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Participation Constraints of Matching Mechanisms

CAMA Working Paper 63/2014
October 2014
Version updated on 4 May 2018

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Abstract

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Keywords

Public goods, Matching mechanisms, Participation constraints

JEL Classification

C78, H41

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Participation Constraints of Matching Mechanisms

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Abstract. Matching mechanisms have been proposed to improve public good provision in voluntary contributions. However, such decentralized subsidizing mechanisms may not be Pareto-improving and may suffer from incomplete information and incredible commitment. This paper examines participation constraints of matching mechanisms, and investigates the existence condition of Pareto-improving outcomes of small matching schemes, and characterizes the condition of desirable matching schemes. Income distributions across players play an important role. If the income distribution ensures an interior equilibrium, there always exist small Pareto-improving matching schemes regardless of preferences. This universal existence is useful for cooperation among heterogeneous players in the context without global information of preferences or at the international level without central governments. However, if the income inequality induces a corner equilibrium, matching schemes work in different ways and have different welfare effects in certain cases, and the existence of Pareto-improving matching schemes is not universal but is possible under a certain condition. In particular, if the corner player unilaterally matches the interior player, both players can be better off, indicating that unilateral action through matching can possibly generate Pareto-improving outcomes.

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1 Introduction

It is well known that public goods are often underprovided in voluntary contributions. Since Pigou (1920), centralized approaches such as taxes and subsidies through government intervention have been widely used to improve public good provision (see, e.g., Roberts 1987, 1992; Boadway, Pestieau and Wildasin 1989; Andreoni and Bergstrom 1996; more comprehensively, see Cornes and Sandler 1996). This subsidizing idea has also been developed in decentralized approaches. Instead of government intervention, agents subsidize each other to reduce public good prices and thus increase public good provision. One example of such decentralized approaches is matching mechanisms (Guttman 1978, 1987). The mechanism works as a two-stage game. At the first stage, each agent announces a matching rate indicating by how much the agent would subsidize public good contributions of all other agents. At the second stage, all agents decide independently how much of the public good they would provide. For example, should one announce a matching rate of 0.1 at the first stage, the agent would provide 0.1 units of the public good as a matching contribution if another agent provides one unit of the public good at the second stage. The idea is still to stick to the non-cooperative mode of public good provision but meanwhile to subsidize individual public good contributions and hence to lower the effective price of the public good. Guttman (1978) shows that the sub-game perfect equilibrium in such a two-stage game of two identical players with quasi-linear preferences is fully efficient. Danziger and Schnytzer (1991) generalize this result to multiple agents with more general preferences. This approach has been refined and applied in various ways.[1]

However, there are both theoretical and practical issues with implementing this mechanism, which have not been much observed or considered until recently. Theoretically, Buchholz et. al (2015) reveals a paradoxical effect of matching schemes that the recipient of matching contributions is worse off than without matching schemes. The current paper also shows that at the sub-game perfect equilibrium of the two-stage matching game, players may be worse off compared to the initial equilibrium although it can achieve the optimal public good provision. Cornes and Sandler (2000) argue that policies that can increase public good supply and improve everyone's well-being are more desirable than policies that simply augment public good provision. In a decentralized context without a central planner, voluntary participation in this mechanism requires that each agent should not be worse off with matching schemes.

Besides, Buchholz, Cornes and Rubbelke (2011) show that a matching scheme does not necessarily entail the desired interior matching equilibrium in which all agents make positive direct contributions to public goods. A matching scheme works as desired only if each agent chooses a public good contribution that equates the marginal rate of substitution between the public and the private good with the price ratio of the two goods modified by matching. This requires interiority of matching equilibria but interiority only emerges for specific income distributions. However, as this paper will show, at a corner equilibrium where one player initially does not provide public good contributions, a matching scheme works in different ways from at an interior equilibrium and have different welfare effects in certain cases.

[1]See Althammer and Buchholz (1993), Varian (1994a, 1994b), Falkinger (1996), Kirchsteiger and Puppe (1997), Rubbelke (2006), Boadway, Song and Tremblay (2007, 2011), Fujita (2013), Buchholz, Cornes and Rubbelke (2011, 2012, 2014, 2015), Buchholz and Liu (2018), Liu (2018), etc.

Practically, it is difficult to obtain complete information which is required for attaining the optimal matching equilibrium. In the presence of public goods, the initial Nash equilibrium is inefficient and is often far away from the optimal matching equilibrium. Agents generally have less information about invisible further-off positions and hence are more uncertain about larger changes from the observable current position. From the perspective of informational requirement, it is much easier to implement small Pareto-improving policy reforms than to attain overall optimal policy because the former requires only information of the current position while the latter requires information of a large range of positions (see, e.g., Deaton 1987; Myles 1995). In practice, the information required for small Pareto-improving policy reforms is often directly observable or can be inferred from the current position, but the information of entire preferences, demands and production possibilities required for optimal policy is generally not obtainable. This information problem becomes even worse in the presence of global public goods because it is much more difficult to obtain aggregate information for countries with heterogeneous agents.

This matching game and, more broadly, such two-stage games are far from implementation not only because of their information requirement but also because of their sophisticated structures. The sub-game perfect equilibria of such multi-stage games can generally implement efficient allocations (see, e.g., Crawford 1979; Moulin 1979, 1981; Moore and Repullo 1988). However, as Moore and Repullo (1988) point out, "...the mechanisms we construct to deal with a general environment are far from simple: agents move simultaneously at each stage and their strategy sets are unconvincingly rich. We present such mechanisms to demonstrate what is possible, not what is realistic." In the matching mechanism, players announce their matching rates simultaneously at the first stage, and decide their public good contributions simultaneously at the second stage and also provide matching contributions at the same time. This sophisticated game structure is difficult to be satisfied in practice.

Moreover, an important but implicit assumption of matching mechanisms is the credibility of commitment that players would provide matching contributions to other players at the second stage. The commitment should not be taken for granted in the absence of a central government (Boadway, Song and Tremblay, 2007). Although direct and matching contributions are both provided at the second stage, they both take time in practice and cannot happen simultaneously. If there is a time gap, the credibility of commitment becomes problematic. The fact that no supranational authority can force agents to implement their commitment makes ambitious matching schemes less applicable in a decentralized context.

To overcome or mitigate all these issues, this paper investigates participation constraints of matching mechanisms and focuses on small matching schemes. A small matching scheme implies a small change from the current position and thus requires less information, and also implies less ambitious commitment and thus make it more credible. The paper distinguishes interior and corner equilibria and shows that matching schemes work in different ways and have different welfare implications in the two cases. The paper does not address how the matching scheme is determined at an equilibrium in a certain context but assumes the matching scheme is exogenously given. It can be determined in different ways,[2] and players may end up with various matching

[2]For example, it can be the outcome of the equilibrium of the two-stage matching game with complete information and credible commitment, or can be the outcome of the equilibrium of the

schemes. Whatever matching schemes, from a normative perspective, participation constraints must be satisfied, or the matching equilibrium must be Pareto-improving. Therefore, the paper aims to show under what conditions which matching schemes are possible from a normative perspective and to provide directions for implementing matching schemes at both interior and corner equilibria.

Income distributions across players play an important role for achieving Pareto-improving outcomes of matching schemes. At an interior equilibrium, i.e., if the income inequality is small so that both players provide positive public good contributions, a matching scheme works as desired where the marginal rate of substitution between the two goods is equal to their marginal rate of transformation modified by matching. In such a case, there always exist small Pareto-improving matching schemes regardless of preferences. This indicates that players at an interior equilibrium can always implement small matching schemes to generate Pareto-improving outcomes. Since implementing small matching schemes only requires local information and limited commitment and does not require side payments, the universal existence of Pareto-improving matching schemes is useful for cooperation among heterogeneous players in the context without global information of preferences or at the international level without central governments.

However, it is more common to encounter corner equilibria than interior equilibria in a pure public good economy. Moreover, corner equilibria are theoretically important in some situations,[3] and are also practically relevant due to large income inequality in the current world. Therefore, we examine corner equilibria separately in great detail as well. At a corner equilibrium where one player initially does not provide public good contributions, a matching scheme does not work as desired. On the one hand, if an interior player announces a matching rate to induce the corner player to provide positive public good contributions, the matching rate must be sufficiently large. Until the matching rate reaches a critical level, the marginal rate of substitution between the two goods does not equal their relative price for the corner player, so the corner player still does not provide public good contributions and the matching rate is not effective. On the other hand, if the corner player provides matching contributions to the interior player, the interior player behaves in the same way as in an interior equilibrium, but the corner player behaves differently because she cannot free ride on her initial (zero) public good contributions, in contrast with the interior case where an interior player reduces her initial contributions when providing matching contributions. In such a corner case, the existence of Pareto-improving matching schemes is not universal but is possible under a certain condition.

The remainder of this paper is organized as follows. Section 2 sets up the model, introduces the aggregative game approach to characterize equilibria, and provides a motivating example. Section 3 explains how matching mechanisms work and shows

matching game with incomplete information or limited commitment (Boadway, Song and Tremblay 2007), or can even be the outcome of an evolutionary process of conditional cooperation (Guttman 2013), or can also be imposed by a central agency or suggested by a third party.

[3]For example, Bergstrom, Blume and Varian (1986) show that large income redistributions may change the contributor set of public goods leaving some players at the corner and thereby break down Warr neutrality (Warr 1982, 1983). Itaya, de Meza and Myles (1997) prove that social welfare can be increased by creating sufficient income inequality where only the rich provide public goods. Cornes and Sandler (2000) argue that it is possible to achieve Pareto-improving redistribution from non-contributors to contributors of public goods.

the universal existence of Pareto-improving matching schemes at interior equilibria, and further characterizes such matching schemes, and provides a graphic illustration, followed by an extension from the two-player model to a multiple-player case. Section 4 examines Pareto-improving matching schemes at corner equilibria, and also extends the two-player corner model to a multiple-player case. Section 5 concludes.

2 The framework

2.1 The model

Consider a pure public good economy with one private good, one pure public good and two players. The utility function of player $i(i = 1, 2)$, represented by $u_i(x_i, G)$ where x_i is the private good consumption and G is the total public good provision, is continuous and differentiable, strictly increasing in both variables and strictly quasi-concave. Both goods are strictly normal, and indifference curves asymptote to the two axes. We consider small matching schemes and assume that players have complete information and credible commitment in the neighbourhood of the initial equilibrium.

Player i has an initial income of w_i units of the private good. The total income is $W = w_1 + w_2$, and their income ratio is $k = w_1/w_2$. The two players have the same linear production function for the public good, and the relative price of the public good in terms of the private good is normalized to one. Player i offers a matching rate $\mu_i(\mu_i \geq 0)$ by which to match the other player's direct public good contribution, and the two matching rates compose a matching scheme (μ_1, μ_2). Player i's contribution to the public good consists of a direct flat contribution y_i, and of an indirect matching contribution that player i makes by matching the other player's flat contribution. The flat contribution is non-negative, i.e., $y_i \geq 0$. Therefore, the budget constraints of the two players are respectively

$$x_1 + y_1 + \mu_1 y_2 = w_1, \quad x_2 + y_2 + \mu_2 y_1 = w_2 \tag{1}$$

The private marginal rate of transformation between the private and the public good is $\pi_i = 1 + \mu_j (j = 1, 2, j \neq i)$. The effective public good price that player i pays for an additional unit of the public good is $p_i = 1/\pi_i$. The public good is aggregated by a summation technology across players, so the total public good provision is

$$G = (1 + \mu_2)y_1 + (1 + \mu_1)y_2 \tag{2}$$

Given preferences, incomes and a matching scheme, a matching equilibrium is defined as follows.

Definition 1. Given a matching scheme (μ_1, μ_2), a tuple $(x_1^*, x_2^*, y_1^*, y_2^*, G^*)$ is a matching equilibrium if each player maximizes her own utility $u_i(x_i, G)$ subject to the individual budget constraint (1) and the public good aggregation (2).

The initial equilibrium without matching is a special case of the matching equilibrium when $\mu_1 = \mu_2 = 0$. Denote the initial equilibrium by $(\bar{x}_1, \bar{x}_2, \bar{y}_1, \bar{y}_2, \bar{G})$ and the matching equilibrium by (x_1, x_2, y_1, y_2, G), and the utility levels of player i at the initial equilibrium and the matching equilibrium by $u_i(\bar{x}_i, \bar{G})$ and $u_i(x_i, G)$ respectively. To examine participation constraints, we define Pareto improvements under matching schemes as below.

Definition 2. A matching scheme (μ_1, μ_2) is Pareto-improving if each player has a higher utility level under the matching scheme than at the initial equilibrium, i.e., $u_i(x_i, G) > u_i(\bar{x}_i, \bar{G})$ for $i = 1, 2$.

One more definition distinguishes interior and corner equilibria.

Definition 3. (i) An interior equilibrium is an equilibrium where each player chooses a positive flat contribution, i.e., $y_i > 0, i = 1, 2$; (ii) A corner equilibrium is an equilibrium where one player chooses zero flat contribution, i.e., $y_i = 0, i = 1$ or 2.

2.2 Aggregative game approach

This paper applies the aggregative game approach (Cornes and Hartley 2003, 2007) to characterize equilibria. While the conventional reaction function method struggles to deal with public good models which involve more than two players, this approach avoids high-dimensional problems when the number of players increases, so it provides great convenience to characterize equilibria particularly in multiple-player cases.

Suppose there are $n(n \geq 2)$ players in the economy. Let $e_i(G, \pi_i)$ denote player i's income expansion path which is a function of the total public good provision G, on which player i's marginal rate of transformation is π_i. At an interior matching equilibrium, the following two conditions must be satisfied

$$x_i = e_i(G, \pi_i)$$

$$G + \sum_{i=1}^{n} e_i(G, \pi_i) = W$$

The first condition holds because, when any player chooses a positive flat contribution, the marginal rate of substitution between the private and the public good must equal the private marginal rate of transformation between the two goods, so that the choice is on the income expansion path. The second condition is the aggregate budget constraint.

Given the assumptions on the utility functions, $e_i(G, \pi_i)$ is increasing in G and decreasing in π_i. We will often use its two derivatives $\partial e_i/\partial G$ and $\partial e_i/\partial \pi_i$. In terms of interpretation, $\partial e_i/\partial G$ captures the preference intensity over the public good relative to the private good, and $\partial e_i/\partial \pi_i$ captures the elasticity of substitution between the two goods. Take a constant elasticity of substitution (CES) utility function $u_i(x_i, G) = (a_i x_i^{\rho_i} + (1 - a_i)G^{\rho_i})^{1/\rho_i}$ as an example where $0 < a_i < 1$ and $\rho_i \leq 1$. The income expansion path is derived as

$$x_i = e_i(G, \pi_i) = \left(\frac{a_i}{1 - a_i} \frac{1}{\pi_i} \right)^{\frac{1}{1-\rho_i}} G$$

The two derivatives at $\pi_i = 1$ are given as

$$\frac{\partial e_i}{\partial G} = \left(\frac{a_i}{1 - a_i} \right)^{\frac{1}{1-\rho_i}}, \quad \frac{\partial e_i}{\partial \pi_i} = -\frac{1}{1 - \rho_i} \left(\frac{a_i}{1 - a_i} \right)^{\frac{1}{1-\rho_i}} G$$

The interpretations are more evident in this example. $\partial e_i/\partial G$ captures the preference intensity between the two goods because, if players have stronger preferences over the public good than over the private good, i.e., $a_i < 0.5$, then $\partial e_i/\partial G < 1$ regardless of

ρ_i, and if $a_i > 0.5$, then $\partial e_i/\partial G > 1$ regardless of ρ_i. $\partial e_i/\partial \pi_i$ captures the elasticity of substitution between the two goods because the term $1/(1-\rho_i)$ is the elasticity of substitution.

2.3 A motivating example

Boadway, Song and Tremblay (2007) show that the optimal matching scheme at the sub-game perfect equilibrium of the two-stage matching game satisfies $\mu_1\mu_2 = 1$. Given a Cobb-Douglas utility function $u_i(x_i, G) = x_i G$, the optimal matching scheme is derived as (see Appendix A)

$$\mu_1 = k, \quad \mu_2 = 1/k$$

Consider an example with $w_1 = 3/4$ and $w_2 = 1$. Table 1 provides the values of all variables at the initial equilibrium and the optimal matching equilibrium for comparison.

Table 1: Comparison between initial equilibrium and optimal matching equilibrium

	x_1	x_2	G	u_1	u_2
Initial equilibrium	7/12	7/12	7/12	49/144	49/144
Optimal matching equilibrium	3/8	4/8	7/8	21/64	28/64

The public good provision $G = 7/8$ at the optimal matching equilibrium is at the optimal level. However, player 1 is worse off compared to the initial equilibrium. This indicates that the sub-game perfect equilibrium may not be Pareto-improving although it achieves the optimal public good provision.

In addition, this optimal matching scheme indicates that if player 1 provides one unit of the public good, player 2 must provide 4/3 units of the public good as matching contributions. This matching rate is too ambitious in a decentralized context either from the perspective of informational requirement or from the perspective of commitment credibility.

3 Pareto improvement at interior equilibrium

This section investigates existence conditions of Pareto-improving matching schemes at interior equilibria. From the Samuelson rule, an interior matching equilibrium is optimal if and only if $p_1 + p_2 = 1$ which implies $\mu_1\mu_2 = 1$, so it is natural to assume $\mu_1\mu_2 < 1$ in our analysis and we focus on small matching schemes and even on marginal matching schemes to provide tractable results.

3.1 Interiority condition

At the initial interior equilibrium, the public good provision is characterized by the aggregate budget constraint as

$$G + e_1(G, 1) + e_2(G, 1) = W \tag{3}$$

If $\lim_{G\to 0} e_i(G,1) = 0$ and $\lim_{G\to\infty} e_i(G,1) = \infty$, the existence of a public good level $\hat{G}$ is implied by the intermediate value theorem. Uniqueness is ensured by the strict monotonicity of the income expansion paths. The following proposition characterizes the interiority condition and provides an important feature of an interior equilibrium.

Proposition 1. Given the total income W in a two-player economy, at an interior equilibrium,

(1) The income ratio must satisfy

$$\frac{e_1(\hat{G},1)}{W - e_1(\hat{G},1)} < k < \frac{W - e_2(\hat{G},1)}{e_2(\hat{G},1)} \tag{4}$$

where $\hat{G}$ is characterized by equation (3);

(2) The private good consumption of both players and the total public good provision are independent of the income distribution;

(3) The individual public good contribution depends on the income distribution, and is increasing in their income share.

Proof: (1) Interiority requires $y_i = w_i - e_i(\hat{G},1) > 0$. Given $k = w_1/w_2$, we have $w_1 = k/(1+k) * W$ and $w_2 = 1/(1+k) * W$. Substituting w_i into the interiority condition yields condition (4).
(2) The independence of the total public good provision on the income distribution trivially follows from equation (3), and hence the private good consumption $x_i = e_i(G,1)$ is also independent of the income distribution.
(3) Since $\hat{G}$ only depends on W and $y_i = w_i - e_i(\hat{G},1)$, the individual public good contribution is increasing in their income share. QED

The range in condition (4) is referred to as the interiority zone denoted by $(\underline{k}, \overline{k})$. Part (2) of this proposition is well known as Warr neutrality (Warr 1982, 1983).

3.2 Matching effects

Buchholz et. al (2015) consider a special matching scheme – unilateral matching, and reveals a paradoxical effect that the matched player is worse off while the matching player is better off. This section provides more effects of matching schemes in a more general context at interior equilibria and then explains broadly how matching schemes work from a different perspective.

Proposition 2. At an interior matching equilibrium with a small matching scheme,

(1) Each player's utility is increasing in the own matching rate and decreases in the opponent's matching rate;

(2) The total public good provision is increasing in both players' matching rates;

(3) The matching contribution is increasing in the own matching rate while the flat contribution is decreasing in the own matching rate to a larger extent, resulting that the individual public good provision is decreasing in the own matching rate;

(4) The matching contribution is decreasing in the opponent's matching rate while the flat contribution is increasing in the opponent's matching rate to a larger extent, resulting that the individual public good provision is increasing in the opponent's matching rate;

(5) The private good consumption is increasing in the own matching rate and is decreasing in the opponent's matching rate.

Proof: See Appendix B.

The key insight of this mechanism is that one player provides matching contributions to induce the opponent to contribute more to the public good but meanwhile decreases her own flat contributions to a larger extent, resulting in lower individual contributions. This can be viewed as a free-riding behavior: players free ride on their own initial flat contributions when providing matching contributions. The total public good provision is increased because the opponent is induced to provide a larger flat contribution. Hence, players are better off through providing matching contributions and have incentives to increase their own matching rates.

To further understand these effects on public good provision, private good consumption and utility, we decompose the overall effect on each of them into components as below. To highlight the matching effect compared to the initial equilibrium, we focus on marginal matching schemes.

3.2.1 Matching effects on total public good provision

To explore how the total public good provision increases, we consider a unilateral matching scheme $(\mu_1, 0)$ where only player 1 provides matching contributions. The aggregate budget constraint is

$$e_1(G, 1) + e_2(G, \pi_2) + G = W, \pi_2 = 1 + \mu_1 \tag{5}$$

This aggregate budget constraint implies (by the implicit function theorem)

$$\frac{dG}{d\mu_1} = -\frac{\partial e_2/\partial \pi_2}{1 + \partial e_1/\partial G + \partial e_2/\partial G} \tag{6}$$

where all derivatives are evaluated at $\mu_1 = 0$ (hereafter when one variable is differentiated at the margin, all derivatives in the equation are evaluated at $\mu_i = 0$ without explicit notation on the evaluation point, as we do in equation (6) for convenience). When player 1 provides matching contributions, player 2's public good price is lower and thus her income expansion path becomes flatter, so player 2 would reduce her private good consumption by $-\partial e_2/\partial \pi_2$ if the total public good provision was not changed. This released resource is then reallocated, but it cannot all go to the public good because, if the total public good increased by $-\partial e_2/\partial \pi_2$, the private good consumption must also increase along each player's income expansion path, which either violates the aggregate budget constraint or does not satisfy optimality conditions. Thus, the released resource must be reallocated between the public good and the private good consumption: a fraction goes to the total public good provision, and a fraction goes to player 1's private good consumption along her expansion path $e_1(G, 1)$, and a fraction goes to player 2's private good consumption along her new expansion path $e_2(G, \pi_2)$, and the allocation must satisfy the aggregate budget constraint (5). Equation (6) shows that the fraction of the released resource allocated to the total public good is $1/(1 + \partial e_1/\partial G + \partial e_2/\partial G)$, and hence the fractions allocated to player 1's private good consumption and to player 2's private good consumption are respectively

$$\frac{\partial e_1/\partial G}{1 + \partial e_1/\partial G + \partial e_2/\partial G}, \quad \frac{\partial e_2/\partial G}{1 + \partial e_1/\partial G + \partial e_2/\partial G}$$

The effect of a bilateral matching scheme (μ_1, μ_2) on the total public good provision is the sum of the effects of two unilateral matching schemes $(\mu_1, 0)$ and $(0, \mu_2)$. More specifically, the change of the total public good provision at a marginal matching scheme is

$$\frac{dG}{d\mu_1} + \frac{dG}{d\mu_2} = -\frac{\partial e_1/\partial \pi_1 + \partial e_2/\partial \pi_2}{1 + \partial e_1/\partial G + \partial e_2/\partial G}$$

3.2.2 Matching effects on private good consumption

The matching effect on the private good consumption can also be decomposed further. We first examine the effects of receiving matching contributions and then the effects of providing matching contributions (with player 1 as an example). When player 1 receives matching contributions, the change of her private good consumption with respective to μ_2 at the margin is

$$\frac{dx_1}{d\mu_2} = \frac{\partial e_1}{\partial G}\frac{dG}{d\mu_2} + \frac{\partial e_1}{\partial \pi_1} \tag{7}$$

This decomposition suggests that there are two effects of receiving matching contributions on the private good consumption. Figure 1 illustrates the two effects in the space of (x, G) where player 1's initial income expansion path is represented by OA and her initial consumption bundle is at point A. First, player 1's individual public good price decreases due to matching, so her income expansion path becomes flatter from OA to OB, and she would reduce her private good consumption from x_A to x_B if the total public good provision was not changed at G_A. Second, player 2's matching rate increases the total public good provision from G_A to G_C, and player 1 would increase her private good consumption along her new income expansion path OB from x_B to x_C. Proposition 2 shows that the negative effect dominates the positive effect so that player 1 reduces her private good consumption from x_A to x_C when receiving matching contributions. To match equation (7) with Figure 1, the following relationships must hold:

$$x_B - x_A = \frac{\partial e_1}{\partial \pi_1}, \quad G_C - G_A = \frac{dG}{d\mu_2}, \quad x_C - x_B = \frac{\partial e_1}{\partial G}\frac{dG}{d\mu_2}, \quad x_C - x_A = \frac{dx_1}{d\mu_2}$$

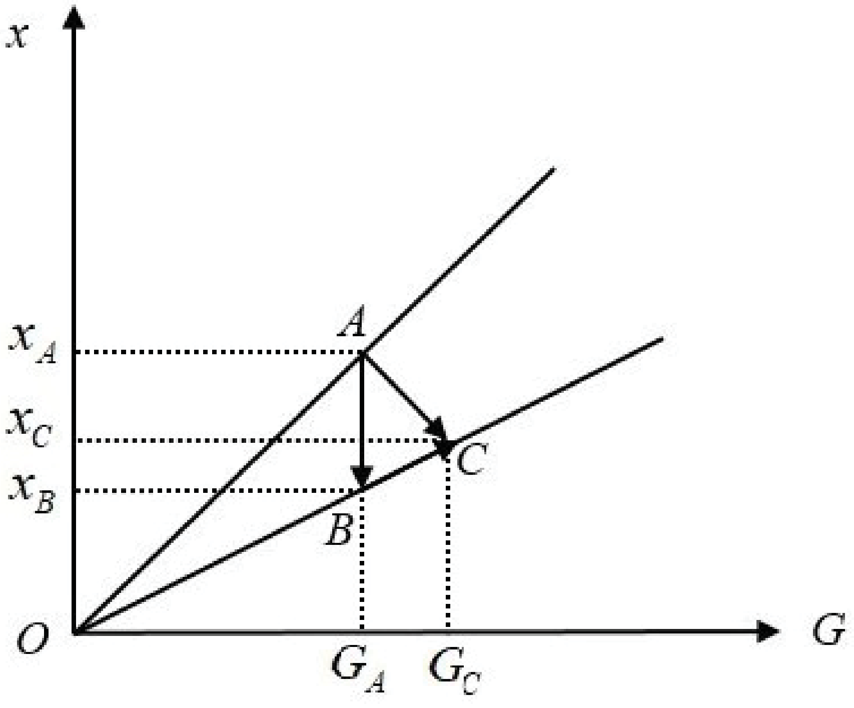

Figure 1: Matching effects on private good consumption

On the other hand, when player 1 provides matching contributions, the change of her private good consumption with respect to μ_1 at the margin is

$$\frac{dx_1}{d\mu_1} = \frac{\partial e_1}{\partial G}\frac{dG}{d\mu_1}$$

There is only one effect of providing matching contributions on the private good consumption, i.e., player 1's own matching rate increases the total public good provision and she would increase her private good consumption along her income expansion path. Her income expansion path does not change when she provides matching contributions. Therefore, player 1 increases her private good consumption when providing matching contributions.

3.2.3 Matching effects on utility

To decompose the matching effect on utility, we also examine the effects of receiving matching and providing matching respectively (with player 1 as an example). When player 1 receives matching contributions, her utility change with respective to μ_2 at the margin is

$$\frac{du_1(x_1,G)}{d\mu_2} = \frac{\partial u_1}{\partial x_1}\left(\frac{\partial e_1}{\partial G}\frac{dG}{d\mu_2} + \frac{\partial e_1}{\partial \pi_1}\right) + \frac{\partial u_1}{\partial G}\frac{dG}{d\mu_2} = \frac{\partial u_1}{\partial G}\left(\left(\frac{\partial e_1}{\partial G} + 1\right)\frac{dG}{d\mu_2} + \frac{\partial e_1}{\partial \pi_1}\right)$$

where the second equality holds because $\partial u_1/\partial x_1 = \partial u_1/\partial G$ at the initial equilibrium. This decomposition suggests that there are three effects of receiving matching contributions on the utility. First, player 1's individual public good price decreases due to matching, so her income expansion path becomes flatter, which has a negative effect on her private good consumption and hence a negative effect on her utility. Second, player 2's matching rate increases the total public good provision and hence has a positive effect on player 1's utility. Third, since the total public good provision increases, player 1 would increase her private good consumption along her new income expansion path, which also has a positive effect on her utility. Proposition 2 shows that the negative effect dominates the two positive effects so that player 1 is worse off when receiving matching contributions.

On the other hand, when player 1 provides matching contributions, her utility change with respect to μ_1 at the margin is

$$\frac{du_1(x_1, G)}{d\mu_1} = \frac{\partial u_1}{\partial x_1}\left(\frac{\partial e_1}{\partial G}\frac{dG}{d\mu_1} + \frac{\partial e_1}{\partial \pi_1}\right) + \frac{\partial u_1}{\partial G}\frac{dG}{d\mu_1} = \frac{\partial u_1}{\partial G}\left(\frac{\partial e_1}{\partial G} + 1\right)\frac{dG}{d\mu_1}$$

This decomposition suggests that there are two effects of providing matching contributions on the utility. First, player 1's own matching rate increases the total public good provision and hence has a positive effect on her utility. Second, since the total public good provision increases, she would increase her private good consumption along her income expansion path, which also has a positive effect. Her income expansion path does not change when she provides matching contributions. These two effects go in the same direction, so player 1 is better off when providing matching contributions.

A caveat with Proposition 2 is that it is built on the assumption of complete information and credible commitment. However, it is often the case that large matching rates induce high uncertainty of outcomes and low credibility of commitment. High uncertainty and low credibility would make large matching rates less favourable, so players would make a trade-off between the two opposing effects of increasing matching rates, which is left for future research. As we argued, at small matching rates, it is not a major problem to assume complete information and credible commitment in the neighborhood around the initial equilibrium which is often observable, so the above proposition should be interpreted with small matching schemes.

As one player's utility is increasing in the own matching rate and decreases in the opponent's matching rate, it implies that not all matching schemes can generate Pareto-improving outcomes. Then two questions arise: (1) Given an initial equilibrium, does there exist a Pareto-improving matching scheme? (2) If there exists, what combinations of matching rates are required?

3.3 Pareto-improving matching schemes

This section shows the universal existence of Pareto-improving matching schemes at interior equilibria in a two-player economy. To prove the existence, we first establish two lemmas. Lemma 1 proves the existence of individual public good prices so that the interior matching equilibrium strictly Pareto dominates the initial equilibrium, and Lemma 2 proves that the individual public good prices can be realized by a matching scheme.

Lemma 1. Given the total income W at an initial interior equilibrium, there exist infinite pairwise individual public good prices $\vec{P} = (p_1, p_2)$ so that the interior matching equilibrium $(x_1(\vec{P}, W), x_2(\vec{P}, W), G(\vec{P}, W))$ strictly Pareto dominates the initial equilibrium.

Proof: See Appendix C.

Lemma 2. Given the total income W at an initial interior equilibrium, if the public good prices $\vec{P} = (p_1, p_2)$ are sufficiently close to the initial prices $\vec{P_0} = (1, 1)$, there exists a matching scheme realizing the public good prices.
Proof: See Appendix C.

Combining Lemma 1 and Lemma 2, the following proposition shows the universal existence of Pareto-improving matching schemes at interior equilibria.

Proposition 3. Given an initial interior equilibrium, there always exist small Pareto-improving matching schemes regardless of preferences.
Proof: See Appendix C.

The intuition of this proposition is captured by the two lemmas. The allocative function of the mechanism is achieved by distorting each player's relative price between the public and the private good through reciprocal subsidization of flat contributions. By distorting the relative prices, players are induced to provide larger contributions and hence the externality of the public good is corrected to some extent, which brings the economy to a more efficient level. If the distortion of each player's relative price is relatively symmetric across players (relative to their preferences), both players can be better off. Given an initial interior equilibrium, since the utility functions are continuous, we can always choose sufficiently small matching rates to ensure that the matching equilibrium is also interior.

To further understand how a bilateral matching scheme generates a Pareto-improving outcome, we consider for comparison a unilateral matching scheme $(0, \mu_2)$ where only player 2 provides matching contributions. This unilateral scheme has two opposing effects on player 1's private good consumption. First, it lowers the effective public good price for player 1 and thus makes her income expansion path flatter, so player 1 would reduce her private good consumption if the total public good provision was not changed. Second, it increases the total public good provision, so player 1 would increase her private good consumption along her new income expansion path. Proposition 2 shows that the first effect dominates the second one and thus the overall effect is that player 1's private good consumption decreases.

In addition to player 2's matching rate, suppose player 1 also provides matching contributions. Player 1's matching rate has only one effect on her own private good consumption, i.e., it increases the total public good provision and hence increases her private good consumption along her income expansion path.

The overall effect of a bilateral matching scheme on the private good consumption is ambiguous compared to the initial equilibrium. However, the total public good provision is increasing in both players' matching rates. The aggregate budget constraint then implies that at least one player decreases her private good consumption. Even if both players decrease their private good consumption, the increase in the total public good provision can possibly compensate both players for their decreases in the private good consumption if the matching scheme is not so asymmetric (relative to their preferences).

Then what combinations of matching rates can induce Pareto-improving outcomes? Intuitively, the two matching rates cannot be too asymmetric. In other words, the ratio of the two matching rates must be bounded in a certain range. If we consider an extremely asymmetric case – a unilateral matching scheme, i.e., the ratio of the matching rates is either zero or infinity, then it is impossible to achieve Pareto-improving outcomes because a unilateral matching scheme only benefits one player at interior equilibria. This indicates that the ratio cannot be too small or too large. The following proposition characterizes Pareto-improving matching schemes at the margin in general preferences, and Section 3.4 will show all Pareto-improving matching schemes in a diagram with a Cobb-Douglas example.

Proposition 4. At an initial interior equilibrium, a marginal matching scheme (μ_1, μ_2) is Pareto-improving if and only if the ratio of the two matching rates satisfies

$$\frac{\partial e_1/\partial G}{1+\partial e_2/\partial G} * \frac{\partial e_2/\partial \pi_2}{\partial e_1/\partial \pi_1} < \frac{\mu_2}{\mu_1} < \frac{1+\partial e_1/\partial G}{\partial e_2/\partial G} * \frac{\partial e_2/\partial \pi_2}{\partial e_1/\partial \pi_1}$$

where all derivatives are evaluated at the margin.

Proof: See Appendix D.

This condition has two general implications:

(1) The income expansion path is a function of the total public good provision and the latter depends on the total income but not on the income distribution at interior equilibria, so the condition in this proposition depends only on the preferences but is independent of the income distribution.

(2) The range is determined by the relative preference intensity and the relative elasticity of substitution of the two players. It can also be interpreted in another way: the range is first determined by their relative preferences as below and then is distorted by their relative elasticity of substitution.

$$\frac{\partial e_1/\partial G}{1+\partial e_2/\partial G} < \frac{\mu_2}{\mu_1} < \frac{1+\partial e_1/\partial G}{\partial e_2/\partial G}$$

To explore the above condition further, we consider again the CES function $u_i(x_i, G) = (a_i x_i^{\rho_i} + (1-a_i)G^{\rho_i})^{1/\rho_i}$. For notation convenience, the income expansion path is rewritten as

$$x_i = \left(\frac{A_i}{\pi_i}\right)^{\frac{1}{1-\rho_i}} G, \quad A_i = \frac{a_i}{1-a_i}$$

The condition in Proposition 4 reads as

$$\frac{1-\rho_1}{1-\rho_2}\frac{{A_2}^{1/(1-\rho_2)}}{1+{A_2}^{1/(1-\rho_2)}} < \frac{\mu_2}{\mu_1} < \frac{1-\rho_1}{1-\rho_2}\frac{1+{A_1}^{1/(1-\rho_1)}}{{A_1}^{1/(1-\rho_1)}} \tag{8}$$

Consider the case where two players have the same elasticity of substitution, i.e., $\rho_1 = \rho_2 = \rho$, this condition further reduces to

$$\frac{{A_2}^{1/(1-\rho)}}{1+{A_2}^{1/(1-\rho)}} < \frac{\mu_2}{\mu_1} < \frac{1+{A_1}^{1/(1-\rho)}}{{A_1}^{1/(1-\rho)}} \tag{9}$$

It is self-evident that the condition only depends on the preferences but not on the income distribution. Besides, the term on the left side is always smaller than one, and the term on the far-right side is always larger than one, so a pair of symmetric matching rates $(\mu_1 = \mu_2)$ can always generate a Pareto-improving equilibrium regardless of the preference intensity over the public good.

Next we provide a couple of extreme cases which can give us some general insights.

(1) **Altruistic Case**

If $a_1 = a_2 \to 0$ and hence $A_1 = A_2 \to 0$, i.e., players have extremely strong preferences over the public good, then condition (9) reads as $0 < \mu_2/\mu_1 < \infty$, so any pair of positive matching rates can generate a Pareto-improving equilibrium. To see this, we consider a matching scheme $(\mu, \beta\mu)$ and decompose its overall effect on the utility at the margin into three components as

$$\frac{du_1(x_1, G)}{d\mu} = \frac{\partial u_1}{\partial G}\left(\frac{\partial e_1}{\partial G}\frac{dG}{d\mu} + \frac{dG}{d\mu} + \beta\frac{\partial e_1}{\partial \pi_1}\right) \tag{10}$$

Given the matching scheme, the aggregate budget constraint implies

$$\frac{dG}{d\mu} = \left(-\beta\frac{\partial e_1}{\partial \pi_1} - \frac{\partial e_2}{\partial \pi_2}\right)\left(1 + \frac{\partial e_1}{\partial G} + \frac{\partial e_2}{\partial G}\right)^{-1} \tag{11}$$

When $a_1 = a_2 \to 0$, $\partial e_1/\partial G = \partial e_2/\partial G = A_i^{1/(1-\rho)} \to 0$. The above equation reduces to

$$\frac{dG}{d\mu} \approx -\beta\frac{\partial e_1}{\partial \pi_1} - \frac{\partial e_2}{\partial \pi_2} \tag{12}$$

This indicates that when players have extremely strong preferences over the public good, if they are faced with lower public good prices, they reduce their private good consumption and reallocate almost all of the released resource to the total public good given the weight on the private good is close to zero. Substituting (12) into (10) yields

$$\frac{du_1(x_1, G)}{d\mu} \approx \frac{\partial u_1}{\partial G}\left(\frac{\partial e_1}{\partial G}\left(-\beta\frac{\partial e_1}{\partial \pi_1} - \frac{\partial e_2}{\partial \pi_2}\right) - \frac{\partial e_2}{\partial \pi_2}\right) \approx -\frac{\partial u_1}{\partial G}\frac{\partial e_2}{\partial \pi_2} > 0$$

When player 1 receives matching contributions, she lowers her income expansion path and hence reduces her private good consumption if the total public good provision was not changed, which has a negative effect on her utility. The released resource is almost all reallocated to the public good, which has a positive effect on her utility. These two effects are offset because the marginal utility of the private good equals the marginal utility of the public good at the margin. On the other hand, when player 1 provides matching contributions, player 2 increases the public good provision, which has a positive effect on player 1's utility. Player 1 would also increase her private good consumption along her income expansion path, but given $\partial e_1/\partial G \to 0$, this effect is negligible. Thus, in this extreme case, player 1's utility change is driven by player 2's contribution to the public good. The same logic applies to player 2.

A general insight of this extreme case is that if players are more altruistic, it is more flexible to implement matching schemes.

(2) **Selfish Case**

If $a_1 = a_2 \to 1$ and hence $A_1 = A_2 \to \infty$, i.e., players have extremely strong preferences over the private good, then condition (9) reads as $\mu_2/\mu_1 \to 1$, so only a pair of symmetric matching rates can generate a Pareto-improving equilibrium. When player 1 is matched at the rate of $\beta\mu$, her income expansion path becomes flatter and she would reduce her private good consumption by $-\beta\partial e_1/\partial \pi_1$ if the total public good provision was not changed. This released resource is reallocated among the total public good, player 1's private good consumption and player 2's private good consumption. Since $\partial e_1/\partial G = \partial e_2/\partial G \to \infty$, the share on the total public good is

$$\frac{1}{1+\partial e_1/\partial G+\partial e_2/\partial G} \approx 0$$

Since the released resource allocated to the total public good is close to zero, the effect of the increase in the total public good is negligible. The shares allocated to the private good consumption are respectively

$$\frac{\partial e_1/\partial G}{1+\partial e_1/\partial G+\partial e_2/\partial G} \approx \frac{1}{2}, \quad \frac{\partial e_2/\partial G}{1+\partial e_1/\partial G+\partial e_2/\partial G} \approx \frac{1}{2}$$

One half of the released resource is reallocated to player 1's private good consumption and the other half goes to player 2's private good consumption. On the other hand, when player 1 provides matching contributions at the rate μ, player 2 would reduce her private good consumption by $-\partial e_2/\partial \pi_2$ and, similarly, half of this released resource is reallocated to player 1's private good consumption and half goes to player 2's private good consumption. Therefore, to make player 1 better off, the half of player 2's released resource must be larger than the half of player 1's released resource, which implies that $\beta \leq 1$. In this extreme case, player 1's utility change is driven by two channels: the change of player 1's income expansion path, and the change of player 1's private good consumption through the public good change. Mathematically, substituting (11) into (10) yields

$$\frac{du_1(x_1,G)}{d\mu} \approx \frac{\partial u_1}{\partial G}\left(-\frac{\partial e_1}{\partial G}\frac{\partial e_2}{\partial \pi_2}+\beta\frac{\partial e_2}{\partial G}\frac{\partial e_1}{\partial \pi_1}\right)\left(1+\frac{\partial e_1}{\partial G}+\frac{\partial e_2}{\partial G}\right)^{-1}$$

where $\partial e_1/\partial G * \partial e_2/\partial \pi_2 = \partial e_2/\partial G * \partial e_1/\partial \pi_1$ given the CES function. To make the utility change positive, $\beta \leq 1$ must hold.

Similarly, from player 2's perspective, $1/\beta \leq 1$ must hold. Thus, $\beta \to 1$ in this extreme case.

A general insight of this extreme case is that if players are more selfish, it is more difficult to implement matching schemes.

(3) **Asymmetric Case**
If $a_1 \to 0$ and $a_2 \to 1$ and hence $A_1 \to 0$ and $A_2 \to \infty$, i.e., player 1 is altruistic and player 2 is selfish, then condition (9) reads as $1 < \mu_2/\mu_1 < \infty$. Given $\partial e_1/\partial \pi_1 \to 0$ and $\partial e_1/\partial G \to 0$, the change of the total public good provision is approximated as

$$\frac{dG}{d\mu} = \left(-\beta\frac{\partial e_1}{\partial \pi_1}-\frac{\partial e_2}{\partial \pi_2}\right)\left(1+\frac{\partial e_1}{\partial G}+\frac{\partial e_2}{\partial G}\right)^{-1} \approx -\frac{\partial e_2/\partial \pi_2}{\partial e_2/\partial G} = \frac{1}{1-\rho_2}G$$

Substituting this into (10) yields

$$\frac{du_1(x_1,G)}{d\mu} = \frac{\partial u_1}{\partial G}\left(-\frac{\partial e_1}{\partial G}\frac{\partial e_2/\partial \pi_2}{\partial e_2/\partial G}-\frac{\partial e_2/\partial \pi_2}{\partial e_2/\partial G}+\beta\frac{\partial e_1}{\partial \pi_1}\right)$$

Since $\partial e_1/\partial G \to 0$ and $\partial e_1/\partial \pi_1 \to 0$, player 1's utility change is dominated by $dG/d\mu$ and is always positive regardless of β.

From player 2's perspective,

$$\frac{du_2(x_2, G)}{d\mu} = \frac{\partial u_2}{\partial G}\left(\frac{\partial e_2}{\partial G}\frac{dG}{d\mu} + \frac{dG}{d\mu} + \frac{\partial e_2}{\partial \pi_2}\right) \tag{13}$$

Substituting (11) into (13) yields

$$\frac{du_2(x_2, G)}{d\mu} = \frac{\partial u_2}{\partial G}\left(-\beta\frac{\partial e_2}{\partial G}\frac{\partial e_1}{\partial \pi_1} - \beta\frac{\partial e_1}{\partial \pi_1} + \frac{\partial e_1}{\partial G}\frac{\partial e_2}{\partial \pi_2}\right)\left(1 + \frac{\partial e_1}{\partial G} + \frac{\partial e_2}{\partial G}\right)^{-1}$$

Since $\partial e_2/\partial G \to \infty$ and $\partial e_2/\partial \pi_2 \to \infty$, the second term in the first bracket is negligible. Given the CES function, $\partial e_2/\partial G * \partial e_1/\partial \pi_1 = \partial e_1/\partial G * \partial e_2/\partial \pi_2$, so $\beta > 1$ must hold to make player 2 better off. In this extremely asymmetric case, the effects of all three channels on player 2's utility are all important.

A general insight of this asymmetric case is that if one player is selfish and the other is altruistic, it is more flexible for the altruistic player but is more demanding for the selfish player to implement matching schemes, and the selfish player's matching rate has to be larger than the altruistic player's matching rate to make the selfish one better off.

Condition (9) also has several implications on the effect of elasticity of substitution. It is empirically plausible to assume that players value the private good more than the public good, i.e., $a_1 > 0.5, a_2 > 0.5$ and hence $A_1 > 1, A_2 > 1$. Given this assumption, we consider different cases for elasticity of substitution.

(1) **Cobb-Douglas Case**
If $\rho = 0$, the private and the public good are neither substitutes nor complements, and the CES function is degenerated to a Cobb-Douglas function $u_i(x_i, G) = x_i^{A_i}G$, and the condition is reduced to

$$\frac{A_2}{1 + A_2} < \frac{\mu_2}{\mu_1} < \frac{1 + A_1}{A_1}$$

The range is determined by the preference intensity between the two goods, and it has a straightforward interpretation in a diagram in Section 3.4.

(2) **Complement Case**
If $\rho < 0$, i.e., the private and the public good are complements, then ${A_i}^{1/(1-\rho)} > A_i$. This indicates that the range of the ratio is smaller in the case of complements than in the Cobb-Douglas case. The intuition is that, if the two goods are complements, when players are faced with lower public good prices due to matching, they would increase their private good consumption ceteris paribus, which tends to make the economy less efficient.

(3) **Substitute Case**
If $\rho > 0$, i.e., the private and the public good are substitutes, ${A_i}^{1/(1-\rho)} < A_i$. This indicates that the range of the ratio is larger in the case of substitutes than in the Cobb-Douglas case and hence larger than in the case of complements. In

contrast to complements, if the two goods are substitutes, when players are faced with lower public good prices, they would decrease their private good consumption ceteris paribus, which tends to make the economy more efficient.

(4) **Asymmetric Case**
If two players have different elasticities of substitution, $\rho_1 \neq \rho_2$, the range in (9) is distorted by their relative elasticity of substitution into (8), and symmetric matching rates may not be Pareto-improving.

In contrast, if players value the public good more than the private good, then the range in the complement case is larger than in the Cobb-Douglas case, which is larger than in the substitute case.

It is worth highlighting that the existence of Pareto-improving matching schemes is guaranteed by the interiority of the initial equilibrium regardless of the preferences although which matching schemes are Pareto-improving depend on the preferences. This indicates that two players at an interior equilibrium can always implement small matching schemes to generate Pareto-improving outcomes regardless of their preferences. Since matching schemes do not require side payments and implementing small matching schemes only requires local information about preferences and limited commitment credibility, the universal existence of Pareto-improving matching schemes is useful for cooperation between heterogeneous players in the context without complete information of global preferences or at the international level without central governments.

3.4 Graphic representation in an example

This section provides a diagram with a Cobb-Douglas example to: (1) illustrate the existence of Pareto-improving matching schemes at interior equilibria; (2) show marginal Pareto-improving matching schemes and their graphic interpretation; (3) show all Pareto-improving matching schemes and illustrate how they change with income distributions; (4) show the optimal matching scheme may not be Pareto-improving, which is one of the motivations of this paper.

Given $u_i(x_i, G) = x_i^{\alpha_i} G$, at an initial interior equilibrium, the income ratio must lie in the interiority zone

$$\frac{\alpha_1}{\alpha_2 + 1} < k < \frac{\alpha_1 + 1}{\alpha_2}$$

If an interior matching equilibrium is Pareto-improving, the following conditions must hold:

$$y_1 > 0 \Rightarrow \mu_2 > \frac{\alpha_1(1+\mu_1)}{k(1+\alpha_2) - \mu_1} - 1 \tag{14}$$

$$y_2 > 0 \Rightarrow \mu_2 < \frac{1/k * (1+\alpha_1)(1+\mu_1) - \alpha_2}{1 + \mu_1 + \alpha_2} \tag{15}$$

$$u_1(x_1, G) > u_1(\bar{x}_1, \bar{G}) \Rightarrow 1 + \mu_1 > \frac{\alpha_2(1+\mu_2)}{(\alpha_1+\alpha_2+1)(1+\mu_2)^{1/(\alpha_1+1)} - \alpha_1 - 1 - \mu_2} \tag{16}$$

$$u_2(x_2, G) > u_2(\bar{x}_2, \bar{G}) \Rightarrow 1 + \mu_2 > \frac{\alpha_1(1+\mu_1)}{(\alpha_1+\alpha_2+1)(1+\mu_1)^{1/(\alpha_2+1)} - \alpha_2 - 1 - \mu_1} \tag{17}$$

The first two conditions ensure the interiority of a matching equilibrium and the last two conditions satisfy participation constraints. Figure 2 illustrates these conditions in a special case ($\alpha_1 = \alpha_2 = 1, k = 1$) in the space of (μ_1, μ_2) where the origin represents the initial equilibrium. The dashed curve Y_2 represents $y_2 = 0$ and the dotted curve Y_1 represents $y_1 = 0$, and they intersect at point E where $\mu_1\mu_2 = 1$ holds. The hyperbola denotes all matching schemes satisfying $\mu_1\mu_2 = 1$, so it passes through point E. Given our focus on small matching schemes, we are only interested in the area under the hyperbola. The area enclosed by Y_2 and Y_1 together with the two axes represents matching schemes that generate interior matching equilibria (hereafter the interiority area). The upper bound of the lens-shaped area, I_1, denotes player 1's indifference curve and the lower bound I_2 denotes player 2's indifference curve at their respective utility levels at the initial equilibrium. Thus, the lens-shaped area represents Pareto-improving matching schemes. The overlapping area between the lens-shaped area and the interiority area represents all matching schemes that generate Pareto-improving outcomes at interior equilibria.

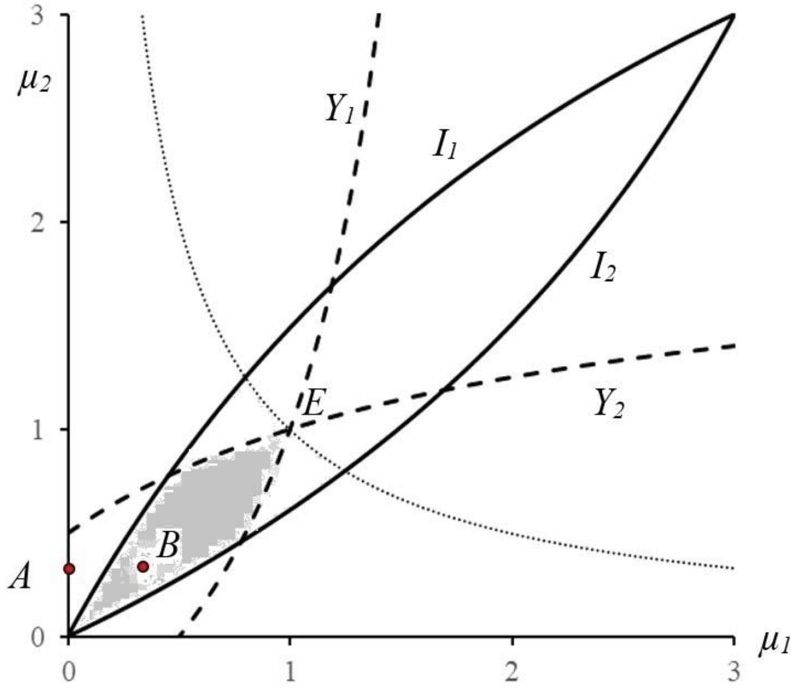

Figure 2: Pareto improvements at interior equilibria with $k = 1$

Consider a symmetric bilateral matching scheme $(\hat{\mu}_1, \hat{\mu}_2)$ where $\hat{\mu}_1 = \hat{\mu}_2 = \hat{\mu} > 0$, represented by point B. To understand how this bilateral matching scheme generates a Pareto-improving outcome, we consider for comparison a unilateral matching scheme $(0, \hat{\mu}_2)$ represented by point A. Given this unilateral scheme, player 2 is better off with utility u_2^A and player 1 is worse off with utility u_1^A, i.e., $u_2^A > u_2(\bar{x}_2, \bar{G})$ and $u_1^A < u_1(\bar{x}_1, \bar{G})$. Compared to the unilateral scheme, player 1 is better off with utility u_1^B and player 2 is worse off with utility u_2^B in the bilateral matching scheme, i.e., $u_1^B > u_1^A$ and $u_2^B < u_2^A$. Overall, player 2 is better off through providing matching contributions but worse off when receiving matching contributions to a smaller extent, resulting in higher utility. The positive effect of providing matching contributions and the negative effect of receiving matching contributions in this symmetric matching scheme is not completely offset because the total public good provision is increasing in both players' matching rates. The same logic applies to player 1.

Now consider different income ratios. The lens-shaped area does not change with the income ratio because the utility functions depend on the private good consumption and the total public good provision and they are both independent of the income distribution at interior equilibria. But the interiority area changes with the income ratio. Given the utility function with $\alpha_1 = \alpha_2 = 1$, the interiority zone is $(0.5, 2)$. If the income ratio decreases from $k = 1$, the interiority area shifts to the left in Figure 2. When $k = 0.5$, the interiority area is tangent to the lens-shaped area at the origin on the left (see Figure 3(a)). If the income ratio increases from $k = 1$, the interiority area shifts to the right in Figure 2. When $k = 2$, the interiority area is tangent to the lens-shaped area at the origin on the right (see Figure 3(b)). Given this monotonic shift of the interiority area, it is clear that at any interior equilibrium there is an overlapping area, i.e., there exist small Pareto-improving matching schemes.

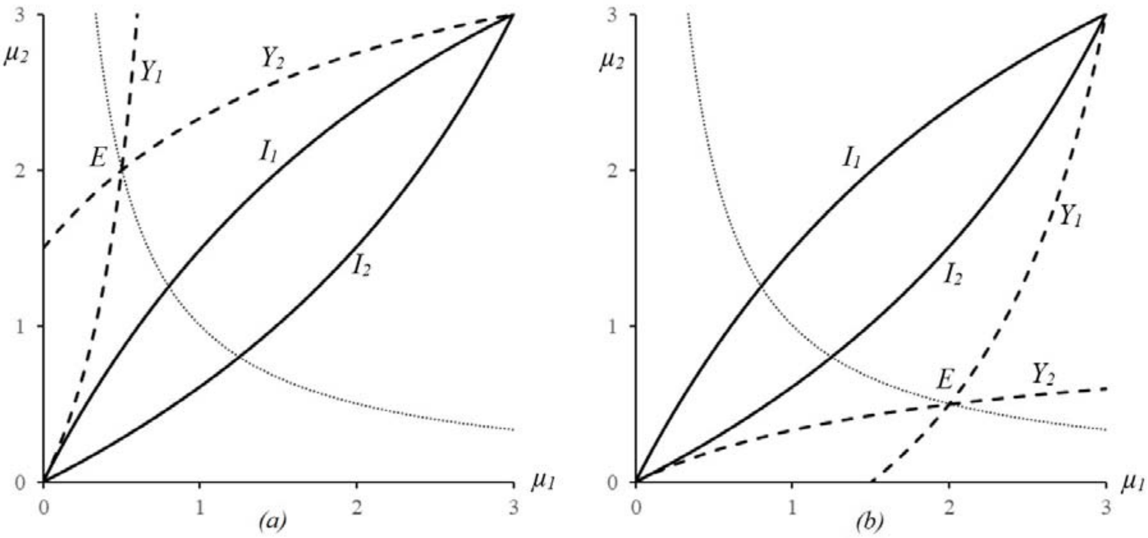

Figure 3: Pareto improvements at interior equilibria: (a) $k = 0.5$; (b) $k = 2$

Figure 2 (and Figure 3) show that Pareto-improving matching schemes are bounded by the lens-shaped area. Particularly, in marginal matching schemes, the ratio of the two matching rates for Pareto-improving outcomes are bounded by the tangent lines of the lens-shaped area at the origin. Given the Cobb-Douglas function, Proposition 4 indicates that, to achieve Pareto-improving outcomes, marginal matching schemes must satisfy

$$\frac{\alpha_2}{1+\alpha_2} < \frac{\mu_2}{\mu_1} < \frac{1+\alpha_1}{\alpha_1} \tag{18}$$

The two bounds of this condition (the bounds are equal to 0.5 and 2 respectively in our example with $\alpha_1 = \alpha_2 = 1$) are the slopes of I_2 and I_1 respectively at the origin. To see this, the two indifference curves are characterized by

$$u_1(x_1, G) = u_1(\bar{x}_1, \bar{G}) \Rightarrow 1 + \mu_1 = \frac{\alpha_2(1+\mu_2)}{(\alpha_1+\alpha_2+1)(1+\mu_2)^{1/(\alpha_1+1)} - \alpha_1 - 1 - \mu_2}$$

$$u_2(x_2, G) = u_2(\bar{x}_2, \bar{G}) \Rightarrow 1 + \mu_2 = \frac{\alpha_1(1+\mu_1)}{(\alpha_1+\alpha_2+1)(1+\mu_1)^{1/(\alpha_2+1)} - \alpha_2 - 1 - \mu_1}$$

By the implicit function theorem, the above two conditions imply that

$$\left.\frac{\mu_2}{\mu_1}\right|_{\mu_1=0,\mu_2=0} = \frac{1+\alpha_1}{\alpha_1}, \qquad \left.\frac{\mu_2}{\mu_1}\right|_{\mu_1=0,\mu_2=0} = \frac{\alpha_2}{1+\alpha_2}$$

To see how the ratio of the two matching rates must be bounded by the slopes of the two tangent lines at the margin, we can approximate the power terms in (16) and (17) by their first-order Taylor expansions because μ_1 and μ_2 are small, and simplify the two conditions as

$$1 + \mu_1 > \frac{\alpha_2(1+\mu_2)}{(\alpha_1+\alpha_2+1)(1+\mu_2/(\alpha_1+1)) - \alpha_1 - 1 - \mu_2}$$

$$1 + \mu_2 > \frac{\alpha_1(1+\mu_1)}{(\alpha_1+\alpha_2+1)(1+\mu_1/(\alpha_2+1)) - \alpha_2 - 1 - \mu_1}$$

Given μ_1 and μ_2 are small, we can further discard the terms of $\mu_1\mu_2$ and then the two conditions immediately lead to (18).

Section 3.3 has shown that if players have strong preferences over the public good, the ratio of the two matching rates in Pareto-improving matching schemes has a large range, while if players have strong preferences over the private good, the ratio has a small range. Figure 4 presents two cases with $\alpha_1 = \alpha_2 = 0.1$ in (a) and $\alpha_1 = \alpha_2 = 10$ in (b) to illustrate this point (For comparison with Figure 3, Figure 4 has the same scale on the two axes at the price of readability of Figure 4(b)).

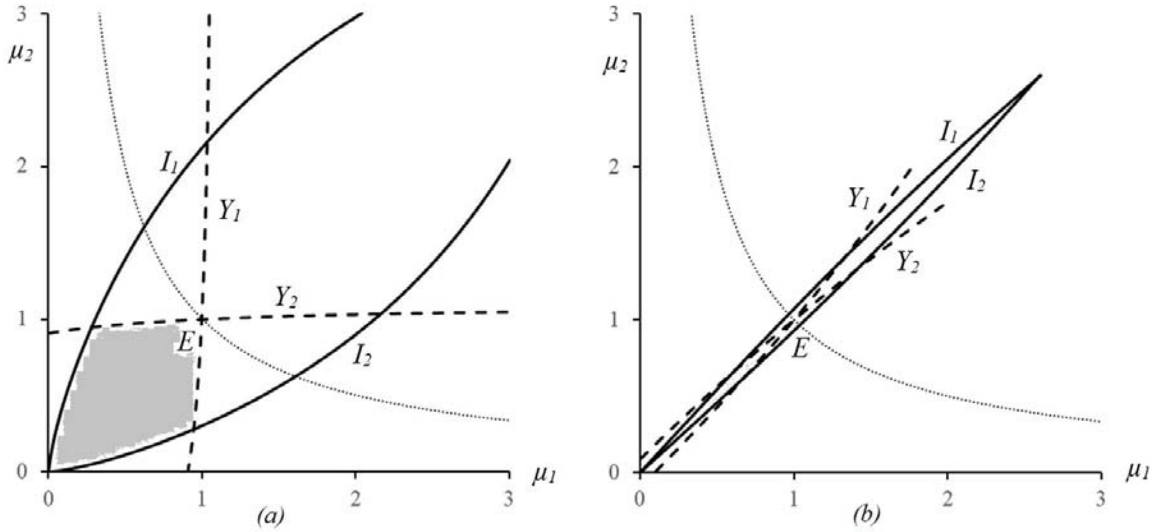

Figure 4: Pareto improvements at interior equilibria: (a) $\alpha_i = 0.1$; (b) $\alpha_i = 10$

In addition, Appendix A indicates that point E is the optimal matching scheme where

$$\mu_1 = \frac{k(1+\alpha_2)}{1+\alpha_1}, \; \mu_2 = \frac{1+\alpha_1}{k(1+\alpha_2)} \tag{19}$$

To make the optimal matching scheme Pareto-improving, i.e., to keep point E inside the lens-shaped area, the income ratio must be bounded in a certain range. To derive the range, we substitute (19) into the conditions of participation constraints (16) and (17) and replace $\alpha_1 = \alpha_2 = 1$, and then obtain the range $0.8 < k < 1.25$. This range is much smaller than the interiority zone $0.5 < k < 2$, indicating that it is difficult to reach the optimal equilibrium if participation constraints must be satisfied.

3.5 Extension in a multiple-player model

This section extends the existence conclusion (Proposition 3) in a two-player case into a multiple-player case. Suppose there are $n (n \geq 2)$ players and player i's income is w_i. The total income is $W = \sum_{i=1}^{n} w_i$ and the income distribution is denoted by $\vec{W} = (w_1, w_2, ..., w_n)$. The utility functions follow the same assumptions in the two-player case and the price of the public good in terms of the private good is still one. At the initial equilibrium, the aggregate budget constraint reduces to

$$\sum_{i=1}^{n} e_i(G, 1) + G = W$$

Given continuity and monotonicity, there exists a unique solution $\hat{G}(W)$. Interiority requires

$$y_i = w_i - e_i(\hat{G}, 1) > 0 \Rightarrow w_i > e_i(\hat{G}, 1)$$

Since $\hat{G}(W)$ is increasing in W, the interiority condition implies that the income share of each player must be above a certain level at an interior equilibrium. Given an initial interior equilibrium, the following proposition extends the existence conclusion to a multiple-player case.

Proposition 5. At an interior equilibrium in an economy with a finite number of players, there always exist small Pareto-improving matching schemes regardless of preferences.

Proof: See Appendix C.

The intuition behind this general existence conclusion is similar to the two-player case. Players subsidize each other through reciprocal matching and thus distort the relative price between the public and the private good, thereby correcting the externality of the public good to some extent. Given an initial interior equilibrium, we can always choose sufficiently small matching rates to ensure the matching equilibrium is also interior. Certainly, the selection of a Pareto-improving matching scheme for multiple players is more complicated than in a two-player case.

Consider a symmetric case where all players have the same preferences and the same income level. In such a case, a small uniform matching rate for all players can generate a Pareto-improving outcome. Even if players have different income levels but they all provide positive public good contributions, a small uniform matching rate can still achieve the same Pareto-improving outcome due to Warr neutrality. To show this, the aggregate budget constraint reduces to (with player index i dropped)

$$n * e(G, \pi) + G = W, \pi = 1 + (n-1)\mu$$

This implies

$$\frac{dG}{d\mu} = -\frac{(n-1)n * \partial e/\partial \pi}{n * \partial e/\partial G + 1}$$

The utility change with respect to μ at the margin is

$$\frac{du(x,G)}{d\mu} = \frac{\partial u}{\partial x}\left(\frac{\partial e}{\partial G}\frac{dG}{d\mu} + (n-1)\frac{\partial e}{\partial \pi}\right) + \frac{\partial u}{\partial G}\frac{dG}{d\mu} = -\frac{(n-1)^2}{n * \partial e/\partial G + 1}\frac{\partial u}{\partial G}\frac{\partial e}{\partial \pi} > 0$$

Liu (2018) examines matching coalitions with a uniform matching rate in a multiple-player economy with the same preferences but different income levels. The above case is a grand coalition (with all players participating in a matching coalition) with a uniform matching rate and it is always Pareto-improving regardless of preferences.

However, if players are heterogeneous in preferences, a uniform matching scheme may not be Pareto-improving and the combinations of all matching rates must be adjusted according to their preferences, as constructed in Appendix C.

4 Pareto improvement at corner equilibrium

There are two reasons for taking corner equilibria into consideration. First, it is generally more common to encounter corner equilibria than interior equilibria in a pure public good economy. For example, given $u_i(x_i, G) = x_i^a G$, the interiority zone is $\alpha/(1+\alpha) < k < (1+\alpha)/\alpha$. It is empirically plausible to assume $\alpha > 1$, so the income ratio must be in a small range to generate an interior equilibrium in contrast with the large income heterogeneity in the current world. Second, this section will show that matching schemes work differently at a corner equilibrium from at an interior equilibrium. There are two important differences. One is that when the interior player provides matching contributions to the corner player, the matching rate must be sufficiently large to induce the corner player to contribute to the public good. Until it reaches a critical level, the matching rate of substitution between the two goods does not equal the relative price of the public good modified by matching for the corner player, so the corner player still does not provide positive contributions, i.e., the matching rate is not effective and does not change players' behaviors. The other is that when the corner player provides matching contributions, she cannot free ride on her initial flat contributions, which produces quite different welfare effects from the interior case where players reduce their initial flat contributions when providing matching contributions. This unilateral matching scheme at the corner can possibly make both players better off, which is in contrast with the interior case where a unilateral matching scheme can only benefit one player.

4.1 Marginal matching schemes

At the initial corner equilibrium, one player (hereafter the corner player or player 1) does not provide public good contribution while the other (hereafter the interior player or player 2) provides positive contributions. There are three possible cases of matching equilibria:

(1) The interior player provides matching contributions to the corner player;

(2) The corner player provides matching contributions to the interior player;

(3) Both players provide matching contributions to each other.

The first two cases are unilateral matching schemes, and the first one implies an interior matching equilibrium while the second one implies a corner matching equilibrium. The third case is a bilateral matching scheme and it implies an interior matching equilibrium.

In the first case, the interior player must offer a sufficiently large matching rate μ_2 such that $w_1 > e_1(\bar{G}, 1+\mu_2)$ to induce the corner player to provide positive public good contributions. Consider the critical matching rate $\hat{\mu}_2$ that satisfies $w_1 = e_1(\bar{G}, 1+\hat{\mu}_2)$ (hereafter the minimum effective matching rate). Since the utility of one player is decreasing in the opponent's matching rate at an interior equilibrium, the corner player would be worse off in a matching scheme $(0, \mu_2)$ where $\mu_2 > \hat{\mu}_2$ than in the matching scheme $(0, \hat{\mu}_2)$. On the other hand, the corner player is indifferent between the matching equilibrium with $(0, \hat{\mu}_2)$ and the initial equilibrium because the player does not provide public good contributions whenever $\mu_2 \leq \hat{\mu}_2$. Therefore, the corner player is worse off if she is induced to provide positive flat contributions in the first case. By the same argument, the corner player is worse off in a bilateral matching scheme $(\hat{\mu}_1, \hat{\mu}_2)$ than in a unilateral matching scheme $(\hat{\mu}_1, 0)$. This indicates that the interior player's matching contributions cannot improve the corner player's welfare, so it is more likely to generate Pareto-improving outcomes in the second case than in the third one.

Given a unilateral matching scheme in the second case, we first characterize the existence condition of marginal Pareto-improving matching rates in the next proposition, and then discuss the case of non-marginal matching rates in Section 4.2. For notation convenience, we denote

$$MRS_i = \frac{\partial u_i/\partial x_i}{\partial u_i/\partial G}$$

Proposition 6. At a corner equilibrium, the corner player (player 1) provides matching contributions to the interior player (player 2) at a marginal matching rate. The interior player is always better off, and the corner player is better off if and only if

$$\left.\frac{dy_2}{d\mu_1}\right|_{\mu_1=0} > \bar{y}_2(MRS_1|_{\mu_1=0} - 1) \tag{20}$$

Proof: See Appendix E.

There are several implications:

(1) The matched player in such a unilateral matching scheme is always better off at a corner equilibrium. This contrasts with the interior case where the matched player is always worse off in a unilateral matching scheme. The intuition behind this difference is that, at an interior equilibrium, when one player provides matching contributions, she reduces her initial contributions, but in the corner case the corner player cannot do this because her initial contribution is zero. Since the corner player provides positive contributions through matching, the interior player is unambiguously better off.

(2) The interpretation of the above condition is that, to make the corner player better off, the interior player must be induced to increase her flat contribution and the increase must be sufficiently large to compensate the corner player for the forgone

private good consumption due to matching contributions. This occurs only if the private and public goods are substitutes for the interior player. To see this, we arrange player 2's budget constraint as

$$x_2 + \frac{G}{1+\mu_1} = w_2$$

Player 2's problem is equivalent to maximizing the individual utility subject to this budget constraint. Compared to the initial equilibrium, player 2 is faced with a lower public good price due to matching and she would decrease her private good consumption if the two goods are substitutes for her. Her individual budget constraint then implies that she would increase her public good contributions, i.e., $\partial y_2/\partial \mu_1|_{\mu_1=0} > 0$ holds. However, it is not sufficient to require the two goods are substitutes for the interior player because $MRS_1|_{\mu_1=0}$ may be far larger than one.

(3) The condition is favoured by strong substitution between the two goods for the interior player and by relative large income of the corner player. The stronger the substitution between the two goods, the larger $dy_2/d\mu_1|_{\mu_1=0}$. The larger the relative income of the corner player, the smaller $MRS_1|_{\mu_1=0}$.

(4) A unilateral matching scheme at the corner can possibly generate Pareto-improving outcomes in contrast with the interior case where only one player benefits. This is because the corner player initially does not contribute to the public good and hence cannot free ride on her flat contributions when providing matching contributions (see Buchholz and Liu (2018) for comprehensive discussion on Pareto improvements under unilateral matching).

4.2 Non-marginal matching schemes

Given a non-marginal matching rate, it can be derived from the budget constraint of the interior player that

$$\frac{dy_2}{d\mu_1} = -\frac{\partial e_2/\partial G * y_2 + \partial e_1/\partial \mu_1}{1 + \partial e_2/\partial G * (1+\mu_1)}$$

The utility change of the corner player at $\mu = \mu_1$ is

$$\frac{du_1(x_1, G)}{d\mu_1} = \frac{\partial u_1}{\partial G}\left\{(1-(MRS_1-1)\mu_1)\frac{dy_2}{d\mu_1} - (MRS_1-1)y_2\right\} := \frac{\partial u_1}{\partial G} * f(\mu_1)$$

where all terms in the curly bracket is denoted by $f(\mu_1)$.

Particularly, at a marginal matching rate, $du_1/d\mu_1|_{\mu_1=0} > 0$ and hence $f(\mu_1)|_{\mu_1=0} > 0$ immediately generates the condition in Proposition 6. We now consider, given a small matching rate μ_1, whether $f(\mu_1)$ is positive or negative to obtain the monotonicity of the utility function with respect to the matching rate.

Suppose $f(\mu_1)|_{\mu_1=0} > 0$ holds, i.e., the corner player is better off at a marginal matching rate. Then y_2 is increasing in μ_1, but $dy_2/d\mu_1$ must decrease since $y_2 < w_2$. As the matching rate increases, the private good consumption of the corner player decreases and hence the marginal rate of substitution decreases at an increasing rate because it is assumed to approach infinity when the private good consumption goes to zero. Thus, the change of MRS_1 dominates the change of all other terms, and it follows that $df(\mu_1)/d\mu_1 < 0$ when μ_1 is sufficiently large. This indicates that, when the matching rate increases, the utility of the corner player is increasing but at a declining rate. When $du_1(x_1, G)/d\mu_1 = 0$, player 1's utility achieves the highest level and then declines if the matching rate continues to increase.

If player 1's matching rate can make both better off, player 2 can also provide a positive matching rate μ_2, but μ_2 must satisfy certain conditions to make this bilateral matching scheme Pareto-improving. First, μ_2 must be larger than the minimum effective matching rate so that it is effective, i.e., player 1 would provide positive flat contributions. Otherwise, the bilateral matching scheme is essentially a unilateral matching scheme. Second, μ_2 must be sufficiently small (relative to the minimum effective matching rate) because player 2's effective matching rate would decrease player 1's utility. If the negative effect of μ_2 is smaller than the positive effect of μ_1, player 1 can still be better off compared to the initial equilibrium, and player 2 is always better off in such a bilateral matching scheme.

On the other hand, if $f(\mu_1)|_{\mu_1=0} < 0$ holds, i.e., the corner player is already worse off at a marginal matching rate. By continuity, $f(\mu_1) < 0$ holds at a small matching rate, so the corner player is further worse off when the matching rate increases.

4.3 A corner example

Consider again the CES function $u_i(x_i, G) = (a_i x_i^{\rho_i} + (1 - a_i)G^{\rho_i})^{1/\rho_i}$. The interiority zone is $(\underline{k}, \bar{k})$ where

$$\underline{k} = \frac{A_1^{1/(1-\rho_1)}}{1 + {A_2}^{1/(1-\rho_2)}}, \quad \bar{k} = \frac{A_2^{-1/(1-\rho_2)}}{1 + {A_1}^{1/(1-\rho_1)}}, \quad A_i = \frac{a_i}{1 - a_i}$$

Without loss of generality, consider the corner case where $k < \underline{k}$. Suppose the corner player provides matching contributions to the interior player. The flat contribution of the interior player is

$$y_2 = w_2 \Big(1 + (1 + \mu_1)^{-\rho_2/(1-\rho_2)} {A_2}^{1/(1-\rho_2)}\Big)^{-1}$$

Therefore,

$$\left.\frac{\partial y_2}{\partial \mu_1}\right|_{\mu_1=0} = w_2 \frac{\rho_2}{1 - \rho_2} {A_2}^{1/(1-\rho_2)} \Big(1 + {A_2}^{1/(1-\rho_2)}\Big)^{-2}$$

At the initial equilibrium,

$$\bar{G} = \bar{y}_2 = \frac{w_2}{1 + {A_2}^{1/(1-\rho_2)}}, \quad MRS_1|_{\mu_1=0} = \frac{a_1}{1 - a_1} \frac{1}{k^{1-\rho_1} \Big(1 + {A_2}^{1/(1-\rho_2)}\Big)^{1-\rho_1}}$$

The condition of achieving Pareto-improving outcomes in Proposition 6 reads as

$$k > k_{\min} = \left(A_1 \Big(1 + {A_2}^{1/(1-\rho_2)}\Big)^{\rho_1} \Big(1 + \frac{1}{1 - \rho_2} {A_2}^{1/(1-\rho_2)}\Big)^{-1}\right)^{1/(1-\rho_1)}$$

Since player 1 is at the corner, then $k_{\min} < \underline{k}$ must hold, which implies $\rho_2 > 0$. This condition is fulfilled when the private and the public good are substitutes for player

2. Therefore, if the two goods are substitutes for the interior player and the income of the corner player is larger than $k_{\min}$, then there exist unilateral Pareto-improving matching schemes.

For example, given $a_1 = a_2 = 0.5$ and $\rho_1 = \rho_2 = 0.5$, then $\underline{k} = 1/2$ and $k_{\min} = 2/9$. If $2/9 < k < 1/2$, both players can possibly be better off when the corner player provides matching contributions to the interior player. Figure 5 presents how player 1's utility changes with her matching rate in this CES example with $k = 0.4$. The horizontal and vertical axes denote respectively the matching rate μ_1 and player 1's utility level. The horizontal line represents the utility level at the initial equilibrium, and the hump-shaped curve denotes the utility level at a given matching rate. The utility at a marginal matching equilibrium is higher than in the initial equilibrium. When the matching rate increases, the utility increases and reaches its highest level and then starts to decline. When the matching rate reaches a critical level, the utility goes below the initial level and the player is no longer better off.

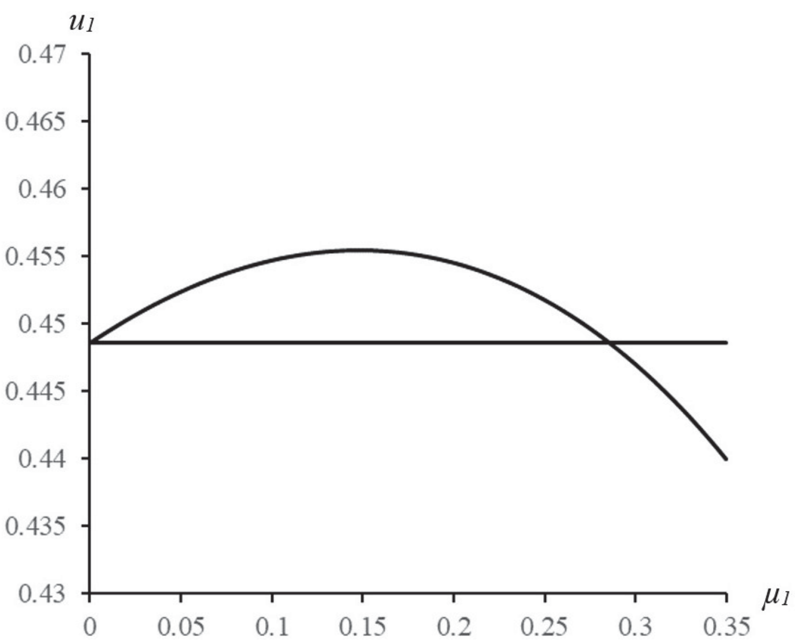

Figure 5: Pareto improvements at corner equilibria with $k = 0.4$

Figure 6 presents two cases with $k = 0.3$ and $k = 0.22$ respectively. Given $k = 0.3$ (and any $k \in (k_{\min}, \underline{k})$), the utility in Figure 6(a) displays the same pattern as in Figure 5, but the range of Pareto-improving matching rates is much smaller. The smaller k, the smaller the range of Pareto-improving matching rates. Figure 6(b) presents the critical case where $k = k_{\min}(\approx 0.22)$. The utility at a marginal matching equilibrium is lower than in the initial equilibrium. When the matching rate increases, the utility decreases monotonically and the player is never better off, so there is no Pareto-improving matching scheme.

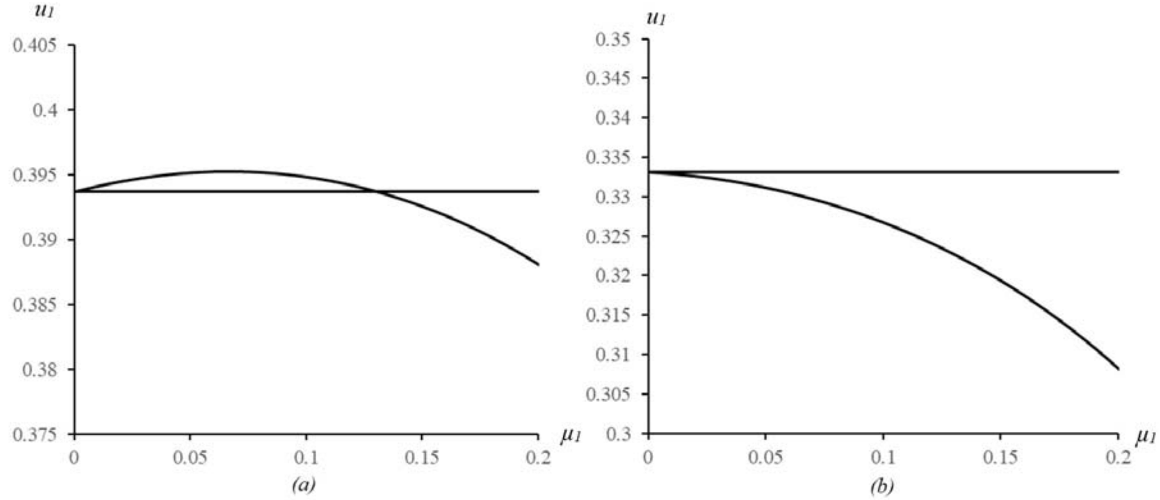

Figure 6: Pareto improvements at corner equilibria: (a) $k = 0.3$; (b) $k = 0.22$

If $\rho_2 = 0$, i.e., the CES function is degenerated to a Cobb-Douglas function, then $k_{\min} = \underline{k}$. This indicates that it is impossible to achieve Pareto-improviing outcomes at corner equilibria given Cobb-Douglas functions. Mathematically, at the corner, given Cobb-Douglas functions, $dy_2/d\mu_1|_{\mu_1=0} = 0$ holds and hence the condition $dy_2/d\mu_1|_{\mu_1=0} > \bar{y}_2(MRS_1|_{\mu_1=0} - 1)$ does not hold. Intuitively, given Cobb-Douglas functions, the private and the public good are neither substitutes nor complements, so the interior player does not change her private good consumption and hence not change her public good contribution although she is faced with a lower public good price when the corner player provides matching contributions. The corner player reduces her private good consumption and provides matching contributions, but does not induce the interior player to provide larger contributions, and thus becomes worse off.

If $\rho_2 < 0$, the private and the public good are complements, so the interior player would increase her private good consumption and hence decrease her public good contributions when she is faced with a lower public good price due to matching from the corner player. The corner player reduces her private good consumption and provides matching contributions, but induces the interior player to reduce her contributions, which is even worse than in the Cobb-Douglas case.

4.4 Extension in a multiple-player model

Suppose there are n players in the economy, and the first m players provide positive contributions and the last $n - m$ players are at the corner at the initial equilibrium. For comparison with the two-player corner case, we assume that all corner players provide matching contributions to all interior players at a uniform marginal matching rate. At the matching equilibrium, the aggregate budget constraint is

$$\sum_{j=1}^{m} e_j(G, \pi) + \frac{G}{\pi} = \sum_{j=1}^{m} w_j, \pi = 1 + (n - m)\mu$$

The following proposition characterizes the conditions for the existence of a Pareto-improving equilibrium in such a matching scheme.

Proposition 7. Suppose there are n players in the economy, and the first m players provide positive contributions and the last $n-m$ players are at the corner at the initial equilibrium. All corners player provide matching contributions to all interior players at a uniform marginal matching rate. Then,

(1) Interior players are better off if and only if

$$\left(1+\frac{\partial e_i}{\partial G}\right)\frac{\bar{G}-\sum_{j=1}^{m}\partial e_j/\partial\pi}{1+\sum_{j=1}^{m}\partial e_j/\partial G}+\frac{\partial e_i}{\partial\pi}>0,\quad i=1,2,...,m \tag{21}$$

(2) Corner players are better off if and only if

$$\sum_{j=1}^{m}\left.\frac{dy_j}{d\mu}\right|_{\mu=0}>(MRS_i|_{\mu=0}-(n-m))\sum_{j=1}^{m}\bar{y}_j,\quad i=m+1,...,n \tag{22}$$

Proof: See Appendix F.

Consider a two-person case with $n=2$ and $m=1$. The second condition immediately reduces to its counterpart in Section 4.1, and the first condition reduces to

$$1+\frac{\partial e_1}{\partial G}\bar{G}>0$$

This condition always holds, so the interior player is always better off. However, in a multiple-player case with $n\geq 3$, condition (21) suggests that an interior player may not be better off if $\partial e_i/\partial\pi$ is large, i.e., if the player has a large elasticity of substitution. The intuition behind this difference is that, when corner players provide matching contributions, interior players are induced to provide more flat contributions but meanwhile they free ride on each other if there are multiple interior players. This free-riding behavior is impossible when there is only one interior player in a two-player corner case.

To highlight the effect of group sizes in comparison with the two-player corner case, we assume that all players have the same preferences, and all m interior players have the same income level and all $n-m$ corner players also have the same income level. For interior players, condition (21) reduces to

$$\left(1+\frac{\partial e_i}{\partial G}\right)\frac{\bar{G}-m*\partial e_i/\partial\pi}{1+m*\partial e_i/\partial G}+\frac{\partial e_i}{\partial\pi}>0,\quad i=1,2,...,m$$

This condition always holds, so all interior players are always better off because they are symmetric. For corner players, condition (22) reduces to

$$\left.\frac{dy_i}{d\mu}\right|_{\mu=0}>(MRS_i|_{\mu=0}-(n-m))\bar{y}_i$$

As in the two-player corner case, this condition is also favoured by strong substitution between the two goods for interior players and by relative large incomes of corner players. Moreover, in this multiple-player case, the condition is also favoured by a large group of corner players. This is straightforward because the more the corner players, the more the matching contributions. Besides, when the number of corner players is large so that $MRS_i|_{\mu=0} < n - m$ holds, the right side of this condition is negative and hence $dy_i/d\mu|_{\mu=0}$ can be negative. This indicates that the two goods do not necessarily have to be substitutes and can be complements to achieve Pareto-improving outcomes at the corner. Therefore, when more corner players provide matching contributions, the requirement on the preferences for Pareto-improving outcomes is less strict and hence it is easier to satisfy participation constraints.

5 Conclusions

Matching mechanisms have been proposed to improve public good provision in voluntary contributions and ideally to reach the optimal equilibrium. Nevertheless, the optimal matching scheme may not be Pareto-improving and is also too ambitious in practice. This paper has investigated Pareto-improving outcomes of small matching schemes. Income distributions play an important role for achieving Pareto-improving outcomes. At an interior equilibrium, there always exist small Pareto-improving matching schemes. This indicates that players at an interior equilibrium can always implement small matching schemes to generate Pareto-improving outcomes regardless of their preferences. This finding is useful for cooperation among heterogeneous players in the context without global information of preferences or at the international level without central governments. However, it is more common to encounter corner equilibria than interior equilibria in a pure public good economy. At a corner equilibrium, a matching scheme works in different ways and have different welfare implications from at an interior equilibrium, and the existence of Pareto-improving matching schemes is not universal but is possible. Particularly, if the corner player unilaterally provides matching contributions to the interior player, the matched player is always better off, which is in sharp contrast with the interior case where the matched player is always worse off. The corner player is also possibly better off under a certain condition, indicating that unilateral action through matching can generate Pareto-improving outcomes.

6 Appendix

A. Optimal matching scheme in the Cobb-Douglas function.
The budget constraints of two players are respectively

$$x_1 + y_1 + \mu_1 y_2 = w_1, \quad x_2 + y_2 + \mu_2 y_1 = w_2$$

At the sub-game perfect equilibrium, the matching rates satisfy $\mu_1\mu_2 = 1$. Multiply the second equation by μ_1 and rewrite the two budget constraints as

$$y_1 + \mu_1 y_2 = w_1 - x_1, y_1 + \mu_1 y_2 = \mu_1(w_2 - x_2)$$

The above two conditions imply

$$w_1 - x_1 = \mu_1(w_2 - x_2)$$

Given $u_i(x_i, G) = x_i^{\alpha_i} G$, at an interior matching equilibrium,

$$x_1 = \frac{\alpha_1}{1+\mu_2} \frac{W}{\alpha_1/(1+\mu_2) + \alpha_2/(1+\mu_1) + 1}, x_2 = \frac{\alpha_2}{1+\mu_1} \frac{W}{\alpha_1/(1+\mu_2) + \alpha_2/(1+\mu_1) + 1}$$

Combining the above three equations yields the optimal matching scheme as

$$\mu_1 = \frac{k(1+\alpha_2)}{1+\alpha_1}, \ \mu_2 = \frac{1+\alpha_1}{k(1+\alpha_2)}$$

Given $\alpha_1 = \alpha_2 = 1$, the optimal matching scheme reduces to $\mu_1 = k$ and $\mu_2 = 1/k$.

B. Proof of Proposition 2.

At an interior matching equilibrium, the public good provision is characterized as

$$G + e_1(G, 1+\mu_2) + e_2(G, 1+\mu_1) = W$$

For $i, j = 1, 2, i \neq j$, the utility change with respect to the matching rate is

$$\frac{du_i(x_i, G)}{d\mu_i} = \frac{\partial u_i}{\partial x_i} \frac{\partial e_i}{\partial G} \frac{dG}{d\mu_i} + \frac{\partial u_i}{\partial G} \frac{dG}{d\mu_i}$$

$$\frac{du_j(x_j, G)}{d\mu_i} = \frac{\partial u_j}{\partial x_j} \left(\frac{\partial e_j}{\partial G} \frac{dG}{d\mu_i} + \frac{\partial e_j}{\partial \mu_i} \right) + \frac{\partial u_j}{\partial G} \frac{dG}{d\mu_i}$$

At an interior matching equilibrium,

$$\frac{\partial u_i / \partial x_i}{\partial u_i / \partial G} = 1 + \mu_j$$

The aggregate budget constraint implies

$$\frac{dG}{d\mu_i} = -\frac{\partial e_j / \partial \mu_i}{1 + \partial e_i / \partial G + \partial e_j / \partial G}$$

As $\partial e_j / \partial \mu_i < 0$, the total public good provision increases in the matching rate. Combining the above equations yields

$$\frac{du_i(x_i, G)}{d\mu_i} = -\frac{\partial u_i}{\partial G} \frac{\partial e_j}{\partial \mu_i} \frac{(1+\mu_j)\partial e_i / \partial G + 1}{1 + \partial e_i / \partial G + \partial e_j / \partial G}$$

$$\frac{du_j(x_j, G)}{d\mu_i} = \frac{\partial u_j}{\partial G} \frac{\partial e_j}{\partial \mu_i} \frac{(1+\mu_j)\partial e_j / \partial G + \mu_j}{1 + \partial e_i / \partial G + \partial e_j / \partial G}$$

As $\partial u_i / \partial G > 0$ and $\partial e_j / \partial \mu_i < 0$, it follows that $du_i / d\mu_i > 0$ and $du_j / d\mu_i < 0$, i.e., the utility increases in the own matching rate and decreases in the opponent's matching rate. Thus, players are better off when providing matching contributions while worse off when receiving matching contributions.

The change of the private good consumption with respect to the own matching rate is

$$\frac{\partial e_i}{\partial \mu_i} = \frac{\partial e_i}{\partial G} \frac{\partial G}{\partial \mu_i} > 0$$

Thus the private good consumption increases in the own matching rate and hence, subject to the individual budget constraint, the individual public good provision decreases in the own matching rate. As the total public good provision increases in the matching rate, the individual public good provision increases in the opponent's matching rate and hence, subject to the individual budget constraint, the private good consumption decreases in the opponent's matching rate.

The flat contribution is solved as

$$y_i = \frac{w_i - x_i - \mu_i(w_j - x_j)}{1 - \mu_i\mu_j}$$

The change of the flat contribution with respect to the matching rate is

$$\frac{dy_i}{d\mu_i} = \frac{-(w_j - x_j - \mu_i \partial e_j/\partial \mu_i)(1 - \mu_i\mu_j) + \mu_j(w_i - x_i - \mu_i(w_j - x_j))}{(1 - \mu_i\mu_j)^2}$$

$$\frac{dy_i}{d\mu_j} = \frac{(-\partial e_i/\partial \mu_j + \mu_i \partial e_j/\partial \mu_j)(1 - \mu_i\mu_j) + \mu_i(w_i - x_i - \mu_i(w_j - x_j))}{(1 - \mu_i\mu_j)^2}$$

At a marginal matching scheme,

$$\left.\frac{dy_i}{d\mu_i}\right|_{\mu_i=0,\mu_j=0} = -(w_j - x_j) < 0, \quad \left.\frac{dy_i}{d\mu_j}\right|_{\mu_i=0,\mu_j=0} = -\frac{\partial e_i}{\partial \mu_j} > 0$$

By continuity, the two conditions also hold at small matching schemes. This indicates that the flat contribution decreases in the own matching rate and increases in the opponent's matching rate. Therefore, the matching contribution $\mu_i y_j$ increases in the own matching rate and decreases in the opponent's matching rate. QED

C. Proof of Proposition 4

This section proves Proposition 5 in a multiple-player case, and Proposition 3 follows as its special case. To proceed, we first prove Lemma 1 and Lemma 2 in a multiple-player case respectively.

Proof of Lemma 1

Let $x_i = h_i(G)$ denote player i's indifference curve passing through $(x_i(\vec{P}, W), G(\vec{P}, W))$ where $\vec{P} = (p_1, p_2, ..., p_n)$. Define

$$W(\tilde{G}) = \sum_{i=1}^{n} h_i(\tilde{G}) + \tilde{G}$$

for any $\tilde{G} \geq G(\vec{P}_0, W)$ where $\vec{P}_0 = (1, 1, ..., 1)$ is the individual public good prices at the initial equilibrium. This function is decreasing in $\tilde{G}$ at $\tilde{G} = G(\vec{P}_0, W)$ since

$$\left.\frac{dW(\tilde{G})}{d\tilde{G}}\right|_{\tilde{G}=G(\vec{P}_0,W)} = \sum_{i=1}^{n} \frac{dh_i(G(\vec{P}_0, W))}{dG} + 1 = n * (-1) + 1 = -(n-1) < 0$$

where the second equality holds because $dh_i(G(\vec{P}_0, W))/dG = -1$ at the initial equilibrium given the relative price of the public good is one. Given the monotonicity, it follows that $W(\tilde{G}) < W$ for $\tilde{G} > G(\vec{P}_0, W)$.

The allocation $(h_1(\tilde{G}), h_2(\tilde{G}), ..., h_n(\tilde{G}), \tilde{G})$ is the matching equilibrium given the public good prices $p_i(\tilde{G}) = -dh_i(\tilde{G})/d\tilde{G}$ and the total income $W(\tilde{G})$. Let $\vec{P}(\tilde{G}) = (p_1(\tilde{G}), p_2(\tilde{G}), ..., p_n(\tilde{G}))$. At the matching equilibrium given $\vec{P}(\tilde{G})$ and $W(\tilde{G})$, all players attain the same utility levels as in the initial equilibrium. If the total income is increased from $W(\tilde{G})$ to W while the public good prices are maintained at $\vec{P}(\tilde{G})$, all players move outwards along their income expansion paths $e_i(G, p_i(\tilde{G}))$ resulting in higher utility levels. Therefore, for all $\tilde{G} > G(\vec{P}_0, W)$ the allocation given $\vec{P}(\tilde{G})$ and W strictly Pareto dominates the initial equilibrium. QED

Proof of Lemma 2.

Given $\vec{P} = (p_1, p_2, ..., p_n)$, we construct a matching scheme by assuming that the flat contributions of player j are matched by each other player $i \neq j$ at the matching rate

$$\mu_{ij}(p_j) = \frac{1/p_j - 1}{n - 1}$$

This indicates that matching contributions flowing to one player are equally distributed among all other players. Furthermore, assume $\mu_{ii}(p_i) = 1$ for all $i = 1, 2, .., n$.

If the matching equilibrium given $\vec{P}$ and W is realized by the above matching scheme, the following system must have a positive solution in flat contributions (see Buchholz, Cornes and Rubbelke 2011).

$$y_i + \sum_{j=1}^{n} \mu_{ij}(p_j) * y_j(\vec{P}, \vec{W}) = w_i - x_i(\vec{P}, W)$$

where $\vec{W} = (w_1, w_2, ..., w_n)$ and $W = \sum_{j=1}^{n} w_j$. Note that the private good consumption and the total public good provision depend on the total income but not on the income distribution, while the flat contributions depend on the total income and also on the income distribution. This system of equations reads in a matrix form as

$$\begin{pmatrix} 1 & \mu_{12}(p_2) & . & . & . & \mu_{1n}(p_n) \\ \mu_{21}(p_1) & 1 & . & . & . & \mu_{2n}(p_n) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{n1}(p_1) & \mu_{n2}(p_2) & . & . & . & 1 \end{pmatrix} \begin{pmatrix} y_1(\vec{P}, \vec{W}) \\ y_2(\vec{P}, \vec{W}) \\ . \\ . \\ . \\ y_n(\vec{P}, \vec{W}) \end{pmatrix} = \begin{pmatrix} w_1 - x_1(\vec{P}, W) \\ w_2 - x_2(\vec{P}, W) \\ . \\ . \\ . \\ w_n - x_n(\vec{P}, W) \end{pmatrix} \tag{23}$$

If $\vec{P}$ is close to $\vec{P}_0$ and thus $\mu_{ij}(i \neq j)$ is close to zero, the determinant of the first matrix is close to one so that the system has a unique solution. Moreover, for any $\vec{P}$ in a small neighbourhood of $\vec{P}_0$, $y_i(\vec{P}, \vec{W})$ must be positive because it is close to $y_i(\vec{P}_0, \vec{W})$ which is positive at an initial interior equilibrium. QED

Proof of Proposition 5.

Consider the matching equilibrium given $\vec{P}(\tilde{G})$ and W as constructed in Lemma 1 which strictly Pareto dominates the initial equilibrium for $\tilde{G} > G(\vec{P}_0, W)$. If the public good provision $\tilde{G}$ goes to $G(\vec{P}_0, W)$, the public good price $p_i(\tilde{G})$ goes to one. It follows from Lemma 2 that, for $\tilde{G}$ which is close to $G(\vec{P}_0, W)$, there exists a matching scheme realizing the matching equilibrium. QED

D. Proof of Proposition 4.

Denote $\mu_1 = \mu$ and $\mu_2 = \beta\mu(\beta \geq 0)$. At an interior matching equilibrium, the public good provision is characterized as

$$G + e_1(G, 1 + \beta\mu) + e_2(G, 1 + \mu) = W$$

The utility change with respect to μ is

$$\frac{du_1(x_1,G)}{d\mu} = \frac{\partial u_1}{\partial x_1}\left(\frac{\partial e_1}{\partial G}\frac{dG}{d\mu} + \beta\frac{\partial e_1}{\partial \pi_1}\right) + \frac{\partial u_1}{\partial G}\frac{dG}{d\mu}$$

$$\frac{du_2(x_2,G)}{d\mu} = \frac{\partial u_2}{\partial x_2}\left(\frac{\partial e_2}{\partial G}\frac{dG}{d\mu} + \frac{\partial e_2}{\partial \pi_2}\right) + \frac{\partial u_2}{\partial G}\frac{dG}{d\mu}$$

At an interior matching equilibrium,

$$\frac{\partial u_1/\partial x_1}{\partial u_1/\partial G} = 1+\beta\mu, \quad \frac{\partial u_2/\partial x_2}{\partial u_2/\partial G} = 1+\mu$$

The aggregate budget constraint implies

$$\frac{dG}{d\mu} = -\frac{\beta\partial e_1/\partial\pi_1 + \partial e_2/\partial\pi_2}{1+\partial e_1/\partial G + \partial e_2/\partial G}$$

Combining the above equations and evaluating at the margin yields

$$\frac{du_1(x_1,G)}{d\mu} > 0 \Rightarrow \beta < \frac{1+\partial e_1/\partial G}{\partial e_2/\partial G}\frac{\partial e_2/\partial\pi_2}{\partial e_1/\partial\pi_1}$$

$$\frac{du_2(x_2,G)}{d\mu} > 0 \Rightarrow \beta > \frac{\partial e_1/\partial G}{1+\partial e_2/\partial G}\frac{\partial e_2/\partial\pi_2}{\partial e_1/\partial\pi_1}$$

E. Proof of Proposition 6.

For the corner player, the utility change with respect to a marginal matching rate is

$$\frac{du_1(x_1,G)}{d\mu_1} = \frac{\partial u_1}{\partial x_1}\frac{\partial x_1}{\partial \mu_1} + \frac{\partial u_1}{\partial G}\frac{dG}{d\mu_1}$$

At the matching equilibrium, $G = (1+\mu_1)y_2$ and $x_1 = w_1 - \mu_1 y_2$, so

$$\frac{dG}{d\mu_1} = y_2 + (1+\mu_1)\frac{dy_2}{d\mu_1}, \quad \frac{dx_1}{d\mu_1} = -\left(y_2 + \mu_1\frac{dy_2}{d\mu_1}\right)$$

Combining the above equations and evaluating at the margin yields

$$\frac{du_1(x_1,G)}{d\mu_1} = \frac{\partial u_1}{\partial G}\left(\frac{dy_2}{d\mu_1} - \bar{y}_2(MRS_1|_{\mu_1=0} - 1)\right)$$

Thus, at the margin,

$$\frac{du_1(x_1,G)}{d\mu_1} > 0 \Rightarrow \left.\frac{dy_2}{d\mu_1}\right|_{\mu_1=0} > \bar{y}_2(MRS_1|_{\mu_1=0} - 1)$$

For the interior player, the utility change with respect to a marginal matching rate is

$$\begin{aligned}\frac{du_2(x_2,G)}{d\mu_1} &= -\frac{\partial u_2}{\partial x_2}\frac{dy_2}{d\mu_1} + \frac{\partial u_2}{\partial G}\left(y_2 + (1+\mu_1)\frac{dy_2}{d\mu_1}\right)\\ &= \left(\frac{\partial u_2}{\partial G} - \frac{\partial u_2}{\partial x_2}\right)\frac{dy_2}{d\mu_1} + \bar{y}_2\frac{\partial u_2}{\partial G}\\ &= \bar{y}_2\frac{\partial u_2}{\partial G} > 0\end{aligned}$$

F. Proof of Proposition 7.
At the matching equilibrium, the total public good provision is characterized by the aggregate budget constraint as

$$\sum_{j=1}^{m} e_j(G, 1+(n-m)\mu) + \frac{G}{1+(n-m)\mu} = \sum_{j=1}^{m} w_j$$

This implies that

$$\frac{dG}{d\mu} = (n-m)\frac{\bar{G} - \sum_{j=1}^{m} \partial e_j/\partial\pi}{1+\sum_{j=1}^{m} \partial e_j/\partial G}$$

(1) Interior players
Interior players ($i = 1, 2, ..., m$) are on their expansion paths so that

$$\frac{\partial u_i/\partial x_i}{\partial u_i/\partial G} = 1+(n-m)\mu$$

The utility change with respect to μ at the margin is

$$\begin{aligned}\frac{du_i(x_i,G)}{d\mu} &= \frac{\partial u_i}{\partial x_i}\left(\frac{\partial e_i}{\partial G}\frac{dG}{d\mu} + (n-m)\frac{\partial e_i}{\partial \pi}\right) + \frac{\partial u_i}{\partial G}\frac{dG}{d\mu} \\ &= \frac{\partial u_i}{\partial G}\left(\left(\frac{\partial e_i}{\partial G}+1\right)\frac{dG}{d\mu} + (n-m)\frac{\partial e_i}{\partial \pi}\right) \\ &= (n-m)\frac{\partial u_i}{\partial G}\left(\left(\frac{\partial e_i}{\partial G}+1\right)\frac{\bar{G} - \sum_{j=1}^{m} \partial e_j/\partial\pi}{1+\sum_{j=1}^{m} \partial e_j/\partial G} + \frac{\partial e_i}{\partial \pi}\right)\end{aligned}$$

The condition for interior players follows from $\dfrac{du_i(x_i,G)}{d\mu} > 0$.

(2) Corner players
The budget constraint of corner players ($i = m+1, ..., n$) is

$$x_i + \mu\sum_{j=1}^{m} y_j = w_i$$

This implies that

$$\frac{dx_i}{d\mu} = -\left(\sum_{j=1}^{m} y_j + \mu\sum_{j=1}^{m}\frac{dy_j}{d\mu}\right)$$

The total public good is

$$G = (1+(n-m)\mu)\sum_{j=1}^{m} y_j$$

This implies

$$\frac{dG}{d\mu} = (n-m)\sum_{j=1}^{m} y_j + (1+(n-m)\mu)\sum_{j=1}^{m}\frac{dy_j}{d\mu}$$

The utility change with respect to μ at the margin is

$$\frac{du_i(x_i,G)}{d\mu} = \frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial \mu} + \frac{\partial u}{\partial G}\frac{dG}{d\mu} = \frac{\partial u_i}{\partial G}\left(MRS_i|_{\mu=0} * \frac{dx_i}{d\mu} + \frac{\partial G}{\partial \mu}\right)$$

$$= \frac{\partial u_i}{\partial G}\left(-MRS_i|_{\mu=0}\left(\sum_{j=1}^{m} y_j + \mu \sum_{j=1}^{m}\frac{dy_j}{d\mu}\right) + (n-m)\sum_{j=1}^{m} y_j + (1+(n-m)\mu)\sum_{j=1}^{m}\frac{dy_j}{d\mu}\right)$$

$$= \frac{\partial u_i}{\partial G}\left(-MRS_i|_{\mu=0}\sum_{j=1}^{m} y_j + (n-m)\sum_{j=1}^{m} y_j + \sum_{j=1}^{m}\frac{dy_j}{d\mu}\right)$$

The condition for corner players follows from $\frac{du_i(x_i,G)}{d\mu} > 0$.

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