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US shocks and the uncovered interest rate parity

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Abstract

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Keywords

Time-varying parameter, Stochastic volatility, Model uncertainty, Exchange rate, Uncovered interest rate parity

JEL Classification

C11, C32, F31, F37

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U.S. shocks and the uncovered interest rate parity

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The literature on uncovered interest rate parity (UIP) shows two empirical puzzles. One is the failure of UIP, and the other is the unstable coefficients in the UIP regression. We propose a time-varying coefficients model with stochastic volatility and US structural shocks (TVC-SVX) to study how US structural shocks affect time-variation in the bilateral UIP relation for twelve countries. An unconditional test and a conditional test for UIP are developed. The former tests if UIP coefficients mean-revert to their theoretical values, whereas the latter tests coefficients at each point in time. Our findings suggest that the failure of UIP results from omitted US factors, in particular US monetary policy, productivity and preference shocks, which are also found to Granger cause local movements of UIP coefficients.

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1. Introduction

Uncovered interest rate parity (UIP) states that, under rational expectation and no arbitrage, expected returns from the investment in two economies should be equal once returns are converted into the same currency. UIP implies that positive (negative) interest rate differentials should predict bilateral nominal exchange rate depreciation (appreciation). A vast literature that empirically tests UIP usually replies on the following regression model of Fama (1984),

$$\Delta s_{t+q} = \alpha + \beta(i_{t+q} - i^*_{t+q}) + \epsilon_{t+q}, \tag{1}$$

where Δs_{t+q} denotes the q-month log return of exchange rate, $i_{t+q} - i^*_{t+q}$ is the q-month interest rate differential and $E_t(\epsilon_{t+h}) = 0$. Based on equation (1), UIP implies a null hypothesis $H_0 : \alpha = 0$ and $\beta = 1$. Statistical evidence in numerous papers points to a nonzero α and negative β (see *e.g.* Engel, 2014 for a review) – a phenomenon many term as *the UIP puzzle*. Other literature including Backus et al. (2013) and Ismailov and Rossi (2018) finds unstable UIP coefficients which can be attributed to time-varying risk premium in the currency market according to the early paper by Fama (1984).

This paper aims to address the role of US structural shocks as common drivers behind the time-variation of UIP coefficients across a panel of countries. Many papers show that there exist some common factors driving the currency risk premium (see *e.g.* Taylor, 2001; Gagnon and Ihrig, 2004; Backus et al., 2013; Corsetti et al., 2008; Rabitsch, 2016; Lustig and Verdelhan, 2007; Lustig et al., 2014), thus affect movements of UIP coefficients. We make explicit these driving forces by letting US shocks partially lead the local movements of coefficients. The basic logic of our modelling is that US dollars dominate cross-border transactions, thus it is natural to conjecture that the spillover of US shocks may drive the time variation in UIP relations.

Different from the majority of studies on UIP which reply on static models, we propose a time-varying coefficient model with stochastic volatility and US structural shocks (TVC-SVX) to study: (i) the time-variation of UIP coefficients; (ii) the common forces driving co-movements in time variations; and (iii) whether or not bilateral UIP relations hold over the whole sample period and at a certain point in time. Briefly, the TVC-SVX model considers UIP coefficients as mean-reverting latent stochastic processes driven by their own past, US structural shocks

as exogenous variables, and a non-US idiosyncratic innovation term. The role of US shocks is threefold. Firstly, controlling for them mitigates endogeneity which can bias coefficient estimate as US nominal and real shocks are found to affect the key currency (*i.e.* US dollars) risk premium (Chinn and Meredith, 2004; Backus et al., 2013). Secondly, they expand the information set for us to extract coefficient time-variations. Thirdly, instead of modelling US shocks as common factors, they explain variation of UIP coefficients additional to their own past in the TVC-SVX model, thus bear a Granger causality interpretation.

deKoning and Straetmans (1997), Craighead et al. (2010) and Ismailov and Rossi (2018) among others use rolling-window least squares to address coefficient instability in Model (1). This method assigns equal weight to observations within each window with a subjectively chosen size, and the inference on time-variation of estimates lacks theoretical guidance. We show that our TVC-SVX model provides an automatic and data-driven weighting scheme for estimating UIP coefficients at each point in time, taking time-varying uncertainty measured by SV, and the common US shocks into account. This framework directly allows for hypothesis testing for UIP coefficients. We develop two UIP tests based on marginal likelihood. One tests for the unconditional means to which coefficients mean-revert, which we term as unconditional UIP test, the other called conditional UIP test addresses coefficient values at a certain point in time. The unconditional test bears different content from the conditional test, as the former looks at the bilateral UIP relation over the whole sample period, whereas the latter checks UIP in a certain environment given the value of volatility, a measure of uncertainty (Bjørnland, 2009; Bloom, 2009), and US shocks.

The remainder of this paper is organized as follows. In Section 2, we introduce our TVC-SVX model and discuss motivations for letting US shocks to partially drive model coefficients, in comparison to the recent literature. In Section 3, we develop two UIP tests and show the model implied observation weighting scheme. In Section 4, we apply the model to 12 currencies and examine the time-variation of UIP relations. We conclude in Section 5.

2. Model and estimation

This section introduces our time-varying coefficients model with stochastic volatility and US structural shocks (TVC-SVX) which we use to study the time variation in bilateral UIP relations

and understand the common foreign forces behind time variations.

2.1. The TVC-SVX model

Let r_t and d_t denote the q-month log return of exchange rate Δs_{t+q} and q-month interest rate differential $i_{t+q} - i^*_{t+q}$, respectively. Throughout the study we focus on country-specific short-term UIP with $q = 3$, so for notational simplicity we suppress the dependence on a particular country and q. Following the UIP regression model of Fama (1984), for $t = 1, ..., T$, our TVC-SVX model is given by

$$\begin{aligned} r_t &= \alpha_t + \beta_t d_t + \exp\left(\frac{h_t}{2}\right)\epsilon_t, \\ f_{t+1} &= \mu_f(1-\phi_f) + \phi_f f_t + \gamma_f' x_t + \sigma\eta_{f,t}, \\ f_1 &\sim N\left(\mu_f, \frac{\sigma^2}{1-\phi_f^2}\right), \qquad f \in \{\alpha, \beta, h\}, \\ (\epsilon_t, \eta_{\alpha,t}, \eta_{\beta,t}, \eta_{h,t})' &\overset{\text{i.i.d}}{\sim} N(0, I_4). \end{aligned} \tag{2}$$

In the TVC-SVX model (2), UIP coefficients α_t, β_t and log-variance h_t are independent autoregressive distributed lag processes of order one, or ADL(1), with k zero-mean predetermined variables $x_t \in \mathbb{R}^k$. We estimate the model using Bayesian methods. We make two remarks.

Remark 1. The innovation variance of α_t, β_t and h_t being a common value σ^2 is a simple remedy to overfitting without imposing tight priors. The smoothness of f_t, $f \in \{\alpha, \beta\}$, depends on their signal-to-noise ratios $SNR_f = \sigma_f^2/\sigma_\epsilon^2$ with $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ in a TVC model. If SNR_f is too large, f_t captures more noise than signal, resulting in overfitting. Adding SV mitigates the issue during high-volatility periods, as $SNR_{f,t} = \sigma_f^2 \exp(-h_t)$ decreases with h_t. Under $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_h^2 = \sigma^2$, the smoothness of posterior paths of α_t and β_t is automatically balanced in a data-driven way. Because the tendency of overfitting or increasing $SNR_{f,t}$ due to big σ^2 leads to overshooting h_t which in return decreases $SNR_{f,t}$. Also, this specification does not restrict our TVC-SVX model where $SNR_{f,t} = \left(\gamma_f E(x_t x_t')\gamma_f' + \sigma^2\right) / \left(\gamma_h E(x_t x_t')\gamma_h' + \exp(h_t)\right)$ follows the *Johnson's SB* distribution (Johnson, 1949) *a priori*. Its first moment does not exist based on our flat priors summarised in Table 1. So we still allow for ample room for data to speak for the posterior behaviour of $SNR_{f,t}$.

Remark 2. Hartmann et al. (2010) and Mikosch and de Vries (2013) among others document

the presence of heavy tails in exchange rate returns and its effect on testing pricing equations including the one used by Fama (1984). Spronk et al. (2013) rationalises heavy-tailed innovations via carry trade activity. It is straightforward to extend TVC-SVX models to allow for heavy tails. One can replace the Gaussian error ϵ_t in (2) with a Student's t error $\epsilon_t^* = \sqrt{w_t}\epsilon_t$ where w_t is an inverse gamma mixing variable. We also estimate this model for all countries in the empirical study and find essentially no changes to our conclusion. Results are available upon request.

2.2. US structural shocks

Since US Dollar has been dominating international cross-border transactions, it is natural to conjecture that the spillover of US shocks may drive the time variation in country-specific UIP relations. To study the role played by US shocks, x_t collects standardised US structural shocks identified by the FRBNY DSGE-DFM model of Gelfer (2019). FRBNY DSGE-DFM augments the dynamic stochastic general equilibrium model used by the Federal Reserve Bank of New York (Del Negro et al., 2013) which builds on Smets and Wouters (2007) and features a financial accelerator as in Bernanke et al. (1999) with a dynamic factor model so that model variables are linear combinations of observables. For example, inflation can load on CPI, PCE, GDP deflator and other series. This data-rich environment robustifies the identified shocks from the original FRBNY DSGE model.

After estimating the model, eight structural shocks that drive the dynamic general equilibrium forces in the US economy are extracted. Figure 1 shows the US shocks that we use in our study. Notice that we do not differentiate between anticipated and unanticipated monetary policy shocks, because the channel of forward guidance during periods of zero-lower bound is muted for most of our sample period.

The ADL(1) specification in UIP coefficients (α_t, β_t) and log-volatility h_t in (2) serves four purposes.

First, US nominal and real shocks are main drivers behind the key currency (*i.e.* US dollars) risk premium, so controlling for them mitigates endogeneity. Implementing the TVC-SVX model country-by-country gives us a factor model which attributes systematic movements in UIP coefficients to US fundamentals. Ignoring SV, one may have a latent factor structure to

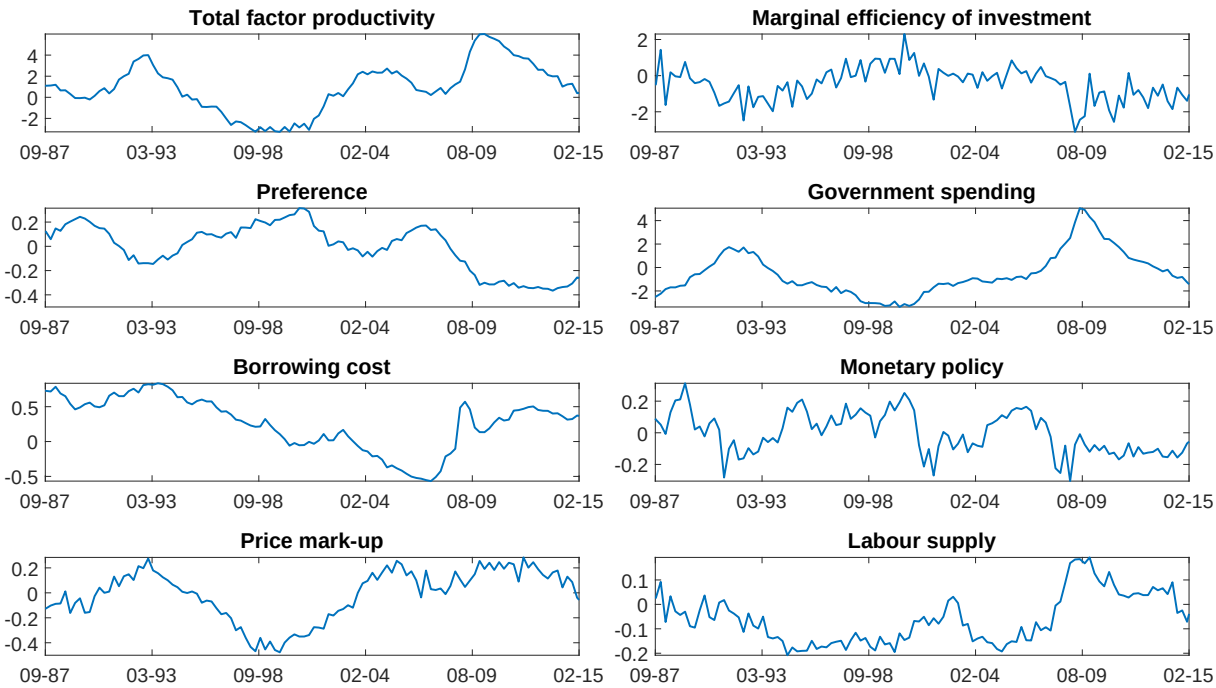

Figure 1: US structural shocks. Posterior median of US structural shocks estimated from the FRBNY DSGE-DFM model of Gelfer (2019).

country-specific UIP relations where factors capture key currency risk premium (Backus et al., 2013) and the world interest rate premium (Greenaway-McGrevy et al., 2018). It follows that

$$\begin{aligned} r_{i,t} &= \alpha_{i,t} + \beta_{i,t} d_{i,t} + \epsilon_{i,t}, \\ f_{i,t} &= \bar{f}_i + \Lambda'_{f_i} l_t + \eta_{f_i,t}, \qquad f \in \{\alpha, \beta\}, \end{aligned} \tag{3}$$

where i indicates a currency pair against US dollars. Understandably, l_t is closely related to US monetary policy, especially in the short-term (Taylor, 2001; Gagnon and Ihrig, 2004; Backus et al., 2013), productivity (Corsetti et al., 2008; Rabitsch, 2016) and consumption growth (Lustig and Verdelhan, 2007; Lustig et al., 2014). Through international channels including monetary policy coordination (Edwards, 2015; Bruno and Shin, 2015) and foreign direct investment (Klein and Rosengren, 1994; Barrell and Pain, 1996), the bilateral interest rate differential $d_{i,t}$ is correlated with l_t, biasing least squares estimates of UIP coefficients (or estimates from Kalman filter in a TVC model). So explicitly controlling for US shocks is not only intuitive, but also necessary to control for endogeneity.

Secondly, the ADL(1) dynamics separates the source of time variation into US shocks $\gamma'_f x_t$ and non-US innovations $\eta_{f,t}$, where the latter can be thought as mainly domestic. It allows for

a Granger causality interpretation (see *e.g.* Tian and Ma, 2010): if $\gamma_f \neq 0$, x_t Granger causes f_{t+1}; that is, US shocks predict UIP coefficients conditional on their own history, which is in hand with discussions on predictability of exchange rate through the lens of UIP and *purchasing power parity* fundamentals; see *e.g.* Rossi (2013) and Engel (2014) for reviews. We discuss two alternatives: (i) a simple regression with interaction terms; and (ii) an unobserved components model. Ignoring SV, we see that the first case arises if we directly follow (3) by attributing changes in α_t and β_t to US shocks only, *i.e.* $l_t = x_t$. This gives us a regression model with interaction terms

$$r_{i,t} = \bar{\alpha}_i + \bar{\beta}_i d_{i,t} + \Lambda'_{\alpha_i} x_t + \Lambda'_{\beta_i}(d_{i,t} x_t) + \epsilon^*_{i,t}, \qquad \epsilon^*_t = \eta_{\alpha_i,t} + \eta_{\beta_i,t} d_{i,t} + \epsilon_{i,t}, \tag{4}$$

which can be estimated via least squares, corrected for heteroskedasticity and autocorrelation. However, the inference on $(\bar{\alpha}_i, \bar{\beta}_i)$ is likely to suffer from endogeneity and small-sample bias found in literature on testing UIP (Chinn and Meredith, 2004; Chinn and Quayyum, 2012; Chen and Tsang, 2013) and such a model tends to generate noisy path of $f_{i,t} = \bar{f}_i + \Lambda'_{f_i} x_t$, $f \in \{\alpha, \beta\}$. In the second case, $f_{i,t}$ consists of an country-specific time-varying component and a US component; that is $f_{i,t} = \bar{f}_{i,t} + \Lambda'_{f_i} x_t$ with some stochastic process $\bar{f}_{i,t}$. This gives us

$$r_{i,t} = \bar{\alpha}_t + \bar{\beta}_t d_{i,t} + (\gamma'_{\alpha_i} + d_{i,t}\gamma'_{\beta_i}) x_t + \epsilon_{i,t},$$

which can be estimated using Kalman filter with time-variant system matrices (Durbin and Koopman, 2012). This model however does not grant US shocks with the Granger causality interpretation and does not allow past shocks to affect current UIP coefficients.

Thirdly, x_t helps identify latent time-variation in UIP coefficients by effectively expanding the information set. Compared with rolling-window least squares estimates of UIP coefficients (deKoning and Straetmans, 1997; Craighead et al., 2010; Ismailov and Rossi, 2018), inference based on TVC models utilise all data information and do not depend on window size and autocorrelation generated by rolling regressions. Although the flexibility of state space models necessarily complicates the identification of latent processes when effective sample size is small (Stock and Watson, 1998; Holston et al., 2017), we partially anchor the variation in UIP coefficients in (2) by x_t and avoid under- or overfitting *a priori*.

Table 1: Prior distributions

Parameter θ	Prior distribution $p_0(\theta)$	Parametrisation of $p_0(\theta)$
μ_α	$N(\alpha_0, v)$	$\alpha_0 = 0,\ v = 10$
μ_β	$N(\beta_0, v)$	$\beta_0 = 1,\ v = 10$
μ_h	$N(h_0, v)$	$h_0 = \log\left(\frac{1}{T}\sum_{t=1}^{T}(r_t - d_t)^2\right),\ v = 10$
$(\phi_f + 1)/2$	$Beta(a, b)$	$a = 20,\ b = 1.5$
γ_f	$N(\gamma_0, \Sigma)$	$\gamma_0 = 0,\ \Sigma = 10I_8$
σ^2	$IG(\vartheta, \delta)$	$\vartheta = 0.5,\ \delta = 0.5$

Subscript $f \in \{\alpha, \beta, h\}$ unless stated otherwise.

Lastly, specifying an ADL(1) model for h_t is motivated by McCurdy and Morgan (1991) and Alexius and Sellin (1999) which find a common volatility component in cross-sectional UIP regressions using GARCH-type models. Our model thus investigates if US shocks can generate comovement in exchange rate volatility, which serves as the TVC-SVX model interpretation of Dornbusch's *Overshooting Hypothesis* (Borgersen and Göcke, 2007; Bjørnland, 2009). Without explicitly modelling correlation terms among $\eta_{\alpha,t}$, $\eta_{\beta,t}$ and $\eta_{h,t}$, the presence of x_t in three latent processes correlates time-varying risk premium with volatility in a natural way.

2.3. Bayesian estimation

The TVC-SVX model (2) is a conditional linear Gaussian state space model with predetermined regressors x_t. This conditional structure enables us to design a Markov chain Monte Carlo (MCMC) algorithm that iteratively samples h_t and (α_t, β_t) to generate the full conditional posterior distribution of UIP coefficients as well as system parameters whose prior distributions are non-informative throughout our empirical study. We summarise the priors $p_0(\cdot)$ in Table 1. Estimation via MCMC is standard and thus left in the supplementary appendix.

3. UIP tests and accounting of time-variation

We propose two tests for UIP using Bayes factor (BF) based on Savage-Dickey density ratio ($SDDR$) (see *e.g.* Kass and Raftery, 1995). An unconditional test and a conditional test are developed to examine UIP in the long-run and at a certain point in time, respectively.

3.1. Two tests based on the TVC-SVX model

The unconditional UIP test builds on the rationale that under rational expectation no arbitrage holds in equilibrium and thus UIP parameters converge to their unconditional means. We thus test

$$H_0 : \mu_\alpha = 0 \text{ and } \mu_\beta = 1 \quad \text{versus} \quad H_1 : \mu_\alpha \neq 0 \text{ or } \mu_\beta \neq 1. \tag{5}$$

Bayes factor calculates data density ratio conditional on the alternative and null hypothesis respectively, or $p(R_T|D_T; H_1)/p(R_T|D_T; H_0)$ with $R_T = (r_1, ..., r_T)$ and $D_T = (d_1, ..., d_T)$. It can be shown that the Bayes factor is equal to the $SDDR$

$$SDDR = \frac{p(\mu_\alpha = 0, \mu_\beta = 1|R_T, D_T)}{N(\mu_\alpha = 0; \alpha_0, v) \times N(\mu_\beta = 1; \beta_0, v)}, \tag{6}$$

which is the ratio of posterior and prior ordinate. The denominator can be readily computed, whereas the numerator is computed nonparametrically by fitting a two-dimensional kernel density to the MCMC samples of (μ_α, μ_β) after burn-in.

The unconditional test is a static test which is similar to the test in a linear regression model. The novelty of the conditional test is that it takes the time-variation in UIP coefficients into account and tells if there is enough statistical evidence to reject UIP at a certain point in time. For $t = 1, ..., T$, we test

$$H_0 : \alpha_t = 0 \text{ and } \beta_t = 1 \quad \text{versus} \quad H_1 : \alpha_t \neq 0 \text{ or } \beta_t \neq 1. \tag{7}$$

The conditional Bayes factor in this case is given by

$$SDDR_t = \frac{p(\alpha_t = 0, \beta_t = 1|R_T, D_T)}{p_0(\alpha_t = 0, \beta_t = 1)}. \tag{8}$$

The posterior ordinate is obtained via a kernel density estimator using posterior samples of (α_t, β_t). The prior ordinate in the denominator is evaluated under the null that UIP holds at time t, given their prior distribution $p_0(\alpha_t, \beta_t)$. To evaluate the prior distribution, we need to integrate out nuisance parameters $\theta^* = (\mu_\alpha, \phi_\alpha, \gamma_\alpha, \mu_\beta, \phi_\beta, \gamma_\beta)$. It follows that

$$p_0(\alpha_t, \beta_t) = \int p_0(\alpha_t, \beta_t|\theta^*) p_0(\theta^*) \mathrm{d}\theta^*, \tag{9}$$

where $p_0(\theta^*)$ is given in Table 1. Because $f_t = \mu_f(1-\phi_f) + f_1 + \sum_{s=2}^{t} \phi^{t-s}(\sigma\eta_{f,s-1} + \gamma_f' x_{s-1})$ for $t \geq 2$ and $f_1 \sim N\left(\mu_f, \frac{\sigma^2}{1-\phi_f^2}\right)$, $f \in \{\alpha, \beta\}$, we have

$$p_0(\alpha_t, \beta_t|\theta^*) \overset{d.}{=} N\left(\mu_\alpha + \mathbb{1}_{t\geq 2}\gamma_f' \sum_{s=2}^{t} \phi_\alpha^{t-s} x_{s-1}, \frac{\sigma^2}{1-\phi_\alpha^2}\right) \times N\left(\mu_\beta + \mathbb{1}_{t\geq 2}\gamma_f' \sum_{s=2}^{t} \phi_\beta^{t-s} x_{s-1}, \frac{\sigma^2}{1-\phi_\beta^2}\right),$$

where $\overset{d.}{=}$ denotes equivalence in distribution and $\mathbb{1}_{\cdot}$ denotes the indicator function. The Monte Carlo integration $\hat{p}_0(\alpha_t, \beta_t) = \frac{1}{J}\sum_{j=1}^{J} p_0\left(\alpha_t, \beta_t|\theta^{*(j)}\right)$ with $\theta^{(j)}$ being the j-th draw from $p_0(\theta^*)$ serves as an unbiased estimator for $p_0(\alpha_t, \beta_t)$ in (9). This is then used to compute the prior ordinate in (8).

The proposed conditional UIP test can also be implemented in TVC, TVC-SV models, and the regime switching model of Ichiue and Koyama (2011). In the TVC-SVX model, the test incorporates model uncertainty via h_t and international forces via x_t when evaluating the posterior distribution of UIP coefficients at t. In comparison, the common practice in rolling window least squares is to conduct event study or give a narrative about the point estimate obtained using a chosen window size (see *e.g.* Chaboud and Wright, 2005, Lothian and Wu, 2011 and Ismailov and Rossi, 2018).

3.2. Accounting of time-variation

Any estimator of time-varying coefficients at a certain point in time, say t, implicitly implies a weighting scheme imposed on observations around t. Take β_t in UIP regressions as an example. We have

$$\hat{\beta}_t = \sum_{j=1}^{T} \omega_{jt} f(r_j, d_j),$$

where the weighting function ω_{jt} denotes the implied observation weight of r_j and d_j, $j = 1, ..., T$, on estimating β_t and where the function $f(\cdot)$ is implied by a chosen econometric model. Sensible accounting of time-variation and statistically valid inference on UIP coefficients at different points in time crucially depend on such weighting schemes.

The proposed TVC-SVX model provides an automatic and data-driven weighting scheme for estimating UIP coefficients, taking time-varying uncertainty measured by SV, and common US shocks into account. We show in the supplementary appendix that a first order approximation

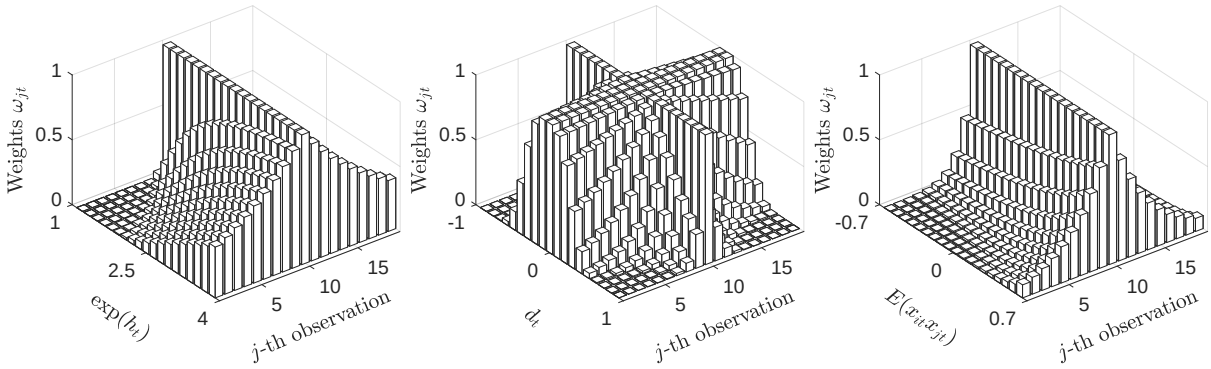

Figure 2: Weighting functions implied by the TVC-SVX model. The left, middle and right figure show the effect of increasing volatility h_t, changing interest rate differential d_t from -1 to 1 and changing cross-sectional correlation in x_t from -0.7 to 0.7 on ω_{jt} for $j = 1, ..., 19$ and $t = 10$.

of w_{jt} in the TVC-SVX model is given by

$$\omega_{jt} \propto \left(\frac{2\sigma_t^4}{\sigma_\beta^2 + \sqrt{\sigma_\beta^4 + 4\sigma_\beta^2\sigma_t^2} + 2\sigma_t^4} \right)^{|t-j|},$$

where $\sigma_\beta^2 = \sigma^2 + \gamma_\beta' E(x_t x_t')\gamma_\beta$ and $\sigma_t^2 = \exp(h_t)/d_t^2$. So remote observations (*i.e.* j is away from t) becomes more important in determining $\hat{\beta}_t$ if period t is associated with (1) high uncertainty or volatility h_t; (2) small interest rate differential d_t; (3) low variation in non-US shocks driving β_t or small σ^2; and (4) low variation in US shocks driving β_t or small $\gamma_\beta' E(x_t x_t')\gamma_\beta$. Notice the $E(x_t x_t')$ in the latter becomes conditional expectation if x_t is subject to heteroskedasticity. In Figure 2, we plot the weighting function for estimating β_{10} with $T = 19$ under increasing volatility, changing interest rate differential and changing corss-sectional correlation in x_t.

In comparison, rolling window regressions (RWR) naïvely assign equal weight to observations in all equal-sized windows. Despite its popularity in empirical studies on UIP, we do not consider it in our context because of: (1) estimation sensitivity to window size which lacks theoretical guidance (see Inoue et al., 2017 for optimal size in predictive regressions); (2) its scarce inferential theory (Cai et al., 2000 gives frequentist's asymptotic results in varying-coefficient models proposed by Hastie and Tibshirani (1993) which assume coefficients to be smooth functions of observable index variables); and (3) extra tuning parameters brought by kernel smoothing or polynomial spline for smoothness in RWR estimates (Henderson et al., 2015).

4. Empirical study

We collect monthly data spanning 1993:M1 to 2018:M9 on exchange rates and interest rates. US strutural shocks are identified by the FRBNY DSGE-DFM model of Gelfer (2019) as in Section 2. Twelve currency pairs against the US dollar are considered: the Australian dollar, the Canadian dollar, the Danish krone, the Euro, the Japanese yen, the New Zealand dollar, the Norwegian krone, the South African rand, the South Korean won, the Swedish krona, the Swiss franc, and the British pound. For Denmark, the EU, Norway, South Africa, Sweden, Switzerland, and the UK, interest rates are the three-month interbank rate. For Australia and New Zealand, we use three-month bank bills rate. For South Korea and Japan, we consider three-month certificates of deposit rate. For Canada and the US, we consider three-month treasury bills rate. Both interest rates and exchange rates are obtained from the CEIC database.

4.1. Estimation result

4.1.1. Parameter estimates

Table 2 shows the posterior estimates of autoregressive coefficients ϕ_α, ϕ_β and ϕ_h for TVC-SVX and TVC-SV models. Without US shocks modelled as explanatory variables in the dynamics of f_t where $f \in \{\alpha, \beta, h\}$, f_t exhibits high autocorrelation. Once US shocks are incorporated, all countries show a clear reduction in autoregressive coefficients, especially in ϕ_α and ϕ_β. This observation suggests a strong Granger effect from US shocks, meaning that US shocks x_t predicts the movement of f_{t+1} by diluting its dependence on f_t. Not surprisingly, based on our finding one can interpret the US structural shocks as the common international shocks that drive bilateral UIP relations as in Lustig et al. (2011) and Greenaway-McGrevy et al. (2018).

Table 3 shows the posterior means of $100\mu_\alpha$, μ_β and μ_h estimated from TVC-SVX and TVC-SV models. We notice that controlling for US shocks tends to increase μ_β, even though x_t is normalised to have zero mean in TVC-SVX models. μ_β's are all positive under TVC-SVX, which solves the UIP puzzle directionally. The positive sign of μ_β for all countries together with the ADL(1) specification of β_t in TVC-SVX models indicate that high interest rate currency mean reverts to be depreciating. While controlling for US shocks leads more countries' credible intervals to cover one, it tends to decrease μ_α, excluding zero from the credible interval for Japan and Norway. We conjecture that controlling for US shocks mitigates the negative correlation

Table 2: Estimates of autoregressive coefficients

TVC-SVX: with US shocks	AU	CA	DK	EU	JP	NZ	NO	ZA	KR	SE	CH	UK
ϕ_α	0.74	0.34	0.37	0.46	0.64	0.73	0.30	0.67	0.39	0.32	0.47	0.87
	(0.11)	(0.19)	(0.18)	(0.17)	(0.14)	(0.16)	(0.17)	(0.10)	(0.13)	(0.17)	(0.19)	(0.05)
ϕ_β	0.67	0.57	0.81	0.80	0.19	0.75	0.76	0.68	0.71	0.64	0.65	0.57
	(0.09)	(0.15)	(0.10)	(0.05)	(0.16)	(0.06)	(0.08)	(0.07)	(0.07)	(0.11)	(0.11)	(0.09)
ϕ_h	0.58	0.69	0.61	0.70	0.63	0.48	0.60	0.68	0.80	0.57	0.60	0.69
	(0.14)	(0.10)	(0.08)	(0.10)	(0.15)	(0.11)	(0.13)	(0.06)	(0.11)	(0.11)	(0.11)	(0.12)
TVC-SV: without US shocks	AU	CA	DK	EU	JP	NZ	NO	ZA	KR	SE	CH	UK
ϕ_α	0.94	0.92	0.95	0.96	0.90	0.92	0.94	0.82	0.84	0.93	0.92	0.92
	(0.04)	(0.04)	(0.03)	(0.02)	(0.07)	(0.07)	(0.05)	(0.11)	(0.09)	(0.04)	(0.05)	(0.04)
ϕ_β	0.86	0.93	0.92	0.91	0.81	0.89	0.88	0.77	0.78	0.90	0.85	0.85
	(0.05)	(0.07)	(0.06)	(0.05)	(0.08)	(0.04)	(0.06)	(0.06)	(0.07)	(0.05)	(0.07)	(0.06)
ϕ_h	0.85	0.89	0.80	0.84	0.78	0.86	0.80	0.86	0.86	0.78	0.81	0.83
	(0.06)	(0.05)	(0.07)	(0.06)	(0.07)	(0.06)	(0.08)	(0.06)	(0.05)	(0.08)	(0.08)	(0.03)

The table reports the autoregressive coefficient ϕ_f in the dynamics of f_t, $f \in \{\alpha, \beta, h\}$. Reported are posterior means and standard deviations (in parentheses), obtained under the TVC-SVX specification $f_{t+1} = \mu_f(1-\phi_f) + \phi_f f_t + \gamma_f' x_t + \sigma\eta_{f,t}$ and the TVC-SV specification $f_{t+1} = \mu_f(1-\phi_f) + \phi_f f_t + \sigma\eta_{f,t}$. Countries are indicated using their two-letter abbreviations.

between relative interest rate and foreign currency risk premium and thus violates the necessary condition behind UIP puzzle (Fama, 1984; Backus et al., 2013). Apparently, expanding the information set via US shocks, we have more confidence in inference as endogeneity is controlled for.

The TVC-SVX model correlates forces that drive α_t, β_t and h_t by US shocks. It is necessary to model such correlations in any TVC models as they can explain the breaches of UIP through the necessary condition in (Fama, 1984) and the overshooting hypothesis (Bjørnland, 2009). We use a simple diagnostic to check if there is empirical evidence on such correlation by computing the implied correlation among innovations using their posterior samples from TVC-SV and TVC-SVX models. Figure 3 shows the distribution of correlation coefficients computed from posterior samples. Under TVC-SV, we observe non-zero implied correlations, $E(\eta_{\beta,t}\eta_{h,t})$ and $E(\eta_{\alpha,t}\eta_{\beta,t})$ in particular, for many countries. Under TVC-SVX however, innovation correlations are uniformly closer to zero. This may suggest that correlations behind TVCs of bilateral UIP relations come from international sources, US in particular, rather than domestic ones. Also, there may not be any need to model correlations if US shocks are present in a TVC model, except for New Zealand which may have minor evidence against zero $E(\eta_{\alpha,t}\eta_{\beta,t})$.

Table 3: Estimates of unconditional means

TVC-SVX: with US shocks	AU	CA	DK	EU	JP	NZ	NO	ZA	KR	SE	CH	UK
μ_α	-0.65	-0.18	-0.24	-0.22	-1.2	-0.09	-0.81	0.62	-0.58	-0.14	-1.86	-0.35
μ_β	0.04	0.46	0.33	0.50	0.63	0.14	0.52	0.15	0.21	0.62	0.07	0.62
μ_h	-6.46	-7.42	-6.53	-6.88	-6.22	-6.55	-6.34	-5.89	-6.92	-6.39	-6.59	-7.27
TVC-SV: without US shocks	AU	CA	DK	EU	JP	NZ	NO	ZA	KR	SE	CH	UK
μ_α	0.31	0.15	-0.08	-0.25	-0.34	0.26	-0.13	0.67	-0.29	0.23	-0.94	-0.57
μ_β	-0.22	0.22	0.08	0.11	-0.05	-0.06	0.12	0.15	0.03	-0.12	-0.27	0.59
μ_h	-6.46	-7.54	-6.51	-6.93	-6.37	-6.59	-6.48	-5.95	-7.12	-6.46	-6.59	-7.34

The table reports the posterior means of $100\mu_\alpha$, μ_β and μ_h. Except for μ_h, entries are shaded if the 90% highest posterior density interval excludes the UIP theoretical values $\mu_\alpha = 0$ and $\mu_\beta = 1$. Posterior samples are obtained under the TVC-SVX specification $f_{t+1} = \mu_f(1-\phi_f) + \phi_f f_t + \gamma_f' x_t + \sigma\eta_{f,t}$ and the TVC-SV specification $f_{t+1} = \mu_f(1-\phi_f) + \phi_f f_t + \sigma\eta_{f,t}$, f_t, $f \in \{\alpha, \beta, h\}$.

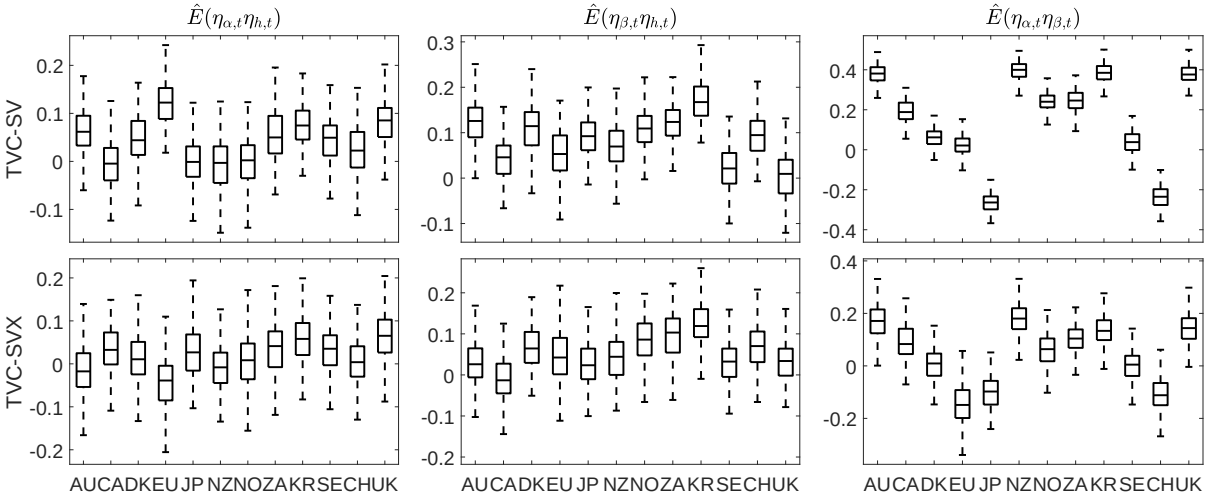

Figure 3: Implied correlation among innovations. The graph shows boxplots of empirical correlation among innovations $\eta_{\alpha,t}$, $\eta_{\beta,t}$ and $\eta_{h,t}$, computed from their posterior samples. Whiskers point to maximum and minimum; whereas the rectangle indicates inter-quartile range with the median drawn in the middle. The upper and the lower panel correspond to those obtained from the TVC-SV and the TVC-SVX model, respectively.

4.1.2. Time-varying UIP coefficients

As discussed previously, the local movements of UIP coefficients are anchored by US shocks which expand the information set. Figure 4 and 5 show the posterior distribution of α_t and β_t, respectively, under both the TVC-SVX and the TVC-SV model.

It is evident that UIP coefficients from the TVC-SVX model show richer local movements. α_t and β_t obtained from TVC-SV models cannot even adequately serve as local means of their TVC-SVX counterparts. For example, right after the GFC, α_t and β_t are seen to have experienced a significant drop and rise, respectively for all countries (except for UK) due to the decreased sensitivity of exchange rates to key currency risk premium (Backus et al., 2013; Lustig et al., 2014). This is however completely missed for most of the countries under the TVC-SV specification. US shocks also affect inference. For example, posterior distributions of α_t for New Zealand and South Africa indicate high certainty, compared with Norway and EU which are precisely estimated. Common patterns across countries include the more spread-out posterior distributions of α_t and β_t during the GFC due to higher uncertainty, and the slightly increased β_t after the GFC for European countries.

We take Canada as an example and study the time-variation extracted from different models. Results for other countries are available upon request. Figure 6 shows different estimates of time-varying UIP coefficients α_t and β_t and volatility $\exp(h_t/2)$ obtained from TVC-SVX, TVC-SV, OLS with interaction terms (4) and a simple rolling window OLS with window size chosen according to Inoue et al. (2017). Both TVC-SV and rolling window OLS miss the drop in α_t in 2009 due to the lack of identifiability using exchange rate r_t and interest rate differential d_t alone. This however does not hold throughout the sample period. Rolling window estimates of α_t decreases while β_t increases from 2012, showing a very different pattern from other models. This means that the equal weighting imposed and autocorrelation generated by rolling windows may bias our inference. Furthermore, both α_t and β_t from OLS with interaction terms overshoot between 2008 and 2012, suggesting that local movements of UIP coefficients can also be misleading if all movements are attributed to US shocks. The TVC-SVX model balances these issues by allowing for extra information from US shocks on top of coefficient autoregressive dynamics that captures non-US factors and utilises SV to discount observations.

To see how TVC-SVX provides a sensible data-driven way of discounting observations when

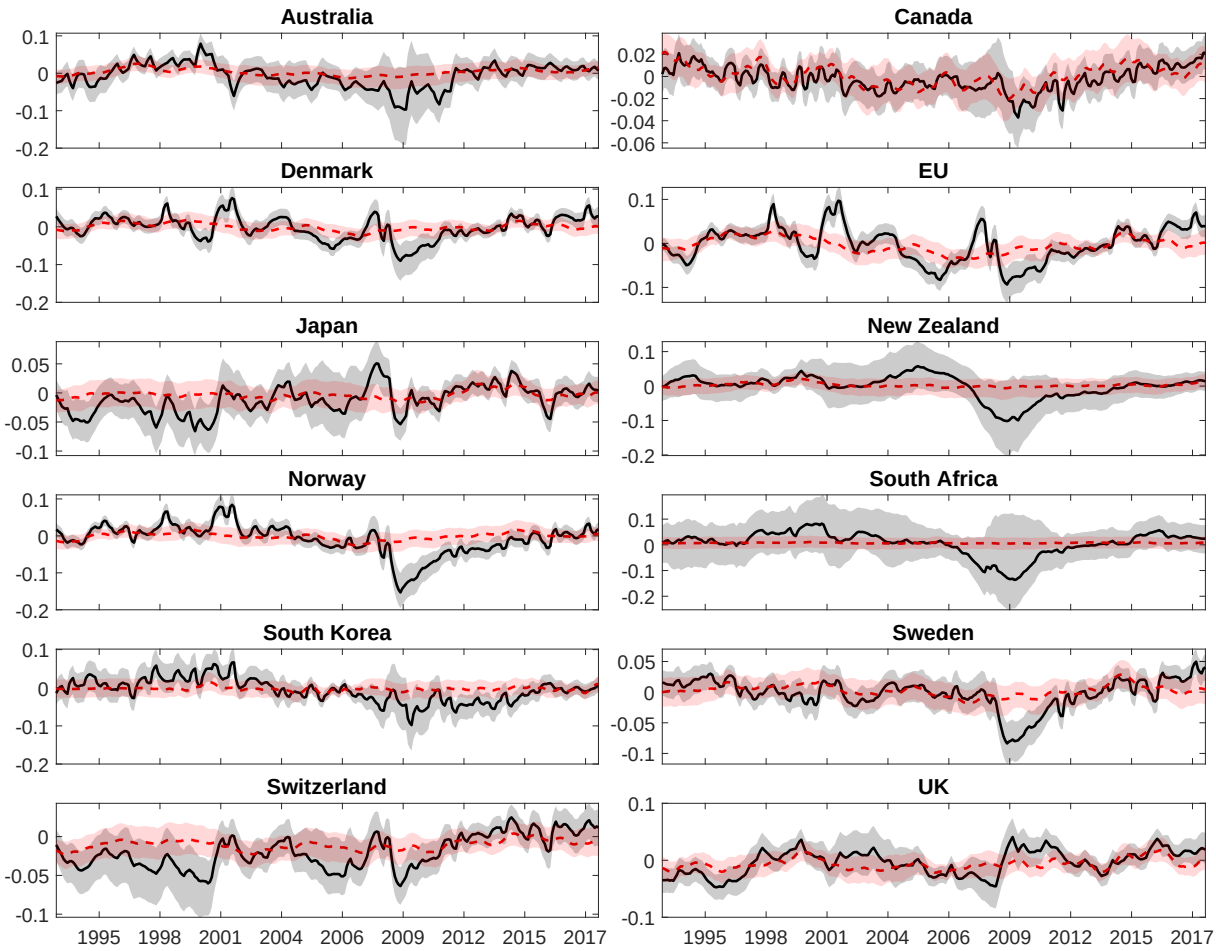

Figure 4: Posterior estimate of α_t. Reported are posterior median indicated by the thick line and the 0.025 lower and upper quantiles indicated by the shaded area of α_t. Black and red indicate estimates obtained from the TVC-SVX and the TVC-SV model, respectively.

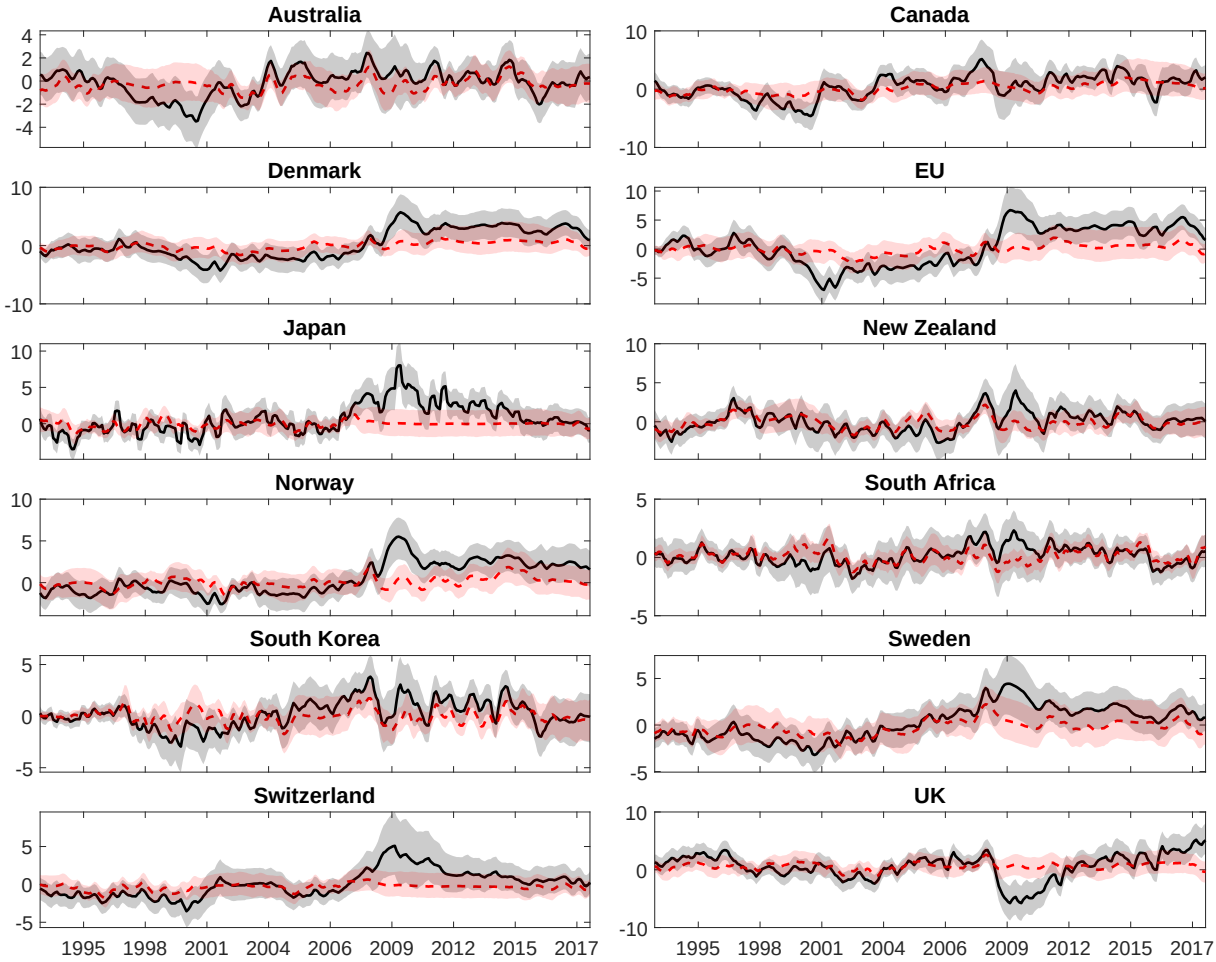

Figure 5: Posterior estimate of β_t. Reported are posterior median indicated by the thick line and the 0.025 lower and upper quantiles indicated by the shaded area of β_t. Black and red indicate estimates obtained from the TVC-SVX and the TVC-SV model, respectively.

estimating UIP coefficients at a certain point in time, we compare its implied observation weights introduced in Section 3.2 with those obtained from TVC-SV, least squares with stochastic volatility (LS-SV) and OLS with interaction terms, taking Canada as an example. Specially, we look at June 2003, a quiet and normal month, and November 2008, 3 months into the GFC when economic uncertainty was high. Results are given in Figure 7. TVC suffers from overfitting with observations tightly around the month of interest overweighted. TVC-SV improves on TVC by taking into account volatility changes so that α_t and β_t capture signal rather than noise. This is supported by the slower decaying weights assigned to observations around November 2008. Intuitively, when economic uncertainty and volatility are high, inference should be based on more observations. Further improved in the TVC-SVX models are the weighting functions, which are driven by a larger information set, *i.e.* US shocks, around the month of interest. During the volatile period, the more spread-out weighting function also displays a non-monotone decaying pattern which is clearly more flexible than TVC-SV.

LS-SV is a constant coefficients model and similar to generalised least squares which weights down observations that have large variances. This however implies that UIP coefficients for Canadian dollars are mainly estimated from sample prior to 2004. OLS with interaction terms can also be seen as a constant coefficients model, if we project exchange rate r_t and interest rate differential d_t in (4) off the space spanned by "control variables" x_t and $d_t x_t$. Define $M = I - Z(Z'Z)^{-1}Z'$ where Z is a matrix with columns collecting US shocks and interaction terms. (4) is equivalent to $M\boldsymbol{r} = \bar{\beta}M\boldsymbol{d} + M\boldsymbol{\epsilon}^*$ where vectors of demeaned r_t, d_t and ϵ_t^* for $t = 1, ..., T$ are indicated in bold. Thus, observation weights from OLS with interaction terms are normalised column sum of M. Contrary to models with SV, it weights up observations during GFC. This counter-intuitive feature leads to the overshooting of UIP coefficients seen in Figure 6, which results from a higher signal-to-noise ratio as US shocks are also more volatile during that period.

4.2. UIP tests

The credible intervals of μ_α and μ_β in Table 3 cannot formally test if UIP coefficients mean revert to their theoretical values. We base the test on Bayes factor and SDDR as introduced in Section 3.1. If we fail to reject the null in (5) using the scale of Kass and Raftery (1995), then

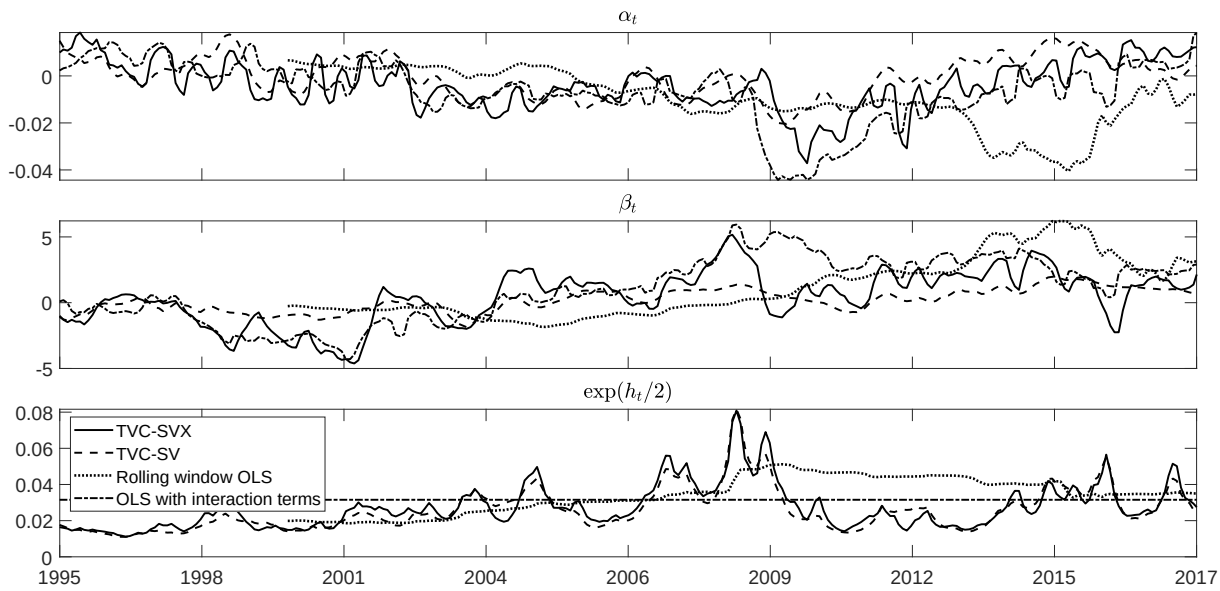

Figure 6: Estimates of time-varying coefficients for Canada under different model specifications.

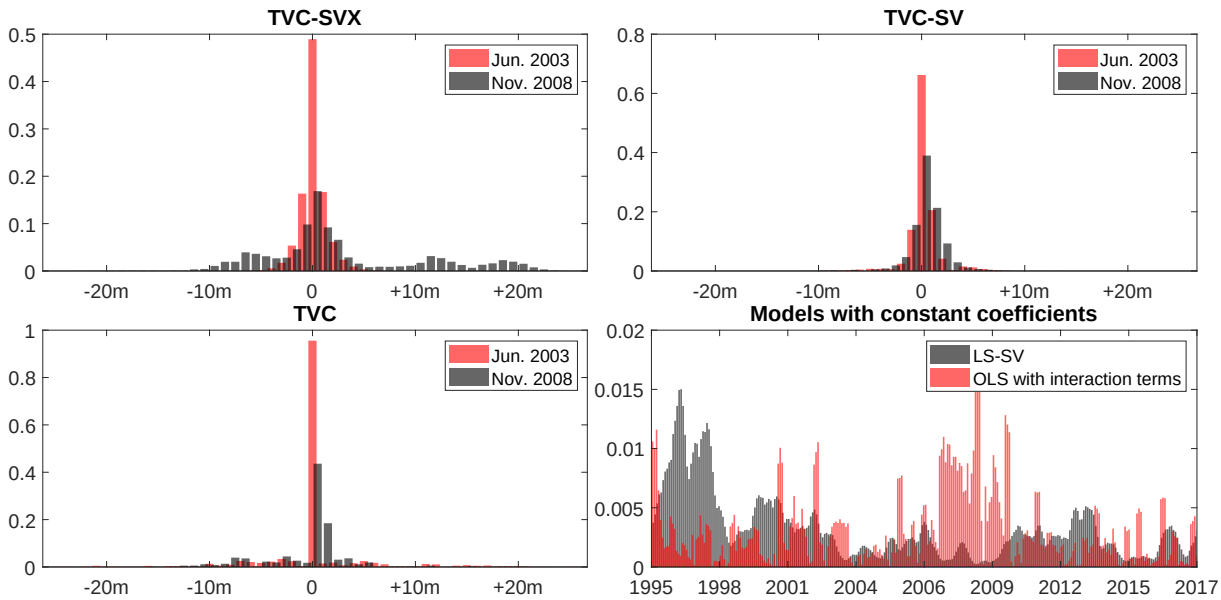

Figure 7: Estimation weighting functions for Canada implied by different model specifications. For the TVC-SVC, the TVC-SV and the TVC model, we compute the observation weights (Durbin and Koopman, 2012, Chapter 4) for estimating the posterior mean of (α_t, β_t) when t indicates Jun. 2003 (red) and Nov. 2008 (black). $-x$m and $+x$m on the x-axis indicate observations that are x months prior to and after t. LS-SV and OLS with interaction terms use all observations for estimation. Rolling window OLS uses equal weight that is reciprocal to the window size and is omitted.

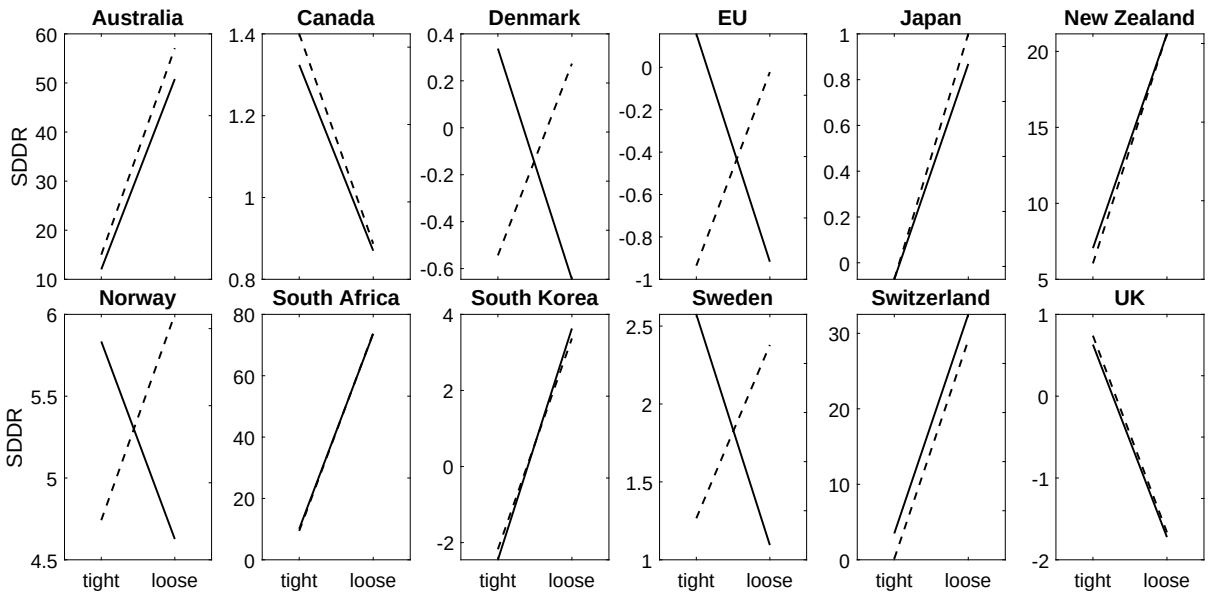

Figure 8: Unconditional UIP test result. Null and alternative of the unconditional test are given by (5). Tests are conducted under TVC-SVX (thick line) and TVC-SV (dashed line), with tight prior for μ_α and μ_β (prior variance $v = 0.01$) and loose prior (prior variance $v = 10$). y-axis shows the logarithm of SDDR as in (6).

there is not enough statistical evidence against unconditional UIP or UIP in the long run.

Figure 8 illustrates the test result from TVC-SVX and TVC-SV models, estimated using either tight or loose priors (prior variance $v = 0.01$ or $v = 10$) for μ_α and μ_β. There are three main findings. First, except for Norway who shows marginal rejection of unconditional UIP, only Australia, New Zealand, South Africa and Switzerland show strong violation of unconditional UIP especially under the loose prior. This is in stark contrast to many studies which find universal rejection (see *e.g.* Engel, 2014 for a review) based on least squares analysis. Second, for these countries US shocks do not have any clear effect on test conclusions (see also Table 3). Because Australian Dollar, New Zealand Dollar and South Africa Rand are commodity curencies, this result agrees with Kohlscheen et al. (2016) which shows that global risk appetite or carry does not drive the link between commodity prices and the exchange rates of commodity currencies. Third, when US shocks make a difference, they drive the unconditional test in the "non-rejection" direction, which suggests that the commonly found rejection may result from omitted variables of international shocks and small sample bias as discussed in Chinn and Quayyum (2012) among others.

The unconditional test ignores the local movement of UIP coefficients as it only looks at their

mean reversion or long-term behaviour. Local movements of α_t and β_t are important in the short-term when one tests bilateral UIP relations at a certain point in time. Rather than giving a narrative about point estimates as in Craighead et al. (2010) and Ismailov and Rossi (2018), we conduct the conditional UIP test introduced in Section 3.1 with null and alternative given by 5. Such a test takes into account model uncertainty through the posterior distribution of (α_t, β_t) which is affected by SV as well as US shocks in TVC-SVX models.

Figure 9 plots the test results over time together with estimated SV $\exp(h_t/2)$ under both TVC-SV and TVC-SVX models. The rolling-window regressions in Ismailov and Rossi (2018) reveal that breaches of UIP are associated with periods of high uncertainty. This narrative however does not always hold in our analysis if uncertainty is proxied by volatility (Bloom, 2009). The figure shows that during the GFC both models show spike of volatility for all countries. Yet the conditional test does not suggest violation of UIP for Canada, South Africa, and South Korea, due to the increased uncertainty of coefficient estimates as seen in 4 and 5. Conversely, there are low-volatility periods when UIP is rejected; take Australia, Canada and UK in the late 90s for example. Intuitively, our conditional test rejects UIP drawing inference from the whole posterior distribution instead of just a point estimate at a point in time. Furthermore, we observe that by expanding the information set US shocks may change the test result in both directions. Under TVC-SVX, New Zealand, Australia and South Africa show smaller probability of rejection around 2004, while stronger violations of UIP can be noticed for many countries in 2009. Puzzling it may seem at first, but it is important to bear in mind that TVC-SV misses local movements of α_t and β_t due to possible underidentification, whereas TVC-SVX fixes it by anchoring movements partially to international forces. Lastly, contrary to Ismailov and Rossi (2018) which shows that uncertainty increase foreruns the violation of UIP, our conditional test finds an ambiguous ordering between the two; for example Switzerland in 2009 versus UK in 2009.

We conclude this section by clarifying that unconditional and conditional UIP tests bear different content. One currency pair may have UIP locally established from time to time, yet the "steady state" or unconditional mean relationship suggests violation; a good example is South Africa. The reverse is also possible; take EU for example.

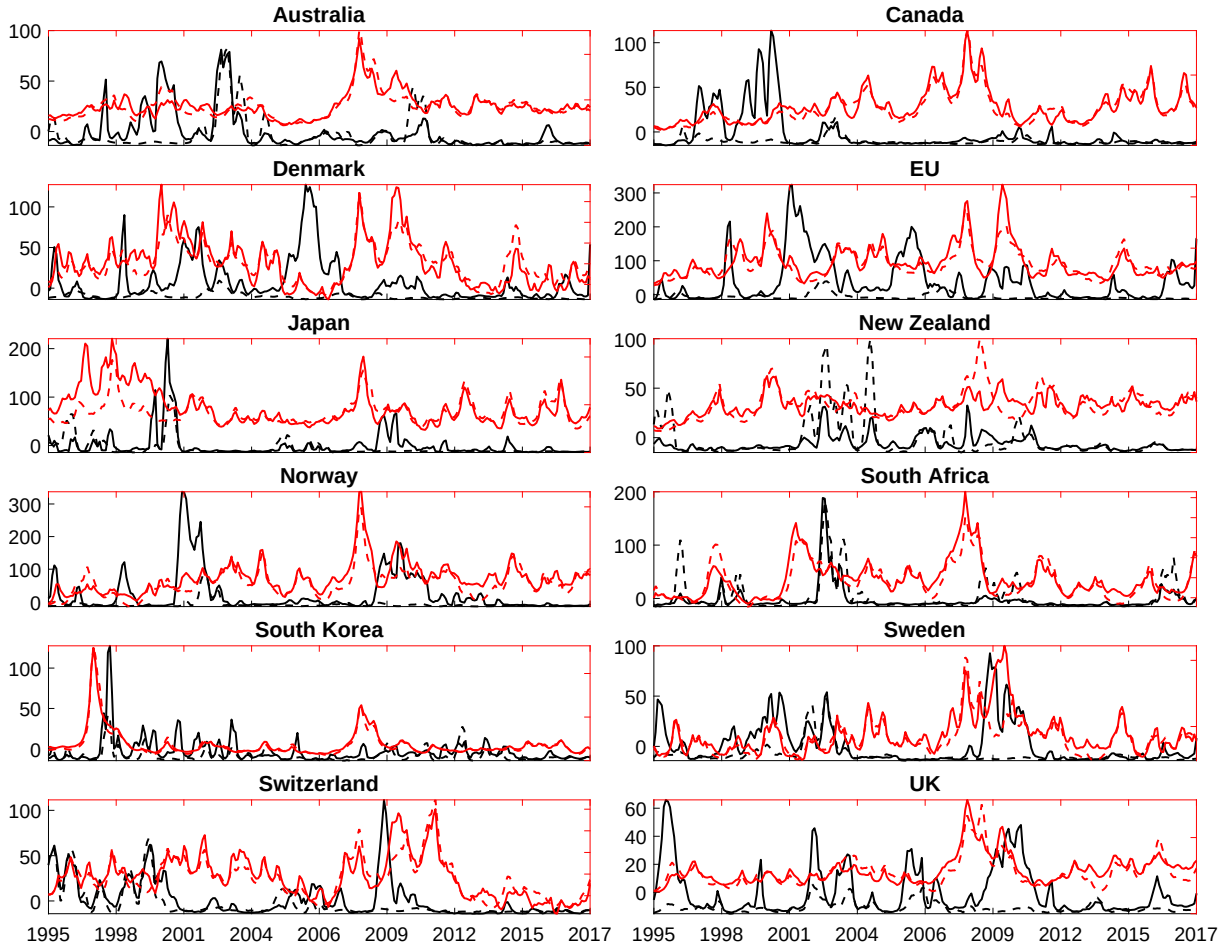

Figure 9: Conditional UIP test result and volatility overtime. Null and alternative of the conditional test are given by (7) . Tests are conducted under TVC-SVX (thick line) and TVC-SV (dashed line). Time evolution of the error term volatility $\exp(h_t/2)$ is plotted in red; for readability their scales are not plotted. The left y-axis corresponds the logarithm of SDDR as in (8) plotted in black.

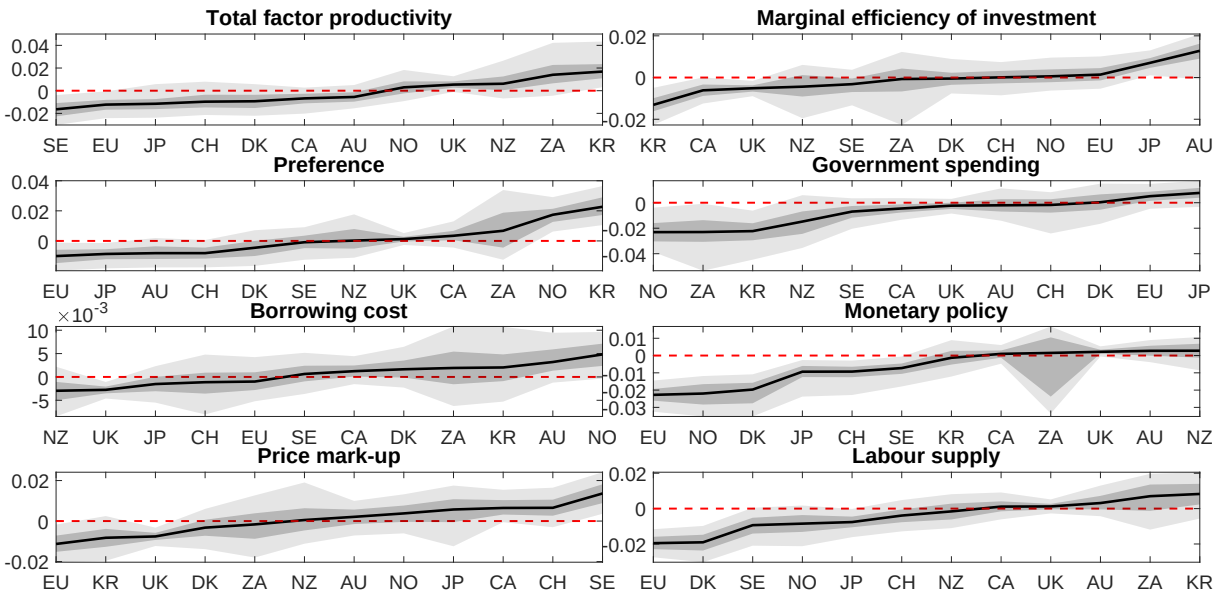

Figure 10: Effect of US shocks on α_t. Reported are the posterior 5-th, 25-th, 50-th, 75-th and 95-th percentiles of elements of γ_α as in (2). Countries are sorted based on the magnitude of the effect each US shock has on α_t.

4.3. The role of US shocks

The time variation in UIP coefficients of different currency pairs may respond to US shocks differently. Figure 10, 11 and 12 summarise the posterior distributions of entries in γ_α, γ_β and γ_h, respectively, sorted country-wise based on their posterior median. Since the shocks are standardised, the effect of a US structural shock is for one standard deviation (std) change.

The most important US shock that Granger causes α_t is the monetary policy shock. Whenever the coefficient is significant, it has a negative sign. For example, for Norway, South Africa and Denmark, a one std US monetary policy shock is expected to decrease α_t by 0.02 conditional on other shocks and α_{t-1}; or equivalently, it is expected to decrease the 3-month return of exchange rate by 2bps conditional on other shocks and the interest rate differential. Similarly, supply-side shocks such as labour supply and government spending negatively Granger cause α_t, except for currencies that respond insignificantly. However, total factor productivity, preference and inflation shocks show non-uniform effect on α_t conditional on α_{t-1}. For example. while preference shock drives up α_t for Norway and South Korea significantly, it does the opposite for EU, Japan, Australia and Denmark.

Different from α_t, US shocks tend to affect the dynamics of slope β_t with same sign. In terms of magnitude, conditional on β_{t-1} total factor productivity and preference shocks negatively and

government spending shocks positively affect β_t almost for countries. Less profoundly are the negative effect of labour supply and borrowing cost shocks, with the latter largely insignificant, and the marginally positive effect of inflation and investment shocks. That borrowing cost shocks do not seem to Granger cause UIP coefficients suggests that US financial condition may only drive movement in country-specific α_t and β_t indirectly through volatility that leads to changes in risk premia (Corte et al., 2016; Verdelhan, 2018), although the evidence that borrowing cost shock affects SV is only marginal for most of the countries considered. Contrary to the significant effect on α_t, most country-specific β_t's do not respond to the US monetary policy shock. Exceptions include Denmark, Norway and EU which show small and positive response but negative response in α_t. This observation implies that for these countries US monetary policy generates negative correlation between UIP coefficients, conditional on their own past and other shocks. Furthermore, US shocks do not seem to play any role in explaining the variation of SV. So the common component in country-specific volatilities, also documented by Alexius and Sellin (1999), must come from non-US factors, conditional on their own past. It is worthwhile noting the estimated country-specific heterogeneous responses to US shocks implies that while theories reviewed in Section 2 may give us clues on the causes of instability in bilateral UIP relations, one should note that the roles of different channels may potentially depend on domestic factors such as exchange rate policy (Backus et al., 2013; Edwards, 2015), financial market sophistication (Burnside, 2014) and currency sensitivity to commodity price (Kohlscheen et al., 2016) which the current study abstracts from.

Figure 13 and 14 show communality of US shocks — the proportion of variation in α_t and β_t net of their past explained by the variation in US shocks over time. We approximate communality by a simple rolling window, holding α_t and β_t, as well as parameters in their ADL(1) specification at posterior median values. For $f \in \{\alpha, \beta\}$, $t = 1, ..., T - \tau$ and window size $\tau = 72$, the communality over time is computed by $\gamma_f'(\sum_{s=t}^{\tau+t-1} x_s x_s')\gamma_f / \sum_{s=t}^{\tau+t-1}(f_{s+1} - \phi_f f_s)$.

Results suggest non-constant explanatory power of US shocks. Across countries, the communality of US shocks increases around 2004 and declines afterwards, and rebounds around 2010. Also, though US shocks do not fully drive changes in f_t, they do play an important role with an average communality of 24% across countries and time. Due to the ADL(1) structure modelled for the UIP coefficients, the driving force of f_t conditional on its own past is decomposed into

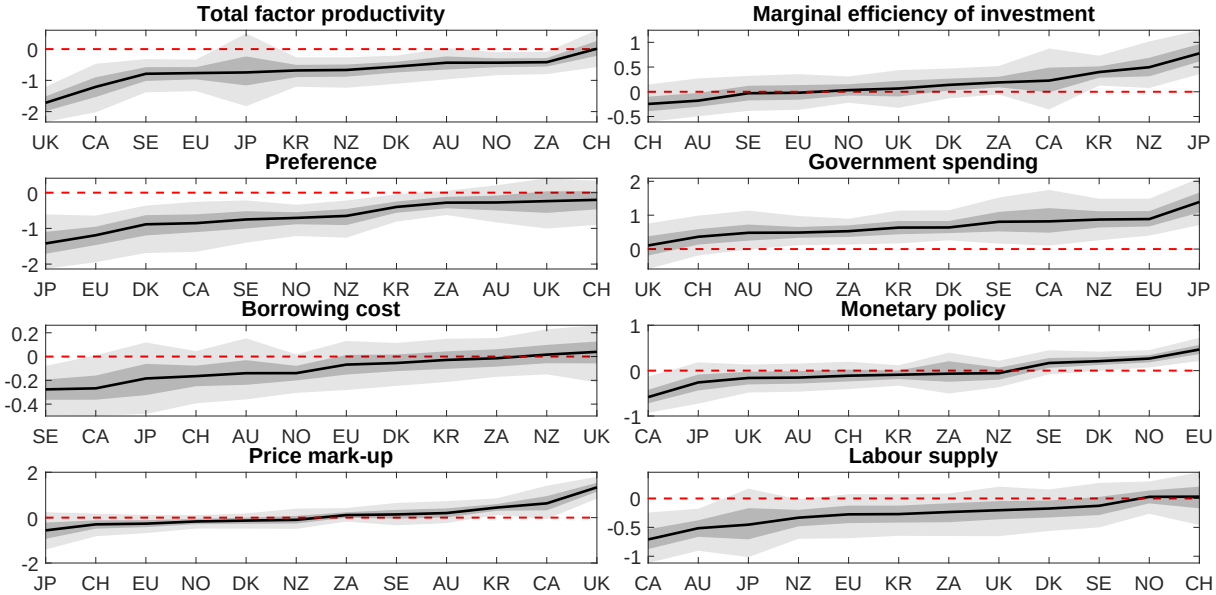

Figure 11: Effect of US shocks on β_t. Reported are the posterior 5-th, 25-th, 50-th, 75-th and 95-th percentiles of elements of γ_β as in (2). Countries are sorted based on the magnitude of the effect each US shock has on β_t.

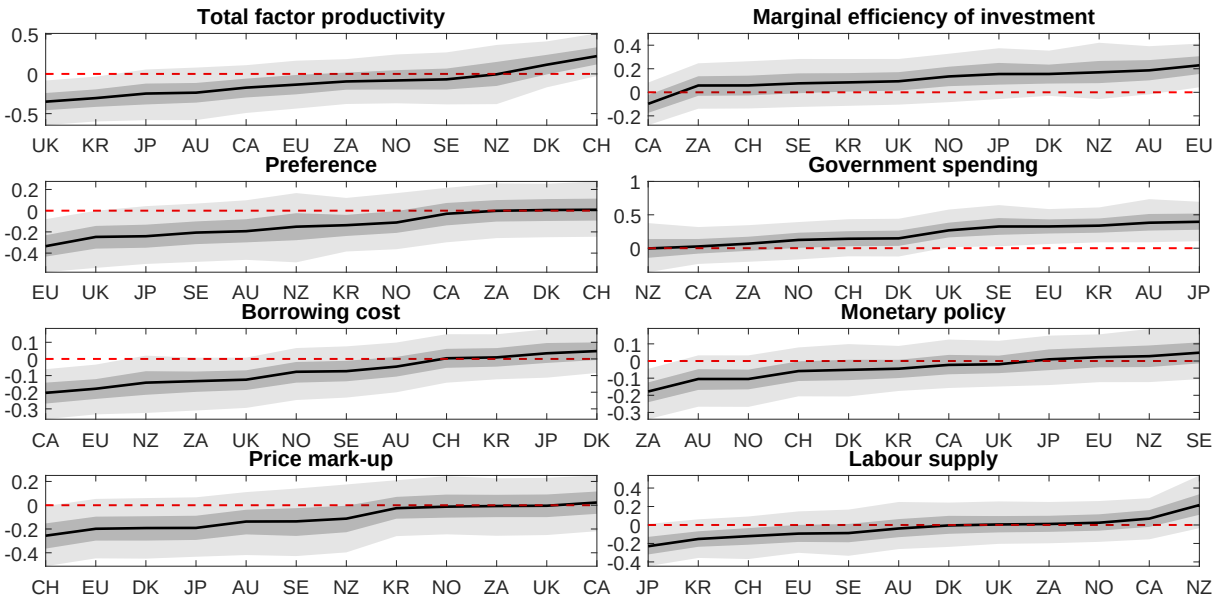

Figure 12: Effect of US shocks on h_t. Reported are the posterior 5-th, 25-th, 50-th, 75-th and 95-th percentiles of elements of γ_h as in (2). Countries are sorted based on the magnitude of the effect each US shock has on h_t.

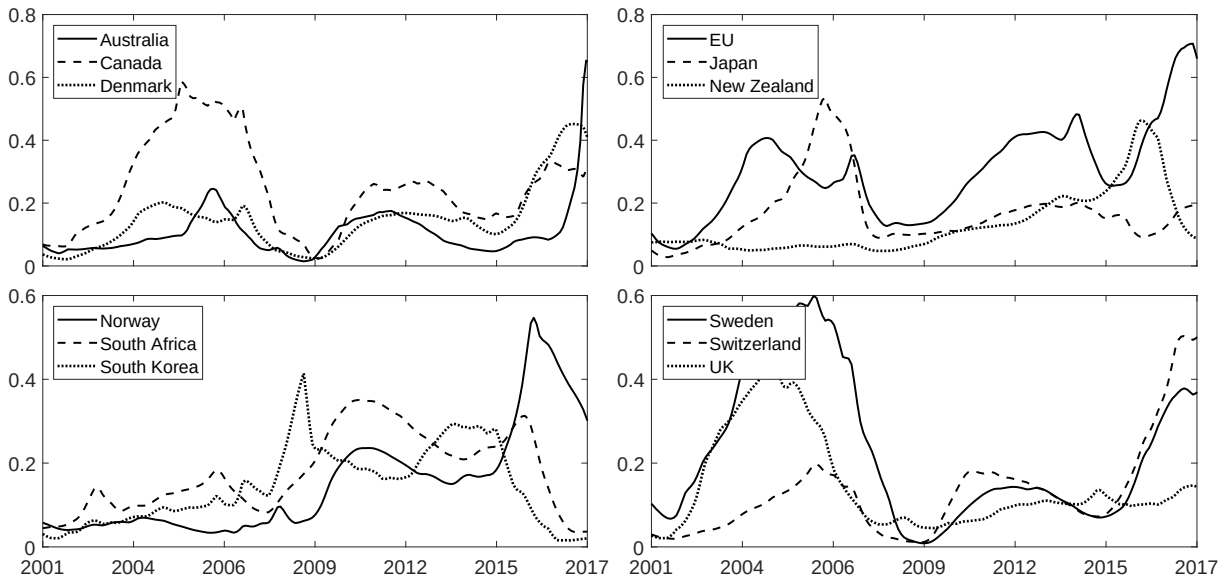

Figure 13: Proportion of variance of $\alpha_t - \phi_\alpha \alpha_{t-1}$ explained by US shocks. Results are obtained by rolling windows with α_t and γ_α fixed at their posterior median level.

a US factor $\gamma_f x_t$ and an idiosyncratic or non-US innovation $\eta_{f,t}$. This exercise shows that both parts are essential for extracting the time variation, compared to OLS with interaction terms (4) which attributes all changes to US shocks and other TVC models that abstracts from such a decomposition.

Lastly, we point out that two-stage methods, such as the one in Cecchetti et al. (2017) which estimates a TVC-SV model in the first stage and regresses the first-stage posterior mean of the TVC on some variables in the second stage, must be treated with caution. Our estimation shows that whether or not one models explanatory variables inside TVC dynamics makes a sizeable difference. Also, inference from two-stage methods has no theoretical guidance as the first stage is a Bayesian point estimate obtained from a restricted information set. In practice, our TVC-SVX model is superior as it leads to correct and simple inference by estimating the TVC conditional on explanatory variables.

5. Conclusion

In this paper, we propose a time-varying coefficient model with stochastic volatility and US structural shocks to study uncovered interest rate parity and understand how US shocks affect the time variation in the parity's coefficients. We develop an conditional test which looks at the

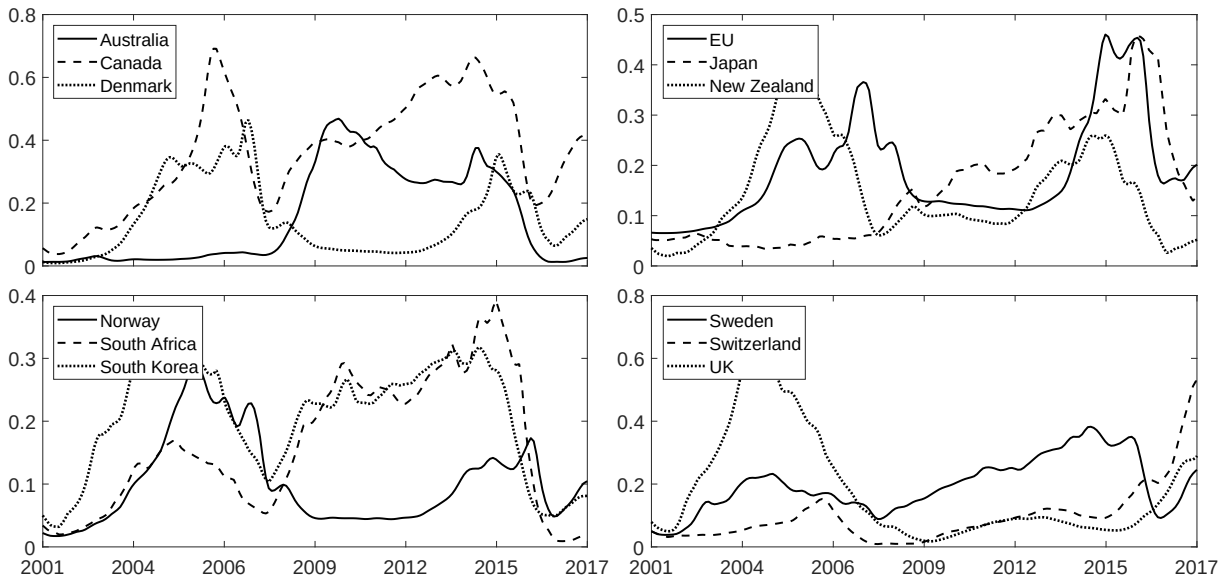

Figure 14: Proportion of variance of $\beta_t - \phi_\beta \beta_{t-1}$ explained by US shocks. Results are obtained by rolling windows with β_t and γ_β fixed at their posterior median level.

uncovered interest rate parity over the whole sample period and a conditional test which tests the parity's coefficients at each point in time, taking into account volatility or uncertainty and US shocks. Our empirical study leads to three main findings. First, there exists significant time-variation of coefficients whose co-movement can be explained by US structural shocks. Second, US shocks mitigates possible endogeneity and expands the information set for identification and tests. Last, the US monetary policy shock plays the most important role in Grander causing the intercept, while total factor productivity, preference and government spending shocks influence the slope.

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Appendix A Bayesian estimation procedure

The MCMC algorithm used for Bayesian inference iterates over the following four blocks:

1. Sample log stochastic volatility process $\log \sigma_t$ from $p(\sigma_t | Y_T, X_T, \{w_t\}_{t=1}^T, \{\alpha_t\}_{t=1}^T, \{\beta_t\}_{t=1}^T, \mu_{\log\sigma}, \phi_{\log\sigma}, \sigma_{\log\sigma})$;
2. Sample the mixing variable w_t from $p(w_t | Y_T, X_T, \{\alpha_t\}_{t=1}^T, \{\beta_t\}_{t=1}^T, \{\sigma_t\}_{t=1}^T, \zeta)$;
3. Sample UIP coefficients from $p(\alpha_t, \beta_t | Y_T, X_T, \mu_\alpha, \mu_\beta, \phi_\alpha, \phi_\beta, \sigma_\alpha, \sigma_\beta, \{\sigma_t\}_{t=1}^T, \{w_t\}_{t=1}^T)$;
4. Sample parameters from $p(\theta | \{\alpha_t\}_{t=1}^T, \{\beta_t\}_{t=1}^T, \{\log\sigma_t\}_{t=1}^T, \{w_t\}_{t=1}^T)$.

In the first block, the conditional posterior of $\log \sigma_t$ comes from the following standard stochastic volatility model with measurement equation

$$\delta_t = \sigma_t \epsilon_t, \qquad \delta_t = \big(\Delta s_{t+h} - \alpha_t - \beta_t (i_{t+h} - i^*_{t+h})\big)/\sqrt{w_t}. \tag{10}$$

The model is equivalent to the linear but non-Gaussian state space model

$$\begin{aligned} \log(\delta_t^2) &= 2\log\sigma_t + \xi_t, \qquad \xi_t \sim \log\chi_1^2, \\ \log\sigma_{t+1} &= \mu_{\log\sigma}(1-\phi_{\log\sigma}) + \phi_{\log\sigma}\log\sigma_t + \sigma_{\log\sigma}\eta_{\log\sigma,t}. \end{aligned} \tag{11}$$

According to Kim et al. (1998), the $\log\chi_1^2$ distribution can be closely approximated using a Gaussian mixture with seven components tabulated by the triple (q_j, m_j, ψ_j), $j = 1, ..., 7$ where q_j is the probability that ξ_t is described by component $N(m_j, \psi_j)$. Thus given a sequence of auxiliary variables $s_t \in \{1, ..., 7\}$ indicating which component is chosen, the measurement equation can be written as

$$\log(\delta_t^2) = m_{j,s_t} + 2\log\sigma_t + \sqrt{\psi_{j,s_t}}\xi_t^*, \qquad \xi_t^* \sim N(0,1). \tag{12}$$

So conditional on $S_T = (s_1, ..., s_T)$, system (11) and (12) form a linear Gaussian state space model with time-variant but pre-determined transition matrices. The simulation smoother of Frühwirth-Schnatter (1994) or De Jong and Shephard (1995) can be used to draw $\log\sigma_t$ for $t = 1, ..., T$ as one block efficiently. The sampling of S_T can be easily done due to the fact $p(s_t = j | \log(\delta_t^2), \log\sigma_t) \propto q_j N(\log(\delta_t^2); m_j + 2\log\sigma_t, \psi_j)$. Importantly, S_T is sampled at the end of each MCMC iteration to ensure it is generated from the correct conditional posterior (Del Negro and Primiceri, 2015).

The mixing component w_t that models the potential heavy-tailedness of exchange rate return in the second block can be sampled based on rewriting (10) as $\tilde{\delta}_t = \sqrt{w_t}\epsilon_t$ where $\tilde{\delta}_t = \big(\Delta s_{t+h} - \alpha_t - \beta_t(i_{t+h} - i^*_{t+h})\big)/\sigma_t$. Thus for $t = 1, ..., T$, w_t is sampled from $IG((\zeta+1)/2, (\zeta + \tilde{\delta}_t^2)/2)$ in parallel.

Given the volatility process σ_t, TVS-SV model (2) becomes a linear Gaussian state space model with pre-determined time-variant system matrices. A simulation smoother is used to generate draws of α_t and β_t for $t = 1, ..., T$ as one block.

For $f = (\alpha, \beta, \log\sigma)$, the unconditional mean μ_f is drawn from a Gaussian distribution $N(\underline{u_f}, \underline{v_f})$ with

$$\underline{v_f} = \left(\frac{1-\phi_f^2}{\sigma_f^2} + \frac{(T-1)(1-\phi_f)^2}{\sigma_f^2} + \frac{1}{v}\right)^{-1},$$
$$\underline{u_f} = \underline{v_f}\left(\frac{u_f}{v} + \frac{(1-\phi_f^2)f_1}{\sigma_f^2} + (1-\phi_f)\frac{\sum_{t=2}^T (f_t - \phi_f f_{t-1})}{\sigma_f^2}\right).$$

The conditional posterior distribution of innovation variance σ_t is $IG(\underline{\gamma_f}, \underline{\delta_f})$ with $\underline{\gamma_f} = \gamma + T/2$ and $\underline{\delta_f} = \delta + \frac{1}{2}(\sum_{t=2}^T (f_t - \phi_f f_{t-1})^2 + (1-\phi_f^2)(f_1 - \mu_f)^2)$. Let $f_t^* = f_t - \mu_f$. Given the $Beta(a, b)$ prior, the conditional posterior distribution of autoregression coefficient is given by

$$p(\phi_f | f_1, ..., f_T, \mu_f, \sigma_f) \propto Beta(\phi_f; a, b)\sqrt{1-\phi_f^2}\exp\left(-\frac{(1-\phi_f^2)f_1^2}{2\sigma_f^2} - \sum_{t=2}^T \frac{(f_t^* - \phi_f f_{t-1}^*)^2}{2\sigma_f^2}\right)$$
$$\propto (1+\phi_f)^{a-1}(1-\phi_f)^{b-1}\sqrt{1-\phi_f^2}\exp\left(\frac{(\phi_f - \underline{a})^2}{2\underline{b}}\right),$$

where $\underline{a} = (\sum_{t=2}^T f_t^* f_{t-1}^*)/\sum_{t=1}^{T-1} f_t^{*2}$ and $\underline{b} = \sigma_f^2/\sum_{t=1}^{T-1} f_t^{*2}$. To sample from this distribution, we apply the Metropolis-Hastings accept-reject algorithm (Chib and Greenberg, 1995) by drawing a candidate ϕ^{new} from $N(\underline{a}, \underline{b})$ truncated between $(-1, 1)$ to ensure stationarity, and the draw is accepted with probability

$$\min\left(\frac{(1+\phi_f^{new})^{a-1}(1-\phi_f^{new})^{b-1}\sqrt{1-\phi_f^{new2}}}{(1+\phi_f)^{a-1}(1-\phi_f)^{b-1}\sqrt{1-\phi_f^2}},\ 1\right).$$

Appendix B Observation weights

Conditional on $\sigma_1, ..., \sigma_T$, *e.g.* an MCMC draw or a posterior estimate, and θ, Kalman filter and smoother can be used to compute the observation weights of the TVC-SV model (2), with

respect to the UIP coefficient β_t. For simplicity, we assume $\alpha_t = 0$ for all t, which is in line with our empirical results. Let $\beta_t^* = \beta_t - \mu_\beta$. The state space model becomes

$$\bar{y}_t = \beta_t^* + \frac{\sigma_t}{\tilde{x}_t}\epsilon_t,$$
$$\beta_{t+1}^* = \phi_\beta \beta_t^* + \sigma_\beta \eta_{\beta,t},$$
$$\beta_1^* \sim N\left(0, \frac{\sigma_\beta^2}{1-\phi_\beta^2}\right),$$

where $\bar{y}_t$ is equal to $\tilde{y}_t/\tilde{x}_t - \mu$. Suppose the system were time-invariant, say $\sigma_1/\tilde{x}_1 = ... = \sigma_T/\tilde{x}_T = \sigma_\epsilon$.[1] Kalman filter outputs reach their steady state quickly (Durbin and Koopman, 2012, Chapter 4). Let a_t and P_t denote the filtering expectation $E(\beta_t|\tilde{Y}_{t-1}, \tilde{X}_{t-1})$ and variance $Var(\beta_t|\tilde{Y}_{t-1}, \tilde{X}_{t-1})$, respectively. The Kalman filter iterates forward over

$$a_{t+1} = \phi_\beta a_t + \frac{P_t}{P_t + \sigma_\epsilon^2}(\bar{y}_t - A_t), \quad P_{t+1} = \frac{P_t \sigma_\epsilon^2}{P_t + \sigma_\epsilon^2} + \sigma_\beta^2.$$

The steady state is given by the fixed-point solution to the second equation, and it is $\bar{P} = (q + \sqrt{q^2 + 4q})/2$ where $q = \sigma_\beta^2/\sigma_\epsilon^2$ is the signal-to-noise ratio. Let b_t denote the smoothed expectation $E(\beta_t|\tilde{Y}_T, \tilde{X}_T)$. The Kalman smoother iterates backward from $r_T = 0$ and $Var(r_t) = N_t$ over

$$r_{t-1} = \frac{\bar{y}_t - a_t}{P_t + \sigma_\epsilon^2} + \frac{\sigma_\epsilon^2}{P_t + \sigma_\epsilon^2} r_t, \quad b_t = a_t + P_t r_{t-1}, \quad N_{t-1} = \frac{1}{P_t + \sigma_\epsilon^2} + \left(\frac{\sigma_\epsilon^2}{P_t + \sigma_\epsilon^2}\right)^2 N_t.$$

Because $\frac{\sigma_\epsilon^2}{P_t + \sigma_\epsilon^2} < 1$, so the steady state of N_t exists and is $\bar{N} = (\bar{P} + \sigma_\epsilon^2)/(\bar{P}^2 + 2\bar{P}\sigma_\epsilon^2)$.

Suppose we can write $b_t = \sum_{j=1}^T \omega_{jt}\bar{y}_j$ with weight ω_{jt} associated with the j-th observation corresponding to t-th smoothed estimate. Then we have $E(b_t\epsilon_j) = \omega_{jt}E(\bar{y}_j\epsilon_j) = \omega_{jt}$, but

$$E(b_t\epsilon_j) = \begin{cases} -Cov(\epsilon_j - E(\epsilon_j|\tilde{Y}_T, \tilde{X}_T), \beta_t - b_t), & \text{for } j < t; \\ -Cov(\beta_t - b_t, \epsilon_j - E(\epsilon_j|\tilde{Y}_T, \tilde{X}_T)), & \text{for } j \geq t. \end{cases}$$

After minor algebraic manipulation, the steady state gives for $j < t$

$$Cov(\epsilon_j - E(\epsilon_j|\tilde{Y}_T, \tilde{X}_T), \beta_t - b_t) = E(\epsilon_j(\beta_t - b_t)) = -\frac{\bar{P}}{\bar{P} + \sigma_\epsilon^2}\left(\frac{\sigma_\epsilon^2}{\bar{P} + \sigma_\epsilon^2}\right)^{t-j} \frac{\bar{P}\sigma_\epsilon^2}{\bar{P}^2 + 2\bar{P}\sigma_\epsilon^2}.$$

[1] One can think of σ_ϵ as the unconditional mean of $\sigma_1/\tilde{x}_1$, because the interest rate differential in the denominator is stationary between developed economies, and the stochastic volatility is assumed to be mean-reverting.

Similarly, for $j \geq t$ we have

$$Cov(\beta_t - b_t, \epsilon_j - E(\epsilon_j | \tilde{Y}_T, \tilde{X}_T)) = -\frac{\bar{P}}{\bar{P} + \sigma_\epsilon^2} \left(\frac{\sigma_\epsilon^2}{\bar{P} + \sigma_\epsilon^2} \right)^{j-t} \frac{\bar{P}\sigma_\epsilon^2}{\bar{P}^2 + 2\bar{P}\sigma_\epsilon^2}.$$

So as j moves away from t, observation $\bar{y}_j$ receives exponentially declining weight proportional to

$$\omega_{jt} \propto \left(\frac{\sigma_\epsilon^2}{\bar{P} + \sigma_\epsilon^2} \right)^{|t-j|} = \left(\frac{2\sigma_\epsilon^4}{\sigma_\beta^2 + \sqrt{\sigma_\beta^4 + 4\sigma_\beta^2\sigma_\epsilon^2} + 2\sigma_\epsilon^4} \right)^{|t-j|}.$$

This result is heuristic in our case because when stochastic volatily is present, the steady state of Kalman filter ceases to exist; but the first-order approximation of the weighting function can be computed by replacing σ_ϵ by $\sigma_t / \tilde{x}_t$. So if σ_t increases or the interest rate differential $\tilde{x}_t$ decreases, ω_{jt} becomes larger. This means that during volatile times, the accounting of UIP coefficient β_t relies on more backward and forward information, and vice versa.