h)) - (1+i_T)}{1+\gamma r_t(h)} \right] = 0. \tag{56}$$

To eliminate the distortion from monopolistic competition in loan market and realize $1+\bar{r} = 1+\bar{i}$, we set $1+\tau_r = \frac{\epsilon\gamma(1+\bar{r})}{\epsilon\gamma(1+\bar{r})-1-\gamma\bar{r}}$.[28]

By log-linearizing Eq. (56), we have a following equation:

$$\frac{\gamma(1-\epsilon)(1+\bar{r})(1+\tau_r)}{\beta\varphi - 1} \widehat{r}_t(h) = \mathrm{E}_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \left[\gamma\epsilon(1+\bar{i})\widehat{i}_T - \gamma\epsilon \widehat{M}_T \right]. \tag{57}$$

Here working groups that are allowed to change their loan rates will set the same loan rate, so the solution of $r_t(h)$ in Eq. (56) is expressed by r_t^*, and so the solution of $\widehat{r}_t(h)$ in Eq. (57) is expressed by $\widehat{r}_t^*$. On the other hand, we have the following evolution of aggregate loan rate index:

$$1+R_t = \varphi(1+R_{t-1}) + (1-\varphi)(1+r_t^*). \tag{58}$$

By log-linearizing Eq. (58) around the efficient steady state, we have

$$\widehat{r}_t^* = \frac{1}{1-\varphi}\widehat{R}_t - \frac{\varphi}{1-\varphi}\widehat{R}_{t-1}.$$

Then, by considering of $\widehat{r}_t^* - \varphi\beta \mathrm{E}_t \widehat{r}_{t+1}^*$ in Eq. (57), we finally have a loan rate curve:

$$\widehat{R}_t = \lambda_1 \mathrm{E}_t \widehat{R}_{t+1} + \lambda_2 \widehat{R}_{t-1} + \lambda_3 \widehat{i}_t - \frac{\lambda_3}{1+\bar{i}} \widehat{M}_t,$$

where $\lambda_1 \equiv \frac{\varphi\beta}{1+\varphi^2\beta}$, $\lambda_2 \equiv \frac{\varphi}{1+\varphi^2\beta}$, and $\lambda_3 \equiv \frac{1+\bar{i}}{1+\bar{r}} \frac{\epsilon}{\epsilon-1} \frac{(1-\varphi\beta)(1-\varphi)}{1+\varphi^2\beta} \frac{1}{1+\tau_r}$.

[28] When $\gamma = 1$, $1+\tau_r = \frac{\epsilon}{\epsilon-1}$.

B Impulse Responses of Other Cases

I show the impulse responses of the cases for the weak cost channel and the flexible loan rate to the marginal cost shock, the productivity shock, and the monetary policy shock in Figures 6 to 8, respectively.

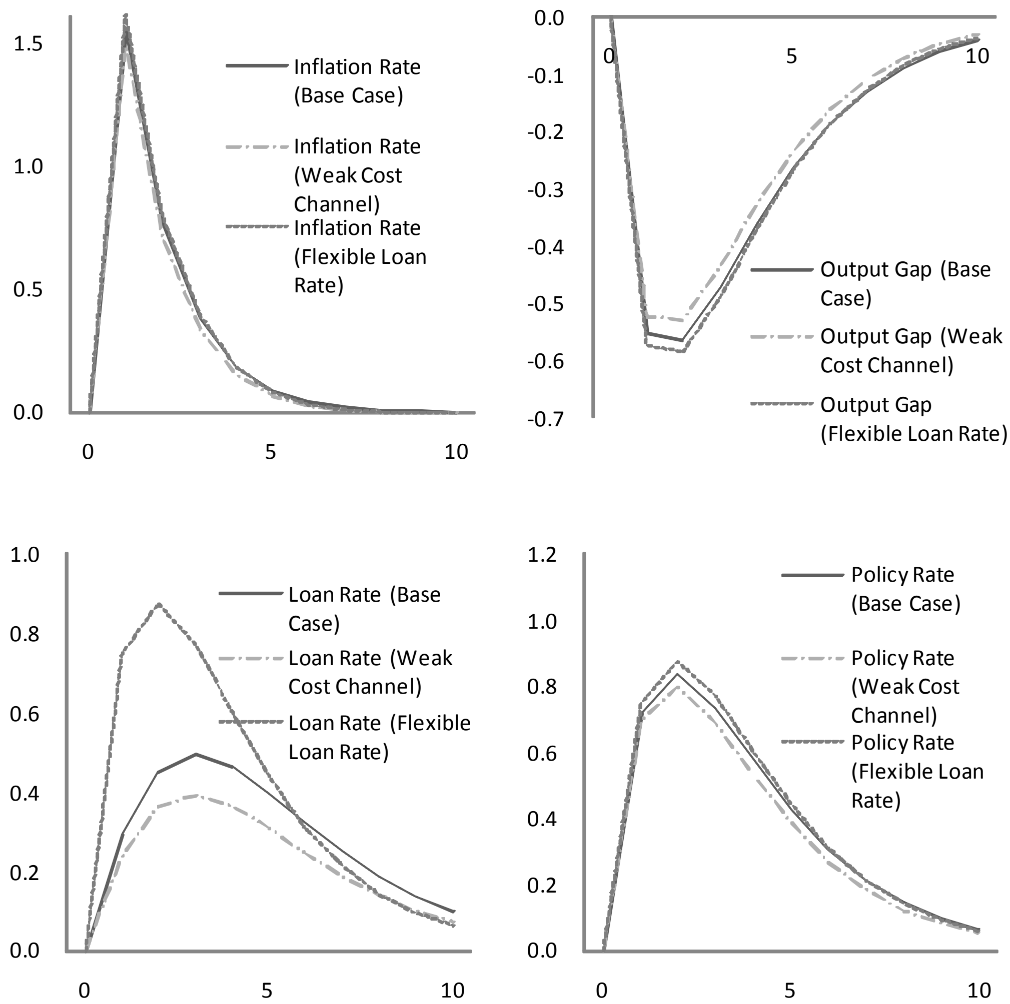

Fig. 6. Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Taylor-type Rule for Marginal Cost Shock.

Notes. In the case of weak cost channel, the model assumes the ratio of the external finance is 0.5. In the case of flexible loan rate, the model assumes the flexible loan rate.

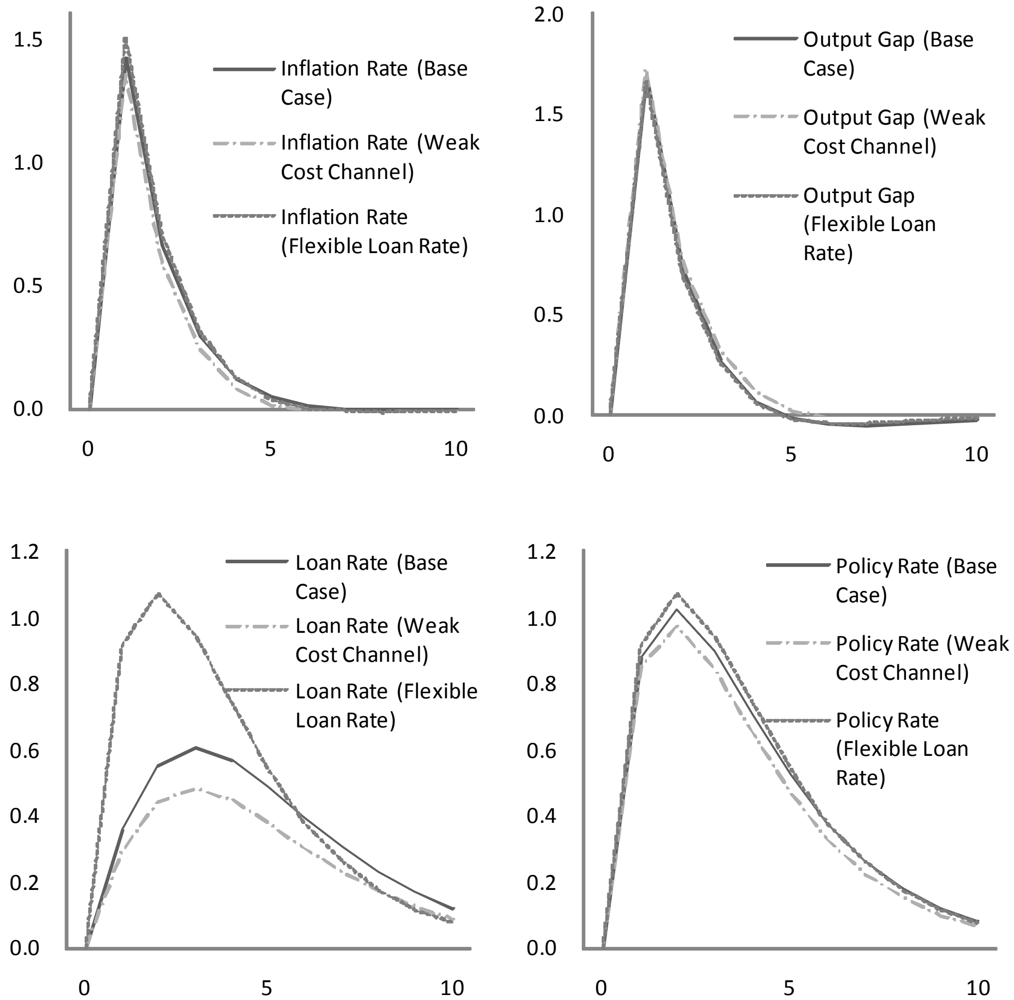

Fig. 7. Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Taylor-type Rule for Productivity Shock.

Notes. In the case of weak cost channel, the model assumes the ratio of the external finance is 0.5. In the case of flexible loan rate, the model assumes the flexible loan rate.

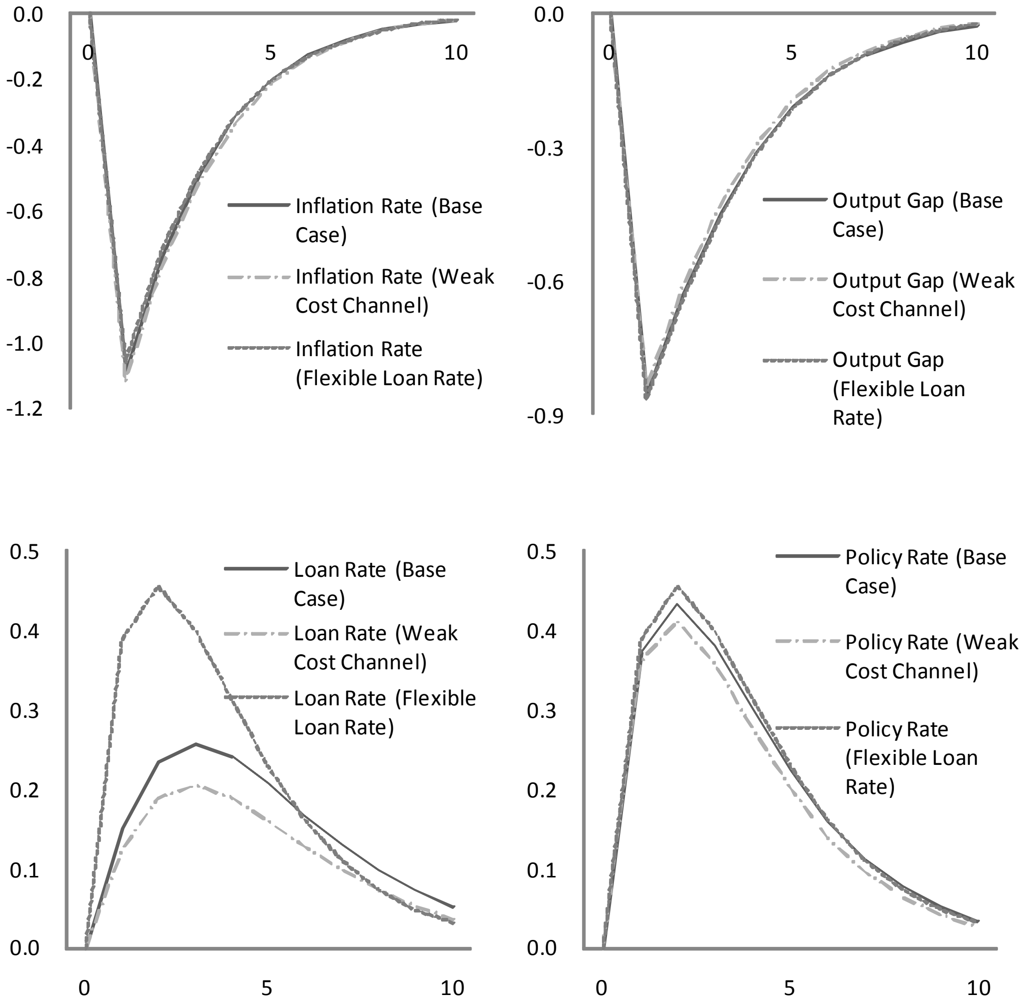

Fig. 8. Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Taylor-type Rule for Monetary Policy Shock.

Notes. In the case of weak cost channel, the model assumes the ratio of the external finance is 0.5. In the case of flexible loan rate, the model assumes the flexible loan rate.

C Derivation of Approximated Welfare Function

In derivation of approximated welfare function, we basically follow the way of Woodford (2003a). Note that we think of the second order approximation around the efficient steady state. Under the situation in which goods supply matches goods demand in every level, $Y_t = C_t$ and $y_t(f) = c_t(f)$ for any f, the welfare criteria of consumer is given by

$$\mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} N_T \right\},$$

where

$$N_t \equiv U(Y_t, \nu_t) - \int_0^1 V(l_t(h), \nu_t) dh, \tag{59}$$

and

$$Y_t \equiv \left[\int_0^1 y_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}.$$

We log-linearize Eq. (59) step by step to derive an approximated welfare function. Firstly, we log-linearize the first term of Eq. (59):

$$\begin{aligned} U(Y_t; \nu_t) &= \overline{U} + U_c \widetilde{Y}_t + U_\nu \nu_t + \frac{1}{2} U_{cc} \widetilde{Y}_t^2 + U_{c\nu} \widetilde{Y}_t + \frac{1}{2} \nu_t' U_{\nu\nu} \nu_t + Order(\| \xi \|^3) \\ &= \overline{U} + \overline{Y} U_c \left(\widehat{Y}_t + \frac{1}{2} \widehat{Y}_t^2 \right) + U_\nu \nu_t + \frac{1}{2} U_{cc} \overline{Y}^2 \widehat{Y}_t^2 + \overline{Y} U_{c\nu} \nu_t \widehat{Y}_t + \frac{1}{2} \nu_t' U_{\nu\nu} \nu_t + Order(\| \xi \|^3) \\ &= \overline{Y} U_c \widehat{Y}_t + \frac{1}{2} \left[\overline{Y} U_c + \overline{Y}^2 U_{cc} \right] \widehat{Y}_t^2 - \overline{Y}^2 U_{cc} g_t \widehat{Y}_t + t.i.p + Order(\| \xi \|^3) \\ &= \overline{Y} U_c \left[\widehat{Y}_t + \frac{1}{2} (1 - \sigma^{-1}) \widehat{Y}_t^2 + \sigma^{-1} g_t \widehat{Y}_t \right] + t.i.p + Order(\| \xi \|^3), \end{aligned} \tag{60}$$

where $\overline{U} \equiv U(\overline{Y}; 0)$, $\widetilde{Y}_t \equiv Y_t - \overline{Y}$, $t.i.p$ means the terms that are independent from monetary policy, $Order(\| \xi \|^3)$ expresses order terms higher than the second order approximation, $\sigma^{-1} \equiv -\frac{\overline{Y} U_{cc}}{U_c} > 0$, and $g_t \equiv -\frac{U_{c\nu} \nu_t}{\overline{Y} U_{cc}}$. To replace $\widetilde{Y}_t$ by $\widehat{Y}_t \equiv \ln(Y_t / \overline{Y})$, we use the Taylor series expansion on $Y_t / \overline{Y}$ in the second line as

$$Y_t/\overline{Y} = 1 + \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + Order(\parallel \xi \parallel^3).$$

Secondly, we log-linearize the second term of Eq. (59) by a similar way:

$$\begin{aligned}
\int_0^1 V(l_t(h);\nu_t)dh &= V_l\overline{L}(E_h\widehat{l}_t(h) + \frac{1}{2}E_h(\widehat{l}_t(h))^2) + \frac{1}{2}V_{ll}\overline{L}^2E_h(\widehat{l}_t(h))^2 + V_{l\nu}\overline{L}\nu_tE_h\widehat{l}_t(h) \\
+t.i.p + Order(&\parallel \xi \parallel^3) \\
&= \overline{L}V_l\left[\widehat{L}_t + \frac{1}{2}(1+\nu)\widehat{L}_t^2 - \nu\widetilde{\nu}_t\widehat{L}_t + \frac{1}{2}(\nu + \frac{1}{\epsilon})var_h\widehat{l}_t(h)\right] + t.i.p + Order(\parallel \xi \parallel^3) \\
&= \overline{L}V_l\left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t\widehat{Y}_t + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ +\frac{1}{2}(\nu + \frac{1}{\epsilon})var_h \ln l_t(h) \end{array}\right] \\
+t.i.p + Order(&\parallel \xi \parallel^3) \\
&= \overline{Y}U_c\left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t\widehat{Y}_t \\ +\frac{1}{2}(\nu + \frac{1}{\epsilon})var_h\widehat{l}_t(h) + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \end{array}\right] \\
+t.i.p + Order(&\parallel \xi \parallel^3), \qquad (61)
\end{aligned}$$

where $\widetilde{\nu}_t \equiv -\frac{V_{l\nu}\nu_t}{\overline{L}V_{ll}}$, $\nu \equiv \frac{\overline{L}V_{ll}}{V_l}$, $q_t \equiv (1+\omega^{-1})a_t + \omega^{-1}\nu\widetilde{\nu}_t$, $a_t \equiv \ln A_t$, $var_h\widehat{l}_t(h)$ is the variance of $\widehat{l}_t(h)$ across all types of labor, and $var_f\widehat{p}_t(f)$ is the variance of $\widehat{p}_t(f)$ across all differentiated good prices. Here the definition of labor aggregator is given by

$$L_t \equiv \left[\int_0^1 l_t(h)^{\frac{\epsilon-1}{\epsilon}}dh\right]^{\frac{\epsilon}{\epsilon-1}},$$

and so we have $\widehat{L}_t = E_h\widehat{l}_t(h) + \frac{1}{2}\frac{\epsilon-1}{\epsilon}var_h\widehat{l}_t(h) + Order(\parallel \xi \parallel^3)$ in the second order approximation. We use this relation in the second line. From the second line to the third line, we use the condition that the demand of labor is equal to the supply of labor as

$$L_t = \int_0^1 L_t(f)df = \int_0^1 f^{-1}(\frac{y_t(f)}{A_t})df,$$

where the linear production function is given by $y_t(f) = A_tL_t(f)$, where $f(\cdot)$ is an increasing and concave function. By taking the second order approximation, we have

$$\widehat{L}_t = \left(\widehat{Y}_t - a_t\right) + \frac{1}{2}\omega_p\left(\widehat{Y}_t - a_t\right)^2 + \frac{1}{2}(1+\omega_p\theta)\theta var_f\widehat{p}_t(f) + Order(\parallel \xi \parallel^3),$$

where we log-linearize the demand function on differentiated goods to derive the relation $var_f \ln y_t(f) = \theta^2 var_f \ln p_t(f)$, which can be derived from the consumer's cost minimization problem under Dixit-Stiglitz aggregator, as

$$y_t(f) = Y_t \left[\frac{p_t(f)}{P_t} \right]^{-\theta},$$

where the aggregate price index is given by $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}}$. Also, we use the relation of $\nu = \omega_w$. To the forth line, we replace $\overline{L}V_l$ by $\overline{Y}U_c$ from Eq. (54), where there is no distortion in the steady state.

Then we can combine Eq. (60) and Eq. (61) as

$$\begin{aligned} N_t &= \overline{Y}U_c \left[\begin{array}{c} \widehat{Y}_t - \frac{1}{2}(\sigma^{-1} + \omega)\widehat{Y}_t^2 + (\sigma^{-1}g_t + \omega q_t)\widehat{Y}_t \\ -\frac{1}{2}\theta(1 + \omega_p\theta)var_f \ln p_t(f) - \frac{1}{2}\left(\nu + \epsilon^{-1}\right) var_h \ln l_t(h) \end{array} \right] \\ &+t.i.p + Order(\parallel \xi \parallel^3) \qquad (62) \\ &= -\frac{1}{2}\overline{Y}U_c \left[(\sigma^{-1} + \omega)x_t^2 + \theta(1 + \omega_p\theta)var_f \ln p_t(f) + \left(\nu + \epsilon^{-1}\right) var_h \ln l_t(h) \right] \\ &+t.i.p + Order(\parallel \xi \parallel^3). \end{aligned}$$

In the second line, we use the log-linearization of Eq. (54) as

$$\widehat{Y}_t^n \equiv \ln(Y_t^n/\overline{Y}) = \frac{\sigma^{-1}g_t + \omega q_t}{\sigma^{-1} + \omega} + Order(\parallel \xi \parallel^2).$$

To evaluate $var_h \widehat{l_t}(h)$, we use the optimal condition of labor supply and the labor demand function given by following equations

$$l_t(h) = L_t \left[\frac{(1 + \gamma r_t(h))w_t(h)}{\Omega_t} \right]^{-\epsilon},$$

$$\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{V_l(l_t(h), \nu_t)}{U_C(C_t, \nu_t)}.$$

where

$$\Omega_t \equiv \left[\int_0^1 ((1 + \gamma r_t(h))w_t(h))^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}}.$$

By log-linearizing these equations, we finally have a following relation:

$$var_h \ln l_t(h) = \frac{\epsilon^2}{(1+\nu\epsilon)^2}\left(\frac{\gamma(1+\overline{r})}{1+\gamma\overline{r}}\right)^2 var_h \ln(1+r_t(h)) + Order(\parallel \xi \parallel^3).$$

Then, Eq. (62) is transformed into

$$N_t = -\frac{1}{2}\overline{Y}U_c \left[\begin{array}{c} (\sigma^{-1}+\omega)x_t^2 + \theta(1+\omega_p\theta)var_f \ln p_t(f) \\ +\frac{\epsilon^2(\nu+\epsilon^{-1})}{(1+\nu\epsilon)^2}\left(\frac{\gamma(1+\overline{r})}{1+\gamma\overline{r}}\right)^2 var_h \ln(1+r_t(h)) \end{array} \right] + t.i.p + Order(\parallel \xi \parallel^3). \tag{63}$$

The remaining work to derive the approximated welfare function is to evaluate $var_h \ln p_t(f)$ and $var_h \ln(1+r_t(h))$ in Eq. (63). Following Woodford (2003a), we define

$$\overline{P}_t \equiv \mathrm{E}_f \ln p_t(f),$$

$$\triangle_t \equiv var_f \ln p_t(f).$$

Then we can make the following relation as

$$\begin{aligned} \overline{P}_t - \overline{P}_{t-1} &= \mathrm{E}_f \left[\ln p_t(f) - \overline{P}_{t-1}\right] \\ &= \alpha \mathrm{E}_f \left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right] + (1-\alpha)\mathrm{E}_f \left[\ln p_t^*(f) - \overline{P}_{t-1}\right] \\ &= (1-\alpha)\mathrm{E}_f \left[\ln p_t^*(f) - \overline{P}_{t-1}\right], \end{aligned} \tag{64}$$

and we also have

$$
\begin{aligned}
\triangle_t &= var_f\left[\ln p_t(f) - \overline{P}_{t-1}\right] \\
&= \mathrm{E}_f\left\{\left[\ln p_t(f) - \overline{P}_{t-1}\right]^2\right\} - (\mathrm{E}_f \ln p_t(f) - \overline{P}_{t-1})^2 \\
&= \alpha \mathrm{E}_f\left\{\left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right]^2\right\} + (1-\alpha)\mathrm{E}_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - \left(\overline{P}_t - \overline{P}_{t-1}\right)^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)\mathrm{E}_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)(var_f(\ln p_t^*(f) - \overline{P}_{t-1}) + \left\{\mathrm{E}_f\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]\right\}^2) - \left(\overline{P}_t - \overline{P}_{t-1}\right)^2 \\
&= \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}\left(\overline{P}_t - \overline{P}_{t-1}\right)^2, \qquad (65)
\end{aligned}
$$

where we use Eq. (64) and $p_t^*(f)$ is an optimal price setting by the agent f following the Calvo (1983) - Yun (1992) framework. It notes that all project groups re-set the same price at time t when they are selected to change prices, because the unit marginal cost of production is same for all project groups. Also, we have a following relation that relates $\overline{P}_t$ with P_t as

$$\overline{P}_t = \ln P_t + Order(\parallel \xi \parallel^2),$$

where $Order(\parallel \xi \parallel^2)$ is order terms higher than the first order approximation. Here we make use of the definition of price aggregator $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$. Then Eq. (65) can be transformed as

$$\triangle_t = \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}\pi_t^2. \qquad (66)$$

From Eq. (66), we have

$$\triangle_t = \alpha^{t+1}\triangle_{-1} + \sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha}{1-\alpha}\right)\pi_s^2,$$

and so

$$\mathrm{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\triangle_T = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\mathrm{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\pi_T^2 + t.i.p + Order(\parallel \xi \parallel^3). \qquad (67)$$

To evaluate $var_h \ln(1 + r_t(h))$, we define $\overline{R}_t$ and $\triangle_t^R$ as

$$\overline{R}_t \equiv \mathrm{E}_h \ln(1 + r_t(h)),$$

$$\triangle_t^R \equiv var_h \ln(1 + r_t(h)).$$

Then, we can make following relations:

$$\begin{aligned} \overline{R}_t - \overline{R}_{t-1} &= \mathrm{E}_h \left[\ln(1 + r_t(h)) - \overline{R}_{t-1} \right] \\ &= \varphi \mathrm{E}_h \left[\ln(1 + r_{t-1}(h)) - \overline{R}_{t-1} \right] + (1 - \varphi) \left[\ln(1 + r_t^*) - \overline{R}_{t-1} \right] \\ &= (1 - \varphi) \left[\ln(1 + r_t^*(h)) - \overline{R}_{t-1} \right], \end{aligned} \tag{68}$$

and

$$\begin{aligned} \triangle_t^R &= var_h \left[\ln(1 + r_t(h)) - \overline{R}_{t-1} \right] \\ &= \mathrm{E}_h \left\{ \left[\ln(1 + r_t(h)) - \overline{R}_{t-1} \right]^2 \right\} - \left(\mathrm{E}_h \ln(1 + r_t(h)) - \overline{R}_{t-1} \right)^2 \\ &= \varphi \mathrm{E}_h \left\{ \left[\ln(1 + r_{t-1}(h)) - \overline{R}_{t-1} \right]^2 \right\} + (1 - \varphi) \left[\ln(1 + r_t^*) - \overline{R}_{t-1} \right]^2 - \left(\overline{R}_t - \overline{R}_{t-1} \right)^2 \\ &= \varphi \triangle_{t-1}^R + \frac{\varphi}{1 - \varphi} \left(\overline{R}_t - \overline{R}_{t-1} \right)^2, \end{aligned} \tag{69}$$

where we use Eq. (68). Also, as in the discussion on price, we have

$$\overline{R}_t = \ln(1 + R_t) + Order(\parallel \xi \parallel^2), \tag{70}$$

where we make use of the definition of the aggregate loan rates $1 + R_t \equiv \int_0^1 \frac{q_t(h)}{Q_t}(1 + r_t(h))dh$. Then, from Eq. (69) and Eq. (70), we have

$$\triangle_t^R = \varphi \triangle_{t-1}^R + \frac{\varphi}{1 - \varphi} \left(\widehat{R}_t - \widehat{R}_{t-1} \right)^2, \tag{71}$$

where $\widehat{R}_t \equiv \ln \frac{1+R_t}{1+\bar{r}}$. From Eq. (71), we have

$$\triangle_t^R = \varphi^{t+1} \triangle_{-1}^R + \sum_{s=0}^{t} \varphi^{t-s} \left(\frac{\varphi}{1 - \varphi} \right) \left(\widehat{R}_s - \widehat{R}_{s-1} \right)^2,$$

and so

$$\mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \triangle_T^R = \frac{\varphi}{(1-\varphi)(1-\varphi\beta)} \mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left(\widehat{R}_T - \widehat{R}_{T-1} \right)^2 + t.i.p + Order(\parallel \xi \parallel^3). \tag{72}$$

From Eq. (63), Eq. (67), and Eq. (72), we finally have

$$\mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} N_T \simeq -\Lambda \mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left(\lambda_\pi \pi_T^2 + \lambda_x x_t^2 + \lambda_R \left(\widehat{R}_T - \widehat{R}_{T-1} \right)^2 \right),$$

where $\Lambda \equiv \frac{1}{2}\overline{Y} u_c$, $\lambda_\pi \equiv \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta(1+\omega_p\theta)$, $\lambda_x \equiv (\sigma^{-1}+\omega)$, and $\lambda_R \equiv \frac{\epsilon^2}{(1+\nu\epsilon)^2} \left(\frac{\gamma(1+\bar{r})}{1+\gamma\bar{r}} \right)^2 \frac{\varphi\left(\nu+\epsilon^{-1}\right)}{(1-\varphi)(1-\varphi\beta)}$.

D Robustness Analysis

D.1 *Robust Analysis for Different Fraction of External Finance for Labour Cost (γ) in Table 2*

To check the robustness of the outcomes in Table 2 for different value of fraction of external finance for labour cost, we set γ as 0.5 alongside the other parameters given in Table 1. Table 3 shows the simulation results. The properties of the results do not significantly differ from ones for the base case. The optimal monetary policy induces a higher autocorrelation of the policy rate than the optimal monetary policy with no loan rate smoothing does for the productivity shock and the marginal cost shock. For the loan rate shock, the optimal monetary policy induces a lower autocorrelation of the policy rate than the optimal monetary policy with no loan rate smoothing does.

Table 3: Autocorrelation of Policy Rate When $\gamma = 0.5$

Shock\Policy	Optimal Monetary Policy	Monetary Policy with No Loan Rate Smoothing
AR(1) of Shock=0.6		
Productivity	0.72	0.62
Marginal Cost	0.87	0.85
Loan Rate	0.1	0.5
AR(1) of Shock=0.4		
Productivity	0.54	0.42
Marginal Cost	0.76	0.71
Loan Rate	-0.02	0.57

Note: The median value of the resulting distribution of AR(1) is calculated by simulating 1000 data series from each model, in which each data length is 100, following Steinsson (2007).

D.2 *Robust Analysis for Different Fraction of External Finance for Labour Cost (γ) in Figures 3 to 5*

To check the robustness of the outcomes in Figure 3 to 5 for different value of fraction of external finance for labour cost, we set γ as 0.5 alongside the other parameters given in Table 1. Figures 9 to 11 show the simulation results when we assume $\gamma = 0.5$. These results do not significantly differ from ones of the base case.

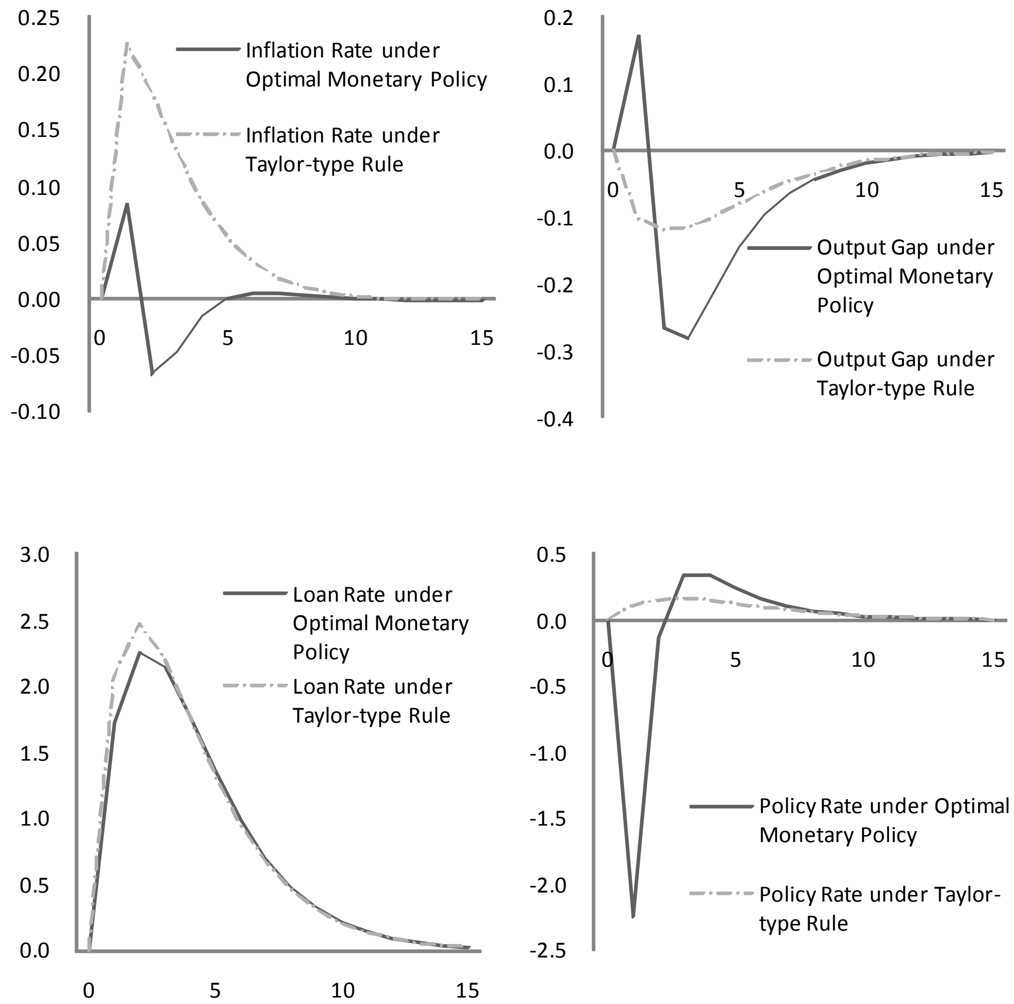

Figure 9: Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Optimal Monetary Policy and Taylor-type Rule for Loan Rate Shock When $\gamma = 0.5$.

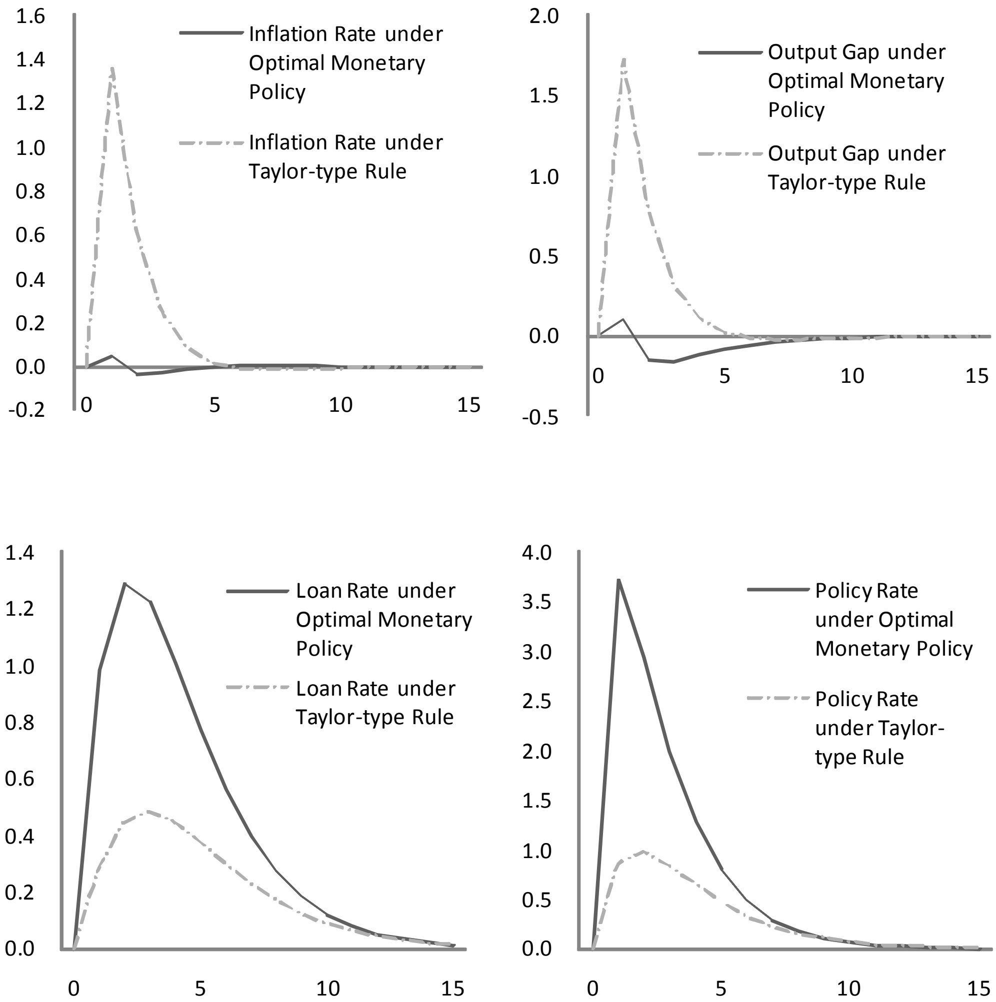

Figure 10: Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Optimal Monetary Policy and Taylor-type Rule for Productivity Shock When $\gamma = 0.5$.

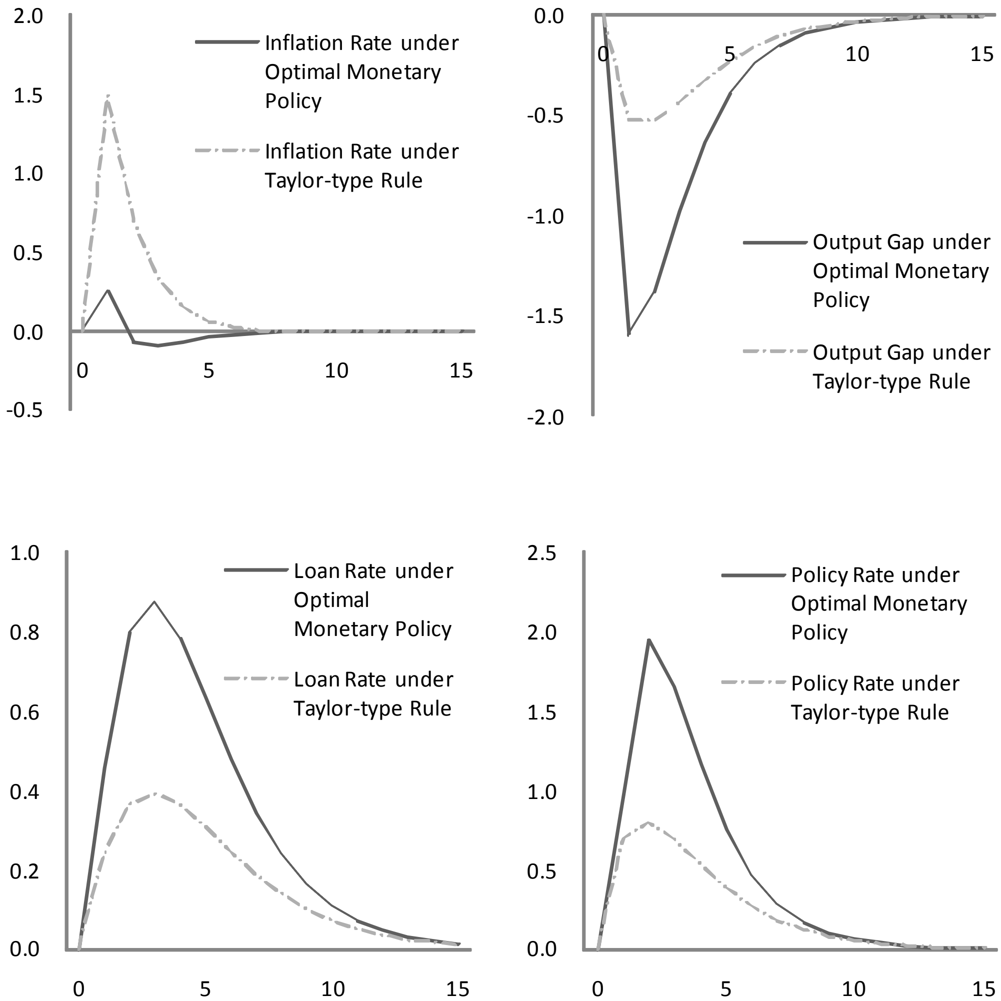

Figure 11: Impulse Responses of Inflation Rate (π_t), Output Gap (x_t), Loan Rate ($\widehat{R}_t$), and Policy Rate ($\widehat{i}_t$) under Optimal Monetary Policy and Taylor-type Rule for Marginal Cost Shock When $\gamma = 0.5$.