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Optimal climate policy with directed technical change, extensive margins and decreasing substitutability between clean and dirty energy

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Abstract

This paper uses a benchmark climate model with endogenous technical change to consider the effects of three extensions on optimal policy under a clean transition. First, the movement of workers between non-energy and energy sectors lowers the cost of abatement by more than an order of magnitude, favouring taxes over subsidies. Second, the free movement of researchers between non-energy and energy sectors increases the power of policy to avert environmental disaster and leads to a period of intense research in the clean sector above the long-run share, as productivity in the clean sector catches up to the non-energy sector. Third, a decreasing elasticity of substitution between clean and dirty inputs as the share of clean energy rises is considered, reflecting the increasing difficulty of integrating intermittent clean energy supply in electricity. A decreasing elasticity increases the initial optimal tax on dirty energy and therefore lowers the subsidies required to direct technical change towards clean energy.

Keywords

Climate change, directed technical change, optimal policy, energy

JEL Classification

O33, O44, Q30, Q54, Q56, Q58

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Optimal climate policy with directed technical change, extensive margins and decreasing substitutability between clean and dirty energy

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This paper uses a benchmark climate model with endogenous technical change to consider the effects of three extensions on optimal policy under a clean transition. First, the movement of workers between non-energy and energy sectors lowers the cost of abatement by more than an order of magnitude, favouring taxes over subsidies. Second, the free movement of researchers between non-energy and energy sectors increases the power of policy to avert environmental disaster and leads to a period of intense research in the clean sector above the long-run share, as productivity in the clean sector catches up to the non-energy sector. Third, a decreasing elasticity of substitution between clean and dirty inputs as the share of clean energy rises is considered, reflecting the increasing difficulty of integrating intermittent clean energy supply in electricity. A decreasing elasticity increases the initial optimal tax on dirty energy and therefore lowers the subsidies required to direct technical change towards clean energy. (JEL O33, O44, Q30, Q54, Q56, Q58)

Keywords: Climate change, directed technical change, optimal policy, energy.

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This paper examines three key questions relating to the transformation of the energy sector towards clean technologies. The first is the role of the extensive labour margin in the energy sector, where workers can move between non-energy and energy sectors. Such flexibility is important as it lowers the cost of transformation of the energy sector and changes optimal policy. The second is the role of the extensive research margin where researchers can move between non-energy and energy sectors, allowing a lagging clean sector to catch up to the non-energy sector. The third question is how a decreasing elasticity of substitution between clean and dirty inputs, epitomized by the increasing difficulty of integrating clean intermittent sources of electricity supply as the clean share rises, may affect optimal policy.

This paper builds on the growth model with endogenous technology described in Acemoglu, Aghion, Bursztyn, and Hemous (2012), hereafter referred to as AABH, which has led to a number of related papers.[1] This growth model has only two sectors: a clean and a dirty sector. Optimal policy relies on both a distortionary carbon tax and costless research subsidies. The strength of this framework is the analytical tractability of the monopolistically competitive framework which allows profits to accrue, and therefore subsidies to be determined.

The first major extension is the introduction of a non-energy sector and the free movement of labour between non-energy and energy sectors (referred to as the extensive labour margin). This extension is important as the transformation of the economy to clean energy is limited to a small proportion of the economy. An extensive labour margin only marginally increases the complexity of the model but the impact is large: the costs of abatement are reduced by a factor approximately equal to the inverse of the share of total labour in the energy sector (20 in this paper)

[1] For example, Greaker and Heggedal (2012), Greaker, Heggedal, and Rosendahl (2018) , Pottier, Hourcade, and Espagne (2014) , Acemoglu, Aghion, and Hémous (2014), Durmaz and Schroyen (2013), Van den Bijgaart (2017) and Lemoine (2017).

while the effects of damages are unchanged. This straightforward result has important implications in determining the optimal balance between subsidies and taxes, which many papers have examined. The findings of Greaker et al. (2018) and Lemoine (2017) that subsidy-only policy out-performs a tax-only policy would likely be reversed if these studies explicitly considered a non-energy sector. The implausibly high abatement costs in the basic energy-only AABH framework, noted by Pottier et al. (2014), are corrected with the addition of the non-energy sector. This is not a criticism of the AABH framework which this paper builds on, but highlights the limitation when using the unaltered framework to weigh policy options where abatement costs affect one instrument (the tax) but not the other (subsidy). The current paper finds that lower abatement costs tend to lead to higher optimal taxes and lower costs for tax-only policy compared with subsidy-only policy, as found by Hart (2019) who also includes a non-energy sector.[2]

The second major extension adds an extensive research margin so that researchers are able to choose between non-energy and energy based on future profits. The corner solution where all research is undertaken in one sector in the long-run[3] does not apply, and research in energy and non-energy asymptote to long-run shares. This distinction means that subsidies are typically applied for longer with the extensive research margin. The stabilization of research shares at long-run rates is due to a low elasticity of substitution between non-energy and energy and such a dynamic is similar to that described in Lemoine (2017).

An extensive research margin also lowers the cost of abatement and increases the power of technical change to avert environmental disaster. Without an extensive research margin, a subsidy alone can only avoid disaster when clean and dirty inputs are strongly substitutable. With an extensive research margin, disaster can

[2] Fried (2018) also considers a three-sector model where an emissions target is set exogenously.

[3] Found in the AABH model and similar subsequent papers.

be avoided for any positive elasticity of substitution. However, this flexibility occurs at the cost of long-run growth for weak substitutes, as the research share in clean energy increases and displaces research in the non-energy sector. The optimal tax is temporary for strong substitutes and permanent for weak substitutes, while the optimal subsidy is always temporary or asymptotes to zero when both instruments are available.

An extensive research margin allows consideration of temporarily using researchers from the non-energy sector to boost the productivity of a lagging clean energy sector under a clean transition. It seems likely that, under optimal conditions, additional researchers would be used to raise clean productivity so that it catches up to non-energy productivity. This intuition is correct: under optimal policy there is a period of clean technology catch-up with the non-energy sector, where the clean research share exceeds the long-run share.

The third major extension in the current paper is consideration of a decreasing elasticity of substitution between clean and dirty energy. Papers investigating optimal climate policy have considered multiple dependencies including the substitutability between clean and dirty inputs, but uncertainty in substitutability is often reflected through sensitivity analysis using different elasticities in an isoelastic production function. However, it is well known that for electricity, the largest component of energy production, variable clean energy sources become harder to integrate as their share increases. For example, variable generation lowers the utilisation rates of dirty generation, and at high penetration rates, curtailment can occur when clean electricity supply exceeds total demand, with some supply therefore wasted.[4] The corresponding effect of a decreasing elasticity of substitution on optimal policy and climate outcomes has not been examined

[4] For example, see Hirth, Ueckerdt, and Edenhofer (2015) and Ueckerdt, Hirth, Luderer, and Edenhofer (2013).

explicitly in a macroeconomic model, and indeed contrasts with some other papers that discuss how the elasticity may increase with time.[5]

This paper uses a bimodal isoelastic production function to induce a decrease in the elasticity of substitution, which is relatively simple to implement and is a reasonable approximation to results from a structural energy model (Wiskich, 2019b). A high elasticity applies for clean to dirty ratios below a switch point w, and a low elasticity above this point. A similar production function is described in Antony (2009) and builds on Jones (2003). While the elasticity of substitution between clean and dirty inputs used in macroeconomic models generally lie between 10 and 1,[6] empirical estimates of the elasticity are typically below 3.[7] Wiskich (2019b) argues for a high elasticity of 3 or above for a clean share below 50 percent, with a lower elasticity of 3 or below for higher clean shares. All numerical results present the bimodal case with high and low elasticities of 4 and 2,[8] with the switch at a clean share of 50%, against isoelastic cases with these high and low elasticities.

A consequence of a decreasing elasticity is that, if clean research occurs immediately, the lower future elasticity means less future environmental quality. Expectations of lower environmental quality in the future raises the initial tax, relative to the high isoelastic case, and therefore lowers the subsidies required to direct clean research. Numerical examples find that while the tax in the bimodal simulation lies between the tax in the high and low isoelastic cases, the bimodal subsidy can be lower than the subsidy in both isoelastic cases due to this effect.

In addition to the major extensions discussed, this paper also considers some sensitivities introduced in other papers. A ‘stepping on toes’ effect, where the

[5] For example, Mattauch, Creutzig, and Edenhofer (2015).

[6] For example see Lanzi and Sue Wing (2011) and Acemoglu et al. (2012).

[7] Examples include Papageorgiou, Saam, and Schulte (2017), Lanzi and Sue Wing (2011) and Stern (2012).

[8] Similar to the isoelastic values of 3 and 1.5 considered by Greaker et al. (2018), and 4 adopted by HART.

marginal returns to research decrease when the share of researchers in a sector becomes high, is considered in the case of an extensive research margin.[9] This effect increases the cost of a tax-only policy as a higher initial subsidy is required, and can induce immediate clean research to lower future 'stepping on toes' costs when clean research is boosted. Extended patent lifetimes are discussed following Greaker et al. (2018) and are found to have little effect, contrasting with Greaker and Heggedal (2012) who find that extended patent lifetimes leads to a diminished or non-existent role for subsidies. Finally, the impacts of policy delay and a costly subsidy are examined.

This paper also goes further than the AABH paper in establishing several interesting interactions between the elasticity of substitution, consumption, profits and taxes. For example, in the presence of a carbon tax, consumption can temporarily decrease as clean productivity rises, particularly so if the elasticity is high, and a drop in elasticity can lead to a temporary boost in consumption. When subsidies have a distortionary cost, optimal taxes are increased by an amount proportional to this cost and roughly independent of the elasticity of substitution and the relative productivities of clean and dirty technologies

The main contributions of this paper are the major extensions to the AABH model and associated insights. Consideration of both an extensive labour margin in production and researchers in an energy context has important policy implications, and the explicit consideration of a decreasing elasticity in a macroeconomic model with the use of the bimodal production function is novel in this field. Extensive margins and a decreasing elasticity affect both the profile of production and the optimal timing of clean research, and while this paper focusses on optimal policy, environmental and economic outcomes are obviously impacted.

[9] I only apply diminishing returns above the long-run share of energy researchers, rather than the functional form typically adopted which implies returns become infinite as researchers go to zero in any sector.

Section 1 describes the model including the bimodal production function and section 2 presents characteristics of the model. I have split numerical examples into section 3, which considers an extensive labour margin, and section 4 which adds an extensive research margin.

I. Model

The model builds on the socially optimal allocation in the AABH framework without exhaustible resources. A representative household maximises

(1) $$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t),$$

where C_t is consumption, S_t is environmental quality, ρ is the discount rate and the instantaneous utility function $u(C_t, S_t)$ satisfies the usual conditions of differentiability, concavity and Inada conditions. Aggregate consumption demands a fixed proportion of energy C_e to non-energy C_0. As energy demand is relatively inelastic, assuming a fixed proportion seems reasonable and is the simplest approach.[10]

(2) $$C_t = \min(aC_{0t}, C_{et}).$$

Total energy production Y_e, some of which is absorbed in the energy production process, is produced competitively using clean and dirty inputs, Y_c and Y_d, according to a bimodal elasticity of substitution production function. For technical

[10] However, omitting the possibility of responding to changes in energy prices by changing energy intensity increases the cost of abatement.

efficiency variable A_e and elasticity of substitution between clean and dirty energy σ, we have:

$$(3) \qquad Y_{et} = A_e \left(\beta Y_{ct}^{\frac{\sigma-1}{\sigma}} + Y_{dt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \text{ where}$$

$$\sigma, A_y, \beta = \begin{cases} \sigma_1, 1, 1 & if\ \bar{Y} < w \\ \sigma_2, \left(w^{\frac{\sigma_1-1}{\sigma_1}} + 1 \right)^{\frac{\sigma_1}{\sigma_1-1} - \frac{\sigma_2}{\sigma_2-1}}, w^{\frac{1}{\sigma_2} - \frac{1}{\sigma_1}} & if\ \bar{Y} > w. \end{cases}$$

The elasticity of substitution σ switches from σ_1 to σ_2 when the ratio of clean to dirty inputs $\bar{Y} := \frac{Y_c}{Y_d}$ exceeds a switch point w. This switch in elasticity generalises the AABH model and allows consideration of a change in the elasticity on model results.

The environmental externality is caused by the production of the dirty input so that the quality of the environment evolves as follows:[11]

$$(4) \qquad S_{t+1} = -\xi Y_{dt} + (1 + \delta) S_t,$$

whenever the right-hand side of (3) is in the interval $(0, \bar{S})$[12], where $\bar{S}$ is the quality of the environment absent any human pollution. The parameter ξ measures the rate of environmental degradation due to the production of dirty inputs, and δ is the rate of environmental regeneration. The shadow price of input j relative to the price of the final good are denoted p_{jt}, with first-order conditions with respect to Y_{ct} and Y_{dt} giving

[11] This paper adopts the relatively simple AABH climate model, as the key insights of the effects of extensive margins and a decreasing elasticity should persist with a more realistic climate model, and this approach allows greater comparability.

[12] Specifically $S_{t+1} = \min(\max(-\xi Y_{dt} + (1 + \delta) S_t; 0); \bar{S})$.

(5) $$p_{ct} = A_e \beta Y_{ct}^{\frac{-1}{\sigma}} \left(\beta Y_{ct}^{\frac{\sigma-1}{\sigma}} + Y_{dt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \text{ and}$$

$$p_{dt}(1+\tau) = A_e Y_{dt}^{\frac{-1}{\sigma}} \left(\beta Y_{ct}^{\frac{\sigma-1}{\sigma}} + Y_{dt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} .$$

These shadow prices satisfy

(6) $$\beta^{\sigma} \left(\frac{p_{ct}}{A_e} \right)^{1-\sigma} + \left(\frac{p_{dt}}{A_e} (1+\tau_t) \right)^{1-\sigma} = 1.$$

The inputs Y_0, Y_c and Y_d are produced using labour and a continuum of sector-specific intermediates:

(7) $$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} \, di,$$

where A_{jit} is the quality of intermediate of type i used in sector j at time t and x_{jit} is the quantity of this intermediate. Total labour supply is normalised to 1:

(8) $$L_{ct} + L_{dt} = L_{et}, \qquad L_{et} + L_{0t} = L_t := 1.$$

Intermediates are supplied by monopolistically competitive firms and cost ψ units of the final good which is normalised to $\psi := \alpha^2$. For simplicity, I assume non-energy intermediates cost ψ units of the non-energy good and energy intermediates cost ψ units of the energy good. Market clearing for the final good implies that

(9.1) $$C_{0t} = Y_{0t} - \psi \int_0^1 x_{0it}\, di \text{ and}$$

(9.2) $$C_{et} = Y_{et} - \psi \left(\int_0^1 x_{cit}\, di + \int_0^1 x_{dit}\, di \right),$$

and consumption is maximised when

(10.1) $$x_{0it} = \left(\frac{\alpha}{\psi} \right)^{\frac{1}{1-\alpha}} A_{0it} L_{0t} \text{ and}$$

(10.2) $$x_{jit} = \left(\frac{\alpha p_{jt}}{\psi} \right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \text{ for } j \in \{c, d\}.$$

Combining (10) with (7) implies

(11.1) $$\mathrm{Y}_{0t} = \left(\frac{\alpha}{\psi} \right)^{\frac{\alpha}{1-\alpha}} L_{0t} A_{0t} \text{ and}$$

(11.2) $$\mathrm{Y}_{jt} = \left(\frac{\alpha}{\psi} p_{jt} \right)^{\frac{\alpha}{1-\alpha}} L_{jt} A_{jt} \text{ for } j \in \{c, d\},$$

where the average productivity in sector j is

(12) $$A_{jt} := \int_0^1 A_{jit}\, di.$$

Labour is divided between non-energy and energy to ensure maximization of consumption (2.1) so $aC_{0t} = C_{et}$, which implies

(13) $$L_{0t} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{\alpha-1}} \frac{C_{et}}{\alpha A_{0t}(1-\alpha)}.$$

The equalisation of wages sets the price of the non-energy sector and the relative prices of clean and dirty inputs as follows:

(14.1) $$p_{0t}\frac{Y_{0t}}{L_{0t}} = \frac{Y_{et}}{L_{et}} \;\; as\; p_{et} = 1 \text{ and}$$

(14.2) $$p_{ct}^{\frac{1}{1-\alpha}} A_{ct} = p_{dt}^{\frac{1}{1-\alpha}} A_{dt}.$$

Intermediate producers maximise profit $\pi_{jit} = (p_{jit} - \psi p_{jt})\, x_{jit}$ subject to (10) which implies a constant markup over marginal cost $p_{jit} = \psi p_{jt}/\alpha$. Equilibrium profits are therefore

(15.1) $$\pi_{0it} = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \; p_{0t}L_{0t}A_{0it} \text{ and}$$

(15.2) $$\pi_{jit} = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \; p_{jt}^{\frac{1}{1-\alpha}} L_{jt}A_{jit} \text{ for } j \in \{c, d\}.$$

Combining (5), (11.2) and (14.2) implies

(16) $$\frac{L_{ct}}{L_{dt}} = \beta^{\sigma}(1+\tau_t)^{\sigma}\bar{A}_t^{\,-\varphi} \text{ where } \; \bar{A}_t := \frac{A_{ct}}{A_{dt}}.$$

Combining the equations above allows prices, labour, output and consumption to be written as functions of productivities and the carbon tax, listed in Appendix A. The ratio of clean to dirty inputs is $\bar{Y} = \beta^{\sigma}(1+\tau_t)^{\sigma}\bar{A}^{\sigma(1-\alpha)}$. For zero taxes this ratio, denoted $\bar{Y}_0$, is $\bar{Y}_0 = \beta^{\sigma}\bar{A}^{\sigma(1-\alpha)} = \bar{A}^{\sigma(1-\alpha)}$ for $\bar{Y} < w$. This ratio is useful to compare the isoelastic and bimodal results because initial clean shares are equal by construction in the numerical examples.

Technology advances due to the research of scientists, and each scientist decides at the start of each period to direct their research to clean or dirty technologies and, when an extensive research margin is considered, non-energy technologies. Scientists are successful in innovation in sector j with probability η, where innovation increases the quality of intermediates by a factor $1+\gamma$. The total number of scientists is normalised to 1:

$$(17) \qquad s_{0t} + s_{ct} + s_{dt} \leq 1.$$

As a sensitivity, I apply a 'stepping on toes' effect where the marginal returns to research decrease when the share of researchers in any sector exceeds the respective long-run share. This approach avoids returns to research approaching infinity as the number of researchers falls to zero, which coupled with a zero cost of subsidies, leads to extremely high and permanent subsidies as found by Greaker et al. (2018). For long-run shares s_j^{LR}, with $s_d^{LR} = s_c^{LR} := s_e^{LR}$ and $s_0^{LR} = 1 - s_e^{LR}$, productivity evolves according to

$$(18) \qquad A_{jt} = \left(1+\gamma\eta s'_{jt}\right)A_{jt-1} \text{ where } s'_{jt} = \begin{cases} \dfrac{s_{jt}}{s_{jt}^{LR}} \; if \; s_{jt} < s_{jt}^{LR} \\ \left(\dfrac{s_{jt}}{s_{jt}^{LR}}\right)^{\omega} \; if \; s_{jt} > s_{jt}^{LR}. \end{cases}$$

A successful scientist obtains a one-period patent in the main scenario, following AABH. As another sensitivity, I consider extended patent lifetimes following Greaker et al. (2018) with patents lasting 20 years, as this duration applies in most patent laws. Expected contemporaneous profits are

(19.1) $$\Pi'_{0t} = (1 + q_{0t})\eta \frac{s'_{jt}}{s_{jt}} (1+\gamma)(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \ p_{0t} L_{jt} A_{jt-1} \text{ and}$$

(19.2) $$\Pi'_{jt} = \left(1 + q_{jt}\right)\eta \frac{s'_{jt}}{s_{jt}} (1+\gamma)(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_{jt}^{\frac{1}{1-\alpha}} L_{jt} A_{jt-1}$$

for $j \in \{c, d\}$ and q_t is a proportional research subsidy. For the extended patent lifetime sensitivity, profits are discounted by a replacement rate $z_{jt} := \eta_j s'_{jt}$ which is the chance that an innovation will be superseded and hence profits become zero, and the discount rate which I approximate as $\rho + \eta\varepsilon$ as η is the long-run growth in the economy:

(20) $$\Pi_{jt} = \sum_{k=0}^{n} \prod_{\pi=1}^{k} \left(\frac{1 - z_{j,t+\pi}}{1 + \rho + \eta\varepsilon}\right) \Pi'_{jt}.$$

Consider a fixed split of researchers between non-energy and energy. In each period, all scientists in the energy sector either research in clean or dirty if $1 + \varphi < 0$, or equivalently if elasticity $\sigma > \frac{2-\alpha}{1-\alpha} = 2.5$ for $\alpha = 1/3$, which holds for an elasticity of 4 discussed in the next section. This polarisation of research in each period occurs due to the dominance of the market size effect ($\Pi_{jt} \sim L_{jt}$) when the elasticity is sufficiently high, implying profits in a sector rise with increasing research in that sector. This leads to multiple equilibria as I describe in a separate paper (Wiskich, 2019a), which proposes either setting $\Pi_{ct}(1) = \Pi_{dt}(1)$ as a lower bound or $\Pi_{ct}(1) = \Pi_{dt}(0)$ as an upper bound. The latter approach is more complex to implement as it introduces counterfactual outcomes, particularly for extended patent lifetimes investigated in this paper. Therefore, I use the former method where the calculated subsidy represents a lower bound for an elasticity of 4.

When researchers are able to choose between non-energy and energy, the relevant comparator of clean profits is the greater of non-energy and dirty profits.[13] This is a key distinction in how the subsidy is determined: in the AABH model (and similar subsequent papers) it is set to equalize profits between clean and dirty sectors, whereas with an extensive research margin the subsidy is set to equalize profits between the clean and non-energy sectors. As clean and non-energy technology grow and dirty technology stagnates under energy transformation, this distinction means that subsidies are typically applied for longer with the extensive research margin.

When a subsidy is used, the total subsidy expenditure $sub_t = q_{ct}\frac{\Pi'_{ct}}{1+q_{ct}}$ required to direct research to clean energy is such that the critical profit ratio $\frac{\Pi_{ct}}{\Pi_{jt}}$ is 1. As a sensitivity, a proportional cost of subsidies χsub_t is extracted from consumption following Acemoglu, Akcigit, Hanley, and Kerr (2016).

II. Model characteristics

Key results from the AABH paper persist with extensive margins and the adoption of a bimodal elasticity of substitution production function. The first proposition relates to three market failures in the economy: (i) the underutilisation of machines due to monopoly pricing; (ii) the environmental externality; and (iii) the knowledge externality in the technology frontier.

PROPOSITION 1: The socially optimal allocation can be implemented using: (i) a tax on dirty input (a "carbon" tax); (ii) a subsidy to clean innovation and, with an extensive research margin, subsidies to the other sectors; and (iii) a subsidy for the

[13] In the numerical examples, non-energy profits are always greater.

use of all machines (all proceeds from taxes/subsidies being redistributed/financed lump sum).

The proof closely follows the proof of proposition 5 in AABH and so is omitted. The only difference is that subsidies to the dirty or non-energy sectors are required when an extensive research margin is considered. It is useful to describe the long-run asymptotic properties of key variables under energy transformation.

LEMMA 1: Under clean energy transformation, long-run properties for dirty and clean inputs and energy consumption, with $\bar{Y} < w$ and $A_{dt} = 1$ for simplicity, are:

$$\mathrm{Y}_{dt} \xrightarrow{t\to\infty} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{{A_{ct}}^{\alpha+\varphi} L_{et}}{(1+\tau_t)^{\sigma}}, \qquad \mathrm{Y}_{et} \xrightarrow{t\to\infty} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_{ct} L_{et} \text{ and}$$

$$\mathrm{C}_{et} \xrightarrow{t\to\infty} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) A_{ct} L_{et}.$$

It is clear from this result that dirty inputs can be driven to zero by a sufficiently large tax, and that this tax has no effect on long-run consumption. The optimal tax rate reflects marginal benefits from environmental quality relative to the marginal benefit of additional consumption. The economic burden of this tax, as well as the cost of directing clean research, is much reduced from the introduction of the non-energy sector and an extensive labour margin.

PROPOSITION 2: For a given small distortionary cost x to the energy sector holding energy labour fixed, the introduction of an extensive labour margin (holding research fixed) reduces aggregate costs to the share of labour in energy times x. The extensive research margin (holding labour fixed) implies aggregate costs are the long-run share of energy researchers times x in the long-run, but the cost is greater during the transformation of the energy sector.

The proof is intuitive: consumption in the non-energy sector is proportional to output, which is linear in labour and technology from (11.1). For small changes, energy output is also linear in labour. Thus, any distortionary cost to the energy sector can be largely balanced by a shift in labour from the non-energy sector. Considering an extensive research margin, in the limit that clean (or dirty) technology dominates and $\sigma > 1$, energy output is also linear in clean (or dirty) technology so a similar result applies. However, before either clean or dirty sources have dominated, output is less than linear in either technology so the aggregate costs are higher.

It is insightful to consider situations where only one policy instrument is available. As lemma 1 shows, increasing the tax to infinity drives dirty input Y_d to zero, environmental disaster can always be avoided with a tax alone. A similar result applies to a clean subsidy alone in the long-run with an extensive research margin, but not when the quantity of energy research is fixed.

PROPOSITION 3: For strong substitutes: a temporary subsidy avoids disaster if $\bar{S}$ is sufficiently high. For weak substitutes $1 < \sigma < 1/(1-\alpha)$, disaster cannot be avoided with a subsidy alone without an extensive research margin. With an extensive research margin, disaster can be avoided with a permanent subsidy alone for any positive elasticity (for sufficiently high $\bar{S}$) at the cost of long-run growth, which is $\frac{\sigma(1-\alpha)}{(1/s_e^{LR}-1)\sigma(1-\alpha)+1}\gamma\eta$.

The proof is in Appendix C. The increased power of directed technical change to avert environmental disaster when an extensive research margin applies is a key result of this paper. The next proposition considers optimal policy when both instruments are applicable.

PROPOSITION 4: With an extensive research margin, researchers approach the long-run share and both non-energy and energy sectors asymptotically grow at the rate $\gamma\eta$. For strong (long-run) substitutes $\sigma > \frac{1}{1-\alpha}$ the tax is temporary, while for weak substitutes $1 < \sigma < 1/(1-\alpha)$ the tax is permanent.

The proof of this proposition for strong substitutes is straightforward. In the long run, energy output under transformation and non-energy output are proportional to $A_c L_e$ and $A_0 L_0$. As long-run consumption from each sector is $(1-\alpha)$ times output, and given flexibility in labour and technology allocation, it is optimal to allocate an energy share of s_e^{LR} for both technology and labour, so that A_c approaches A_0. The case for weak substitutes is interesting as both a tax or subsidy can avoid disaster in the long-run. As lemma 1 describes, the long-run impact of the tax on consumption disappears. As described in proposition 3, a subsidy can push the research share in clean energy above s_e^{LR} at a cost to growth. Thus, the tax instrument is used permanently while the subsidy approaches zero, and researchers approach the long-run share and the economy grows at the rate $\gamma\eta$.

As technology in both energy sectors starts below the non-energy sector in the numerical model, the following corollary follows.

COROLLARY 1: With an extensive research margin, if clean technology initially lags non-energy, research in energy must exceed the long-run level for a period.

Except for the elasticity of substitution, this paper adopts AABH parameters for the numerical examples discussed in section 3, shown in Appendix A. [14] The high and low isoelastic cases can be considered as bimodal with limits $w \to \infty$ and $w = 0$ respectively. The next proposition discusses the behaviour of energy output available for consumption, C_{et}, with increasing clean productivity, in the presence of taxes and a decreasing elasticity.

PROPOSITION 5: If taxes are zero, energy consumption strictly increases with advancing clean technology, and a switch to a lower elasticity decreases consumption relative to the isoelastic case. In the presence of taxes, consumption can decrease as clean productivity rises, particularly if the elasticity is high, and consumption can increase from a switch to a lower elasticity. The same results apply to total energy output as well as output available for consumption.

The proof of this proposition, and proofs for all further results in this section, are in Appendix C. When taxes are zero, a strictly increasing consumption with rising clean productivity is intuitive. Perhaps less intuitive is a declining consumption with rising clean productivity in the presence of a tax, the extent of which increases with the elasticity, and a potential increase in consumption from a fall in elasticity. Figure 1 demonstrates these effects for a tax rate of 0.5 for illustration, with an elasticity of 4 (and low elasticity of 2 for the bimodal case) consistent with the numerical examples in the next section.

[14] Further explanations of the parameter and functional choices are outlined in AABH.

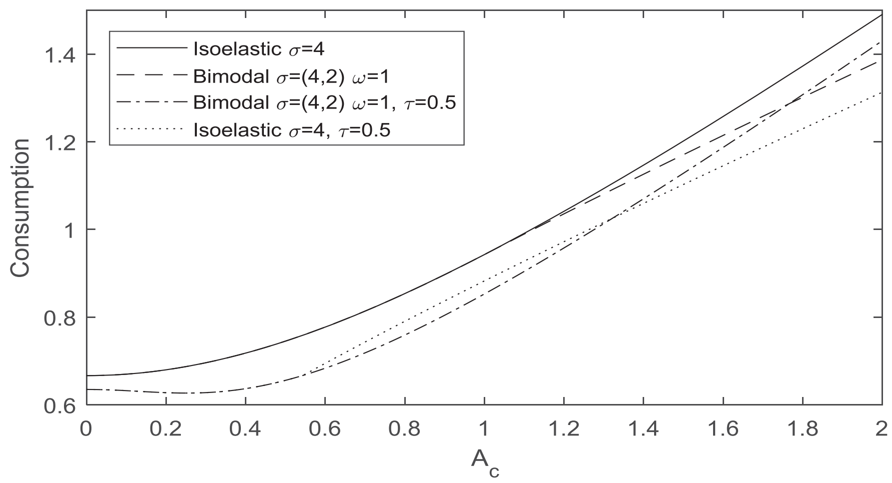

FIGURE 1: ENERGY OUTPUT AS CLEAN PRODUCTIVITY A_c INCREASES, WITH FIXED DIRTY PRODUCTIVITY $A_d = 1$

Proposition 5 highlights the intuition that, when taxes are zero, a decreasing elasticity lowers output, and thereby consumption, due to reduced substitutability between inputs. Therefore, for the bimodal case, the weight of the initial high elasticity period in the discounted utility function is lower than for the high isoelastic case. In this respect, the effect of a decreasing elasticity on intertemporal optimisation is similar to a decrease in the discount rate.

When a tax is present, consumption in the bimodal case is higher than the isoelastic case for a range of productivity values. Thus, given a tax rate, a fall in elasticity can lead to an increase in consumption for a range of productivities. As clean productivity rises beyond this range, consumption in the high isoelastic case surpasses the bimodal case as flexibility in the production function dominates. The proof of proposition 5 in Appendix C also highlights the continuity of consumption at the switch point, a property which extends to output, inputs and prices.[15] Understanding the interaction between elasticity, consumption, subsidies and taxes

[15] The second derivative of prices are also continuous, and the second derivative of output is continuous when taxes are zero.

helps explain numerical results, and the next lemma describes the marginal effects of taxes on consumption.

LEMMA 2: For small tax rates, the marginal cost to consumption is proportional to the tax rate. If the share of clean energy is also small, the marginal cost of taxes is a function of α and independent of the elasticity. The low tax approximation in the isoelastic cases is:[16]

$$(21) \qquad \frac{1}{L_e}\frac{\partial C_e}{\partial \tau} \xrightarrow{low\ \tau} -\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_d}{(\bar{A}^{-\varphi}+1)^{\frac{1}{\varphi}+2}}\left(\frac{\alpha}{1-\alpha}+2\sigma\bar{A}^{-\varphi}\right)\tau.$$

Marginal costs to consumption are discussed further in Proposition 7. To facilitate comparison between and within simulations with differing elasticities, the ratio of clean to dirty energy in the absence of a tax, $\bar{Y}_0 = \bar{A}^{\sigma(1-\alpha)}$, provides a useful measure of the relative productivity between clean and dirty inputs. The next proposition discusses the magnitude of the subsidy or tax required to direct clean research.

PROPOSITION 6: For a low clean share at zero taxes $\bar{Y}_0$, the required subsidy increases with the elasticity of substitution, while the tax required decreases with the elasticity. Subsidy and small tax approximations are as follows:

$$(22) \qquad q_t = \frac{\gamma_c^{1+\varphi}}{(1+\tau_t)^{\sigma}\bar{Y}_{0t-1}^{1-1/\sigma}} - 1 \text{ and } \tau_t \xrightarrow{low\ \tau} \frac{1}{\bar{Y}_{0t-1}^{1/\sigma}} - 1.$$

[16] Time subscripts omitted.

Actual values using the parameter choices are shown in Figure 2 and demonstrate how the required subsidy or tax varies with the elasticity. As described previously, I use $\bar{Y}_0$ on the x-axis to ease comparison across different elasticities which have different starting productivity levels. Thus, for a small initial clean share and low taxes, the subsidy required to direct technical change will increase with the elasticity. However, if a subsidy is not applicable and the tax is used to direct technical change initially, the required tax decreases with the elasticity.

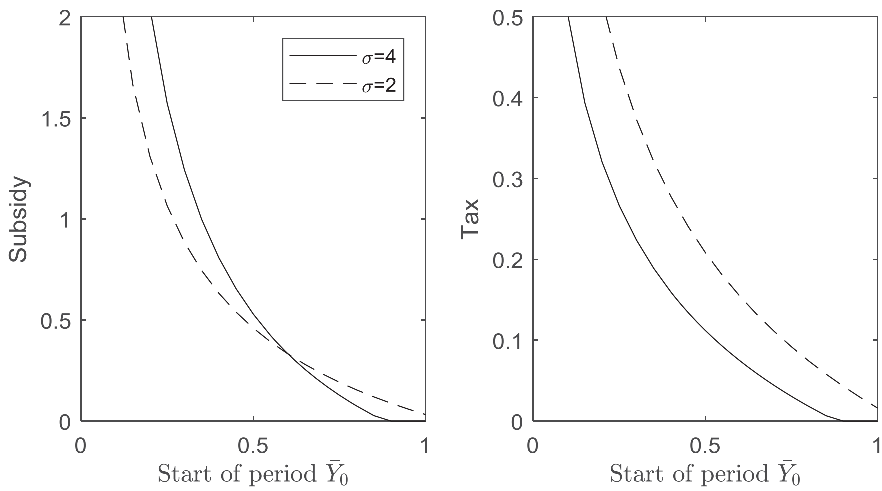

FIGURE 2: SUBSIDY AND TAX REQUIRED TO DIRECT CLEAN RESEARCH

REMARK 1: The subsidy (or tax) required in isolation to direct technical change increases without bound as the ratio of clean to dirty technology approaches zero. However, the required subsidy expenditure is bounded as total clean profits approach zero.

Figure 3 shows the required subsidy expenditure to output ratio and the tax revenue to output ratio to direct technical change. As discussed in Appendix C, subsidy expenditure is bounded above by $\frac{(1+\gamma)(1-\alpha)\eta}{(1+\tau)}$, and the tax revenue to output ratio is roughly half the tax rate or less.

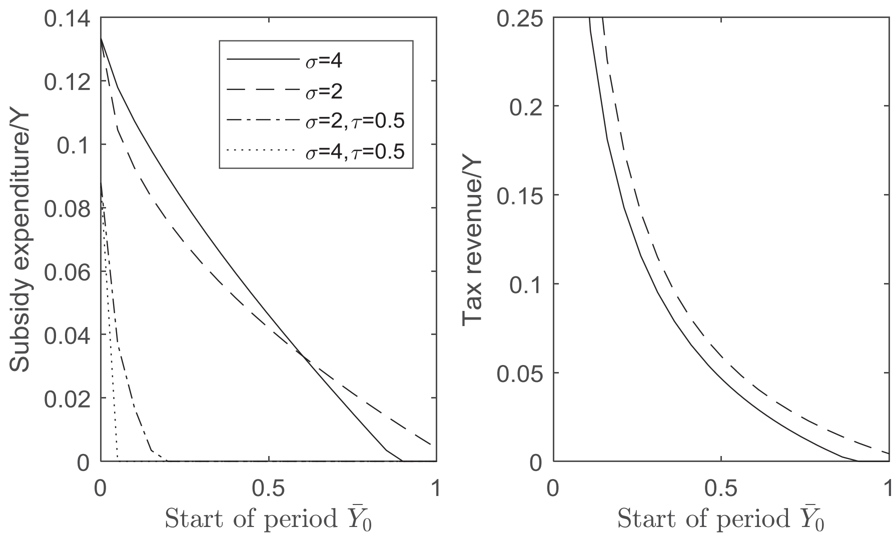

FIGURE 3: SUBSIDY EXPENDITURE AND TAX REVENUE RATIOS (WITH OUTPUT) REQUIRED TO DIRECT CLEAN RESEARCH

If subsidies have a distortionary cost as taxes do, then the optimal tax will increase until the extra marginal cost from the tax, described in lemma 2, is balanced by the marginal benefit from reduced subsidy costs, described in the next lemma.

LEMMA 3: If subsidies have a distortionary cost, the marginal consumption loss from subsidies, χsub, for small tax rates is independent of the tax rate and is as follows:

$$(23)\ \frac{1}{L_e}\frac{\partial(\chi sub)}{\partial\tau} \xrightarrow{small\ \tau} -\frac{\chi\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma)(1-\alpha)\eta A_d}{(1+\bar{A}^{-\varphi})^{\frac{1}{\varphi}+2}}\left(\frac{1-\bar{A}^{-\varphi}}{1-\alpha}+2\sigma\bar{A}^{-\varphi}\right).$$

Fortunately, from a policy perspective, the degree to which the tax increases due to costly subsidies is approximately independent of the elasticity and the relative productivity of clean technology, as the next proposition describes.

PROPOSITION 7: When subsidies have a proportional distortionary cost, the additional optimal tax rate, which helps direct clean research, is roughly independent of the productivity of technology and the elasticity of substitution. The optimal additional tax $\Delta\tau$ is roughly proportional to the subsidy cost proportion (χ) as follows:

$$\Delta\tau \approx (1+\gamma)(1-\alpha)\eta\left(2-\frac{1}{\sigma}\right)\chi. \quad (24)$$

Thus, the additional tax is proportional to the subsidy cost χ and insensitive to clean productivity and the elasticity, if clean technology does not lag too far behind dirty and the elasticity is high enough so that $2 \gg \frac{1}{\sigma}$. Figure 4 shows the marginal costs to consumption of the tax and the marginal benefits from reduced subsidies. The intersection of these lines roughly corresponds to the additional optimal tax to direct clean research.[17] As clean technology advances, the consequent rise in the marginal cost of consumption (at a given tax level) is balanced by a greater reduction in subsidies.

[17] The benefit from reduced environment externality also needs to be considered, which will push the tax slightly higher. However, for a non-zero tax base under costless subsidies, the increasing distortionary cost with the tax rate will tend to reduce the tax increment.

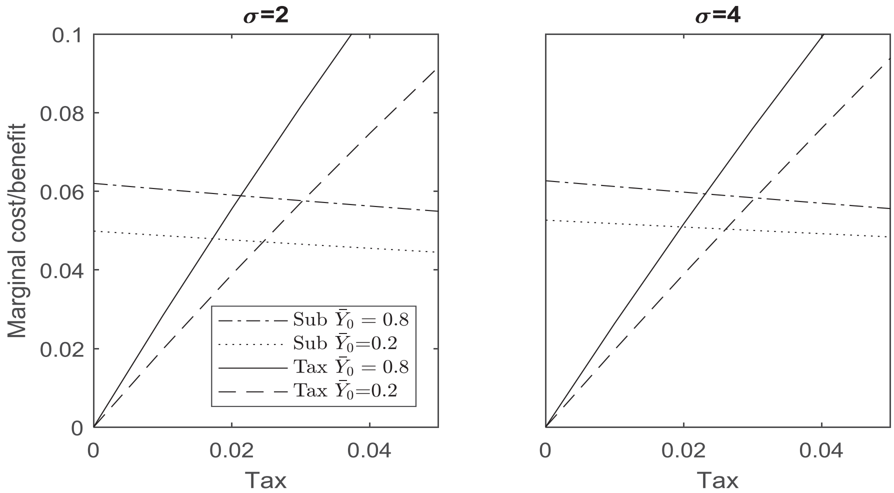

FIGURE 4: MARGINAL EFFECTS ON CONSUMPTION FROM TAXES WHEN USING POLICY TO DIRECT CLEAN RESEARCH, FOR DIFFERENT ELASTICITIES AND CLEAN RATIO WITHOUT TAX $\bar{Y}_0$

Notes: Marginal costs from the tax are shown with marginal benefits from the reduced subsidy required to direct clean research.

Figure 5 shows how the intersection changes with $\bar{Y}_0$. The additional optimal tax is just above 0.02 for the parameters used in the examples in the next section and a subsidy cost of 10 percent, noting that the tax rate is proportional to the subsidy cost rate.

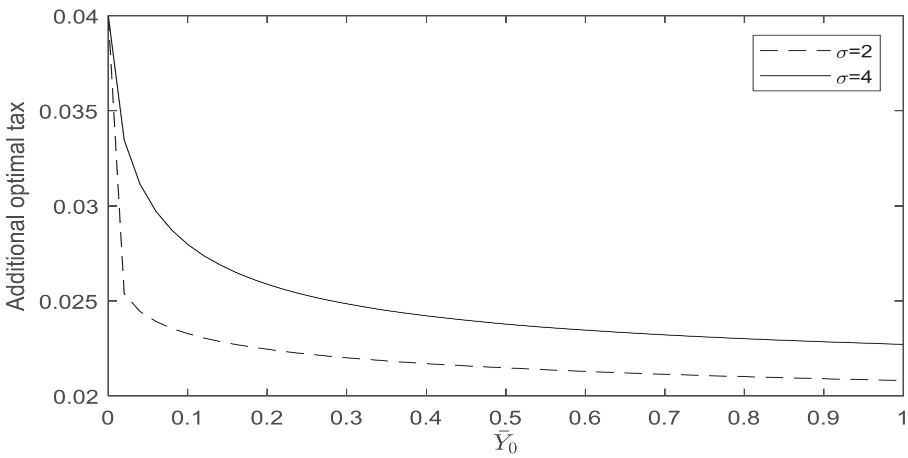

FIGURE 5: ADDITIONAL OPTIMAL TAX WHEN SUBSIDIES HAVE A DISTORTIONARY COST OF 10 PERCENT

III. Numerical examples with an extensive labour margin

This section uses numerical examples to demonstrate the effects of an extensive labour margin, keeping the split of energy and non-energy researchers fixed at long-run shares. The model characteristics described in the previous section help to explain results. Results for both a high discount rate of 1.5% and a low discount rate of 0.1% are discussed, consistent with AABH. First, optimal policy is considered, with the bimodal case compared against high and low isoelastic cases. Next, second-best policy is discussed, along with sensitivities of costly subsidies and extended patent lifetimes.

Optimal policy

Research in clean energy occurs immediately in both high and low discount rates, shown in Figure 6. The subsidy starts relatively small with a low elasticity (proposition 6) but falls slower as clean productivity increases. The subsidy in the bimodal case mirrors the high elasticity case, as the subsidy is only needed during the high elasticity regime. However, as the tax is boosted due to higher long-term emissions, the subsidy is lowered, so much so in the low discount rate case that the subsidy in the bimodal case is lower than both isoelastic cases.

Taxes are higher in the low discount rate case as future benefits of abatement have a higher weight. The tax drops to zero with a high elasticity but climbs throughout the whole simulation period in the bimodal and low elasticity cases.

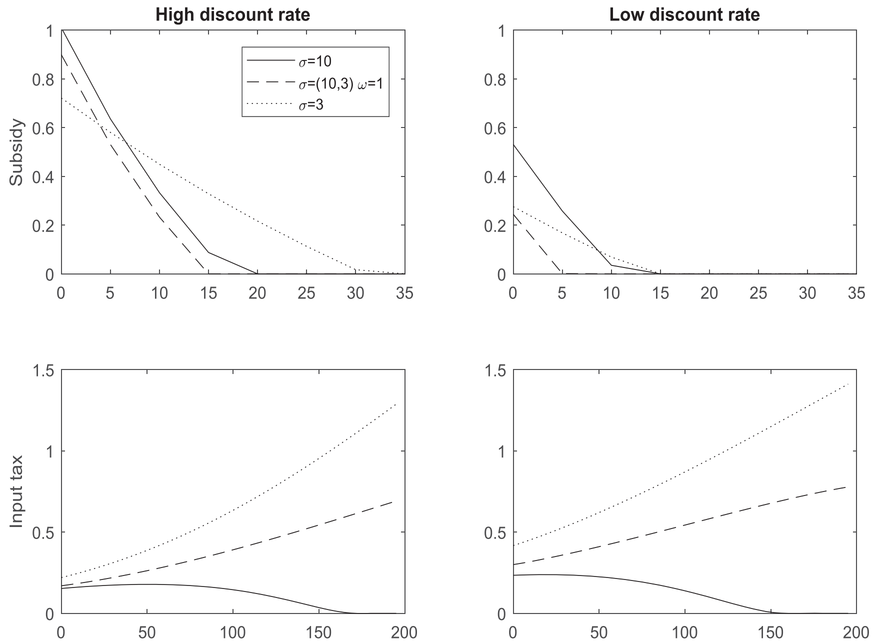

FIGURE 6: OPTIMAL POLICY FOR HIGH AND LOW DISCOUNT RATES

Further results for the high discount rate case are shown in Figure 7, including the transformation of the energy sector. Above a share of 0.5, the rate of transformation in the bimodal case drops off, reflecting the fall in elasticity, and a lower rate of increase in consumption growth results, as described in proposition 5. While the long-run growth rate is $\gamma\eta := 2\%$, the laissez-faire cases also start below this rate.[18] The consumption cost of optimal policy is the gap between the laissez-faire and optimal policy cases. Long-run temperature outcomes vary widely.

[18] The bimodal laissez faire case is identical to the high isoelastic case as the switch in elasticity never occurs.

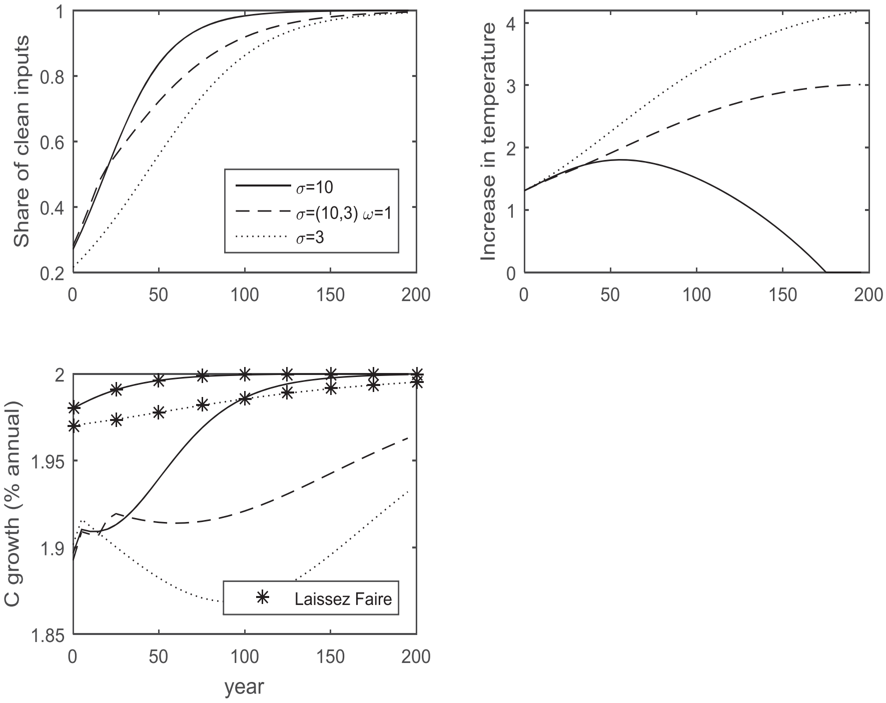

FIGURE 7: HIGH DISCOUNT RATE RESULTS

Second-best policy and sensitivities

Two types of sub-optimal or second-best policy are examined. The first assumes that only one instrument is used and the second delays optimal policy. Sensitivities assuming costly subsidies and extended patent lifetimes are also discussed.

One instrument only— As Table 2 shows, the welfare costs of using just a tax are very low at less than 0.1%. Interestingly, tax-only costs are lowest for the bimodal case because, compared with the high isoelastic case, the high initial tax required to direct clean research has greater future environmental benefits due to higher temperature outcomes. Extending patent lifetimes makes very little difference to the cost as the required subsidies are similar. For the isoelastic cases, the welfare cost of relying solely on a tax is smaller when the elasticity is high, as a lower tax is required to direct clean research (proposition 6). The cost of using subsidy-only policy is larger for a high elasticity, and clean technology advances alone cannot avert disaster in the bimodal and low elasticity cases.

Delayed policy— Delays of one, two and three decades are considered. The costs of delay increase with the elasticity of substitution due to a lower long-term contribution from advances in the dirty technology before the optimal policy is implemented. However, costs are more sensitive to the length of delay than the elasticity.

Costly subsidies—In the previous optimal policy section there is no cost of directing technical change using a subsidy, but there is a cost of doing so with a carbon tax. Following Acemoglu et al. (2016), I introduce a distortionary cost of using a subsidy, proportional to the subsidy expenditure, of 10 per cent. The cost is subtracted from consumption and implies a greater role for a carbon tax in directing technical change.

Figure 8 shows the effect on optimal tax rates compared with the cases where subsidies have no distortionary cost. When patent life is one period, the markup in tax rate is roughly 0.02 in all cases while the subsidy is present, consistent with proposition 7. While the increase in tax is similar across all cases, the reduction in the subsidy is greater for the high isoelastic and bimodal cases due to the starting high elasticity. A noteworthy outcome of extended patent life (not shown) is that the tax increase persists beyond the end of the subsidy, as these future taxes still have an impact on profits due to the extended patent life.

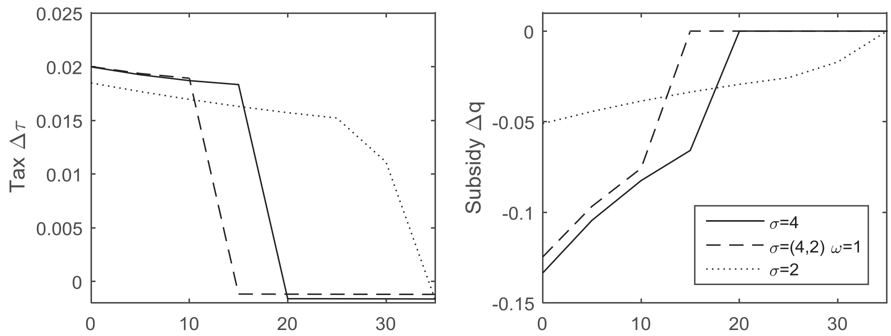

FIGURE 8: THE EFFECT OF COSTLY SUBSIDIES ON OPTIMAL TAXES AND SUBSIDIES FOR A HIGH DISCOUNT RATE

IV. Numerical examples with extensive margins in energy production and research

While the previous section restricted research in the energy sector to the long-run share, in this section researchers can move between sectors, like labour in production, so profits are equalised between the non-energy sector and the clean sector. The flexibility in research allocation improves welfare outcomes, detailed in Table 1.

TABLE 1 — WELFARE GAIN FROM EXTENSIVE RESEARCH MARGIN

Elasticity of substitution	$4\ (w \to \infty)$	$4/2\ (w = 1)$	$2\ (w = 0)$
Welfare gain (%)	0.42	0.46	0.67

Notes: Percentage increase in utility relative to research shares fixed at the long-run share.

As profits in the non-energy sector begin higher than both the dirty and clean sectors, research in the dirty sector would begin below the long-run share under laissez-faire. To provide a clearer representation of the policy needed to transform the energy sector, I introduce a costless subsidy to the energy sector that leads to research in the dirty sector at the long-run share under laissez-faire. The subsidy shown in the figures to come is then the additional subsidy above this general energy subsidy.[19]

Figure 9 shows results for the high discount rate case only, as the low discount rate results are similar. Interestingly, it is optimal to delay clean research before dedicating more than the long-run share of researchers, as corollary 1 outlines. Optimal taxes are somewhat lower but have a similar profile to the previous section. The boost in clean research transforms the sector sooner, lowering maximum temperatures, and leads to a dip in consumption growth.

[19] This detail makes little difference when the elasticity is 4 and reduces the required subsidy by up to a quarter when the elasticity is 2.

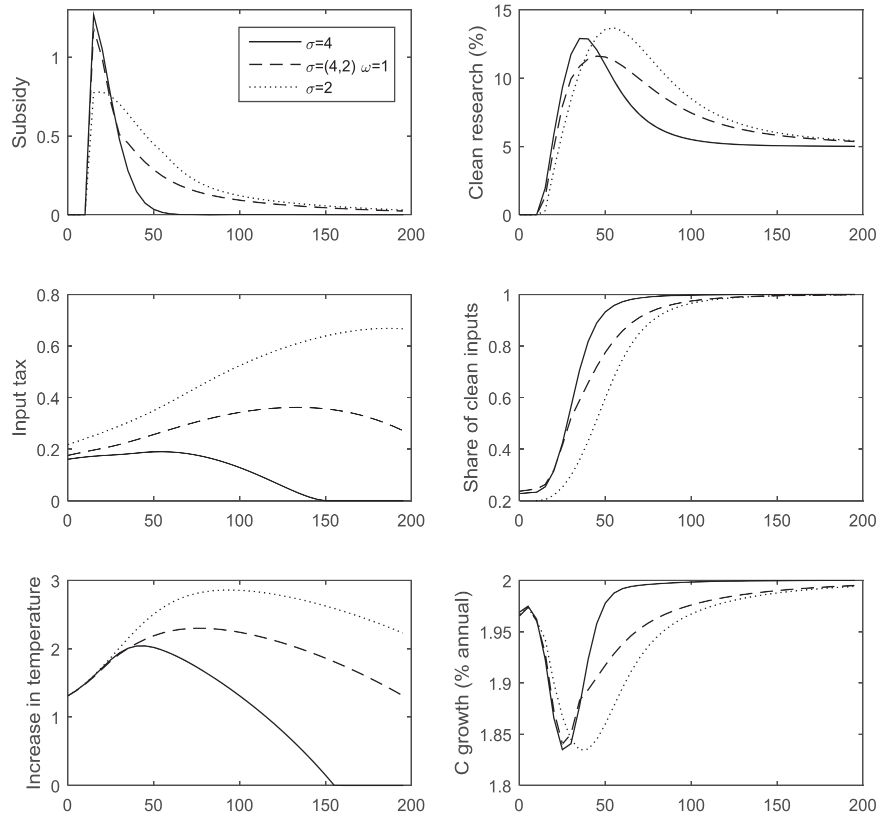

FIGURE 9: HIGH DISCOUNT RATE RESULTS WITH AN EXTENSIVE RESEARCH MARGIN

One instrument only— As Table 2 shows, tax only policy becomes more costly with the extensive research margin. The intuition is quite simple: the extensive margin leads to variation in research shares and directing technical change as a consequence, which is costly using a tax.

Stepping on toes — Figure 10 shows that assuming a 'stepping on toes' effect tends to induce immediate clean research at the long-run rate, in anticipation of losses from the stepping on toes effect to come when research is boosted to catch up with non-energy technology. A higher initial subsidy is required as a result, implying a higher welfare cost of the tax-only scenario.

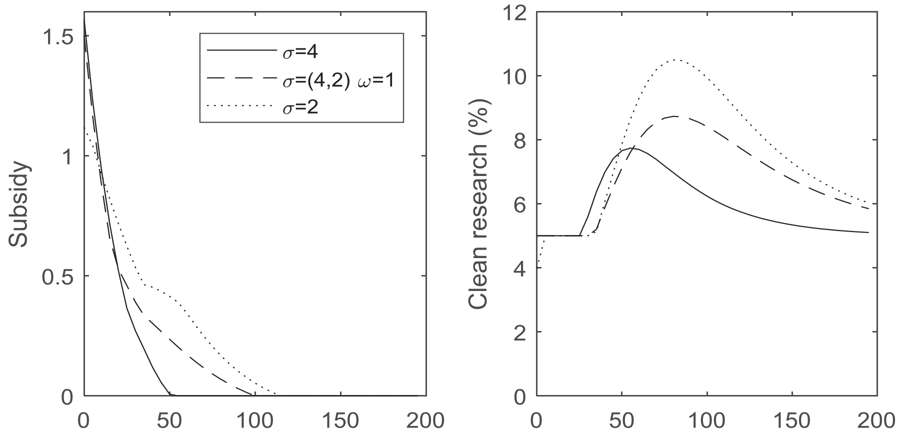

FIGURE 10: HIGH DISCOUNT RATE RESULTS WITH AN EXTENSIVE MARGIN OF ENERGY RESEARCH AND STEPPING ON TOES

TABLE 2 — WELFARE COSTS OF RELYING SOLELY ON CARBON TAX OR SUBSIDY (%)

Elasticity of substitution	4 ($w \to \infty$)	4/2 ($w = 1$)	2 ($w = 0$)
Tax only	0.039	0.030	0.078
Tax only 20 year patent	0.038	0.028	0.076
Subsidy only	0.177	disaster	disaster
Costly subsidy only	0.197	disaster	disaster
Delay of 10 years	0.30	0.24	0.20
Delay of 20 years	0.60	0.51	0.48
Delay of 30 years	0.94	0.85	0.89
Extensive research margin			
Tax only	0.177	0.389	0.573
Tax only 20 year patent	0.161	0.280	0.435
Subsidy only	0.148	0.217	0.310
Costly subsidy only	0.151	0.227	0.310
Extensive research margin with 'stepping on toes'			
Tax only	0.485	0.688	0.865
Tax only 20 year patent	0.359	0.519	0.652
Subsidy only	0.142	0.210	0.351
Costly subsidy only	0.141	0.208	0.337

Notes: Percentage reductions in utility relative to optimal policy.

V. Conclusion

This paper investigated three key questions relating to the transformation of the energy sector towards clean technologies, building on the growth model with endogenous technology described in Acemoglu et al. (2012). These questions relate to the extensive margins in the energy sector, where labour in production and research can move between energy and non-energy sectors, and a decreasing elasticity of substitution between clean and dirty inputs which is anticipated as the clean share rises in electricity supply. The structure and dynamics of the model with these extensions are described. Two key findings of AABH persist: (i) optimal policy involves both taxes and subsidies; and (ii) for sufficient substitutability between clean and dirty inputs, sustainable long-run growth can be achieved using policy intervention that is temporary or approaches zero in the long-run.

Considering a non-energy sector reduces abatement costs by more than an order of magnitude. Flexibility in research also plays an important role: clean research is delayed in this paper as in AABH, but maximum temperature outcomes are much lower and below 3°C in all cases; and the cost of a tax-only policy is increased. With an extensive research margin, clean research rises above the long-run share for a period, and the power of subsidy-only policy in averting disaster is increased.

The novel examination of a decreasing elasticity uses a bimodal production function with two isoelastic regimes. An advantage of this approach is that it is straightforward to adapt an existing model with an isoelastic nest between clean and dirty inputs, as demonstrated in this paper. When clean research occurs immediately, a decreasing elasticity of substitution implies a greater role for a carbon tax over subsidies, due to expectations of lower environmental quality in the future.

This paper makes a first step in investigating some potential implications of a decreasing elasticity of substitution between clean and dirty inputs and future quantitative exercises involving a transition to clean energy could consider this approach. Extensive margins should be considered in future exercises due to the magnitude of their effect, and the framework described in this paper allows this extension without a great cost in complexity.

APPENDIX A – PARAMETERS AND ASSUMPTIONS

TABLE 3 — PARAMETER AND FUNCTIONAL ASSUMPTIONS.

Number of years in a period		5
Discount rate	ρ	0.015 and 0.001 per annum
Low clean share elasticity of substitution	σ_1	10
High clean share elasticity of substitution	σ_2	3
Switch point	w	∞ (high CES), 0 (low CES), 1 and 0.5
Share of machines in production	α	1/3
Size of innovation	γ	1
Probability of success in research	η	0.02 per annum
Initial world emissions (from 2002 to 2006)	$Emit_0$	17.48251782 ppm
Initial production of clean energy (2002 to 2006)	Y_{c0}	307.77
Initial production of dirty energy (2002 to 2006)	Y_{d0}	1893.25
Rate of environmental degradation	ξ	$\frac{Emit_0}{Y_{d0}}$
Rate of environmental regeneration	δ	$\frac{Emit_0}{2S_0}$
Utility function	$u(C_t, S_t)$	$\frac{(\theta(S_t)C_t)^{1-\varepsilon}}{1-\varepsilon}, \theta(S) = \frac{(\Delta_{disaster} - \Delta(S))^{\lambda} - \lambda\Delta_{disaster}^{\lambda}(\Delta_{disaster} - \Delta(S))}{(1-\lambda)\Delta_{disaster}^{\lambda}}$
Parameter to match Nordhaus damage function	λ	0.1443
Intertemporal elasticity of substitution	ε	2
Increase in temperature since preindustrial times	Δ	$3\log_2\left(\frac{Emit}{280}\right)$
Disastrous increase in temperature	$\Delta_{disaster}$	6 degrees Celsius
Environmental quality	S	$S = Emit_{disaster} - \max(Emit, 280)$
Initial environmental quality	S_0	379 ppm
Proportional cost of subsidies	χ	0 (0.1)
Long-run share of researchers in energy	s_e^{LR}	5%
Ratio of energy to non-energy consumption	a	$s_e^{LR}/(1 - s_e^{LR})$

Diminishing returns to research parameter	ω	1 (0.8)

APPENDIX B – FURTHER EQUATIONS

$$\text{(B.1)} \quad p_{dt} = \frac{A_e}{\left(\beta^{\sigma}\bar{A}_t^{-\varphi} + (1+\tau_t)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}, p_{ct} = \frac{A_e}{\left(\beta^{\sigma} + (1+\tau_t)^{1-\sigma}\bar{A}_t^{\varphi}\right)^{\frac{1}{1-\sigma}}}.$$

$$\text{(B.2)} \quad \frac{L_{ct}}{L_{et}} = \frac{\beta^{\sigma}(1+\tau_t)^{\sigma}}{\left(\bar{A}_t^{\varphi} + \beta^{\sigma}(1+\tau_t)^{\sigma}\right)}, \quad \frac{L_{dt}}{L_{et}} = \frac{1}{\left(1 + \beta^{\sigma}(1+\tau_t)^{\sigma}\bar{A}_t^{-\varphi}\right)}$$

$$\text{(B.3)} \quad \frac{Y_{ct}}{L_{et}} = \left(\frac{A_e\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{(\beta(1+\tau_t))^{\sigma}A_{ct}}{\left(\beta^{\sigma} + (1+\tau_t)^{1-\sigma}\bar{A}_t^{\varphi}\right)^{\frac{\alpha}{\varphi}}\left(\bar{A}_t^{\varphi} + (\beta(1+\tau_t))^{\sigma}\right)}$$

$$\text{(B.4)} \quad \frac{Y_{dt}}{L_{et}} = \left(\frac{A_e\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{dt}}{\left(\beta^{\sigma}\bar{A}_t^{-\varphi} + (1+\tau_t)^{1-\sigma}\right)^{\frac{\alpha}{\varphi}}\left(1 + (\beta(1+\tau_t))^{\sigma}\bar{A}_t^{-\varphi}\right)}$$

$$\text{(B.5)} \quad \frac{Y_{et}}{L_{et}} = \left(\frac{A_e\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_eA_{dt}(1+\tau_t)^{\sigma}}{\left(\beta^{\sigma}\bar{A}_t^{-\varphi} + (1+\tau_t)^{1-\sigma}\right)^{\frac{1}{\varphi}-1}\left(1 + (\beta(1+\tau_t))^{\sigma}\bar{A}_t^{-\varphi}\right)}$$

$$\text{(B.6)} \quad \frac{C_{et}}{L_{et}} = \left(\frac{A_e\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_eA_{dt}\left(1 - \alpha + \frac{\tau_t}{(\beta(1+\tau_t))^{\sigma}\bar{A}_t^{-\varphi} + 1}\right)}{\left(\beta^{\sigma}\bar{A}_t^{-\varphi} + (1+\tau_t)^{1-\sigma}\right)^{\frac{1}{\varphi}}}.$$

$$\text{(B.7)} \quad \frac{\Pi_{ct}}{\Pi_{dt}} = (1+q_t)\beta^{\sigma}(1+\tau_t)^{\sigma}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{-\varphi-1}\bar{A}_{t-1}^{-\varphi}.$$

$$\text{(B.8)} \quad \frac{\Pi_{ct}}{L_{et}} = Z_t(1+q_t)\eta\beta^{\sigma}(1+\tau_t)^{\sigma}\bar{A}_t^{-(1+\varphi)}A_{ct-1}, \frac{\Pi_{dt}}{L_{Yt}} = Z_t\eta A_{dt-1}$$

$$\text{where } Z_t = \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma)(1-\alpha)A_e^{\frac{1}{1-\alpha}}}{\left(\beta^{\sigma}\bar{A}_t^{-\varphi} + (1+\tau_t)^{1-\sigma}\right)^{\frac{1}{\varphi}}\left(1 + \beta^{\sigma}(1+\tau_t)^{\sigma}\bar{A}_t^{-\varphi}\right)} \text{ and } s_{ct} = 1$$

$$(\mathrm{B.9}) \qquad \frac{sub_t}{\mathrm{L}_{et}} = \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma)(1-\alpha)\eta\left(A_{dt-1} - (1+\tau_t)^{\sigma}\bar{A}_t^{-(1+\varphi)}A_{ct-1}\right)}{\left(\bar{A}_t^{-\varphi} + (1+\tau_t)^{1-\sigma}\right)^{\frac{1}{\varphi}}\left(1+(1+\tau_t)^{\sigma}\bar{A}_t^{-\varphi}\right)}$$

APPENDIX C – PROOFS

Proposition 4

From (13) and lemma 1,

$$L_{0t} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{\alpha-1}}\frac{C_{et}}{aA_{0t}(1-\alpha)} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{\alpha-1}}\frac{Y_{et}}{aA_{0t}} = \frac{A_{ct}L_{et}}{aA_{0t}}$$

$$1 - L_{et} = \frac{A_{ct}L_{et}}{aA_{0t}} \; so \; L_{et} = \frac{aA_{0t}}{aA_{0t} + A_{ct}}$$

For a zero long-run tax and no extensive research margin, A_{0t} and A_{ct} grow at the same rate so we retain the AABH result: dirty inputs will fall in the long run if $\alpha + \varphi < 0$ or equivalently $\sigma > 1/(1-\alpha)$.

For an extensive research margin, if $\sigma > 1/(1-\alpha)$ then A_c approaches A_0 in the long-run (proposition 3). If $\sigma < 1/(1-\alpha)$, then L_e must shrink and thus A_c will grow faster than A_0. For $A_{ct} \gg A_{0t}$ we have $\mathrm{L}_{et} \xrightarrow{t\to\infty} \frac{aA_{0t}}{A_{ct}}$. Then in the long-run $\mathrm{Y}_{dt} \sim A_{ct}{}^{\alpha+\varphi-1}A_{0t}$ and for clean technology growth g_c and non-energy growth g_0, dirty inputs will fall if $g_0 < \sigma(1-\alpha)g_c$. As $s_e^{LR}g_c + (1-s_e^{LR})g_0 = \gamma\eta$, then $g_0 < \frac{\sigma(1-\alpha)}{(1/s_e^{LR}-1)\sigma(1-\alpha)+1}\gamma\eta$. As $A_{ct} > A_{0t}$ for weak substitutes, from (14.1) $p_{0t} > 1$ and hence $\Pi'_{0t} > \Pi'_{ct}$ if $q_{ct} = 0$. Hence $q_{ct} > 0$ in the long-run so the clean subsidy is permanent.

Proposition 5

The following proves that consumption is lower after a decrease in elasticity when taxes are zero. Omitting the time subscript for clarity and assuming the tax is zero, the following holds.

$$\bar{Y} = \beta^{\sigma_2}\bar{A}^{\sigma_2(1-\alpha)} \geq w \text{ and } \beta^{\sigma_2} = w^{1-\frac{\sigma_2}{\sigma_1}} \text{ so } w \leq \bar{A}^{\sigma_1(1-\alpha)}, \text{and}$$

$$A_e^{\frac{1}{1-\alpha}} = \left(w^{\frac{\sigma_1-1}{\sigma_1}} + 1\right)^{\frac{\sigma_2}{\varphi_2}-\frac{\sigma_1}{\varphi_1}}.$$

Let energy consumption above the switch point be C_{low}, while for the isoelastic case with high elasticity it would be C_{high}. Then we have:

$$\frac{C_{low}}{c} = \frac{A_e^{\frac{1}{1-\alpha}}}{\left(w^{1-\frac{\sigma_2}{\sigma_1}} + \bar{A}^{\varphi_2}\right)^{\frac{1}{\varphi_2}}} \text{ for c} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} A_c L_e$$

$$= \frac{\left(w^{\frac{\sigma_1-1}{\sigma_1}} + 1\right)^{\frac{\sigma_2}{\varphi_2}-\frac{\sigma_1}{\varphi_1}}}{\bar{A}\left(\bar{A}^{-\varphi_2} w^{\frac{1-\sigma_2}{\sigma_1}} w^{\frac{\sigma_1-1}{\sigma_1}} + 1\right)^{\frac{1}{\varphi_2}}}$$

$$\leq \frac{\left(w^{1-\frac{1}{\sigma_1}} + 1\right)^{\frac{\sigma_2}{\varphi_2}-\frac{\sigma_1}{\varphi_1}}}{\bar{A}\left(w^{1-\frac{1}{\sigma_1}} + 1\right)^{\frac{1}{\varphi_2}}} = \frac{\left(w^{1-\frac{1}{\sigma_1}} + 1\right)^{-\frac{1}{\varphi_1}}}{\bar{A}}$$

$$\leq \frac{(\bar{A}^{-\varphi_1} + 1)^{-\frac{1}{\varphi_1}}}{\bar{A}} = \frac{1}{(1 + \bar{A}^{\varphi_1})^{\frac{1}{\varphi_1}}} = \frac{C_{high}}{c}.$$

The following proves that consumption can fall as clean productivity rises. The derivative of consumption with respect to $\bar{A}^{-\varphi}$, when $\bar{A}^{-\varphi} = 0$, is:

$$\frac{1}{L_e}\frac{\partial C_e}{\partial \bar{A}^{-\varphi}}\bigg|_{\bar{A}^{-\varphi} = 0} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_e{}^{\frac{1}{1-\alpha}} A_d \beta^{\sigma} (1+\tau_t)^{\sigma - \frac{1}{1-\alpha}}}{(-\varphi)} \left(1 - \frac{\alpha}{1+\tau_t} + \tau_t \varphi\right)$$

The derivative is positive when the tax is zero, and less than zero if:

$$\frac{\partial C_e}{\partial \bar{A}^{-\varphi}}\bigg|_{\bar{A}^{-\varphi} = 0} < 0 \rightarrow\ 1 - \frac{\alpha}{1+\tau_t} + \tau_t \varphi < 0$$

Thus, consumption declines if the tax is sufficiently large and $\sigma > 1$.

Lemma 2

Omitting time subscripts, when taxes are low the marginal change in consumption from a tax is approximately the following:

$$\frac{1}{L_e}\frac{\partial C_e}{\partial \tau} \xrightarrow{low\ \tau} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_e{}^{\frac{1}{1-\alpha}} A_d}{(\beta^{\sigma} \bar{A}^{-\varphi} + 1)^{\frac{1}{\varphi}+2}} \left(\frac{\alpha}{\alpha - 1} - 2\sigma\beta^{\sigma}\bar{A}^{-\varphi}\right)\ \tau$$

Proposition 6

Considering $\bar{Y} < w$ and small taxes,

$$\frac{\Pi_{ct}(1)}{\Pi_{dt}(1)} = (1+q_t)(1+\tau_t)^{\sigma} \frac{\bar{A}_{t-1}^{-\varphi}}{\gamma_c{}^{1+\varphi}} \text{ and } \frac{\bar{A}_{t-1}^{-\varphi}}{\gamma_c{}^{1+\varphi}} = \frac{\bar{Y}_{0,t-1}^{1-1/\sigma}}{\gamma_c{}^{1+\varphi}} = \frac{\bar{A}_t^{-\varphi}}{\gamma_c}.$$

Thus, the subsidy required is

$$q_t = \frac{{\gamma_c}^{1+\varphi}}{(1+\tau_t)^{\sigma}\bar{Y}_{0,t-1}^{1-\frac{1}{\sigma}}} - 1.$$

and, for low $\bar{Y}_0$, increases with the elasticity. If there are no subsidies, the tax required is roughly a function of $\bar{Y}_{0,t-1}^{1/\sigma}$ and thus decreases with the elasticity for a given $\bar{Y}_0$: $\tau_t \xrightarrow{low\ \tau} \frac{1}{\bar{Y}_{0,t-1}^{1/\sigma}} - 1$.

Remark 1

Using (23), assuming $\bar{Y} < w$ and omitting time subscripts:

$$\frac{sub}{L_e} \sim \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma)(1-\alpha)\eta A_d}{(\bar{A}^{-\varphi} + (1+\tau_t)^{1-\sigma})^{\frac{1}{\varphi}}(1+\bar{A}^{-\varphi}(1+\tau_t)^{\sigma})}(1 - (1+\tau_t)^{\sigma}\bar{A}^{-\varphi})$$

The subsidy expenditure and output for small $\bar{A}^{-\varphi}$ are as follows:

$$\frac{sub}{L_e} \xrightarrow{\bar{A}^{-\varphi}\to 0} \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma)(1-\alpha)\eta}{(1+\tau)^{\frac{1}{1-\alpha}}} \text{ and } \frac{Y_e}{L_e} \xrightarrow{\bar{A}^{-\varphi}\to 0} \left(\frac{\alpha}{\psi(1+\tau)}\right)^{\frac{\alpha}{1-\alpha}}.$$

Thus, for the parameters used in this paper, the subsidy expenditure to output ratio is

$$\frac{sub}{Y_e} \xrightarrow{\bar{A}^{-\varphi}\to 0} \frac{(1+\gamma)(1-\alpha)\eta}{1+\tau}.$$

From (17) the ratio of dirty inputs to output is:

$$\frac{Y_d}{Y_e} = \frac{\bar{A}^{-\varphi}}{(1+\tau)^{\sigma}(\bar{A}^{-\varphi} + (1+\tau)^{1-\sigma})^{\frac{\sigma}{\sigma-1}}}$$

Tax revenue to output ratio is $\frac{\tau Y_d}{Y_e}$, and for $1 + \tau_t = \frac{1}{\bar{A}_{t-1}^{-\varphi/\sigma}}$:

$$\frac{Y_d}{Y_e} = \frac{\bar{A}_{t-1}^{-\varphi}}{\left(\bar{A}_t^{-\varphi} + \bar{A}_{t-1}^{-\varphi\left(\frac{1-\sigma}{\sigma}\right)}\right)^{\frac{\sigma}{1-\sigma}}} \xrightarrow{high\ \sigma} \frac{1}{(1+\gamma\eta)^{-\varphi} + 1} < \frac{1}{2}.$$

Lemma 3

Omitting time subscripts and considering $\bar{Y} < w$, using (23) with a small tax the subsidy expenditure is approximately:

$$\frac{sub}{L_e} \approx \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma)(1-\alpha)\eta A_d}{(1+\bar{A}^{-\varphi})^{\frac{1}{\varphi}+1}}\left(1 - \bar{A}^{-\varphi} + \tau\frac{\frac{1-\bar{A}^{-\varphi}}{\alpha-1} - 2\sigma\bar{A}^{-\varphi}}{1+\bar{A}^{-\varphi}}\right).$$

The marginal consumption loss from subsidies, χsub, for small tax rates is independent of the tax rate:

$$\frac{1}{L_e}\frac{\partial(\chi sub)}{\partial\tau} \xrightarrow{small\ \tau} -\frac{\chi\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma)(1-\alpha)\eta A_d}{(1+\bar{A}^{-\varphi})^{\frac{1}{\varphi}+2}}\left(\frac{1-\bar{A}^{-\varphi}}{1-\alpha} + 2\sigma\bar{A}^{-\varphi}\right).$$

Proposition 7

As the additional tax equalises the marginal cost to consumption and marginal benefit from reduced subsidies, lemmas 2 and 3 imply:

$$\frac{\partial C_e}{\partial \tau} = \frac{\chi \partial sub}{\partial \tau} \text{ if } \tau \approx \frac{\chi(1+\gamma)(1-\alpha)\eta\left(\frac{1 - \bar{A}^{-\varphi}}{1-\alpha} + 2\sigma\bar{A}^{-\varphi}\right)}{\left(\frac{\alpha}{1-\alpha} + \sigma\bar{A}_t^{\,-\varphi}\right)}.$$

The approximation lies in the interval $\chi(1+\gamma)(1-\alpha)\eta\left(\frac{1}{\alpha}, 2 - \frac{1}{\sigma}\right)$. If $\bar{Y}_0(\approx \bar{A}^{-\varphi})$ is large enough so that $\sigma\bar{A}^{-\varphi} \gg \frac{\alpha}{1-\alpha}$, then

$$\tau \approx \chi(1+\gamma)(1-\alpha)\eta\left(2 - \frac{1}{\sigma}\right).$$

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