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Abstract

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Keywords

Climate change, tipping points, optimal policy, optimal taxes, global warming potential.

JEL Classification

H23, O44, Q30, Q40, Q54, Q56, Q58

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Weighing cows and coal: optimal taxes for methane and carbon from a tipping risk

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Different optimal tax paths for short-lived methane and long-lived carbon arise in a cost-benefit framework with an unknown temperature threshold where severe and irreversible climate impacts, called a tipping point, occurs. Tax paths are compared with a cost-minimising approach where an upper-temperature limit is set. In both approaches, the weight (ratio) of methane to carbon taxes converge to the same value by the end of the peak temperature stabilisation period. Numerical results from the cost-benefit framework suggest: the optimal weight is close to the current United Nations policy of a 100-year Global Warming Potential; and the time-frame should decrease to align with the expected end of peak temperature. (JEL H23, O44, Q30, Q40, Q54, Q56, Q58)

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Global warming potentials (GWP) are used to compare the climate effects of emissions of different greenhouse gases. The 100-year GWP was adopted by the United Nations Framework Convention on Climate Change and its Kyoto Protocol and is now used widely as the default metric. The approach has been subject to the criticism that it neglects economic considerations of how relative weights between gases should change over time.[1]

The GWP of a gas is the time-integrated radiative forcing from a pulse emission, relative to an equal mass of CO_2, and thus results for short-lived versus long-lived gases depend on the choice of time horizon. For example, methane has a 100-year GWP of 28 and a 20-year GWP of 84 (IPCC, 2014). Papers have highlighted the suboptimality of assuming constant weights between gases in a cost-minimisation (or cost-effectiveness) framework where an upper bound of temperature or emissions concentration is exogenously imposed, either at a point in time or for all time.[2]

The key finding of the cost-minimisation literature is that the optimal weight of methane to carbon increases by more than an order of magnitude as peak temperature (or the designated point in time) approaches because early on the short-lived methane emissions contribute less to peak temperature.[3] The cost-minimisation approach for a temperature target at a point in time aligns with an alternative to the GWP: the Global Temperature Change Potential (GTP), which quantifies the temperature change at some given time after a pulse emission (Shine, Fuglestvedt, Hailemariam, & Stuber, 2005).

Use of an integrated assessment model that includes a damage function allows a cost-benefit analysis where a utility function is maximised. Most studies that investigate different gases using such an approach focus on initial prices rather than

[1] For example, Shine (2009).

[2] Cost-minimisation references include Manne and Richels (2001), O'Neill (2003), Aaheim, Fuglestvedt, and Godal (2006), and Johansson, Persson, and Azar (2006). A growing price ratio as a target stock of emissions is approached was perhaps first illustrated by Michaelis (1992).

[3] However, if a limit to the rate of temperature increase is set due to a perceived risk from a rapidly changing temperature, the profile flattens.

the change in relative prices over time.[4] Due to different rates of decay between gases, the marginal cost of methane relative to carbon is highly sensitive to the choice of the discount rate, but this cost ratio is often time-invariant. For example, the model described by Golosov, Hassler, Krusell, and Tsyvinski (2014) implies a constant expected optimal tax to income ratio, so the relative optimal tax between methane and carbon would also be constant.[5]

Thus, there is a divergence in the relative optimal tax paths for methane and carbon depending on whether a cost-minimisation or cost-benefit approach is taken, which are abbreviated to CM and CB hereafter. The ratio of optimal taxes for methane and carbon, hereafter referred to as the (optimal) weight, starts very low in a CM framework and increases by more than an order of magnitude, whereas it is relatively flat in a CB approach. Tol, Berntsen, O'Neill, Fuglestvedt, and Shine (2012) discuss the relationships between these different approaches and appropriate metrics for each, noting that the GWP is a special case of a solution to a CB approach. Consequently, they argue that this metric is inconsistent with a temperature target. The current paper helps to link the CB and CM approaches and provides further insight into what metric may be appropriate for weighing different gases. Optimal tax paths under both frameworks are discussed with some numerical results, as a guide to policymakers as they weigh uncertainties in the policy goal and metrics such as the discount rate.[6]

The key difference between this paper and previous CB papers is the treatment of uncertainty of climate damages: I assume no climate effect unless a permanent and irreversible tipping point is triggered, with the tipping risk increasing with

[4] Cost-benefit references include Waldhoff, Anthoff, Rose, and Tol (2011), Hope (2005) and Tol (1999).

[5] Marten and Newbold (2012) find that the social cost of methane relative to carbon rises by up to 50 percent by 2050, due in part to their climate model where the marginal forcing of methane decreases slower than carbon with the increasing atmospheric stock.

[6] Most previous papers only show results from one framework, with Goulder and Mathai (2000) an exception.

temperature. Many papers have considered such threshold environmental effects, going back to Cropper (1976). Damage is a function of temperature and awareness of tipping is immediate, similar to Lemoine and Traeger (2014). The dependence of the risk of tipping on the temperature increase leads to some commonality between the CB and CM approaches which has not been discussed in the literature. For example, the carbon price falls to zero following peak temperature, and both CB and CM weights converge to the same value as the end of peak temperature approaches.

The framework in this paper is close to the model I developed in Wiskich (2019), which also incorporates a tipping point into the same economic model but only considers the optimal tax profile of carbon. The focus of the current paper is the optimal weight between methane and carbon under different learning assumptions. While the numerical exercises in this paper are limited in scope, they suggest a couple of interesting policy insights. First, current policy using a 100-year GWP to weight methane is close to optimal. Second, the time horizon should reduce to correspond with an early estimate of the end of the stabilisation period for peak temperature, which is relatively simple to calculate. Another, and perhaps greater, contribution is how the tipping framework described captures the sensitivity to temperature outcomes in a similar way to a CM approach.

I. Model

The model builds on Wiskich (2019), which builds on the completely characterised model described in Golosov et al. (2014). A global representative household has logarithmic preferences over consumption with discount factor β, and thus maximises the following over discrete decadal time periods:

$$\text{(1)} \qquad \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \text{ where } U(C_t) := \log(C_t).$$

Final output is a Cobb-Douglas specification of sector '0' capital $K_{0,t}$ and labour $N_{0,t}$, oil $E_{1,t}$ and other energy E_t, and a multiplicative exponential damage function of atmospheric temperature T_t above pre-industrial, as follows:

$$Y_t = \mathrm{F}_{0,t}(N_{0,t}, K_{0,t}, \boldsymbol{E}_t, T_t) = e^{-\gamma_t T_t} A_{0,t} {K_t}^{\alpha} {N_{0,t}}^{1-\alpha-\nu} {E_{1,t}}^{\omega} {E_t}^{\nu-\omega}. \tag{2}$$

Damages are an exponential function of temperature which is consistent with some recent evidence (Burke, Hsiang, & Miguel, 2015). Climate dynamics for carbon and methane are taken from Shine et al. (2005) and account for climate-system inertia. Temperature dynamics are a function of radiative forcing R_t:

$$H \frac{dT_t}{dt} = R_t - \frac{T_t}{\lambda}, \tag{3}$$

where H is the heat capacity of the system and λ is a climate sensitivity parameter. For carbon, radiative forcing and temperature responses at time t after an emissions pulse are

$$R_t^c := \frac{\partial R_t}{\partial E_{f,0}} = a_0 + \sum_{i=1}^{4} a_i e^{-\frac{t}{\alpha_i}} \text{ and} \tag{4}$$

$$T_t^c := \frac{\partial T_t}{\partial E_{f,0}} = \frac{A_c}{H} \left\{ \zeta a_0 \left(1 - e^{-\frac{t}{\zeta}}\right) + \sum_{i=1}^{4} \frac{a_i \left(e^{-\frac{t}{\alpha_i}} - e^{-\frac{t}{\zeta}}\right)}{(\zeta^{-1} - {\alpha_i}^{-1})} \right\}, \tag{5}$$

where a_i are coefficients which sum to 1, α_i reflect gas lifetimes in years, ζ is by definition the constant λH in years, and A_c is the radiative forcing due to a 1-kg change in carbon dioxide. For methane the equations are simpler:

(6) $$R_t^m = A_m e^{-\frac{t}{\alpha_m}} \text{ and}$$

(7) $$T_t^m = \frac{A_m}{H(\zeta^{-1} - \alpha_m{}^{-1})}\left(e^{-\frac{t}{\alpha_m}} - e^{-\frac{t}{\zeta}}\right).$$

Temperature is a linear function of previous emissions, including historical emissions, as follows:

(8) $$T_t = \sum_{i=-\infty}^{t} T_{t-i}^c E_{f,i}.$$

The first panel of Figure 1 shows the radiative forcing from emissions pulses of carbon and methane. The impulse function for methane is normalised so that the non-discounted sum over a time horizon of 100 years is the same as for carbon, corresponding to equal integrated radiative forcing (GWP) over 100 years. The second panel shows the GTP, highlighting the sharp temperature response to the methane pulse relative to the carbon pulse. The third panel shows GWP and a similar metric, integrated GTP (iGTP), which integrates total temperature changes up to a point in time.[7] As the time horizon decreases, the weight for methane increases relative to carbon and the metrics GWP and iGTP are almost identical.

[7] This notation is taken from Peters, Aamaas, Berntsen, and Fuglestvedt (2011) and is equivalent to the sustained GTP concept discussed in Shine et al. (2005).

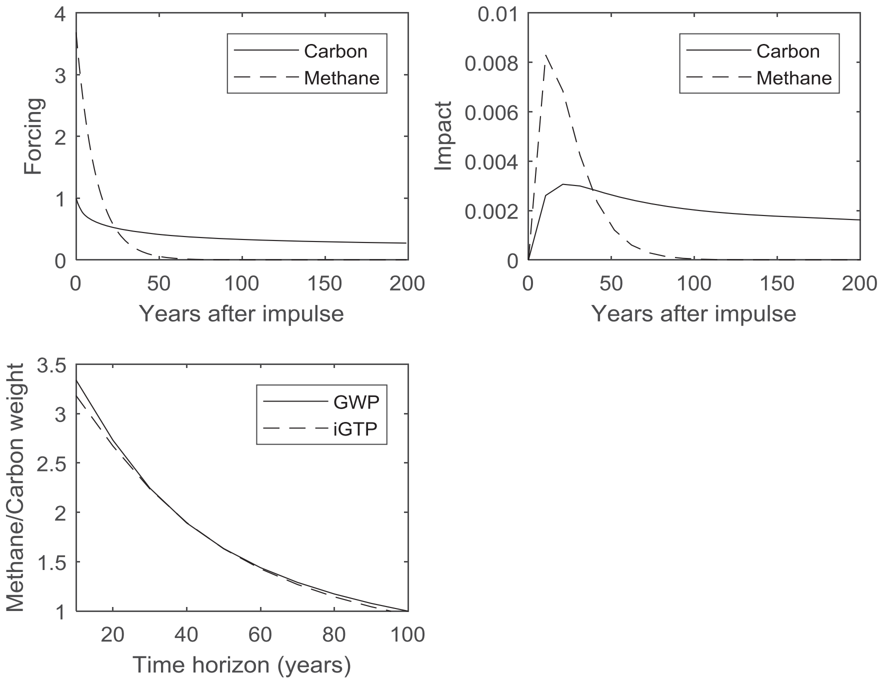

FIGURE 1: FORCING AND TEMPERATURE IMPACT FROM A PULSE EMISSION OF CARBON AND METHANE, AND RELATED WEIGHTS FOR DIFFERENT GWP AND IGTP TIME HORIZONS

Other energy E_t is a composite isoelastic function of coal $E_{2,t}$ and clean $E_{3,t}$,

(9) $$E_t = \left(\kappa E_{2,t}^{\rho} + (1-\kappa) E_{3,t}^{\rho}\right)^{1/\rho}.$$

Dirty (fossil) energy $E_{f,t}$, which contributes to carbon emissions, is the sum of oil and coal energy. As methane is a minor component of total greenhouse gas emissions, it is not explicitly included in the output equation (2), but the optimal tax can be derived using (7) as discussed in the optimal tax section below. Oil can be extracted at zero cost and is in finite supply $X_t \geq 0$,

(10) $$E_{1,t} = X_{t-1} - X_t.$$

Coal and renewable sectors require only labour in production

$$E_{i,t} = A_{i,t} N_{i,t} \text{ for } i = 2{,}3 \text{ where } N_{0,t} + N_{2,t} + N_{3,t} = N. \tag{11}$$

Oil prices follow a hotelling rule corrected for the carbon tax to GDP ratio $\widehat{\Lambda}_t$:

$$\frac{\omega}{E_{1,t}} - \widehat{\Lambda}_t = \beta\, \mathbb{E}_t \left(\frac{\omega}{E_{1,t+1}} - \widehat{\Lambda}_{t+1} \right). \tag{12}$$

Coal and renewable prices are set by wages in sector '0' as follows:

$$A_{2,t} \left(\frac{(\nu - \omega)\kappa}{E_{2,t}^{1-\rho} E_t^{\rho}} - \widehat{\Lambda}_t \right) = \frac{1 - \alpha - \nu}{N_{0,t}} \text{ and } A_{3,t} \frac{(\nu - \omega)(1 - \kappa)}{E_{3,t}^{1-\rho} E_t^{\rho}} = \frac{1 - \alpha - \nu}{N_{0,t}}. \tag{13}$$

Tipping point and damages

To implement the concept of a tipping point, I use a probability distribution for the tipping point threshold's location that is uniform in temperature, as used by Lemoine and Traeger (2014). I assume that the tipping point has not been reached to date and hence lies between the initial temperature T_0 and an upper limit T_{upper}, set at 6^{O}C warming. If tipping is triggered, damages and awareness occur immediately. The damages function γ_t in (2) is the product of a constant parameter γ and the hazard rate p_t which is a function of temperature, $\gamma_t := \gamma p_t$. Due to the assumption of a tipping threshold and irreversibility, the expected hazard rate is a simple function of expected temperature conditional on no tipping. Let $\mathbb{E}_t$ be an expectation operator at time t assuming no tipping prior to t, and let $\widehat{\mathbb{E}}_t(x_{t+j})$ signify the expectation at time t of variable x_{t+j} conditional on no tipping prior to period $t + j$. The probability of tipping is of the following form:

$$(14) \quad \mathbb{E}_t(p_{t+j}) = max\left(min\left(\frac{\widehat{\mathbb{E}}_t(T^H_{t+j}) - T_s}{T_{upper} - T_s}, 1\right), 0\right), \; T^H_{t+j} := \max_{k \le t+j}(T_k).$$

The constant damage parameter γ is calibrated to a 30% loss of GDP from warming of $6^{O}C$. This formulation leads to an expected damage function that increases rapidly as the probability of tipping increases, shown in Figure 2. The damage at high temperatures is much greater than Golosov et al. (2014) but less than damage functions used in other papers.[8]

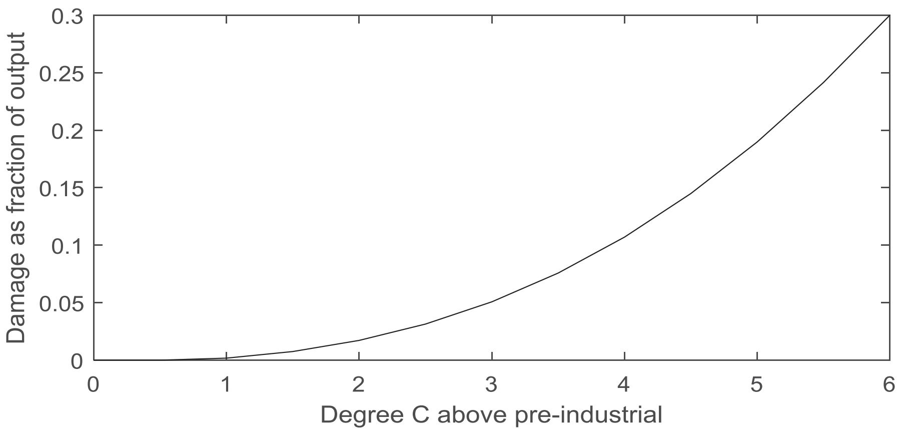

FIGURE 2: EXPECTED DAMAGES

Optimal tax under cost-benefit

The derivation of optimal taxes follows Wiskich (2019) and is discussed in Appendix B. Although methane is not included explicitly in the economic model,

[8] For example, Weitzman (2010), Rezai and Van Der Ploeg (2017) and Acemoglu, Aghion, Bursztyn, and Hemous (2012).

the optimal tax can be derived based on the temperature response function (7). The optimal tax to output ratios for carbon (c) and methane (m) are

(15) (15.1) $$\widehat{\Lambda}_t^g := \frac{\Lambda_t^g}{Y_t} = \sum_{j=0}^{\infty} \beta^j T_j^g \mathbb{E}_t \left(\frac{\partial (\gamma_{t+j} T_{t+j})}{\partial T_{t+j}} \right).$$

(15.2)
$$= \sum_{j=0}^{\infty} \beta^j T_j^g \left(\gamma \mathbb{E}_t (p_{t+j}) + \gamma \mathbb{E}_t \left(T_{t+j} \frac{\partial p_{t+j}}{\partial T_{t+j}} \right) \right) \text{ for } g \in \{c, m\}.$$

The term $\gamma \mathbb{E}_t (p_{t+j})$ corresponds to the exogenous (hazard rate) component of the tax, while the term $\gamma \mathbb{E}_t \left(T_{t+j} \frac{\partial p_{t+j}}{\partial T_{t+j}} \right)$ is due to the endogeneity of the hazard rate. This endogenous term is zero if tipping has occurred, in which case the optimal tax is constant

(16) $$\widehat{\Lambda}_t^{g,tip} = \gamma \mathbb{E}_t \left(\sum_{j=0}^{\infty} \beta^j T_j^g \right).$$

Increasing tax rates as the discount rate decreases follow from (16). When the discount rate is high, the weight of methane to carbon is high due to greater weight on the rapid temperature effect of a methane pulse. As the discount rate decreases, the optimal methane to carbon weight drops. This sensitivity to the discount rate has been discussed previously in the literature.[9]

In the numerical example in the next section, the endogenous term in (15) dominates and the increase in temperature is limited. Such results can be understood

[9] For example, Tol (1999).

by considering damage from tipping which is a constant fraction of output and a constant marginal hazard rate $\frac{\partial p_t}{\partial T_t} = p'$, so that for the onset of peak temperature in period τ, using (15) the tax ratio becomes

$$\widehat{\Lambda}_t^g(17) \qquad \widehat{\Lambda}_t^g = \gamma p' \sum_{j=0}^{\tau-t} \beta^j \mathrm{T}_j^g. \tag{17}$$

REMARK 1: For a distant peak temperature and parameters set such that the primary role of the tax is to reduce the risk of a tipping event, the tax ratio of methane will be roughly flat initially.

This remark follows from (17) and the short-term temperature response for methane. Remark 1 is symmetrical to the key result of a constant tax ratio if the probability of severe climate sensitivity is constant, as described in GHKT. In both cases the exponent of the damage multiplier is linear in temperature: for GHKT the damage is linear in temperature and the hazard rate is constant, while for (17) the hazard rate rises linearly with temperature and the damage ratio is constant.

Optimal tax under cost-minimisation

As an alternative approach to CB, CM sets a maximum temperature increase exogenously and the tax is optimised to minimise costs. While some papers set a maximum temperature at a point in time, this paper allows the model to endogenously determine the onset and end of peak temperature. The damage function is excluded and the optimisation problem is then

(18) $$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C_t) \text{ such that } T_t \leq T_{max} \text{ where}$$

$$Y_t = A_{0,t} {K_t}^{\alpha} {N_{0,t}}^{1-\alpha-\nu} {E_{1,t}}^{\omega} {E_t}^{\nu-\omega}.$$

Given the onset of peak temperature at period τ, the optimal tax is simply

(19) $$\widehat{\Lambda}_t^{CM,g} = \sum_{j=\tau-t+1}^{\infty} \beta^j \tilde{\lambda}_{t+j}^{CM} T_j^g.$$

where $\tilde{\lambda}^{CM}$ are the Lagrange multipliers for cost-minimisation, similar to the multipliers used in cost-benefit (see Appendix B for details). Computation is comparable to the CB method, with peak onset manually iterated to ensure optimisation, which is simple to verify as the CM approach is deterministic. While the benefits of abatement are discounted in the CB approach due to the damages function, they are not in the CM framework and the optimal tax therefore increases much faster. If the temperature effect was flat (or if the constraint was cumulative emissions rather than a temperature limit), the price would follow a Hotelling rule. Considering the form of the temperature pulse (5), the following proposition applies to the initial growth in the tax.

PROPOSITION 1: The optimal tax ratio for cost-minimisation initially grows faster than the discount rate if peak temperature is far enough in the future so that the slope of the temperature pulse is negative at peak temperature.

Proof: A negative slope of temperature pulse simply means that peak temperature is 30 or more years in the future for the climate dynamics used in this paper, and indicates $T_j^g < T_{j-1}^g$. The tax in period 1 is as follows:

$$(20) \qquad \widehat{\Lambda}_1^{CM,g} = \sum_{j=\tau}^{\infty} \beta^j \tilde{\lambda}_{j+1}^{CM} T_j^g > \sum_{j=\tau}^{\infty} \beta^j \tilde{\lambda}_{j+1}^{CM} T_{j+1}^g = \frac{1}{\beta} \sum_{j=\tau+1}^{\infty} \beta^j \tilde{\lambda}_j^{CM} T_j^g = \frac{\widehat{\Lambda}_0^{CM,g}}{\beta}.$$

Once peak temperature has been reached, taxes in both CB and CM approaches are functions of the Lagrange multipliers as per (B.6) and (19).

PROPOSITION 2: For both cost-benefit and cost-minimisation approaches, the optimal methane to carbon weight in the period prior to the end of peak temperature is equal to the GTP with a one-period time horizon, GTP(1) or equivalently iGTP(1).

This result follows easily from (19), as $\widehat{\Lambda}_{n-1}^g = \frac{\tilde{\lambda}_n T_1^g}{\beta^{n-1}}$ the ratio $\frac{\widehat{\Lambda}_{n-1}^m}{\widehat{\Lambda}_{n-1}^c} = \frac{T_1^m}{T_1^c}$ which is GTP(1) by definition. Conceptually, the sole objective of the tax in both CB and CM approaches in the period prior to the end of peak temperature is to keep the temperature constant in the next period, and the relative effectiveness of abatement of carbon and methane is given by their temperature response ratio one period ahead. Previous papers have highlighted the CM nature of the GTP metric for a point-in-time temperature limit. For a cost-minimisation approach which sets a temperature limit for all time and in the presence of a prolonged period of temperature stabilisation, as in this paper, the condition only holds at the end of peak temperature. Prior to this point, all future periods where the temperature is maintained at peak and the stabilisation constraint binds are included in (19). As the ratio of the methane to carbon temperature effect is greatest one period in advance in the climate model used in this paper, the optimal weight is lower than GTP(1) prior to this point.

II. Numerical examples

Parameters are shown in Table 1 which are largely taken from Golosov et al. (2014) and Shine et al. (2005). Historical emissions go back a century and induce an initial warming at 2020 of 1.11^{0}C, aligning with the centre of the range of IPCC estimates (IPCC, 2014). Initial decadal global GDP is set to \$800 trillion. Projections show a future path where no tipping occurs, but of course the optimal tax considers uncertainty about the future.

TABLE 1: CALIBRATION PARAMETERS

gA_0 (%/year)	gA_2 (%/year)	gA_3 (%/year)	$A_{2,0}$	$A_{3,0}$	ρ	ν	ω	κ
1.3	0	2	8792	1498	0.5	0.04	0.0215	0.1786
β (annual)	δ (%/decade)	$gTFP$ (%/year)	N	a_0	a_1	a_2	a_3	a_4
0.985	100	1.3	1	0.1756	0.1375	0.1858	0.2423	0.2589
T_0 (^{0}C)	X_0 (GtC)	γ	$10^{14}Y_0$ (\$)	α	α_1	α_2	α_3	α_4
1.11	253.8	0.05945	8	0.3	421.093	70.5965	21.4216	3.4154
E_f(-10:-1)			Period 0	A_c	H	A_m/A_c	α_m	
[10,10,10,20,30,40,50,60,80,100]			2020	1.98	4.2	3.687	12	

As shown in Panels A and B of Figure 3, carbon and methane taxes have a similar profile but the tax for methane climbs faster prior to peak temperature. Panels C and D demonstrate that the bulk of the tax is due to an increased probability of tipping rather than increase in damages if tipping was to occur. The relative increase in methane tax near peak temperature is due to this endogenous component, and the tax ratio of methane is flat initially as described in remark 1. The profile of the carbon tax leads to the hump shape in coal emissions, while oil emissions are driven by the hotelling rule and fall throughout the period. Peak temperature occurs for almost a century as a result.

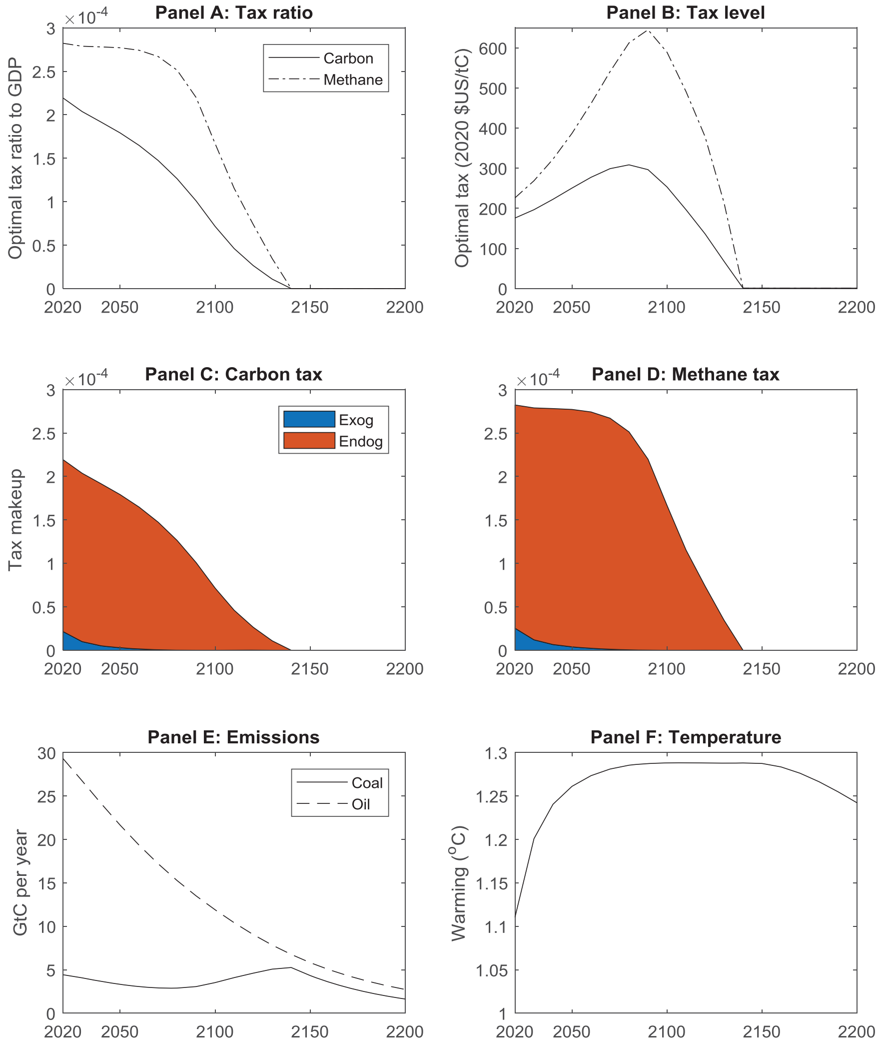

FIGURE 3: OPTIMAL TAX AND ENVIRONMENTAL RESULTS

Figure 4 compares the optimal tax under a CM framework, assuming the same target peak temperature which results endogenously in the CB framework. As expected intuitively and consistent with proposition 1, the optimal tax for carbon starts lower than the tax under CB and, as peak temperatures are identical by construction, surpass the CB price prior to peak temperature. For methane, the tax under CB is comparable initially with the carbon tax, while under CM it starts much lower and increases as peak temperature approaches, consistent with profiles found in the CM literature such as Manne and Richels (2001). The optimal tax for methane under CB is flat initially, consistent with remark 1.

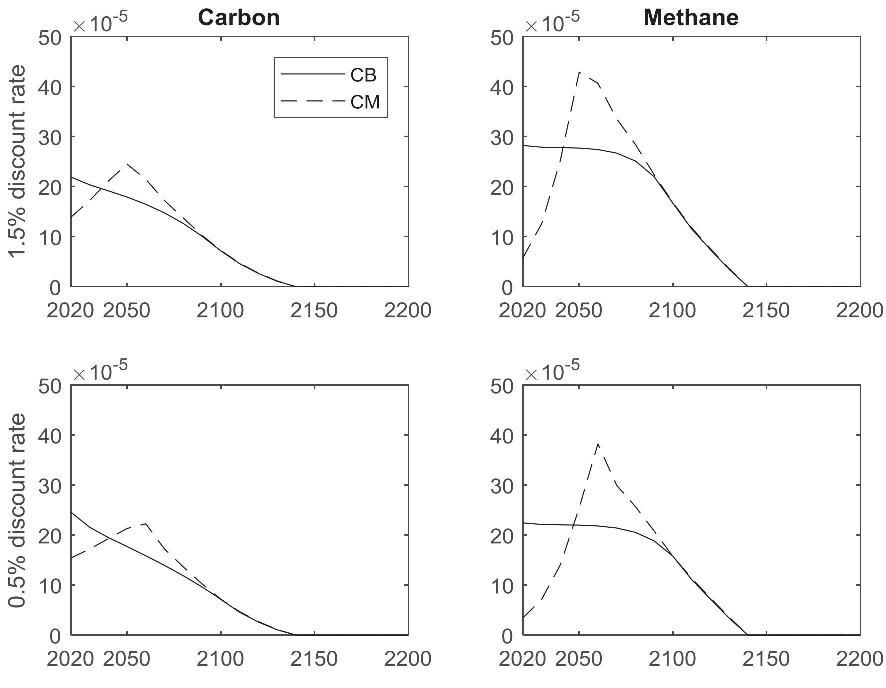

FIGURE 4: OPTIMAL TAX TO GDP RATIOS FOR CARBON AND METHANE UNDER COST-BENEFIT AND COST-MINIMISATION

Results using a lower discount rate of 0.5% are also shown in Figure 4. Again, the same peak temperature is imposed for comparison, by reducing the damage parameter γ by around a third. To identify the impact of the discount rate on relative taxes of methane and carbon, the use of oil is exogenously fixed to be the same as the 1.5% discount rate case. For completeness, results which allow oil use to change with the discount rate are included in Appendix A.

Although the profiles of optimal taxes shown in Figure 4 are interesting, the focus of this paper is the optimal weight of methane to carbon. Figure 5 shows increasing weights of methane to carbon as the end of peak temperature approaches. Also shown are adjusted iGTP metrics where the time horizon decreases from 100 to correspond to the time until both the onset and end of peak temperature.[10] The graphs show that a starting ratio of 1 implied by the current 100 year GWP time horizon is close to the optimal starting weight under CB. Given a range of estimates for the end of peak temperature stabilisation, adjusting the GWP to reflect an early estimate would do a reasonable job of replicating the optimal weight profile. In contrast, the CM weight starts much lower as others have found, but then reaches the same point just prior to the end of the stabilisation period as proposition 2 describes.

[10] As the onset and end of peak temperature are endogenously determined, these periods differ in each simulation.

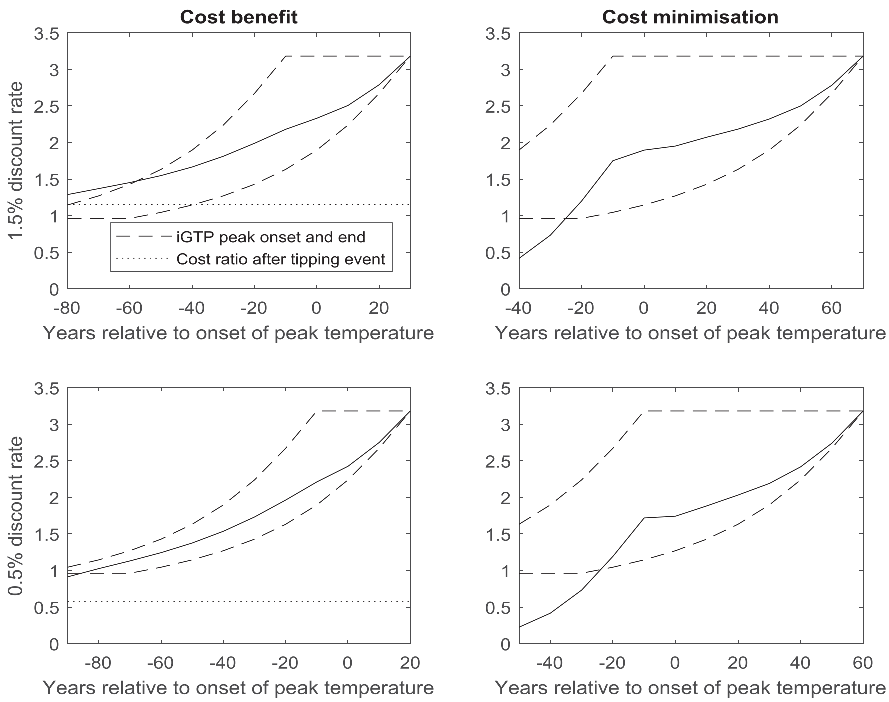

FIGURE 5: OPTIMAL WEIGHTS OF METHANE TO CARBON AGAINST ADJUSTED IGTP METRICS

When the discount rate is low, the weight under CM is not changed significantly relative to the onset of peak temperature. In contrast, the weight under CB in the event of tipping is roughly halved under a low discount rate. The effect on the weight under CB if tipping has not occurred lies in between, reflecting shared characteristics with the CM approach.

III. Conclusion and Discussion

This paper examines optimal policy under in the presence of a tipping point with an endogenous hazard rate. The paths of the optimal tax to output ratios for carbon and methane differ due to different temperature responses, as methane decays much faster than carbon. For a given peak temperature, the cost-minimisation tax starts below the cost-benefit tax and rises faster than the discount rate. In a decadal discrete-time framework, ratios in both approaches converge to the same value just prior to the end of the stabilisation period for peak temperature: approximately equal to the Global Warming Potential (GWP) with a 10-year time horizon.

The clearest recommendation for a 100-year GWP period adopted by the IPCC is that this time period broadly represents the time scale over which a significant fraction of CO_2 is removed from the atmosphere (Fuglestvedt et al., 2003). As noted by WMO (1992), this period also roughly corresponds to the anticipated maximum change in temperature. The limited results in this paper suggest a 100-year GWP does well in generating the optimal weight of methane to carbon today in a cost-benefit framework. A key finding is that adjusting the time horizon so that it corresponds to an early estimate of the anticipated end of peak temperature goes most of the way to generating the optimal weight profile. This approach is also quite simple.

Some have argued that a cost-effective metric such as the Global Temperature Pulse (GTP) is most appropriate for a temperature target.[11] As demonstrated in this paper, such an approach would imply very low taxes on methane today, rising as peak temperature approaches. However, there are several reasons to impose a higher weight on methane in the short term. Uncertainty in climate impacts is an obvious one, including risks associated with the rate of increase in temperature. This paper highlights that if the probability of tipping rises with temperature, and if tipping is irreversible, then this constitutes another reason to have a higher initial weight for methane. The optimal weight in this tipping approach shares a common destination and general profile with the cost-minimisation path. However, due to the higher initial tax for methane in the CB approach, the numerical results

[11] For example, Shine, Berntsen, Fuglestvedt, Skeie, and Stuber (2007) and Tol et al. (2012).

considered in this paper, although limited in scope, support an integrated approach (GWP or iGTP) rather than point in time (GTP).

The tipping framework described provides a novel way of determining the suitability of the use of GWP in quantifying weights between gases. The framework leads to greater comparability between results under cost-benefit and cost-minimisation, which will hopefully help policy debate. A natural question is the welfare implications of keeping GWP fixed according to current policy. Many have found that, although the GWP metric is suboptimal, the additional costs are minor compared with other suboptimal approaches (IPCC, 2014). This paper does not consider welfare effects which I leave as a potential future exercise.

Appendix A – Results for a low discount rate with endogenous oil extraction

This section provides results for the low discount rate case allowing the extraction of oil to be determined endogenously according to (12). The effect of oil use leads to quite different tax profiles. For a given peak temperature, one would expect higher initial taxes with a low discount rate for carbon, as shown in Figure 4 in the main paper. However, oil extraction is much flatter under a low discount rate regime, which means lower taxes are needed to stay below the temperature target. While the difference in the oil extraction profile changes the level of optimal taxes, the weights of methane to carbon are similar to those shown in Figure 5.

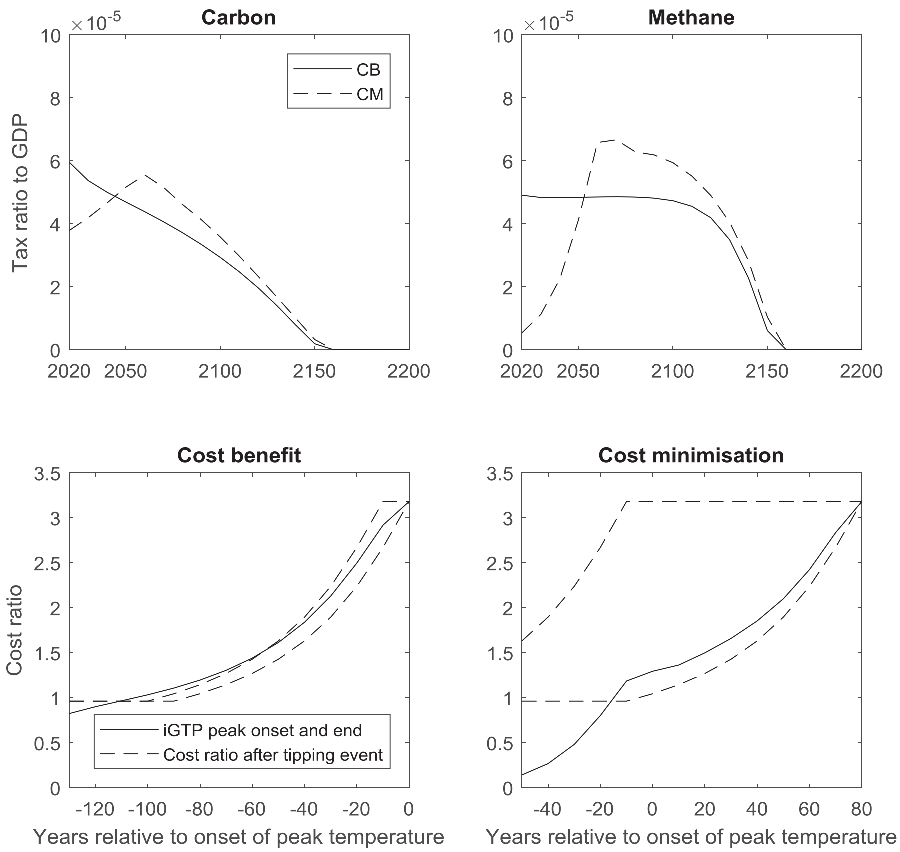

FIGURE 6: OPTIMAL CARBON TAX TO GDP RATIO AND METHANE TO CARBON WEIGHTS UNDER COST-BENEFIT AND COST-MINIMISATION FOR 0.5% DISCOUNT RATE AND ENDOGENOUS OIL EXTRACTION

APPENDIX B – DERIVING THE OPTIMAL TAX UNDER COST-BENEFIT

The derivation closely follows Wiskich (2019). Consider an (infinitesimal) additional energy component E_4 associated with methane emissions. The Lagrangian maximizes (1) subject to production and temperature constraints as follows:

$$(\mathrm{B.1})\quad \mathcal{L}(C_t, \boldsymbol{N}_t, K_{t+1}, K_{t+1}, X_t, \boldsymbol{E}_t, T_t) = \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

$$+\sum_{t=0}^{\infty}\lambda_{0,t}\left(\mathrm{F}_0\left(N_{0,t},K_t,\boldsymbol{E}_t,T_t\right)-C_t-K_{t+1}\right)+\sum_{t=0}^{\infty}\sum_{i=1}^{3}\lambda_{i,t}\left(\mathrm{F}_i\left(N_{i,t}\right)-E_{i,t}\right)$$

$$+\sum_{t=0}^{\infty}\lambda_{N,t}\left(N_t-\sum_{i=1}^{3}N_{i,t}\right)+\sum_{t=0}^{\infty}\lambda_{T,t}\left(T_t-\sum_{i=-\infty}^{\mathrm{t}-1}\mathrm{T}_{t-i}^{c}E_{f,i}-\sum_{i=-\infty}^{\mathrm{t}-1}\mathrm{T}_{t-i}^{m}E_{4,i}\right)$$

$$+\mu_1\left(X_0-\sum_{t=1}^{\infty}E_{1,t}\right).$$

The optimal condition describing the marginal costs and benefits of producing a unit of energy of type *i*, in terms of final consumption good at time *t*, is

$$(\mathrm{B.2.1})\qquad \frac{\lambda_{i,t}}{\lambda_{0,t}}+\frac{\mu_{i,t}}{\lambda_{0,t}}+\frac{1}{\lambda_{0,t}}\sum_{j=0}^{\infty}\lambda_{T,t+j}\mathrm{T}_j^{c}=\frac{\partial F_{0,\mathrm{t}}}{\partial E_{\mathrm{i,t}}}\ \text{where } g=\begin{cases}c\ for\ i=1,2\\ m\ for\ i=4\end{cases}\ \text{and}$$

$$(\mathrm{B.2.2})\qquad \lambda_{T,t}(\mathrm{B.2.2})\qquad \lambda_{T,t}=-\lambda_{0,t}\frac{\partial F_{0,\mathrm{t}}}{\partial T_{\mathrm{t}}}\ \text{and}\ \lambda_{0,t}\mathrm{C}_t=\beta^t.$$

The costs in (B.2.1) include the cost of input use $\frac{\lambda_{i,t}}{\lambda_{0,t}}$, the scarcity cost $\frac{\mu_{i,t}}{\lambda_{0,t}}$, and the marginal externality damage. This last cost is the optimal Pigouvian tax and from (B.2.2) and (2), the optimal carbon tax to output ratio is as follows:

$$(\mathrm{B.3})\qquad \widehat{\Lambda}_t=\sum_{j=0}^{\infty}\beta^j\mathrm{T}_j^{g}\,\mathbb{E}_t\left(\frac{\partial(\gamma_{t+j}T_{t+j})}{\partial T_{t+j}}\right).$$

Expectations of the derivative of future probability and tipping enter the tax to output ratio. The expected derivative is

$$(\mathrm{B.4})\qquad \mathbb{E}_t\left(\frac{\partial(\gamma_{t+j}T_{t+j})}{\partial T_{t+j}}\right)=\gamma\mathbb{E}_t(p_{t+j})+\gamma\mathbb{E}_t\left(T_{t+j}\frac{\partial p_{t+j}}{\partial T_{t+j}}\right).$$

Equation (B.4) can be calculated from expectations of temperature conditional on no tipping, as can oil extraction. Thus, although tipping can happen at any period prior to peak temperature, the optimal tax can be determined by temperature outcomes conditional on tipping not occurring. This result follows from the form of the damages function coupled with key assumptions made in the GHKT model that lead to a constant tax ratio: logarithmic utility, a multiplicative exponential damage function of temperature which is a linear function of energy use, and constant savings rates. Computation is therefore simpler, as the tax profile can be determined by one future outcome rather than having to handle all possible outcomes. The first component of (B.4) is the tax that would apply given an exogenous probability profile corresponding to the expected emissions for the scenario.[12] The second component is due to the endogeneity of the hazard rate.

To handle the discontinuity in the derivative of the probability of tipping, a time of onset of peak temperature (τ) is imposed with constraints ensuring temperature after period τ does not exceed T_τ: $\lambda_{M,t}(T_\tau - T_t) \geq 0$ for $t > \tau$. Equation (B.2.1) becomes $\lambda_{T,t} = -\lambda_{0,t}\frac{\partial F_{0,t}}{\partial T_t} + \lambda_{M,t}$ and the tax becomes

$$(\mathrm{B.5}) \qquad \hat{\Lambda}_t^g = \mathbb{E}_t\left(\sum_{j=0}^{\varsigma}\beta^j\frac{\partial(\gamma_{t+j}T_{t+j})}{\partial T_{t+j}}T_j^g + \sum_{j=\varsigma+1}^{\infty}\left(\beta^j\gamma_\tau + \frac{\tilde{\lambda}_{t+j}}{\beta^t}\right)T_j^g\right)$$

$$\text{where } \varsigma = \max(\tau - t, -1) \text{ and } \tilde{\lambda}_{t+j} := \lambda_{M,t+j}\frac{C_t}{Y_t}.$$

[12] This sum is not the optimal tax which would apply in a separate scenario with the same time-dependent hazard rates, as the lower tax would lead to higher temperature outcomes.

The multiplier $\tilde{\lambda}_{t+j}$ is $\lambda_{M,t+j}$ adjusted by the constant consumption rate. Taxes following peak temperature are set so that temperature does not exceed T_τ and these peak temperature taxes are equal to:

$$(\text{B.}6) \qquad \widehat{\Lambda}_t^g = \sum_{j=0}^{\infty} \frac{\tilde{\lambda}_{t+j}}{\beta^t} T_j^g .$$

The multipliers are derived from the taxes needed to stabilize temperature: the carbon tax in the period prior to the end of peak temp is given by $\widehat{\Lambda}_{n-1}^c = \gamma_L \sum_{j=0}^{\infty} \beta^j T_j^c + \frac{\tilde{\lambda}_n T_1^c}{\beta^{n-1}}$ which gives $\tilde{\lambda}_n$, the tax in the prior period is $\widehat{\Lambda}_{n-2}^c = \gamma_L \sum_{j=0}^{\infty} \beta^j T_j^c + \frac{\tilde{\lambda}_{n-1} T_1^c}{\beta^{n-2}} + \frac{\tilde{\lambda}_n T_2^c}{\beta^{n-2}}$ which gives $\tilde{\lambda}_{n-1}$ and so on. The choice of τ is determined through manual iteration: for a high τ, peak temperature occurs prior to this value, and thus the value of τ is reduced until it corresponds with peak temperature.

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