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# Role of Expectations in a Liquidity Trap

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## Abstract

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**Keywords**

Expectations, Liquidity Trap, Monetary Policy

**JEL Classification**

E31, E52, E58, E61

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# Role of Expectations in a Liquidity Trap*

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**Abstract**

A number of previous studies suggest that inflation expectations are important in considering the effectiveness of monetary policy in a liquidity trap. However, the role of inflation expectations can be very different, depending on the type of monetary policy that a central bank implements. This paper reveals how a private agent forms inflation expectation affects the effectiveness of monetary policy under the optimal commitment policy, the Taylor rule, and a simple rule with price-level targeting. We examine two expectation formations: (i) different degrees of anchoring, and (ii) different degrees of forward-lookingness. We show that how to form inflation expectations is less relevant when a central bank implements the optimal commitment policy, while it is critical when the central bank adopts the Taylor rule or a simple rule with price-level targeting. Even for the Japanese economy, the effects of monetary policy on economic dynamics significantly change according to expectation formations under rules other than the optimal commitment policy.

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# 1 Introduction

A number of previous studies suggest that inflation expectations are important when considering the effectiveness of monetary policy in a liquidity trap. In particular, Eggertsson and Woodford (2003b) and Jung, Teranishi, and Watanabe (2005) analyze optimal monetary policy in situations in which a central bank is limited in terms of reducing the policy interest rate, and conclude that an optimal commitment policy is the most effective.[1] Such a commitment policy can reduce the real interest rate and stimulate the economy by controlling inflation expectations. Their conclusions, however, fully depend on two important assumptions, i.e., rational expectation formation and optimal commitment monetary policy. The role of inflation expectations can be very different depending on the type of monetary policy a central bank implements. Moreover, the effectiveness of monetary policy can also vary widely depending on how inflation expectations are formed.

Particularly in Japan, as Kuroda (2017) noted, inflation expectations are important in monetary policy, since inflation expectations have been trapped at a low level by prolonged deflation. To escape from the liquidity trap and mitigate this situation, the Bank of Japan introduced an inflation target policy under the Quantitative and Qualitative Easing (QQE) Policy in April 2013. In terms of anchoring inflation expectations, Kuroda (2017) states: "In practice, QQE has produced its intended effects. Inflation expectations climbed notably after the introduction of QQE. This demonstrates that a strong determination by the central bank pushes up people's forward-looking inflation expectations." Thus, the Bank of Japan was partially successful in anchoring inflation expectations.[2] Moreover, whether inflation expectations are anchored to a target level or not remains a central issue in the implementation of monetary policy. In terms of inflation expectation formation, Kuroda (2017) emphasizes: "The rate of change in the consumer price index (CPI) recently has been around 0% and there is still a long way to go until the price stability target of 2% is achieved. [....] Analysis by the Bank of Japan

[1]See also Adam and Billi (2006, 2007) and Nakov (2008).

[2]For example, Kamada, Nakajima, and Nishiguchi (2015) support this point.

suggests that, as a result of prolonged deflation, the backward-looking, or adaptive, component in the formation of inflation expectations continues to be much stronger in Japan than in Europe and the United States." Thus, how inflation expectations are formed is also an issue crucial to monetary policy in Japan.[3] This paper introduces these issues into a model and evaluate the effectiveness of monetary policy.

In this paper, we relax the assumption of a purely forward-looking rational expectation and show the role of expectation formation in a liquidity trap under a standard New Keynesian model. We assume two states in expectation formation. First, we change the degree of how much expectations are anchored. Some papers argue that inflation expectations are well anchored under an inflation targeting policy. Beechey, Johannsen, and Levin (2011) show that long-run inflation expectations in the euro area are well anchored, while in the United States, the expected inflation rate is not firmly anchored. Such a difference derives from the ECBs communication strategy in which a goal of price stability is specified by setting a target inflation rate. Using survey data of the inflation expectations for 36 developed and developing countries, Davis (2014) argues that inflation expectations tend to be anchored for periods after the introduction of an inflation targeting policy. We describe this situation simply by fixing a fraction of expected inflation at a target level. Second, we change the degree of how much expectation formation is forward-looking, and describe it by assuming that expectations depend on a weighted average between rational expectation and a current inflation rate. Numerical simulations reveal how these different expectation formations change the effects of monetary policy in a liquidity trap.

Moreover, we also relax the assumption of optimal commitment policy. In addition to optimal commitment policy, we introduce two realistic monetary policy rules; i.e., the Taylor rule, and a simple rule with price-level targeting.[4] Several papers (e.g., Smets and

[3]For example, Bank of Japan (2018) supports adaptive learning for inflation expectation formation in Japan.

[4]Many previous studies, such as Benhabib, Schmitt-Grohe, and Uribe (2001), suggest that the Taylor rule can produce multiple equilibria in a liquidity trap. Moreover, whether or not to anchor inflation expectations should be deeply related to the existence of multiple equilibria in a liquidity trap.

Wouters, 2003, 2007), assume a Taylor rule to fit a theoretical model to data. Regarding a simple rule with price-level targeting, Eggertsson and Woodford (2003b) and Eggertsson and Woodford (2003a) reveal that this rule is very effective in a liquidity trap due to history-dependent monetary easing. They argue that a price-level targeting policy is a simple and realistic rule for replicating a feature of optimal monetary policy.

We obtain significantly different outcomes according to monetary policy rules in numerical simulations. Under an optimal commitment policy, the effect of monetary policy does not change markedly with different expectation formations for an inflation rate. One reason why is that optimal monetary policy includes a feature of history-dependent easing, and so there can be considerable management of expectations even though there is little leeway for this. Thus, the role of expectation formation for an inflation rate is not so important for optimal monetary policy.

On the other hand, under the Taylor rule, reductions in an inflation rate and the output gap for some periods at first become sufficiently smaller, as the degree to which inflation expectations are anchored becomes stronger. This also holds true as the degree of forward-lookingness in expectation formation become stronger. The Taylor rule does not involve a history-dependency, and does not work on expectations in a forward-looking model. Thus, anchored expectations and forward-looking expectations compensate for a drawback of the Taylor rule. In a forward-looking economy, there is an innate mechanism to realign the economy to a steady state when a negative shock disappears. Thus, when the degree of forward-lookingness decreases, this mechanism weakens and the economic slowdown can be severe. The role of expectation formation is non trivial under the Taylor rule.

A simple rule with price-level targeting can mitigate large drops in inflation rate and output gap. As explained in previous papers analyzing monetary policy in a liquidity trap, this is because targeting a price level gives a policy maker control of expectation formation by promising future monetary easing, as in the optimal commitment policy. This remains effective, even though there is limited room for managing expectations. Moreover, unlike the Taylor rule, the inflation rate and output gap become less sensitive to expectation formations. Thus, the role of expectation formation is not so serious a problem for a simple rule with price-level targeting.

Moreover, we show that whether or not the economy is in a liquidity trap is critical when considering the different roles that inflation expectations play in different monetary regimes by running simulations without the zero lower bound on the nominal interest rate. We observe a larger difference for monetary policy effectiveness under different expectation formations in a liquidity trap. However, the output gap and inflation rate drop less when we do not assume a liquidity trap.

We estimated to what degree expectations are anchored to a targeting level, and to what degree expectation formation is forward-looking in the Japanese economy. In Japan, expectations have been partially anchored since the Bank of Japan introduced a price stability target of 2% in January 2013. Moreover, expectation formation is not perfectly forward-looking, but depends on the present inflation rate, which implies adaptive expectation. However, even in the Japanese case, expectation formation is not a topic discussed in monetary policy, regardless of whether or not expectations are anchored, or if the Bank of Japan can implement an optimal commitment policy. Optimal monetary policy with strong history-dependent easing can control expectation formation in the Japanese economy.

Under the Taylor rule, compared to a perfectly anchored case, we observe drops in the inflation rate and output gap for other expectation formations. However, these drops are largely mitigated when inflation expectations are partially anchored. Thus, even under a weak anchoring of inflation expectations, the Taylor rule can mitigate serious deflation. The drops are larger in the case in which inflation expectations are based on the current inflation rate compared to where inflation expectations are partially anchored. Moreover, by committing to a simple history-dependent rule like a price-level targeting rule, the Bank of Japan can further mitigate the effect of weak anchoring for expectations and a lack of forward-lookingness in expectation formation.[5]

[5]We introduce the same expectation formations for the output gap as for an inflation rate and implement a simulation analysis in a longer working paper version of the Hasui, Nakazono, and Teranishi (2018) paper.

Our paper is related to two strands in the literature. First, it is related to studies of the optimal monetary policy in a liquidity trap. Eggertsson and Woodford (2003b) and Jung, Teranishi, and Watanabe (2005) analyze the optimal commitment policy in a liquidity trap, and show that a central bank needs to continue a zero-interest-rate policy even after the natural rate turns positive. Adam and Billi (2006, 2007) and Nakov (2008) analyze the optimal commitment policy and discretionary policy in a liquidity trap under a stochastic shock.[6] Our paper relaxes the assumption of forward-looking rational expectation in these papers and analyzes the role of expectation formation in a liquidity trap.

Second, our paper is related to the formation of expected inflation with empirical assessment and a forward-guidance puzzle. Recently, the sluggish response of an expected inflation rate was described with the New Keynesian model. Coibion and Gorodnichenko (2012, 2015) show the state of a sluggish response in an expected inflation rate using the U.S. survey data. Pfajfar and Žakelj (2018) introduce the expectation formation based on a laboratory experiment into the New Keynesian model, and analyze the design of monetary policy when the expectations are not perfectly rational.[7] Wiederholt (2014) and Andrade, Gaballo, Mengus, and Mojon (2015) build a model that includes the sluggish response of an expected inflation rate by extending the New Keynesian model with a heterogeneous belief. Some papers focus on expectation formation to solve the forward-guidance puzzle identified by Del Negro, Giannoni, and Patterson (2012). They show that forward guidance is unrealistically powerful in the New Keynesian model. For example, Andrade, Gaballo, Mengus, and Mojon (2015) show that pessimistic expecta-

[6]Many papers analyze monetary policy in a liquidity trap. Werning (2011) analyzes fiscal policy and monetary policy in a liquidity trap in a continuous New Keynesian model. Billi (2011) analyzes the optimal long-run U.S. inflation rate in a stochastic framework with a robust control technique. Fujiwara, Nakajima, Sudo, and Teranishi (2013) extend the model to the open economy, and show an optimal zero-interest-rate policy in a global liquidity trap.

[7]The approach of introducing expectations from the laboratory is also shown in Marimon and Sunder (1994) and Bernasconi and Kirchkamp (2000) for analyzing the effects of monetary policy. Recently, Adam (2007) shows the persistent responses of output and inflation rate by introducing the expectations from the experiment in a laboratory to a simple cash-in-advance model.

tions weaken the effects of forward guidance. Our paper is related to these in focusing on the effects of expectation formation on monetary policy and economic dynamics.

The remainder of the paper is structured as follows. In Section 2, we define expectation formation and show the empirical evidence. We set up the model in Section 3. Section 4 shows numerical results for expectation formation of an inflation rate. In Section 5, we calibrate a model for the Japanese economy, and show the effects of expectation formation on monetary policy in Japan. Section 6 concludes the paper.

# 2 Expectation Formation and Empirical Evidences

Before examining theoretical model and numerical simulations, this section discusses how we relax an assumption of purely forward-looking rational expectation by a private agent, and provide some evidence supporting the variants of expectation formation. Figure 1 and Figure 2 show longer-term inflation forecasts (5-10 years ahead forecasts of Consensus Forecasts (CF) from Consensus Economics) and short-term inflation forecasts (1-year ahead forecasts of CF from Consensus Economics), respectively. These figures suggest that inflation expectations are partially anchored by the 2% target level set by the Bank of Japan in January 2013, or partially depend on the current inflation rate.

We assume two cases for expectation formations for the inflation rate as follows.

$$\pi^e_{t+1} := \begin{cases} \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\bar{\pi}, & \text{(a)} \\ \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\pi_t, & \text{(b)} \end{cases} \tag{1}$$

where $\pi^e_{t+1}$ is the expected inflation rate, $\mathrm{E}_t\pi_{t+1}$ is a rational inflation expectation defined in the next section, $\gamma_\pi$ is a parameter satisfying $0 \leq \gamma_\pi \leq 1$, and $\bar{\pi}$ is anchored inflation at a targeting level.

In case (a), we change the degree to which expectations are anchored to a targeting level set by a central bank in an inflation targeting policy. The expected inflation rate is given by a weighted average between a rational inflation expectation and an anchored inflation at a constant number. When $\gamma_\pi$ is one, expectation formation follows a rational expectation, as in a standard New Keynesian model. On the other hand, when $\gamma_\pi$ is zero, expectations for a future inflation rate are strongly anchored at constant $\bar{\pi}$.

In case (b), we change the degree to which the expectation formation is forward-looking and describe it by assuming that expectations depend on a weighted average between rational expectation and a current inflation rate. In this case, economic agents reflect current inflation to form inflation expectations. As $\gamma_\pi$ decreases, the degree of forward-lookingness in expectation formation decreases. Note that the inflation expectation $\pi^e_{t+1}$ is reduced to become perfectly forward-looking, i.e. $\pi^e_{t+1} = \mathrm{E}_t\pi_{t+1}$ when $\gamma_\pi = 1$.

We estimate the degrees and examine whether these assumptions are empirically supported. In order to estimate $\gamma_\pi$, we arrange equation (a) into the following equation by using the definition of the forecast error,[8]

$$\pi_{t,t+k} - \pi^e_{t,t+k} = \beta(\pi^e_{t,t+k} - \bar{\pi}) + \varepsilon_{t,t+k}, \tag{2}$$

where

$$\beta = \frac{1-\gamma_\pi}{\gamma_\pi},$$

and

$$\varepsilon_{t,t+k} = \pi_{t,t+k} - \mathrm{E}_t\pi_{t,t+k}.$$

$\pi^e_{t,t+k}$ is defined as an inflation expectation over $k$-periods ahead, and is formed at time $t$. $\varepsilon_{t,t+k}$ denotes the forecast error and should not be predictable from information in time $t$ under a rational expectation. As a result, we can test whether $\beta = 0$. When $\gamma_\pi = 1$, expectation formation follows a rational expectation as in a standard New Keynesian model. When $\gamma_\pi < 1$, agents put some weight on $\bar{\pi}$. Following the introduction of "Price Stability Target" of 2% by the Bank of Japan in January 2013, we set $\bar{\pi} = 2\%$ after the first quarter of 2013. $\bar{\pi}$ is set to be $= 0\%$ before the Bank of Japan announced that 1% inflation was desirable in mid-February 2012, and subsequently, $\bar{\pi}$ is set to be $= 1\%$ before the introduction of the "Price Stability Target" of 2% in equation (a). By

[8] This estimation strategy follows Ichiue and Yuyama (2009).

rewriting equation (2), we estimate the following equation:

$$\begin{aligned}\pi_{t,t+k} - \pi^e_{t,t+k} =& \beta^A(\pi^e_{t,t+k} - 0\%) \times D_1 + \beta^B(\pi^e_{t,t+k} - 1\%) \times D_2 \\ &+ \beta^C(\pi^e_{t,t+k} - 2\%) \times (1 - D_1 - D_2) + \varepsilon_{t,t+k},\end{aligned} \tag{3}$$

where dummy variable $D_1$ takes one value from 2001Q2 to 2011Q4 and dummy variable $D_2$ takes one value from 2012Q1 to 2012Q4. All other values of $D_1$ and $D_2$ are equal to zero. When estimating equation (3), we set $k$ to be four and the inflation expectation, $\pi^e_{t,t+4}$,[9] is quarterly forecast on inflation rates over four-quarter ahead about Japan at time $t$. Thus, one period in the equation corresponds to one quarter. The data on inflation expectations is obtained from the CF, collected by Consensus Economics.[10] We use the year-on-year rate of change in the CPI (excluding perishables) at time $t+4$ as $\pi_{t,t+4}$. The data covers from 2001:Q2[11] to 2016:Q2.

Table 1 shows the estimation results in equation (3), namely $\beta^i$ and $\gamma^i_\pi = \frac{1}{\beta^i+1}$ $(i = A, B, C)$. While $\gamma^A_\pi$ and $\gamma^B_\pi$ are above one but insignificant before the introduction of an inflation target: $\gamma^C_\pi$ becomes statistically non-zero at approximately 0.64 in a 10% interval after that. It is suggested that inflation expectations are weakly and partially anchored after the new inflation target at 2% is introduced in January 2013.[12] This evidence is consistent with the literature, which documents unstable inflation expectations in recent years.[13]

[9] When $k$ is set to be four, $\pi^e_{t,t+4}$ corresponds to inflation forecasts over one-year ahead.

[10] The Consensus Forecast (CF) is one of the longest surveys regarding inflation expectations in Japan. CF is a monthly survey, published by Consensus Economics, on developed and developing countries for professional forecasters such as economists. CF publishes quarterly forecasts in the third month of each quarter. As for the inflation outlook, forecasters submit year-on-year changes for the CPI (all items).

[11] The sample period starts from 2001:Q2, because the Bank of Japan did not announce an explicit inflation target before adopting the Quantitative Easing (QE) Policy in March 2001. In a meeting on 19 March 2001, the Bank of Japan introduced the commitment policy, which promised that the QE Policy would continue in place until the CPI (excluding perishables) inflation rate registered stably at 0% or an increase year on year.

[12] Figure 1 provides more evidence that even longer-term inflation forecasts are not anchored to 2% after 2013.

[13] Lyziak and Paloviit (2017), Nautz and Strohsal (2013), and Strohsal, Melnick, and Nautz (2016) report that inflation expectations are de-anchored after the onset of a global financial crisis.

Next, in order to estimate $\gamma_\pi$ in case (b), we arrange equation (b) into the following equation,

$$\pi_{t,t+k} - \pi^e_{t,t+k} = \beta(\pi^e_{t,t+k} - \pi_{t-k,t}) + \varepsilon_{t,t+k}, \tag{4}$$

where

$$\beta = \frac{1-\gamma_\pi}{\gamma_\pi},$$

and

$$\varepsilon_{t,t+k} = \pi_{t,t+k} - \mathrm{E}_t\pi_{t+k}.$$

$\varepsilon_{t,t+k}$ also denotes the forecast error and should be white noise from information set in time $t$ under a rational expectation. As a result, we can test a null hypothesis of $\beta = 0$. In estimating equation (4), $k$ is set to be four. The data covers from 1994:Q1 to 2016:Q2.

Table 2 shows the estimation results in equation (4), namely $\beta$ and $\gamma_\pi = \frac{1}{\beta+1}$. The estimate of $\gamma_\pi$ is approximately 0.8; the expectation formation basically follows a rational expectation, but forecasters put small weight on realized inflation rates at time $t$ ($\pi_{t-4,t}$). This indicates that when expectations are formed, an adaptive response to *current* inflation rates impedes the formation of a rational expectation.[14]

# 3 Model Setup

## 3.1 Model

The model is a New Keynesian model proposed by Woodford (2003). The macroeconomic structure is given by the following three equations.

$$x_t = x^e_{t+1} - \sigma(i_t - \pi^e_{t+1} - r^n_t), \tag{5}$$

$$\pi_t = \beta\pi^e_{t+1} + \kappa x_t, \tag{6}$$

$$r^n_t = \rho_r r^n_{t-1} + \epsilon_t, \tag{7}$$

[14]Figure 2 suggests that realized inflation rates and inflation forecasts are closely related to each other. This also implies that inflation forecasts are affected by the most recent inflation rates.

where $x_t$, $\pi_t$, $i_t$, and $r_t^n$ denote the output gap, inflation rate, nominal interest rate, and natural interest rate, respectively. For arbitrary variable $z$, $z^e$ denotes the expectation of $z$. $\epsilon_t$ denotes an i.i.d. disturbance with standard deviation $\sigma_\epsilon$. $\sigma$, $\kappa$, and $\rho_r$ are parameters satisfying $\sigma > 0$, $\kappa > 0$, and $0 \leq \rho_r < 1$. Equation (5) is a forward-looking IS curve, which is derived by households' intertemporal decisions regarding consumption. Equation (5) shows that the current output gap depends on an expected output gap and deviation of the real interest rate from the natural interest rate. Equation (6) is the New Keynesian Phillips curve (henceforth, NKPC), which is derived by firms' optimal price setting with price stickiness. Equation (6) shows that the current inflation rate depends on the current output gap and expected inflation rate.[15]

The slope of equation (6), $\kappa$, consists of deep parameters as follows.

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\sigma^{-1}+\omega}{1+\omega\theta},$$

where $\alpha$, $\omega$ and $\theta$ denote the rate of the fixing price, elasticity of marginal cost, and elasticity of demand for goods.

In this paper, we assume three cases for monetary policy rules: optimal commitment policy; the Taylor rule; and a simple rule with price-level targeting.

First, we describe a case of optimal commitment policy. The central bank minimizes the intertemporal loss function.

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda_x x_t^2), \tag{8}$$

where $\lambda_x \equiv \kappa/\theta > 0$. Note that the targeted inflation rate is zero in this model. Moreover, the anchored inflation rate is also given a value of zero. The central bank faces a non-negativity constraint on the nominal interest rate.

[15] Note that this Phillips curve becomes a discounted Phillips curve in the sense that a parameter for an expected inflation rate is discounted by $\gamma_\pi$ when the expectation formation is partially anchored to a targeting level, i.e., case (a). When the expectation formation depends on a weighted average between a rational expectation and a current inflation rate, i.e., case (b), this Phillips curve again becomes a discounted Phillips curve. In this case, a parameter for the output gap becomes greater than $\kappa$. See Gabaix (2016) for a different justification for a discounted Phillips curve.

$$i_t \geq 0. \tag{9}$$

The central bank minimizes an intertemporal loss function (8) subject to equations (5), (6), and (9).

When inflation expectations are partly anchored (case (a) of equation 1), the first order conditions under the optimal commitment policy are given as follows:

$$\pi_t - \beta^{-1}\sigma\gamma_\pi\phi_{1t-1} + \phi_{2t} - \gamma_\pi\phi_{2t-1} = 0, \tag{10}$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0, \tag{11}$$

$$i_t\phi_{1t} = 0, \;\; \phi_{1t} \geq 0, \;\; i_t \geq 0. \tag{12}$$

When an agent reflects a current inflation to form inflation expectations (case (b) of equation 1), the first order conditions under the optimal commitment policy are given as follows:

$$\pi_t - \beta^{-1}\sigma\gamma_\pi\phi_{1t-1} - \sigma(1-\gamma_\pi)\phi_{1t} + \phi_{2t} - \gamma_\pi\phi_{2t-1} - \beta(1-\gamma_\pi)\phi_{2t} = 0, \tag{13}$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0, \tag{14}$$

$$i_t\phi_{1t} = 0, \;\; \phi_{1t} \geq 0, \;\; i_t \geq 0. \tag{15}$$

Here, $\phi_{1t}$ and $\phi_{2t}$ denote Lagrange multipliers associated with equations (5) and (6), respectively. Equations (10) and (13) show the first order conditions with respect to the inflation rate and equations (11) and (14) show the first order conditions with respect to the output gap. Equations (12) and (15) show the first order conditions with respect to the nominal interest rate in considering the non-negativity constraint on the nominal interest rate. If the nominal interest rate is zero, the Lagrange multiplier $\phi_{1t}$ becomes positive, and vice versa.

Second, we define the Taylor rule. We set the following interest rate rule with the non-negativity constraint on the nominal interest rate.

$$i_t = \max[0, \psi_\pi\pi_t + \psi_x x_t], \tag{16}$$

where $\psi_\pi$ and $\psi_x$ are parameters satisfying $\psi_\pi > 0$ and $\psi_x > 0$.

Third, we introduce a simple rule with price-level targeting as follows.

$$i_t = \max[0, \psi_p(\ln P_t - \ln P^*) + \psi_x x_t], \tag{17}$$

where $\ln P^*$ is the steady state value of $\ln P_t$ and $\psi_p > 0$.

### 3.2 Baseline Calibration

We set the baseline quarterly parameters as in Table 3. We set $\sigma = 6.25$ following Woodford (2003). Meanwhile, we set $\alpha = 0.875$, $\beta = 0.995$, $\omega = 2.149$, $\theta = 6.0$, $\sigma_\epsilon = 0.102$, and $\rho_r = 0.892$ following Sugo and Ueda (2008), which estimated parameters for the Japanese economy.[16] For a baseline calibration, we set the parameter of weights on inflation expectations as $\gamma_\pi = 0.5$, and anchored levels of the inflation rate as $\bar{\pi} = 0$.

## 4 Baseline Simulations

In this section, we reveal the role of expectations in a liquidity trap by numerical simulations following the expectation formation for an inflation rate, as shown in equation (1). We show cases under different parameters for expectation formation and different monetary policies.

### 4.1 Optimal Commitment Policy

We assume that a one-time shock to the natural interest rate occurs at period 0[17]; we give a negative 0.75% (annually 3%) quarterly shock to a natural interest rate. Figure 3 shows the impulse responses under the optimal commitment policy. Note that we use

[16]Sugo and Ueda (2008) estimate preference shock for the natural rate shock. We use the estimated value of preference shock as that of the natural rate shock. Regarding $\sigma$, a smaller value cannot secure a convergence in this simulation of a liquidity trap. Even when we set $\sigma = 2$, which is closer to the value in Sugo and Ueda (2008), our conclusion does not change, as shown in Appendix A.

[17]The details of the numerical algorithms are given in Appendix B.

three different scales for figures.[18] Solid lines denote a case when inflation expectations are purely forward-looking (i.e., $\pi^e_{t+1} = \mathrm{E}_t\pi_{t+1}$); dashed lines denote a case when the inflation expectations are perfectly anchored (i.e., $\pi^e_{t+1} = \bar{\pi} = 0$); chained lines denote a case where the inflation expectations are partly anchored (i.e., $\gamma_\pi \mathrm{E}_t\pi_{t+1} + (1-\gamma_\pi)\bar{\pi}$); and dotted lines denote a case in which the degree of forward-lookingness decreases (i.e., $\gamma_\pi \mathrm{E}_t\pi_{t+1} + (1-\gamma_\pi)\pi_t$, where $\gamma_\pi = 0.5$).

Figure 3 shows some observations. The response of the inflation rate, output gap, nominal interest rate, and real interest rate do not change much according to the inflation expectation formation (Figures 3a, b, e, f). Note that responses of the nominal interest rate and real interest rate are almost identical for all cases. These observations show that, under optimal commitment policy, the formation of inflation expectations is a trivial problem. The power of commitment is a key point for this result. Under optimal commitment policy, the power of controlling expectations is strong due to history-dependent monetary easing. Thus, the response of an economy does not change drastically even when the room for managing expectations is limited by anchored inflation expectations and by less forward-looking inflation expectation.[19]

We were interested to show whether or not the economy being in a liquidity trap is critical when considering the role of different inflation expectation formations on monetary policy. Without a liquidity trap, a policy maker can perfectly stabilize the economy against the natural rate shock under an optimal commitment policy, regardless of the expectation formation, as implied by Jung, Teranishi, and Watanabe (2005) and Adam and Billi (2006). Thus, the existence of the zero lower bound is crucial to evaluating the roles of expectation formations on monetary policy effectiveness.

[18] Across different figures, we use three different scales, 'small', 'medium', and 'large'. We plot Figures 3, 9, and A1 for optimal commitment policy with the 'small' scale, Figures 4, 5, 6, 10, and A2 for the Taylor rule with the 'large' scale, and Figures 7, 8, 11, and A3 for a simple rule with price-level targeting with the 'medium' scale. The scale is noted in the caption of each figure.

[19] We analyze a case when expectation formation follows $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t\pi_{t+1} + (1-\gamma_\pi)\pi^e_t$. We obtain more persistent responses of the inflation rate, but responses of the output gap, nominal interest rate, and real interest rate do not change sufficiently.

## 4.2 The Taylor Rule

Next, we look at impulse responses under the Taylor rule. We set $\psi_\pi = 1.5$ and $\psi_x = 0.5$. Figure 4 shows the results. Responses change drastically in comparison to those under the optimal commitment policy on two points.[20]

First, in cases of perfectly and partly anchored inflation expectations, monetary policy achieves smaller drops in the inflation rate and output gap (dashed lines and chained lines in Figures 4a, b). This is because of the low real interest rate. The real interest rate stays at a lower level, as shown in Panel (f) in Figure 4, since inflation expectations are anchored. Consequently, monetary policy can avoid large drops in the inflation rate and output gap. Thus, under the Taylor rule, anchoring inflation expectations plays an important role in stabilizing the economy. In other words, the effects of monetary policy significantly change according to different inflation expectation formations under the Taylor rule. This is because the Taylor rule does not have a history-dependency, and cannot work on expectations in a forward-looking model. Anchored expectations can compensate for these drawbacks of the Taylor rule.

How great a degree of anchoring is required to stabilize the economy can still be important. To answer this question, we show impulse responses by changing $\gamma_\pi$ in $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\bar{\pi}$. Figure 5 plots the responses under the Taylor rule for $\gamma_\pi = 0$, 0.4, 0.8, and 1. As $\gamma_\pi$ decreases, inflation expectations are more strongly anchored. The results show that the output gap becomes larger and the real interest rate smaller as $\gamma_\pi$ becomes larger. An important point is that drops in the inflation rate and the output gap are sufficiently mitigated even for the small weight of an anchored inflation rate such as $\gamma_\pi = 0.8$. This means that the benefit of anchoring inflation expectations exists, even though the inflation expectations are not anchored strongly. Thus, partly anchoring inflation expectations is effective for stabilizing an economy in a liquidity trap.

[20] Even when we assume alternative parameters such as ($\psi_\pi = 5$ and $\psi_x = 0.5$) and ($\psi_\pi = 5$ and $\psi_x = 0.75$) by following Fujiwara, Nakajima, Sudo, and Teranishi (2013) and ($\psi_\pi = 3$ and $\psi_x = 0.25$) by following Erceg and Lindé (2014), the Taylor rule is far from optimal policy, and gives a similar outcome to that in Figure 4.

Second, an impulse response changes when the degree of forward-lookingness in the expectation formation changes. The output gap and inflation rate decrease more as the degree of forward-lookingness becomes smaller. Panels (a) and (b) in Figure 4 indicate this observation: dotted lines decrease sufficiently in all responses. When partial inflation expectations reflect a current inflation rate, it is difficult to control economic dynamics through monetary policy, since the Taylor rule cannot work on expectations in a forward-looking model. A forward-looking expectation can compensate for this drawback of the Taylor rule. In a forward-looking economy, there is an innate mechanism to align an economy to a steady state where negative shock disappears. When the degree of forward-lookingness decreases, this tendency weakens.

We would like to investigate how crucial the existence of the zero lower bound is when considering the role of different inflation expectation formations under the Taylor rule. Figure 6 shows impulse responses without the zero lower bound under the Taylor rule. Compared to Figure 4 (a case with the zero lower bound), different inflation expectations make much smaller differences in the economic dynamics in Figure 6. Moreover, the output gap and inflation rate drop less in a simulation without the zero lower bound than with this bound. This is because of the real interest rate, due to the existence of the zero lower bound. The real interest rate is negative, and therefore not so different for different inflation expectation formations without the zero lower bound. On the other hand, the real interest rate is not negative and responds differently for different inflation expectation formations in a liquidity trap. These results show that whether the economy is in a liquidity trap or not is critical when considering the role of inflation expectations under the Taylor rule.

## 4.3 Simple Rule with Price-level Targeting

Some previous papers (Eggertsson and Woodford, 2003b; Nakov, 2008; Fujiwara, Nakajima, Sudo, and Teranishi, 2013) show that a simple rule with price-level targeting is effective in a liquidity trap. We investigate the effectiveness of a simple rule with price-level targeting under different inflation expectation formations. In simulations, we set

$\psi_p = 1.5$.

Figure 7 shows impulse responses. As observed in previous results for the Taylor rule, the response of an economy changes according to the expectation formation for an inflation rate (Figure 7b). However, compared to the case when applying the Taylor rule, all responses do not change markedly (Figure 7a, e, f). Although the output gap decreases significantly, it remains higher under a simple rule with price-level targeting than under the Taylor rule. Though inflation rates show some differences, the output gaps are not so different according to the expectation formation. Under a simple rule with price-level targeting, such targeting provides the power to control the expectation formation due to history-dependent easing. Thus, the expectation formation is not as serious a problem for a simple rule with price-level targeting.

Figure 8 shows impulse responses without the zero lower bound under a simple rule with price-level targeting. The responses of an economy become less severe in any expectation formation than in Figure 7 (a case with the zero lower bound), since the nominal interest rate is not constrained by the zero lower bound. Thus, whether the economy is in a liquidity trap or not is critical when considering the effect of monetary policy against different inflation expectation formations under a simple rule with price-level targeting.

# 5 Simulations for the Japanese Economy

In this section, we use estimated parameters for expectation formations for the Japanese economy in section 2.

We show impulse responses under the estimated value of $\gamma_\pi$ in Tables 1 and 2. We assume three cases for monetary policy: the optimal commitment policy; Taylor rule; and a simple rule with price-level targeting. Figures 9, 10, and 11 show impulse responses under these three monetary policies, respectively. Dashed lines in figures denote the case when inflation expectations are perfectly anchored. Chained lines denote the case when $\gamma_\pi = 0.643$ in $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\bar{\pi}$. Dotted lines denote the case when $\gamma_\pi = 0.803$ in $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\pi_t$.

Figure 9 shows that responses of the inflation rate, output gap, and interest rates do not change much under the commitment policy. This implies that expectation formation is not a topic in monetary policy if the Bank of Japan can implement an optimal commitment policy. Optimal monetary policy with strong history-dependent easing can control expectation formation in the Japanese economy. Figure 10 shows a case when applying the Taylor rule. Compared to a perfectly anchored case, we observe drops in the inflation rate and output gap for other expectation formations for the inflation rate. However, these drops are largely mitigated when inflation expectations are partially anchored, by approximately 36%. Thus, even under a weak anchoring in inflation expectations, the Taylor rule can stop a serious deflation though optimal monetary policy and a simple rule with price-level targeting shows much better outcomes than the Taylor rule does. Moreover, these drops are larger when inflation expectations are based on a current inflation rate (by approximately 20%) compared to when inflation expectations are partially anchored. Figure 11 shows the case of a simple rule with price-level targeting, where the differences in impulse responses are smaller than under the Taylor rule. Therefore, by committing to a simple history-dependent rule like a price-level targeting rule, the Bank of Japan can further mitigate the effect of a weak anchoring in expectations and a lack of forward- lookingness in expectation formation.

# 6 Concluding Remarks

This paper shows that expectation formation has different outcomes for monetary policy in a liquidity trap. Such a difference is trivial for optimal monetary policy. However, for simple and realistic rules such as the Taylor rule and a rule with price-level targeting, we observe significant difference in the effect of monetary policy on economic dynamics according to expectation formation. Therefore, a central bank can play an important role in helping an economy to escape from a liquidity trap by managing expectations.

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# SUPPLEMENTARY APPENDIX

## A Additional Analysis Under Low Elasticity of Intertemporal Substitution

This section presents numerical results when $\sigma = 2$, which is closer to the value in Sugo and Ueda (2008).[21] See Figures A1–A3.

## B Numerical Algorithm

We solve the central bank's optimization problem by calculating the solution for equations (5) – (6) and equations (10) – (15). Since the zero lower bound (ZLB) introduces nonlinearity into the model, we employ a numerical technique that approximates expected variables.

First, we specify the grids for three state variables, $r_t^n$, $\phi_{1t-1}$, and $\phi_{2t-1}$. Let $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$ denote the vector of grids for $r_t^n$, $\phi_{1t-1}$, and $\phi_{2t-1}$, respectively. A tensor of these grid vectors, defined as $\mathbf{S} \equiv \mathbf{S}_1 \otimes \mathbf{S}_2 \otimes \mathbf{S}_3$, determines the combination of all grids. The size of $\mathbf{S}$ is $N = n_1 \times n_2 \times n_3 = 9261$. As for $\mathbf{S}_1$, we put relatively larger number of grids near the kink point stemming from the ZLB with the aim of mitigating the expected approximation error. The *p.d.f.* for the natural interest rate is discretized by Gaussian quadrature.

Notice that we can rewrite the complementarity conditions regarding the ZLB, equations (12) and (15), as

$$\min(\max(\sigma\phi_{1t}, -i_t), \infty) = 0. \tag{A.1}$$

In order to employ an algorithmic solution that is basically designed for differentiable functions, we approximate equation (A.1) by a semismooth function, in a so-called Fis-

[21] The estimated value by Sugo and Ueda (2008) is $\sigma = 0.8006$.

cher's equation:

$$\psi^{-}(\psi^{+}(\sigma\phi_{1t}, -i_t), \infty) = 0,$$

where $\psi^{\pm}(u, v) = u + v \pm \sqrt{u^2 + v^2}$ (c.f., Miranda and Fackler, 2004).

Let $\mathbf{h}_t \equiv (x_t, \pi_t, \phi_{2t})$ denote the vector of forward-looking variables at time $t$. We must obtain $\mathbf{h}_t$, $i_t$, and $\phi_{1t}$ by solving the central bank's optimization problem, taking state variables as given. In order to calculate the expectation terms, we approximate the time-invariant function for forward-looking variables, $\mathbf{h}$, by a collocation method. Our solution procedure is summarized as follows:

1. Given a particular set of grids for state variables, denoted by $\mathbf{S}^j$, and the initial guess of the functional form for $\mathbf{h}(\mathbf{S}^j)$, denoted by $\mathbf{h}^0(\mathbf{S}^j)$, compute $\mathbf{h}^1(\mathbf{S}^j)$, $i_t$, and $\phi_{1,t}$ as a solution for equations (5) to (6) and equations (10) to (15). A cubic-spline function is used to interpolate $\mathbf{h}(\mathbf{S^j})$.

2. Repeat step 1 for all $j = 1, \ldots N$.

3. Stop if $\|\mathbf{h}^1 - \mathbf{h}^0\|_\infty / \|\mathbf{h}^0\|_\infty < 1.5 \times 10^{-6}$. Otherwise, update the initial functional form as $\mathbf{h}^0 \equiv \mathbf{h}^1$ and go to step 1.

Euler residuals from first order conditions are in the order of $10^{-3}$, which is concentrated mostly around the ZLB. Computation time is around 4 hours. We used Matlab software, CPU is Xeon with 3.60GHz, and Memory is 32GB.

Table 1: The degree of expectation anchoring: Case (a)

$$\pi^e_{t,t+4} = \gamma_\pi \mathrm{E}_t \pi_{t,t+4} + (1-\gamma_\pi)\bar{\pi}$$

$$\pi_{t,t+4} - \pi^e_{t,t+4} = \beta^A(\pi^e_{t,t+4} - 0\%) \times D_1 + \beta^B(\pi^e_{t,t+4} - 1\%) \times D_2 + \beta^C(\pi^e_{t,t+4} - 2\%) \times (1 - D_1 - D_2) + \varepsilon_{t,t+4}$$

| | $\beta^A$ | $\gamma^A_\pi$ | $\beta^B$ | $\gamma^B_\pi$ | $\beta^C$ | $\gamma^C_\pi$ | Observations |
|---|---|---|---|---|---|---|---|
| Equation (3) | −0.407 (0.371) | 1.686 | −0.331 (0.231) | 1.495 | 0.556* (0.304) | 0.643 | 61 |

Note: The data on inflation forecasts is obtained from Consensus Economics, and covers 2001:Q2 to 2016:Q2. We use core inflation rates for $\pi$. $\bar{\pi}$ is set to be 0% before the Bank of Japan announced that 1% inflation was desirable in mid-February 2012, 1% before the introduction of an inflation target, and 2% after that. $D_1$ and $D_2$ take one from 2001Q2–2011Q4 and from 2012Q1–2012Q4, respectively, otherwise zero. Standard errors in parentheses are calculated by the Newey-West (1987) estimator. Here, * indicates 10% significance.

Table 2: The degree to which expectation formation is forward-looking: Case (b)

$$\pi^e_{t,t+4} = \gamma_\pi \mathrm{E}_t \pi_{t,t+4} + (1-\gamma_\pi)\pi_{t-4,t}$$

$$\pi_{t,t+4} - \pi^e_{t,t+4} = \beta(\pi^e_{t,t+4} - \pi_{t-4,t}) + \varepsilon_{t,t+4}$$

| | $\beta$ | $\gamma_\pi$ | Observations |
|---|---|---|---|
| Equation (4) | 0.246* (0.130) | 0.803 | 90 |

Note: The data on inflation forecasts is obtained from Consensus Economics, and covers 1994:Q1 to 2016:Q2. We use core inflation rates for $\pi$. Standard errors in parentheses are calculated by the Newey-West (1987) estimator. Here, * indicates 10% significance.

Table 3: Parameter values

| Parameters | Values | Explanation |
|---|---|---|
| $\beta$ | 0.995 | Discount Factor |
| $\sigma$ | 6.25 | Elasticity of Output Gap to Real Interest Rate |
| $\alpha$ | 0.875 | Price Stickiness |
| $\theta$ | 6 | Elasticity of Goods Demand |
| $\omega$ | 2.149 | Elasticity of Marginal Cost |
| $i^*$ | 0.5 | Steady State Nominal Interest Rate |
| $\sigma_\epsilon$ | 0.102 | Standard Deviation of Natural Rate Shock |
| $\rho_r$ | 0.892 | Persistence of Natural Rate Shock |
| $\psi_\pi$ | 1.5 | Coefficient of Inflation Rate in Taylor Rule |
| $\psi_x$ | 0.5 | Coefficient of Output Gap in Taylor Rule |
| $\psi_p$ | 1.5 | Coefficient of Price Level in Price-level Targeting Rule |
| $\bar{\pi}$ | 0 | Anchored Level of Inflation Rate |

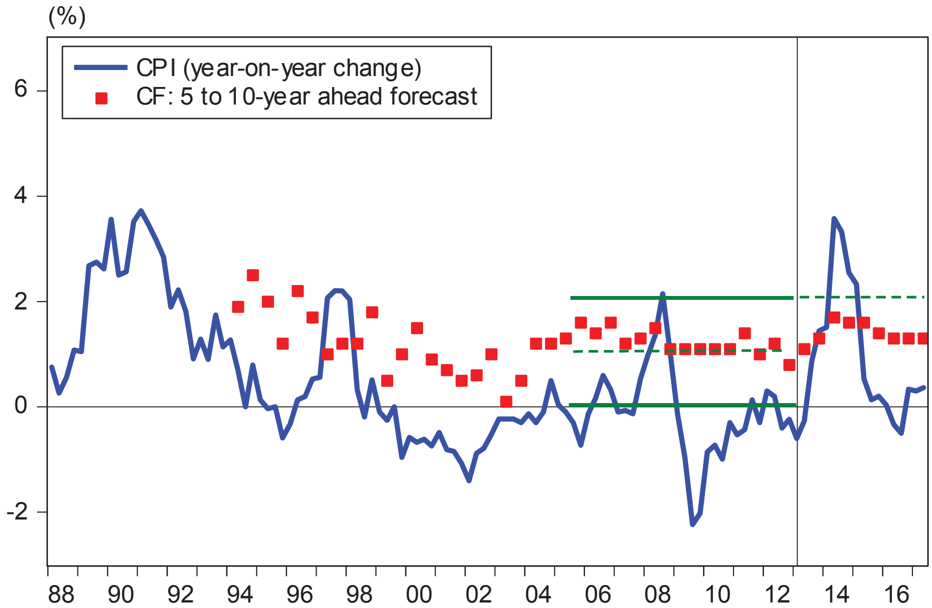


Figure 1: Longer-term inflation forecasts (5–10 years ahead forecasts of CF from Consensus Economics) and inflation rates in Japan. The solid and dashed lines are defined as the upper and lower limits of inflation targets and the point targets or middle points, respectively. The vertical lines show when inflation targets are introduced.

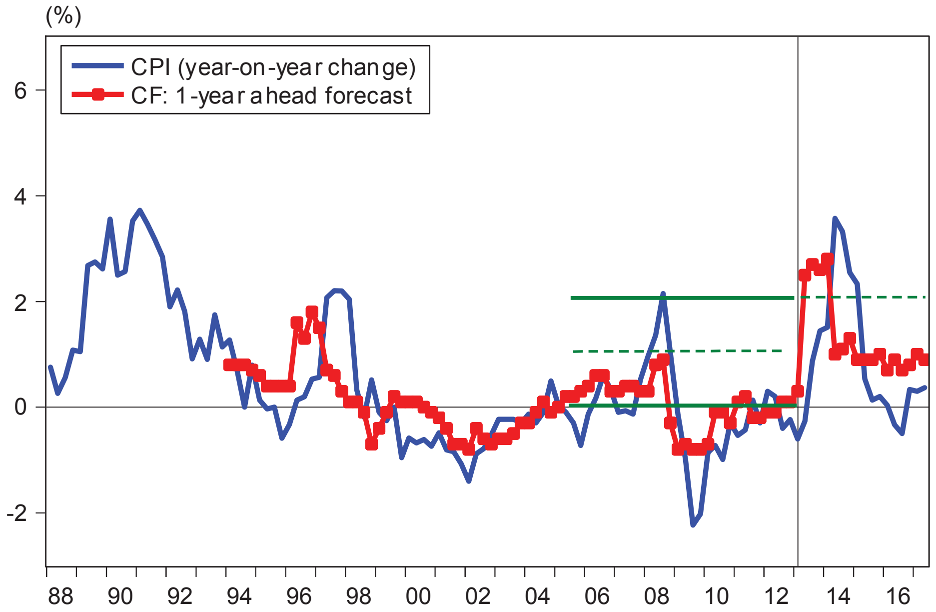


Figure 2: Short-term inflation forecasts (1-year ahead forecasts of CF from Consensus Economics) and inflation rates in Japan. The solid and dashed lines are defined as the upper and lower limits of inflation targets and the point targets or middle points, respectively. The vertical lines show when inflation targets are introduced.

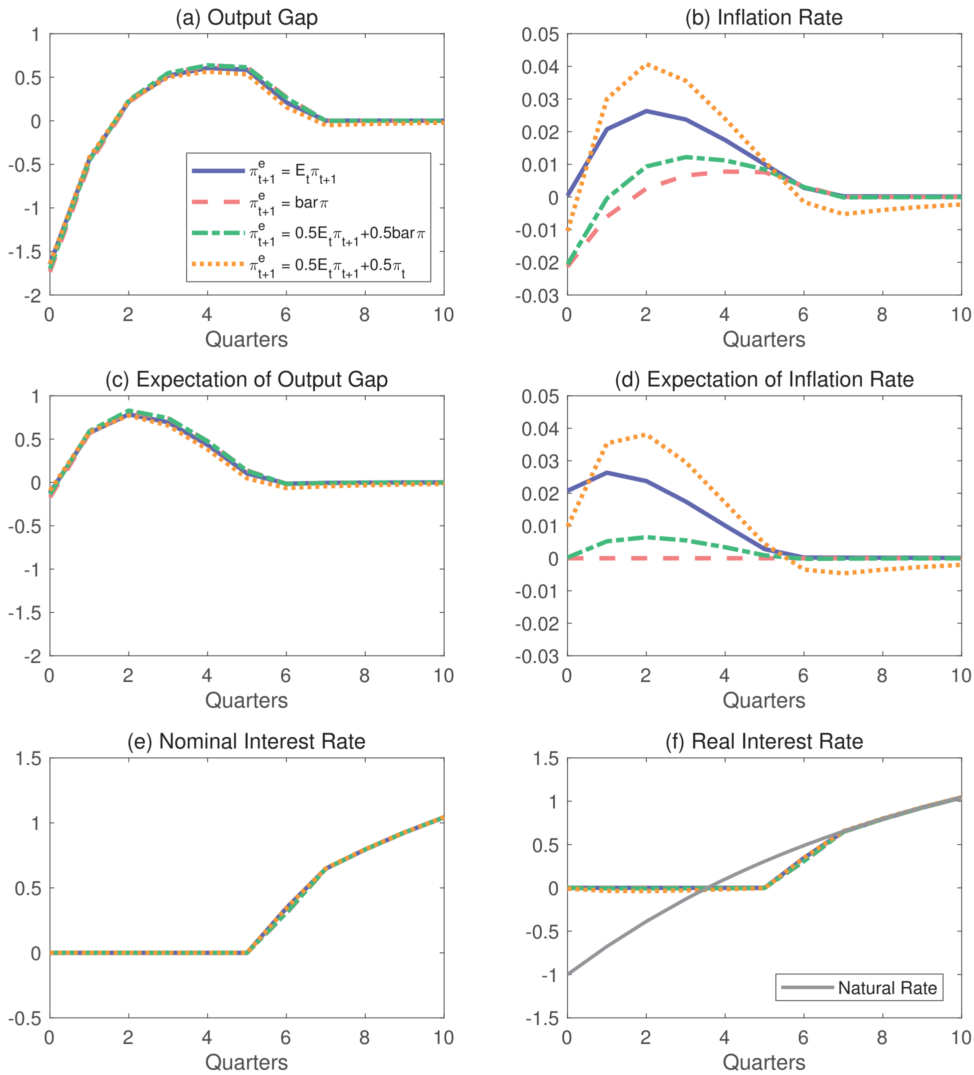


Figure 3: Impulse responses to an annual $-3\%$ natural rate shock under optimal commitment policy for different expectation formations for an inflation rate. Note: the scale of the figure is 'small'.

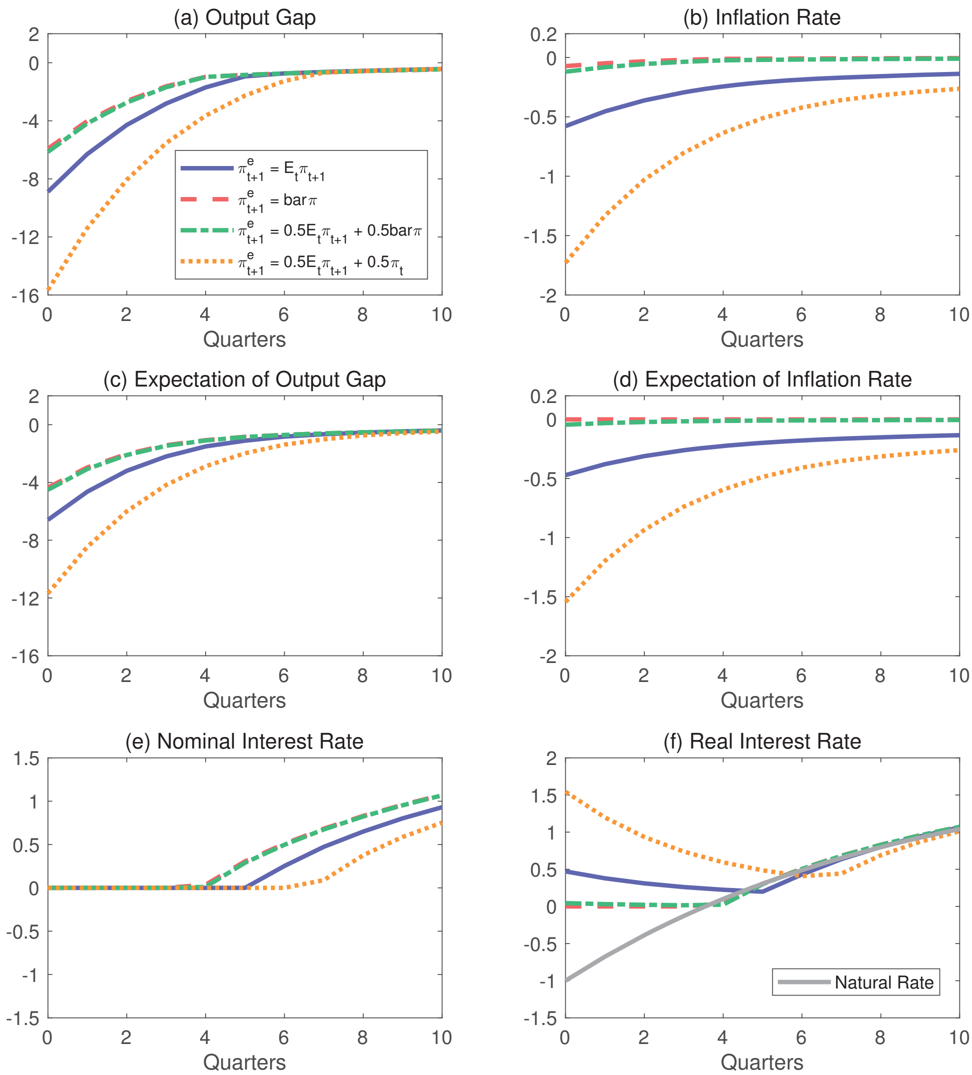


Figure 4: Impulse responses to an annual $-3\%$ natural rate shock under the Taylor Rule for different expectation formations for an inflation rate. Note: the scale of the figure is 'large'.

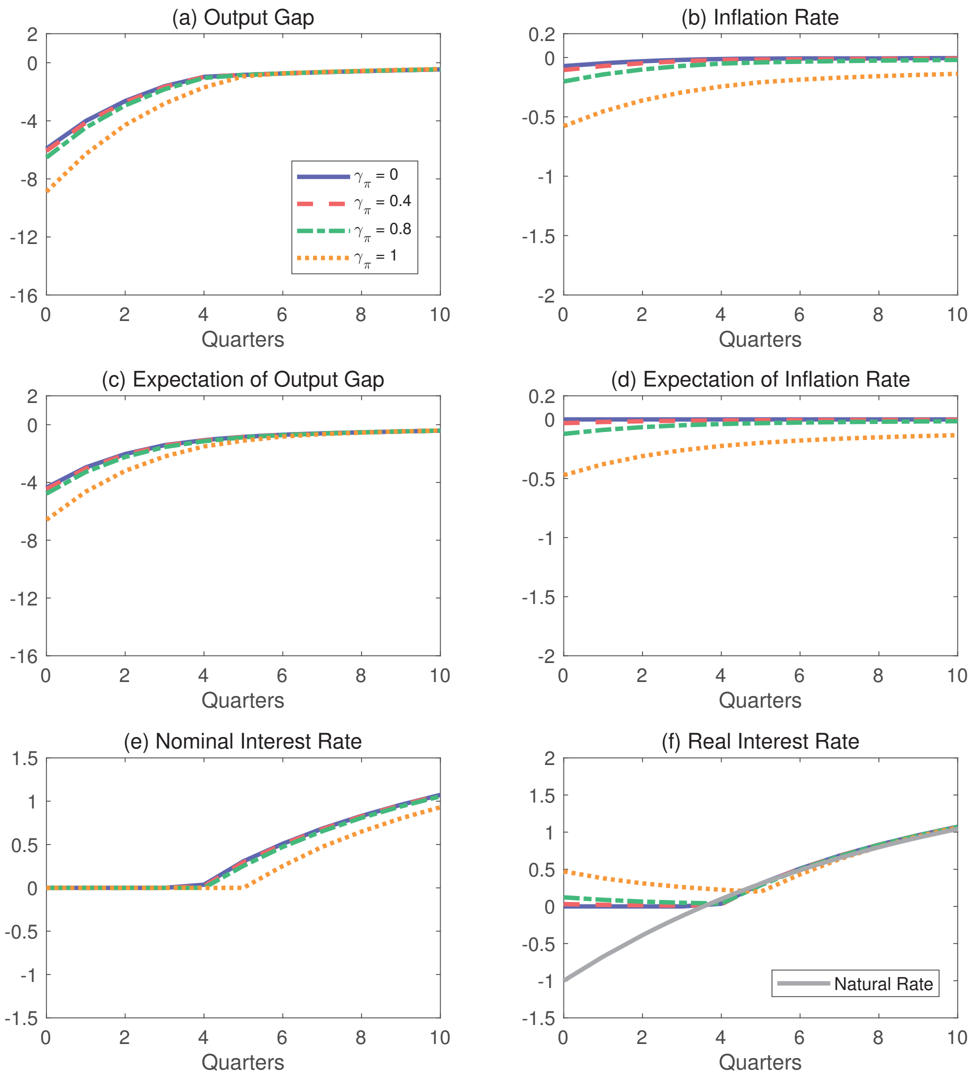


Figure 5: Impulse responses to an annual $-3\%$ natural rate shock for various values of $\gamma_\pi$ in $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1 - \gamma_\pi)\bar{\pi}$ under the Taylor Rule. Note: the scale of the figure is 'large'.

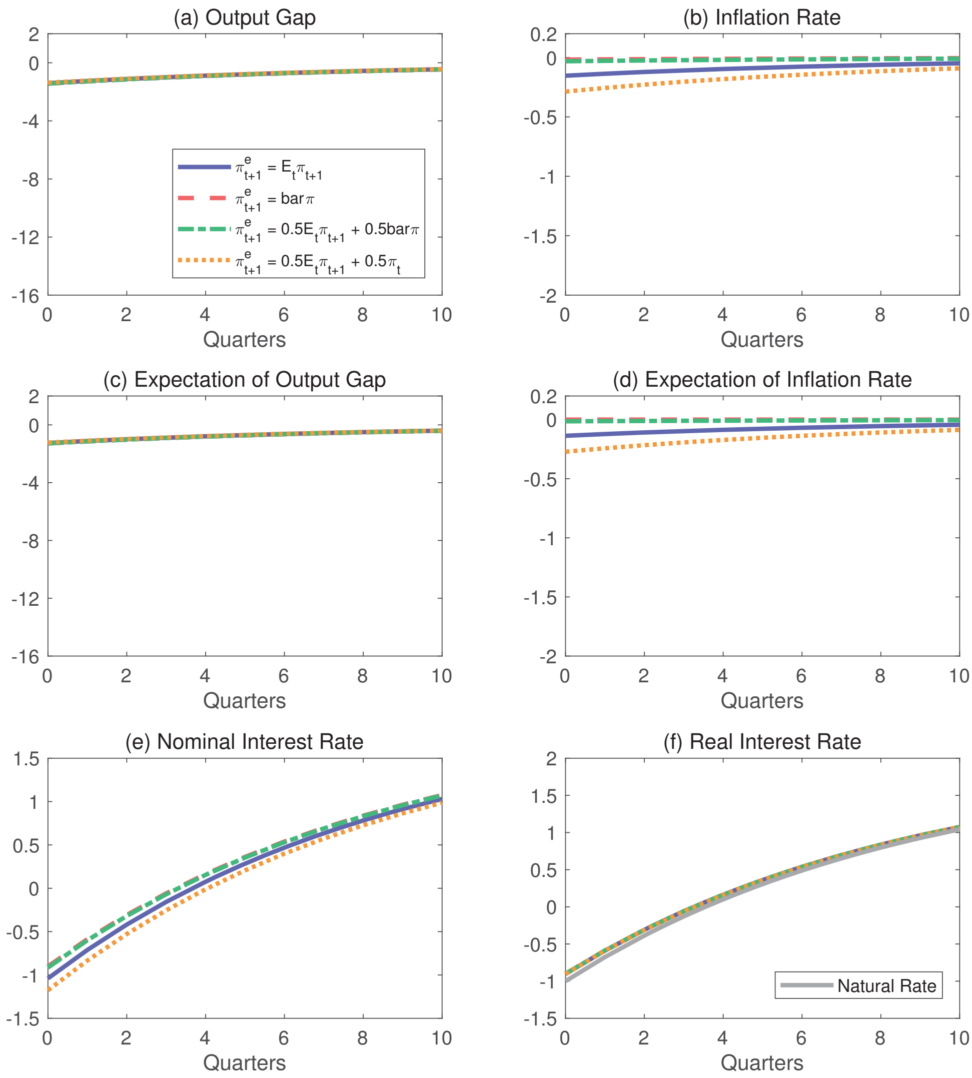


Figure 6: Impulse responses to an annual −3% natural rate shock under the Taylor Rule for different expectation formations for an inflation rate without the zero lower bound. Note: the scale of the figure is 'large'.

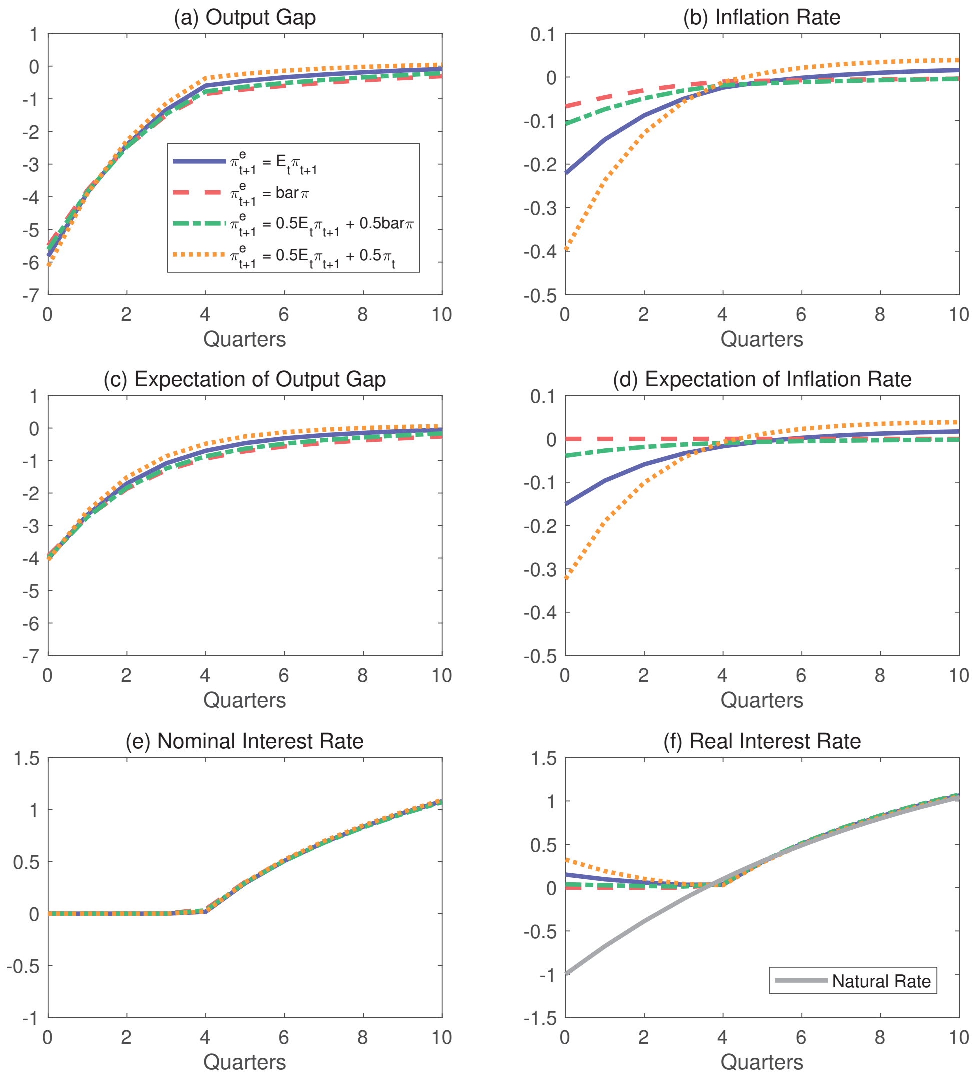


Figure 7: Impulse responses to an annual −3% natural rate shock under a simple rule with price-level targeting for different expectation formations for an inflation rate. Note: the scale of the figure is 'medium'.

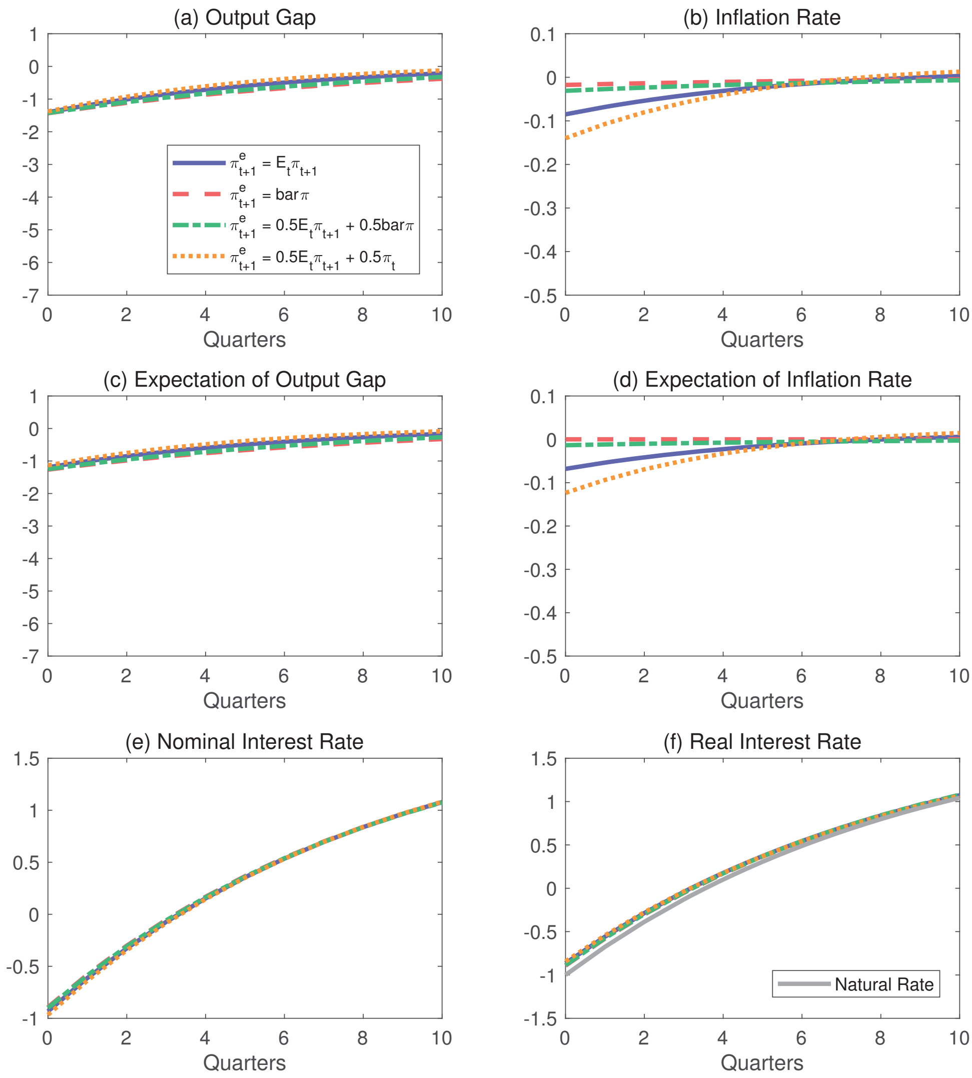


Figure 8: Impulse responses to an annual −3% natural rate shock under a simple rule with price-level targeting for different expectation formations for an inflation rate without the zero lower bound. Note: the scale of the figure is 'medium'.

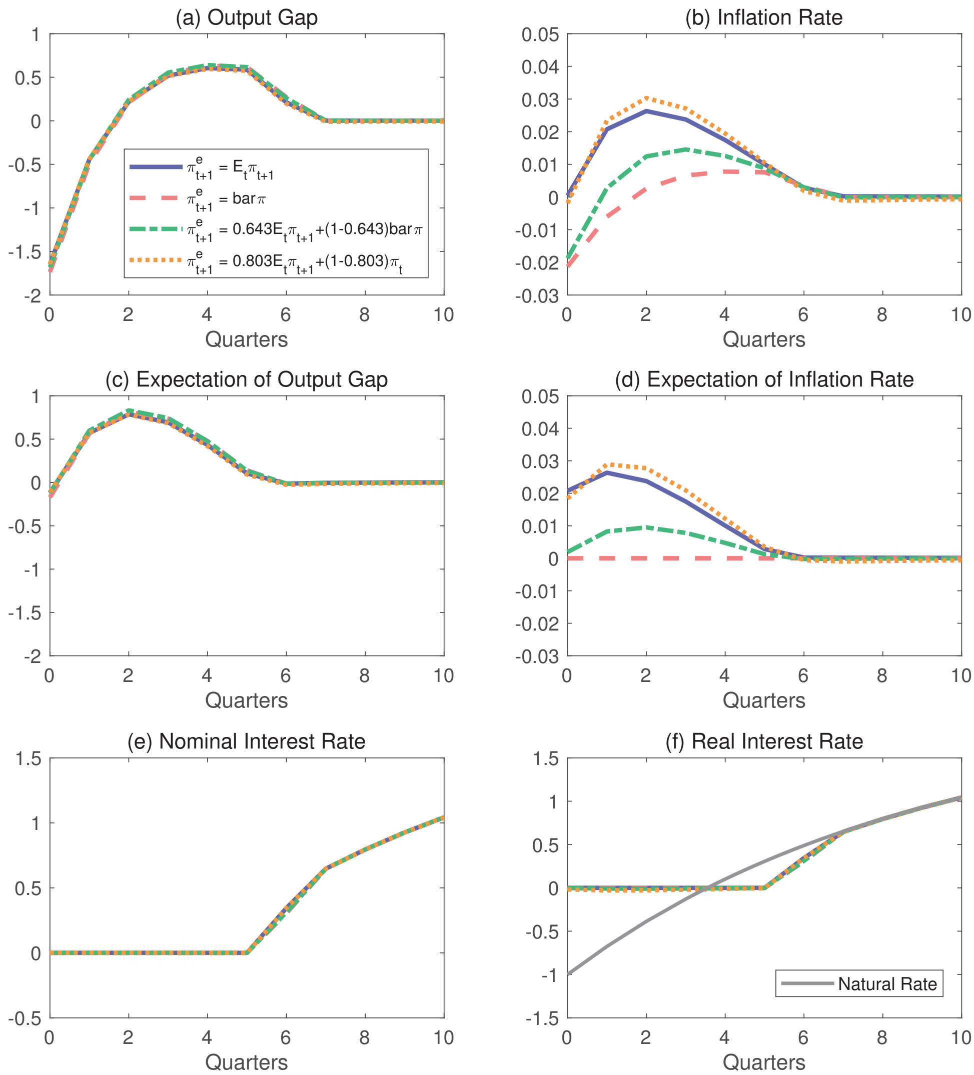


Figure 9: Impulse responses to an annual $-3\%$ natural rate shock under optimal commitment policy with estimated value of $\gamma_\pi$ for the Japanese economy. Note: the scale of the figure is 'small'.

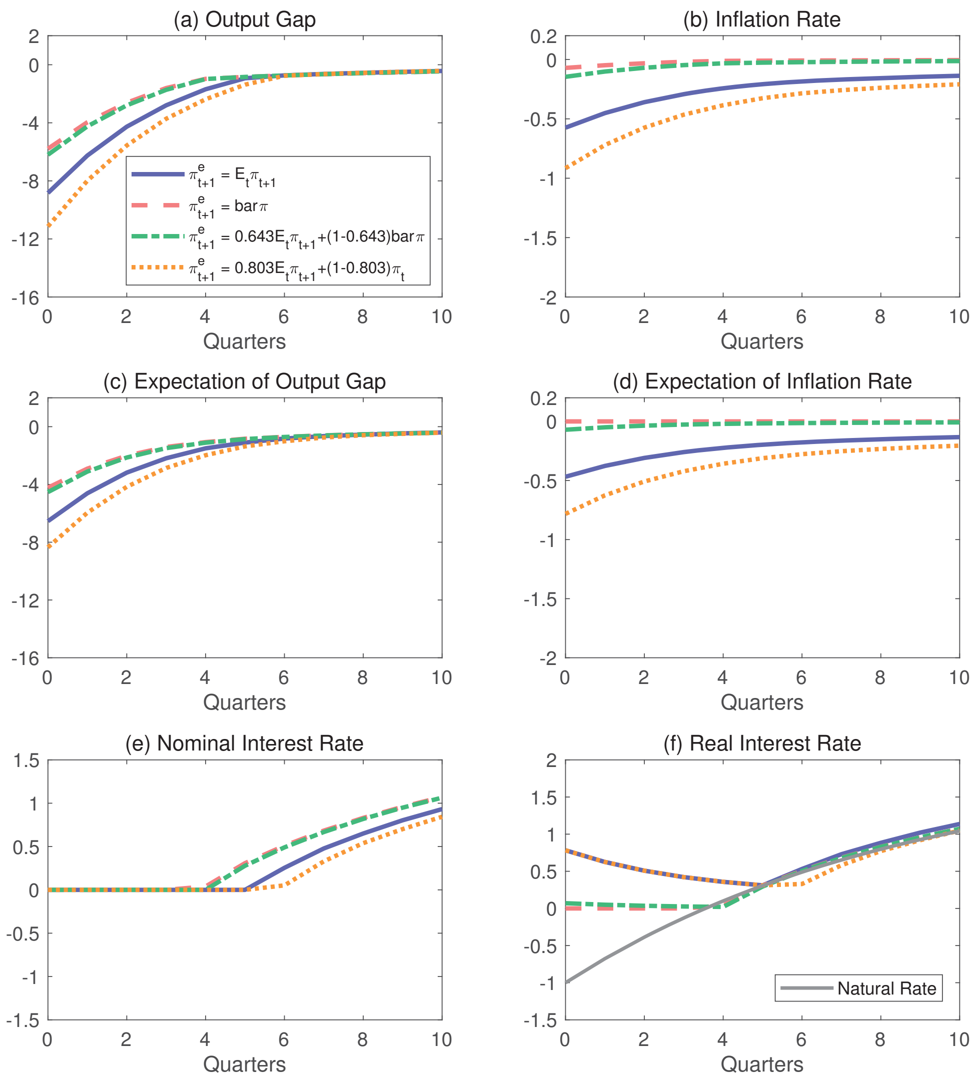


Figure 10: Impulse responses to an annual $-3\%$ natural rate shock under the Taylor rule with estimated value of $\gamma_\pi$ for the Japanese economy. Note: the scale of the figure is 'large'.

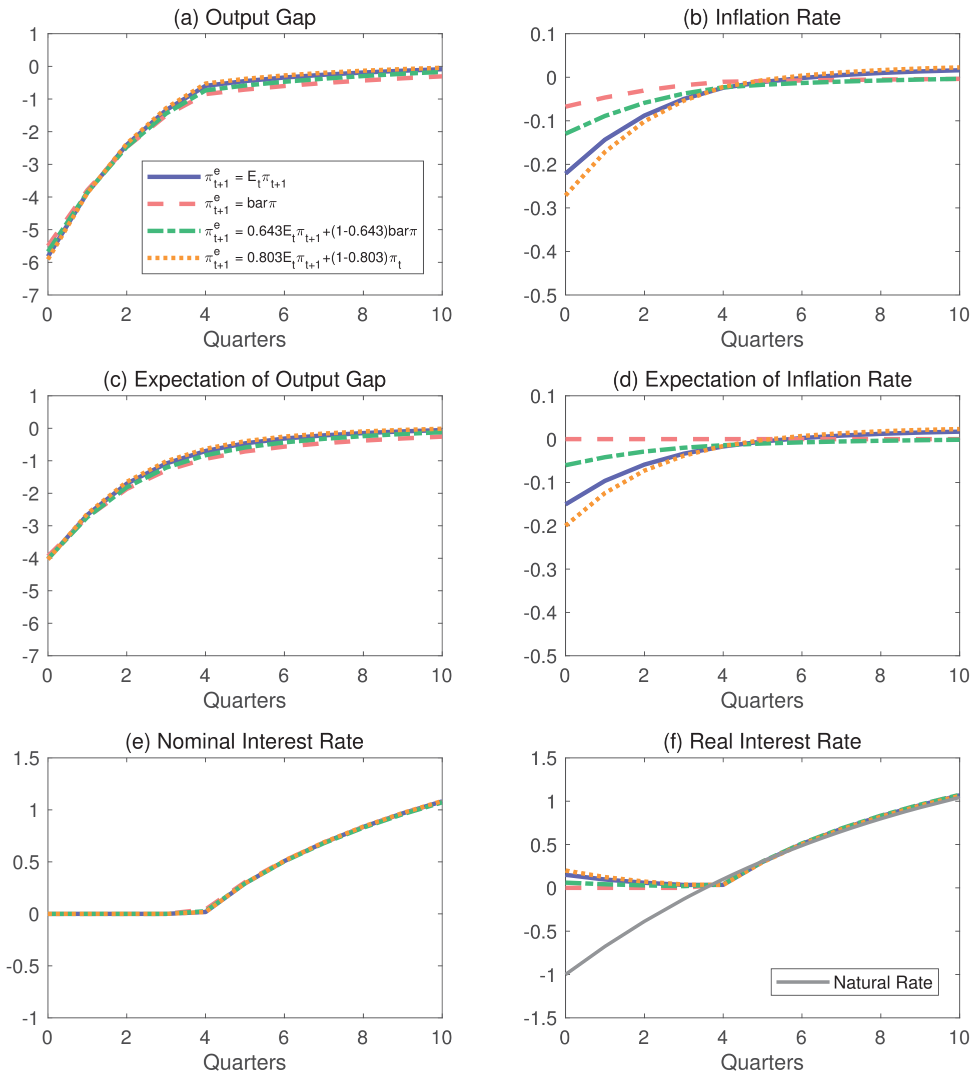


Figure 11: Impulse responses to an annual $-3\%$ natural rate shock under a simple rule with price-level targeting with estimated value of $\gamma_\pi$ for the Japanese economy. Note: the scale of the figure is 'medium'.

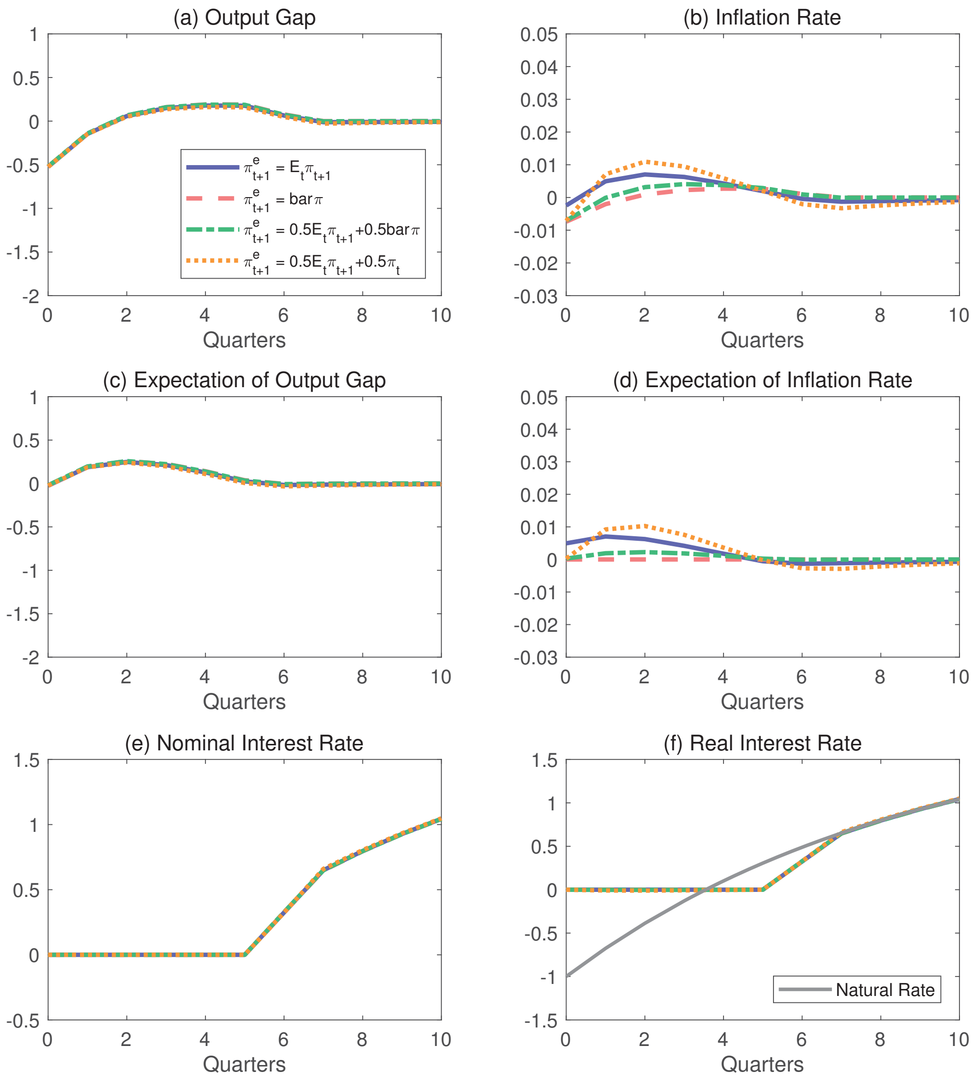


Figure A1: Impulse responses to an annual $-3\%$ natural rate shock under optimal commitment policy for different expectation formations for an inflation rate when we set $\sigma = 2$. Note: the scale of the figure is 'small'.

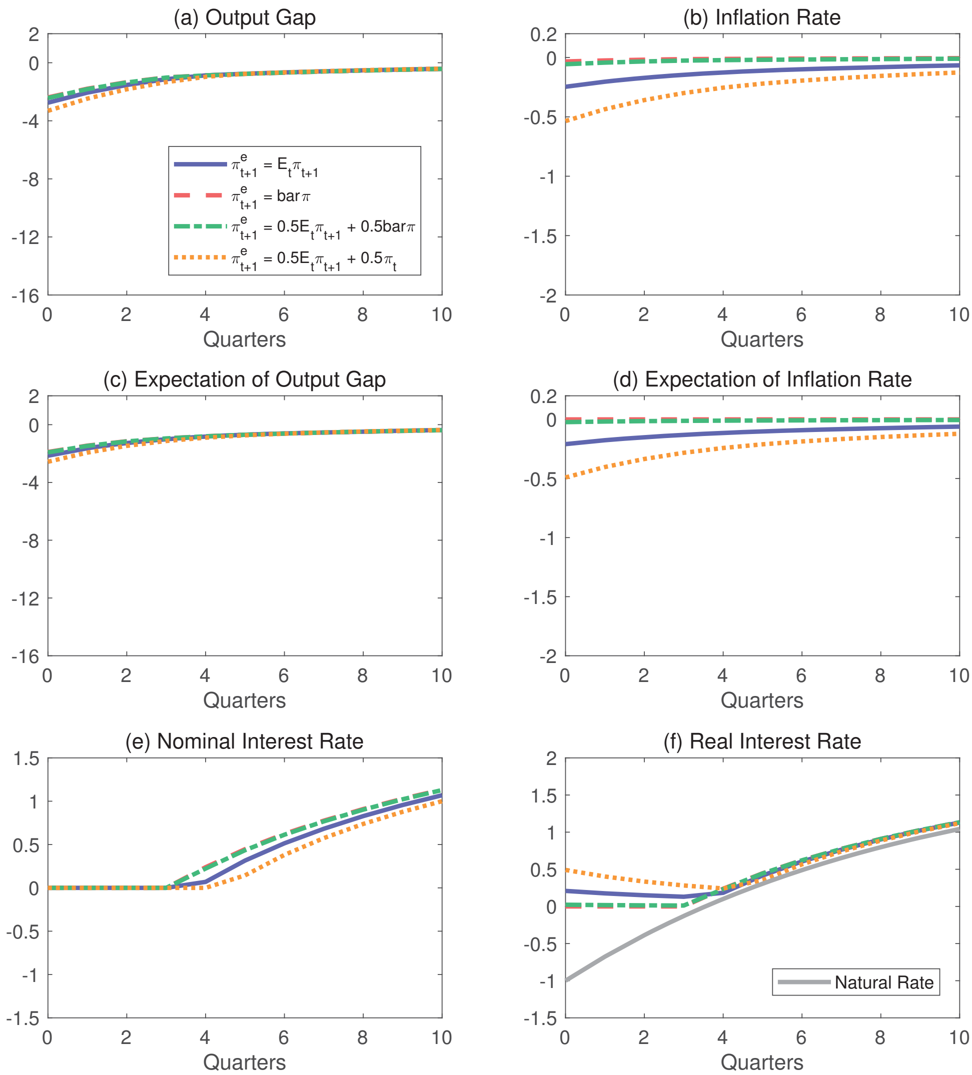


Figure A2: Impulse responses to an annual $-3\%$ natural rate shock under the Taylor Rule for different expectation formations for an inflation rate when we set $\sigma = 2$. Note: the scale of the figure is 'large'.

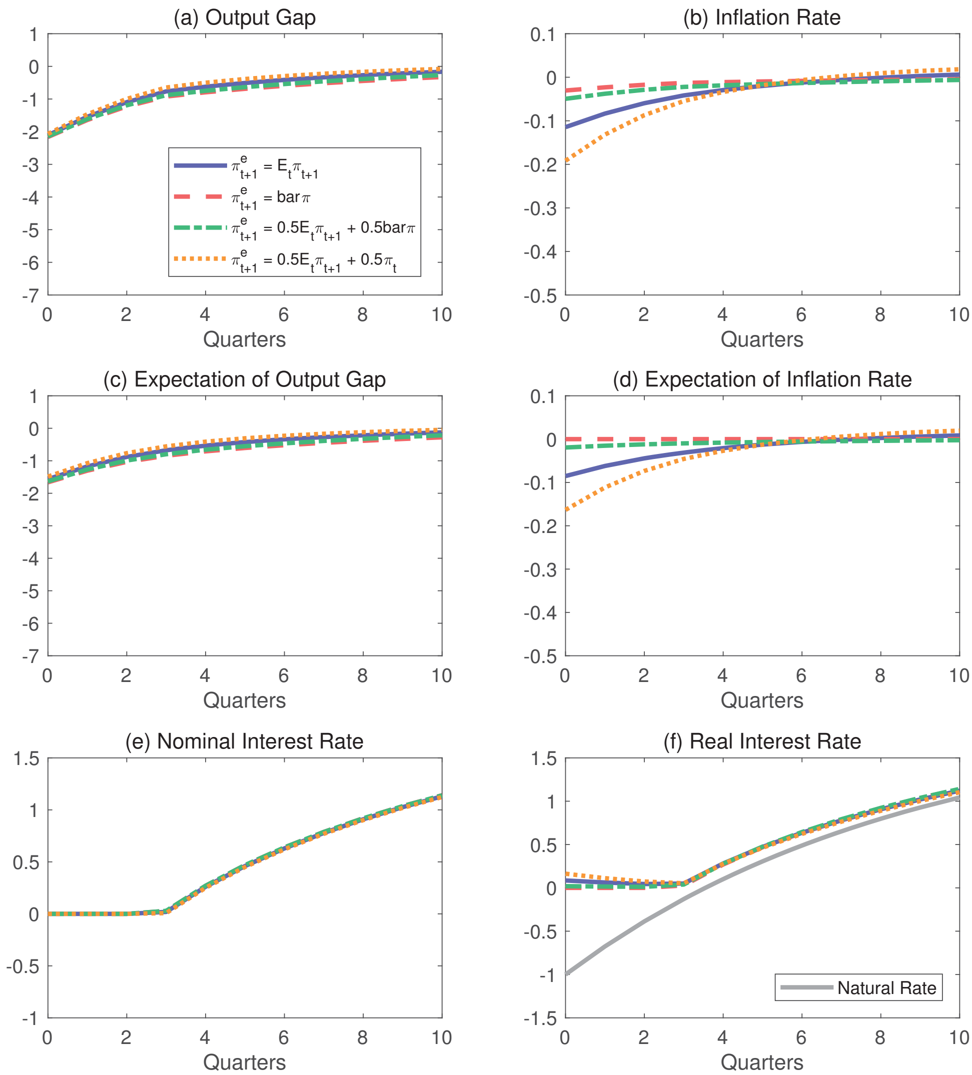


Figure A3: Impulse responses to an annual $-3\%$ natural rate shock under a simple rule with price-level targeting for different expectation formations for an inflation rate when we set $\sigma = 2$. Note: the scale of the figure is 'medium'.