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CAMA Working Paper 64/2019
September 2019

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Abstract

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JEL Classification

H23, O44, Q30, Q40, Q54, Q56, Q58

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ISSN 2206-0332

The optimal carbon tax with a tipping climate and peak temperature

By ANTHONY WISKICH*

This paper describes an integrated assessment model with an unknown temperature threshold where severe and irreversible climate impacts, called a tipping point, occurs. The possibility of tipping leads to the following linked outcomes: a prolonged period of peak temperature; a rebound in emissions prior to and during peak temperature; and a fall in the optimal carbon tax as a ratio of output prior to and during peak temperature. Although tipping can occur in any period where temperature rises to a new maximum, the optimal carbon price can be calculated from future temperature outcomes conditional on no tipping. Learning that tipping has not occurred lowers the tax. (JEL H23, O44, Q30, Q40, Q54, Q56, Q58)

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International agreements have committed nations to limit global warming to 2 degrees Celsius (^{O}C). Specifying a temperature limit seems a reasonable approach in the presence of considerable uncertainty of climate impacts[1], particularly as risks

[1] A recent review of the subject highlights criticism of Integrated Assessment Models and discusses the merits of using models that incorporate uncertainty (Farmer, Hepburn, Mealy, & Teytelboym, 2015).

are thought to increase with temperature. Tipping points can capture this risk: a tipping point in this paper is defined as an irreversible, permanent switch to a climate state with a higher sensitivity to temperature. It may, therefore, seem natural that optimal policy would look quite different before and after peak temperature in a model with uncertain climate impacts.

In contrast, the integrated assessment model described by Golosov, Hassler, Krusell, and Tsyvinski (2014), hereafter GHKT, implies that the optimal tax is not a function of temperature at all. In this framework, the climate sensitivity parameter is uncertain and the optimal carbon tax is constant as a proportion of output, depending only on the discount rate, the expected damage elasticity and the structure of carbon depreciation in the atmosphere. Stochastic values of future output, consumption, emissions concentration, technology growth, and population do not influence the optimal tax rate. Various extensions of this benchmark model have already been made.[2] The current paper builds on the GHKT framework and introduces a tipping point, which creates an endogenous link between peak temperature and optimal policy.

Many papers have considered such threshold environmental effects, going back to Cropper (1976). The current paper adopts tipping characteristics similar to those described in D. Lemoine and Traeger (2014), hereafter LT: each threshold temperature increase may trigger a tipping point with equal probability; there exists a trigger point between the current temperature and an upper bound; and both Bayesian learning and non-learning scenarios are considered. The probability of tipping in a period is referred to as the hazard rate. Whereas LT and many other papers[3] that consider endogenous tipping build on the DICE model (Nordhaus, 2008), this paper builds on the GHKT model, allowing investigation of technology

[2] For example, Traeger (2015) includes a richer climate model and Epstein-Zin preferences in the GHKT model, maintaining the closed-form solution of the optimal carbon tax which remains constant as a proportion of output.

[3] For example, Cai, Judd, and Lontzek (2013).

growth and energy substitutability in the optimal policy choice, and simplifying analysis and computation as described below.

Engström and Gars (2016) also build on the GHKT framework which leads to constant savings rates and no role for precautionary savings discussed in other papers[4]. However, Engström and Gars (2016) adopt a climate model that does not allow temperature to peak and use a simpler energy structure. In the current analysis, climate dynamics follow Shine, Fuglestvedt, Hailemariam, and Stuber (2005) and peak temperature plays a key role. The separation of coal and oil leads to a potential rebound in coal use.

The existence of a tipping point increases the carbon price in two ways. First, expected damages rise from a given temperature increase. This effect dominates the literature and is the only effect considered in deterministic studies, or when the hazard rate is exogenous as considered by Gerlagh and Liski (2018) in a similar analytical climate-economy model. Second, the endogeneity of the hazard rate raises the tax, as described by Van der Ploeg (2014) in a partial equilibrium framework. Numerical exercises have found that an endogenous hazard rate can significantly boost today's optimal carbon price.[5] This paper also finds this result.

In addition to increasing today's carbon price, the endogenous hazard rate changes the profile of the tax as the component due to endogeneity goes to zero by the end of peak temperature. Other studies have found an optimal tax to income ratio that varies[6], including some that also consider tipping points.[7] The current

[4] Recent examples include van der Ploeg and de Zeeuw (2019) and van der Ploeg and de Zeeuw (2017).

[5] Lontzek, Cai, Judd, and Lenton (2015) include an endogenous hazard rate in the DICE model with a phased impact from tipping. They find today's optimal carbon price is increased by around 50 to 200 per cent, depending on the probability and severity of tipping. LT find a tipping point increases the near-term optimal carbon price by between 25 and 40 per cent, with a small component due to endogeneity. In a later paper, they include multiple tipping points in a stochastic dynamic version of the DICE model and find this nearly doubles today's optimal carbon tax (D. Lemoine & Traeger, 2016).

[6] Literature without tipping points include Rezai and Van Der Ploeg (2017), Ploeg and Withagen (2014), Jensen and Traeger (2014) and D. Lemoine (2017).

[7] For example Gerlagh and Liski (2018), Lontzek et al. (2015), D. Lemoine and Traeger (2016), Engström and Gars (2016) and Cai and Lontzek (2019).

paper discusses links between learning and the concavity of future temperature with growth in the tax ratio. A novel quantitative outcome relates to the prolonged stabilization of peak temperature.

This paper is the first to show that, taking into account climate inertia, a declining optimal carbon price can lead to prolonged stabilisation at peak temperature and a rebound in emissions. [8] Such behaviour is similar to the emissions overshooting described in D. Lemoine and Rudik (2017), where the emissions path minimises costs with a temperature limit. Thus, although a cost-benefit framework is used in this paper, the results using this approach share some similar characteristics with a cost-minimisation framework due to the combination of the threshold tipping framework and the severe damages in a post-tipping world. I explore this similarity further in Wiskich (2019b).

Learning is shown to reduce growth in the optimal carbon tax. Gerlagh and Liski (2018) find that the tax ratio can increase without learning and decrease if learning occurs, although in their quantitative exercise this is not material before 2100. while it may either grow or shrink with learning depending on the concavity of the no-tipping temperature profile. While the current paper finds that learning lowers the tax in the absence of a tipping event, the tax ratio can increase or decrease initially depending on parameter settings. If peak temperature is low and occurs soon, the tax ratios will fall in both learning and non-learning scenarios, as demonstrated in the numerical examples in this paper. However, for a distant peak temperature, the tax ratio will grow if there is no learning and may grow or shrink with learning, depending on the concavity of the temperature profile. For a linearly increasing temperature profile, the growth in tax ratio with learning will be roughly half of the scenario without learning.

[8] D. M. Lemoine and Traeger (2012) also report optimal policy that keeps the temperature constant over most of the next century, but only when inertia in the climate system is not considered.

The framework used in this paper does not exhibit sensitivity to fat-tailed risks of catastrophic outcomes (Weitzman, 2009). Due to the threshold tipping framework and climate dynamics, the probability of tipping can be reduced to zero with a finite tax, depending on historical emissions. In this case, even if expected damages from tipping were infinite, the optimal tax would be finite.

This paper makes some methodological contributions related to the inclusion of a tipping point with an endogenous hazard rate into the GHKT model, rather than the DICE model. Optimal tax equations are explicitly derived, making the impact of endogeneity transparent, and computation is straight forward as the optimal carbon price can be calculated with temperature outcomes conditional on tipping not occurring. Thus, although the introduction of endogenous tipping means the optimal tax becomes dependent on expectations of temperature outcomes and so is more complex than the simple form discussed in GHKT, computation is still straight forward as only one future outcome is needed to derive the tax.[9]

I. Model

The model modifies the completely characterised model described in GHKT. A global representative household has logarithmic preferences over consumption with discount rate β, and thus maximises the following:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \text{ where } U(C_t) := \log(C_t)\,. \tag{1}$$

Damages are an exponential function of temperature, as used by Gerlagh and Liski (2018), rather than emissions concentration as in GHKT. There is

[9] Engström and Gars (2016) also report this result with a simpler energy model. In the current paper, the extraction of oil is solved intertemporally but the Cobb-Douglas form implies this dynamic can also be solved with only the no-tipping outcome.

considerable uncertainty around the form of the damages function, but an exponential function of temperature is consistent with some recent evidence (Burke, Hsiang, & Miguel, 2015). Final output is a Cobb-Douglas specification of capital K_t, sector '0' labour $N_{0,t}$, oil $E_{1,t}$ and other energy E_t, and a multiplicative exponential damage function of atmospheric temperature T_t above pre-industrial, as follows:

$$(2) \qquad Y_t = \mathrm{F}_{0,t}\left(N_{0,t}, K_{0,t}, \boldsymbol{E}_t, T_t\right) = e^{-\gamma_t T_t} A_{0,t} {K_t}^{\alpha} {N_{0,t}}^{1-\alpha-\nu} {E_{1,t}}^{\omega} {E_t}^{\nu-\omega}.$$

Oil has been separated from other energy (coal and clean) so that a lower elasticity of substitution can be applied, reflecting evidence of high substitutability between clean and dirty inputs in electricity (Wiskich, 2019a). The Cobb Douglas formulation also leads to easier computation under uncertainty as described below.

Climate dynamics for carbon are taken from Shine et al. (2005) and account for climate-system inertia. Temperature dynamics are a function of radiative forcing R_t:

$$(3) \qquad H\frac{dT_t}{dt} = R_t - \frac{T_t}{\lambda},$$

where H is the heat capacity of the system and λ is a climate sensitivity parameter. For carbon, radiative forcing and temperature responses at time t after an emissions pulse are

$$(4) \qquad R_t^c := \frac{\partial R_t}{\partial E_{f,0}} = a_0 + \sum_{i=1}^{4} a_i e^{-\frac{t}{\alpha_i}} \text{ and}$$

$$(5) \qquad T_t^c := \frac{\partial T_t}{\partial E_{f,0}} = \frac{A_c}{H}\left\{\tau a_0\left(1 - e^{-\frac{t}{\zeta}}\right) + \sum_{i=1}^{4} \frac{a_i\left(e^{-\frac{t}{\alpha_i}} - e^{-\frac{t}{\zeta}}\right)}{\left(\zeta^{-1} - {\alpha_i}^{-1}\right)}\right\},$$

where a_i are coefficients which sum to 1, α_i reflect gas lifetimes in years, ζ is by definition the constant λH in years, and A_c is the radiative forcing due to a 1-kg change in carbon dioxide. A standard assumption in the literature, as used by the DICE model and GHKT, is that steady-state temperature is a logarithmic function of the emissions stock, which is a linear function of dirty energy (emissions flow). This paper follows Gerlagh and Liski (2018) and assumes temperature is a linear function of dirty energy, consistent with some physical science literature (Allen et al., 2009). For temperatures below 3 degrees, the logarithmic function is almost linear in any case, shown in Figure 1. The response in temperature to an emissions pulse peaks at 20 years in this paper, much shorter than the 60-year peak in the climate model of Gerlagh and Liski (2018), although the general response profiles are similar.

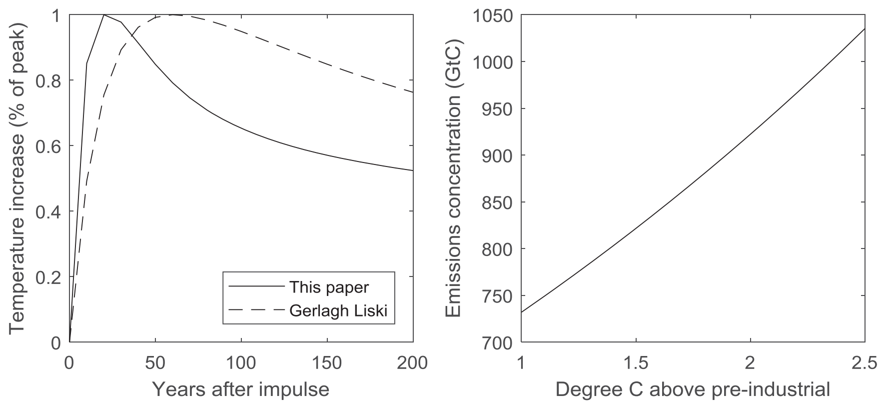

FIGURE 1: TEMPERATURE RESPONSES TO AN EMISSIONS PULSE AND THE NEAR-LINEAR LOGARITHMIC RELATIONSHIP BETWEEN EMISSIONS AND TEMPERATURE USED IN DICE

Temperature is a linear function of previous dirty energy use, including historical use, as follows:

$$T_t = \sum_{i=-\infty}^{\mathrm{t}} T_{t-i}^{c} E_{f,i} \tag{6}$$

Other energy E_t is a composite isoelastic function of coal $E_{2,t}$ and clean $E_{3,t}$,

$$E_t = \left(\kappa E_{2,t}^{\rho} + (1-\kappa) E_{3,t}^{\rho}\right)^{1/\rho}. \tag{7}$$

Oil can be extracted at zero cost and is in finite supply $X_t \geq 0$,

$$E_{1,t} = X_{t-1} - X_t. \tag{8}$$

Dirty (fossil) energy $E_{f,t}$, which contributes to carbon emissions, is the sum of oil and coal energy. Coal and renewable sectors require only labour in production

$$E_{i,t} = A_{i,t} N_{i,t} \text{ for } i = 2{,}3 \text{ where } N_{0,t} + N_{2,t} + N_{3,t} = N. \tag{9}$$

Oil prices follow a hotelling rule corrected for the carbon tax to GDP ratio $\widehat{\Lambda}_t$:

$$\frac{\omega}{E_{1,t}} - \widehat{\Lambda}_t = \beta\, \mathbb{E}_t \left(\frac{\omega}{E_{1,t+1}} - \widehat{\Lambda}_{t+1}\right). \tag{10}$$

Coal and renewable prices are set by wages in sector '0' as follows:

$$A_{2,t}\left(\frac{(\nu-\omega)\kappa}{E_{2,t}^{1-\rho} E_t^{\rho}} - \widehat{\Lambda}_t\right) = \frac{1-\alpha-\nu}{N_{0,t}} \text{ and } A_{3,t}\frac{(\nu-\omega)(1-\kappa)}{E_{3,t}^{1-\rho} E_t^{\rho}} = \frac{1-\alpha-\nu}{N_{0,t}}. \tag{11}$$

Tipping point, damages and scenarios

To implement the concept of a tipping point, I use a probability distribution for the tipping point threshold's location that is uniform in temperature, as used by LT. I assume that the tipping point has not been reached to date and hence lies between the initial temperature T_0 and an upper limit T_{upper}, set at 6^{O}C warming.[10] The temperature sensitivity of the damages parameter γ_t in (2) incorporates the hazard rate as a function of temperature $p_t(T)$. Due to the assumption of a tipping threshold and irreversibility, the expected hazard rate is a simple function of temperature conditional on no tipping. Let $\mathbb{E}_t$ be an expectation operator at time t assuming no tipping prior to t, and let $\widehat{\mathbb{E}}_t(x_{t+j})$ signify the expectation at time t of variable x_{t+j} conditional on no tipping prior to period $t+j$. The probability of severe climate sensitivity associated with a post-tipping state at time $t+j$ is of the following form:

$$(12) \quad \mathbb{E}_t(p_{t+j}) = max\left(min\left(\frac{\widehat{\mathbb{E}}_t(T^H_{t+j}) - T_s}{T_{upper} - T_s}, 1\right), 0\right), \; T^H_{t+j} := \max_{k \le t+j}(T_k).$$

Two values of the variable s are explored under different scenarios. One scenario considers immediate awareness of tipping after crossing the threshold, in which case $s=t$. Another scenario excludes learning, or equivalently assumes delayed awareness in the extreme case that it is unknown whether a tipping point has been triggered for the entire simulation period, so $s=0$. These scenarios are named *Learning* and *No learning* and both scenarios assume immediate impacts from tipping. While delayed effects, as considered in Lontzek et al. (2015), could be modelled as part of either scenario, assuming immediate impacts is simple and

[10] This upper bound temperature is the midpoint used in LT, as they vary bounds between 3^{O}C and 9^{O}C.

allows easy identification of the effects of learning. For comparison, I also show a *No tipping* scenario where there is no learning and a fixed probability of severe climate sensitivity, so the tax ratio is constant. I use a probability of 4 per cent which roughly corresponds to the prior probability of tipping in the *Learning* and *No learning* scenarios.

Due to irreversibility, expectations of the derivative $\frac{\partial p_{t+j}}{\partial T_{t+j}}$ are non-zero only if tipping has not occurred,

$$(13) \quad \mathbb{E}_t\left(\frac{\partial p_{t+j}}{\partial T_{t+j}}\right) = \begin{cases} \dfrac{1}{T_{upper} - \mathrm{x}} & \text{with probability } 1 - \mathbb{E}_t(p_{t+j}) \\ 0 & \text{otherwise} \end{cases}$$

$$\text{where x} = \begin{cases} \widehat{\mathbb{E}}_t(T_{t+j}) & \text{for } Learning \\ T_0 & \text{for } No\ learning, \end{cases}$$

and the following lemma applies, which will be used in the next section.

LEMMA 1: The expected probability of tipping is a function of future temperatures conditional on no tipping, and the expected temperature-derivative of the hazard rate is non-zero only under no tipping.

GHKT use two estimates of the damage function parameter from Nordhaus (2008): one central estimate consistent with a loss of 0.48% of GDP from a 2.5^{O}C warming and another consistent with the catastrophic outcome of a 30% loss of GDP from warming of 6^{O}C. These numbers calibrate moderate and catastrophic parameters γ^L and γ^H, with GHKT assuming a fixed probability of 6.8% of the latter. This paper adopts the same derivation of moderate and catastrophic (severe) parameters in the tipping framework, so that $\gamma_t := \gamma^L + (\gamma^H - \gamma^L)p_t$. Expected damages as a function of temperature are shown in Figure 2. The formulation of the damage function lies below the GHKT damage function for low warming, but then increases more rapidly as the probability of tipping increases. The damage at high temperatures is much greater than DICE-2007, but less than damage functions used in other papers.[11]

In this paper, climate sensitivity in the severe regime after tipping is high relative to the moderate regime. For comparison, LT represent a climate feedback tipping point as increasing climate sensitivity to 4^{O}C, 5^{O}C or 6^{O}C rather than the standard assumption of 3^{O}C. As damages are assumed to increase with the square of temperature increase, the triggering of the tipping point in the LT framework therefore increases damages by either 80%, 180% or 300%. In contrast, the tipping point from the severe sensitivity in this paper corresponds to over a 30-fold increase in damages.

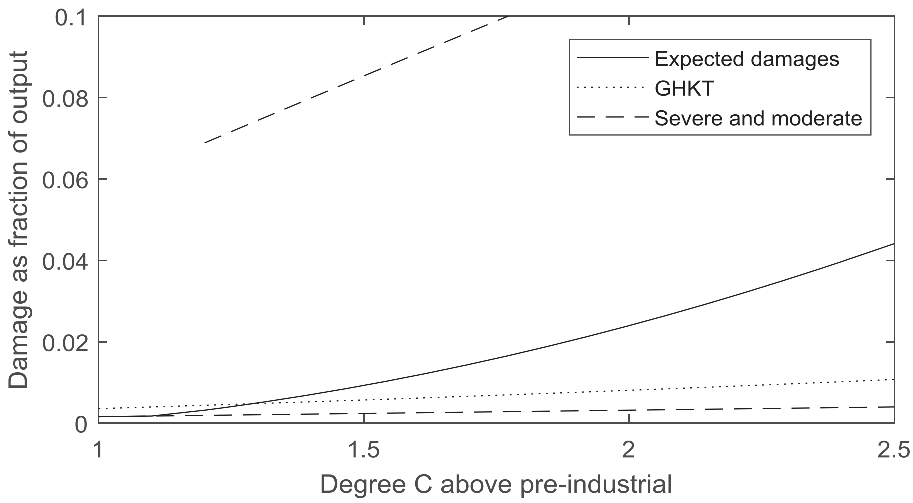

FIGURE 2: EXPECTED, SEVERE AND MODERATE DAMAGES COMPARED WITH GHKT

[11] For example, Weitzman (2010), Rezai and Van Der Ploeg (2017) and Acemoglu, Aghion, Bursztyn, and Hemous (2012).

II. Optimal tax

The Lagrangian maximizes (1) subject to production and temperature constraints as follows:

$$(14)\quad \mathcal{L}(C_t, \boldsymbol{N}_t, K_{t+1}, K_{t+1}, X_t, \boldsymbol{E}_t, T_t) = \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

$$+\sum_{t=0}^{\infty} \lambda_{0,t}\left(\mathrm{F}_0\left(N_{0,t}, K_t, \boldsymbol{E}_t, T_t\right) - C_t - K_{t+1}\right) + \sum_{t=0}^{\infty}\sum_{i=1}^{3} \lambda_{i,t}\left(\mathrm{F}_i\left(N_{i,t}\right) - E_{i,t}\right)$$

$$+\sum_{t=0}^{\infty} \lambda_{N,t}\left(N_t - \sum_{i=1}^{3} N_{i,t}\right) + \sum_{t=0}^{\infty} \lambda_{T,t}\left(T_t - \sum_{i=-\infty}^{t-1} \mathrm{T}_{t-i}^{c} E_{f,i}\right) + \mu_1\left(X_0 - \sum_{t=1}^{\infty} E_{1,t}\right).$$

The optimal condition describing the marginal costs and benefits of producing a unit of energy of type *i*, in terms of final consumption good at time *t*, is

$$(15.1)\qquad \frac{\lambda_{i,t}}{\lambda_{0,t}} + \frac{\mu_{i,t}}{\lambda_{0,t}} + \frac{1}{\lambda_{0,t}}\sum_{j=0}^{\infty} \lambda_{T,t+j}\mathrm{T}_j^c = \frac{\partial F_{0,\mathrm{t}}}{\partial E_{\mathrm{i,t}}} \text{ where}$$

$$(15.2)\qquad \lambda_{T,t} = -\lambda_{0,t}\frac{\partial F_{0,\mathrm{t}}}{\partial T_{\mathrm{t}}} \text{ and } \lambda_{0,t}\mathrm{C}_t = \beta^t.$$

The costs in (15.1) include the cost of input use $\frac{\lambda_{i,t}}{\lambda_{0,t}}$, the scarcity cost $\frac{\mu_{i,t}}{\lambda_{0,t}}$, and the marginal externality damage. This last cost is the optimal Pigouvian tax (Λ_t) and from (15.2) and (2) is as follows:

$$(16.1)\quad \Lambda_t = \mathbb{E}_t\left(\sum_{j=0}^{\infty} \frac{\lambda_{T,t+j}}{\lambda_{0,t}}\mathrm{T}_j^c\right) = \mathbb{E}_t\left(\sum_{j=0}^{\infty} \frac{\beta^{t+j} C_t Y_{t+j}}{\beta^t C_{t+j}} \frac{\partial(\gamma_{t+j}T_{t+j})}{\partial T_{t+j}}\mathrm{T}_j^c\right).$$

$$\text{(16.2)} \quad \widehat{\Lambda}_t := \frac{\Lambda_t}{\mathrm{Y}_t} = \sum_{j=0}^{\infty} \beta^j \mathrm{T}_j^c \mathbb{E}_t \left(\frac{\partial(\gamma_{t+j} T_{t+j})}{\partial T_{t+j}} \right).$$

In the GHKT model γ is constant, leading to a constant tax (to output) ratio $\widehat{\Lambda}$. Gerlagh and Liski (2018) consider an exogenous γ_t, so the tax ratio is pre-determined and independent of temperature outcomes. By assuming an endogenous link between the hazard rate and temperature, the tax ratio in this paper is a function of expected temperature outcomes. In the event of tipping in the *Learning* scenario, the damage parameter is the constant γ_H and therefore the tax ratio after tipping is constant. The expected derivative in (16.2) is

$$\text{(17)}\ \mathbb{E}_t \left(\frac{\partial(\gamma_{t+j} T_{t+j})}{\partial T_{t+j}} \right) = \gamma^L + (\gamma^H - \gamma^L) \mathbb{E}_t(p_{t+j}) + (\gamma^H - \gamma^L) \mathbb{E}_t \left(T_{t+j} \frac{\partial p_{t+j}}{\partial T_{t+j}} \right).$$

Following from lemma 1, this expression can be calculated from future temperatures conditional on no tipping. However, oil extraction is a function of expectations as (10) describes. The expectation term in (10), $\mathbb{E}_t \left(\frac{\omega}{E_{1,t+1}} - \widehat{\Lambda}_{t+1} \right)$, is given by

$$\text{(18)} \qquad \mathbb{E}_t(p_{t+1}) \left(\frac{\omega}{\mathrm{E}_{1,t+1}^{tip}} - \widehat{\Lambda}_{t+1}^{tip} \right) + \left(1 - \mathbb{E}_t(p_{t+1})\right) \left(\frac{\omega}{\mathrm{E}_{1,t+1}^{no\ tip}} - \widehat{\Lambda}_{t+1}^{no\ tip} \right).$$

As the tipping tax ratio $\widehat{\Lambda}_{t+1}^{tip}$ is constant, $\mathrm{E}_{1,t+1}^{tip}$ is determined by the following:

$$\text{(19)} \qquad \frac{\omega}{\mathrm{E}_{1,t+1}^{tip}} = \beta \frac{\omega}{\mathrm{E}_{1,t+2}^{tip}} + (1 - \beta) \widehat{\Lambda}^{tip}.$$

Although (19) is not amenable to an analytical solution, it is self-contained leading to the following lemma.

LEMMA 2: Oil use does not depend on temperature outcomes if tipping occurs.

The combination of lemmas 1 and 2 lead to the following key result.

PROPOSITION 1: The optimal tax ratio can be calculated by temperature outcomes conditional on no tipping only: outcomes where tipping occurs are not needed.

Thus, although tipping can happen at any period prior to peak temperature, the optimal tax can be determined by a single future outcome where tipping does not occur. This result follows from the form of the damages function coupled with key assumptions made in the GHKT model that lead to a constant tax ratio: logarithmic utility, a multiplicative exponential damage function of temperature which is a linear function of energy use, and constant savings rates.[12] Computation is consequently much simpler, as the tax profile can be determined by one future outcome rather than having to handle all possible outcomes as previous studies have done.[13] Proposition 1 would also hold for other tipping point constructs, such as a within-period hazard rate that is a function of current temperature only, as used by Gerlagh and Liski (2018).[14]

The first component of (17), $\gamma^L + (\gamma^H - \gamma^L)\mathbb{E}_t(p_{t+j})$, is the tax that would apply given an exogenous probability profile corresponding to the no-tipping temperature

[12] As already noted, GHKT use emissions in place of temperature but the result would still hold in this case.
[13] For example, D. Lemoine and Traeger (2014) and Lontzek et al. (2015).
[14] Complexity would increase using this approach as the cumulative hazard rate is path-dependent.

outcomes for the scenario.[15] The second component is due to the endogeneity of the hazard rate and is equal to

$$(20)\quad \mathbb{E}_t\left(T_{t+j}\frac{\partial(p_{t+j})}{\partial T_{t+j}}\right)=\begin{cases}\widehat{\mathbb{E}}_t\left(T_{t+j}\right)\dfrac{1-\mathbb{E}_t\left(p_{t+j}\right)}{T_{upper}-\mathrm{x}} & \text{if } \widehat{\mathbb{E}}_t\left(T_{t+j+1}\right)\geq\widehat{\mathbb{E}}_t\left(T^H_{t+j}\right)\\ 0 & \text{otherwise}\end{cases}$$

$$\text{where x}=\begin{cases}\widehat{\mathbb{E}}_t\left(T_{t+j}\right) & \text{for } Learning\\ T_0 & \text{for } No\ learning.\end{cases}$$

For all scenarios considered in the numerical exercises except *Laissez-faire*, peak temperature starts prior to 2300. A computationally challenging feature of (20) is the discontinuity in the derivative of the probability of tipping. To handle this discontinuity, a time of onset of peak temperature (τ) is imposed with constraints ensuring temperature after period τ does not exceed T_τ: $\lambda_{M,t}(T_\tau - T_t)\ \geq 0$ for $t > \tau$. Equation (15.1) becomes $\lambda_{T,t} = -\lambda_{0,t}\frac{\partial F_{0,t}}{\partial T_t} + \lambda_{M,t}$ and the tax becomes

$$(21)\qquad \widehat{\Lambda}_t=\mathbb{E}_t\left(\sum_{j=0}^{\varsigma}\beta^j\frac{\partial\left(\gamma_{t+j}T_{t+j}\right)}{\partial T_{t+j}}T^c_j+\sum_{j=\varsigma+1}^{\infty}\left(\beta^j\gamma_\tau+\frac{\tilde{\lambda}_{t+j}}{\beta^t}\right)T^c_j\right)$$

$$\text{where } \varsigma=\max\left(\tau-t,-1\right)\ \text{ and }\ \tilde{\lambda}_{t+j}:=\lambda_{M,t+j}\frac{C_t}{Y_t}.$$

The multiplier $\tilde{\lambda}_{t+j}$ is $\lambda_{M,t+j}$ adjusted by the constant consumption rate. If tipping has not occurred, taxes following peak temperature are set so that temperature does not exceed T_τ and these peak temperature taxes are equal to:

[15] This sum is not the optimal tax which would apply in a separate scenario with the same time-dependent hazard rates, as the lower tax would lead to higher temperature outcomes.

$$(22) \qquad \widehat{\Lambda}_t = \sum_{j=0}^{\infty} \left(\beta^j \gamma_\tau + \frac{\tilde{\lambda}_{t+j}}{\beta^t} \right) T_j^c.$$

The multipliers are derived from the (numerically determined) taxes needed to stabilize temperature: the tax in the period prior to the end of peak temp is given by $\widehat{\Lambda}_{n-1} = \gamma_L \sum_{j=0}^{\infty} \beta^j T_j^c + \frac{\tilde{\lambda}_n T_1^c}{\beta^{n-1}}$ which gives $\tilde{\lambda}_n$, the tax in the prior period is $\widehat{\Lambda}_{n-2} = \gamma_L \sum_{j=0}^{\infty} \beta^j T_j^c + \frac{\tilde{\lambda}_{n-1} T_1^c}{\beta^{n-2}} + \frac{\tilde{\lambda}_n T_2^c}{\beta^{n-2}}$ which then gives $\tilde{\lambda}_{n-1}$ and so on. The choice of τ is determined through manual iteration: for a high τ, peak temperature occurs prior to this value, and thus the value of τ is manually reduced until it corresponds with peak temperature. Any further reduction in τ implies the tax in this period is higher than would be the case without the upper limit constraint applying (holding the tax in all other periods constant), implying suboptimality.

The main scenario assumes the hazard rate is a linear function of temperature, consistent with damages. Convex functions could also be modelled as in Van der Ploeg (2014). An advantage of the analytical expression (17) is that it is clear how the slope of the hazard rate with respect to temperature, $\frac{\partial p_{t+j}}{\partial T_{t+j}}$, enters the tax equation. In the numerical examples in the next section, the endogenous tax component dominates. Therefore, a quadratic function with a much lower initial slope should greatly reduce the optimal tax, and this is confirmed with a sensitivity in the next section.

The effects of learning

To understand what drives changes in the tax ratio initially, consider a prolonged period of peak temperature and small temperature changes relative to the upper limit such that the probability of tipping is small, $1 \gg \mathbb{E}_t(p_{t+j})$. Ignoring the multipliers in (21), and using (12) (17) and (20), the tax ratio prior to peak temperature is approximately given by:

$$(23) \qquad \widehat{\Lambda}_t \sim \gamma^L \sum_{j=0}^{\infty} \beta^j T_j^c + \frac{(\gamma^H - \gamma^L)}{T_{upper} - T_0} \mathbb{E}_0 \sum_{j=0}^{\tau - t} \beta^j \left(2T_{t+j} - T_s\right) T_j^c.$$

The absolute growth in initial tax ratios for each scenario $\Delta\widehat{\Lambda}_0^{Learn}$ and $\Delta\widehat{\Lambda}_0^{No\ learn}$ are then

$$(24) \quad \Delta\widehat{\Lambda}_0^{Learn} \sim \frac{(\gamma^H - \gamma^L)}{T_{upper} - T_0} \mathbb{E}_0 \left(\sum_{j=0}^{\tau-1} 2\beta^j \Delta T_j T_j^c - \beta^\tau T_\tau T_j^c - \sum_{j=0}^{\infty} \beta^j \Delta T_0\, T_j^c \right)$$

$$\Delta\widehat{\Lambda}_0^{No\ learn} \sim \frac{(\gamma^H - \gamma^L)}{T_{upper} - T_0} \mathbb{E}_0 \left(\sum_{j=0}^{\tau-1} 2\beta^j \Delta T_j T_j^c - \beta^\tau T_\tau T_j^c \right).$$

The first term $\sum_{j=0}^{\tau-1} 2\beta^j \Delta T_j T_j^c$ is positive and increases with the growth in temperature prior to the peak. The second term $-\beta^\tau T_\tau T_j^c$ is negative and increases with the proximity of peak temperature. Thus, if peak temperature is low and occurs soon, the tax ratios will fall in both learning and non-learning scenarios, as demonstrated in the numerical examples in this paper. However, for a distant peak temperature, the tax ratio will grow if there is no learning and may grow or shrink with learning, depending on the third term. This third term implies growth in the *Learning* scenario is lower than the *No learning* scenario.

PROPOSITION 2: For temperatures well below the upper bound, learning that tipping has not occurred reduces growth in the optimal carbon tax.

The condition that the temperature is well below the upper bound is needed because, if tipping has not occurred, the optimal tax approaches infinity as the temperature approaches the upper bound. This follows due to the certain existence of a threshold below the upper bound and should be considered merely as an artefact of this modelling assumption. While an interesting implication is that the tax can rise above the optimal post-tipping tax if tipping has not occurred, this behaviour is not explored in the numerical examples.

Consider a distant peak temperature (τ approaches infinity) where $\Delta T_{j+1} = a\Delta T_j$ for all j and constant temperature response T^c, then the change in tax rates can be approximated by:

$$\Delta\widehat{\Lambda}_0^{Learn} \sim \frac{(\gamma^H - \gamma^L)T^c}{T_{upper} - T_0}\left(\frac{2}{1-\beta a} - \frac{1}{1-\beta}\right)\Delta T_0 \tag{25.1}$$

$$\Delta\widehat{\Lambda}_0^{No\ learn} \sim \frac{(\gamma^H - \gamma^L)T^c}{T_{upper} - T_0}\left(\frac{2}{1-\beta a}\right)\Delta T_0. \tag{25.2}$$

Thus for $a\sim 1$, the tax ratio for *Learning* will grow at roughly half the rate of *No learning*, while for a highly concave steady-state emissions profile ($a < 0.5$) the *Learning* tax ratio will shrink.

REMARK 1: Consider a distant peak temperature (τ approaches infinity) and slowly growing temperature. The *No learning* tax will grow initially while growth in the *Learning* tax depends on the concavity of temperature profile. For an almost linear temperature profile, the *Learning* tax will grow at roughly half the rate of the *No learning* tax.

Catastrophic damages

Recent literature has suggested that climate policy is highly sensitive to 'fat-tailed' risks of catastrophic outcomes (Weitzman, 2009). As noted in the previous section, the framework used in this paper considers a known high climate sensitivity in the severe regime after tipping. Consider that the severe climate sensitivity γ^H is unknown and moderate damages are zero, and the tipping framework is as already described. Uncertainty of γ^H is independent of uncertainty related to the future probability of a tipping event. The economic framework used in this paper does not exhibit sensitivity to fat-tailed risks, as the key term in the optimal tax equation (17) is simply a function of the expectation of γ^H as follows:[16]

$$(26)\quad \mathbb{E}_t\left(\frac{\partial(\gamma_{t+j}T_{t+j})}{\partial T_{t+j}}\right) = \mathbb{E}_t(\gamma^H)\mathbb{E}_t(p_{t+j}) + \mathbb{E}_t(\gamma^H)\mathbb{E}_t\left(T_{t+j}\frac{\partial p_{t+j}}{\partial T_{t+j}}\right).$$

Now consider a finite probability of infinite climate sensitivity $\gamma^H \to \infty$. If the probability of tipping is non-zero, equation (26) indicates that the tax ratio will also be infinite, as will the tax level.[17] Due to the climate model and tipping framework, the probability of tipping is zero in the *Learning* scenario if the onset of peak temperature has occurred, and thus the tax will be finite.

Consider that an infinite tax in period t leads to zero fossil energy use in that period. The temperature may rise or fall in a period with zero emissions, depending on the profile of historical emissions according to (6). For the parameter settings in

[16] While Weitzman (2009) considers a coefficient of relative risk aversion greater than one, in this paper utility is a logarithmic function of consumption.

[17] To consider the behaviour of the tax level as the tax ratio approaches infinity, note that the energy composite approaches a finite value if the elasticity of substitution is greater than 1: $E_t \xrightarrow{\widehat{\Lambda}_t \to \infty} (1-\kappa)^{\frac{1-\rho}{\rho}}$ if $\rho > 0$. Assuming an infinite tax level and constant oil extraction in (10) leads to zero oil extraction and $E_1 \xrightarrow{\widehat{\Lambda} \to \infty} \omega/\widehat{\Lambda}$. Thus output approaches zero with $\widehat{\Lambda}^{-\omega}$ and the tax level also approaches infinity if the probability of tipping is non-zero.

this paper, the initial temperature rises in 2020 even if energy use is zero due to the recent ramp up in emissions, implying an infinite tax if there is a finite probability of infinite damages. However, for later periods the temperature would fall as the temperature response from an emissions pulse drops after two decades, as shown in Figure 1. Thus from 2030 the optimal tax will be finite as the hazard rate becomes zero. More formally, from (6) a decreasing temperature, $T_{t+1} < T_t$ leads to the following remark as $E_{f,t} \to 0$.

REMARK 2: If the expected damages from tipping are infinite but tipping has not yet occurred, the optimal tax in period t is finite if:

$$\sum_{j=-\infty}^{t} \left(T_{t+1-j}^{c} - T_{t-j}^{c}\right) E_{f,j} < 0. \tag{27}$$

Under a *No learning* scenario, the hazard rate becomes finite permanently if the temperature rises above the starting point, which occurs in 2020 even if energy use is zero as discussed above. Hence the tax will always be infinite in this scenario if the expected damages from tipping are infinite. In the next section, the effects of catastrophic damages are further explored by choosing extreme climate sensitivity in a post-tipping world.

III. Numerical examples

Table 1 shows parameters for the main simulation, taken from GHKT and Shine et al. (2005). Historical emissions go back a century and induce initial warming at 2020 of 1.11^0C, aligning with the centre of the range of IPCC estimates (IPCC, 2014). Initial decadal global GDP is set to \$800 trillion. Key differences in parameter choices between this paper and GHKT are the elasticity of substitution and assumed growth in coal technology. This paper assumes a higher elasticity of 2, which GHKT consider as a sensitivity, and reduced growth in coal technology, reflecting the relative maturity of dirty technology.[18] As sensitivities, I investigate coal productivity growth at 2 per cent as in GHKT, a lower elasticity of substitution between coal and clean of 1.5, a quadratic probability in temperature, and catastrophic damages under tipping.

Due to the separation of oil and other energy, some parameters are recalibrated as follows. Using initial values of energy inputs from GHKT $\widetilde{\mathrm{E}}_{1,0}$, $\widetilde{\mathrm{E}}_{2,0}$ and $\widetilde{\mathrm{E}}_{3,0}$[19] the ratio of renewable to coal is

$$\frac{1-\kappa}{\kappa}\left(\frac{\widetilde{\mathrm{E}}_{3,0}}{\widetilde{\mathrm{E}}_{2,0}}\right)^{\rho-1} = 5.87, \tag{28}$$

and the ratio of the oil to coal price is

$$\frac{\omega}{\kappa(\nu-\omega)}\left(\frac{\widetilde{\mathrm{E}}_{0}}{\widetilde{\mathrm{E}}_{2,0}}\right)^{\rho}\frac{\widetilde{\mathrm{E}}_{2,0}}{\widetilde{\mathrm{E}}_{1,0}} = 5.87. \tag{29}$$

TABLE 1: CALIBRATION PARAMETERS

gA_0 (%/year)	gA_2 (%/year)	gA_3 (%/year)	$A_{2,0}$	$A_{3,0}$	ρ	ν	ω	κ
1.3	1	2	8792	1498	0.5	0.04	0.0215	0.1786
β (annual)	δ (%/decade)	X_0 (GtC)	N	a_0	a_1	a_2	a_3	a_4
0.985	100	253.8	1	0.1756	0.1375	0.1858	0.2423	0.2589
T_0 (^{0}C)	γ_L	γ_H	10^{14} Y_0 (\$)	α	α_1	α_2	α_3	α_4
1.11	0.00160	0.05945	8	0.3	421.09	70.597	21.422	3.4154
$E_f(-10:-1)$			Period 0	A_c	H			
[10,10,10,20,30,40,50,60,80,100]			2020	1.98	4.2			

[18] Coal technology grows at 1% per annum while clean technology grows at 2 per cent per annum.

[19] These values in GtC/yr are 3.43, 3.75 and 2.3 respectively.

Results are described for the two tipping scenarios *Learning* and *No learning*, the fixed probability of severe damage scenario *No tipping,* and a *Laissez-faire* scenario where the tax is zero. Projections for the *Learning* scenario show a future path where no tipping occurs, but the optimal tax naturally considers uncertainty about the future. After all, we are interested in how a potential tipping point affects optimal policy. I also show today's expected tax path for the *Learning* scenario in figures.

Main simulation

As shown in Panel A of Figure 3, the *Learning* tax ratio starts at about the same level as *No learning* but falls faster, consistent with proposition 2. Ratios in both scenarios drop until the end of peak temperature (panel G) and real taxes flatten (panel B) coinciding with peak temperature. Following peak temperature, the tax grows with income. Today's expected tax ratio in the *Learning* scenario is also shown, which ends up marginally higher than the *No learning* tax ratio. Endogeneity of the hazard rate makes up the majority of the optimal tax (panels C and D). Coal emissions spike with a falling tax ratio (panel E), while oil use is almost identical between scenarios (panel F). The benefits of reduced damages on consumption and the costs from the distortionary effects on the other energy composite E_t are shown in panel H for the *Learning* scenario relative to the *Laissez-faire* scenario. For simplicity, reduced damages correspond to the difference in damage multipliers for moderate climate sensitivity, $\exp(\gamma^L \mathrm{T}_t^{Learn}) - \exp(\gamma^L \mathrm{T}_t^{LF})$, which excludes the benefits of reduced risks and damages from tipping. The tax distortion captures the distortionary effect in the other energy composite E_t holding oil use and capital stock constant. These two effects, in addition to the effect on oil extraction (not shown), contribute to the total change in consumption.

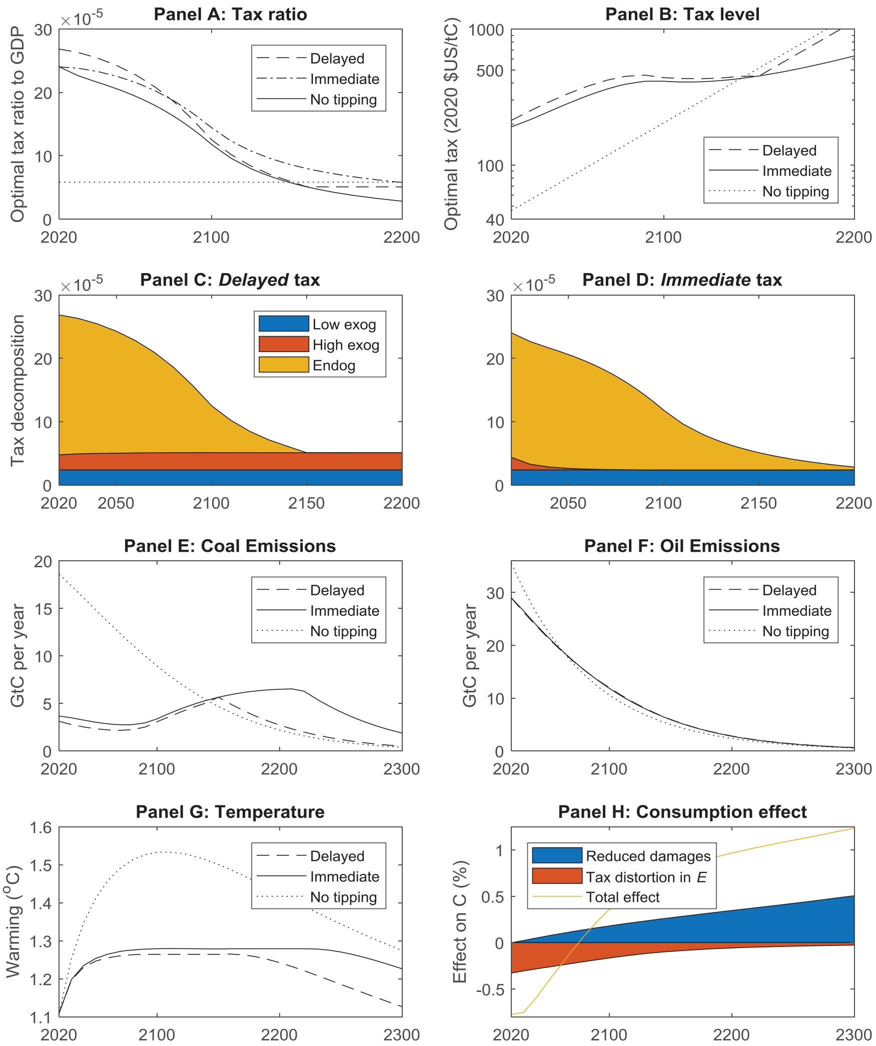

FIGURE 3: RESULTS FOR THE MAIN SIMULATION

Scenarios *Learning*, *No learning* and *No tipping*. E(Learning) is the initial expected tax profile in the *Learning* scenario. The total consumption effect in panel H is the effect of the tax in the *Learning* scenario relative to a *Laissez-faire* scenario. Reduced damages correspond to the difference in damage multipliers with moderate climate sensitivity, and the tax distortion captures the distortionary effect in the other energy composite E_t holding oil use and capital stock constant.

While GHKT find that coal – rather than oil – is the main threat to economic welfare, panel F indicates that oil dominates over coal for contribution to peak temperature. Amongst other assumptions, this result depends on the substitutability of energy and technology growth, discussed in the next section as sensitivities.

Sensitivities

Figure 4 shows the results from various sensitivities. The first row shows results when coal technology grows by 2 per cent per annum, in line with clean technology. The second reduces the elasticity between coal and clean energy to 1.5. The third shows results when the probability of tipping is quadratic in temperature change, so that $\mathbb{E}_t\left(p_{t+j}\right) \sim \frac{\left(\hat{\mathbb{E}}_t\left(T^H_{t+j}\right)\right)^2 - {T_S}^2}{{T_{upper}}^2 - {T_S}^2}$ in equation (12). The fourth shows catastrophic damages where γ_H is increased by a factor of 10 so that damages reach 30 per cent at 2^{O}C warming. Stabilisation of peak temperature occurs in all sensitivities.

As the technology growth rates for coal and clean energy are the same in the first sensitivity, the marginal cost of abatement is flat after oil is depleted. This contrasts with the main simulation where the marginal cost of abatement declines exogenously, and in previous studies using the DICE model. The comparable temperature outcomes between the first sensitivity and the main result indicate that the tax is dominant in determining energy inputs, rather than productivity growth.

As one would expect, outcomes are highly sensitive to the elasticity of substitution between coal and clean inputs. A reduction in elasticity to 1.5 increases temperature outcomes and the tax. Changing the slope of the hazard rate in the third sensitivity eliminates most of the endogenous component of the tax. Catastrophic damages lead to very high taxes and a peak temperature in 2030. The drop in the tax is sufficient to dominate the discount rate in the hotelling formula for oil extraction (10), leading to peak oil use in 2030 rather than 2020.

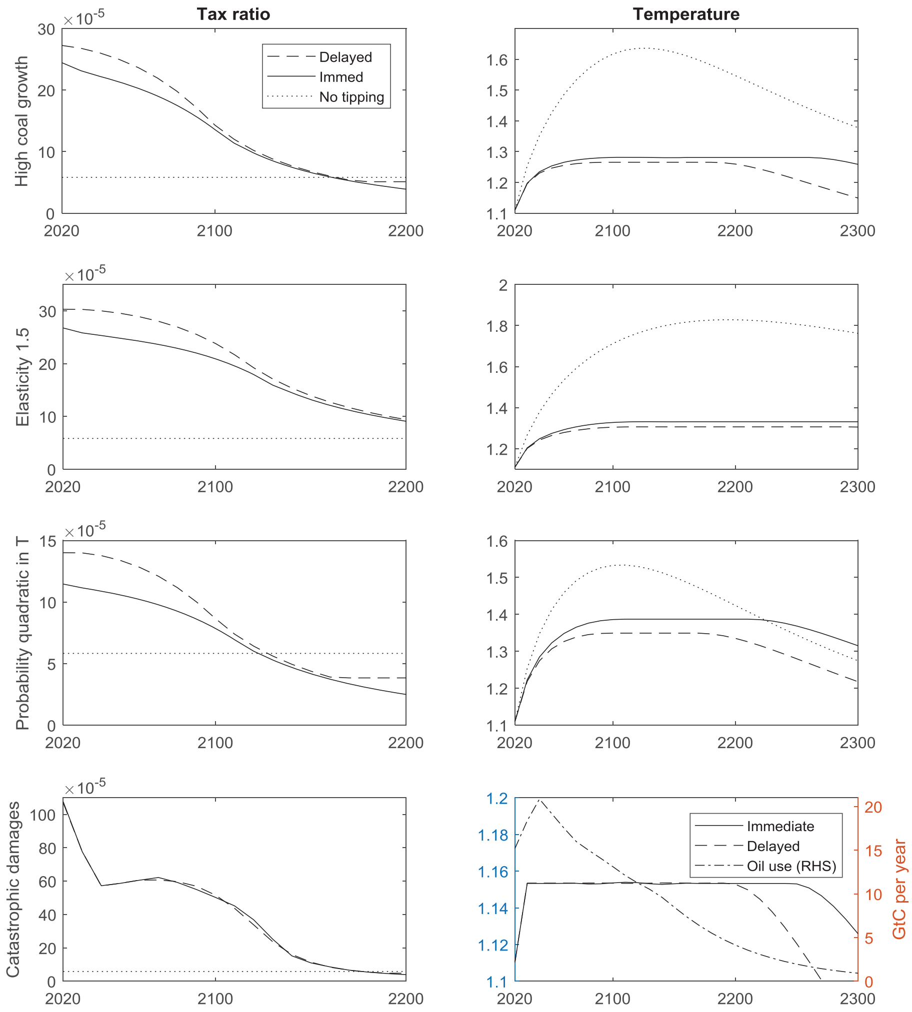

FIGURE 4 RESULTS FOR THE SENSITIVITIES

Table 2 shows some results for the above simulations plus Laissez-faire simulations where the tax is zero. Peak temperature does not occur before 2300 for the Laissez-faire simulations.

TABLE 2: ALL RESULTS

Simulations		Λ_{2020} (\$2020)	$\widehat{\Lambda}_{2020}$ (10^{-5})	$\widehat{\Lambda}_{2100}$ (10^{-5})	$\widehat{\Lambda}_{2200}$ (10^{-5})	Peak onset (τ)	Peak end	Peak T (^{0}C)	T_{2100} (^{0}C)	T_{2200} (^{0}C)
Main	Learn	191	24	11.8	2.8	2110	2230	1.28	1.28	1.28
	No learn	213	26.8	12.5	5.1	2110	2160	1.26	1.28	1.24
	No tip	46.3	5.8	5.8	5.8	2120	2120	1.53	1.53	1.42
Coal growth	Learn	194	24.4	13.6	3.9	2110	2270	1.28	1.28	1.28
	No learn	216	27.2	14.3	5.1	2110	2190	1.27	1.28	1.26
	No tip	46.3	5.8	5.8	5.8	2140	2140	1.64	1.62	1.55
Low elasticity	Learn	212	26.8	20.9	9.1	2130	2420	1.33	1.33	1.33
	No learn	240	30.3	23.8	9.4	2130	2300	1.31	1.33	1.31
	No tip	46.2	5.8	5.8	5.8	2200	2200	1.83	1.71	1.83
Quadratic probability	Learn	91.3	11.5	7.8	2.5	2120	2220	1.39	1.39	1.39
	No learn	112	14	8.6	3.8	2110	2180	1.35	1.39	1.33
Catastrophic damages	Learn	842	108	52	3.9	2040	2250	1.15	1.15	1.15
	No learn	837	107	51	4.8	2040	2200	1.15	1.15	1.15

		Main	Coal growth	Low elasticity
Laissez-faire	T_{2100} (^{0}C)	2.4	3.7	2.6
	T_{2200} (^{0}C)	3.4	22	4.8

Scenarios shown are: *Learning* (Learn), *No learning* (No learn) and *No tipping* (No tip).

III. Conclusion

This paper examines optimal policy where there is an unknown temperature threshold of severe and irreversible climate tipping. Most of the initial carbon price is due to the endogeneity of the probability of tipping, so using an exogenous probability of tipping may materially underestimate the starting optimal tax rate. The tax ratio drops prior to and during peak temperature, leading to a prolonged stabilisation at peak temperature and a rebound in emissions.

A decline in the optimal tax ratio merits greater awareness not only for the implications for temperature and emissions outcomes, but also for the potential effect on public perceptions of a carbon price. Such a decline may alleviate public resistance to a carbon price due to the temporary nature of high tax levels. In addition, temperature stabilisation resulting from the decline conceptually links the political strategy of setting temperature targets with an optimal tax from an integrated assessment framework.

Without the risk of tipping, or if tipping has already occurred, the optimal carbon tax is constant as a ratio of output and independent of expected temperature. When there is a chance of tipping, the optimal carbon tax becomes a function of future temperature, which is sensitive to the structure of energy and technology growth. However, due to the structure of the model, only future temperature outcomes without tipping need to be considered in deriving the tax ratio. This makes consideration of tipping points easy to incorporate compared with the DICE model, as used by previous studies.

Along with this relative advantage, the model used in this paper allows investigation of energy technology growth and energy substitutability on the optimal policy choice. The derived explicit tax equations provide general insights which future studies can build on and apply. For example, in another paper I use these equations to derive the optimal weights between methane (or aerosols associated with geoengineering which are also short-lived) and carbon due to different decay profiles (Wiskich, 2019b).

REFERENCES

Acemoglu, D., Aghion, P., Bursztyn, L., & Hemous, D. (2012). The environment and directed technical change. *American economic review, 102*(1), 131-166.

Allen, M. R., Frame, D. J., Huntingford, C., Jones, C. D., Lowe, J. A., Meinshausen, M., & Meinshausen, N. (2009). Warming caused by cumulative carbon emissions towards the trillionth tonne. *Nature, 458*, 1163. doi:10.1038/nature08019
https://www.nature.com/articles/nature08019#supplementary-information

Burke, M., Hsiang, S. M., & Miguel, E. (2015). Global non-linear effect of temperature on economic production. *Nature, 527*(7577), 235.

Cai, Y., Judd, K. L., & Lontzek, T. S. (2013). *The social cost of stochastic and irreversible climate change*. Retrieved from

Cai, Y., & Lontzek, T. S. (2019). The social cost of carbon with economic and climate risks. *Journal of Political Economy, 127*(6), 000-000.

Cropper, M. L. (1976). Regulating activities with catastrophic environmental effects. *Journal of Environmental Economics and management, 3*(1), 1-15.

Engström, G., & Gars, J. (2016). Climatic tipping points and optimal fossil-fuel use. *Environmental and Resource Economics, 65*(3), 541-571.

Farmer, J. D., Hepburn, C., Mealy, P., & Teytelboym, A. (2015). A third wave in the economics of climate change. *Environmental and Resource Economics, 62*(2), 329-357.

Gerlagh, R., & Liski, M. (2018). Carbon prices for the next hundred years. *The Economic Journal, 128*(609), 728-757.

Golosov, M., Hassler, J., Krusell, P., & Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica, 82*(1), 41-88.

IPCC. (2014). Climate Change 2014: Synthesis Report. . *Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*.

Lemoine, D. (2017). The climate risk premium: how uncertainty affects the social cost of carbon.

Lemoine, D., & Rudik, I. (2017). Steering the climate system: using inertia to lower the cost of policy. *American economic review, 107*(10), 2947-2957.

Lemoine, D., & Traeger, C. (2014). Watch your step: optimal policy in a tipping climate. *American Economic Journal: Economic Policy, 6*(1), 137-166.

Lemoine, D., & Traeger, C. P. (2016). Economics of tipping the climate dominoes. *Nature Climate Change, 6*(5), 514.

Lemoine, D. M., & Traeger, C. P. (2012). *Tipping points and ambiguity in the economics of climate change*. Retrieved from

Lontzek, T. S., Cai, Y., Judd, K. L., & Lenton, T. M. (2015). Stochastic integrated assessment of climate tipping points indicates the need for strict climate policy. *Nature Climate Change, 5*(5), 441.

Nordhaus, W. D. (2008). A question of balance: economic modeling of global warming. In: Yale University Press New Haven.

Ploeg, F., & Withagen, C. (2014). Growth, renewables, and the optimal carbon tax. *International Economic Review, 55*(1), 283-311.

Rezai, A., & Van Der Ploeg, F. (2017). Abandoning fossil fuel: how fast and how much. *The Manchester School, 85*(S2).

Shine, K. P., Fuglestvedt, J. S., Hailemariam, K., & Stuber, N. (2005). Alternatives to the global warming potential for comparing climate impacts of emissions of greenhouse gases. *Climatic Change, 68*(3), 281-302.

Traeger, C. P. (2015). Analytic integrated assessment and uncertainty.

Van der Ploeg, F. (2014). Abrupt positive feedback and the social cost of carbon. *European Economic Review, 67*, 28-41.

van der Ploeg, F., & de Zeeuw, A. (2017). Climate tipping and economic growth: Precautionary capital and the price of carbon. *Journal of the European Economic Association, 16*(5), 1577-1617.

van der Ploeg, F., & de Zeeuw, A. (2019). Pricing carbon and adjusting capital to fend off climate catastrophes. *Environmental and Resource Economics, 72*(1), 29-50.

Weitzman, M. L. (2009). On modeling and interpreting the economics of catastrophic climate change. *The Review of Economics and Statistics, 91*(1), 1-19.

Weitzman, M. L. (2010). What Is The" Damages Function" For Global Warming—And What Difference Might It Make? *Climate Change Economics, 1*(01), 57-69.

Wiskich, A. (2019a). Decreasing substitutability between clean and dirty energy. *CAMA Working Paper No. 72/2019*. Retrieved from https://ssrn.com/abstract=3458743

Wiskich, A. (2019b). Weighing cows, geoengineering and coal under a climate tipping risk and a temperature target. *CAMA Working Paper No. 65/2019*. Retrieved from https://ssrn.com/abstract=3446458