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Climate Change, Strict Pareto Improvements in Welfare and Multilateral Financial Transfers

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JEL Classification

H23, F18

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1 Introduction

The Paris agreement establishes the 'principle of common but differentiated responsibility and respective capabilities, in the light of different national circumstances' (Art. 2.2). This principle calls primarily for developing countries to take the lead by undertaking 'economy-wide absolute emission reduction targets' (Art. 4.4) but also the obligation to '*provide financial resources to assist developing country Parties*' (Art. 9.1, emphasis added).[1] Arguably, there is a strong equity argument behind the provision of financial transfers that compensate the countries that adopt emission mitigation strategies (possibly through taxing carbon). These transfers could come in different guises (tied — in the sense of being conditional on specific pollution mitigation actions — or untied) and would be necessary for universal participation in a Climate Change Agreement, and could be substantial. Though precise estimates are difficult to come by, Jacoby et al. (2008), for example, estimate that if developed countries were to fully compensate developing countries for the costs of mitigation then their welfare costs, if shared equally, will be around 2% in 2020, rising to some 10% in 2050, with the implied financial transfers being over $400 billion per year in 2020 rising to around $3 trillion in 2050. Implicit in the discussion behind financial assistance is the premise that financial transfers can deliver, in addition to other instruments (trade and carbon taxes) that can be used, welfare improvements for all participating countries.[2]

In a world economy in which there are trade and pollution distortions, the latter arising from inefficient carbon pricing, the question arises as to whether there exist multilateral financial transfers (income) that are strictly Pareto-improving in welfare.[3] And, if they do exist, under what conditions? This is the theme of the paper: to explore conditions under which there exist international transfers of income that result in strict welfare gains to all participating countries, thereby partially ameliorating the negative welfare effects of carbon price distortions in the presence of global emissions externalities. The key task in this paper is therefore to elaborate on the conditions that are necessary and/or sufficient to hold in the initial equilibrium for multilateral transfers to general strict Pareto improvements in welfare. This is a deceptively simple question but, as the analysis will show, with an answer that is surprisingly not simple. On a more practical note, the analysis highlights, as noted in the preceding paragraph, an issue that has

[1] For a critical review of the Paris agreement see Streck et al. (2016).

[2] Recently, the EU as part of the European Green Deal (COM(2019) 640 final) action plan anounced 'The Just Transition Mechanism' to address the social and economic effects of the transition to a carbon neutral EU economy. The mechanism provides targeted support of at least 150 billion Euros over the period of 2021-2027 in the regions most affected by this transition. Part of this financial support constitutes tied financial transfers with the sole surpose to compensate those that are most affected by the move towards the green economy.

[3] Financial transfers and income will be used interchangeably throughout. As will be shown below, and from a modeling perspective, these are transfers of the numeraire good.

been prominent in current discussions in climate change negotiations, and in particular with the nature of the responsibilities and actions countries need to take in relation to financial transfers to compensate for enhanced climate action.

There is, of course, another way to state the problem and, as it will be seen shortly, this is the approach that will be taken to address the issues at hand. If, for some reason, the global economy is constrained from setting carbon and trade tax instruments at their Pareto efficient values, can we nevertheless ensure that the use of multilateral transfers can generate welfare gains to all participating countries? This is the objective of this paper: to identify conditions under which this is, or is not, the case. To address the issue, use is made of a general equilibrium model of a trading world comprising many countries and goods in which production generates (carbon) emissions that result in global negative externalities on households. Within this framework three types of policy instruments are considered: trade taxes (tariffs), carbon taxes and multilateral income transfers. Taking trade and carbon tax settings as given, attention is turned to the role of multilateral income transfers in yielding strict Pareto improvements in welfare.

To anticipate the results that follow, what emerges, unsurprisingly, is that Pareto-improving income transfers exist only if there are initial trade and/or carbon tax distortions. In this case, international transfers of income can be used to generate a strict Pareto improvement in welfare, even though the policy instrument being used is different from the ones that are causing the distortion. The existence of strict Pareto-improving transfers is shown to depend on the violation of a generalization of a Hatta normality condition, the generalization taking into account the global externality created by carbon emissions and its general equilibrium impact on world prices and households. Strict Pareto improvements do not exist if trade taxes and carbon taxes are set at their Pareto optimal levels. Nor do they exist if all goods are normal in all countries and the world substitution matrix (which accounts for the effect of global emissions on compensated demands in all countries) exhibits net substitutability.[4] However, assuming that trade taxes are Pareto optimally set (at zero) while carbon prices are not optimally set, it is proved that multilateral transfers are able to yield strict Pareto improvements in welfare. This theoretical possibility is illustrated through numerical examples.

The analysis builds on two strands of literature: one that has discussed trade and pollution reforms and one that has discussed the possibility of immiserizing transfers arising when a recipient becomes worse off when the donor gives them resources (the latter taking place in a framework within which distortions from emissions are assumed away or, indeed, not part of the framework). The first strand of literature includes theoretical contributions that have addressed the linkages between climate (environmental, more

[4] The perspective taken here reinforces, in some sense, a plausibly held belief that multilateral transfers might not deliver strict Pareto improvements — particularly under climate change conditions.

generally) and trade policies. Some of these studies have focused on non-cooperative policy formation, characterizing nationally optimal trade and environmental policies and the interplay between them. See, for instance, the studies by Markusen (1975), Baumol and Oates (1988), Copeland (1996), Panagariya et al. (2004), Copeland (2011) and Ishikawa and Kiyono (2006). Other studies have focused on desirable directions of reform — whether for small or large economies — when one policy instrument, environment or trade, is for some reason constrained away from its optimal level. See, for example, studies by Copeland (1994), Hoel (1996), Turunen-Red and Woodland (2004), Neary (2006), Vlassis (2013), Kotsogiannis and Woodland (2013), Michael and Hatzipanayotou (2013), Keen and Kotsogiannis (2014), and more recently Tsakiris and Vlassis (2020).

Part of this latter literature has, in particular, characterized Pareto-efficient allocations in which potentially three sets of policy instruments may be deployed: international lump-sum transfers, carbon pricing, and trade tariffs.[5] The first set of these policy instruments is naturally directed to equity concerns, moving the world around its utility possibly frontier; the second set is naturally targeted to controlling emissions; and the third set would have no role if the other two instruments were optimally deployed. Attention has thus focused on the implications of various constraints on these policy instruments for the setting of the other policy instruments to achieve constrained Pareto-efficient or Pareto-improving welfare outcomes. However, these analyses have been undertaken without delving into the conditions required for international financial (untied) transfers amongst participating countries to deliver (or not) strict Pareto improvements in welfare. To put it differently, many contributions in the literature (as in the ones referred to above) are implicitly using the conditions identified here, but none has explicitly characterize the conditions required for international transfers to exist (or not). This is the focus of the current paper.

The analysis here also relates to a second strand of literature, namely the international trade literature that has analyzed the transfer problem, as in Turunen-Red and Woodland (1988). They considered the age-old question of whether a transfer of income between two countries necessarily benefits the recipient at the expense of the donor. In a general equilibrium model with many countries, they showed that interesting paradoxes can occur and, in particular, that it may be possible for multilateral transfers of income to improve the welfare of every country in the world, provided that there are trade distortions in the initial equilibrium. The transfers thus exploit the trade tax distortions to generate strict Pareto improvements. Their model, however, did not (and did not need to) consider the possibility of environmental distortions. In the present paper, we explicitly model global

[5] See also the related work of Keen and Wildasin (2004), who characterize Pareto efficient taxation (commodity taxes and trade taxes), with and without lump sum transfers, in a world economy that does not incorporate environmental externalities. More recent work emphasises the role of taxes. On this see Michael et al. (2015).

environmental externalities and show that strict Pareto-improving income transfers may exist when there are carbon tax distortions but no trade tax distortions. The analysis does not consider 'tied' income transfers, that is, income transfers conditional on the commitment by the recipient country to take action regarding a reduction in carbon emissions (either through the use of more efficient technology or through more efficient carbon pricing) and/or trade distortions. Nor does it consider the possibility of income transfers inducing an endogenous (untied) carbon/trade policy response. This is not because these issues are unimportant (to the contrary, they are), but simply because the same condition as the one identified here would be at work whenever the policy instruments are constraint to be away from their Pareto efficient level.

The plan of this paper is as follows. Section 2 sets out the model of a world economy comprising many countries and goods and embodying global carbon emission externalities. As a preliminary analysis, Section 3 examines the basic issues concerning the welfare effects of policy reform, while Section 4 provides a general characterization of the necessary and sufficient conditions required for strict Pareto-improving international income (untied) transfers to exist. Section 5 then characterizes conditions required for international lump sum transfers not to exist. Section 6 provides a numerical example that further illustrates the mechanism at work. Finally, Section 7 provides some brief concluding remarks.

2 The structure of the model

The model is based upon those of Turunen-Red and Woodland (2004) and Keen and Kotsogiannis (2014). It is a standard perfectly competitive general equilibrium model of international trade in which there are J countries, indexed by the superscript j, that trade in N commodities the production of which generates pollution. The N-vector of international commodity prices is denoted by p. International trade is subject to trade taxes (or subsidies), the vector of which is denoted in country j by τ^j. If $\tau_i^j > 0$ ($\tau_i^j < 0$) and commodity i is being imported by country j, then τ_i^j is an import tariff (import subsidy); and if $\tau_i^j > 0$ ($\tau_i^j < 0$) and commodity i is being exported by country j, then τ_i^j is an export subsidy (export tax). The domestic commodity price vector in country j is thus given by the N-vector $p^j = p + \tau^j$.[6,7]

The production of each commodity generates some pollutant with the N-vector z^j denoting emissions in country j.[8] This formulation allows for emissions to be distinguished

[6]The framework is consistent with the most-favoured nation principle, in the sense that each country applies the same tariff rates to all other countries.

[7]Consumption taxes do not feature in the model as their inclusion does not offer any additional insights.

[8]This is, of course, a rather specific form of emissions (best suited to the concentration of greenhouse

by the industry (product) of origin. Total emissions in country j are thus given by $1_N^{\intercal} z^j$, where 1_N is the N-vector of 1s and the superscript $^{\intercal}$ indicates transposition. Global emissions, on which damage in each country depends, are thus given by the scalar

$$k = 1_N^{\intercal} \sum_{j=1}^{J} z^j. \tag{1}$$

Pollution discharges in country j are subject to pollution taxes, given by the N-vector t^j. Such pollution taxes are, in general, permitted to be sector (product)-specific.

The production sector in country j is competitive and characterized by a revenue function, which takes the form

$$G^j(p^j, t^j) = \max_{y^j, z^j} \{p^{j\intercal} y^j - t^{j\intercal} z^j \quad : \quad f^j(y^j, z^j) \leq 0\}, \tag{2}$$

where $f^j(\cdot)$ is the implicit production possibility frontier in country j, with y^j being the vector of net outputs of traded goods.[9] This revenue function has the standard properties of linear homogeneity, convexity and differentiability in the price vector (p^j, t^j).[10] Notice, following from (2) — and as an envelope property — that[11]

$$G_p^j(p^j, t^j) \equiv \frac{\partial G^j(p^j, t^j)}{\partial p^j} = y^j, \tag{3}$$

$$G_t^j(p^j, t^j) \equiv \frac{\partial G^j(p^j, t^j)}{\partial t^j} = -z^j. \tag{4}$$

The consumption sector in country j is characterized by the (restricted) expenditure function

$$E^j(p^j, u^j, k) = \min_{x^j} \{p^{j\intercal} x^j : U^j(x^j, k) \geq u^j\}, \tag{5}$$

which is concave and linearly homogeneous in prices, increasing in utility u^j and in-

gasses in the global atmosphere). Generalizing this to include many types of pollutants (expressed, appropriately modified, by k being a vector) that may have differential impacts across countries (as in Turunen-Red and Woodland, 2004) is feasible at a cost of some additional notation. This generalization, however, is not pursued here as it is beyond the focus of the analysis.

[9] Throughout, the dependence of the revenue function on the vector of (non-polluting) endowments for brevity (and being fixed) is suppressed. Any abatement technology that might be available to countries is implicit in the description of the production sector.

[10] Linear homogeniety in prices implies the identity

$$(p^{j\prime}, t^{j\prime}) \begin{pmatrix} G_{pp}^j & G_{pt}^j \\ G_{tp}^j & G_{tt}^j \end{pmatrix} \equiv (0', 0'),$$

where, for example, $G_{pp}^j = \frac{\partial G_p^j}{\partial p'}$ is the second derivative of G^j with respect to p. The convexity condition implies that the second derivative matrix in the above identity is positive semi-definite.

[11] Throughout, we use subscripts to denote derivatives as in the expressions below.

creasing in global emissions k.[12] The last property implies that higher global pollution requires greater expenditure on goods to maintain the level of utility (the utility function is decreasing in k). Shephard's lemma implies that the price gradient $E_p^j(p^j, u^j, k) \equiv \frac{\partial E^j(p^j,u^j,k)}{\partial p^j}$ gives the vector of compensated demands, whereas the scalar $E_k^j(p^j, u^j, k) \equiv \frac{\partial E^j(p^j,u^j,k)}{\partial k}$ is the compensation required for a marginal increase in global emissions, that is the marginal willingness to pay for pollution reduction.

It will prove convenient to make use of the net revenue function, denoted by $S^j(p^j, t^j, u^j, k)$ and defined as the difference between national revenues $G^j(p^j, t^j)$ and expenditures $E^j(p^j, u^j, k)$. The net revenue function is

$$S^j\left(p^j, t^j, u^j, k\right) \equiv G^j\left(p^j, t^j\right) - E^j\left(p^j, u^j, k\right), \tag{6}$$

with the gradient vector with respect to product prices

$$S_p^j\left(p^j, t^j, u^j, k\right) = G_p^j\left(p^j, t^j\right) - E_p^j\left(p^j, u^j, k\right), \tag{7}$$

giving country j's compensated net-export vector and the gradient vector with respect to carbon taxes

$$S_t^j = G_t^j = -z^j, \tag{8}$$

giving the pollution N-vector in country j.[13] Since u^j and k only appear in the household expenditure function, the affects of marginal changes in these variables upon S^j are given by $S_u^j = -E_u^j < 0$ and $S_k^j = -E_k^j < 0$. We assume, without loss of generality, that $S_u^j = -E_u^j = -1$, and hence $-S_{pu}^j = E_{pu}^j$ may be interpreted as the income derivative of country j's Marshallian demand functions at the initial equilibrium.[14]

Assuming that countries impose arbitrary fixed tariffs on their net imports and carbon taxes on pollution emissions, the world equilibrium is characterized by the system of

[12] Linear homogenity of the expenditure function in prices implies the identity $p^{j\prime}E_{pp}^j \equiv 0$, while concavity implies that E_{pp}^j is a negative semi-definite matrix.

[13] For the properties of these functions, see Woodland (1982).

[14] This follows from the fact that $E_p^j\left(p^j, u^j\left(p^j, m^j, k\right), k\right) = x^j\left(p^j, m^j, k\right)$, which implies that $E_{pu}^j\left(p^j, u^j\left(p^j, m^j, k\right), k\right) \partial u^j/\partial m^j = x_m^j$. Since $\partial u^j/\partial m^j = E_u^j$. With $E_u^j = 1$, it then follows that $E_{pu}^j\left(p^j, u^j\left(p^j, m^j, k\right), k\right) = x_m^j$. Expression x_m^j is therefore the income derivative of country j's Marshallian demand functions.

equations

$$p^{\intercal} S_p^j(p^j, t^j, u^j, k) = b^j, \ j = 1, ..., J, \tag{9}$$

$$\sum\nolimits_{j=1}^{J} S_p^j \left(p^j, t^j, u^j, k\right) = 0_N, \tag{10}$$

$$\sum\nolimits_{j=1}^{J} b^j = 0, \tag{11}$$

$$-1_N^{\intercal} \sum\nolimits_{j=1}^{J} S_t^j \left(p^j, t^j\right) = k, \tag{12}$$

where, as noted earlier, $p^j = p + \tau^j$.

Equation (9) is the national budget constraint in every country j, stating that the value (at world prices) of net exports (the trade balance) must be equal to a constant b^j. If $b^j = 0$ then country j has a zero trade balance, while b^j different from zero implies a financial transfer to or from country j to the rest of the world.[15] When $b^j > 0$, country j makes a transfer to the rest of the world; when $b^j < 0$, country j receives a transfer from the rest of the world. Since world trade must balance in value, equation (11) must hold. The N equations in (10) are the world market equilibrium conditions for tradeable commodities, stating that the world excess supplies for these goods must be zero. Equation (12) indicates how aggregate world pollution depends on the national vectors of pollution, which depend on domestic prices for products and carbon taxes. Given the tariff vectors τ^j, $j = 1, ..., J$, the carbon tax vectors t^j, $j = 1, ..., J$, and the vector of multilateral financial transfers $b = (b^1, ..., b^J)^{\intercal}$ satisfying (11), the market equilibrium conditions (10), the national budget constraints (9) and the world pollution equation (12) determine the competitive equilibrium world price N-vector for tradeable commodities, p, the world pollution level, k, and the vector of country utilities $u = (u^1, ..., u^J)^{\intercal}$.[16]

3 Preliminary analysis

Prior to undertaking a complete analysis of the comparative statics of the model, it is instructive to undertake a preliminary examination of the model. Perturbation of

[15] Equation (9) can also be written as $E^j \left(p^j, u^j, k\right) = G^j \left(p^j, t^j\right) - b^j + R^j \left(p^j, t^j, u^j, k\right)$, $j \in J$, where the revenues (trade and carbon taxes) in country j are given (following (7) and (8)) by $R^j \left(p^j, t^j, u^j, k\right) = -\tau^{j\intercal} S_p^j - t^{j\intercal} S_t^j$. This implies that expenditure in country j (for given global emissions k) is equal to GDP less any financial transfer b^j to the rest of the world, plus any additional tax revenue R^j returned to the consumer in that country in a lump sum fashion.

[16] Some useful notation: If $x = (x_1, ..., x_N)^{\intercal}$ then $x \gg 0$ means $x_n > 0$ for all $n = 1, ..., N$; $x > 0_N$ means $x_n \geq 0$ for all $n = 1, ..., N$, and $x \neq 0$; and $x \geq 0$ means $x_n \geq 0$ for all $n = 1, ..., N$. The existence of a competitive equilibrium solution with $p \gg 0$ is assumed, and no restrictions are imposed on this equilibrium.

equation (9) for country j with respect to world prices, transfers and global emissions, while keeping tariffs and carbon taxes unchanged, reveals that[17]

$$p^\intercal E^j_{pu} du^j = -db^j + S^{j\intercal}_p dp - \left(\tau^{j\intercal} S^j_{pp} + t^{j\intercal} G^j_{tp}\right) dp - p^\intercal E^j_{pk} dk. \tag{13}$$

Equation (13) shows how utility for a country j is affected by an income transfer, b^j, a change in world prices, p, and a change in global emissions, k. There are thus several effects upon utility in country j. The first one, given by $-db^j$, is the direct effect on utility in country j of an income transfer received by country j: if $p^\intercal E^j_{pu} > 0$, as is assumed under the Hatta (1977) normality condition, a reduction in the transfer b^j to the rest of the world (or an increase in the transfer received by j) confers a utility gain to this country. The second effect, given by $S^{j\intercal}_p dp$, is the familiar terms-of-trade effect: if a change in international prices increases the terms-of-trade $(S^{j\intercal}_p dp > 0)$ for country j, then its welfare increases under the assumed Hatta (1977) normality condition. The third effect, given by $-\left(\tau^{j\intercal} S^j_{pp} + t^{j\intercal} G^j_{tp}\right) dp$, gives the change in trade and carbon tax revenues — respectively, $-\tau^{j\intercal} S^j_{pp} dp$ and $-t^{j\intercal} G^j_{tp} dp$ — as the international price N-vector p changes, keeping utility, transfers and global emissions constant. If tariff/tax revenue increases as a result of the price change, then welfare increases. The final term, given by $-p^\intercal E^j_{pk} dk$, indicates that utility is reduced by an increase in global emissions if $p^\intercal E^j_{pk} > 0$, meaning that an increase in global emissions requires the consumer in j to increase the value, at world prices, of consumption to maintain the given level of utility. Overall, under the assumption of Hatta (1977) normality that $p^\intercal E^j_{pu} > 0$, a welfare gain requires world prices and global emissions to change in response to the change in the income transfer to make the right hand side of (13) positive.

Since global emissions are a function of world prices, given tariffs and carbon taxes, the above expression for the change in utility can be re-expressed in terms of the change in transfers and world prices alone. Specifically, global emissions given by (1) and (8) are a function of world prices, given tariffs and carbon taxes, can be expressed by the global emissions function

$$\kappa(p) = -1^\intercal_N \sum\nolimits_{j=1}^{J} S^j_t, \tag{14}$$

with price derivative

$$\kappa^\intercal_p = -1^\intercal_N \sum\nolimits_{j=1}^{J} S^j_{tp}. \tag{15}$$

[17] This derivation makes use of the homogeneity property of the $S^j(\cdot)$ function, the definition of $S^j(\cdot)$ as the difference between the revenue and expenditure functions, together with the definition of country j's prices, $p^j = p + \tau^j$.

This implies that (13) can be conveniently written as[18]

$$p^{\intercal}E^{j}_{pu}du^{j}=-db^{j}+S^{j\intercal}_{p}dp-\left[\tau^{j\intercal}\left(S^{j}_{pp}+S^{j}_{pk}\kappa^{\intercal}_{p}\right)+t^{j\intercal}G^{j}_{tp}\right]dp-E_{k}\kappa^{\intercal}_{p}dp. \tag{16}$$

The negative of the term within the square brackets in (16) gives the change in tariff/carbon tax revenue arising from a change in world prices at unchanged utility, taking into account the impact of this change upon global emissions and the impact of this upon tariff/carbon tax revenue. The term $S^{j}_{pp}+S^{j}_{pk}\kappa^{\intercal}_{p}$ is the pollution-augmented net substitution matrix in country j, which gives the responses in the net exports to changes in the terms of trade, when consumer compensated demands respond to the endogenous change in global pollution emissions; multiplication by $-\tau^{j\intercal}$ yields the effect on carbon tax revenues. The term G^{j}_{tp} gives the pollution emissions response in country j due to a change in its terms of trade; multiplication by $-t^{j\intercal}$ gives the effect on tariff revenue. The final term on the right hand side of (16) provides the additional expenditure on goods required to maintain utility following the effect of the change in world prices on global emissions; an increase in global emissions arising from the price change will reduce utility since $E_{k}>0$ by assumption.

Equation (16) is at the heart of the analysis that follows as it identifies all welfare effects that are associated with the transfers and their summation that determines total welfare. What this suggests is that, at least in principle, a multilateral transfer of purchasing power across countries (even in the absence of trade distortions) could reduce the welfare of every country. Assuming, as we shall, that the welfare function is total world utility — and making use of (10) and (11) — equation (13) summed over all J countries gives

$$\sum\nolimits_{j=1}^{J}p^{\intercal}E^{j}_{pu}du^{j}=\left\{-\sum\nolimits_{j=1}^{J}\left[\tau^{j\intercal}\left(S^{j}_{pp}+S^{j}_{pk}\kappa^{\intercal}_{p}\right)+t^{j\intercal}G^{j}_{tp}\right]\right\}dp-\left\{\left[\sum\nolimits_{j=1}^{J}E^{j}_{k}\right]\kappa^{\intercal}_{p}\right\}dp. \tag{17}$$

Clearly, (17) shows that if $p^{\intercal}E^{j}_{pu}>0$, $j=1,...,J$, then a necessary condition for a strict Pareto improvement is for the right hand side of (17) to be positive. The first term on the right hand side is the increase in compensated (keeping utility constant) global tariff and carbon tax revenue arising from the world price change following the multilateral income transfer. The second term on the right hand side is the global effect on households' expenditures needed to compensate for the change in global emissions $\kappa^{\intercal}_{p}dp$.

It is instructive for the interpretation of (17) to consider two special cases. First, in the absence of any global emissions effect (the second term vanishes), (17) implies that for global welfare to increase it is necessary that the world price vector alters so that the compensated world tariff and carbon tax revenue increases (the first term is positive). If,

[18] This derivation makes use, again, of (8) and country j' definition, $p^{j}=p+\tau^{j}$, and the fact that, following the homogeneity property of the $S^{j}(\cdot)$ function, $p^{j\intercal}S^{j}_{pk}\kappa^{\intercal}_{p}=0^{\intercal}$.

in addition, all tariffs and carbon taxes are zero in the initial equilibrium the first term, on the right hand side of (17) is zero making the whole right hand side zero. In this case, the tax revenue channel for a global welfare improvement disappears. Second, in the absence of any global tax revenue effect (the first term vanishes), (17) implies that for global welfare to increase it is necessary that the world price vector alters so that global emissions fall ($dk = \kappa_p^\intercal dp < 0$) under the assumption that $\sum_{j=1}^J E_k^j > 0$ (greater emission require more global expenditure to maintain utility levels). Again, if global emissions do not alter, or the effect on global expenditures is zero, the second term vanishes and so the global emission reduction channel for a global welfare gain disappears.

If the right hand side of (17) is zero — which would be the case if all countries trade freely and levy zero carbon taxes and there was no effect on global emissions — then a necessary (but not sufficient) condition for a strict Pareto improvement reform ($du^j > 0,\ j = 1, 2, ..., J$) is that *not all* $p^\intercal E_{pu}^j,\ j = 1, ..., J,$ terms be of the same sign. The implication of this is that some commodities must be *inferior* in *some* countries but *normal* in *others*.

In summary, the above discussion has highlighted several channels through with strict Pareto improvements in welfare may arise from a multilateral transfer of income. Distortions created by trade taxes or carbon taxes may be partially corrected via the changes in world prices so that global tariff revenue or carbon tax revenue increases. Also, the world price adjustments may reduce global emissions. Expression (17) shows that at least one of these channels is necessary for a welfare improvement.

We next turn to the formal characterization of necessary and sufficient conditions under which there exits a multilateral transfer of income that will raise the welfare in every country.

4 Existence of strict Pareto-improving multilateral transfers

The analysis that follows proceeds by characterizing the conditions under which there exists a multilateral transfer of income such that there is a Pareto improvement in welfare, assuming that all tariff and carbon tax rates are given but taking into account the general equilibrium impacts of the income transfer upon world prices, world pollution and national utility levels. Formally, the analysis proceeds by using Motzkin's Theorem of the Alternative to characterize the necessary and sufficient conditions under which a Pareto-improvement exists when multilateral transfers may be endogenously chosen.

The system (9)-(12) can be differentiated with respect to utility levels, world prices, world pollution and income transfers at the initial equilibrium. This yields the differential system

$$Adu + Bdp + Cdb + Ddk = 0_{J+N+2}, \tag{18}$$

where the matrices A, B, C and D are defined by[19]

$$A \equiv \begin{bmatrix} p^\intercal S^1_{pu} & 0 & \cdots & 0 \\ 0 & p^\intercal S^2_{pu} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & p^\intercal S^J_{pu} \\ S^1_{pu} & S^2_{pu} & \cdots & S^J_{pu} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B \equiv \begin{bmatrix} p^\intercal S^1_{pp} + S^{1\intercal}_p \\ p^\intercal S^2_{pp} + S^{2\intercal}_p \\ \vdots \\ p^\intercal S^J_{pp} + S^{J\intercal}_p \\ S_{pp} \\ 0^\intercal \\ 1_N^\intercal S_{tp} \end{bmatrix},$$

$$C \equiv \begin{bmatrix} -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \\ 0_N & 0_N & \cdots & 0_N \\ 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad D \equiv \begin{bmatrix} p^\intercal S^1_{pk} \\ p^\intercal S^2_{pk} \\ \vdots \\ p^\intercal S^J_{pk} \\ S_{pk} \\ 0 \\ 1 \end{bmatrix}, \tag{19}$$

and the vectors of change are

$$du \equiv \begin{bmatrix} du^1 \\ du^2 \\ \vdots \\ du^J \end{bmatrix}, \quad dp \equiv \begin{bmatrix} dp_1 \\ dp_2 \\ \vdots \\ dp_N \end{bmatrix}, \quad db \equiv \begin{bmatrix} db^1 \\ db^2 \\ \vdots \\ db^J \end{bmatrix}. \tag{20}$$

In these expressions,

$$S_{pp} \equiv \sum\nolimits_{j=1}^{J} S^j_{pp}, \tag{21}$$

is the world substitution matrix, which gives the aggregate world (compensated) substitution effects on net excess supply of changes in international prices, p, in the absence of environmental changes, and

$$S_{tp} \equiv \sum\nolimits_{j=1}^{J} S^j_{tp}, \tag{22}$$

gives the negative of the change in global emissions due to changes in international prices, p, and

$$S_{pk} \equiv \sum\nolimits_{j=1}^{J} S^j_{pk}, \tag{23}$$

gives the change in the compensated net supply vectors across all countries as a conse-

[19] Matrix A is of dimension $(J+N+2) \times J$, B is of dimension $(J+N+2) \times N$, C is of dimension $(J+N+2) \times J$, and D is of dimension $(J+N+2) \times 1$.

quence of changes in global emissions, k.

Before proceeding to analyze these comparative statics equations, it is useful to define a pollution augmented world substitution matrix $\tilde{S}_{pp}$ that will play a pivotal role in the subsequent analysis. Recalling from (14) that the impact of world prices on global emissions, given the trade taxes and carbon taxes, is given by the vector

$$\kappa_p^\intercal = -1_N^\intercal \sum\nolimits_{j=1}^{J} S_{tp}^j = -1_N^\intercal S_{tp}, \tag{24}$$

the matrix defined by

$$\tilde{S}_{pp} \equiv S_{pp} + S_{pk}\kappa_p^\intercal, \tag{25}$$

is the pollution-augmented world net substitution matrix, which gives the responses in the world net exports to changes in the terms of trade, when consumer compensated demands respond to changes in pollution emissions arising from the world price change. According to (25), changes in world prices affect excess supply of products directly via S_{pp} and indirectly via the change in consumption plans resulting from changes in global emissions.[20] Notice also that this pollution-augmented world substitution matrix can be written in alternative forms as[21]

$$\tilde{S}_{pp} = \begin{bmatrix} \tilde{S}_{p1} & \tilde{S}_{pq} \end{bmatrix} = \begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{1q} \\ \tilde{S}_{q1} & \tilde{S}_{qq} \end{bmatrix} = \begin{bmatrix} S_{11} + S_{1k}\kappa_1 & S_{1q} + S_{1k}\kappa_q^\intercal \\ S_{q1} + S_{qk}\kappa_1 & S_{qq} + S_{qk}\kappa_q^\intercal \end{bmatrix}, \tag{26}$$

where we use the partition $\kappa_p^\intercal = \begin{pmatrix} \kappa_1 & \kappa_q^\intercal \end{pmatrix}$. It will be assumed that the sub-matrix corresponding to non-numeraire goods $j = 2, ..., N$ defined by

$$\tilde{S}_{qq} \equiv S_{qq} + S_{qk}\kappa_q^\intercal, \tag{27}$$

is of full rank and therefore invertible.[22]

We now proceed to establish conditions under which there exists a multilateral transfer of income that will raise the level of welfare in every country j. A strict differential Pareto-improving multilateral income transfer is defined as a transfer, db, such that (du, dp, db, dk) solves the differential system (19) with $du \gg 0$. That is, the income transfer, along with the general equilibrium changes in world prices and global emissions, yields an increase in the utility level for every country (du is strictly positive). The

[20] To see this in a clear way first notice that $S_{pk}^j = -E_{pk}^j$ and hence that $S_{pk}^j\kappa_p^\intercal = -E_{pk}^j\kappa_p^\intercal$. The summation of this expression over all J countries gives $\sum_{j=1}^J S_{pk}^j\kappa_p^\intercal = -\sum_{j=1}^J E_{pk}^j\kappa_p^\intercal$, which implies that $S_{pk}\kappa_p^\intercal = -E_{pk}\kappa_p^\intercal$. This, as stated in the text above, is the reduction in world consumption of goods due to the change in global emissions caused by the change in world prices for goods.

[21] This follows from using (6) and (12), after using (8).

[22] $\tilde{S}_{qq}$ is an important matrix and will be central in the analysis that follows. Invertibility of this matrix is a regularity assumption so equation (10) can be solved for prices.

conditions for the existence of such a multilateral transfer of income are established using Motzkin's Theorem of the Alternative (Mangasarian, 1969, p.34).

We perform a conventional normalization by fixing the price of commodity 1, ignore the market equilibrium condition for that commodity, following Walras' Law, and assume without loss of generality that the numeraire good is freely traded. These assumptions imply that $\tau_1^j = 0$ and $p_1^j = 1$, $j = 1, ..., J$, and thus that $p^\intercal = (1, q^\intercal)$ where $q^\intercal = (p_2, \ldots, p_N)$. Equipped with this, we arrive at the following result (all proofs are relegated to the Appendix).

Lemma 1 *A strict Pareto-improving multilateral income transfer (with $p_1 = 1$) exists if and only if there is no vector $y = \begin{pmatrix} y_2^\intercal & y_3 \end{pmatrix}^\intercal \in R^N$ such that*

$$y_2^\intercal S_{qu}^j + y_3 p^\intercal S_{pu}^j \leq 0\ ,\ j = 1, \ldots, J, \tag{28}$$

$$y_2^\intercal \widetilde{S}_{qq} + y_3 p^\intercal \widetilde{S}_{pq} = 0_{N-1}^\intercal, \tag{29}$$

with (28) holding with strict inequality for at least one country j.

While Lemma 1 provides the necessary and sufficient conditions for a strict Pareto-improving multilateral transfer of income to exist, there is an alternative characterization that is more readily interpreted from an economics viewpoint. Following Lemma 1, one can define the country-specific scalars

$$\widetilde{\beta}^j \equiv p^\intercal S_{pu}^j - p^\intercal \widetilde{S}_{pq} \widetilde{S}_{qq}^{-1} S_{qu}^j,\ j = 1, \ldots J. \tag{30}$$

Using this definition, we can establish the following characterization of the existence conditions.

Proposition 1 *Let the pollution augmented world net substitution matrix $\tilde{S}_{qq} \equiv S_{qq} + S_{qk}\kappa_q^\intercal$ have full rank, and let $p_1 = 1$. Then, a strict Pareto-improving differential multilateral income transfer exists if and only if there is no scalar $y_3 \in R$ such that*

$$y_3 \widetilde{\beta}^j \leq 0\ ,\ j = 1, \ldots J, \tag{31}$$

and with strict inequality for at least one country j, where $\widetilde{\beta}^j$ is given by (30).

Proposition 1 has a clear interpretation. It is evident from the inequalities in (31) that if all $\widetilde{\beta}^j$ have the same sign — either positive or negative — then a $y_3 \in R$ that solves (31) does exist (and it can be either positive or negative, but not zero). In these cases, there are no multilateral income transfers that can generate strict Pareto improvements in welfare.[23]

[23] The necessary and sufficient conditions in Proposition 1 involve a variable y_3 that can be thought

On the other hand, strict Pareto-improving multilateral income transfers exist when there does not exist a scalar y_3 that solves (31). This case occurs when at least two countries have non-zero $\widetilde{\beta}^j$, $j = 1, \ldots J$, terms that differ in sign. What this implies in practice is that the existence of strict Pareto-improving multilateral income transfers has been narrowed down to the sign structure of easily recognizable quantities defined by the country-specific scalars $\widetilde{\beta}^j$, $j = 1, \ldots J$, in (30). This result may be formalized as in the following corollary to Proposition 1.

Corollary 1 *Under the assumptions of Proposition 1, a strict Pareto-improving transfer of income exists if, and only if, at least two of the scalars $\widetilde{\beta}^j$, $j = 1, \ldots J$, differ in sign.*

It has been just demonstrated that a sufficient condition for Pareto-improving multilateral income transfers to exist is that there exist at least two countries whose $\widetilde{\beta}^j$ scalars differ in sign. This condition has an intuitive interpretation, and one that relates to the well-known Hatta Normality Condition. To see this notice that for a small open economy j the Hatta Normality Condition is

$$p^{\intercal} S^j_{pu} < 0 \qquad (p^{\intercal} E^j_{pu} > 0), \tag{32}$$

which implies that all income effects on net exports (weighted by the world N-vector of international commodity prices) are strictly negative.[24] In the present framework, however, things are different in the sense that our normality condition for country j is

$$\widetilde{\beta}^j \equiv \widetilde{p}^{\intercal} S^j_{pu} < 0 \qquad (\widetilde{\beta}^j \equiv \widetilde{p}^{\intercal} E^j_{pu} > 0), \tag{33}$$

which is equivalent to the definition for $\widetilde{\beta}^j$ in (30), where

$$\widetilde{p}^{\intercal} \equiv p^{\intercal} - \begin{pmatrix} 0 & p^{\intercal} \widetilde{S}_{pq} \widetilde{S}^{-1}_{qq} \end{pmatrix}. \tag{34}$$

As demonstrated in Appendix B, this last expression can be written alternatively as

$$\widetilde{p}^{\intercal} = p^{\intercal} \left[I - \begin{pmatrix} 0 & p^{\intercal} \widetilde{S}_{pq} \widetilde{S}^{-1}_{qq} \end{pmatrix} \right] \tag{35}$$

$$= \left[p_1 - p_1 \widetilde{S}_{1q} \widetilde{S}^{-1}_{qq} \right]. \tag{36}$$

of as the implicit social marginal value of income, evaluated at the Pareto-efficient allocation being characterized, common across countries. This interpretation follows from the formalities in the proof of Proposition 1 on noting that the conditions expressed there are equivalent to those of maximizing a social welfare function $W(u)$ with marginal weights $W^{\intercal}_u = y^{\intercal} Z_u$ (where Z is a matrix with elements from matrices A and D) with the typical elements being (after appropriate substitutions) $W_{u^j} = y_3 \widetilde{\beta}^j$ (where $\widetilde{\beta}^j$ is given by (30)). This implies that $y_3 = W_{u^j} / \widetilde{\beta}^j$.

[24] Notice that in this case (given the homogeneity property of the S^j function) $p^{\intercal} S^j_{pu} = -E^j_u$. This implies that $y_3 = -W_{u^j}/E^j_u = -W_{u^j}$ (following from the fact that $E^j_u = 1$).

We call (33) the **Generalized Hatta Normality Emissions Condition** (GHNEC) for country j. Here the income effects on net exports are weighted by a 'shadow price vector', $\widetilde{p}$, which accounts for the general equilibrium impacts of product prices on emissions and their subsequent effect on prices. These indirect general equilibrium effects operate through matrix κ_p, which indicates how price changes affect pollution, and S_{pk}, which gives the subsequent effect of the change in pollution on net exports.

The significance of Proposition 1 (and its corollary) is that it provides a straightforward condition that can be checked when evaluating the effects of international transfers. This requires knowledge of the matrix $\widetilde{S}_{pq} = S_{pq} + S_{pk}\kappa_q^{\intercal} = \sum_{j=1}^{J}\left[G_{pq}^{j} - E_{pq}^{j} - E_{pk}^{j}\left(1_N^{\intercal}\sum_{l=1}^{J}G_{tq}^{l}\right)\right]$ together with S_{pu}^{j} at the initial perfectly competitive equilibrium. In principle, these matrices are observable marginal responses to prices and emissions.

We turn now to the identification of conditions for the non-existence of strict Pareto-improving reforms.

5 Non-existence of strict Pareto-improving multilateral income transfers

Proposition 1 established necessary and sufficient conditions for the existence of strict Pareto-improving multilateral transfers of income. Intuition suggests that if both policy instruments (tariffs and carbon taxes) are set at their Pareto efficient levels then no strict Pareto-improving multilateral income transfers would exist. This intuition is confirmed by the following corollary to Proposition 1, in which we make use of the Pareto efficient characterization of Pareto efficient tariffs and carbon taxes obtained by Keen and Kotsogiannis (2014, Proposition 2, p. 122), under the assumption that multilateral transfers of income are available.

Corollary 2 *Suppose that tariffs and carbon taxes in every country j, $j = 1, ..., J$, are set at their Pareto efficient levels given by $\tau^j = 0$ and $t^j = \sum_{i=1}^{J} E_k^i 1_N = t1_N$, where $t \equiv \sum_{i=1}^{J} E_k^i$. Then, if each good is normal in every country in the sense that $S_{pu}^j < 0$, $j = 1, ..., J$, a strict Pareto improvement in welfare arising from transfers does not exist.*

This corollary shows that strict Pareto-improving welfare outcomes from a multilateral transfer of income can only exist if tariffs and/or carbon taxes are set at non-optimal levels. Besides this special case, the question that arises is: under what other conditions will a strict Pareto-improving transfer of income not exist? This is the question to which we now turn.

Following the analysis of the preceding section, if all countries satisfy GHNEC then there is no multilateral transfer of income that can yield a strict Pareto improvement in welfare. Can we identify special cases of these conditions that have clear economic interpretations? In the following, we identify two such special cases.

The first special case is obtained by assuming that a change in global emissions does not affect the household's compensated demands for commodities at the margin and in the aggregate. If compensated demands (on the aggregate) are unresponsive to global emissions in the sense that

$$S_{pk} \equiv \sum_{j=1}^{J} S_{pk}^{j} = 0, \tag{37}$$

then the shadow price vector $\widetilde{p}$ defined by (34) takes the special form

$$\widetilde{p}^{\intercal} \equiv p^{\intercal} - \begin{pmatrix} 0 & p^{\intercal} S_{pq} S_{qq}^{-1} \end{pmatrix}. \tag{38}$$

For this case, we arrive at the following result.

Proposition 2 *Assume that compensated net exports are unresponsive to global emissions in the sense that $S_{pk} = 0$, that the world substitution matrix S_{qq} is of full rank, that each good is normal in every country in the sense that $S_{pu}^{j} < 0$, $j = 1, ..., J$, and that all goods are world net-substitutes at the initial equilibrium. Then, a strict Pareto-improving transfer of income does not exist.*

Proposition 2 re-confirms a result in Turunen-Red and Woodland (1988) in the present context: in the absence of pollution effects on compensated demands and if all goods are normal in all countries and there is sufficient substitutability in the world substitution matrix then income transfers across all countries do not generate a strict Pareto improvement.

Our second special case goes in a quite different direction. Suppose now, going to the opposite extreme to the circumstances of Proposition 2, that production and consumption, on the aggregate, are unresponsive to international prices. In this case, we have the following result.

Proposition 3 *Assume that there is no substitution in production or consumption in any of the J countries (and so, in the aggregate, $S_{pp} = 0$), that the matrix $S_{qk}\kappa_q^{\intercal}$ has full rank and exhibits net substitutability and that each good is normal in every country in the sense that $S_{pu}^{j} < 0$, $j = 1, ..., J$. Then, a strict Pareto-improving transfer of income does not exist.*

Beyond its technical content, there is a practical element behind Proposition 3 and one that relates to the climate change discussions referred to at the outset.[25] In the presence

[25] Proposition 3 can be thought of as the generalization of Proposition 2 in Turunen-Red and Woodland (1988). Things, however, are somewhat more complicated here due to the presence of global pollution, which affects compensated demands.

of inefficiencies from carbon pricing, normality and sufficient substitutability rule out the possibility of strict Pareto-improving welfare gains from financial transfers. Negative off-diagonal elements of matrix $S_{qk}\kappa_q^\intercal$ (net substitutability) imply that the effects of a change in the price of l^{th} good, through a change in global emissions, on compensated demands for good i is negative. That is

$$\frac{dE_{p_i}}{dp_l} = \frac{\partial E_{p_i}}{\partial k}\frac{\partial \kappa}{\partial p_l} < 0. \tag{39}$$

To give an example, suppose that the price of air-conditioning equipment increases (good l) and as a consequence more of this product is being supplied. Assume further that, as a result, pollution intensity (on the aggregate) increases ($\frac{\partial \kappa}{\partial p_l} > 0$). If this increase in global emissions induces consumers to decrease aggregate compensated demand for goods other than air-conditioners ($\frac{\partial E_{p_i}}{\partial k} < 0$), then $\frac{dE_{p_i}}{dp_l} < 0$ implying net-substitutability. What Proposition 3 requires is that compensated demand for goods other than air-conditioners (good i) — either for each country j or for the aggregate — decreases.[26]

6 An illustrative example

This section presents a 2-country, 2-tradeable-goods illustrative example to demonstrate the multilateral income transfer mechanism at work in creating a strict Pareto improvement in welfare when carbon taxes are not set at their Pareto optimal levels. Appendix C extends this example to 3-countries and 3-tradeable-goods following a more general specification of production and consumption.

Consider a world comprising two countries, producing two products under conditions of free trade and with zero carbon taxes that are clearly not Pareto optimal. One product is clean while the other is dirty in the sense that its production generates carbon emissions that adversely affect the climate. Consumers have preferences that are, in turn, adversely affected by the level of global emissions. In this context, the numerical example demonstrates that both countries can benefit in welfare from a financial transfer from one country to the other.

The model has a simple specification designed to provide a clear illustration of the possibility of a strict Pareto improvement in welfare. Both countries are assumed to have the same technology and endowments, so they have the same production possibility frontier and revenue function. The revenue function takes the constant elasticity of transformation form, which satisfies all of the theoretical conditions for a revenue function. This

[26]Notice that this does not preclude the possibility that the technology is one of fixed emissions (per unit of output), in the sense that $G_t^j = -z^j = -Ay^j$, where A is a matrix with off-diagonal elements 0 and the diagonal elements α_i^j giving the emission of good i in country j. Since $y^j = G_p^j$ with $y_p^j = G_{pp}^j$ and so $G_{tp}^j = -AG_{pp}^j$.

functional form is symmetric in the net producer prices for the two products and the carbon tax on emissions, and is given by

$$G(p,t) = \left(\sum_{i=1}^{2}(p_i - \alpha_i t)^2 + t^2\right)^{0.5}, \tag{40}$$

where $t = 0$ is a scalar carbon tax rate and α_i is the emissions produced by one unit of product i.[27] It will be assumed that $\alpha_1 = 0, \alpha_2 = 1$; product 1 is clean, while product 2 is dirty with carbon emissions equal to output. The net producer price for product i is given by $p_i - \alpha_i t$, which subtracts the cost of emissions, $\alpha_i t$, from the market price received per unit of output. Following (40), net emissions of the production sector are given by

$$z = -\frac{\partial G(p,t)}{\partial t} = \sum_{i=1}^{2}\alpha_i \frac{p_i - \alpha_i t}{G(p,t)} - \frac{t}{G(p,t)}, \tag{41}$$

whereas net output for good i is given by

$$y_i = \frac{\partial G(p,t)}{\partial p_i} = \frac{p_i - \alpha_i t}{G(p,t)}. \tag{42}$$

Following (42), (41) can be written as

$$z = \sum_{i=1}^{2}\alpha_i y_i(p,t) - z^a(p,t). \tag{43}$$

The first (summation) term is nothing else but gross emissions of carbon, which are proportional to industry outputs $y_i(p,t)$, while $z^a(p,t) = \frac{t}{G(p,t)}$ represents abatement of emissions. Since, by assumption, $t = 0$, production decisions are unaffected by emissions of carbon and no abatement activity occurs.

While the two countries share the same technology, they have different preferences. Preferences are of the Cobb-Douglas variety, supplemented by an environmental damage function. The expenditure function takes the functional form

$$E(p,u,k) = \left(\prod_i {p_i}^{a_i}\right)(u + D(k)) = e(p)(u + D(k)), \tag{44}$$

where $e(p)$ is the Cobb-Douglas unit expenditure function, u is the level of utility, k is the state of climate (global emissions) and $D(k)$ is the environmental damage function, which is increasing in k. An increase in k requires the consumer to spend more on goods to achieve a given level of utility, u.

[27] This function does not include explicit reference to factor endowments since they are constant and have been subsumed into the function.

While both countries share the same family of preferences, actual preferences differ by country because of different national parametric values. In the example, it will be assumed that $a_1 = 0.9, a_2 = 0.1$ for country 1, while $a_1 = 0.1, a_2 = 0.9$ for country 2. Thus, country 1 consumers have a strong preference for the clean good over the dirty good (has a very high marginal propensity to consume the clean good), while country 2 consumers have a strong preference for the dirty good. Given the symmetry of the technology, this means that country 1 exports the dirty good (product 2), while country 2 exports the clean good under free trade.

The impact of climate change upon utility depends on the damage function, assumed common to both countries, which is specified as

$$D(k) = \theta k^{\gamma}/\gamma \tag{45}$$

with parameters $\theta > 0$ and $\gamma \geq 1$. The example assumes that $\theta = 15$ and $\gamma = 2$, so global emissions ($k = z$) strongly affect welfare.

The example continues as follows. We first compute the initial free trade equilibrium with no transfers. Then the perfectly competitive equilibrium for this initial situation is computed and the existence of a strict Pareto-improving income transfer is verified using the methodology developed further above in this paper. This verification, with the details being given further below, resulted in the conclusion that a strict Pareto-improving income transfer did indeed exist. Following this, we then compute a new competitive equilibrium with transfers – Country 2 being the donor, transferring \$0.1 in units of the numeraire product, product 1, to the recipient, Country 1. This income transfer is shown below to yield a strict Pareto-improvement in welfare; both countries gain from the transfer.

Table 1 provides the numerical results.

Table 1: Equilibria Pre and Post Financial Transfer

	Initial equilibrium	New equilibrium	Change
Transfers	$b_1 = b_2 = 0$	$b_2 = -b_1 = 0.1$	
Product price p_1	1	1	0
Product price p_2	1	0.943	−0.057
Utility u_1	−14.986	−14.112	0.874
Utility u_2	−14.986	−14.113	0.872
Climate state k	1.414	1.372	−0.042

The table shows that the transfer of \$0.1 of income from the donor Country 2 to the recipient Country 1 results in a drop in the world price of product 2 – the dirty product

– from 1 to 0.943. The increase in the relative price of the clean good, and the resulting switch away from the production of the dirty good, results in global emissions falling by 0.042 units. The overall effect on welfare is for utility levels in both countries to increase by approximately 0.87. Thus, both countries gain from the income transfer, taking into account all general equilibrium and externality effects.

Each country in this example experiences several welfare effects. At unchanged world prices, the recipient country experiences a gain in welfare from the higher disposable income. Given our assumptions regarding preferences above, Country 1 spends most of this income transfer on the clean good. Country 2, the donor, suffers a loss in welfare since it has less disposable income and, under the assumptions on preferences, cuts down primarily on the consumption of the dirty good. The overall result is an increase in the world excess supply of the dirty good and an increase in the excess demand for the clean good, resulting in an increase in the relative world price of the clean good (now permitting world prices to adjust). This constitutes a deterioration of the terms of trade for the recipient country (which exports the dirty good), causing a reduction in welfare. The donor country, on the other hand, exports the clean good and so experiences an increase in its terms of trade and, hence, welfare. Thus, each country experiences both positive and negative effects on its welfare.

The terms of trade changes result in both countries moving around their production possibility frontiers to produce more of the clean good and less of the dirty good. This, in turn, reduces global emissions. Lower global emissions of carbon reduce k and hence reduce the environmental damage to consumers in both countries, imparting a positive welfare effect in both countries. If this reduction in environmental damage is sufficiently strong, its positive welfare effect can ensure that the overall welfare effect of the income transfer from the donor to the recipient is strictly positive. The numerical example developed provides such a case.

To connect this example more directly with the theoretical results in the paper, and as indicated further above, we computed[28] the shadow world price vector $\widetilde{p}$ and the $\widetilde{\beta}$ vector at the initial equilibrium. To do this, we evaluated numerically the various required national and world substitution matrices at the initial equilibrium with no income transfers. Using the formulae in (33) and (34), the resulting numerical values for the shadow world price vector $\widetilde{p}$ and the $\widetilde{\beta}$ vector at the initial equilibrium are

$$\widetilde{p}^{\intercal} = \begin{pmatrix} 1 & -0.999 \end{pmatrix}, \tag{46}$$

$$\widetilde{\beta}^{\intercal} = \begin{pmatrix} -80.009 & 79.915 \end{pmatrix}. \tag{47}$$

[28] Computations for this example, and for the example of Appendix C, have been performed using Gauss, version 19, and routine eqSolvemt.

Since $\widetilde{\beta}$ has both positive and negative elements, Corollary 2 indicates that a Pareto-improving transfer of income exists. The results presented above in Table 1 provide a concrete confirmation of this existence result. The country exhibiting a negative value for $\widetilde{\beta}$ (Country 1, with $\widetilde{\beta}^1 = -80.0$) is the recipient country, while the country with the positive $\widetilde{\beta}$ value (Country 2, with $\widetilde{\beta}^2 = 79.9$) is the donor.[29] The strong positive welfare effects of the reduction in global carbon emissions ensure that both countries gain from the income transfer.

It should be evident from our methodology, of course, that other such examples may be readily constructed — examples that are based on primitives, such as in the example presented above, and examples based on the methodology discussed in Appendix C. Moreover, it is straightforward to construct examples that do not yield strict Pareto-improving welfare gains. In such examples the vector $\widetilde{\beta}$ has all elements of the same sign, so that some countries will gain and some countries will lose from an international income transfer.

7 Concluding remarks

Discussions in climate change negotiations inevitably raise (as it has in climate change negotiations) the issue of financial compensation for countries undertaking costly environmental policy actions. This paper has explored the role of multilateral financial/income (untied) transfers in achieving *strict* welfare gains amongst countries, focusing in particular on identifying conditions under which their use is warranted, or not, when there is a global environmental externality and where there may be nationally set carbon prices and tariff impediments to international trade. The analysis has shown Pareto-efficiency does require international financial transfers when either carbon or trade taxes in some countries are constrained: its purpose then is to account for the impact on emissions of inappropriate carbon pricing and the trade distortions that exist. Importantly, it is shown that a strict Pareto-improving multilateral financial transfer exists if and only if a *generalized normality condition* is violated. In the special case where trade taxes are set at their Pareto-efficient levels (zero), there may exist multilateral financial transfers that are strict Pareto-improving in welfare when carbon taxes are sub-optimally set. To illustrate this possibility, we provided a numerical example of such a case when carbon taxes are set to zero in every country and there is free trade.

The analysis here is, of course, limited in several respects. Factors of production have been assumed internationally immobile, for example, precluding the possibility of carbon

[29] In a model with unchanged world prices such as a small open economy, recall that Hatta normality implies negative β terms. In such a situation, the donor country loses. Here the donor has a postive $\widetilde{\beta}$ value that takes account of world price adjustments and pollution externalities to yield a welfare gain.

leakage through location choices that is a major concern in policy debates (elements of this appear in Kotsogiannis and Woodland (2013)). The preceding analysis, however, suggests that a similar set of conditions will emerge. Income transfers have also been assumed to be unconditional and not tied to a reduction in carbon emissions — either through the use of more efficient technology or through more efficient carbon pricing — and/or to trade distortions. Nor has the analysis considered the possibility of income transfers inducing an endogenous (untied) carbon/trade policy response. It will be interesting to explore these issues in the context of policy reforms (and their direction) when these reforms are conditional on tied income transfers. In this context too, it is expected that a *generalized normality condition* will emerge as a condition that needs to be violated for the reforms to be (strictly) Pareto efficient, but the characterization of these reforms will most likely differ from the ones identified by the existing literature.

Appendices

Appendix A: Proofs of propositions and lemmas

Proof of Lemma 1 Omitting the market equilibrium for commodity 1 (the numeraire), the system can be written as

$$A^* du + B^* dq + C^* db + D^* dk = 0, \tag{A.1}$$

where the matrices are

$$A^* \equiv \begin{bmatrix} p^\intercal S^1_{pu} & 0 & \cdots & 0 \\ 0 & p^\intercal S^2_{pu} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & p^\intercal S^J_{pu} \\ S^1_{qu} & S^2_{qu} & \cdots & S^J_{qu} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \; B^* \equiv \begin{bmatrix} p^\intercal S^1_{pq} + S^{1\intercal}_q \\ \vdots \\ p^\intercal S^J_{pq} + S^{J\intercal}_q \\ S_{qq} \\ 0^\intercal \\ -\kappa^\intercal_q \end{bmatrix}, \tag{A.2}$$

$$C^* \equiv \begin{bmatrix} -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \\ 0_{N-1} & 0_{N-1} & \cdots & 0_{N-1} \\ 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \; D^* \equiv \begin{bmatrix} p^\intercal S^{1\intercal}_{pk} \\ p^\intercal S^{2\top}_{pk} \\ \vdots \\ p^\intercal S^{J\top}_{pk} \\ S_{qk} \\ 0 \\ 1 \end{bmatrix},$$

the vectors of change are

$$du \equiv \begin{bmatrix} du^1 \\ du^2 \\ \vdots \\ du^J \end{bmatrix}, \; dq \equiv \begin{bmatrix} dp_2 \\ \vdots \\ dp_N \end{bmatrix}, \; db \equiv \begin{bmatrix} db^1 \\ db^2 \\ \vdots \\ db^J \end{bmatrix}, \tag{A.3}$$

and

$$S_{qq} \equiv \sum\nolimits_{j \in J} S^j_{qq}, \tag{A.4}$$

$$\kappa^\intercal_q \equiv -1^\intercal_N \sum\nolimits_{j \in J} S^j_{tq}, \tag{A.5}$$

$$S_{qk} \equiv \sum\nolimits_{j \in J}^{J} S^j_{qk}. \tag{A.6}$$

A strict differential Pareto improvement exists if and only if there are du, dq, db, dk satisfying (A.1) and $du \gg 0$. Applying Motzkin's Theorem of the Alternative (Mangasarian, 1969, page 34), there exists du, dq, db, dk satisfying (A.1) if and only if there is no vector $y \in R^{J+N+1}$ such that

$$y^\intercal A^* < 0_J^\intercal, \tag{A.7}$$

$$y^\intercal [B^*, C^*, D^*] = 0_{J+N+1}^\intercal \ . \tag{A.8}$$

It is convenient to partition the vector y as $y = (y_1^\intercal, y_2^\intercal, y_3, y_4)^\intercal$ where $y_1 \in R^J, y_2 \in R^{N-1}, y_3 \in R, y_4 \in R$. It is now straightforward to show that $y^\intercal C^* = 0^\intercal$ implies that $y_1 = y_3 1_J$, where 1_J is the $J-$dimensional vector of ones. (A.7) and (A.8) can then be written as

$$y_2^\intercal S_{qu}^j + y_3 p^\intercal S_{pu}^j \leq 0 \ , \ j = 1, \dots J, \tag{A.9}$$

$$y_2^\intercal S_{qq} + y_3 p^\intercal S_{pq} - y_4 \kappa_q^\intercal = 0_{N-1}^\intercal \ , \tag{A.10}$$

$$y_2^\intercal S_{qk} + y_3 p^\intercal S_{pk} + y_4 = 0 \ , \tag{A.11}$$

with (A.9) holding with strict inequality for at least one j (in (A.10), (10) has also been used). Solving equation (A.11) for y_4 and substituting into (A.9) and (A.10) one obtains

$$y_2^\intercal S_{qu}^j + y_3 p^\intercal S_{pu}^j \leq 0, \ j = 1, \dots J, \tag{A.12}$$

$$y_2^\intercal \left(S_{qq} + S_{qk}\kappa_q^\intercal\right) + y_3 p^\intercal \left(S_{pq} + S_{pk}\kappa_q^\intercal\right) = 0_{N-1}^\intercal. \tag{A.13}$$

If (A.12) and (A.13) have a solution $y_2 \in R^{N-1}, y_3 \in R$, then (A.7) and (A.8) have a solution with

$$y_1 = y_3 1_J, \tag{A.14}$$

$$y_4 = -y_2^\intercal S_{qk} - y_3 p^\intercal S_{pk}, \tag{A.15}$$

and conversely, (A.7) and (A.8) have a solution if and only if (A.12) and (A.13) have a solution with (A.14) and (A.15). □

Proof of Proposition 1. The proof of Proposition **1** utilizes Lemma 1. Using the assumption that the matrix $\tilde{S}_{qq} \equiv S_{qq} + S_{qk}S_{sq}$ has full rank (and so its inverse exists), it follows that (28) implies

$$y_2^\intercal = -y_3 p^\intercal \left(S_{pq} + S_{pk}\kappa_q^\intercal\right)\left(S_{qq} + S_{qk}\kappa_q^\intercal\right)^{-1}. \tag{A.16}$$

Substituting this into (A.9) gives

$$y_3 \left[p^\intercal S_{pu}^j - p^\intercal \left(S_{pq} + S_{pk}\kappa_q^\intercal\right)\left(S_{qq} + S_{qk}\kappa_q^\intercal\right)^{-1} S_{qu}^j\right] \leq 0, \ j = 1, \dots J, \tag{A.17}$$

and with strict inequality for at least one country j. Recalling the definition of the country-specific scalars as

$$\widetilde{\beta}^j \equiv p^\intercal S^j_{pu} - p^\intercal \left(S_{pq} + S_{pk}\kappa_q^\intercal)\right) \left(S_{qq} + S_{qk}\kappa_q^\intercal\right)^{-1} S^j_{qu},\ j = 1, \dots J, \tag{A.18}$$

(A.17) reduces to

$$y_3 \widetilde{\beta}^j \leq 0,\ j = 1, \dots J, \tag{A.19}$$

with strict inequality for at least one country j.

Close inspection of (A.19) reveals that the only feasible solution is $y_3 \neq 0$. The reason for this is as follows. If all $\widetilde{\beta}^j$ have the same sign, say positive, then $y_3 < 0$. If they are all negative then $y_3 > 0$. If some are negative and some positive then y_3 is not admissible as it violates the inequalities. This implies that if (A.19) has a solution it must be $y_3 \neq 0$. This implies that y_2 has a non-trivial solution given by (A.16), that $y_1 = y_3 1_J$ and that $y_4 = -y_2^\intercal S_{qk} - y_3 p^\intercal S_{pk}$, $j = 1, ..., J$. □

Proof of Corollary 2 The proof of Corollary **2** utilizes the proof of Proposition 1 in showing that (34) reduces to $\widetilde{p}^\intercal \equiv p^\intercal \gg 0$ if $\tau^j = 0$ and $t^i = \sum_{j=1}^J E^j_k 1_N \equiv t 1_N$ and so, if $S^j_{pu} < 0$, $j = 1, ..., J$, then $p^\intercal S^j_{pu} < 0$. This implies that there is $y_3 \neq 0$ that satisfies (A.19). To see that $\widetilde{p}^\intercal \equiv p^\intercal \gg 0$, notice that (34) implies

$$p^\intercal \left(S_{pp} + S_{pk}\kappa_p^\intercal\right) = p^\intercal \left[\sum\nolimits_{j=1}^J S^j_{pp} - \sum\nolimits_{j=1}^J S^j_{pk} \left(1_N^\intercal \sum\nolimits_{l=1}^J S^l_{tp}\right)\right]. \tag{A.20}$$

The homogeneity property of the S^j function implies, since $p^{j\intercal} = p^\intercal$, that $p^\intercal S^j_{pp} + t^{j\intercal} S_{tp} = 0^\intercal$ and that $p^\intercal S^j_{pk} = -E^j_k$ and so $-p^\intercal \sum_{j=1}^J S^j_{pk} = \sum_{j=1}^J E^j_k$. Accordingly, (A.20) can be written as

$$-t1_N^\intercal \sum\nolimits_{j=1}^J S^j_{tp} + \sum\nolimits_{j=1}^J E^j_k \left(1_N^\intercal \sum\nolimits_{l=1}^J S^l_{tp}\right) = -t1_N^\intercal \sum\nolimits_{j=1}^J S^j_{tp} + t1_N^\intercal \sum\nolimits_{j=1}^J S^j_{tp} = 0^\intercal. \tag{A.21}$$

Using (34), this implies that $\widetilde{p}^\intercal \equiv p^\intercal$ as required. □

Proof of Proposition 2 The proof follows the proof of Proposition 1. Since $S_{pk} = 0$ by assumption, the shadow price vector $\widetilde{p}$ takes the special form given by (38) and

$$p^\intercal S_{pq} S_{qq}^{-1} = \begin{pmatrix} p_1 & q^\intercal \end{pmatrix} \begin{pmatrix} S_{1q} \\ S_{qq} \end{pmatrix} S_{qq}^{-1} = p_1 S_{1q} S_{qq}^{-1} + q^\intercal. \tag{A.22}$$

These results imply that the shadow price vector is now

$$\widetilde{p}^\intercal \equiv \begin{pmatrix} p_1 & q^\intercal \end{pmatrix} - \begin{pmatrix} 0 & p^\intercal S_{pq} S_{qq}^{-1} \end{pmatrix} = \begin{pmatrix} p_1 & -p_1 S_{1q} S_{qq}^{-1} \end{pmatrix}. \tag{A.23}$$

If S_{pp} exhibits net substitutability (all off-diagonal elements are negative) then $S_{1q} \ll 0$

and $S_{qq}^{-1} \geq 0$. The implication of this is that $\widetilde{p}^{\intercal} > 0$. Since all goods are assumed normal and so $S_{pu}^{j} = -E_{pu}^{j} \ll 0$, $j = 1, ..., J$, it follows that $\widetilde{\beta}^{j} < 0$ for all countries $j = 1, ..., J$. Thus, by Corollary 1 to Proposition 1 there does not exist a strict Pareto-improving transfer of income. □

Proof of Proposition 3 The proof follows the proof of Proposition 1. Since $S_{pp} = 0$ by assumption, the shadow price vector defined by (34) may be expressed as

$$\widetilde{p}^{\intercal} \equiv p^{\intercal} - \begin{pmatrix} 0 & p^{\intercal} \left(S_{pk} \kappa_{q}^{\intercal} \right) \left(S_{qk} \kappa_{q}^{\intercal} \right)^{-1} \end{pmatrix}, \tag{A.24}$$

which, in turn, implies that

$$p^{\intercal} \left(S_{pk} \kappa_{q}^{\intercal} \right) \left(S_{qk} \kappa_{q}^{\intercal} \right)^{-1} = \begin{pmatrix} p_1 & q^{\intercal} \end{pmatrix} \begin{pmatrix} S_{1k} \kappa_{q}^{\intercal} \\ S_{qk} \kappa_{q}^{\intercal} \end{pmatrix} \left(S_{qk} \kappa_{q}^{\intercal} \right)^{-1} = p_1 S_{1k} \kappa_{q}^{\intercal} + q^{\intercal}, \tag{A.25}$$

and thus that

$$\widetilde{p}^{\intercal} \equiv \begin{pmatrix} p_1 & q^{\intercal} \end{pmatrix} - \begin{pmatrix} 0 & p^{\intercal} \left(S_{pk} \kappa_{q}^{\intercal} \right) \left(S_{qk} \kappa_{q}^{\intercal} \right)^{-1} \end{pmatrix} = \begin{pmatrix} p_1 & -p_1 \left(S_{1k} \kappa_{q}^{\intercal} \right) \left(S_{qk} \kappa_{q}^{\intercal} \right)^{-1} \end{pmatrix}. \tag{A.26}$$

If $S_{pk} \overline{S}_{tp}$ exhibits net substitutability (all off-diagonal elements are negative) then $S_{1k} \kappa_{q}^{\intercal} \ll 0$ and $\left(S_{qk} \kappa_{q}^{\intercal} \right)^{-1} \geq 0$. The implication of this is that $\widetilde{p}^{\intercal} > 0$. Since all goods are assumed normal and so $S_{pu}^{j} \ll 0$ $j = 1, ..., J$, it follows that $\widetilde{\beta}^{j} < 0$ for all countries $j = 1, ..., J$. Thus, by Corollary 1 to Proposition 1 there does not exist a strict Pareto-improving transfer of income. □

Appendix B: Structure of the shadow price vector, and alternative expressions

As noted in the text, and in (36), the shadow price vector $\widetilde{p}$ (re-written here for convenience) is defined by

$$\widetilde{p}^\intercal = \left[\begin{array}{cc} p_1 & -p_1 \widetilde{S}_{1q} \widetilde{S}_{qq}^{-1} \end{array}\right], \tag{B.1}$$

and, following the definition of $\widetilde{S}_{qq}^{-1}$ from (27), therefore as

$$\widetilde{p}^\intercal \equiv p^\intercal - \left(\begin{array}{cc} 0 & p^\intercal \left(S_{pq} + S_{pk}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \end{array}\right), \tag{B.2}$$

which can be written as

$$\widetilde{p}^\intercal = \left[\begin{array}{cc} p_1 & -p_1 \left(S_{1q} + S_{1k}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \end{array}\right]. \tag{B.3}$$

To show this write the shadow price as

$$\widetilde{p}^\intercal = p^\intercal A, \tag{B.4}$$

where

$$A \equiv I - \left(\begin{array}{cc} 0 & \left(S_{pq} + S_{pk}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \end{array}\right) \tag{B.5}$$

$$\equiv I - \left(\begin{array}{cc} 0 & C \end{array}\right). \tag{B.6}$$

Matrix C simplifies in structure to the expressions

$$C \equiv \left(S_{pq} + S_{pk}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1}, \tag{B.7}$$

$$= \left[\begin{array}{c} S_{1q} + S_{1k}\kappa_p^\intercal \\ S_{qq} + S_{qk}\kappa_p^\intercal \end{array}\right] \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1}, \tag{B.8}$$

$$= \left[\begin{array}{c} \left(S_{1q} + S_{1k}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \\ \left(S_{qq} + S_{qk}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \end{array}\right], \tag{B.9}$$

$$= \left[\begin{array}{c} \left(S_{1q} + S_{1k}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \\ I_{N-1} \end{array}\right], \tag{B.10}$$

$$\equiv \left[\begin{array}{c} d \\ I_{N-1} \end{array}\right], \tag{B.11}$$

where the equalities in (B.7)-(B.10) follow from decomposing the matrix $S_{pq} + S_{pk}\kappa_p^\intercal$, where (B.11) follows from defining

$$d \equiv \left(S_{1q} + S_{1k}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1}. \tag{B.12}$$

Making use of (B.6) into (B.4) the shadow price vector becomes

$$\begin{aligned}
\widetilde{p}^{\intercal} &= p^{\intercal}\left[I - (0, C)\right] && \text{(B.13)}\\
&= p^{\intercal}\left[\begin{bmatrix} 1 & 0 \\ 0 & I_{N-1} \end{bmatrix} - \begin{bmatrix} 0 & d \\ 0 & I_{N-1} \end{bmatrix}\right] \\
&= p^{\intercal}\begin{bmatrix} 1 & -d \\ 0 & I_{N-1} - I_{N-1} \end{bmatrix} \\
&= p^{\intercal}\begin{bmatrix} 1 & -d \\ 0 & 0_{N-1} \end{bmatrix} \\
&= p_1\begin{bmatrix} 1 & -d \end{bmatrix} \\
&= \begin{bmatrix} p_1 & -p_1\left(S_{1q} + S_{1k}\kappa_p^{\intercal}\right)\left(S_{qq} + S_{qk}\kappa_p^{\intercal}\right)^{-1} \end{bmatrix}, && \text{(B.14)}\\
&= \begin{bmatrix} p_1 & -p_1\widetilde{S}_{1q}\widetilde{S}_{qq}^{-1} \end{bmatrix}, && \text{(B.15)}
\end{aligned}$$

where the last equality follows from the definition in (27). This completes the proof. □

This shows that the sign structure of $\widetilde{p}$ depends crucially upon the sign structure of the $(N-1)$-vector $d \equiv \left(S_{1q} + S_{1k}\kappa_p^{\intercal}\right)\left(S_{qq} + S_{qk}\kappa_p^{\intercal}\right)^{-1}$. This vector depends upon the full substitution matrix through the second inverse matrix. The first term is a vector expressing the total effects of a change in world prices upon the net exports of the numeraire good, taking into account the impact of world prices on world emissions and hence on net exports as well as the direct price effects. □

Appendix C: A 3-country, 3-traded-goods world economy

The model for this example comprises three countries engaged in perfectly competitive international trade in three tradeable commodities — with commodity 1 being taken as the numeraire with world price $p_1 = 1$. As with the example presented in the text, referred to here as Example 1, to focus on the plausibility that multilateral income transfers ameliorate the welfare-reducing distortionary effects of global emissions (and to simplify matters), it will be assumed that all countries trade freely facing no tariffs and impose zero carbon taxes. This implies that $\tau^j = 0,\ t^j = 0$ and $p^j = p = 1$ for $j = 1, 2, 3$. The example demonstrates a situation where a set of financial transfers raise the welfare levels for each of the three countries. Unlike, however, the example developed in the text, the numerical example in this appendix does not specify functional forms for the expenditure and revenue functions for the three countries. Rather, it assumes that the countries have valid expenditure and revenue functions and that there is a competitive equilibrium for the model specified in Section 2. This is sufficient for the illustration of the propositions and it simplifies the exposition.

Following the analytics developed in the text, only matrices representing aggregate effects (consisting of sums of national effects) are required, since the propositions are expressed in terms of these (with the exception only of the country-specific income effects). By construction, these matrices satisfy all the required conditions imposed by economic theory, and adopted in the model of Section 2, and are, therefore, valid for an illustration. To put it differently, the strategy in this example is to construct matrices that satisfy the properties of the functions discussed in the propositions, and in turn show that for these functions there are shadow prices for which the propositions are satisfied. Consider an international trade equilibrium with the aggregate matrix structures for $S_{pp},\ S_{pt},\ S_{tt},\ S_{pk}$ and a matrix of national income effects $S_{pu} = \begin{bmatrix} S_{pu}^1 & S_{pu}^2 & S_{pu}^3 \end{bmatrix}$ given, respectively, by

$$S_{pp} = \begin{bmatrix} 4.716 & -1.928 & -2.788 \\ -1.928 & 1 & 0.928 \\ -2.788 & 0.928 & 1.860 \end{bmatrix}, \tag{C.1}$$

$$S_{pt} = S_{tp}^{\intercal} = \begin{bmatrix} 1.212 & -0.243 & 0.341 \\ -0.514 & 0.055 & -0.535 \\ -0.698 & 0.189 & 0.193 \end{bmatrix}, \tag{C.2}$$

$$S_{tt} = \begin{bmatrix} 1.313 & -0.195 & -0.489 \\ -0.195 & 1.040 & -0.402 \\ -0.489 & -0.402 & 2.439 \end{bmatrix}, \tag{C.3}$$

$$S_{pk} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \tag{C.4}$$

$$S_{pu} = \begin{bmatrix} S_{pu}^1 & S_{pu}^2 & S_{pu}^3 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -4 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}. \tag{C.5}$$

These matrices satisfy all theoretical conditions imposed by the general model. The results presented below are based upon these matrices.

Recall from the discussion in the text that S_{pp} is the world net substitution matrix (aggregate of national net substitution matrices), which gives the effects of world price changes upon global net exports. The matrix specified in (C.1) indicates that goods 2 and 3 are net substitutes with good 1, while goods 2 and 3 are net complements with each other. The matrix S_{pt} describes the global effects of increases in carbon taxes t upon net exports of goods. The sign structure of the assumed matrix in (C.2) shows that an increase in the carbon tax on emission j has a positive effect on the net output of some products and a negative effect on the net outputs of other products.

Matrix S_{tt} describes the global effects of increases in carbon taxes t upon net carbon emission reductions. The specified matrix in (C.3) shows that an increase in the carbon tax in sector j reduces emissions in that sector but raises emissions in all other sectors. The equal and negative elements in the assumed vector S_{pk} in (C.4) indicate that global emissions have the same effects on world net exports for each good; greater global emissions k negatively affect utility in each country and so increase global compensated demands and reduce global compensated net exports of goods. The matrix S_{pu} of national income effects, specified in (C.5), has negative elements, meaning that all goods are normal in consumption in every country; the large negative term of -4 shows that country 1 has a very high marginal propensity to consume the second good out of income.

Based on these assumptions about the initial equilibrium, the shadow price vector $\widetilde{p}$, given in (34), and the vector $\widetilde{\beta}$ with element j given by $\widetilde{\beta}^j = \widetilde{p}^{\intercal} S_{pu}^j$ (and given in (33)) are computed to be

$$\widetilde{p}^{\intercal} = \begin{bmatrix} 1 & -1.394 & 1.933 \end{bmatrix}, \tag{C.6}$$

$$\widetilde{\beta}^{\intercal} = \begin{bmatrix} 2.645 & -1.539 & -1.539 \end{bmatrix}. \tag{C.7}$$

Since the vector $\widetilde{\beta}$ in (C.7) has elements of different sign, Corollary 1 implies that there exists a multilateral transfer of income that is strictly Pareto-improving in welfare for this world economy. The direction of financial transfers is from country 1 to countries 2 and 3. All countries gain in welfare from the income transfers. □

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