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Three Questions Regarding Impulse Responses and Their Interpretation Found from Sign Restrictions

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Abstract

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Keywords

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Three Questions Regarding Impulse Responses and Their Interpretation Found from Sign Restrictions

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Abstract

When sign restrictions are used in SVARs impulse responses are only set identified. If sign restrictions are just given for a single shock the shocks may not be separated, and so the resulting structural equations can be unacceptable. Thus, in a supply demand model, if only signs are given for the impulse responses to a demand shock this may result in two supply curves being in the SVAR. One needs to find the identified set so that this effect is excluded. Granziera el al's (2018) frequentist approach to inference potentially suffers from this issue. One also has to recognize that the identified set should be adjusted so that it produces responses to the same size shock. Finally, because researchers are often unwilling to set out sign restrictions to separate all shocks, we describe how this can be done with a SVAR/VAR system rather than a straight SVAR.

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1 Introduction

Finding the impulse responses for shocks identified by suitable parametric restrictions is a well defined operation in structural vector autoregression models (SVARs). Doing so with sign restrictions is more problematic because the impulse responses that result are only set identified, *i.e.*, there is more than one impulse response to consider. There have been numerous proposals to choose a single member of the set by using extra criteria, such as choosing the largest. But generally the identified set or characteristics of it – such as the median or the spread between some quantiles – are presented. In this paper we argue that three questions arise over how one finds and interprets the impulse responses when existing methods to find identified sets are used.

To appreciate that there can be a problem consider the typical way that an identified set is generated. An initial matrix of impulse responses R is selected with the property that $R'R$ equals the covariance matrix of the data (or the VAR residuals) and then new impulse responses are generated as RQ, where Q is orthogonal. This means that the new responses also match the covariance matrix. By simulating many Q, and retaining the impulse responses RQ that satisfy the sign restrictions, one eventually ends up with the "identified set".

One well known issue with this approach was set out in Baumeister and Hamilton (2015). They observed that a characteristic of the identified set such as the median could be changed by varying the way that Q is simulated. Mostly however Q has been generated from a uniform density. When Q is taken to be a Givens matrix it is a function of cosine and sine terms that depend upon a parameter λ that lies between zero and π, so simulating it from a uniform density over $(0, \pi)$ seems unexceptional. In Ouliaris and Pagan (2016) we noted this point of Baumeister and Hamilton's and suggested that one use the maximum and minimum of the responses, as that would be the limits of the identified set. One would expect this to be useful. Baumesiter and Hamilton (2018) however point out there are still many papers which report a characteristic of the identified set such as quantiles, even though they feel that "...a researcher should not be reporting point estimates or quantiles of a distribution, but should instead describe ... the "identified set." Because every QR replicates the data there is no reason to choose one quantile over another, although from the frequency of use by researchers it is clearly believed that there is some useful information in computing them.

So the question is how to find an identified set when there is just a single shock to be isolated? In the description above one recovers as many shocks as observed variables. So, when there is a SVAR with n variables, there are n shocks and $M = RQ$ is an $n \times n$ matrix.[1] When a single shock is of interest, traditionally only sign restrictions on the impulse responses for it are stated and used to determine the Q to be retained. Thus, calling the first shock "demand" in a two variable SVAR arranged as price and quantity would produce the restrictions $M_{11} > 0, M_{21} > 0$ for a positive demand shock, where M_{ij} are the contemporaneous impulse responses. But do we know that the first shock in an SVAR is demand? Maybe it is the second shock. This raises the question addressed in Section 3 – where in the SVAR is the single shock we are looking for, and does it matter?

A more important question comes up in Section 4 relating to the focus on impulse responses. In Ouliaris and Pagan (2016) we argued that one might want to work with the SVAR structural equations, *i.e.*, $Ay_t = \varepsilon_t$. Given coefficients for these one can find $M = A^{-1}$. Some of the coefficients in A can be simulated and others estimated, and M can then be found and retained if the sign restrictions are satisfied This method was called SRC – *sign restrictions from coefficients* – versus the one above that was termed SRR – *sign restrictions by recombining impulse responses*. To relate the two we have $A = M^{-1} = (RQ)^{-1} = QR^{-1}$, so we can think about simulating some of the coefficients of the structural equations by using QR^{-1}.

The SRC perspective makes one ask what do the structural equations for a given M look like, and are they acceptable? So the emphasis is not just on the signs of impulse responses. In particular, we might ask what are the structural equation implications if the M have multiple shocks with the same signs. This "multiple shocks" problem was raised by Fry and Pagan (2011). It does not come up if a complete set of sign restrictions are set out to separate the nominated shocks in the system, but when only the signs are given for a single shock it can clearly arise. The reaction to this criticism seems to have been a bit like Uhlig (2017, p. 113) "For those that have bothered to read their paper: there really is no "multiple shocks" or "multiple models" issue, given the Bayesian approach".

To see that there is an issue, we consider the structural equations that can emerge when there are multiple shocks. In the context of a 2 equation demand/supply model, we show that when there are two demand shocks there

[1] All our discussion will be the contemporaneous impact of shocks so we ignore dynamics.

must be two supply curves in the model, and this seems to be an economic issue that is not solved by Bayesian analysis.[2] It occurs because it is not enough to simply state sign restrictions for a single shock. Rejecting the Q that produces this outcome would seem to be important and this is done by looking at the underlying structural equations. If one rejects such Q there is a different identified set, and we show by simulation methods that this can provide quite different information about the range of possible values for impulse responses.

Section 5 asks a question that was the concern of our 2016 paper. Impulse responses found from sign restrictions are to one standard deviation shocks. So what is the magnitude of these standard deviations? As we observed in Ouliaris and Pagan (2016) the standard deviation varies with Q. Accordingly, until one chooses a single model (a single Q), a big impulse response may just mean a big shock. One presumably wants to correct for the size of shocks when presenting impulse responses to those advising policy makers – if you have a target to achieve you want to find how great a variation in the instrument is needed, and that requires knowledge of the impact of a one-unit shock. Doing this will produce a different identified set. Therefore, there are a number of identified sets depending on what information is being conditioned on.

This paper initially uses a simple experiment involving a two variable demand-supply model. Two versions of this model are employed. Following Granziera *et al.* (2018) the first has a negative covariance between quantity and price in the data generation process (DGP), while the second by design has a positive covariance. In the first case, although only the signs of one shock are given, there is shock separation owing to the nature of the chosen R, so the issue raised in Section 3 disappears. But it is present in the second model we use. It would seem desirable that one gets shock separation without depending on a particular R and Q. This involves setting out a complete set of sign restrictions.

Section 6 examines a three variable New Keynesian type SVAR and finds the same results as we found in the demand-supply models. However, now it is much more complex to decide on what structures might be rejected. It is shown that what would seem to be plausible signs for the *structural equation*

[2] Of course these may be two upward sloping demand curves or one of those and a supply curve. However, we will generally refer to this case as giving rise to two supply curves or unacceptable structural equations.

coefficients are often incorrect when one does not have a complete set of sign restrictions. Inspired by how parametric models manage to separate shocks when only one is of interest, we utilize the SRC perspective to suggest a similar procedure, which accommodates researchers' wariness about providing a complete set of sign restrictions. Lastly, the conclusion discusses which of the identified sets is the most appropriate and what might be the best way to proceed.

2 The Demand-Supply Structure Used

Granziera *et al.* (2018) consider inference in sign restricted SVARs for a single shock in a demand/supply context. So we focus on their paper. Their model is one of demand and supply, and the data are arranged as price and quantity in a VAR with shocks u_t. They write these as[3]

$$u_t = \Sigma_{tr} Q \varepsilon_t \tag{1}$$

where Q is an arbitrary orthogonal matrix (they call this Ω_ε). The ε_t have unit variances and are structural shocks. These are what we will call *listed shocks.* Their description is only as the first and second shocks of the SVAR.

Then we can write the MA representation for the VAR as

$$y_t = \sum_{j=0}^{\infty} D_j u_{t-j},$$

where D_j are the responses of the variables to the VAR shocks. Using (1) the VAR becomes

$$y_t = \sum_{j=0}^{\infty} D_j \Sigma_{tr} Q \varepsilon_{t-j},$$

and so the structural impulse responses to listed shocks are $D_j \Sigma_{tr} Q$. They then proceed by assuming that the first SVAR shock is a positive demand shock. Now a demand shock is what we will refer to as a *named shock,* and it would be expected to have positive effects on both price and quantity. The second (named) shock is supply. Their mapping between listed and named shocks is that the first listed shock is demand. Hence, with M being the

[3] We later just follow their experiment 1 where there are no lagged variables, but we still refer to these as VAR shocks.

contemporaneous responses of variables to shocks, they require $M_{11} > 0$ and $M_{21} > 0$ and that the demand shock is positive.[4] From this they produce a spread of impulse responses $M = \Sigma_{tr} Q$ by simulating Q from a uniform density. To get Σ_{tr} they use the impulse responses found by ordering price before quantity in an SVAR.[5] We will refer to this as the listed shocks strategy, and the identified set found from it, *i.e.*, the retained impulse responses that have $M_{11} > 0$ and $M_{21} > 0$, as the *listed shock identified set* (LIS).

In order to be precise about the issues it is useful to set up and work with a demand/supply system in the form used by the SRC method. The demand equation will be the first one and supply the second. Using assumed coefficients for the demand and supply curves, the system has the form

$$Ay_t = \eta_t \tag{2}$$

$$A = \begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix}, \tag{3}$$

where η_t are named demand and supply shocks given the nature of the structural equations. The contemporaneous impulse responses to the named shocks are $M = A^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ 0.75 & 0.25 \end{bmatrix}$, and these are clearly separated.

2.1 Where Are the Named Shocks and Does that Matter?

In the demand/supply system with named shocks the first one is demand and the second is supply. Is this true of the SVAR system? Testing only those Q that produce $M_{11} > 0$ and $M_{21} > 0$ is in effect assuming they are. But is this correct?

To look further at this we compute the covariance matrix for price and quantity, $V = A^{-1}A^{-1\prime}$, for our supply/demand model. Ordering price before quantity, we obtain an initial set of responses Σ_{tr} using a Cholesky decomposition of V to simulate 10 Q's, and thereby compute 10 contemporaneous

[4] Unless stated otherwise, we will always be assuming positive shocks in what follows.

[5] We need to make it clear that the SVAR we are dealing with has price and quantity as the first two variables. They are *arranged* that way, and this arrangement stays throughout the paper. Ordering is just a convenient way of saying how one gets a variety of values for Σ_{tr}.

impulse responses $M = \Sigma_{tr}Q$.[6] The *modus operandi* for getting an identified set is to accept those Q for which the signs are satisfied. Granziera *et al.*, use a listed shocks strategy to construct a LIS by retaining a Q only if $M_{11} > 0$ and $M_{21} > 0$. But is that what we would get if we looked at named shocks, *i.e.*, the *named shock identified set (NIS)*?

Turning to the first of the 10 simulated $Q's$ we get $M = \begin{bmatrix} 0.29 & 0.02 \\ -0.12 & 0.73 \end{bmatrix}$. This does not produce $M_{11} > 0$ and $M_{21} > 0$, so it would not be retained. The problem is that the demand shock is not the first listed shock in the SVAR but the second. So we need to search over the columns of M in order to see if there is an SVAR shock with the required signs. That is, does the demand shock appear in some column other than the first? If so, we then *rotate* M so that the demand shock appears in the first column of M. Hence the NIS is based on finding impulse responses such that *either* $M_{11} > 0, M_{21} > 0$ *or* $sgn(M_{12}) = sgn(M_{22})$. This means that the NIS may be different from the LIS, as it has more valid impulse responses.

Counting the number of acceptances of the 10 $Q's$, including the two cases of two demand shocks, the LIS will have six members while the NIS has ten. So valid impulse responses are being dropped under LIS. The question becomes what is the quantitative impact of doing so? At best it could just mean that many more $Q's$ need to be generated for the LIS to match the NIS. However, it may also be that one is throwing away valid impulse responses, and this can fundamentally affect characteristics of the two identified sets.

We did an experiment with 100000 $Q's$ for our demand and supply model in (2)-(3). The lengths of the LIS and NIS are equal, but the 20th percentile for LIS is .31 while that for the NIS is 41. So, if quantiles are important, one needs to rotate M to find where in the SVAR the demand shock is.

2.2 Do Multiple Shocks Matter: Thinking in Terms of Structures

In the simulations the second of the 10 simulated Q produces $M = \begin{bmatrix} 0.35 & 0.03 \\ 0.30 & 0.73 \end{bmatrix}$. This satisfies $M_{11} > 0$ and $M_{21} > 0$, but there are two shocks here that could be called demand, so which do we choose? For the 10 Q this happens twice. The quantitative differences in the columns are quite marked, so it may mat-

[6] Q was simulated using a QR decomposition as in Arias *et al.* (2018).

ter which one we keep. It would seem that that researchers would retain the first of these and we will do that here as well, although one would wonder if using an SVAR with two demand shocks is problematic.

This issue has been recognized by a number of authors. In this context, Kilian and Luktepohl (2018) say "Simply choosing the first structural shock to be the demand shock is arbitrary." and conclude that "...we need to verify that the signs in the second column are the complement of the signs in the first column." This is the opposite to what Uhlig (2017, p. 119) says, namely "They require, though, that additional shocks can be named and their impact on the variables at hand can be signed. Recall the first two Principles 1 and 2: if you know it, impose it; otherwise do not!". So Kilian and Luktepohl favour a complete sign matrix, not just describing those for a single shock, while Uhlig seems less convinced. Kilian and Luktepohl go on to argue that the issue is not as important in higher-dimensional models because the other columns giving extra sign restrictions imply than the shock of interest could have many different signs. But an inability to be able to prescribe signs for all the shocks does not mean that the issues coming from multiple shocks fade away.

What are the structural equation implications of having two demand shocks? With two demand shocks the generated M has $sgn(M) = \begin{bmatrix} + & + \\ + & + \end{bmatrix}$. The structural equations can be found from M^{-1}. For one of these to be a demand curve, the signs of the coefficients in one of the rows must be the same. However with the M above we get

$$sgn(M^{-1}) = sgn(det(M)) \begin{bmatrix} + & - \\ - & + \end{bmatrix}.$$

Thus the signs of the coefficients in the rows of M^{-1} are different. This means that the two structural equations are actually supply curves. In order to have two demand curves one needs the columns of M^{-1} to have the same signs. The only way to ensure there is one demand curve and one supply curve is to insist on enough signs to separate the shocks, as argued in Fry and Pagan (2011) and Kilian and Lutkepohl above. Uhlig's claim that multiple models are not an issue for Bayesians seems odd.

Now the length of LIS/NIS for our demand/supply model DGP in (2)-(3) when two demand shocks are permitted, *i.e.*, where there are two supply curves curves, is 0 to .77. When we eliminate the structures with two supply curves we get a new identified set - LIS/ES - of length .35 to .77. Here the identifier ES signifies "excluding invalid structural equations". So the identified set changed a great deal in this case.

What happens if we look at the model used in Granziera *et al.* (2018)? We find there is no difference in the LIS length or characteristics of it such as the median. To understand why this happens consider what their $\Sigma_{tr}Q$ looks like. In their experiment they set Σ_{tr} to

$$\Sigma_{tr} = \begin{bmatrix} \Sigma_{11}^{tr} & 0 \\ \Sigma_{21}^{tr} & \Sigma_{22}^{tr} \end{bmatrix} = \begin{bmatrix} 0.597 & 0.0 \\ -0.205 & 0.812 \end{bmatrix}.$$

With a Q generated by a Givens matrix,

$$Q = \begin{bmatrix} \cos\lambda & -\sin\lambda \\ \sin\lambda & \cos\lambda \end{bmatrix},$$

the impulse responses they focus on will be

$$\begin{aligned} M_{11} &= 0.597cos(\lambda) \\ M_{21} &= -0.205\cos(\lambda) + 0.812\sin(\lambda). \end{aligned}$$

If $0 < \lambda < \frac{\pi}{2}$, $M_{11} > 0$. while $M_{12} > 0$ for $0.08\pi \leq \lambda \leq \frac{\pi}{2}$. Consequently, over the latter range for λ,

$$\begin{aligned} M_{12} &= -0.597\cos(\lambda) < 0 \\ M_{22} &= 0.205\sin(\lambda) + 0.812\cos(\lambda) > 0. \end{aligned}$$

That is, $\Sigma_{tr}Q$ always has the signs $\begin{bmatrix} + & - \\ + & + \end{bmatrix}$ and the shocks are therefore separated by their choice of Σ_{tr}.

For our supply/demand system in (2)-(3) the shocks are *not separated by design*, as we have seen from simulations of M. This implies that there is a need to retain only those Q that separate the shocks to avoid situations like multiple supply curves. Uhlig's contention that one does not need to do that would raise questions regarding the underlying economics of the system being used to identify the impulse response set.

The key difference between our demand/supply system and that used in Granziera *et al.* (2018) is in the price/quantity covariance matrices. In our case it is $\begin{bmatrix} 0.125 & 0.125 \\ 0.125 & 0.625 \end{bmatrix}$, while in their case it is $\begin{bmatrix} 0.3564 & -0.1224 \\ -0.1224 & 0.7014 \end{bmatrix}$.

So the covariance between price and quantity is positive in one case and negative in the other.

In general one cannot assume that specifying the signs for a single shock will in fact imply that the shocks can be separated. While it did happen with the supply/demand system used in Granziera *et al.* (2018), it need not happen in others, such as ours. But if you don't separate the shocks, the magnitude of impulse responses found from the identified sets can be quite different. Moreover, one can see that this will be an important issue when the number of variables in the model increases, since specifying a complete set of signs can become very difficult in larger models.[7]

Granziera *et al.* (2018) constructed their experiment in order to illustrate their method for doing frequentist inference with sign restrictions. The first step of this was to find quantiles in the identified set, and to then invert them to get a percentile range for what was essentially λ. This was then used to infer a bound on the percentile range for the estimated impulse response functions. This methodology works in their example because they have shock separation owing to their choice of Σ_{tr}.

However, the method is meant to be general and not to be specific to their case. So we look at how it would work if the DGP was our demand/supply model. We start with a Σ_{tr} that comes from price being ordered before quantity, which is what they used.[8] Then we look at (i) the 20th percentile of the identified set when one only uses signs from the demand shock and (ii) what it is when one deletes two supply curves. These are .31 and .50 respectively, so they are quite different, meaning that first step of the frequentist approach used in Granziera *et al.* (2018) depends on whether one filters out systems with two supply curves and no demand curve, even though the focus is on demand shocks. It would seem better that they specify enough signs to separate the shocks in the first stage of their approach.

[7] Liu *et al.* (2018) used a DSGE model - the Reserve Bank of Australia's Multi-sector Model (MSM) - to do this. They were interested in comparing what the impulse responses from the point estimates of the MSM model were with the identified set.

[8] Ordering quantity before price to get a Σ_{tr} produces the same results.

3 The Impulse Responses Need To Be To The Same Size Shocks

To understand the issue suppose we start with p ordered before q. This means fitting the recursive model

$$\begin{aligned} p_t &= e_{1t} = \sigma_1^e \varepsilon_{1t} \\ q_t &= \alpha p_t + e_{2t} \\ &= \alpha p_t + \sigma_2^e \varepsilon_{2t} \end{aligned}$$

where $\{\varepsilon_{jt}\}_{j=1}^2$ have unit standard deviations. Notice that when one changes ε_{1t} by one unit you are administering a shock of σ_1^e, *i.e.*, a one standard deviation shock. *The shocks that produce the impulse responses found from sign restrictions are for one standard deviation shocks not to one-unit shocks.* This was very clear from Uhlig's original work but seems to be ignored by most researchers, who treat them as responses to unit shocks. Fry and Pagan (2011) argued that one needed to know the standard deviations.

Ouliaris and Pagan (2016) observed that the standard deviation of shocks can be found by using parametric restrictions and normalizing equations. So they proposed doing the same with the structures underlying the sign restricted impulses. Consequently, for a system that had

$$M = \begin{bmatrix} 0.3985 & -0.3637 \\ 1.4835 & 1.0964 \end{bmatrix},$$

the first equation of the structure implied by M^{-1} is

$$1.1228 p_t + 0.3725 q_t = \varepsilon_{1t}.$$

Normalizing on price we get a standard deviation for the demand shock of $\frac{1}{1.1228} = 0.891$. Of course one could also normalize on quantity, and that will give a different answer. But this situation is no different than in any parametric model. The standard deviation of the shock of an equation is in the context of a model.

So every Q we simulate implies a specific standard deviation for the shocks. This means that, when assessing the magnitude of impulse responses, we need to control for the fact that the standard deviations of the shocks differ with each draw. In the context of the Granziera *et al.* (2018) model, the length of the identified set for size-corrected impulse responses is 0 to .72 versus 0 to .82 for non size-corrected responses. Because their DGP and choice of Σ_{tr} performs shock separation, the length and characteristics of the set remain the same.

Turning to our supply/demand model we find that the length of the identified set with no size correction is 0 to .79, while it becomes 0 to 1.25 when there is size correction. Moreover, the 20th percentile of this changes from .34 to .54 if we separate the shocks. Hence if we were to use Granziera *et al.'s* strategy for doing frequentist inference we would need to do their first step with an identified set for size-corrected shocks that are separated with enough sign restrictions.

4 Looking At A New Keynesian Type SVAR

The implications of the above would seem to be to use enough signs to separate shocks, to ask whether the structural equations underlying the resulting aimpulse responses seem appropriate, and to size-adjust the responses. It is therefore of interest to investigate what type of issues might arise with these recommendations when there are more than two variables is the SVAR.

In Ouliaris and Pagan (2016) we looked at a three variable system in the output gap, the inflation rate and the Federal Funds rate. With the variables arranged that way we used the following complete set of sign restrictions on the impulse responses for positive shocks

$$sgn(M) = \begin{bmatrix} + & + & - \\ + & - & - \\ + & + & + \end{bmatrix}. \tag{4}$$

The third structural shock is the monetary shock and so $M_{13} < 0, M_{23} < 0$ and $M_{33} > 0$. Often only the monetary shock is of interest, and so the signs of the third column are used to describe that shock. Stating only that condition, however, would not differentiate the shocks.

4.1 Shock Separation and Structural Equations

When only the signs for one shock in (4) are given we need to ask what this means for the structural equations. Multiple shocks can clearly arise, and this can create the equivalent of two supply curves in the system. But even if there are not multiple shocks there may be shocks thar are hard to interpret, and these can impact in a similar way.

To do some analysis of this issue we need a covariance matrix for the data. In Ouliaris and Pagan (2016) we used a data set from Cho and Moreno (2006) and we do so again here. The variables are arranged as the output gap, inflation and the Federal Funds interest rate. Turning to what the structural equations might look like, for a New Keynesian DSGE model one would expect their coefficients to have signs like

$$sgn(A) = \begin{bmatrix} + & - & + \\ - & + & + \\ - & - & + \end{bmatrix},$$

(where we normalize on the variables as they are arranged). That is, the first equation has output depending positively on inflation and negatively on the nominal interest rate; the second equation has inflation rising with the output gap and declining with the interest rate; while the third one has a standard monetary rule that the nominal rate rises in response to inflation and the output gap. So there should be a monetary rule like the first column in the structural system. However, the other equations are trickier since the standard NK DSGE structural equations are not the same as the structural equations in its underlying SVAR, because the DSGE model has forward-looking expectations, and these are a function of the current variables.

Hence the signs of A in the implied SVAR from a DSGE model are more complex. Indeed, Pagan and Robinson (2016) found that the SVAR coming from the MSM's external sector had the interest rate with a *positive* coefficient in the inflation equation. Accordingly, $sgn(A_{23})$ could be negative rather than positive. It might be set to zero, but then one is introducing a parametric restriction. That can easily done with SRC, but we want to focus only on signs, so we will allow $sgn(A_{23})$ to be positive or negative.

A key insight from above, however, is that one *should not have the same signs for structural equation coefficients* in either of the first two rows. If any of these two rows has the same sign for all coefficients then it would be neither a Phillips nor an IS curve, and we would presumably want them in the system.

To see why this is important we look at the impulse responses found with one simulated Q:

$$M = \begin{bmatrix} 2.8 & 1.1 & -1.65 \\ 0.82 & -1.89 & -.56 \\ 1.04 & 0.15 & 0.37 \end{bmatrix}.$$

Clearly, based on the signs, the shocks are *separated.* $M_{13} < 0, M_{23} < 0$ and $M_{33} > 0$ makes the third one the monetary shock. The first shock looks like a demand shock, but the second one is harder to interpret, unless we decide that a positive productivity shock can have a positive effect on the Federal Funds rate, *i.e.*, even though inflation has declined the Federal Reserve raises rates due to the rise in output. [9]

To understand what these impulse responses mean for the underlying structural equations we invert M to get $A = M^{-1}$, producing

$$A = M^{-1} = \begin{bmatrix} 0.100 & 0.107 & 0.608 \\ 0.144 & -0.448 & -0.035 \\ -0.340 & -0.118 & 1.008 \end{bmatrix}.$$

After allowing for positive normalization, the sign pattern for the structural equation coefficients is

$$sgn(A) = \begin{bmatrix} + & + & + \\ - & + & + \\ - & - & + \end{bmatrix}.$$

Consequently, there is a question mark about the first equation of the structure. Here the shocks are separated, but they certainly are not interpretable along the line of (4). It is this failure which produces a system that might not be acceptable.

It was considerations such as this that led us in our 2016 paper to propose that one should set up and estimate A directly and then check that A^{-1} had the desired signs. One could impose sign restrictions on the elements of A (by using SRC) in order to make the structural equations acceptable. As we said

[9] If one allows M_{12} to be either value this can create shock separation issues when the monetary shock signs are not completely specified. Thus in Uhlig's original work the only signs for the monetary policy shock were $M_{23} < 0$ and $M_{33} > 0$ i.e. a negative effect on inflation, and nothing provided for the output gap (although he did apply these to more than a contemporaneous response). So, if the marginal cost shock was $M_{22} < 0, M_{23} > 0$ and $M_{12} =?$, it could also be a monetary policy shock. Again this would be an example of a failure to separate shocks.

(p. 611 - 612) "It is worth observing here that, rather than looking at impulse responses, one might use sign restrictions on the structural parameters... An even more complex restriction might be to ensure that the Taylor principle for stability held."

When we look at the results for M_{13} based simply on the monetary shock being the third in the SVAR, we find that the LIS length is 0 to -2.11, and this is also true of the NIS that allows the shock to be elsewhere in the SVAR. As before, it is simply a matter of how many simulations one does. When we delete structural equations whose coefficients have the same signs then the LIS/ES length is slightly changed. However, the median goes from -.99 to -1.2, and this shows up in the quantiles. So the characteristics of the LIS/ES are significantly changed. If one imposes a complete set of signs to ensure shock separation, one finds that the median of M_{13} is -1.29, quite different to the LIS. So, just as in the 2 variable case, using a complete set of signs would seem the best way to proceed.

Lastly, size-correction can also be done, since the standard deviation of the interest rate shock can be found by normalizing on the interest rate in the monetary rule. In this case the LIS and the NIS lengths change radically – from 0 to -2.11 to 0 to -.6. Once we also use a complete set of sign restrictions this shrinks further – now from 0 to -.2.

4.2 An Analogue of Parametric Estimation

Reference to uninterpretable shocks above suggests that one might not want to construct responses using RQ. The latter produce responses to uncorrelated shocks and those are essentially structural shocks. It is this that enables us to ask if they are interpretable.

If they were correlated, however, then that would not be the case, which points to the fact that we may want to generate M so that only one shock is interpretable. This seems more in the spirit of what those concentrating on a single shock are trying to do. Basically, this is what happens when we find a single shock using parametric restrictions. Consequently, we ask if that solution can be emulated, *i.e.*, we wish to find a single shock whose impulse responses agree with only a partial list of signs. We therefore do not want complete shock separation.

The SRC orientation is a good way to think about this. Suppose we write down a system composed of a structural equation that is a monetary rule and the VAR equations for the gap and inflation. The VAR errors for the latter are $\{e_{jt}\}_{j=2}^{3}$ and they can be written as $e_{jt} = \rho_j \varepsilon_{1t} + v_{jt}$, where ε_{1t} is the monetary shock and v_{jt} are shocks uncorrelated with ε_{1t} but themselves correlated. The system is then a semi-structural VAR of the form[10]

$$
\begin{aligned}
i_t &= \alpha y_t + \beta \pi_t + \varepsilon_{1t} \\
y_t &= \rho_1 \varepsilon_{1t} + v_{2t} \\
\pi_t &= \rho_2 \varepsilon_{1t} + v_{3t},
\end{aligned}
$$

with i_t as the interest rate, π_t the inflation rate and y_t the output gap. If α and β are simulated the remaining parameters can be estimated, as this is then an exactly identified system. Simulating α and β is like simulating RQ, so we get a set of impulse responses.

Once values of these parameters are found we can get impulse responses to the shocks ε_{1t}, v_{2t} and v_{3t}. Specifically, the response of the gap to a unit monetary shock is ρ_1. It is important to observe here that there is only one known structural equation in the system. We would reject simulations of α and β that are not positive, as that would mean a structural equation that is an unacceptable monetary rule.

So what is the difference between this and what SRR produces? In the SRR procedure the RQ are simulated *under the assumption that all shocks are uncorrelated*. In the method above however, although ε_{1t} is made uncorrelated with v_{jt}, the other two shocks v_{jt} are not uncorrelated.[11] Hence, they are not structural shocks, and the system is just a single structural equation augmented by VAR equations. In such a formulation one does not work with a sign matrix to completely separate shocks and so that seems closer to the agnosticism of researchers.

To see how the approach works we apply it to the covariance matrix of the Cho-Moreno data, and find that the response of the gap to a one-unit monetary shock lies in the range 0 to -.6, just as we found for LIS and NIS in the previous sub-section. If we look at the identified set for this parameter when the shock is not size corrected it is now 0 to -1.96, shorter than the 0 to -2.11 before. Simulating α and β without restricting them to be positive one gets the longer set, so some of the impulse responses in the latter come from having an unacceptable monetary rule in the system.

[10] If there are dynamics one just adds lags into these equations.

[11] Where there are only two variables there are only two shocks and so they must be structural. They just need to be separated, and this is done in SRC by ensuring that there is a demand and a supply curve.

5 Conclusion

The paper has shown that one needs to be precise about the identified parameter set, as typically there is more than one. Which then is the most relevant?

In the Granziera *et al.* (2018) paper a method is provided for doing frequentist inference on sign restrictions in SVARs. They first begin by finding the identified set for Q based on the sign restrictions for a single listed shock, namely $M_{11} > 0$ and $M_{21} > 0$, and a given Cholesky factor Σ_{tr}. Once they have that they do frequentist inference on M_{12}. It is certainly true that they recover the identified set that is consistent with this information. However, if we change the information set so as to separate the shocks, the identified set changes and its length is quite different.

Consequently, our belief is that the identified set they work with is not the correct choice for empirical researchers. It is a strategy that may work for some DGP's, such as in the specific demand/supply model they had, but in others it does not. It did not work for ours. We would argue that their method of frequentist inference requires one to work with an identified set that is robust to the unknown DGP.

More broadly, what do we recommend regarding the problems that can arise from multiple shocks of the same type? These show up as unacceptable structural equations in the SVAR. One way to address this problem is to eliminate multiple shocks by using enough sign restrictions to separate the structural shocks. However, researchers seem loath to do this. Consequently, we have suggested that the system to be worked with should not be a complete SVAR, but composed of (a) structural equations only for the shocks whose responses researchers are prepared to sign and (b) VAR equations for the remaining variables. This system can be implemented easily using the SRC method in our 2016 paper. It enables one to ensure that there are acceptable structural equations as well as impulse responses with the desired signs.

Another issue that we highlighted was that those advising policy makers need to know the size of the shocks embedded in a model, and one needs to size correct the shocks before looking for an identified set. This is also true of those doing (say) cross country comparisons of impulse responses. It may be that the standard deviations of (say) the fiscal shocks are different in one country than another, and so the different magnitudes of impulse responses may just reflect that. As shown in the examples, size correction can make a big change to the quantitative magnitudes. It therefore does seem to us that one would want to construct identified sets that correct for size.

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