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What do the Fama-French factors add to C-CAPM?

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Abstract

This study extends standard C-CAPM by including two additional factors related to firm size (SMB) and book-to-market value ratio (HML) – the Fama-French factors. CCAPM is least able to price firms with low book-to-market ratios. The explanation of these returns, as well as the returns on the SMB and HML portfolios, is significantly improved by the inclusion of the HML factor. The component of the risk premia explained by consumption varies across size. We suggest that a possible explanation for the role of HML is its association with the investment growth prospects of firms.

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Abstract

This study extends standard C-CAPM by including two additional factors related to firm size (SMB) and book-to-market value ratio (HML) – the Fama-French factors. C-CAPM is least able to price firms with low book-to-market ratios. The explanation of these returns, as well as the returns on the SMB and HML portfolios, is significantly improved by the inclusion of the HML factor. The component of the risk premia explained by consumption varies across size. We suggest that a possible explanation for the role of HML is its association with the investment growth prospects of firms.

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1. Introduction

It is well established that most of the cross-section variation in stock returns can be captured empirically by the three Fama-French factors, Fama and French (1993). Moreover, these factors dominate beta in the CAPM model and consumption growth and inflation in the C-CAPM model. In these models the three-factors are ad hoc additional variables thought to represent systematic risk associated with the macroeconomy, Ludvigson (2012). Although implying a formal rejection of CAPM and C-CAPM, there remains the possibility that these factors are consistent with a no-arbitrage framework for asset pricing. This is the question addressed in this paper using the more flexible SDF approach to asset-pricing.

Despite its flexibility, the SDF approach implies strong restrictions if the no-arbitrage condition is to hold over a cross-section of asset returns. The Fama-French model involves the returns on 25 portfolios obtained by sorting by the size (market capitalization) of each asset and their book-to-market ratio (BM). There are two additional returns based on portfolios that highlight the divergence between stocks based on the returns of small-minus-big capitalizations (SMB) and high-minus-low returns (HML). We test whether all 27 of these returns satisfy the no-arbitrage restrictions implied by the SDF approach. We use as factors the returns on SMB and HML together with consumption growth and inflation. Hence, the returns on SMB and HML are both variables to be explained and explanatory variables for the other 25 portfolios. As the model reduces to C-CAPM if these two factors are excluded we are able to assess the additional contributions of the two factors. The individual results for the 25 portfolios can be used to determine the effects of size on the explanatory power of the model and hence on their risk premia. Our econometric model is a multivariate generalized autoregressive conditional heteroskasticity in mean model as in Smith and Wickens (2002).

Our main findings are that consumption growth, inflation and HML are significantly-priced sources of risk for all portfolios but SMB is never significant. As a result, C-CAPM is rejected in favour of a model that also includes HML as a factor. The explanatory power of C-CAPM is best for portfolios that either have higher book-to-market ratios or are of larger size, but not both. HML improves the fit of portfolios with low BM or a larger size. The no-arbitrage restrictions of the model that includes the two significant factors - consumption growth and HML - tend to be rejected much more for portfolios with low BM and small size. Although SMB is not significant in explaining the portfolio returns, as in Liew and Vassalou (2000), it is found to have significant explanatory power for inflation, while HML is able to forecast consumption growth.

The negative relation between firm size and average return (size effect), and the positive relation between the ratio of a firm's book value of common equity to its market value (book-to-market ratio) and average return (value effect), have long been recognized as "anomalies" within asset pricing literature. This was reported by Banz (1981) and Fama and French (1992) for the capital asset pricing model (CAPM), and in the consumption-based CAPM (C-CAPM) with a power utility framework (standard C-CAPM) by Mankiw and Shapiro (1986) and Breeden, Gibbons, and Litzenberger (1989). In the CAPM context, the seminal study by Fama and French (1993) introduced a pricing model that includes, along with the market return, two additional variables related to size (SMB) and book-to-market value ratio (HML). The Fama and French three-factor model can explain the cross-section of equity returns much better than the CAPM.

This study extends the standard C-CAPM in much the same way as was done to the CAPM in Fama and French (1993). Without seeking its general equilibrium representation, the augmented C-CAPM that includes consumption, SMB, and HML as risk factors (hereafter the consumption three-factor model) can be viewed as a particular version of the affine multi-factor stochastic discount factor (SDF) model. Unlike Fama and French (1993), given that SMB and HML are themselves equity returns, they have to satisfy their no-arbitrage conditions under the SDF framework as well as other portfolio returns. As a result, the mispricing theory is ruled out as risk premia for SMB and HML are due to their riskiness.

As in Smith and Wickens (2002), we use the multivariate generalized autoregressive conditional heteroskedasticity in mean model (MGM) to estimate the standard C-CAPM and the consumption three-factor model for the 25 portfolios formed on the intersection of size and book-to-market ratio. We find that in addition to consumption, HML but not SMB can determine equity returns. The explanatory power of HML is as strong as consumption. However, the standard C-CAPM performs well with most of the portfolios that have a not too low book-to-market ratio. The inclusion of HML improves only the fit of the low book-to-market portfolios, SMB, and HML that are not correctly priced in the standard C-CAPM.

A time-varying comparison shows that consumption is the main source of volatilities for the small growth and big value portfolios, with the small growth portfolio having more volatility. From 2000 to 2002, the risk premium for the small growth portfolio decreases sharply, while for the big value portfolios it increases. This movement comes from the fact that during this period, SMB covariance for the small growth (big value) portfolio increases (decreases) while consumption and HML covariances for the small growth (big value) portfolio decrease (increase).

As in Fama and French (2005), the value premium is similar across size, and averages about 5-6% per annum. On the other hand, the relation between HML premium and size is contradictory to the size effect with small portfolios having a higher negative HML premium. The inability of the standard C-CAPM to explain the returns on the portfolios in the two lowest book-to-market quintiles is due to the fact that the consumption covariances exhibit little variation across book-to-market ratio and the risk premia for these portfolios are heavily dependent on HML, where about 40% of their total risk premia comes from HML.

The VAR matrix in the MGM shows that, as in Liew and Vassalou (2000), SMB and HML have information about future macroeconomic variables that is not available through other macroeconomic variables. Indeed, SMB can predict inflation while HML is able to forecast consumption and industrial production. The lag of the excess return on the small growth portfolio can predict inflation and industrial production, but information about inflation contained in the small growth portfolios is similar to that contained in SMB.

We also examine the behavior of average returns across industry as the performance of different industries is expected to vary across the business cycle. The standard C-CAPM cannot explain the industry returns that have a relatively low level of book-to-market ratio and small firm size, but including SMB and HML does not improve the fit of these portfolios either. The inability of the consumption three-factor model to price industry returns is consistent with other related studies (Fama and French, 1997; Ferson and Locke, 1998; and Pastor and Stambaugh, 1999). As size and book-to-market ratio for each industry changes over time, it is therefore difficult to measure the share of SMB and HML correctly. In addition, the behavior of the time-varying risk premia for high-technology (HiTec) and utilities (Utils) are similar to those for small growth and big value stocks respectively, as HiTec has a consistently lower book-to-market ratio while Utils has a larger market common equity.

As the choice of HML is empirically motivated, several studies have attempted to establish the connection between HML and more fundamentally determined factors. Fama and French (1995) suggest that the value premium is due to financial distress. Vassalou and Xing (2004) point out that although HML contain default-risk information, HML contains important price information unrelated to default risk. Our results suggest that financial distress and default risk may not be the reason that HML can explain the equity returns as the relation between HML and size indicates that small firms are less risky than big firms.

One possible explanation is that HML may be associated with the investment growth prospect of firms. Low book-to-market ratio firms may be expected to have higher rates of growth while, to a lesser extent, small firms may also be expected to behave similarly. This interpretation is consistent with Brennan, Wang, and Xia (2004) where they proposed an ICAPM with time-varying investment opportunities that explains SMB and HML well. In addition, Li, Vassalou, and Xing (2006) proposed a sector investment growth model that can explain the cross-section of equity returns, including the small growth portfolio that cannot be priced by most pricing models.

Recent studies attempt to explain the cross-section of equity returns with the modified versions of the standard C-CAPM. By asserting that there are some alternative factors missing from the standard C-CAPM, and taking into account these factors through either conditioning variables (e.g. Lettau and Ludvigson, 2001) or alternative related consumption factors (e.g. Parker and Julliard, 2005; and Yogo, 2006), these modified versions of the standard C-CAPM can explain the cross-section of equity returns as good as (or better than) the Fama and French three-factor model. We take a different approach by using the MGM to directly measure the underlying source of risk premium. This is in contrast to most of the econometric models of equity in the literature that are univariate. Smith, Sorensen, and Wickens (2008) followed this approach and employed the SDF model to generate models involving macroeconomic variables.

In Section 2, we discuss the asset pricing theoretical framework. Sections 3 and 4 describe the econometric methodology and data used in this paper respectively. In Section 5, we report the estimates for all portfolio returns. Section 6 looks at industry portfolios, and Section 7 summarizes the findings in this study.

2. Theoretical Framework

2.1 Stochastic Discount Factor Approach

The SDF approach is based on a proposition that the return on any asset satisfies

$$E_t\left[M_{t+1}R_{t+1}\right]=1 \tag{1}$$

where M_{t+1} is the stochastic discount factor for period $t+1$ and R_{t+1} is the gross nominal return on an asset. If $m_{t+1}=\ln M_{t+1}$, $i_{t+1}=\ln R_{t+1}$, and the logarithm of the nominal risk free rate (i_t^f) are jointly normally distributed, then the expected nominal excess return on the asset is given by

$$E_t(i_{t+1} - i_t^f) + \frac{1}{2}V_t(i_{t+1}) = -Cov_t(m_{t+1}, i_{t+1}) \tag{2}$$

where the term of the right-hand side is the risk premium. The term $V_t(i_{t+1})$ is the Jensen effect and may be omitted if the assumption of log-normality is not made.

For a general linear factor model where $z_{it}(i = 1,...,n-1)$ are $n-1$ factors that are jointly log normally distributed with asset returns

$$m_t = -\sum_{i=1}^{n-1} \alpha_i z_{i,t}$$

and the no-arbitrage condition becomes

$$E_t(i_{t+1} - i_t^f) + \frac{1}{2}V_t(i_{t+1}) = -\sum_{i=1}^{n} \alpha_i Cov_t(z_{i,t+1}, i_{t+1}) \tag{3}$$

The choice of the factors distinguishes between most models.

2.2 C-CAPM

For C-CAPM with power utility

$$m_t = -\gamma \Delta c_t - \pi_t$$

where Δc_t is real consumption growth, π_t is inflation and γ is the coefficient of relative risk aversion (CRRA). This implies that

$$E_t(i_{t+1} - i_t^f) + \frac{1}{2}V_t(i_{t+1}) = \gamma Cov_t(\Delta c_{t+1}, i_{t+1}) + Cov_t(\pi_{t+1}, i_{t+1}) . \tag{4}$$

2.3 Fama and French Three-Factor Model

CAPM does not assume log-normality and has one factor: the nominal market rate of return on equity i_t^m . This gives the SDF no-arbitrage equation

$$E_t(i_{t+1} - i_t^f) = \delta_t Cov_t(i_{t+1}^m, i_{t+1}) \tag{5}$$

where $\delta_t = E_t(i_{t+1}^m - i_t^f) / V_t(i_{t+1}^m)$ is the market price of risk which, in general, is time varying and can be interpreted as the CRRA (Merton (1980b)). In CAPM the aim is to maximize wealth rather than the expected present value of utility derived from consumption as in C-CAPM. The solution for C-CAPM implies that consumption is proportional to wealth. It follows that the rate of growth of aggregate nominal consumption is equal to the nominal market rate of return on equity. CAPM then implies that

$$E_t(i_{t+1} - i_t^f) = \delta_t [Cov_t(\Delta c_{t+1}, i_{t+1}) + Cov_t(\pi_{t+1}, i_{t+1})] \tag{6}$$

Fama and French (1993) extended CAPM by including the two additional factors: SMB and HML. This implies the SDF no-arbitrage equation

$$E_t(i_{t+1} - i_t^f) = \beta_1 Cov_t(i_{t+1}^m, i_{t+1}) + \beta_2 Cov_t(SMB_{t+1}, i_{t+1}) + \beta_3 Cov_t(HML_{t+1}, i_{t+1}) \tag{7}$$

2.4 General Three-Factor Model

A general model that encompasses all of these specifications may be obtained by including the two Fama-French factors in the SDF model for C-CAPM. This gives the basic asset pricing equation used in this paper

$$E_t(i_{t+1} - i_t^f) + \frac{1}{2}V_t(i_{t+1}) = \beta_1 Cov_t(\Delta c_{t+1}, i_{t+1}) + \beta_2 Cov_t(SMB_{t+1}, i_{t+1}) + \beta_3 Cov_t(HML_{t+1}, i_{t+1}) + Cov_t(\Delta\pi_{t+1}, i_{t+1}) \tag{8}$$

2.5 Rational Pricing

Fama and French apply their model to the returns on 25 portfolios sorted by 5 size categories and 5 book-to-market categories. We assume that our general model, equation (8), applies simultaneously to the returns on each of these portfolios. Moreover, as SMB and HML are the returns on two more portfolios, equation (8) should also apply to them. We note that SMB and HML are then both returns i_{t+1} and act as factors. In total we have 27 portfolio returns. If there are no arbitrage opportunities across these 27 portfolios then each of the system of equations based on (8) should have the same coefficients. The system may be written

$$E_t(i_{t+1}^{s,b}-i_t^f)=-\frac{1}{2}V_t(i_{t+1}^{s,b})+\beta_1 Cov_t(\Delta c_{t+1},i_{t+1}^{s,b})+\beta_2 Cov_t(SMB_{t+1},i_{t+1}^{s,b})+\beta_3 Cov_t(HML_{t+1},i_{t+1}^{s,b})+Cov_t(\Delta\pi_{t+1},i_{t+1}^{s,b})$$

$$E_t(SMB_{t+1})=(\beta_2-\frac{1}{2})V_t(SMB_{t+1})+\beta_1 Cov_t(\Delta c_{t+1},SMB_{t+1})+\beta_3 Cov_t(HML_{t+1},SMB_{t+1})+Cov_t(\Delta\pi_{t+1},SMB_{t+1})$$

$$E_t(HML_{t+1})=(\beta_3-\frac{1}{2})V_t(HML_{t+1})+\beta_1 Cov_t(\Delta c_{t+1},HML_{t+1})+\beta_2 Cov_t(HML_{t+1},SMB_{t+1})+Cov_t(\Delta\pi_{t+1},HML_{t+1})$$

where s = 1, 2,…, 5 and b = 1,2,…, 5 indicate in ascending order the size and book-to-market ratio groups that characteristics portfolios belong to respectively. For the industry portfolios, the same set of restrictions apply, but this time to the 10 industry returns where *s,b* is replaced by the industry name defined by the SIC code. The different asset models can be obtained by placing different restrictions on β_i. C-CAPM imposes the restrictions that $\beta_2=\beta_3=0$.

3. Econometric Methodology

We assume that the each excess asset return together with the factors can be modeled jointly as a multivariate generalized autoregressive conditional heteroskedasticity in mean model (MGM) that satisfies the no-arbitrage condition under the SDF framework as in Smith and Wickens (2002). We include 5 factors: consumption growth, inflation, industrial production, SMB and HML. Thus we assume that the n-vector

$$\mathbf{x_{t+1}}=(i_{i,t+1}-i_t^f, SMB_{t+1}, HML_{t+1}, \pi_{t+1}, \Delta c_{t+1}, \Delta q_{t+1})'$$

can be written as the MGM model

$$\mathbf{x}_{t+1}=\alpha+\mathbf{\Gamma}\mathbf{x}_t+\left(\boldsymbol{\lambda}\otimes\mathbf{H_{t+1}}\right)\mathbf{1_n}+\varepsilon_{t+1},$$

$$\boldsymbol{\varepsilon}_{\mathbf{t+1}}\,|\,I_t\sim N(0,\mathbf{H_{t+1}}),$$

where, $\boldsymbol{\alpha}$ is an $n\times 1$ vector of constants, $\mathbf{\Gamma}$ and $\boldsymbol{\lambda}$ are $n\times n$ matrices of coefficients and $\boldsymbol{\varepsilon}_{\mathbf{t+1}}$ is an $n\times 1$ vector of errors and $\otimes$ denotes a Hadamard product.In order to reduce the number of parameters to be estimated we use the definition of $\mathbf{H_{t+1}}$ of Ding and Engle (2001) which specifies the conditional covariance matrix is to be a vector diagonal model with variance targeting. Hence,

$$\mathbf{H_{t+1}}=\mathbf{H_0}(\mathbf{ii}'-\mathbf{aa}'-\mathbf{bb}')+\mathbf{aa}'\otimes(\boldsymbol{\varepsilon}_\mathbf{t}\boldsymbol{\varepsilon}_\mathbf{t}')+\mathbf{bb}'\otimes\mathbf{H_t}$$

where $\mathbf{H_0}$ is the observed sample covariance matrix and $\mathbf{a}$ and $\mathbf{b}$ are $n\times 1$ vectors. The MGM is restricted to satisfy the no-arbitrage restrictions implied by the system defined above. This affects the first three rows. The second three rows define a VAR in the macroeconomic and finance factors The MGM can be written as

$$\begin{pmatrix} i_{i,t+1}-i_t^f \\ SMB_{t+1} \\ HML_{t+1} \\ \pi_{t+1} \\ \Delta c_{t+1} \\ \Delta q_{t+1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 \\ \phi_1 & \phi_2 & \phi_3 & \phi_4 & \phi_5 & \phi_6 \\ \tau_1 & \tau_2 & \tau_3 & \tau_4 & \tau_5 & \tau_6 \end{pmatrix} \begin{pmatrix} i_{i,t}-i_{t-1}^f \\ SMB_t \\ HML_t \\ \pi_t \\ \Delta c_t \\ \Delta q_t \end{pmatrix} + \left[\begin{pmatrix} -\frac{1}{2} & \beta_2 & \beta_3 & 1 & \beta_1 & 0 \\ 0 & \beta_2-\frac{1}{2} & \beta_3 & 1 & \beta_1 & 0 \\ 0 & \beta_2 & \beta_3-\frac{1}{2} & 1 & \beta_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \mathbf{H}_{\mathbf{t+1}} \right] \cdot \mathbf{1}_{\mathbf{n}} + \begin{pmatrix} \varepsilon_{t+1}^{r_{i,t+1}} \\ \varepsilon_{t+1}^{SMB_{t+1}} \\ \varepsilon_{t+1}^{HML_{t+1}} \\ \varepsilon_{t+1}^{\pi_{t+1}} \\ \varepsilon_{t+1}^{\Delta c_{t+1}} \\ \varepsilon_{t+1}^{\Delta q_{t+1}} \end{pmatrix}_{\mathbf{t}}$$

where the coefficients in the $\boldsymbol{\lambda}$ matrix are restricted to satisfy the system no-arbitrage conditions. The no-arbitrage conditions may be tested by comparing the restricted MGM model above with less restricted versions chosen to reflect the particular alternative hypothesis under consideration.

4. Data

Tables 1 and 2 show the monthly data on portfolios returns and macroeconomic variables from 1960.2 to 2004.11 for the US (538 observations). The return on the market portfolio is the value-weighted return on all stocks. The return on a risk-free asset is the one-month Treasury bill rate. There are two datasets of portfolio returns consisting of the 25 value-weighted portfolios formed by the intersections of 5 size and book-to-market quintiles and the 10 industry portfolios defined by the SIC codes. *sb* is used to defined the 25 portfolios according to their size and book-to-market groups. Portfolio 11 refers to the portfolio in the lowest book-to-market and smallest size quintiles. Real non-durable growth consumption is from the Federal Reserve Bank of St. Louis. CPI inflation and the volume index of industrial production are both from Thomson Reuters Datastream.

The descriptive statistics for the excess returns of the 25 portfolios in Table 1 are similar to those in Fama and French (1993b) for the period 1963-1991. This indicates a stronger value effect and relatively weak size effect. For the 10 industry portfolios shown in Table 2, the telecommunications industry (Telcm) has the highest average book-to-market ratio and largest firm size. The Hi-technology industry (HiTec) has the highest standard deviation and the lowest average excess return. In general, most of the excess returns and macroeconomic variables appear to have negative skewness, excess kurtosis, and non-normality, except the risk-free rate, SMB, HML, and inflation that display positive skewness and show volatility persistent.

Table 1

Descriptive Statistics: 25 Size and Book-to-Market Portfolios

The table presents descriptive statistics for the excess returns on the 25 portfolios formed as the intersections of the five size and book-to-market ratio groups. Data and full definition of the returns can be found on http://mba.tuck.dartmouth.edu/pages/faculty/kenfrench/data_library.html.The returns are monthly value-weighted from 1960.2 to 2004.11, 538 observations. t-stat is the test statistics for zero mean hypothesis. $\rho\,(x_t, x_{t-i})$ represents the autocorrelation coefficients over the time interval i month (s).

Size Quintiles	Book-to-Market Equity Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Mean					Standard deviation				
Small	-0.07	0.54	0.66	0.90	0.97	8.20	6.98	5.97	5.56	5.85
2	0.10	0.47	0.72	0.82	0.89	7.48	6.07	5.36	5.14	5.73
3	0.18	0.58	0.57	0.73	0.83	6.86	5.44	4.92	4.75	5.36
4	0.34	0.38	0.63	0.75	0.70	6.04	5.15	4.83	4.61	5.35
Big	0.30	0.39	0.46	0.47	0.49	4.80	4.54	4.29	4.19	4.78
	Skewness					Excess Kurtosis				
Small	-0.53	-0.46	-0.60	-0.59	-0.58	2.72	3.38	3.72	4.35	4.20
2	-0.70	-0.89	-0.92	-0.81	-0.76	2.34	4.03	4.56	4.23	4.32
3	-0.65	-0.99	-0.95	-0.59	-0.80	2.07	4.52	3.85	3.12	4.63
4	-0.49	-0.96	-0.75	-0.32	-0.52	1.99	4.93	3.86	1.82	2.72
Big	-0.46	-0.62	-0.53	-0.15	-0.36	1.89	2.60	3.18	1.23	1.17
	Normality					t-statistics for zero mean				
Small	72.7	110.0	111.1	144.2	137.7	-0.18	1.78	2.56	3.74	3.84
2	50.7	90.1	104.7	107.4	117.5	0.29	1.81	3.13	3.72	3.60
3	44.8	94.7	79.9	84.4	124.4	0.60	2.48	2.68	3.58	3.59
4	46.9	112.3	98.4	46.5	73.3	1.29	1.73	3.02	3.77	3.05
Big	44.2	62.0	92.6	27.5	23.8	1.45	2.00	2.50	2.62	2.37
	Average firm size					Average book-to-market ratio				
Small	37	39	38	34	26	0.28	0.57	0.78	1.03	1.85
2	173	175	177	176	172	0.28	0.54	0.76	1.005	1.70
3	413	421	421	424	431	0.27	0.54	0.75	1.004	1.66
4	1068	1063	1070	1079	1075	0.27	0.55	0.75	1.03	1.70
Big	9511	7119	6166	5052	4643	0.26	0.53	0.75	1.004	1.50
	Average percent of market value					Average number of firms				
Small	0.65	0.44	0.43	0.46	0.56	492	312	315	376	603
2	0.94	0.69	0.69	0.63	0.48	152	110	109	99	77
3	1.71	1.27	1.18	1.00	0.71	115	84	78	66	46
4	3.72	2.79	2.38	1.98	1.31	97	73	62	51	34
Big	36.21	16.87	11.29	7.43	4.17	106	66	51	41	25
	$\rho(x_t, x_{t-1})$					$\rho(x_t, x_{t-3})$				
Small	0.20	0.18	0.20	0.20	0.24	-0.06	-0.09	-0.05	-0.04	-0.04
2	0.16	0.16	0.17	0.16	0.15	-0.07	-0.05	-0.05	-0.05	-0.05
3	0.12	0.15	0.16	0.16	0.14	-0.05	-0.01	-0.05	-0.02	-0.04
4	0.11	0.13	0.11	0.08	0.07	-0.04	-0.04	-0.02	0.01	-0.04
Big	0.06	0.04	0.00	-0.02	0.06	0.03	-0.01	-0.02	0.02	-0.01
	$\rho(x_t, x_{t-6})$					$\rho(x_t, x_{t-12})$				
Small	0.02	0.03	0.03	0.02	-0.01	0.00	0.02	0.06	0.08	0.13
2	0.02	0.01	0.02	0.02	-0.01	-0.03	0.03	0.05	0.08	0.10
3	0.02	0.01	0.02	-0.01	-0.01	-0.03	0.03	0.02	0.04	0.08
4	0.02	0.01	-0.03	-0.03	-0.03	-0.03	0.00	0.03	0.06	0.06
Big	-0.03	-0.06	-0.04	-0.06	0.02	0.05	0.01	0.02	0.02	0.02

Table 2
Summary Statistics: 10 Industry Portfolios and Explanatory Variables

The table presents descriptive statistics for the returns on the 10 industry-sorted portfolios and explanatory variables. The returns are monthly value-weighted from 1960.2 to 2004.11, 538 observations. The NYSE, AMEX, and NASDAQ stocks are assigned to an industry portfolio based on its four-digit SIC code. $i_{m,t+1}$ and i_t^f are the returns on the market portfolios and one-month Treasury bill rate respectively. Consumption growth, inflation, and industrial production growth are represented by Δc_{t+1}, $\Delta \pi_{t+1}$, and Δq_{t+1} respectively. Std. Dev is the standard deviation. t-stat is the t-statistic for zero mean hypothesis. t-stat is the test statistics for zero mean hypothesis. $\rho\ (x_t\ , x_{t-i})$ represents the autocorrelation coefficients over the time interval i month(s). BM denotes book-to-market equity ratio. Firm size, book-to-market equity ratio, percent of the market, and number of firms are in average terms. Data and full definition of 10 industries can be found on http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Panel A: Industry Portfolios

	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
Mean	0.55	0.43	0.31	0.52	0.30	0.31	0.47	0.52	0.33	0.46
Std. Dev.	4.53	5.46	4.86	5.07	6.68	4.93	5.37	5.13	4.05	5.08
Skewness	-0.56	-0.42	-0.74	-0.19	-0.49	-0.37	-0.65	-0.24	-0.12	-0.61
Excess Kurtosis	2.48	2.51	4.21	1.69	1.79	1.92	3.75	2.07	0.98	2.14
Normality	60.13	69.52	113.02	43.80	39.58	47.97	104.03	59.17	18.68	46.84
t-stat	2.80	1.82	1.46	2.37	1.05	1.47	2.05	2.33	1.90	2.10
Firm Size	796	1260	657	1228	602	2133	490	844	1058	492
BM	0.49	0.64	0.60	0.72	0.36	0.81	0.47	0.27	0.96	0.80
No. of firms	327	141	721	196	633	77	461	273	159	1336
% of Market	0.087	0.060	0.159	0.081	0.128	0.055	0.076	0.077	0.056	0.220
$\rho(x_t, x_{t-1})$	0.14	0.09	0.06	0.00	0.07	0.04	0.15	0.02	0.05	0.11
$\rho(x_t, x_{t-3})$	-0.04	-0.03	-0.01	0.02	0.04	0.12	-0.04	-0.05	0.01	-0.03
$\rho(x_t, x_{t-6})$	-0.03	-0.02	-0.05	-0.05	0.04	0.07	-0.07	-0.07	-0.05	-0.03
$\rho(x_t, x_{t-12})$	0.06	0.01	0.01	0.03	0.00	-0.01	0.03	0.04	0.04	0.03

Panel B: Explanatory Variables

	$i_{m,t+1}$	i_t^f	Δc_{t+1}	$\Delta \pi_{t+1}$	Δq_{t+1}	SMB_t	HML_t
Mean	0.94	0.46	0.23	0.35	0.25	0.20	0.44
Std. Dev.	4.41	0.23	0.73	0.30	0.75	3.18	2.89
Skewness	-0.46	1.04	-0.04	0.99	-0.62	0.50	0.10
Excess Kurtosis	1.90	1.70	1.37	1.68	2.98	8.36	5.39
Normality	44.85	98.95	33.56	82.25	75.70	216.33	80.17
$\rho(x_t, x_{t-1})$	0.06	0.95	-0.36	0.64	0.36	0.06	0.13
$\rho(x_t, x_{t-3})$	0.00	0.90	0.14	0.53	0.27	-0.08	0.04
$\rho(x_t, x_{t-6})$	-0.02	0.84	0.01	0.52	0.09	0.08	0.06
$\rho(x_t, x_{t-12})$	0.02	0.72	-0.07	0.44	-0.04	0.12	0.04

Correlations

	$i_{m,t+1}$	i_t^f	Δc_{t+1}	$\Delta \pi_{t+1}$	Δq_{t+1}	SMB_t	HML_t
i_t^f	-0.04	1.00					
Δc_{t+1}	0.15	-0.09	1.00				
$\Delta \pi_{t+1}$	-0.14	0.54	-0.20	1.00			
Δq_{t+1}	-0.03	-0.16	0.14	-0.10	1.00		
SMB_t	0.29	-0.06	0.14	-0.04	-0.02	1.00	
HML_t	-0.41	0.04	-0.03	0.04	0.03	-0.28	1.00

5. Estimates of 25 Size and Book-to-Market Portfolios

5.1 C-CAPM

Table 3 reports the estimates for C-CAPM. The conditional covariances of returns with consumption for all portfolios are highly significant. However, their sizes that range from 127.98 to 174.61 imply implausibly large CRRAs. This is a common feature of consumption-based models (Campbell, 2002; Yogo, 2006; Smith, Sorensen, and Wickens, 2008). We do not observe a systematic relation in these consumption coefficients across size or the book-to-market ratio. Nonetheless, the likelihood ratio statistics support the hypothesis that the consumption coefficients for the three returns in each of the 25 systems are the same. This result implies that the no-arbitrage condition under the standard C-CAPM is satisfied. It suggests that the cross-sectional variation in each portfolio return, SMB, and HML exists due to cross-sectional variation in their conditional covariances with consumption.

Table 3

C-CAPM: 25 Size and Book-To-Market Portfolios

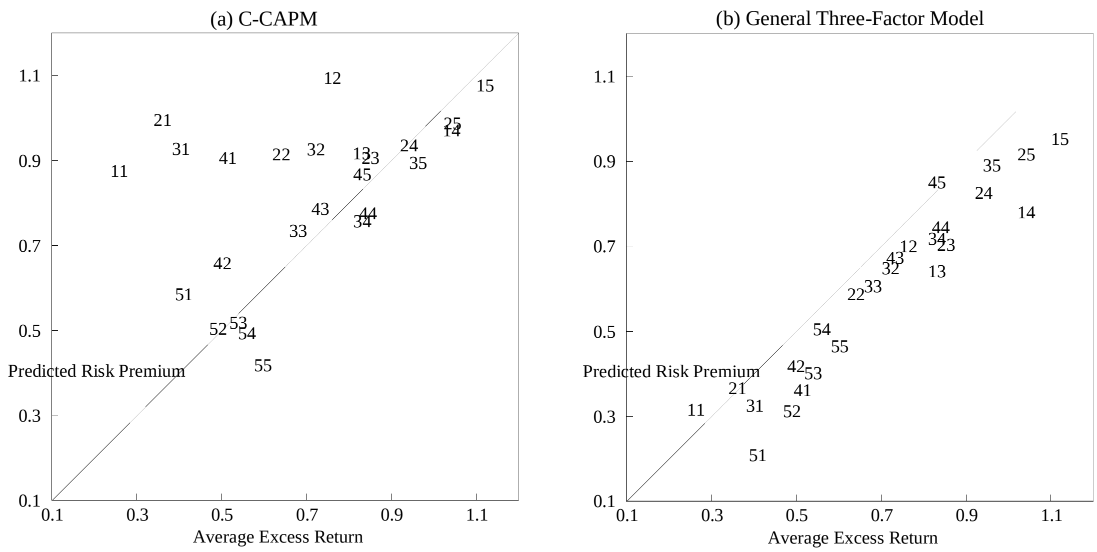
The table presents the estimates of the standard C-CAPM for the 25 size and book-to-market portfolios: 1960.2-2004.11, 538 observations. The model is estimated by the multivariate GARCH in mean model. γ and $t(\gamma)$ denote the coefficient relative risk aversion and its corresponding t–statistics respectively. The mean residual is computed by subtracting the predicted excess return from their historical value. The no arbitrage restriction is tested using the log-likelihood ratio test. The corresponding p-value is denoted by *p-value*.

Size Quintile	Book-to-Market Quintile									
	Low	2	3	4	High	Low	2	3	4	High
	γ					$t(\gamma)$				
Small	127.98	148.76	165.23	175.10	161.25	4.01	4.55	4.50	4.93	5.15
2	141.34	135.24	158.15	161.09	146.88	4.42	4.23	4.62	4.80	4.73
3	151.19	140.07	142.99	142.85	152.55	4.64	4.65	4.34	4.58	5.18
4	164.48	136.44	138.48	149.59	135.84	5.05	4.16	4.76	5.06	5.00
Big	174.61	151.04	152.14	132.89	144.75	5.72	4.57	4.84	4.55	4.34
	Mean Excess Return Residual					Mean SMB Residual				
Small	-0.62	-0.34	-0.09	0.06	0.04	-0.13	-0.17	-0.19	-0.23	-0.19
2	-0.64	-0.28	-0.06	-0.01	0.04	-0.15	-0.14	-0.19	-0.19	-0.16
3	-0.52	-0.20	-0.06	0.07	0.06	-0.19	-0.14	-0.14	-0.15	-0.19
4	-0.40	-0.16	-0.06	0.06	-0.04	-0.20	-0.14	-0.15	-0.18	-0.15
Big	-0.18	-0.01	0.02	0.06	0.19	-0.22	-0.17	-0.20	-0.14	-0.16
	Mean HML Residual					$p-value$				
Small	0.44	0.44	0.45	0.44	0.45	0.98	0.99	0.84	0.29	0.54
2	0.44	0.44	0.44	0.45	0.45	0.95	0.90	0.60	0.71	0.86
3	0.44	0.44	0.45	0.44	0.45	0.62	0.91	0.85	0.70	0.66
4	0.44	0.44	0.44	0.44	0.44	0.74	0.92	0.93	0.84	0.97
Big	0.44	0.44	0.44	0.44	0.44	0.80	0.98	0.89	0.85	0.84

Figure 1

Cross-Sectional Fit: 25 Size and Book-to-Market Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 25 size and book-to-market portfolios. The estimated models are (a) standard C-CAPM and (b) the general three-factor model. The average excess returns are adjusted for the Jensen effect. The 25 portfolios are defined using two numbers, *sb*. *s* = 1,…, 5 and *b* = 1,…, 5 indicate size and book-to-market groups that portfolios are in respectively. The numbers are in ascending order of magnitude. For example, the smallest (largest) size group is denoted by *s* = 1 (5) while the lowest (highest) book-to-market groups is represented by *b* = 1 (5).



5.2 General Three-Factor Model

Table 4 reports estimates of the general three-factor model. The model has four factors: consumption growth, SMB, HML and inflation but only the coefficients for the first three are estimated as that for inflation is restricted to unity by the no-arbitrage condition.. As with C-CAPM, all of the consumption coefficients are significantly different from zero at conventional levels, and their magnitudes range from 114.06 to 207.92. The inclusion of SMB and HML as additional risk factors does not affect the way consumption determines asset returns. SMB plays no role in explaining the equity returns as none of its coefficients is significant. On the other hand, HML appears to be able to explain asset returns. All of the coefficients for the conditional covariances of returns with HML are more than three standard errors larger than zero. The explanatory power of HML is as strong as that of consumption. These HML coefficients are also similar to one another, having an average value of 5.44. Therefore, the differences in the HML risk premium across portfolios arise from the differences in their conditional covariances with HML. Tests of the no-arbitrage restrictions for the three returns in each of the 25 systems finds few rejections (mostly of large firm returns) at conventional significance levels.

Table 4
General Three-Factor Model: 25 Size and Book-to-Market Portfolios

The table presents the estimates of the general three-factor SDF model for the 25 size and book-to-market portfolios: 1960.2-2004.11, 538 observations. The model is estimated by the multivariate GARCH in mean model. β_1, β_2, and β_3 are slope coefficients on consumption, SMB, and HML factors respectively. The mean residual is computed by subtracting the predicted excess return from their historical value. The test for the restrictions that the model places on the coefficients of the portfolio returns and the HML and SMB return equations shown in equation (9) are in the final panel, the corresponding p-value denoted by *p-value*.

Size	Book-to-market equity quintile									
quintile	Low	2	3	4	High	Low	2	3	4	High
			β_1					$t(\beta_1)$		
Small	114.06	149.98	182.42	207.92	171.56	2.62	3.45	3.39	3.81	4.04
2	136.49	134.27	170.65	172.99	152.22	3.25	3.37	3.72	3.91	3.82
3	157.17	141.59	142.31	146.19	158.69	3.52	3.80	3.40	3.62	4.05
4	175.63	136.88	137.89	156.57	129.87	4.19	3.12	3.83	4.10	3.78
Big	187.08	156.45	153.40	127.71	142.82	4.46	3.55	3.73	3.53	2.97
			β_2					$t(\beta_2)$		
Small	0.96	0.06	-0.74	-1.73	-0.55	0.47	0.03	-0.36	-0.80	-0.29
2	0.34	0.10	-0.68	-0.65	-0.19	0.18	0.05	-0.35	-0.33	-0.10
3	-0.56	0.24	0.24	-0.06	-0.28	-0.27	0.13	0.12	-0.03	-0.14
4	-0.81	0.10	0.22	-0.41	0.48	-0.43	0.05	0.12	-0.22	0.27
Big	-0.69	-0.18	-0.01	0.60	0.34	-0.37	-0.09	-0.01	0.33	0.17
			β_3					$t(\beta_3)$		
Small	5.73	5.46	5.30	4.89	5.15	3.74	3.54	3.42	3.11	3.29
2	5.64	5.48	5.46	5.42	5.38	3.58	3.49	3.53	3.43	3.35
3	5.15	5.95	5.59	5.35	5.44	3.22	3.93	3.61	3.42	3.47
4	5.39	5.75	5.71	5.12	5.42	3.50	3.66	3.68	3.30	3.47
Big	5.45	5.56	5.37	5.41	5.40	3.57	3.58	3.38	3.43	3.43
			Mean excess return residual					Mean SMB residual		
Small	-0.06	0.05	0.18	0.26	0.17	-0.04	-0.04	-0.03	-0.03	-0.04
2	-0.01	0.05	0.14	0.11	0.12	-0.03	-0.01	-0.03	-0.02	-0.02
3	0.08	0.07	0.07	0.11	0.07	-0.03	-0.02	-0.03	-0.03	-0.04
4	0.14	0.08	0.06	0.09	-0.02	-0.02	-0.01	-0.03	-0.04	-0.04
Big	0.20	0.18	0.14	0.05	0.13	-0.05	-0.03	-0.07	-0.05	-0.06
			Mean HML residual					$p-value$		
Small	0.01	0.01	0.01	0.01	0.02	0.86	1.00	0.21	0.07	0.51
2	0.00	0.00	-0.01	0.00	0.01	0.88	0.63	0.28	0.08	0.53
3	0.01	-0.03	0.00	0.01	0.00	0.99	0.39	0.05	0.17	0.13
4	-0.01	-0.01	-0.01	0.02	0.02	0.57	0.22	0.12	0.05	0.02
Big	0.00	-0.01	0.01	0.02	0.02	0.09	0.07	0.41	0.18	0.51

Figure 1(b) shows the cross-sectional fit of the general three-factor model for the 25 portfolios. Most portfolios appear to earn average excess returns higher than the model predicts. Although, the largest model residual (0.26% per month) is much lower than in the case of the C-CAPM (0.64%), the general three-factor model explains better than C-CAPM only the returns on 11 portfolios (8 portfolios are in the first two lowest book-to-market quintiles). These 11 portfolios also include the two small growth portfolios (Portfolios 1,1 and 1,2) that previously have the largest residuals in t C-CAPM. The average residuals for these two portfolios in the three-factor model are only -0.06% and -0.01% per month.

On the other hand, C-CAPM explains the returns on 13 portfolios better than the general three-factor model with 9 portfolios having the average residual smaller than 0.07% (in absolute term) per month. Apart from portfolios with a low book-to-market ratio, C-CAPM appears to do a good job in explaining the equity returns. Including SMB and HML improves mainly the cross-sectional fit of the low book-to-market portfolios. However, the general three-factor model can capture the variation in SMB and HML. The biggest SMB residual (-0.07% per month) in this model is lower than that in C-CAPM (-0.23% per month). For HML, the biggest HML residual is -0.03 % per month, which is significantly smaller than 0.44%-0.45% per month in C-CAPM. The ability of the general three-factor model to price SMB and HML is consistent with Brennan, Wang, and Xia (2004) where they propose an ICAPM with time-varying investment opportunities that explains well the returns on SMB and HML.

Table 5
Average Covariances: 25 Size and Book-to-Market Portfolios

The table presents the average covariances of the returns on the 25 size and book-to-market portfolios with consumption, SMB, and HML from the estimation of the consumption three-factor model. The process of the conditional covariances is assumed to follow the multivariate GARCH in mean model.

Size quintile	Book-to-market equity quintile					
	Low	2	3	4	High	Low-High
	Mean consumption covariance					
Small	0.0069	0.0074	0.0056	0.0056	0.0067	0.0002
2	0.0071	0.0068	0.0058	0.0059	0.0068	0.0003
3	0.0062	0.0066	0.0052	0.0053	0.0059	0.0003
4	0.0055	0.0049	0.0057	0.0052	0.0064	-0.0009
Big	0.0034	0.0034	0.0034	0.0039	0.0029	0.0005
Small-Big	0.0035	0.0040	0.0024	0.0017	0.0038	
	Mean SMB covariance					
Small	0.1815	0.1612	0.1330	0.1216	0.1239	0.0576
2	0.1494	0.1203	0.1006	0.0922	0.1045	0.0449
3	0.1241	0.0845	0.0689	0.0602	0.0744	0.0497
4	0.0878	0.0563	0.0454	0.0449	0.0510	0.0368
Big	0.0233	0.0171	0.0112	0.0052	0.0142	0.0091
Small-Big	0.1582	0.1441	0.1218	0.1164	0.1097	
	Mean HML covariance					
Small	-0.1118	-0.0752	-0.0509	-0.0353	-0.0238	0.0880
2	-0.1139	-0.0619	-0.0372	-0.0227	-0.0165	0.0974
3	-0.1107	-0.0507	-0.0246	-0.0095	-0.0045	0.1062
4	-0.0989	-0.0426	-0.0209	-0.0090	-0.0000	0.0989
Big	-0.0746	-0.0374	-0.0216	0.0040	0.0106	0.0852
Small-Big	0.0372	0.0378	0.0293	0.0313	0.0344	

Table 5 shows the conditional covariances of the 25 portfolio returns with consumption, SMB, and HML. The consumption covariances decline as size increases while little variation is observed across book-to-market ratio. On the other hand, we observe the systematic movement in the covariances of SMB and HML. All returns positively co-move with SMB. Small firms have higher SMB covariances than large firms, but the spreads in the SMB covariances across size decrease as book-to-market ratio increases. The differences in SMB covariances between the smallest and biggest size quintiles in the lowest to highest book-to-market quintiles are 0.1582, 0.1441, 0.1218, 0.1164, and 0.1097 respectively.

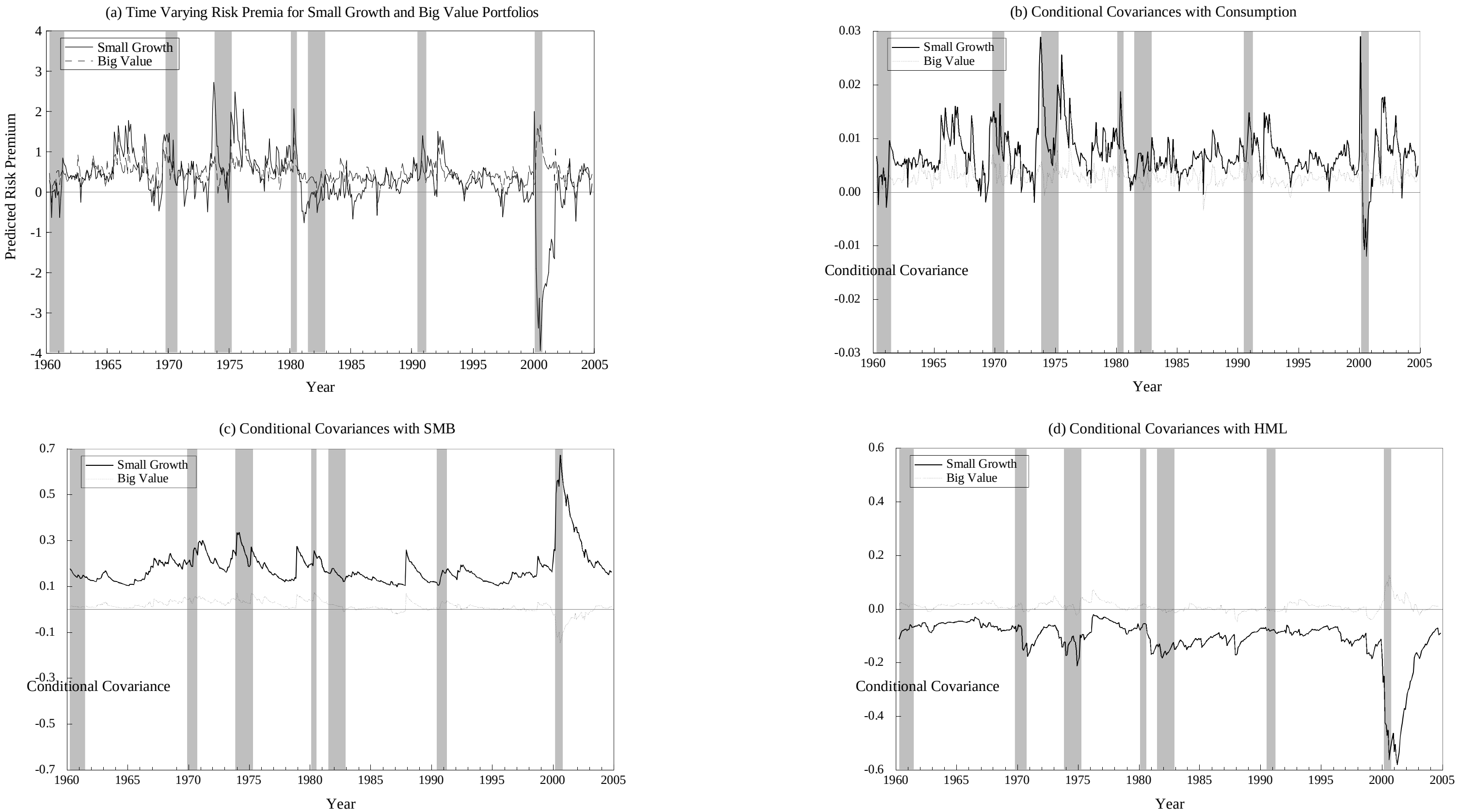
Most portfolios seem to co-move negatively with HML. The exceptions are large portfolios with a high book-to-market ratio: Portfolios 5,4 and 5,5). Low book-to-market portfolios have higher negative HML covariances than high book-to-market portfolios and small growth portfolios have more negative covariances with HML than large growth portfolios. These systematic effects across the 25 portfolio returns, and the SMB and HML returns suggest that sorting portfolios based on size and the book-to-market ratio improves the explanation of the cross-section of equity returns.

A comparison of the variation over time of the risk premia for small growth (Portfolio 1,1) and large value (Portfolio 5,5) portfolios and their conditional covariances with the SMB and HML factors is given in Figure 2. Taken together, the graphs suggest that fluctuations in the two risk premia are due to the covariances with consumption and the HML factor, with the risk premium for the small growth portfolio the more volatile. Striking differences between the two risk premia occur during the burst of the dotcom bubble between 2000 and 2002 when HML returns increased considerably; the risk premium for the small growth portfolio decreases sharply, while the risk premium for the large value portfolio increases. Differences in the behavior of the conditional covariances with SMB and HML also occur during this period. As the conditional covariances of HML with the 6 portfolios in the two highest book-to-market quintiles and the three biggest size quintiles increase during this period, it indicates that these large value stocks became riskier and that investors therefore required a higher risk premium to hold these portfolios.

We find that the HML factor is much more significant than the SMB factor. The choice of both is empirically motivated. Several studies have attempted to establish the connection between HML and more fundamentally determined factors. Fama and French (1995) suggested that the value premium was due to financial distress. Low book-to-market ratio is typical of firms with high returns on capital, while high book-to-market ratio is typical of firms that are relatively distressed. Size is also related to earnings. Controlling for book-to-market ratio, small stocks tend to have lower earnings on book equity. Vassalou and Xing (2004) pointed out that, even though HML contain default-risk information, HML contains important price information unrelated to default risk.

Figure 2
Small Growth and Large Value Portfolios

The figure compares time-varying risk premia and conditional covariances of the returns with the factors between the small growth (Portfolio 11) and big value (Portfolio 55) portfolios from the general three-factor model. The figures are (a) time-varying risk premia, (b) conditional covariances of the returns with consumption, (c) conditional covariances of the returns with SMB, and (d) conditional covariances of the returns with HML. The sample period is 1960:2-2004:11. Shaded areas are recessions as defined by NBER.



Our results suggest that financial distress and default risk may not be the reason why the HML factor contributes to the explanation of equity returns. The relation between HML premium and size indicates that small firms are less risky than big firms. Even though the size effect arising from earnings is thought in the literature to be captured by SMB in this study we find that across size consumption and HML dominate, and that consumption is more important than HML.A possible explanation for why HML predicts that small stocks are less risky than big stocks may be related to an association between HML and the investment growth prospects of firms. Low book-to-market firms may be expected to have higher rates of growth, and this may also be true to a lesser extent for small firms. Li, Vassalou, and Xing (2006) proposed a sector investment growth model in order to explain the cross-section of equity returns and why the return on the small growth portfolio cannot be priced by most asset pricing models.

The remaining feature of the general three-factor model yet to be discussed is dynamic structure implied by the VAR model. In Table 6 we report estimates of the VAR matrix for the small growth portfolio with and without the SMB and HML factors in the model. The lag in inflation has predictive power for all of macroeconomic variables. The lags in consumption and industrial production help forecast inflation and themselves. Whether the SMB and HML factors are included or excluded appears to make little difference to these lag coefficients. However, including the two factors seems to provide information about the macroeconomic variables that is not otherwise contained in other macroeconomic variables: SMB helps to predict inflation, while HML helps forecast consumption and industrial production. The significance in the consumption equation of the coefficients for lagged inflation, consumption, industrial production and the HML factor implies that the conditional covariances of returns with unexpected consumption are priced.

Table 6
The VAR Matrix: Small Growth Portfolio

The table presents the estimates of the VAR parameters in the multivariate GARCH in mean model. The more restricted VAR places restrictions on the coefficients for SMB and HML in the VAR.

Dependent variables	Constant	i_t	$\Delta\pi_t$	Δc_t	Δq_t	SMB_t	HML_t
	Panel A: VAR coefficients						
$\Delta\pi_{t+1}$	0.001262	-0.002014	0.5897	0.0271	-0.0165	0.0125	-0.003923
	(8.06)	(-1.06)	(17.98)	(2.12)	(-1.27)	(3.38)	(-1.02)
Δc_{t+1}	0.003973	-0.003463	-0.2979	-0.3611	0.0958	-0.008741	-0.0242
	(8.21)	(-0.63)	(2.80)	(-9.15)	(2.54)	(-0.66)	(-2.39)
Δq_{t+1}	0.002672	0.010453	-0.2006	-0.0173	0.3258	-0.003845	0.0243
	(5.71)	(2.11)	(-1.94)	(-0.45)	(7.49)	(-0.33)	(2.50)

The lag of the excess return on the small growth portfolio – the lag of SMB - helps to predict inflation in the more restricted VAR but is insignificant in the less restricted VAR. This suggests that information about inflation contained in the small growth portfolios is similar to that contained in SMB. This finding is only present in low book-to-market and small sized portfolios - Portfolios (1,1), (1,2), (2,1), and (3,1). , We also note here that the small growth portfolio helps predict industrial production in both models, indicating that the small growth portfolio, like HML, contains information about industrial production that is unrelated to that contained in other variables. The explanatory power of the portfolio return in predicting future industrial production is unique to the small growth portfolio. As before, including portfolio returns in the VAR does not provide additional information about future macroeconomic variables.

These VAR results can be related to the findings in Liew and Vassalou (2000) who found that even in the presence of several business cycle variables (including, for example, industrial production growth), SMB and HML are able to predict future Gross Domestic Product (GDP) growth. This suggests that an asset pricing model that includes a factor that captures news related to future GDP growth, along with the market factor, may perform as well as the Fama and French three-factor model (Vassalou, 2003). A possible explanation for this is that SMB and HML are reflecting the investment component of GDP. This is consistent with our previous suggestion that investment growth prospect of firms may be the underlying source of the risk associated with the HML factor.

6. Industry Portfolios

Previously, the cross-section of equity returns is categorized on portfolios that have different values of size and book-to-market ratio. This is because we want to examine whether the pricing models can explain a large dispersion in the returns among these portfolios. We now extend this analysis to industry returns. Although the dispersion of average returns for the industry portfolios is relatively small and no systematic pattern is present in these returns, the performance of different industry groups varies through time as an economy passes through different stages of the business cycle. The analysis in this section therefore examines the robustness of the results found above. Industry returns can be classified into two groups based on their sensitivity to macroeconomic shocks (Bodie, Kane, and Marcus, 2002). A cyclical industry, e.g. consumer goods (Durbl) or capital goods (Manuf), is particularly sensitive to macroeconomic conditions while a defensive industry, e.g. non-durable consumer goods (NoDur) and public utilities (Utils), has little sensitivity to the business cycle.

6.1 C-CAPM

Estimates of C-CAPM for the 10 industry portfolios are given in Table 8. All consumption coefficients are highly significant, and range from 128.07 to 173.51, implying a very large CRRA, as in the estimation of the characteristics portfolios. Apart from Hlth, cyclical industries appear to have higher consumption coefficients than defensive industries. Figure 3 (a) shows that C-CAPM provides a particularly poor fit for 4 industry returns: the average residuals for Hitec, Hlth, Shops, and Manuf are -0.30%, 0.21%, -0.20%, and -0.18% per month respectively. The common characteristics of these industries are relatively low levels of the book-to-market ratio and small firm size. This is similar to the previous results where C-CAPM is not able to price portfolios that are in the low book-to-market ratio quintiles. The risk premia for these portfolios are heavily dependent on HML, and consumption exhibits little variation across the book-to-market ratios. On the other hand, C-CAPM is able to successfully price NoDur, Enrgy, Telecommunication (Telcm), Utils, and Other industries as their residuals are less than 0.12% per month. As was shown above, for the 25 characteristics portfolios, the industries that can be priced by C-CAPM generally have higher average book-to-market ratios.

6.2 General Three-Factor Model

The estimates in Table 7 show that all consumption coefficients for this model are significantly different from zero and somewhat smaller than for C-CAPM. We find that SMB plays no role in explaining the industry returns. On the other hand, all of the HML coefficients are highly significant, and their values range from 5.27 to 6.69, which is generally higher than for the 25 size and book-to-market portfolios.

Figure 3(b) shows that the three-factor model explains the spread of average returns better than C-CAPM. C-CAPM prices the Hlth and HiTec industries that have low book-to-market portfolios better than the three-factor model. The inability of this model to price these industries too may be due to the uncertainty about risk factors, as indicated in previous studies of the industry cost of capital (see, for example, Fama and French, 1997; Ferson and Locke, 1998; and Pastor and Stambaugh, 1999). This is possibly due to the fact that the values of size and the book-to-market ratio for each industry change over time. It is difficult to measure the HML risk sensitivity of these industry portfolios precisely over time. However, in general, as in the case of the 25 portfolios, the inability of C-CAPM to price industry returns seems to come from the fact that the model omits another dimension of risk associated with HML.

A time-varying comparison between HiTec (cyclical) and Utils (defensive) industries is shown in Figure 4. In the sample period, the HiTec industry has a consistently low book-to-market ratio while the Utils industry has a relatively high book-to-market ratio. For their firm sizes, the Utils industry has a larger market common equity than the HiTec industry. The risk premium for HiTec is much more volatile and is mainly caused by the movement of the consumption covariance. The average consumption covariances for the HiTec and Utils industries are 0.0047 and 0.0022 respectively (Table 7). The HiTec industry positively co-moves with SMB while the Utils industry seems not to be affected by SMB. The average SMB covariances for the HiTec and Utils industries are 0.0776 and 0.0005 respectively. Similarly, the HML covariance for the Utils industry is also close to zero throughout the sample period, while that for the HiTec industry is always negative. The average HML covariances for the HiTec and Utils industries are -0.1119 and 0.0100 respectively.

Opposite movements occur in the risk premia during the dotcom bubble burst. From 2000 to 2002, the risk premium for the HiTec industry decreases while that for the Utils industry increases. The consumption covariances for the HiTec industry decreases sharply while that for the Utils industry increases, but the reduction of consumption covariances for the HiTec industry is not as strong as the decrease for the small growth portfolio at the same period. The movement of the HML covariance is similar to consumption. The HML covariance for the HiTec industry decreases sharply during this period while that of the Utils industry increases slightly. In contrast, during the same period, the SMB covariance for the HiTec industry increases and that for the Utils industry decreases. According to the behavior of the consumption, SMB and HML covariances, the HiTec industry behaves like the small growth portfolios while the Utils industry behaves similarly to the big value portfolios.

Table 7
Estimates of 10 Industry Portfolios

The table presents the estimates for C-CAPM and the general three-factor model for the 10 industry portfolios. γ denotes the coefficient relative risk aversion. β_1 , β_2 and β_3 are slope coefficients on consumption, SMB, and HML respectively. The mean residual is computed by subtracting the risk premium from their historical value. The *p-value* tests the no-arbitrage restrictions of the general three-factor model.

	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
	Panel A: C-CAPM									
γ	155.84	134.87	128.07	158.56	173.51	147.96	129.20	167.05	139.49	139.80
$t(\gamma)$	4.94	4.55	4.36	4.95	5.56	4.31	4.77	4.92	4.22	4.66
Mean Return Residual	0.06	-0.14	-0.18	0.07	-0.30	0.05	-0.20	0.21	0.10	-0.11
$p-value$	0.96	0.87	0.99	0.95	0.69	0.60	0.86	0.53	0.94	0.94
	Panel B: General Three-Factor SDF Model									
β_1	134.92	69.59	116.65	170.87	180.39	118.88	126.84	125.25	81.72	115.97
$t(\beta_1)$	2.81	1.89	2.79	3.76	3.80	2.51	3.29	2.53	1.79	2.57
β_2	0.52	2.31	0.90	-1.09	-0.56	0.64	0.75	0.29	1.71	0.72
$t(\beta_2)$	0.26	1.33	0.47	-0.58	-0.28	0.32	0.40	0.15	0.88	0.37
β_3	5.96	6.69	5.91	5.27	5.81	6.25	5.79	6.75	6.59	6.01
$t(\beta_3)$	3.64	3.99	3.41	3.06	3.40	3.60	3.52	3.89	3.98	3.58
$Cov_t(r_{t+1}, \Delta c_{t+1})$	0.00375	0.00514	0.00468	0.00366	0.00466	0.00248	0.00630	0.00265	0.00220	0.00500
$Cov_t(r_{t+1}, SMB_{t+1})$	0.0270	0.0304	0.0410	0.0079	0.0776	0.0161	0.0483	0.0173	0.0005	0.0406
$Cov_t(r_{t+1}, HML_{t+1})$	-0.0273	-0.0374	-0.0407	-0.0168	-0.1119	-0.0419	-0.0486	-0.0620	0.0100	-0.0307
Mean Return Residual	0.16	0.26	0.03	0.07	0.28	0.28	0.10	0.29	-0.09	0.00
$p-value$	0.01	0.77	0.25	0.62	0.62	1.00	0.07	0.04	1.00	0.16

Figure 3
Cross-Sectional Fit: 10 Industry Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 industry portfolios for C-CAPM the three-factor model. The average excess returns are adjusted for Jensen effect.

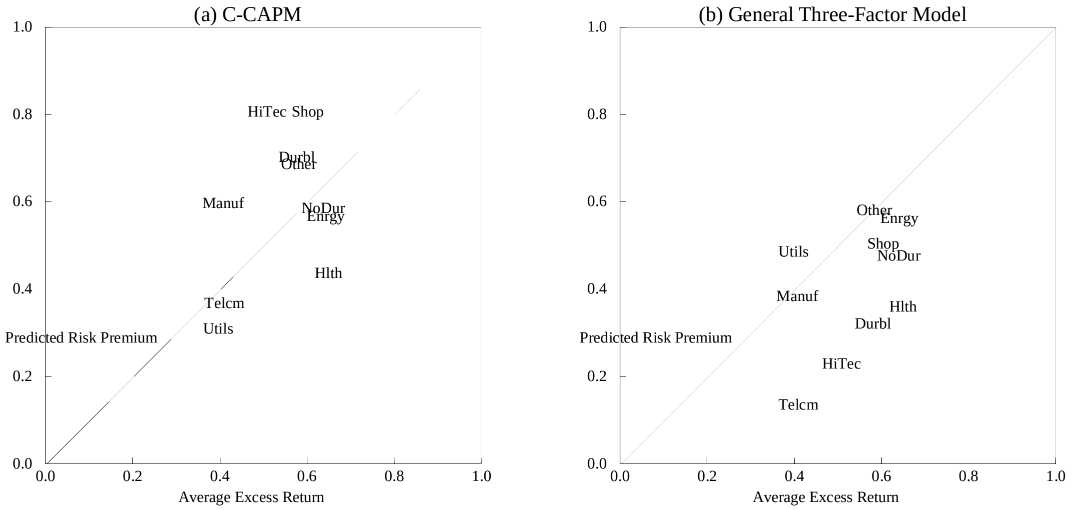
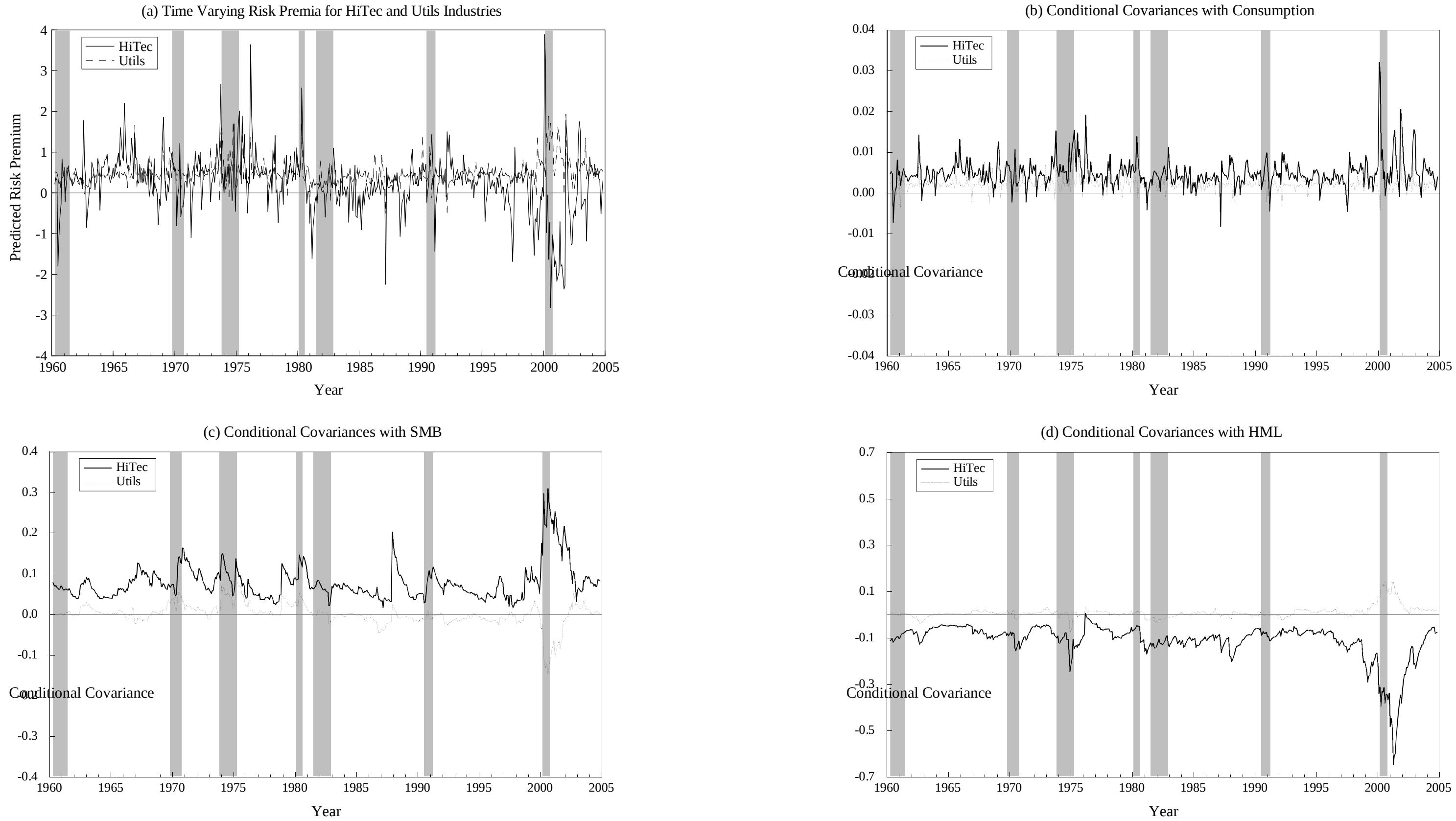


Figure 4
Hi-Technology and Utilities Industries

The figure compares time-varying risk premia and conditional covariances of the returns with the factors between Hi-technology and Utility industries from the general three-factor model. The figures are (a) time-varying risk premia, (b) conditional covariances of the returns with consumption, (c) conditional covariances of the returns with SMB, and (d) conditional covariances of the returns with HML. The sample period is 1960:2-2004:11. Shaded areas are recessions as defined by NBER.



7. Conclusions

We have extended standard C-CAPM by including two additional factors associated with Fama and French (1993) in their study of CAPM: size (SMB) and the book-to-market ratio (HML). As both SMB and HML are themselves portfolios returns, we have incorporated equations for them, together with portfolio returns sorted by size and their book-to-market ratios, to provide a system of no-arbitrage equations derived from the SDF pricing model. This is then estimated, together other macroeconomic factors, in a multivariate GARCH in mean model that satisfies no-arbitrage conditions.

We find that, in addition to consumption, HML, but not SMB, helps to determine equity returns. C-CAPM performs well for most of the portfolios provided they do not have low book-to-market ratios, and that including HML mainly improves the fit of low book-to-market portfolios. HML also helps explain SMB and HML returns. The poor explanatory power of C-CAPM for the portfolio returns in the two lowest book-to-market quintiles reflects the lack of variation of the consumption conditional covariances across book-to-market ratios. In contrast, HML explains about 40% of their total risk premia.

Our findings for industry returns are consistent with those for the portfolios sorted by size and book-to-market ratios. C-CAPM cannot explain the returns of industries having a relatively low book-to-market ratios and a small firm size. However, including SMB and HML as additional factors does not improve the fit for these portfolios either. The inability of the general three-factor model to price industry returns is consistent with other related studies (Fama and French, 1997; Ferson and Locke, 1998; and Pastor and Stambaugh, 1999). Perhaps this is because changes over time in the size and the book-to-market ratios of the industriesmake it more difficult to measure the contributionse of SMB and HML correctly. The behaviour of the time-varying risk premia for High-technology (HiTec) and Utilities (Utils) are similar to those for small growth and large value stocks, respectively. This is because HiTec has a consistently lower book-to-market ratio, while Utils has a larger market common equity.

As the choice of HML is empirically motivated, several studies have attempted to establish the connection between HML and more fundamentally determined factors. Our results suggest that financial distress (Fama and French, 1995) and default risk (Vassalou and Xing, 2004) may not be the reason why HML helps explain equity returns. The relation between HML and size indicates that small firms are less risky than large firms. A possible explanation is that HML is associated with the investment growth prospects of firms. Firms with low book-to-market ratios may be expected to have higher rates of growth. To a lesser extent, small firms may be expected to behave similarly. This interpretation is consistent with Brennan, Wang, and Xia (2004) and Li, Vassalou, and Xing (2006) who suggested that investment-based asset pricing models may be able to explain the cross-section of equity returns.

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