Abstract

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JEL Classification
E1, E2, O4, N1, N30, P1

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ISSN 2206-0332

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Is Inequality Increasing in $r-g$? Piketty’s Principle of Capitalist Economics and the Dynamics of Inequality in Britain, 1210-2013

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Abstract. In his 2014 book, Thomas Piketty argues that wealth inequality is sharply increasing in $r-g$ and refers to $r > g$ as ‘the central contradiction of capitalist economics’, where $r$ is asset returns and $g$ is real income growth. To assess whether inequality is increasing in the $(r-g)$-gap this paper: 1) constructs unique annual data on asset returns for a balanced portfolio and several other variables for Britain over the period 1210-2013, and 2) examines whether the dynamics in the wealth-income ratio, $W/Y$, and capital’s income share, $S^W$, are governed by $(r-g)$. It is shown that $r$ and $g$ are robust and significant determinants of wealth and income inequality and that they have been the major forces behind the large inequality waves over the past eight centuries.

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1. Introduction

Central to Piketty’s (2014) Capitalism in the 21st Century is that the growth in wealth inequality is governed by the gap between the returns to wealth, $r$, and growth in economy-wide net national income, $g$. Piketty (2014) refers to $r > g$ as “the central contradiction of capitalist economics” (p. 398).

Assuming that real asset returns remain fairly constant, Piketty (2014) predicts that the $(r-g)$-gap will widen over the rest of this century because of reduced population and per capita income growth – a prediction that has been met with a storm of resistance from the economics profession, mostly because

1 Comments and suggestions from Hal Hill, Takatoshi Ito, Chris Meissner, Antonio Minniti, Solmaz Moslehi, Cyn-Young Park, Thomas Piketty, Andrew Rose, Holger Strulik, Harald Uhlig, Francesco Venturini, Yves Zenou and, particularly, Nicola Gennaioli (editor) and three anonymous referees are gratefully acknowledged. Comments and suggestions from seminar participants at University of Western Australia, Queensland University of Technology, University of Southern Denmark, University of Melbourne, Australian Economic Society, Kiel Institute of World Economics, Paris School of Economics, Australian National University, University of Aix-Marseille, University of Queensland, and participants at the Monash Macro/Finance Workshop, November 2016, the NBER Asian Seminar, July 2017, are also gratefully acknowledged. I am also grateful to the Australian Research Council for financial support (grants DP150100061 and DP170100339).
the \((r-g)\)-gap does not necessarily increase in response to a reduced \(g\) in standard canonical Euler equations, and because, in standard growth models, the inequality \(r > g\) ensures dynamic efficiency and is consistent with constant steady-state capital-income ratios (Abel et al., 1989). Piketty’s hypothesis that wealth inequality is increasing in the \((r-g)\)-gap is closely related to what is sometimes referred to Piketty’s ‘third law’, which says that inequality tends to diverge when \(r > g\).

Despite being central to the inequality debate, hardly any empirical work has been undertaken to investigate the relationship between inequality and the \((r-g)\)-gap, reflecting, to some extent, the absence of continuous long data on wealth/income inequality and asset returns, not to mention the difficulties associated with construction of a composite measure of \(r\) that includes all asset classes. Annual data on the wealth-income, \(W/Y\), ratio distributed on asset classes since 1970 have only recently become available for a few countries and there have been very few attempts, if any, to construct a composite \(r\) containing all asset classes. In his casual inspection, Piketty (2014, Figures 10.10 and 10.11) shows that a large positive and relatively constant \((r-g)\)-gap prevailed from Antiquity to the early 20th century in the world, and he argues that this was responsible for a high \(W/Y\) ratio and, consequently, high wealth inequality. However, Piketty’s (2014) analysis is based on very few observations, proxies \(r\) by bond rates, and fixes the interest rate at a constant level of 4.5% before the period 1700-1820, constructs that are hard to square with the finding in this paper.

Relying mostly on recent data, Acemoglu and Robinson (2015) and Goes (2016) are two of the few attempts to test whether inequality is increasing in the \((r-g)\)-gap. Acemoglu and Robinson (2015) fail to find significant positive effects of the \((r-g)\)-gap on top 1% income shares, and they conclude that inequality is driven by more important factors than the \((r-g)\)-gap, such as institutions. Almost the same conclusion is reached by Goes (2016) who finds that capital’s income share, as a proxy for inequality, is negatively related to the \((r-g)\)-gap for more than 75% of his country sample. However, the data used by Acemoglu and Robinson (2015) and Goes (2016) are highly problematic as discussed in Section 3.1.

This paper constructs an extensive annual macro dataset and examines whether wealth and income inequality in Britain has been driven by the \((r-g)\)-gap and, therefore, the extent to which the

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3 Piketty (2014) does not use the term “third law of capitalist economics”. Acemoglu and Robinson (2015) and Ray (2015) refer to \(r > g\) as “Piketty’s third law”. Acemoglu and Robinson’s definition of Piketty's Third Law is: “whenever \(r > g\); there will be a tendency for inequality to diverge” (2015, online Appendix p. 9).

4 Piketty and Zucman (2014) construct long historical \(W/Y\) data for the UK (1700-2010), France (1700-2010), the US (1770-2010) and Germany (1870-2010); however, their \(W/Y\) data are not detailed sufficiently on the wealth categories that are needed to use them as weights is a composite measure of asset returns. Furthermore, their data are heavily interpolated, often with 50 year intervals or more, and their historical wealth estimates are mostly based on discounted profit flows with constant discount factors, which, as argued by Madsen (2016), are likely to lead to biased estimates of wealth.
(r-g)-gap can be used as an analytical tool for inequality dynamics and inequality forecasting. More precisely, the paper examines the effects of r and g on the W-Y ratio and capital’s income share, SW, for Britain over the past eight centuries, where r is measured as a weighted average of after-tax real returns on non-human wealth. Capital gains on non-reproducibles are included in capital’s income share in most of the regressions in which capital’s income share is the dependent variable, because these capital gains increase permanent income and, therefore, form a part of capital income. This concept is consistent with Haig-Simons classical definition of income, which states that income is what we can consume while keeping our real wealth intact (see, for discussion, Roine and Waldenström, 2015).

The paper makes the following contributions to the literature. To enable tests of whether inequality is increasing in the (r-g)-gap, a unique annual historical dataset is constructed for Britain over the period 1210-2013 containing several control variables such as returns to assets, wealth and income inequality proxies, tariff rates, saving, openness, taxes, constraints on executive, contract-intensive money, and food prices among other variables. Asset returns are estimated as the average real returns to non-residential fixed capital, agricultural land, housing, government debt, net foreign assets, gold, silver, farm buildings, and livestock, weighted by the share of each asset in total wealth. In Sections 3.6.1 and 3.6.2 it is argued that the W-Y ratio and SW are sound proxies for wealth and income inequality and encompass key dimensions of inequality that are highly influential for inequality but that are not contained in conventional measures of inequality such as capital gains, omission of income of the very rich and inflation-induced changes in relative wealth positions; sources that have been and will probably always be highly influential for inequality as shown below.

The data cover several economic epochs in history, which not only enables assessment of whether the nexus between the (r-g)-gap and inequality applies to all modes of production, but also the extent to which the great historical waves in the W-Y ratio and SW are related to the (r-g)-gap. The analysis covers the late medieval period when agriculture was the dominant mode of production, the First and the Second British Industrial Revolutions where manufacturing steadily took over as the leading sector of growth, and the third phase, starting around the first oil price shock in 1973/74, in which information and communication technology services, ICT-services, have become the main growth promoting sector.

It is imperative to use ultra-long data in the analysis because the W-Y ratio, and the growth therein, and SW move at very low frequencies that even extend beyond a century, driven by slow-moving intergenerational wealth accumulation and low-frequency waves in asset prices. The finding that W-Y growth and SW fluctuate at low frequencies around a constant level in the ultra-long run, implies that estimates covering short periods will be sensitive to the trajectory of W-Y and SW in the
particular estimation period and, consequently, reduces the value of short-term analyses. The empirical estimates in Section 4.3, for example, indicate that only 2% of a disequilibrium in the $W-Y$ ratio is eliminated a year; thus, further suggesting that short-term analyses may not uncover a genuine relationship between $r$, $g$ and inequality. Furthermore, long historical data enable one to gain insight into the dependency of wealth accumulation on the source of returns; e.g. whether the returns are predominantly derived from dividend yields, as during the industrial epoch, or real capital gains on land, as during the pre-industrial period as well as the post-industrial period in which urban land is the principal component of wealth accumulation. Econometrically, long data increase the efficiency of the parameter estimates and reduce their bias. Davidson and McKinnon (2006), for example, show that instrument variable parameter estimates can be severely biased in small samples.

As another contribution, this paper tests whether the $W-Y$ ratio and $SW$ can be explained by $r$, $g$ and saving rates. Inflation, tariffs, and real food prices are used as instruments for $r$, $g$ and saving rates to deal with endogeneity and measurement errors. Furthermore, several control variables, which are often stressed as important determinants of inequality, are included in the inequality regressions to ensure that the coefficients of $r$ and $g$ are not capturing the impact of omitted variables on inequality. These are contract intensive money, constraints on executive, real prices of fixed capital, openness, old age dependency, skill premium, urban-rural wage gap, and taxes; variables that cater for globalization waves, institutional quality, demographics, inequality between wage earning groups, and labor-saving technological progress.

The empirical exercise gives three principal insights. First, three large waves in $r$ and the $(r-g)$-gap are identified over the past eight centuries and graphical evidence suggests that these waves coincide with the waves in inequality. The waves in the data are an outcome of political struggles between capitalists, the landed class and workers, and major shocks such as wars and epidemics; factors that result in inflations and deflations, fluctuations in tariff rates, and real food prices fluctuations. In the ultra-long run, however, the $W-Y$ ratio and $SW$ converge towards a constant, as predicted by extant models of economic growth. Second, it is shown that a large fraction of $r$ has been driven by real capital gains on non-reproducible and that these have been a great source of the evolution of inequality since 1210. Third, it is shown that $r$ and $g$ are highly significant and robust determinants of the $W-Y$ ratio and $SW$, and have the signs predicted by Piketty’s theory.

The rest of the paper is organized as follows. The nexus between the $(r-g)$-gap, inequality and economic theory and the main criticism of Piketty’s $r > g$-hypothesis are briefly discussed in Section 2, while data construction, data reliability, and graphical analyses are reviewed in Section 3. Regression results and robustness checks are presented in Sections 4 and 5, and Section 6 concludes.
2. The (r-g)-Gap, Inequality and Growth

2.1 The functional relationship between inequality and the (r-g)-gap.

According to Piketty (2014, 2015a, 2015b) inequality is increasing in \( r \) because it increases wealth accumulation, thus amplifying the initial heterogeneity of the wealth distribution, and it is decreasing in \( g \) because it reduces the value of existing wealth relative to the new wealth generated in the economy. The claim that \( g \), ceteris paribus, dilutes the \( W-Y \) ratio is consistent with the predications of the Solow model in which per capita capital in efficiency units is diluted by technological progress and population growth, and Kuznets’ (1955) hypothesis that wealth concentration and the \( W-Y \) ratio are diluted in high fertility regimes because of the reduction of the fraction of high-saving and low-fertility wealthy households.

However, Piketty’s proposition that \( r > g \) will lead to increasing inequality has been met with criticism. As stressed by Acemoglu and Robinson (2015) and Mankiw (2015), \( r > g \) holds in steady state in standard growth models, and yet, the capital-output ratio, \( K-Y \), remains constant. Throughout the paper \( K \) denotes fixed capital and \( W \) denotes total wealth. As a central model in macroeconomics, for example, the dynastic model poses the following steady state relationship:

\[
r = \gamma g^A + \rho, \tag{1}
\]

where \( r \) is real asset returns; \( g^A \) is productivity growth; \( \gamma \) is the inverse intertemporal elasticity of substitution; and \( \rho \) is the consumer’s time preference. If, for example, \( \gamma = 1 \), then \( r - g^A = \rho \) and it follows automatically that \( r > g^A \) in steady state.

Furthermore, there has been some confusion in the literature as to whether Piketty (2014) argues that (r-g)-gap has growth or level effects on inequality. The criticism of Mankiw (2015), among others, relates to some of the statements by Piketty (2014) that a sufficiently large (r-g)-gap has permanent growth effects on wealth inequality; partly reflecting the mixed signals given by Piketty.\(^5\)

\(^5\) Piketty (2014), for example, suggests that the (r-g)-gap has permanent growth effects on inequality by the following statement: “If the difference \( r - g \) surpasses a certain threshold, there is no equilibrium distribution: inequality of wealth will increase without limit, and the gap between the peak of the distribution and the average will grow indefinitely” (p. 258). Similarly, a higher saving rate will lead to permanently growing inequality: “If one saves more, because one’s fortune is large enough to live well while consuming somewhat less of one’s annual rent, then one’s fortune will increase more rapidly than the economy, and inequality of wealth will tend to increase even if one contributes no income from labor” (p. 351). This reasoning gains support from Solow (2014) who states that: “This is Piketty’s main point, and his new and powerful contribution to an old topic: as long as the rate of return exceeds the rate of growth, the income and wealth of the rich will grow faster than the typical income from work” (p. 1). However, in other places in his 2014 book and in later writings, Piketty suggests that the (r-g)-gap has level effects on inequality. For example, Piketty (2014) notes that “the distribution of wealth tends toward a long-run equilibrium and that the equilibrium level of inequality is an increasing function of the gap \( r - g \)” (p. 258). Furthermore, Piketty (2016) argues that the (r-g)-gap impacts on the level of inequality. For example, he remarks that “a central property of this large class of models is that for a given structure of shocks, the long-run magnitude of wealth inequality will tend to be magnified if \( r-g \) is higher” (p. 1).
As shown below and in the online Appendix, standard macro models predict a level-level relationship between the \((r-g)\)-gap and inequality proxied mostly by the \(W-Y\) ratio and \(S^W\). Another concern is whether changes in \(r\) and \(g\) have the same impact on inequality and, therefore, whether the \((r-g)\)-gap is the right metric to assess the forces that are guiding inequality. As shown in the empirical section, changes to \(g\) and \(r\) have quite different effects on inequality. To overcome these concerns the empirical analysis below includes \(r\) and \(g\) individually and allows \(r\) and \(g\) to have growth as well as level effects on inequality.

2.2 Is the \((r-g)\)-gap increasing or decreasing in \(g\)?

Piketty’s (2014) prediction of increasing inequality throughout the 21st century rests on the assumption that the \((r-g)\)-gap will decrease in response to a reduction in \(g\) in the 20th century without any counterbalancing response in \(r\). However, intertemporal utility-maximizing models predict that \(r\) is increasing in \(g\). Eq. (1), for example, shows that a reduced growth rate also reduces \(r\). If \(\gamma > 1\), then reduced \(g\) will result in a decreasing \(W-Y\) ratio because \(r\) declines more than proportionally to the decrease in \(g\). This is a well-known result and, as shown in the online Appendix, \(r\) responds approximately proportionally to changes in \(g\) in most of the oft-used growth models such as the Ramsey model; the dynastic model; and Diamond’s OLG model. Furthermore, Krusell and Smith (2014) show that saving rates beyond the Golden Rule and high intertemporal substitution rates are required to generate a negative relationship between the \((r-g)\)-gap and \(g\).

However, the nexus between the \((r-g)\)-gap and \(g\) are, for six reasons, much less clear-cut when utility-maximizing models are combined with endogenous growth models, for different values of the elasticity of substitution between capital, \(\sigma\), and labor in the Solow model, and when capital adjustment costs are allowed for, as shown in the online Appendix. First, the nexus between the \((r-g)\)-gap and \(g\) is negative in steady state, as predicted by Piketty, in the Solow model if \(\sigma\) is higher than 0.85 as an upper limit for the parameters included in the model. Second, most intertemporal models apply to modern industrialized economies with no uncertainty, perfectly functioning credit markets and in which even borrowing against future generations can take place; assumptions that are particularly difficult to maintain in the pre-industrial era. Third, Barro and Sala-i-Martin (1992) show that \(r\) is independent of \(g\) in steady state in the most influential first-generation models of endogenous economic growth (learning-by-doing models, the AK model, and product variety models). Fourth, Barro and Sala-i-Martin (1992) show that \(r\) may even be diminishing in \(g\) when investment adjustment costs are allowed for, as in all Tobin’s \(q\) models of investment. An increase in time preference, for example, shifts the preference as well as the production schedule up in the \(r-g\) space, resulting in an increase in \(r\) but a
decrease in $g$. Fifth, in Schumpeterian growth models in which growth is driven by creative destruction, returns to fixed capital are reduced by the rate the growth in creative destruction in steady state because new technology reduces the real value of the existing capital stock; thus establishing a negative relationship between growth and $r$.

Sixth, the inclusion of land in the production function further complicates the relationship between $r$ and $g$. In agricultural economies, as Great Britain before the 19th century and most developing countries today, one would expect a positive relationship between $r$ and population growth, $g^d$, because population growth drives up returns to land when the supply of land is inelastic, provided that the economy is relatively closed. The link between $g^d$ and $r$ is more complex in agrarian economies because a large fraction of consumers may be trapped in a Malthusian equilibrium and the extra earnings to land-holders from the growing land productivity depends on the elasticity of demand for agricultural produce. Finally, the link between $r$ and $g$ weakens as the economy financially opens up to the outside world and $r$ and $g$ become almost independent for a small economy under perfect capital mobility and fixed exchange rates, where fixed exchange rates have approximately prevailed in Britain over the period 1210-1973, except for the short-lived suspensions of the Gold Standard in the periods 1914-1925 and 1932-1948.

Using the data constructed in this paper the regressions in the online Appendix fail to show a significantly positive relationship between $r$ and $g$; a result that is consistent with the findings in the literature (see, for references of this literature, the online Appendix). In any event, the extent to which $g$ influences $r$ is immaterial for the results in this paper since $r$ and $g$ are simultaneously included in the inequality regressions.

3. Data construction

Most of the discussion in this section centers on the principal variables: $S^W$, the $W$-$Y$ ratio, $g$ and, particularly $r$, since, together with $g$, $r$ is the focus variable in the analysis and the data for $r$ in this paper, offers a significant advance over the proxies for $r$ used in the existing literature. Here, $S^W$ and the $W$-$Y$ ratio are used as proxies for inequality as discussed in depth in Section 3.6. Asset returns, $r$, are computed as a weighted average of real after-tax returns on non-human assets, where the weights are the share of each individual asset in the total portfolio. Following Piketty (2014) and Piketty and Zucman (2014), total wealth, $W$, is defined as total non-human private wealth at market prices and, therefore, includes productive and non-productive wealth and real capital gains on non-reproducibles.6

6 The definition of wealth used by Piketty (2014), Piketty and Zucman (2014) and in the analysis here, deviates from standard analyses in two respects. First, Piketty values wealth at market prices, which conflates the distinction between
The broad principles behind the data construction are presented in this section. Data sources, further details of data construction, data reliability, and the construction of control variables and instruments are discussed in depth in the online Appendix.

3.1 Asset returns

Asset returns on a private portfolio are constructed annually as the weighted average of the real returns to government debt, fixed non-residential capital, agricultural land, dwellings, livestock, gold and silver, and net foreign assets, where the weights are their respective shares in total private wealth computed as a chain index:

\[ r_t = \alpha_t^B r_t^B + \alpha_t^K r_t^K + \alpha_t^T r_t^T + \alpha_t^H r_t^H + \alpha_t^{G&S} r_t^{G&S} + \alpha_t^{LS} r_t^{LS} + \alpha_t^{FB} r_t^{FB} + \alpha_t^{NFA} r_t^{NFA}, \]  

(2)

where \( \alpha_t^X = \) asset \( X \)'s share in total wealth and \( r_t^X \) is asset \( X \)'s real expected after-tax returns;

- \( B = \) government debt/bonds;
- \( K = \) real non-agricultural non-residential fixed capital;
- \( T = \) agricultural land (crop and pastoral land);
- \( H = \) dwellings (value of residential structures plus urban land);
- \( G&S = \) gold and silver;
- \( LS = \) livestock and working animals;
- \( FB = \) farm buildings; and
- \( NFA = \) net foreign assets.

The after-tax returns for each individual asset are computed as:

\[ r_t^B = (1 - \tau_t^Y) r_t^{NB} - \pi_t^{CPI}, \]  

(3)

\[ r_t^K = (1 - \tau_t^Y) \left( \frac{\text{Div}}{p^S} \right)_t + g_t^S - \pi_t^{CPI}, \]  

(4)

\[ r_t^T = (1 - \tau_t^T) (1 - \tau_t^Y) \left( \frac{\text{Rent}^T}{p^T} \right)_t + g_t^T - \pi_t^{CPI}, \]  

(5)

\[ r_t^H = \left[ \theta_t (1 - \tau_t^H) + (1 - \theta_t) (1 - \tau_t^Y) \right] \left( \frac{\text{Rent}^H}{p^H} \right)_t + g_t^H - \pi_t^{CPI}, \]  

(6)

\[ r_t^{G&S} = g_t^{G&S} - \pi_t^I, \]  

(7)

\[ r_t^{FB} = (1 - \tau_t^Y) r_t^{RC} - \pi_t^{CPI}, \]  

(8)

\[ r_t^{NFA} = (1 - \tau_t^Y) \left( \frac{\text{Div}^{NFA}}{NFA} \right)_t + g_t^S - \pi_t^{CPI}, \]  

(9)

\[ r_t^{LS} = (1 - \tau_t^Y) r_t^{RC} + \pi_t^{LS} - \pi_t^{CPI}, \]  

(10)

changes in quantities and relative prices. This has implications for the dynamics of inequality and the mapping between Piketty’s framework and the predictions of growth models for which relative prices are generally assumed constant. Second, Piketty (2014) uses the terms ‘wealth’ (\( W \)) and ‘capital’ (\( K \)) synonymously, which has caused some confusion in the literature because the accumulation of distinctive types of wealth has very different implications for the dynamics and measurement of inequality as well as the feedback effects of the \( W/Y \) ratio on \( r \). In almost all growth models capital refers to fixed non-residential non-agricultural reproducible capital, \( K \), and an increasing \( K \), \textit{ceteris paribus}, is associated with a reduced \( r \) unless the elasticity of substitution between \( K \) and \( L \) (labor), exceeds one. As mentioned above, the letters \( K \) and \( W \) are strictly used here to distinguished wealth, \( W \), and fixed non-residential capital stock, \( K \).
where \( \tau^Y \) = income tax rate; \( r^B = \) real after-tax returns on government; \( r^{NB} = \) nominal bond rate on government debt; \( \pi^{CPI} = \) consumer price inflation rate; \( \text{Div} = \) dividends per stock; \( P^S = \) stock prices; \( g^S = \) capital gains on stocks; \( \tau^T = \) agricultural land taxes; \( \text{Rent}^T = \) land rent per hectare; \( P^T = \) price per hectare of agricultural land; \( g^T = \) capital gains on agricultural land; \( \tau^H = \) property taxes on dwellings; \( \theta = \) share of dwelling wealth that is owner occupied; \( \text{Rent}^H = \) unit housing rent; \( P^H = \) unit dwelling prices; \( g^H = \) capital gains on dwellings; \( g^{G&S} = \) capital gains on gold and silver; \( r^{RC} = \) nominal rent charges; \( \text{Div}^{NFA} = \) dividends on net foreign assets; \( NFA = \) value of net foreign assets; and \( \pi^{LS} = \) livestock price inflation (weighted average of prices of cattle, horses, oxen, pigs and sheep).

In most cases returns to each individual asset are self-explanatory; however, the returns to foreign assets, bonds and fixed capital require some discussion. Returns to foreign assets are calculated to proxy the sum of after-tax dividend yields on foreign assets plus capital gains on domestic stocks, where the dividend yield is estimated as the net income from abroad as a percentage of the net foreign asset position. Capital gains on domestic stocks are used for capital gains on foreign assets because the data on the growth in the net foreign asset position are not of a sufficiently good quality to be used; even the most recent data.\(^7\)

Sold at least as early as the late 12th. rent charges were the most important nominal debt instruments before the late 18th century (Clark, 1988) are also used to backdate nominal bond returns, \( r^{NB} \), before 1775. Rent charges are perpetual fixed nominal obligations that give the owner of the rent charge the right to receive a specified payment each year in perpetuity. Since they were secured by land, houses and other property they were considered as safe debt instruments with low default probabilities and were not affected by medieval usury laws because they were regarded as a sale in advance of rent from property and not as a loan (Clark, 1988). Selling rent charges were a popular way of raising capital until the late 18th century and these were sold at least as early as the late 12th century (Clark, 1988). As argued by Clark (1988) rent charges are probably the closest one can come to the long historical cost of capital. Although interest rates on bonds and other credit instruments are repeatedly used as proxies for asset returns in the literature, they are particularly biased measures of returns on balanced portfolios before well into the 17th century (see, for an in-depth analysis of

\[ g_t^F = g_{t-1}^{FA} - \left( \frac{CA}{FA}_{t-1} \right) \]

where \( CA \) is current account on the balance of payments, and \( g^{FA} \) is the growth in the value of the net foreign asset position. While this approach is theoretically correct, the generated capital gains are implausibly high during the Napoleonic War and, particularly, after 1969. Using this method capital gains apparently exceeded 600% each year over the period 1974-1976, in spite of falling global asset prices, and the annual capital losses exceeded 22% each year over the ten years up to 2013; a period during which the global capital markets experienced large capital gains.

\(^7\) I initially computed capital gains from net foreign assets using the following model:

\[ g_t^F = g_{t-1}^{FA} - \left( \frac{CA}{FA}_{t-1} \right) \]

where \( CA \) is current account on the balance of payments, and \( g^{FA} \) is the growth in the value of the net foreign asset position. While this approach is theoretically correct, the generated capital gains are implausibly high during the Napoleonic War and, particularly, after 1969. Using this method capital gains apparently exceeded 600% each year over the period 1974-1976, in spite of falling global asset prices, and the annual capital losses exceeded 22% each year over the ten years up to 2013; a period during which the global capital markets experienced large capital gains.
estimates of interest rates before the 18th century, online Appendix Section A4, and the discussion in Section 3.7).

The returns to non-residential non-agricultural fixed capital, $r^K$, are estimated as real after tax stock returns (see, Eq. (4)) because corporate earnings data are unavailable before 1900 and, more importantly, because stock returns adequately account for differential tax treatments of dividends and capital gains that are the result of additional fixed capital investment or share-buy-backs. Furthermore, in contrast to corporate earnings in national accounts, stock returns implicitly account for negative capital gains resulting from creative destruction in which new technologies replace older ones and, consequently, reduce the real value of the existing capital stock. The data on dividends and stock prices are available back to 1695 and real rental charges are used as proxies for returns to fixed capital before 1695.

The share of each asset in total wealth, $\alpha^X$, is based on the wealth estimates of Madsen (2016). The guiding principle behind the construction of the wealth data is that 1) non-residential non-agricultural fixed capital is valued by the volume of non-residential non-fixed capital stock multiplied by Tobin’s $q$; 2) urban and agricultural land are valued at market prices and are estimated as the market value of the average unit of asset times its quantity; and 3) the value of farm structures, gold and silver and livestock are valued at acquisition costs.8

The advantage of the approach used here is that it is transparent, is based on annually changing weights (chain index), covers the main assets in a balanced portfolio, includes capital gains on each individual asset, and allows for movements of the relative prices of capital/wealth and consumption (see, on the last point, Caselli and Feyrer, 2007). An alternative method, which is used by Acemoglu and Robinson (2015) and Goes (2016), is to recover real returns from the equation $r^{SK} = S^W \cdot Y/W - \delta$, where $r^{SK}$ is real returns recovered from capital’s income share and $\delta$ is the depreciation rate; however, this method requires data for $W$ that are only sparsely available, mostly do not include important assets such as land, livestock, foreign assets, gold and silver, and requires weighted averages of depreciation rates that vary substantially for asset classes and over time as the production structure changes.9 Problematic for this method is also that taxes cannot be adequately allowed for because they

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8 An oft-used method to generate wealth before WWII, is created from earnings records that are converted to wealth by multiplying by a fixed number; the so-called ‘years of purchase’ that is typically set to 20. In other words, this method assumes that earnings are perpetual discounted with a constant discount rate regardless of whether earnings are atypical and discount rates change over time and, as such, is a very rough approximation (see, for a critical assessment, Madsen, 2016).

9 Goes (2016) uses the post corporate-tax sovereign bond rate minus contemporary inflation. It is not clear, however, why the corporate tax rate as opposed to the direct tax rate, is used to compute the post-tax interest rate. In the robustness section he estimates returns as capital’s share times the $Y$-$K$ ratio from the Penn World Tables, where $K$ is total fixed capital. There are several troubles with his approach; most importantly that $S^W$ includes labor income of self-employed, wealth in the
are specific to each asset and type of return (dividend yield versus capital gain) and that capital gains on non-reproducible assets are not included in \( S^W \). Thus, \( r^{SK} \) is likely to be a severely biased proxy for the real post-tax returns on a balanced portfolio.

The downside of the estimates undertaken here is that returns on individual assets may underestimate those of wealthy individuals (Piketty, 2014) and returns to subsoil wealth are unaccounted for. Data on the value of and the returns to mines are not available and even current methods do not provide wealth estimates for mines but estimate subsoil wealth by discounting rents by a constant discount rate (see, for discussion, Caselli and Feyrer, 2007). Finally, although some of the data are interpolated on decennial frequencies this should not affect the results since the estimates are undertaken in 10-year intervals.

### 3.2 The \( W-Y \) ratio

The \( W-Y \) ratio is estimated as the ratio of private wealth at market values and net national income. Private wealth is computed as:

\[
W_t = q_t P^K K_t + P^K K_{FB} + P^K K_{Res} + P^K K_{Dur} + P^K T^{AGR} + P^K T^U
\]

\[
+ P^K T^U + G&S_t + NFA_t + Debt^G_t - P^K K_t^G, \tag{11}
\]

where \( K \) = stock of fixed non-residential and non-agricultural capital; \( P^K \) = price of fixed capital (non-residential investment price deflator); \( q \) = Tobin’s \( q \); \( K_{Res} \) = stock of residential structures; \( P^K \) = price of residential structures (investment deflator); \( K_{Dur} \) = stock of durable goods; \( K_{FB} \) = residential farm building stock; \( P^T \) = price of agricultural land; \( T^{AGR} \) = agricultural land area (arable plus pastoral land); \( T^U \) = urban land area; \( P^U \) = price of urban land; \( L^S \) = livestock; \( P^L^S \) = price of livestock; \( G&S \) = value of monetary and non-monetary gold and silver; \( NFA \) = value of net foreign assets; and \( Debt^G \) = net government debt.

While the components in the wealth estimate are self-explanatory, one may wonder why the value on fixed non-agricultural and non-residential capital stock is not estimated as the stock price times \( K \), as occasionally done in the literature, as opposed to \( q_t P^K K_t \). The trouble with the ‘conventional’ approach, however, is that real stock prices are not claims on the same units of capital but capital that is expanding over time through retained earnings. In other words, real stock prices do not increase in the long run because of capital gains but solely because of retained earnings – earnings that are not paid out as dividends but retained within the company for further expansion of the capital denominator of the \( Y-W \) ratio based on fixed capital, and returns are measured as gross returns; i.e. is inclusive capital depreciation.
stock or for share buybacks (see, for an exposition, Madsen and Davis, 2006). Share prices deflated by consumer prices of the firm that pays out all earnings tend to decrease over time as a result of creative destruction and investment-specific technological progress. Since Tobin’s $q$ normalizes real stock prices in units of capital, $q_t P^K_t$ is, to a first approximation, a good proxy for the market value of fixed non-agricultural, non-residential non-agricultural capital stock.

3.3 Saving

Private saving is estimated as follows:

$$ S^P_t = I^{Tot}_t + I^{LS}_t + CA_t + I^{G&S}_t, \quad (12) $$

where $S^P$ is gross private saving; $I^{Tot}_t$ is total private nominal gross fixed gross investment at acquisition costs; $I^{LS}_t$ is nominal investment in breeding and working livestock at acquisition costs; $CA_t$ is the current account balance at current prices; and $I^{G&S}_t$ is nominal investment in monetary gold and silver. The saving given by Eq. (12) extends conventional saving estimates by including investment in livestock, and gold and silver to make saving consistent with the wealth estimates. Furthermore, investment in livestock, and monetary gold and silver is part of saving because they are generated from income and are not yet consumed.

Total fixed gross investment is available from Feinstein (1988) after 1760. Before then, $I^{Tot}_t$ is computed as:

$$ I^{Tot}_t = I^{Ind}_t + I^{Ships}_t + I^{Res}_t + I^{FB}_t + I^{Land}_t, \quad (13) $$

where $I^{Ind}_t$ is industrial investment (mining and manufacturing); $I^{Ships}_t$ is investment in ships; $I^{Res}_t$ is investment in residential structures; $I^{FB}_t$ is investment in farm buildings (barns/stalls/byres); and $I^{Land}_t$ is investment in land improvement, enclosure and fencing. Investment in sector $X$, $I^X_t$, where $X = Ind$, Ships, Dwel, Agr and Land, is recovered from the equation, $I^X_t = K^X_t - (1 - \delta^X)K^{X}_{t-1}$, where $\delta^X$ is the depreciation rate for sector $X$’s fixed capital stock.

3.4 Share of income going to capital

Two measures of capital’s income share are used in the regressions as proxies of inequality; a conventional one and one in which capital gains on non-reproducibles are included:

$$ S^{W^U}_t = 1 - \Psi_{1855} \frac{w_t^{UL}(\Phi^A_t w_t^{UA} + (1-\Phi^A_t)(0.5+0.5w_t^{SU}))}{\gamma_t} P^{POP}_t, \quad (14) $$

and
\[ S_t^{WCG} = S_t^W + \left[ (g_t^T - \pi_t^{CP})\alpha_t^T + (g_t^H - \pi_t^{CP})\alpha_t^T \right] / Y_t, \]

where \( S_t^W \) = capital’s income share of net national income \textit{excluding} capital gains on non-reproducibles wealth (conventional case); \( S_t^{WCG} \) = capital’s income share \textit{including} capital gains on non-reproducible wealth; \( Pop \) = population; \( w_{UL} \) = annual wages of unskilled urban labor; \( wp^{UA} \) = the ratio of daily wages of unskilled and agricultural labor; \( wp^{SU} \) = the ratio of daily wages of skilled and unskilled labor; \( \Phi^A \) = the employment share of agriculture in total employment; \( Y \) = nominal net national income; \( g^H \) = capital gains on dwellings (underlying land plus structure); and \( \Psi_{1855} \) = a conversion factor to realign the level of \( S_t^W \) to capital’s share derived from national accounts in 1855, when national account data become available.

Compensation to employees (second term in Eq. (14)) is computed as employment times annual average earnings per worker, where workers’ average earnings are annual earnings of unskilled labor adjusted by the share of different types of labor and their hourly wages relative to that of unskilled labor, assuming that the ratio of hourly and annual wages are the same (annual income are only available for unskilled urban labor). The wage data are constructed by Humphries and Weisdorf (2016) and offer a marked improvement over previous wage data because they allow for variations in the annual number of working days, which have changed substantially over time and reliable estimates of annual hours worked are not currently available (see, for discussion, Madsen and Murtin, 2017). After 1855 \( S_t^W \) is estimated as one minus the share of compensation to employees in net national income, where labor earnings of the self-employed are imputed into compensation to employees (earnings from labor of the self-employed and working family members are counted as profits in national accounts). Nominal net national income is used in the denominator to ensure that \( S_t^W \) is based on earnings after capital depreciation.

Capital gains on urban and agricultural land are included in the second measure of capital’s share, \( S_t^{WCG} \), because they add directly to wealth inequality and, indirectly to income inequality when the assets are sold, borrowed against, or passed on to the next generation in the form of gifts or bequests, as argued in detail in Section 3.6.1. While capital gains on non-reproducibles are not included in national account estimates of \( S_t^W \), capital gains on fixed capital, in the form of capital gains on stocks, are implicitly contained in \( S_t^W \) as corporate earnings in steady state, since, as discussed in Section 3.2, real stock prices only increase in the long run because of retained earnings, which are included in \( S_t^W \).

3.5 Income
The real and nominal GDP data are from Clark (2010) (real net national income) over the period 1210-1270; from Broadberry et al. (2015) over the period 1270-1820 and spliced to Bank of England’s GDP
data after 1820. The data constructed by Broadberry et al. (2015) constitutes the most thorough and probably the most credible reconstruction of long historical GDP data undertaken thus far.

3.6 Factor income shares and the W-Y ratio as measures of inequality

There are considerable difficulties associated with the measurement of wealth and income inequality. The existing wealth inequality data are generally of low quality, cover relatively short time periods, have breaks, and contain a lot of missing observations, suggesting that alternative measures of wealth inequality are called for (see, for a thorough discussion of the existing wealth inequality data, Roine and Waldenström, 2015). Similarly, there are several breaks in the top income inequality data and one needs to splice various top deciles to get continuous series. The only relatively continuous long data available are top 0.05% income share-tax units covering the period 1908-1994. Thus, while data on top 10% or top 1% income shares can be constructed over the period 1908-2013, they need to be spliced from data of many different top income deciles (see Roine and Waldenström, 2015). This section argues that $S^W$, $S^{WCG}$, and the $W$-$Y$ ratio not only overcome the relatively short length of the existing data, they are also solid and reliable indicators of inequality.

3.6.1 Factor income shares and inequality

The share of income going to capital is generally accepted as a good proxy for income inequality and the literature tends to find a strong positive relationship between inequality and the share of income going to capital (Giovannoni, 2010; Jacobson and Occhino, 2012; Adler and Schmid, 2013; Bengtsson and Waldenström, 2017). However, factor shares may recently have weakened as indicators of inequality because the share of high income earners among employees has increased (Piketty, 2015a, 2015b) and there is little, if any, empirical evidence on the correlation between factor shares and inequality before the 19th century. This raises the questions of whether factor shares are good proxies for inequality before the 20th century and whether the recent increase in income dispersion between employees has diluted income shares as indicators of inequality. To this end we need to establish how inequality and factor income sources relate and use this analysis to determine the validity of the share of capital as a proxy for inequality.

As shown more formally in the online Appendix Section A1, the share of income going to capital is a good inequality proxy if, 1) earnings between employees are compressed while the income

10 For the UK wealth inequality data have been constructed by Atkinson and Harrison (1978) and extended by Atkinson (see Roine and Waldenström, 2015) by adjusting tax data of the deceased by the inverse mortality rates for different age, sex and social groups. The data are available for 1800, 1810, 1911, 1923-1930, 1936, 1938, 1950-2005 except 1963 and 2004 (Roine and Waldenström, 2015).
variation between wealth holders varies substantially; 2) employees derive little income from wealth; 3) there is either little unemployment or the ratio of unemployment benefits to average wages (replacement rate) is close to one; and 4) the correlation between earnings of high-income employees and capital income is high. These factors are not mutually exclusive: an increasing wage dispersion between employees, for example, may not weaken the relationship between income inequality and factor shares if earnings of the high-income employees are highly correlated with capital income. Furthermore, a large gap between employees will not affect the value of factor shares in regression analyses if the gap remains constant over time since the identifying variation in regression analysis stems from the variation in inequality and not from the level of inequality.

The share of income going to capital, $S^W/S^{WCG}$, is likely to be a good indicator of personal income and wealth distribution well into the 20th century when there was a strong and clear divide between owners of land and fixed capital and workers. The working and the landless classes were representative of almost the entire wage-earning population and this class derived little income from wealth (Prados de la Escosura, 2008). Similarly, Kuznets (1955) finds that the middle class was very small in pre-industrial societies. From the 18th century and, presumably earlier, up until WWI approximately 90% of the English land was cultivated by tenants (Offer, 1991), suggesting that wealth was highly concentrated and that employees drew little income from capital. Furthermore, the ratio between the variability in $S^{WCG}$ and the variability in the skill premium (building craftsmen versus building labor) and the urban premium (urban building labor versus agricultural labor) is 97.8 and 16.8 over the period 1211 and 1869 (the data terminate in 1869), suggesting identifying variation in income inequality originates predominantly from variations in factors shares and not from variation between the main wage-earning groups.

More recently, factor shares may have weakened as indicators of inequality because the share of high income earners among employees has increased (Piketty, 2015a, 2015b) and because the share of agricultural land rent in total capital income has diminished over time. However, several recent studies suggest that the positive correlation between the personal and the functional income distribution has remained strong in the post-WWII period as high income earning employees have experienced large real capital gains on their assets and have large pension funds that are invested in assets (Roine and Waldenström, 2012). Furthermore, Gennaioli et al. (2014) show that financial income is a function of intermediated wealth, which, in conjunction with financial sector employees being a large fraction of high income earners, gives an indirect, but strong, link between employees’ inequality and the $W-Y$ ratio and $S^W$. For the UK, Bell and Van Reenen (2014) find that bankers’ bonuses account for two-thirds of the increase in the top 1% share over the period 1999-2008; thus confirming a strong link between profits and high-income employees.
Capital gains are significant sources of inequality yet they are unaccounted for in conventional inequality proxies such as top income and wealth shares and Gini coefficients. In contrast to real capital gains on non-reproducibles, real capital gains on stocks are implicitly accounted for in factor shares, $S^W$, because they, at least in steady state, are a result of retained company earnings that increase the capital base of the company. Here, $S^{WCG}$ is used as the preferred measure of inequality because it includes capital gains on non-reproducibles, noting that capital gains have historically been significant sources of asset returns of the landed class in the pre-industrial period and of homeowners in the post-industrial period. Furthermore, capital gains are larger sources of post-tax inequality than dividend income because they were tax exempt before the Finance Act in 1965 and since then been subject to tax rates that are well below income tax rates.

Conventional income inequality measures will often give misleading pictures of capital gain induced movements in income and wealth inequalities. According to the permanent income hypothesis, wealth mirrors permanent income over peoples’ lifespans, therefore capital gains on non-reproducibles should be included in any measure of income dispersion. This result holds in dynastic models where consumption is spread over future generations as well as in economies with low social mobility and in which inheritance is a large fraction of income. As shown for France by Piketty (2014), inheritance is a large fraction of income and tends towards approximately 20% in the long run. Examining the period 1270-2010, Clark (2014) finds a low social mobility in England over the period 1170-1800 with only a slight increase after 1800. The intergenerational correlation of underlying social status jumps from 0.83 before 1800 to 0.73 thereafter, suggesting that the stubbornly low social mobility that prevailed in mediaeval England has almost persisted to the present day. The low social mobility implies a persistent wealth distribution and that the relative income position of an individual will be echoed in the individual’s relative wealth position, particularly when capital gains on non-reproducible assets are allowed for in the estimates of factor shares.

Capital gains have remained a large source of the income of top-income earners throughout the last century in the advanced countries. Based on data for Sweden, Roine and Waldenström (2012) find that the top 1% income share in total income was, on average, 40% higher over the period 1990-2008 when realized capital gains are counted as income. For the US over the period 1916-1998, Piketty and Saez (2003) find that 18% of the income of the top 1% share, on average, is attributed to capital gains. Capital gains, furthermore, amplify the income inequality between employees as high-income employees in the finance sector receive income from bonuses that are often linked to capital gains on real estate, earnings of real estate agents are often set in a fixed constant proportion to property values, and CEO share options. For Britain, the real capital gains share of total real asset returns has fluctuated between 15% and 30% over the period 1210-2013 and has been above 20% over the past two centuries.
if the real capital losses on government bonds are omitted from the estimates (see online Appendix for data sources).

3.6.2 $W$-$Y$ ratio and inequality

There are several reasons why the $W$-$Y$ ratio is a good proxy for wealth and income inequality and why the $W$-$Y$ ratio serves as a fundamental complement/substitute for other inequality measures; particularly wealth inequality. First, since inequality in wealth holdings is higher than that of income, the overall income inequality will increase in response to an increase in the $W$-$Y$ ratio provided that the relative dispersions of $W$ and $Y$ are preserved (Krussel and Smith, 2015). Second, asset values directly affect earnings of high income employees through bankers’ bonuses, CEO share options, owners of rental accommodation where the rent is linked to property values, and earnings of real estate employees that are linked to property values.

Third, agricultural and urban land are large sources of inequality through the $W$-$Y$ ratio because they are concentrated in few hands and are large fractions of total wealth. The finding of Offer (1991) that approximately 90% of the English land, at least over the 18th and 19th centuries, was cultivated by tenants is strong evidence of land wealth being concentrated in a few hands. Similarly, Lindert (1986) estimates that, on average, over the period 1670-1875, 86% of total net wealth in England was in the hands of the 10% wealthiest (preferred estimates, Table 4). Furthermore, the wealthy landed class is likely to have enjoyed higher capital gains on their lands than poor farmers: Not only did the wealthy own the most fertile land they were also disproportionally more affected by movements in the $W$-$Y$ ratio than people with little or no wealth because the variance in the $W$-$Y$ ratio was dominated by fluctuations in agricultural land prices before the 20th century (Offer, 1991). The large landholders also tended to have scale advantages and to implement land-improving new technologies at faster rates than smallholders because they had easier access to credit and larger saving propensities (Offer, 1991).

Urban land values have long been significant sources of wealth inequality as evidenced by low homeownership ratios at least up to the mid-20th century, combined with large variations in urban real estate values. The homeownership ratio was probably approximately 20% over the period from 1798 to 1939, increased to approximately 30% from 1939-1961 and it was first after 1970 that the homeownership ratio climbed above 50%, suggesting a very unequal distribution of housing wealth throughout history, amplified by a highly unequal distribution of housing wealth between home owners (Bethan and Dorling, 2004; Gyourko et al., 2013; the homeownership data, which are not available before 1798, are listed in the online Appendix).

The 15-fold increase in real house prices over the period 1950-2007 has contributed substantially to the increasing inequality since 1982 and, conversely, prevented inequality from
decreasing more than it did in the post-WWII period up to 1982. Furthermore, the housing wealth of the wealthy in the post-WWII period has increased proportionally more than that of the underprivileged class because the wealthy live in the more sort-after areas and in wealthier cities where urban land supply elasticities are significantly lower than those of the poorer neighborhoods and cities (Bethan and Dorling, 2004; Gyourko et al., 2013). Building on a model with overlapping generations of heterogeneous altruistic households, Borri and Reichlin (2015) show that Baumol’s cost disease in the building industry is partly responsible for the increasing housing share in wealth, wealth-to-income ratios and wealth inequality, especially since housing comprises a sizable share of intergenerational bequests.

Fourth, the $W/Y$ ratio impacts directly on income inequality through the share of income going to capital, $S^{WCG}$, following Piketty’s (2014) first law of capitalist economics, viz., $S^W = r \times W/Y$. Provided that returns to fixed capital are not significantly affected by the $W/Y$ ratio, it follows that the $W/Y$ ratio is positively related to inequality as the share of income derived from assets is a steeply positive function of income, as argued above (see also Piketty, 2015a). A potential problem associated with this reasoning, as often pointed out in the literature, including Piketty (2015a, 2015b), is that $S^W$ is not positively related to the $W/Y$ ratio if an investment-induced decline in the marginal productivity of capital dominates the positive effects of the capital-induced increase in the $W/Y$ ratio. This is the case if the elasticity of substitution between labor and capital is below one, which seems to be the consensus in the literature (Jones, 2003; Ray, 2015), though recently challenged by Karabarbounis and Neiman (2014) and Piketty (2015a, 2015b). It cannot be ruled out, however, that the estimates of elasticity of substitution do not adequately capture long-run adjustments, noting Jones (2003) shows that that the elasticity of substitution is small in the short run but approaches one in the long run because the long-run elasticity of substitution is governed by the distribution of ideas; i.e. the ease with which new ideas appropriate for a different input mix can be discovered.

Importantly, however, this line of reasoning only applies to reproducible factors of production that are subject to diminishing returns and for which real capital gains are zero in the long run. Reproducibles, such as livestock and fixed capital, however, are only a relatively small fraction of $W$. Increases in the $W/Y$ ratio induced by non-reproducible wealth, such as agricultural and urban land, come almost entirely from real capital gains since the supply of agricultural and urban land is quite inelastic. Similarly, the returns to precious metals and foreign assets are independent of the quantity of these assets held by the public. Using the estimates in this paper, the value of fixed capital has, on average, only been 19% of total wealth over the period 1210-2013, and 17% in the period 1750-2013. Thus, in response to $W/Y$ expansions that preserve the distribution of wealth on asset types, capital’s
Income share will only increase if the elasticity of substitution between capital and labor is extremely low, suggesting that capital’s income share is, on average, increasing in the $W-Y$ ratio.\footnote{For the CES production function, $Y^R = \left[\alpha K^{(\sigma-1)/\sigma} + (1-\alpha) L^{(\sigma-1)/\sigma}\right]^{(\sigma-1)/\sigma}$, the rate of returns on non-residential and non-agricultural fixed capital is given by $r^K = \alpha (K/Y)^{-1/\sigma}$, where $\sigma$ is the elasticity of substitution between non-residential and non-agricultural fixed capital and labor and $Y^R$ is real income. This implies that capital’s income share is given by $S^K = r^K \cdot K/Y = \alpha (K/Y)^{\sigma/(\sigma-1)}$, where $S^K$ is the share of income going to non-residential. The share of income going to capital, $S^W$, is given by $S^W = S^K + S^T = r^K \cdot K/Y + r^T \cdot T/Y = \alpha \left(\frac{W}{Y}\right)^{\sigma/\sigma} + r^T (1-\varphi) W/Y$, where $\varphi$ is the share of fixed capital in total wealth; and $T$ is the value of land. Thus, the change in the share of income going to wealth holders in response to a change in wealth that keeps $\varphi$ constant, is given by: $\frac{\partial S^W}{\partial (W/Y)} = \alpha \varphi \left(\frac{W}{Y}\right)^{\sigma/\sigma} + r^T (1-\varphi)$, Using the approximate average values for Britain over the period 1210-2013, $\alpha = 0.3$, $\varphi = 0.2$, and $W/Y = 10$ (so $K/Y = 2$), and setting $\sigma = 0.5$, then $S^W$ is a positive function of the $W-Y$ ratio if $\frac{\partial S^W}{\partial (W/Y)} = -0.015 + r^T > 0.0187$, where 0.0187 is approximately a third of the returns to land estimated here. Here it is assumed that the returns to land are fixed and independent of investment in fixed capital stock; however, it is likely that the returns to land are an increasing function of investment in land improvement and structures. For the given parameter values this result holds for any value of the elasticity of substitution $\sigma > 0.006$ for $W-Y = 5$.

One may also question the assumption of diminishing returns to fixed capital in a globalized economy. The increasing globalization, starting during the early industrialization, may have partly counterbalanced diminishing returns to fixed capital through foreign direct investment including outsourcing, overseas portfolio investments and immigration flows. Under fixed exchange rate systems, which prevailed in Britain for most of the period 1210-1971, and perfect capital mobility, the returns to capital would be equalized across the world and counter diminishing returns. The massive British overseas investment during the second half of the 19th century and up until the outbreak of WWI (O’Rourke and Williamson (2002), is a testimony of how overseas investment opportunities counter diminishing returns to domestic capital in periods of high saving rates. The regressions in Section 5.1 do not give any evidence of diminishing returns to fixed capital, suggesting that globalization, population growth dynamics under a possible Malthusian regime, and cross-border movements of labor were effective means of counteracting diminishing returns for Britain.

Finally, the model of Kesten (1973) gives an explicit mapping between $r$, $s^W$, and $g$ and the $W-Y$ ratio (see, also Acemoglu and Robinson, 2015, online Appendix). The law of motion of individual $i$’s assets, $W_i$, is given by:

$$ W_{i,t+1} = (s^W + r + \varepsilon_{it})W_{i,t} + Z_{it}, $$

where $s^W$ is the marginal propensity to save out of wealth, estimated as total private saving divided by private wealth in the empirical section (not to be mistaken for shares of income going to wealth, $S^W$); $Z$ = stochastic labor income; and $r + \varepsilon_{it}$ = the stochastic rate of asset return (see, for a complete...
derivation, the appendix of Acemoglu and Robinson, 2015, online Appendix). Individual $i$ is assumed to be infinitely lived.

Dividing this model by $Y_t$ and normalizing the $W/Y$ ratio to one, $w$, yields:

$$w_{t+1} = q w_t + z_t,$$

where $q = \frac{r + s^W + e_{it}}{1+g}$. Provided that $E(q) < 1$, this model predicts that the distribution of $W$ converges to a stationary distribution with a Pareto tail:

$$B(W) = 1 - \Pr[W_{t+1} \leq W] = \Gamma^{-\nu},$$

where $\nu$ is the Pareto exponent, $\nu \geq 1$, and inequality decreases with $\nu$. If the entire distribution is Pareto, then the top $k$ percentile’s wealth share is $(k/100)^{(1-\nu)/\nu}$. For example, if $\nu = 2$ it follows that the fraction of wealth belonging to the top 1% is 10%.

Assuming that $q$ is log normally distributed with a mean of $[\ln(1 - \bar{z}) - \sigma^2/2]$ and a variance of $\sigma^2$, Acemoglu and Robinson (2015, online Appendix) show that:

$$\nu \approx 1 - \frac{\ln(r-g + s^W)}{\sigma^2/2},$$

from which it follows that $\nu$ is decreasing in $r - g + s^W$ and increasing in $\sigma^2$. Wealth inequality is a positive function of $(r - g + s^W)$ as wealth capitalizes at a faster rate and is less likely to be overtaken by growth. This analysis suggests that $(W/Y)_t$ is governed by $r$, $g$, $s^W$, $\sigma^W/Y$ and $(W/Y)_{t-1}$ — a model specification that guides the empirics presented below.

3.6.3 Correlation between various inequality proxies

Table 1 displays the correlation between $S^w$ and the $W/Y$ gap and the often-used proxies for inequality (top 0.5% income, the Gini coefficient and top 10% wealth share). Capital gains on non-reproducibles are not included in $S^w$ because they are not accounted for in the considered inequality measures. The sample period covers the maximum period at which almost continuous data for top 0.5%, the income Gini, and top 10% wealth are available and the numbers in parentheses are $t$-ratios.

Considering first the $S^w$ column, the correlation coefficients are statistically and economically highly significant, suggesting large commonalities between $S^w$ and conventional inequality measures. These results are consistent with the findings of a strong positive relationship between personal and

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12 A similar relationship can be derived even if $q$ is not log normally distributed (see, for an exposition, Acemoglu and Robinson, 2015, online Appendix).
functional income distributions for other countries (see Giovannoni, 2010, for the OECD countries in the Post-WWII period; Jacobson and Occhino, 2012, for the US; Adler and Schmid, 2013, for Germany; and Bengtsson and Waldenström, 2017, for OECD countries over the last century).

Remarkably, the correlation between the $W/Y$ ratio and $S^W$ is highly significant (note the long data period and the high $t$-statistics). This result has the important implication that the potential response of $r$ to a change in the $W/Y$ ratio is either absent or not sufficiently large to counteract the effects of the change in the $W/Y$ ratio on $S^W$, which may be due to one or more of the following: 1) that the elasticity of substitution between labor and fixed capital exceeds one; 2) that changes in the $W/Y$ ratio have been dominated by real capital gains on non-reproducibles and less so by changes in fixed capital to which diminishing returns applies; 3) that diminishing returns to fixed capital have been counteracted by a highly elastic supply of labor in a globalized economy or, more likely, that, at least into the early 20th century, there has been a highly elastic supply of rural labor readily available to operate the machinery in the urban areas; 4) that overseas direct investment has been a large fraction of total fixed investment and diminishing returns have been overcome by the sheer size of the foreign markets; and/or 5) that the positive relationship between $S^W$ and the $W/Y$ ratio is partly driven by feedback effects from $S^W$ to the $W/Y$ ratio through saving. These are important issues, which I will return to in several places below.

Table 1. Correlation coefficients between inequality measures

<table>
<thead>
<tr>
<th>Dep Var.</th>
<th>$S^W$</th>
<th>$W/Y$</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 0.5% Income</td>
<td>0.94(17.7)**</td>
<td>0.60(4.10)**</td>
<td>1918-2013</td>
</tr>
<tr>
<td>Income Gini</td>
<td>0.55(-5.09)**</td>
<td>0.36(-1.97)**</td>
<td>1948-2013</td>
</tr>
<tr>
<td>$W/Y$</td>
<td>0.57(15.6)**</td>
<td>1.00</td>
<td>1221-2013</td>
</tr>
<tr>
<td>Top 10% Wealth</td>
<td>0.53(4.76)**</td>
<td>0.64(7.09)**</td>
<td>1933-2003</td>
</tr>
</tbody>
</table>

Notes. The numbers in parentheses are $t$-statistics and are based on heteroscedastic and serial correlation consistent standard errors. The data are measured in 10-year overlapping averages following most of the estimates below. See the online Data Appendix for data sources. The wealth inequality data are interpolated between the following years: 1931-1936 (linearly interpolated), 1936-1938 (linearly interpolated), 1938-1950 (interpolated using top 1% income shares); and 1962-1964 (linearly interpolated). *** = significant at the 1% level; ** = significant at the 5% level.

Turning to the $W/Y$ column in Table 1, the top 0.5% income share and the income Gini are positive and statistically significantly correlated with the $W/Y$ ratio; as expected, however, not nearly as significant as their correlation with $S^W$. A statistically much more significant relationship is found between the $W/Y$ ratio and the top 10% wealth share and this is likely to have been much higher had the wealth inequality data not recently been flat although there are strong reasons to believe that wealth inequality has been increasing in the UK since 2000 (Alvaredo et al., 2016). These results are highly indicative of the $W/Y$ ratio being a strong proxy for wealth inequality but less so for income inequality – as intended in the regression analysis below.
Overall, capital’s income share and the $W-Y$ ratio are likely to serve as excellent proxies for inequality because 1) they are significantly correlated with conventional inequality measures; 2) they are available on an annual basis over long periods; 3) real capital gains on non-reproducibles are allowed for in $S^{WCG}$ and the $W-Y$ ratio; 4) they capture the fact that high-income earners derive a significant part, if not the greater part, of their income from wealth or income that originates from profits or capital gains; 5) $W-Y$ affects inequality through Piketty’s second law; and the $W-Y$ ratio includes assets that are rarely accounted for in conventional wealth inequality proxies, such as gold and silver. Econometrically, since $S^W$ and the $W-Y$ ratio are used as dependent variables, their measurement errors will only bias the parameter estimates of the key variables in the structural equations to the extent to which their measurement errors are correlated with the regressors. This is unlikely to be the case to the extent that within-group income and wealth dispersions are uncorrelated with $r$ and $g$.

3.7 Reliability of the data

This section briefly discusses the quality of the historical data vis-à-vis modern data presented here and the extent to which the data offer advances over previous attempts to construct asset returns. Pre-industrial data are often thought of as being of lower quality than contemporary data because the they were collected less frequently, had a narrower geographic coverage, and some data were lost in fires and wars, or were discarded. While this is true, these issues have to be held up against the fact that the economy back in time was substantially simpler and easier to measure than now because production, consumption and investment consisted of fewer and simpler and homogeneous goods in which the quality hardly changed over time and new products were infrequently introduced. However, this simplicity has been exponentially eroded since the onset of the industrialization. Industrialization accelerated the number and the quality of new products, created intangibles for which returns cannot be easily measured, resulted in an increasing heterogeneity of the housing market that has complicated the measurement of house price dynamics, and expanded the asset portfolio through an explosion of debt instruments and resulted in an increase in a large variety of bond types (partly through options). Furthermore, tax havens, trusts, foreign investment etc. have obscured after-tax returns on assets.

The accelerating range and quality of new products have complicated the decomposition of nominal accounts into volume and price indexes; particularly for investment goods in which most technological advances since industrialization have not been adequately incorporated (see, e.g., Gordon, 1990; Karabarbounis and Neiman, 2014). Similarly, housing wealth is substantially harder to measure today than in the past because an increasing variety of structures and heterogeneity of urban land prices. This has become an increasingly important issue because housing has taken over from
agricultural land as the most important asset in total wealth. Thus, the compositional change of wealth since circa 1870, when agricultural land was the dominant source of wealth, has increased the share of assets for which values and returns have become increasingly difficult to measure.

Comparatively, this study offers advances over previous studies in terms of data quality by being probably the first study to construct a composite measure of asset returns, noting that Piketty and Zucman (2014) constructed wealth aggregates, not returns. Very few, if any, composite measures of asset returns have been constructed and most researchers, including Piketty (2014) and Acemoglu and Robinson (2015), rely on real bond returns or returns to fixed capital. Real after-tax bond returns are well-known to be well below the returns to stocks and most other non-fixed income assets and to be sensitive to unexpected inflation while the returns to land, precious metal, livestock and fixed capital, at least if relative prices/wages remain constant, are hedged against inflation. Thus, while the returns estimated here contain measurement errors, they offer big improvements over previous asset return proxies; particularly those of Piketty (2014, Figure 10.9), where real after-tax asset returns over the past 2000 years are based on a few observations for bond rates after 1700 and assumed to be 4.5% over the period 0-1700.

Since asset returns, $r$, is a key variable in this study and the magnitudes and time-profiles of interest rates over the period from the 13th to the 17th century have been highly controversial, the online Appendix A4 presents an in-depth analysis of interest rates during this period. Growth, $g$, as the other key explanatory variable, has been much less controversial and its validity has been discussed in depth by Broadberry et al. (2015). In the absence of source material on asset returns across a broad range of debt instruments in the medieval period, the historical literature has predominantly relied on returns on risky short-term debt instruments, which tend to give misleading pictures of returns to the most important private assets: Land and fixed capital. In their encyclopedia of the history of interest rates, Homer and Sylla (2005, p. 89) show that the scant evidence on interest rates in medieval England, points towards prohibitively high interest rates on short-term debt: In 12th century England the annual compound interest rate on short-term credit was over 52% for safe loans and 80-120% on loans with poor security.

In an influential study, McCloskey and Nash (1984) infer medieval net short-term interest rates of well over 30% from seasonal grain prices in England over the period 1260-1399 under the assumption that grain storage reflects the opportunity costs of credit. Since the arithmetic average of the rate of inflation over the period 1211-1400 was approximately one percent, inflation is not the major source of the differential interest rates used by Piketty (2014) and those estimated by McCloskey and Nash (1984), noting that Piketty (2014) does not refer to any source material nor does he explain why he arrives at a 4.5% interest rate. As argued in the online Appendix A4, the method of McCloskey
and Nash (1984) is highly problematic: the method yields results that are highly sensitive to the choice of cereal type, period, geographic coverage, and the length of storage and, more seriously, yields standard errors that are mostly multiples of the estimated interest rates, pointing toward confidence intervals spanning well into negative ranges. Furthermore, interest rates in excess of 30% and even rates above approximately 15%, are shown to be inconsistent with macro theory, other evidence and the time preference theory of Becker and Mulligan (1997).

Although crucial for this paper, the evidence on returns to long-term debt instruments up to the 17th century is even more scant. Before the mid-17th century, approximately 90% of private assets were composed of agricultural land and fixed capital (barns, land improvement, dwellings, ships, livestock, small-scale industry and enclosure); thus indicating the need to focus on long-term debt instruments. The evidence presented in the online Appendix A4, points towards the nominal returns on long-term loans secured by property of 8-10%, which are comparable with average nominal returns of 8.5% estimated here over the period 1210-1400.

3.8 Summary statistics

Table 2 provides summary statistics of the most important variables. Most noticeable is the high standard deviations of \( r \) and \( g \), reflecting the significant shocks of the past, such as the Black Death and occasional prolonged crop failures, that often resulted in reduced growth and accelerating prices. The high, usually unexpected, price volatility translated into large swings in capital-gains adjusted income shares, \( S^{\text{WCG}} \), that occasionally go well beyond the limits of the income-only factor shares, \( 0 < S^{W} < 1 \).

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S^{W} )</td>
<td>0.545</td>
<td>0.074</td>
<td>0.304</td>
<td>0.699</td>
<td>0.052</td>
<td>0.104</td>
<td>-0.326</td>
<td>0.575</td>
</tr>
<tr>
<td>( S^{\text{WCG}} )</td>
<td>0.588</td>
<td>0.402</td>
<td>-0.885</td>
<td>4.054</td>
<td>0.007</td>
<td>0.057</td>
<td>-0.292</td>
<td>0.239</td>
</tr>
<tr>
<td>( W/Y )</td>
<td>5.437</td>
<td>1.152</td>
<td>3.307</td>
<td>9.465</td>
<td>0.024</td>
<td>0.024</td>
<td>-0.063</td>
<td>0.173</td>
</tr>
</tbody>
</table>

Notes. SD = standard deviation. The data are measured annually and in decimal points. The data period is 1211-2013 (\( N = 803 \)).

3.8 Graphical analysis

3.8.1 Evolution in asset returns

First consider asset returns, \( r \) (the evolution of \( g \) is analyzed in the next subsection). Figure 1 identifies trends in \( r \) at three different frequencies: 1) An ultra-long-run slow moving trend; 2) three long-term waves typically of 100-150 years; and 3) fluctuations below 100 years duration influenced by business-cycles, wars, crop failures and asset market run-ups. The movements in the average returns are
remarkably consistent with the time-profile of real short-term interest rates over the past eight centuries, as detailed in the online Appendix A4. The trend in real asset returns is surprisingly constant in the ultra-long run, fluctuating around the sample average of 5.2%. Regressing \( r \) on a time-trend, yields a coefficient of the time trend of -0.001(-0.44), where the figure in parenthesis is the \( t \)-value and \( N = 804 \).

Notes. The trend-line in Figure 1 is computed from the HP-filter with a smoothing factor of 100,000. The other lines in Figure 1 are computed as 25-years centered moving averages of the actual data. Here \( g \) is the annual growth in total real GDP and \( r \) is the annual real after-tax returns to assets.

Three waves in \( r \) can be identified during the approximate periods 1300-1400, 1560-1660, 1780-1870, and a half wave starting from 1960. The first wave was not unique to medieval UK but was widespread across Western Europe and it appears that a high time-preference was the underlying cause (see, e.g., Marshall, 1890, pp. 680-681; Clark, 1988; Homer and Sylla, 2005) or that usury laws and moral values discouraged lending and, consequently, generated an excess demand for loans. It is likely that the time-preference was heightened by the extraordinary high mortality rates during this period because consumers were induced to place less weight on utility in all future periods; an effect shown formally in the model of Becker and Mulligan (1997). The Hundred-Years’ War, 1337-1453, also contributed to the high returns to capital in response to the Crown’s high demand for credit to finance the war. The high asset returns that prevailed before 1400 were gradually reduced after 1400 as the population continued to decline, following the aftermath of the Black Death in 1348, which gradually reduced the demand for agricultural products, and \( r \) didn’t start to increase again until the population started to pick up two centuries later (for population growth, see Figure 2). The decline in asset returns after 1400 is consistent with the timing of declining interest rates on long-term debt instruments in continental Europe (Homer and Sylla, 2005, Chs. 8 and 9) and short-term interest rates (online Appendix A4).

However, why did the population decline that occurred over the first few decades after the Black Death not reduce the marginal productivity of fixed capital? First, land holders likely prevented the marginal productivity of land from declining in the first decades following the Black Death because
farmers switched from labor intensive arable cultivation to less labor intensive pasture (Clark, 2016). Second, the analysis of Becker and Mulligan (1997) shows that a one percentage point decrease in survival rates is associated with more than a one percentage point increase in the time preference, suggesting an upward pressure on asset returns following the Black death of 1-2 percentage point (see online Appendix A4 for computations).

The second wave in $r$ during the approximate period 1560-1660 was a result of high returns to fixed capital, partly brought about by the scientific and educational revolutions that increased the knowledge stock per worker and led to increasing returns to fixed capital (Madsen and Murtin, 2017). The landed class also enjoyed higher returns brought about by the three-fold increase in real food prices over the period 1570-1709 and significant technological progress in agriculture (Ang et al., 2013). Agricultural productivity was enhanced by increasing crop rotation, technological innovations, improved drainage methods brought about by innovations, and the increasing spread of agricultural technical manuals (Ang et al., 2013). Provided that per capita income was kept constant for the working class in a Malthusian regime, the technological progress would automatically increase the income of the wealthy and, therefore, inequality.

A reversal of the high asset returns occurred over the next century, 1660-1750, as a result of a reduction in macro tariff rates from approximately 50% to 20% (see Figure 7 below) that lowered land rent through lower prices on imported grain. Furthermore, North and Weingast, (1989) argue that the Glorious Revolution of 1688, as probably the most important milestone in British institutional history, resulted in lower interest rates on public debt which also ensured a positive externality for the parallel development of a market for private debt and underpinned the institutional structure for pooling savings and for intermediation between borrowers and lenders.

The third, and the most pronounced wave in $r$ occurred in the first half of the 19th century and was predominantly a result of extraordinarily high returns to agricultural land, housing, and fixed capital. The fundamental forces behind this increase were 1) large productivity advances in agriculture that increased the returns to land; 2) flat real wages that failed to keep up with the large productivity advances, presumably because of the strong population pressure; 3) ineffective unions and lack of political representation made it difficult for workers to get a greater share of income; and, most

13 Land cultivation was not only highly labor intensive during the harvest but also during the periods of plowing, weeding, fertilizing the soil etc. In other words, labor and crop land were gross complements, while labor and pastoral land were gross substitutes, the Plague-induced labor shortage gave landowners an incentive to substitute animal production for corn production. This scenario is consistent with the time-profile of real wages: farmworkers’ real wages increased by 100% over the period 1347-1350 and fell 33% over the period 1350-1352, yielding a net increase of approximately 33% (see online Data Appendix for sources). Using an alternative dataset to the one used here, Clark (2016) finds that real wages of agricultural workers did not increase at the onset of the Black Death and immediately thereafter.
importantly, 4) the Corn Laws over the period 1815-1846 resulted in increasing real prices of agricultural products that were already driven up to a high level during the Napoleonic wars.

The high returns during the first part of the 19th century begs the question, why returns were not driven down by capital deepening and increasing demand for land. Since educational attainment and mortality rates were not much different from those of the previous century, the time-preference, according to the Becker-Mulligan (1997) model, should have been approximately unaltered. The high returns on fixed capital are likely to have remained high because of high adjustment costs of capital, partly kept high by limited access to credit. Furthermore, land prices did not respond to the higher land rents during the first half of the 19th century; thus, preventing the dividend yields from reverting back to their pre-1800 level. Land prices may not have increased because landowners may have expected the high tariffs to be temporary.

The Corn Laws imposed import barriers and steep tariff rates on imports of corn were not designed to safeguard farmers from distress but rather to preserve the extraordinarily high profits that prevailed during the Napoleonic war-years (Fletcher, 1961). The real returns to land, were on average, no less than 17.5% over the period 1815-1846 during which the Corn Laws were in place. The introduction of the Corn Laws is a good example of how influential institutions can be for inequality, as advocated by Acemoglu and Robinson (2015) and Piketty (2014, 2015a, 2015b).

The Trade Union Act of 1871, which legislated trade unions, and the Great Agricultural Depression over the period 1873-1896 marked the end of the high asset returns enjoyed during most of the 19th century. WWI signaled a further blow to the high asset returns experienced during the 19th century and asset returns declined to an average low of 3.1% during the period 1915-1982; a return that even underperforms the average low of 3.4% that prevailed during the period 1350-1560. Falling real prices of agricultural produce, starting in the early 1920s and the often negative corporate profits during the Great Depression forced asset returns down during the 1930s. Finally, following the predictions of the model of Becker and Mulligan (1997), the mortality transition over the period 1880-1930 likely also contributed to the reduced asset returns through reduced time-preference.

The three inflationary episodes, 1915-1920, 1940-1952 and 1973-1982 markedly reduced the real after-tax returns to bonds because the inflations were, to some extent, unexpected, and not all bond-holders understood the implications of inflation on bond returns. The effect of the low and negative returns to bonds on the overall returns was compounded by increasing weight of government bonds in the overall portfolio. The share of government debt in total wealth increased from 7% in 1913 to approximately 40% during WWI and fluctuated around this level up until the late 1950s, and first reached the pre-WWI level in 1977. Since the increasing inflation was only fractionally passed on to higher nominal interest rates and the top-income tax rates increased to historical heights during WWII
and remained high up to the early 1980s, the resulting negative post-tax real bond returns were major contributors to the low returns during this period (Roine et al., 2009; Piketty, 2014).

Signified by increasing unionization and strike activity, the increasing union power following the Trade Union Act of 1871, came to a halt in 1982, which marked the start of the most recent wave in $r$. Weakening union power, disinflation, high returns to non-reproducible assets and reduced top income taxes have all contributed to the upswing in asset returns since 1982 (Roine et al., 2009; Piketty, 2014).

A final, but pressing, question is the extent to which $r$ has been driven by $g$ as predicted by intertemporal consumer-biased models. Empirically, there is not a statistically significant positive relationship between $r$ and $g$ for Britain over the period 1222-2013 (see online Appendix for regression analysis). Eyeballing Figure 1, it is also difficult to pinpoint any positive association. Asset returns were well above their trend despite approximately zero growth rates up to the early 15th century, $r$ and $g$ moved in reverse directions during most of the 16th century, and the high growth period, 1920-1980 coincided with a low $r$. This finding could be a result of the blurred relationship between $r$ and $g$ as implied by some growth models analyzed in Barro and Sala-i-Martin (1992), or that diminishing returns were avoided through high capital mobility and the fixed exchange rate system that prevailed in Britain over most of the period considered here. Malthusian population dynamics in the pre-industrial period could also have prevented diminishing returns to capital because population would respond endogenously to capital deepening.

3.8.2 The income growth path
As seen from Figure 1, the trend growth rate was close to zero from 1210 until the mid-16th century because population growth rates, apart from the marked negative population shock 1348-1351 (Figure 1), were close to zero and slow productivity growth rates driven predominantly by modest advances in technology and education (Madsen and Murtin, 2017). The growth rates picked up substantially during the later 16th century. The spread of the movable printer, which reduced book prices substantially during the 16th and the 17th centuries, was pivotal for the educational expansion in Britain and the dissemination of knowledge which, coupled with the scientific revolution in the 17th century and increasing innovative activity, lead to productivity advances not experienced earlier in recorded British history (Galor, 2005; Madsen et al., 2010; Madsen and Murtin, 2017).

The high population growth rates during the 16th and the 17th centuries additionally increased $g$ directly; however, the total effect on $g$ was reduced by the population growth drag introduced by land as a semi-fixed factor of production (Madsen et al., 2010; Madsen and Murtin, 2017). Assuming
that the output elasticity of land was 0.5 and capital’s income share was a third, the net effect of population growth on \( g \) was only 0.25%.

The productivity advances during the first phase of the British Industrial Revolution were associated with increasing population pressure (Figure 2) because the quantity effect remained strong relative to the quality effect in the fertility decision (Galor, 2005). The fertility transition first occurred later during the British Industrial Revolution as parents substituted quality for quantity in response to increasing returns to education and increased opportunity costs of having children triggered by a reduced gender wage gap (Galor, 2005). The fertility transition coupled with slow productivity advances over the period 1910-1928, reduced \( g \) substantially. Subsequently, the educational and scientific advances over the period 1929-2007 resulted in strong productivity advances that kept \( g \) high despite low population growth rates (Madsen et al., 2010).

Finally, the largest growth wave over the entire timespan considered here, occurs in the approximate period 1935-2000, and is heavily influenced by productivity advances driven predominantly by increasing R&D and human capital, and the fertility transition that was associated with increasing female labor force participation, reduced age dependency and enhanced saving (Madsen, 2010).

3.8.3 Evolution in \( W-Y \) ratio and \( S^{WCG} \)

The growth in the \( W-Y \) ratio and capital’s income share, \( S^{WCG} \), are displayed in Figures 3 and 4. The growth and not the level of the \( W-Y \) ratio is displayed because the \( W-Y \) ratio is highly persistent – the lagged value of the dependent variable in the \( W-Y \) regressions below is approximately 0.8, pointing towards highly persistent effects of shocks to the \( W-Y \) ratio. Common for the \( W-Y \) growth and \( S^{WCG} \) is that they oscillate around a constant ultra-long run trend, again at medium-term frequencies of 100-150 years. The constancy in the \( S^{WCG} \) indicates that the elasticity of substitution between capital, broadly defined, and labor tends towards one in the ultra-long run, but periodically deviates from its long-run equilibrium of one through changing mark-ups, skill-biased technological progress and changing relative prices of reproducible capital (Karabarbounis and Neiman, 2014).\(^{14}\) The periods 1300-1400/1450, 1550-1650, and 1800-1870 are associated with growth in the \( W-Y \) ratio and the level of \( S^{WCG} \) above their ultra-long trends and these periods tend to coincide with the waves in \( r \) and \( g \).

\(^{14}\) Consider the Cobb-Douglas production function, in which technological progress is always Hicks-neutral, \( Y = AK^{\alpha}T^{\beta}L^{1-\alpha-\beta} \), where \( T \) = land. Under perfect competition and profit maximization, the share of income going to capital is constant and determined by output elasticities:

\[
S^W = \frac{r^W}{Y} = \frac{r^K + r^T}{r^K + r^T + w_L} = \alpha + \beta.
\]
Starting from the first wave, a high time preference and the increasing demand for credit during the Hundred Years’ War (1337-1453) kept returns, \( r \), high and the switch from crop to pastoral production, as discussed in the previous sub-section, and prevented the marginal productivity of land from declining in the first decades following the Black Death. The marked negative population growth rates (see Figure 2) were also likely influential for the 1300-1400/1450 wave in \( W-Y \) growth and \( S^{WCG} \) as the shrinking population increased the concentration of wealth and earnings derived from wealth. The technological advances during the second wave, 1550-1650, increased returns to fixed capital and land. Although we would expect capital deepening to reduce these excess returns on fixed capital relatively quickly, the adjustment process was slowed down considerably by the absence of well-developed credit markets.

The third wave in \( W-Y \) growth and \( S^{WCG} \) was predominantly driven by large productivity advances in agriculture, the Corn Laws that upheld the high real corn prices achieved during the Napoleonic wars, and an expanding gap between real wages and the general productivity advances.

Notes. The data are 25-year centered moving averages. Capital’s income share, \( S^{WCG} \), is computed using Eq. (15).

The marked decline in \( S^{WCG} \) and the growth in the \( W-Y \) ratio, over the period from 1833 to 1910 is exceptional and deserves some discussion. Half of the decline is a gravitation towards the ultra-long-run trend and, as discussed above, capital’s extraordinarily high share over the period 1800-1840 was predominantly a result of extraordinarily high land rents driven up by a lucrative market for agricultural produce during the Anglo-French wars 1794-1815 (including the Napoleonic war) and kept artificially high by the Corn Laws. Land rents were deemed to fall as soon as the Corn Laws were repealed. Capital’s income shares and, to some extent, the \( W-Y \) ratio were driven down by a sequence of events such as the repeal of the Corn Laws in 1846, and the Great Agricultural Depression over the period 1873-1896 (see, e.g., Fletcher, 1961). The large population pressure in the 19th century furthered the reduction in the \( W-Y \) ratio through wealth dilution effects and, potentially, also through a reduction the
fraction of the high-saving, low-fertility and high-income earning families as stressed by Kuznets (1955).

Capital’s share and the growth in the $W-Y$ ratio, remained low up to the 1970s as a result of inflationary spells that eroded the real value of nominal claims, high productivity growth rates, and increasing marginal tax rates and increased unionization, noting that the temporary increase in the $W-Y$ ratio during the approximate period 1916-1960 was caused by the markedly increasing government debt incurred during the world wars. The upward turn in $SWCG$ and the $W-Y$ growth rate from the 1980s has been associated with low growth, $g$, and weakening of unions, as signified by a fall in the union membership rate from its historical peak of 46% in 1979 to 22% in 2013. This eased the wage pressure and increased the profit rate in the corporate sector. The ascending real prices of urban and agricultural land induced by increasing real food prices, reduced real and nominal interest rates and increasing urbanization have also been influential for the increasing inequality.

Consistent with most growth theories, Piketty’s first two laws and Kaldor’s (1957) stylized facts, $SWCG$, $SW$ and the $W-Y$ ratio have tended toward constants in the long run, noting that the growth in the $W-Y$ ratio has tended towards zero in the long-run.\(^{15}\) Piketty’s first two laws yield the prediction that $SW = WY = rs^N/g$, where $s^N$ is the net saving rate (net saving out of NNI). Piketty (2014) and Piketty and Zucman (2014) argue that $SK$ and $W-Y$ both gravitate towards steady states determined by $r$, $s^N$ and $g$, and that deviations from these steady states are only temporary. Groth and Madsen (2016) show that $SW$ may be driven away from its equilibrium by adverse supply shocks that lead to dampened internal oscillations and hump-shaped impulse-response functions around the steady state, because of a high degree of real-wage rigidity combined with a low elasticity of factor substitution between capita and labor. The wage shocks immediately after WWI and WWII and in the 1970s, for example, pushed real wages above their long-run equilibrium and, consequently, prevented capital’s income share from gravitating towards its steady state. These adverse wage shocks had two immediate effects; substitution towards more capital-intensive production methods and reductions in Tobin’s $q$, which in turn led to reduced capital investment. The resulting excess unemployment reduced wage growth and capital’s income share gradually recovered from its 1970s low.

The U-shaped path in $W-Y$ growth and $SWCG$ over the past two centuries is also approximately consistent with optimizing models of factor shares in an industrial economy (see, e.g., Karabarbounis

\(^{15}\) Dickey-Fuller tests suggest that $SW$, $SWCG$ and the $W-Y$ ratio are mean-reverting. The Dickey-Fuller test with constant terms included yield the values of -3.88 ($SW$), -30.5 ($SWCG$), -7.11($W-Y$), indicating that the null hypothesis of unit roots is strongly rejected at conventional levels in all cases, where annual data are used over the period 1211-2013. The critical value for the Dickey-Fuller test with a constant included is -3.43 at the 1% level when the number of observations exceeds 500.
Apart from a temporary dip during the interwar period the price of investment in machinery and equipment divided by the GDP-deflator was relatively flat before 1950 and has since declined; thus resulting in a capital-induced increase in the share of income going to capital provided that the elasticity of substitution is higher than one in the private corporate sector, as maintained by Karabarbounis and Neiman (2014).

In the pre-1800 period we can only speculate the extent to which the path of $W-Y$ growth and $S^{WCG}$ were influenced by biased technological progress and relative prices of reproducible working capital. The decrease in real prices of draft animals of approximately 50% over the period 1667-1815, may have contributed to the increase in $S^{WCG}$ over the same period. The major innovations in agriculture, such as increased field crop rotation, enclosure, and better draining methods, (Ang et al., 2013) before 1800 probably did not affect the relative demand for labor and factor shares. The gradual improvement of the plough may, however, have reduced the demand for labor.

3.9 Dynamic efficiency

Several economists have challenged Piketty’s (2014) claim that $r > g$ leads to an ‘endless inegalitarian spiral’ from the perspective that: 1) increasing wealth concentration, which is driven by an increasing $K-Y$ ratio, will eventually drive $r$ below $g$; and 2) the condition $r > g$, modified to apply to fixed capital, is a natural steady-state condition as long as the saving rate does not exceed a level at which the capital stock is pushed beyond the Golden Rule level. Dynamic efficiency requires that $r^K > g^K$, where $r^K$ = the real after-tax returns to non-residential fixed capital and $g^K$ = the growth in the volume of non-residential fixed capital (Abel et al., 1989). If $r^K < g^K$ we are in the problematic situation of dynamic inefficiency in the sense that capital is over-accumulated and all generations can be better off by reducing their saving and that firms can increase stockholder value by paying earnings out rather than retaining and reinvesting them.

The time-profiles of $r^K$ and $g^K$ are displayed in Figure 5. Focusing on the long-run trends in $r^K$ and $g^K$, the British economy has, by a large margin, been dynamically efficient. Dynamic efficiency is also evident over the period 1960-1984 - a result that is consistent with the results of Abel et al. (1989) for the English economy between 1960 and 1984. The $(r^K - g^K)$-gap has narrowed slightly since the onset of the First British Industrial Revolution in 1760 and has, on average, been 3.6% over the period 1760-2013 and 4.0% over the period 1900-2013. The narrowing of the $(r^K - g^K)$-gap after 1760 has been a result of an increase in $g^K$ and not a reduction in $r^K$, where the historically high growth in non-residential fixed capital stock has not resulted in a decline in $r^K$ because $g^K$ has been predominantly driven by population growth and technological progress. These factors have out weighted the $r^K$-reducing effects of a potentially increasing $K-Y$ ratio through a higher saving rate.
Notes. Returns, $r^K$, are 25-year centered moving averages of post-tax returns on non-residential fixed capital and $g^K$ is a 25-year centered moving average of the growth in real non-residential fixed capital. The $r^K$-trend double line is computed from the HP-filter with a smoothing factor of 100,000.

As long as the economy is dynamically efficient, an increasing $K-Y$ ratio is compatible with Piketty’s hypothesis that the $K-Y$ ratio is an increasing function of the $(r^K-g^K)$-gap. If, on the other hand, the economy is dynamically inefficient we are in a situation where further wealth accumulation may reduce the share of income going to capital. Furthermore, an increase in the $W-Y$ ratio, which is not driven by fixed capital accumulation, will not drive a dynamic efficient economy into dynamic inefficiency since dynamic efficiency applies to the $K-Y$ ratio and not the $W-Y$ ratio and, therefore, requires $r^K > g^K$ and not strictly $r > g$. For Britain, $K$ is approximately 20% of $W$ and this number has been relatively constant over time and $r^K$ has historically often been quite different from $r$. A feature of fixed capital is that if $r^K$ exceeds its steady state level determined by the Euler equation, it follows that capital accumulation will automatically reduce $r^K$ because of diminishing returns to capital. Returns to bonds, agricultural land and housing, by contrast, can remain below $g$ without the economy staying overcapitalized.

As a consequence, the $(r^K-g^K)$-gap has not been significantly related to the $W-Y$ ratio because $K$ is a low fraction of $W$ and because investors allocate their portfolio where the returns are highest subject to the riskiness of the assets. Between 1210 and the end of the 15th century the $W-Y$ ratio was well above its long-run trend, while the $(r^K-g^K)$-gap was above and not below its ultra-long trend, as would have been the case if a high and increasing $W-Y$ ratio were incompatible with dynamic efficiency (compare Figures 3 and 5). Focusing on the post-1760 period, the $W-Y$ ratio peaked in the first half of the 19th century without having any noticeable impact on the $(r^K-g^K)$-gap. Regressing $(W/Y)$ on $(r^K-
\( g^K \) over the period 1222-2013 yields a coefficient with a \( t \)-value of -0.45; where the correlation remains insignificant if the coefficients of \( r^K \) and \( g^K \) are not restricted to be the same, but of opposite sign. Coupled with the finding below that the \( W/Y \) ratio is significantly positively related to \( r^K \) and \( -g^K \), these results further emphasize that inferences about Piketty’s \( (r-g) \)-hypothesis derived from dynamic efficiency conditions are untenable. In other words, wealth accumulation that is driven by non-reproducibles without significantly affecting the \( K-Y \) ratio will not jeopardize the dynamic efficiency condition.

4. Regression analysis

4.1 Model specification

Two baseline regressions are used to test Piketty’s \( (r-g) \)-hypothesis and, more broadly, the extent to which wealth and income inequalities are determined by \( r, g \), the saving propensity and the persistence of inequality to shocks. The models are stochastically specified as:

\[
\ln(W/Y)_t = \varrho_0 + \varrho_1 \ln(W/Y)_{t-1} + \varrho_2 r_t + \varrho_3 g_t + \varrho_4 s^W_t + X_t \theta' + \varepsilon_{1,t},
\]

\[
S^W_{t} = \zeta_0 + \zeta_1 S^W_{t-1} + \zeta_2 r_t + \zeta_3 g_t + X_t \zeta' + \varepsilon_{2,t},
\]

where \( W \) = total private nominal wealth; \( Y \) = nominal net national income (GNP minus depreciation and plus net foreign income); \( S^W \) = the share of income going to wealth including capital gains on non-reproducible wealth (\( S^K \) is used in some regressions); \( r \) = the real post-tax return to wealth; \( g \) = the growth in real net national income; \( s^W \) = the gross propensity to save out of wealth (private nominal saving divided by private nominal wealth); \( X \) = a vector of control variables; and \( \varepsilon \) = a stochastic error term. The coefficients of the variables are expected to have the signs as follows: \( r \) positive; \( g \) negative; and \( s^W \) positive. The models are estimated over the period 1222-2013 in 10-year intervals to smooth out erratic movements in the data, to allow for some of the dynamic adjustment towards steady state, and to reduce business-cycle influences on the estimates.

Lagged dependent variables are included in the models 1) to capture the effects of omitted variables that are serially correlated; 2) to allow for slow adjustment towards long-run equilibrium; 3) to capture mean-reverting dynamics and persistent effects in the dependent variable that may be endogenous to inequality; 4) to reduce the bias of the coefficients of the focus variables, which is introduced by confounding variables that push \( S^W \) and the \( W-Y \) ratio out of their steady-state trajectories, such as crises, epidemics, social unrest, war, etc.; 5) because it is not clear from the discussion in the literature whether \( r \) and \( g \) have temporary or permanent effects on the growth in the \( W-Y \) ratio and \( S^W \), (permanent growth effects of \( r \) and \( g \) follow from the limited case where \( \tau_1 \) and \( \tau_1 \) are one, as predicted by the Kesten (1973) model discussed in Section 3.6.2); and 6) bequests creates
wealth persistence because a large fraction of wealth stems from inheritance (Piketty, 2011, and 2014, p. 268). Piketty (2011) shows, for a sufficiently large \((r - g)\)-gap, that in steady state the bequest-income ratio is approximately equal to \((W/Y)/Gen\), where \(Gen\) is the average length of a generation (the average age at which parents have children). Since \(Gen\) is relatively constant, it follows that bequests create persistence in the \(W-Y\) ratio. Further lags were included in the initial regressions; however, they were excluded from the regressions because they were either insignificant or of low significance and the coefficients of \(r\) and \(g\) were unaffected by their exclusion.

The saving propensity out of wealth, \(s^W\), is included in Eq. (16) because it is a crucial determinant of wealth accumulation (see, e.g., Kuznets, 1955; Kesten, 1973; Piketty, 2014; Piketty and Zucman, 2014) and in the representative-agent framework, where each family needs to reinvest the fraction \(g/r\) of its capital income to keep the \(W-Y\) ratio constant. Furthermore, \(s^W\) is a key determinant of wealth inequality in the models of Kesten (1973) and Jones (2015).

The OLS and IV estimators are used to keep the exposition simple and transparent. More advanced techniques such as ARDL and VECM approaches could not be applied because the null hypotheses of unit roots were consistently rejected (see, e.g., footnote 14). Based on Piketty’s Second Law, in which the \(W-Y\) ratio converges towards \(s^n/g\) in steady state, one could argue that \(W-Y\) and \(s^n/g\) form a cointegration relationship; however, again, neither the \(W-Y\) ratio nor \(s^n/g\) contain a unit root.

### 4.2 Identification

In the IV regressions, \(r\), and \(g\) are treated as endogenous because of potential feedback effects from the dependent variables and confounding factors. Real food price inflation, consumer price inflation, and macro tariff rates are used as instruments for \(r\) and \(g\). From the onset, however, it is important to stress that, although instruments are reasonably exogenous, the exclusion restrictions may not be fully satisfied for all instruments as discussed further below.\(^{16}\)

High real food price inflation is likely to be associated with high \(r\) and low \(g\) through a combination of high earnings and low output growth in agriculture, particularly in the pre-industrial era in which the wealth associated with agricultural production, such as agricultural land, buildings and livestock, was the dominant source of national wealth (Madsen, 2016). However, real food price

\(^{16}\)Educational attainment and mortality were also considered as instruments in an early draft of the paper following the predictions of the model of Becker and Mulligan (1997) in which the time preference is shown to be a negative function of educational attainment and life expectancy. Educational attainment (from Madsen and Murtin, 2017) was insignificant in all the first-stage model specifications covering the period 1270-2013 and, consequently, not included in the instrument set. Life expectancy, (also from Madsen and Murtin, 2017) has the expected negative sign and a \(t\)-value of about 3, depending on model specification, estimation period etc.; however, it was not included in the instrument set because of its low significance and because the data are first available from 1510.
Inflation has recently regained some of its influence on total returns, along with a markedly increasing share of housing in total private wealth since housing wealth is linked to real food prices through the price of land: The higher are real prices of food, the higher are the earnings in the agricultural sector, which in turn will tend to be capitalized in agricultural land prices. Farmland prices will influence urban land prices through ripple effects starting from urban land expansion at the fringes of cities (Madsen, 2009). Food prices are likely to be exogenous because they have, to a large extent, been determined by weather conditions, crop disease, population pressure, tariffs and world food prices (O’Rourke and Williamson, 2002); forces that are quite independent of the \( W-Y \) ratio. Furthermore, the exclusion restriction is likely to be satisfied in that food price shocks have macroeconomic effects on the \( W-Y \) ratio through population growth (starvation and poor health) and asset returns (land rent).

Inflation is influential for real returns when inflation is unanticipated as it lowers the real value of contractual nominal payments, such as housing rent, agricultural land rent, and interests on government bonds. Furthermore, Modigliani and Cohn (1979) argue that inflation reduces real returns to equity because 1) growing inflation, \textit{ceteris paribus}, reduces accounting earnings as it increases nominal interest payments on debt, however, investors fail to account for the fact that inflation erodes the real value of the firm’s debt; and 2) investors tend to discount future earnings by nominal interest rates. Furthermore, the level and the change in inflation can be costly for owners of fixed-interest rate government debt; particularly during the approximate period 1940-1980 when marginal income tax rates were well above their long-run trend (Roine et al., 2009). The major inflationary periods, 1549-1575, 1799-1813, 1915-1920, 1940-1952, and 1968-1982, as identified in Figure 6, have been associated with low or negative real after-tax returns. Conversely, the two major deflationary periods, 1318-1339 and 1814-1823, were associated with high real returns.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Inflation, \%}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Tariff Rate, \%}
\end{figure}

\textbf{Notes.} The trend inflation is estimated from the Hodrick-Prescott filter with a smoothing factor of 10,000. The thin line in Figure 6 is a 19-year centered moving average of the rate of inflation. See online Appendix for sources.
Macro tariff rates, calculated as the ratio of import duties and imports, are used as instrument for \( r \) and \( g \) because they have played a major role in the history of the political economy of rent seeking and, at the same time, they reduce income through various channels. Probably the best example of successful rent seeking through tariffs and trade barriers in British history is the marked increase in tariff rates following the introduction of the Corn Laws in 1815, as discussed above. The effects of the Corn Laws on tariff rates, first repealed in 1846, are highly visible in Figure 7. The macro tariff rates were a whopping 50% during this period, noting that macro tariff rates underestimate the effective tariff rates because of substitution away from products most affected by the tariffs. Similarly, capitalizing on the widening trade deficit after the wars between France and Britain in the period 1689-1713, protectionists engineered the imposition of high tariffs on a range of imports from France (Nye, 2007).

However, tariff rates may not necessarily only be increased in response to low returns to land, but that they are a function of a complex political process related to increasing (Corn Laws), decreasing (tariffs during the Great Depression and WWII), or unaltered (entrance to the Common Market in 1973) wealth positions of the landed class. Thus, tariff rates are likely to be independent of land rent. Furthermore, the exclusion restriction is likely to be satisfied since tariffs impact on the \( W - Y \) ratio through land rent because the tariff-induced changes to returns result in increasing wealth accumulation through saving and capital gains.

### 4.3 \( W-Y \) regression results

The results of regressing Eq. (16) are shown in Table 3, where the IV estimator is used in the regressions in columns (4)-(6) and the OLS estimator is used in the other regressions in the table. Consider first the OLS regressions in columns (1)-(3). In the regression in column (1) in which \( s^W \) is excluded, \( r \) and \( g \) are both statistically highly significant and have their expected signs. In the estimates in column (2) the coefficients of \( r \) and \( g \) remain significant and of the expected signs when the propensity to save out of wealth, \( s^W \), is added to the model, and \( s^W \) is significantly positive, as predicted by some of the models presented in the online Appendix (the models of Kesten, 1973, and Jones, 2015, Mankiw, 2015, and Solow). The statistical significance of the coefficients of \( r \) and \( g \) remain high and the magnitudes of the coefficients are close to those of the baseline regression in column (2), when non-overlapping data are used in the regressions (column (3)). Common for all the regressions in columns (1)-(3) (and subsequent regressions) is that the coefficients of the lagged dependent variable are all highly significant and approximately 0.8, indicating shifts in \( g \) and \( r \) have persistent; but not permanent growth effects on the \( W-Y \) ratio and, therefore, wealth inequality. From these estimates it can be concluded that approximately 2% of the \( W-Y \) disequilibrium is eliminated every year, which is consistent with the slow-moving long waves in the \( W-Y \) ratio.
### Table 3. Parameter estimates of Eq. (16).

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(W/Y)_t</td>
<td>OLS</td>
<td>OLS</td>
<td>OLS/Ex Overlap</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>OLS/Post: 1800</td>
<td>OLS/Pre: 1600</td>
<td>OLS/Post: 1600</td>
<td>Growth decom.</td>
</tr>
<tr>
<td>ln(W/Y)_{t-1}</td>
<td>0.78*** (29.7)</td>
<td>0.89*** (24.2)</td>
<td>0.84*** (9.09)</td>
<td>0.73*** (27.9)</td>
<td>0.73*** (18.7)</td>
<td>0.37*** (3.95)</td>
<td>0.98*** (22.6)</td>
<td>0.58*** (9.77)</td>
<td>0.86*** (24.1)</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>1.85*** (7.01)</td>
<td>2.47*** (8.08)</td>
<td>2.69*** (3.15)</td>
<td>0.92*** (2.52)</td>
<td>0.79*** (2.07)</td>
<td>4.41*** (5.44)</td>
<td>2.54*** (7.36)</td>
<td>3.23*** (6.06)</td>
<td>2.34*** (6.67)</td>
<td>2.38*** (7.94)</td>
</tr>
<tr>
<td>b</td>
<td>-5.81 (9.30)</td>
<td>-7.35*** (10.3)</td>
<td>-7.07*** (4.26)</td>
<td>-3.37*** (4.25)</td>
<td>-2.61*** (2.23)</td>
<td>-12.1*** (8.79)</td>
<td>-4.32*** (5.29)</td>
<td>-7.88*** (7.11)</td>
<td>-9.23*** (12.8)</td>
<td></td>
</tr>
<tr>
<td>$\beta^g_1$</td>
<td>-8.63*** (11.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta^r_1$</td>
<td>-4.27*** (6.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s^w_t$</td>
<td>2.53*** (4.64)</td>
<td>1.64 (1.18)</td>
<td>-0.56 (0.79)</td>
<td>0.65 (0.75)</td>
<td>12.8*** (4.51)</td>
<td>-1.14*** (1.89)</td>
<td>2.34*** (4.31)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.65</td>
<td>0.66</td>
<td>0.62</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>0.37</td>
<td>0.58</td>
<td>0.72</td>
<td>0.67</td>
</tr>
<tr>
<td># Obs</td>
<td>792</td>
<td>792</td>
<td>79</td>
<td>792</td>
<td>792</td>
<td>214</td>
<td>378</td>
<td>414</td>
<td>792</td>
<td></td>
</tr>
</tbody>
</table>

**Notes.** The numbers in parentheses are absolute $t$-statistics based on serial correlation and heteroscedasticity consistent standard errors. Constant terms are included in the regressions but not reported. $g^t = \text{productivity growth}; g^n = \text{population growth}; wp^{SU} = \text{wage premium of skilled workers over unskilled workers}; wp^{IU} = \text{wage premium of unskilled labor over agricultural workers.}$ The data are measured in 10-year overlapping intervals except the regression in column (3) (OLS/Ex Overlap), in which the units of observations are in 10-year non-overlapping intervals. The following instruments are used in the IV regressions in columns (4)-(6), CPI inflation, real food price inflation, and macro tariff rates. *: Significant at 10%; **: Significant at 5%; ***: Significant at 1%.

Turning to the IV regressions, the $F$-tests of excluded instruments in the first-stage regressions all exceed the value of 158 for all endogenous variables, suggesting that the potential bias of the coefficients of the regressors relative to OLS estimates is well below 1%. In the second-stage regressions in columns (4)-(6), the coefficients of $g$ and $r$ are statistically highly significant and the absolute long-run elasticities of $r$ and $g$ are lower than those of the OLS regressions, which may, as far as $r$ is concerned, be associated with a negative feedback effect from the $W-Y$ ratio to $r$ for reproducibles.

Eq. (16) is regressed over the periods 1600-2013, 1800-2013, and 1222-1600 in columns (7)-(9). The coefficients of $r$ and $g$ are consistently highly significant and their magnitudes are remarkably stable over time, suggesting that the nexus between the $W-Y$ ratio and $r$ and $g$ is independent of the mode of production, that the relationship between these variables is robust, and that the pre-industrial data are of decent quality. The coefficient of $s^w$, however, is only significant before 1600, perhaps because almost all wealth accumulation was through saving, conventionally defined, and not capital gain, while the primary source of wealth accumulation the post-1600 period has been capital gain; an issue I will return to below.

Growth, $g$, is decomposed into population growth, $g^n$, and productivity growth, $g^t$, in the regression in column (10) in Table 3. The parameter estimates of $r$ and $s^w$ are close to those of the baseline regression in column (2). The coefficients of $g^n$ and $g^t$ are both highly significant; however, the coefficient of $g^t$ is twice as high as that of $g^n$, which is inconsistent with theories predicting that $g^n$
and $g^A$ have the same effects on the $W-Y$ ratio and inequality. However, to the extent that $g^n$ dilutes wealth concentration through inheritance it will affect the $W-Y$ ratio with a time-lag. If $g^n$ is lagged 10 years, for example, the coefficients of $g^n$ and $g^A$ become even more significant than those in the regression in the last column in Table 3 and the coefficients are of approximately the same magnitude, suggesting that $g^n$ and $g^A$ are equally influential for inequality in the long run (the regression is not shown).

In summary, the coefficients of $r$ and $g$ are economically highly significant in all the regressions in Table 3. Referring to the baseline regression in column (2) in Table 3, a permanent one percentage point increase in $r$ is associated with an approximately 2.5% increase in the $W-Y$ ratio in the short run and 22.5% in the long run, with a half-cycle of approximately half a century. Though the adjustment towards steady state is slow, the large low-frequency movements in $r$ over history have been influential for the evolution of the $W-Y$ ratio. The absolute effect of an increase in $g$ is twice as big as that of an increase in $r$, implying that an $(r-2g)$-gap, as opposed to the $(r-g)$-gap, is a more precise determinant of the $W-Y$ ratio and hence wealth inequality. Assuming that $r$ and $g$ are independent, the 2-percentage point increase in $g$ from the stationary economy in the pre-1600 period to the post-1820 period, has contributed to an approximately 50% increase in the $W-Y$ ratio. Thus, overall, the movements in $r$ and $g$ have been influential for the $W-Y$-path over the past eight centuries.

Are the results in Table 3 consistent with the predictions of standard macro models? Most economic models imply a positive and often complex non-linear relationship between the $W-Y$ ratio and $r$, $g$ and $s^W$, where the coefficients of $r$, $g$ and $s^W$ depend on parameters that are model specific. The only restriction imposed in some models is that the coefficients of $s^W$, $r$ and $g$ are of the same magnitude, but of opposite sign; restrictions that are easily rejected even at the 1% level, in the regressions in Table 3. In absolute terms, the coefficient of $g$ is too large relative to that of $r$, which, in turn, is too large relative to that of $s^W$ in relation to the predictions of extant macro models. These results may be an outcome of measurement errors and omitted variables.

4.4 Capital’s share regressions

The results of regressing Eq. (17), where capital’s income share is the dependent variable, are shown in Table 4. The coefficients of $r$ and $g$ are both highly significant and have the expected signs in the baseline regressions in the first two columns. These results remain intact in the non-overlapping 10-year interval regression in column (3). Comparing the $s^W$ and the $S^{WCG}$ estimates in columns (1) and (2) there are distinct differences. First, changes to $r$ and $g$ have much more persistent effects on factor shares in the $s^W$ estimates than the $S^{WCG}$ estimates, as signified by the coefficients of the lagged
dependent variables of 0.84 in the $S^W$ regression and 0.41 in the $S^{WCG}$ regression, reflecting a low persistence in capital gains.

Second, the statistical significance of $r$ and $g$ are higher in the $S^{WCG}$ regression than the $S^W$ regression, probably because a large fraction of the significance of $r$ and $g$ has been driven down by the highly significant coefficient of the lagged variable in the $S^W$ regression. It is well-known that lags of highly persistent dependent variables, such as $S^W$, remove a lot of the statistical significance of the other regressors even if they are relevant and their statistical significance increases substantially when the lag of $S^W$ is omitted (results are not shown). Third, the absolute and the relative values of the coefficients of $r$ and $g$ are much closer in magnitude in the $S^{WCG}$ regression in column (2) than the $S^W$ regression in column (1). Although the restriction $\zeta_2 = -\zeta_3$ ($\chi^2(1) = 4.33$) is rejected at conventional significance levels in the $S^{WCG}$ regression, it has to be noted that this rejection is partly driven by the extraordinarily high statistical significance of $r$ and $g$. In the regression in column (3) with non-overlapping data, for example, the restriction $\zeta_2 = -\zeta_3$ cannot be rejected at conventional significance levels even though the gap between $\hat{\zeta}_2$ and - $\hat{\zeta}_3$ is larger than that of its counterpart in column (2) with overlapping data.

Table 4. Parameter estimates of Eq. (17).

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>1</th>
<th>2 $S^{WCG}_{t-1}$</th>
<th>3 $S^{WCG}_{t-1}$</th>
<th>4 $S^{WCG}_{t-1}$</th>
<th>5 $S^{WCG}_{t-1}$</th>
<th>6 $S^{WCG}_{t-1}$</th>
<th>7 $S^{WCG}_{t-1}$</th>
<th>8 $S^{WCG}_{t-1}$</th>
<th>9 $S^{WCG}_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>OLS/Ex OL</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>OLS/Ex OL</td>
</tr>
<tr>
<td>$S^{W}<em>{t-1}$/$\Delta S^{WCG}</em>{t-3}$</td>
<td>0.84(9.1)***</td>
<td>0.41(9.6)***</td>
<td>0.36(7.9)***</td>
<td>0.52(10.3)***</td>
<td>0.20(6.28)***</td>
<td>0.13(3.04)***</td>
<td>0.35(7.28)***</td>
<td>0.37(6.59)***</td>
<td>0.14(4.15)***</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.42(5.04)***</td>
<td>3.76(21.4)***</td>
<td>4.22(10.2)***</td>
<td>1.48(8.59)***</td>
<td>3.85(16.3)***</td>
<td>2.66(19.8)***</td>
<td>4.30(15.8)***</td>
<td>4.27(11.8)***</td>
<td>3.61(19.1)***</td>
</tr>
<tr>
<td>$g_t$</td>
<td>-1.41(3.15)***</td>
<td>-2.97(8.12)***</td>
<td>-3.20(3.44)***</td>
<td>-6.80(9.60)***</td>
<td>-8.66(15.2)***</td>
<td>-0.74(2.05)***</td>
<td>-2.53(5.68)***</td>
<td>-3.23(4.76)***</td>
<td>-0.66(3.35)***</td>
</tr>
</tbody>
</table>

| Notes. | See notes to Table 3. $S^W$ = capital’s income share (including land rent) excluding capital gains on non-reproducibles; $S^{WCG}$ = capital’s income share including capital gains on non-reproducibles; $wp^{SU} = $ wage premium of skilled workers over unskilled workers; $wp^{UA} = $ wage premium of unskilled labor over agricultural workers. Constant terms are included in the regressions but not shown. Returns are measured in 10-year moving averages and the growth in real NNI, $g$, is measured in 10-year differences. The variables are measured in decimal points and growth, $g$, and returns, $r$, are annualized. OLS/Ex OL = ten-year non-overlapping observations. $\chi^2(1) = $ Wald test of the restriction $\zeta_2 = -\zeta_3$ (coefficients of $r$ and $g$), distributed as $\chi^2(1)$ under the null hypothesis that the restriction is satisfied. The coefficients of $r$ and $g$ remain statistically and economically highly significant in the IV regressions in columns (4) and (5), indicating that the significance of $r$ and $g$ in the OLS regressions is not driven by feedback effects from $S^W$ or $S^{WCG}$ to $r$ and $g$. The long-run coefficients of $r$ and $g$ are, in absolute terms, significantly higher in the IV than the OLS regressions, suggesting the absence of positive feedback effects from the $W$-$Y$ ratio to $r$ and $g$ in the OLS estimates. For $r$ this means that
factor shares that are driven by changes in the $W-Y$ ratio are not countered by potentially adverse responses in $r$, as is often stressed in the criticism of Piketty’s first law.

The results of regressing the $SWCG$ model over the periods 1222-1600, 1600-2013, and 1800-2013 are shown in columns (6)-(8) in Table 4. The regressions covering the period 1600-2013 (column (7)) give results that are close to the regression results covering the entire period (column (2)) in terms of statistical significance and magnitude of the long-run coefficients. Conversely, the coefficients of the lagged dependent variable are substantially lower in pre-1600 regression than the post-1600 regression. This may reflect that the large swings in factor income shares were partly driven by large fluctuations in output prices and quantities combined with sticky nominal wages, as were characteristic for the predominantly agricultural society in the pre-1600 period; thus automatically reducing the persistence in factor shares (these swings are not visible in Figure 4 because the fluctuations are smoothed out).\(^{17}\) The coefficients of $r$ and $g$ remain statistically highly significant in the pre-1600 period, however, the magnitude of the coefficient of $g$ is well below its post-1600 counterparts. This may be because of Malthusian feedback effects in which technological progress could increase inequality under a Malthusian regime because the real per capita income of the poor stays constant while the wealth and income of the landed class increases.

The wage premium of skilled workers over unskilled workers, $wp^{SU}$, and that of unskilled workers over agricultural workers, $wp^{UA}$, are included in the regression in the last column in Table 4 to control for earnings inequality between employees, which will create a wedge between inequality across all income groups and factor shares, as shown formally in the online Appendix Section A1. Intuitively, if the mean preserving income gaps between main income groups widens, *ceteris paribus*, capital’s income share would remain unaffected even though the overall income inequality has increased. Consequently, if these income gaps are omitted from the regressions, the coefficients of $r$ and $g$ would be biased to the extent that these income gaps are correlated with $r$ and $g$. The estimation period terminates in 1869 because data for the skill-premium data used here are not available after 1869 and, more importantly, because earnings inequality between other wage earning groups becomes much more important for the inequality path after this period. The coefficient of $wp^{UA}$ is negative and highly significant as predicted by the models in the online Appendix. The negative coefficient of $wp^{UA}$ is consistent with the predictions of the model: A mean-preserving increase in $wp^{UA}$ induced by $r$ or $g$, is associated with increasing inequality. However, since $SWCG$ is unaffected by this increase, the $wp^{UA}$ term will capture the effects of $r$ and $g$ on inequality. Finally, the coefficients of $r$ and $g$ remain highly

\(^{17}\) Regressing consumer price inflation on its lagged value using annual data over the period 1222-1600, yields a coefficient of lagged inflation of 0.24(5.03), where the value in parenthesis is the $t$-statistic. For wage inflation of unskilled workers, the corresponding estimate is 0.64(17.0), indicating a much higher degree of wage than price stickiness.
significant and of the expected signs, indicating that they are unaffected by the inclusion of the wage gaps in the estimates.

Common for the regressions in Table 4 is that the economic significance of the coefficients of $r$ and $g$ are generally high. Referring to the baseline regression in column (2) in Table 4, a one percentage point increase in $r$ is associated with a 3.8% increase in capital’s share, $SWCG$, in the short run and 6.4% in the long run. To take an extreme event, the 5-percentage point reduction in the trend in $r$ that took place over the period 1830-1950 resulted in a long-run reduction in capital’s share of almost 32%, suggesting that the declining $r$ was a massive force behind the reduced inequality over this period. A one percentage point increase in $g$ is associated with a 3.0% increase in capital’s share, $SWCG$, in the short run and 5.0% in the long run. The shift in the growth trend from approximately 0.7% before 1800 to 2.1% thereafter, resulted in a 7% long-run reduction in $SWCG$, suggesting that the historically larger shifts in $r$ have been a greater source of inequality fluctuations than $g$. A similar conclusion is reached when the effects of a one standard deviation change in $r$ and $g$ are considered.

A one standard deviation increase in $r$ ($SD_r = 10.40$) results in a 67.0% ($10.4 * 3.8 / (1 - 0.41)$) increase in capital’s share. The corresponding number is - 29.3 ($5.76 * 3.0 / (1 - 0.41)$) for a one standard deviation increase in $g$ ($SD_g = 5.76$).

In terms of theory consistence, the negative coefficient of $g$ is inconsistent with the post-Keynesian models (see, e.g., Kaldor, 1955; Pasinetti, 1962) in which increasing demand, $g$, is associated with increasing mark-ups and, therefore, an increasing share of income going to capital. The results are only consistent with standard growth models to the extent that $g$ dilutes capital per effective unit of labor. To the extent that the restriction $\zeta_2 = -\zeta_3$ cannot be rejected, as is the case for the model in column (3) in Table 4, the results are consistent with the Ramsey model of Mankiw (2015) in which the ratio of each capitalist’s consumption over that of each worker, $c_k / c_w$, as an approximation of $SW$, is proportional to the $(r-g)$-gap.

The finding that the coefficient of $r$ is significantly smaller than that of $g$ in absolute values suggests that the restriction $\zeta_2 = -\zeta_3$ in Piketty’s analysis is too stringent and that the $(r-2g)$-gap is a more precise determinant of $SWCG$; a result that is consistent with the results of the $W-Y$ regressions, as shown in the last two paragraphs of Section 4.3, and consistent with some theoretical models. As shown by De Nardi et al. (2015), for example, the restriction, $\zeta_2 = -\zeta_3$, may not be met for various reasons depending on the setup of the model describing inequality. De Nardi et al. (2015) show that $\zeta_2 < -\zeta_3$ because a lower rate of population growth is associated with a higher ratio of deaths to births and, consequently, a higher average bequest size. Since bequests are luxury goods, population growth
will have a more significant impact on wealth concentration at the right tail of the wealth distribution than will an increase in $r$.

5 Extensions and further robustness checks

To gain more insight into the drivers and sources of wealth and income inequality the models in this section are modified in the following capacities: 1) $W$ is limited to the value of fixed non-residential capital measured at market values or at acquisition costs, denoted as $W^{FK,M}$ or $W^{FK,A}$; 2) the models are augmented to encompass the predictions of the Solow model; 3) returns, $r$, are decomposed into dividend yields and real capital gains; and 4) the models are extended with variables that are likely to influence factor shares and the $W/Y$ ratio and, at the same time, might be correlated with the explanatory variables. As will become apparent, the regressions below give important insights into inequality dynamics and the fundamental drivers of inequality.

5.1 Non-residential fixed capital

Wealth has thus far been measured as the sum of reproducible and non-reproducible capital; however, economic growth theories and most of the debate surrounding Piketty’s laws apply to non-residential reproducible capital. To cater for this, the $W/Y$ ratio is first measured as non-residential, non-agricultural fixed capital at market values, $W^{FK,M}$, divided by net national income in the regression in the first column in Table 5. The economic and statistical significance of the coefficients of $g$ and $r$ are substantially lower than those of the baseline regressions, reinforcing that $r$ and $g$ are much more important drivers of the $W/Y$ ratio than the $W^{FK,M}/Y$ ratio. This result suggests that the strength of Piketty’s ($r-g$)-hypothesis hinges on the inclusion of non-reproducibles in wealth.

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>$\ln(W^{FK,M}/Y)_t$</th>
<th>$\ln(W^{FK,A}/Y)_t$</th>
<th>$\ln(W/Y)_t$</th>
<th>$\ln(W/Y)_{t-1}$</th>
<th>$\ln(W/Y)_{t-2}$</th>
<th>$\ln(W/Y)_{t-3}$</th>
<th>$\ln(W/Y)_{t-4}$</th>
<th>$S^{WCG}_{t}$</th>
<th>$S^{WCG}_{t-1}$</th>
<th>$S^{WCG}_{t-2}$</th>
<th>$S^{WCG}_{t-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Dep Var.)</td>
<td>OLS Fixed capital</td>
<td>OLS Fixed capital</td>
<td>OLS Div Y.</td>
<td>OLS All controls</td>
<td>OLS Sign. controls</td>
<td>OLS Incl. old age</td>
<td>OLS Div Y.</td>
<td>OLS Incl. all controls</td>
<td>OLS Incl. sig. controls</td>
<td>OLS Incl. sig. controls</td>
<td>OLS Incl. sig. controls</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.90*** (15.2)</td>
<td>0.83*** (28.2)</td>
<td>0.78*** (23.5)</td>
<td>0.94*** (25.2)</td>
<td>0.91*** (26.3)</td>
<td>0.73*** (22.3)</td>
<td>0.45*** (11.0)</td>
<td>0.11*** (2.85)</td>
<td>0.13*** (3.67)</td>
<td>0.70*** (18.9)</td>
<td>0.29*** (5.61)</td>
</tr>
<tr>
<td>$g_t$</td>
<td>-0.02 (1.62)</td>
<td>-7.87*** (12.5)</td>
<td>-8.95 (10.8)</td>
<td>-8.82*** (9.92)</td>
<td>-11.1*** (14.7)</td>
<td>-3.62*** (9.75)</td>
<td>-0.84*** (3.95)</td>
<td>-0.91*** (4.18)</td>
<td>-0.46*** (3.54)</td>
<td>-1.97*** (4.77)</td>
<td></td>
</tr>
<tr>
<td>$s_t$</td>
<td>1.47*** (4.44)</td>
<td>0.78*** (1.67)</td>
<td>0.96 (1.27)</td>
<td>0.78*** (2.64)</td>
<td>0.96 (1.72)</td>
<td>0.96 (1.27)</td>
<td>0.78*** (1.67)</td>
<td>0.96 (1.27)</td>
<td>0.96 (1.27)</td>
<td>0.96 (1.27)</td>
<td></td>
</tr>
<tr>
<td>$r^C_C$</td>
<td>0.59*** (4.02)</td>
<td>0.66*** (2.64)</td>
<td>0.02 (1.27)</td>
<td>0.02 (1.27)</td>
<td>0.02 (1.27)</td>
<td>0.02 (1.27)</td>
<td>0.02 (1.27)</td>
<td>0.02 (1.27)</td>
<td>0.02 (1.27)</td>
<td>0.02 (1.27)</td>
<td></td>
</tr>
<tr>
<td>$\ln(g_t + \delta_t)$</td>
<td>0.19*** (6.21)</td>
<td>0.19*** (6.21)</td>
<td>0.21*** (2.40)</td>
<td>0.21*** (2.40)</td>
<td>0.28*** (4.28)</td>
<td>0.28*** (4.28)</td>
<td>0.28*** (4.28)</td>
<td>0.28*** (4.28)</td>
<td>0.28*** (4.28)</td>
<td>0.28*** (4.28)</td>
<td></td>
</tr>
<tr>
<td>$r^C_{M}^{(s)}$</td>
<td>4.76***</td>
<td>5.40***</td>
<td>4.76***</td>
<td>5.40***</td>
<td>4.76***</td>
<td>5.40***</td>
<td>4.76***</td>
<td>5.40***</td>
<td>4.76***</td>
<td>5.40***</td>
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</table>

Table 5. Robustness checks and extensions
The regression in column (2) in Table 5 is closer in spirit of the Solow model than the models considered thus far. Here, I regress the steady state condition of the Solow model, \(\ln(K/Y) = \ln(W^{FK.A}/Y) = \ln(s^Y/(g + \delta))\), augmented with \(r^K\), where \(\delta\) is the depreciation rate for non-residential fixed capital; \(W^{FK.A}\) is measured as the value of fixed non-residential, non-agricultural capital stock at acquisition costs; \(r^K\) is the real after-tax returns to non-residential fixed capital following Eq. (4); and \(s^Y\) is the propensity to save out of NNI where saving is measured as private gross saving. The coefficients of \(s^Y\) and \(r^K\) are positive and significant at the 1% level, while the coefficient of \((g + \delta)\) is insignificant. The restriction that the long run coefficient of \(\ln(s^Y/(g + \delta))\) equals one, as predicted by the Solow model, is rejected at conventional significance levels \(\chi(2) = 123\). The regression results remain unaffected when \(r^K\) is excluded from the regression (the results are not shown).

The significantly positive coefficient of \(r^K\) suggests that an increasing \(W^{FK.A}/Y\) ratio is not associated with lower returns to non-residential fixed capital because of a negative feedback effect
from $W^{FK-A-Y}$ to $r^K$ as is often argued in the literature (see, for discussion. Piketty, 2015a; Ray, 2015). This result suggests that foreign investment and effective foreign and domestic labor supply may have countered diminishing returns to fixed reproducible capital or that the elasticity of substitution between labor and capital exceeds one, as found by Karabarbounis and Neiman (2014). In fact, the historical evidence suggests that migration flows and cross-border capital movements have been high in the UK for centuries (Madsen and Murtin, 2017). Furthermore, the positive coefficient of $r^K$ indicates that the positive relationship between $r$ and the $W-Y$ ratio is neither driven by non-reproducibles nor by short-term real capital gains.

5.2 Decomposing $r$ into capital gains and dividend yields

To gain deeper insight into the principal sources of wealth accumulation, asset returns are decomposed into dividend yields, $r^{DY}$, and real capital gains on agricultural and urban land, $r^{CG}$, in the regressions in columns (3) and (7) in Table 5. Dividend yield is a real magnitude because implicit price deflators are included in the numerator (dividends) as well as the denominator (asset value) and, therefore, it does not need be transformed to a real value by deducting consumer price inflation. Real capital gains on fixed assets (stocks) are not included in $r^{CG}$ because, in steady state, capital gains on stocks are driven by retained earnings and share buy-backs, otherwise Tobin’s $q$ would be permanently, and at an increasing rate, diverted from its steady state equilibrium.

For both the $W-Y$ regression and $S^{WCG}$ regressions, the coefficients of $r^{DY}$ and $r^{CG}$ are highly significant; however, the coefficients of $r^{CG}$ are three times as large as those of $r^{DY}$, showing that capital gains are much more important sources of wealth accumulation and inequality than dividend yields. These results imply that the marginal propensity to consume out of dividend yields exceeds that of capital gains, probably because 1) credit constraints prevent landholders from borrowing against capital gains on land; 2) capital gains are not expected to be permanent and, therefore, not expected to increase permanent income; 3) landholders have imperfect knowledge of the real asset appreciation; and/or 4) habit persistence in consumption among landholders results in increasing saving in response to real capital gains on assets since dividend yields are more stable sources of capital income than capital gains.

Consequently, the higher the share of land in total wealth, the larger is the potential for an increasing $W-Y$ ratio and wealth inequality as experienced up to 1870s and since the early 1980s. Conversely, the gradual erosion of land’s share in total wealth as industrialization unfolded contributed to a reduction in $W-Y$ ratio during the approximate period 1875-1975. The declining share of non-reproducibles in total wealth from the 1870s to the 1970s was associated with declining wealth
inequality, whereas the declining land wealth share has been reversed along with the expanding housing wealth since the early 1980s.

The finding that capital gains on non-reproducibles are much more important sources of variations in wealth inequality than dividend yields suggests that under modes of production in which non-reproducibles are a high share of total wealth, such as in the pre-industrial society (agricultural land) and the post-industrial society (urban property), capital gains on land are much more influential for the evolution in inequality than rental income in general. For measurement of wealth and income inequality, it implies that conventional measures of inequality insufficiently capture increases in inequality in periods of secular increases in land values; a conclusion also reached by Roine and Waldenström (2012). The aforementioned flat wealth inequality Gini coefficient since 2000 is a clear example of how standard wealth inequality measures fail to capture potentially large secular changes in inequality. Over the periods of sustained increasing inequality, such as the periods 1741-1871 and 1978-2013, real capital gains have been 40.0% higher than dividend yields, suggesting that the distortion in standard measures of inequality is even higher in periods of increasing inequality.

5.3 Inclusion of control variables

Government debt, institutional quality, import propensity, the variance of the dependent variable, the propensity to save out of income, old age dependency, relative prices of fixed non-residential capital, top income taxes, and institutional quality are included as controls in the regressions in columns (4)-(5) and (8)-(11) because they may simultaneously affect the dependent variables, r and g; thus, potentially leading to biased the coefficients of r and g. These control variables have often been stressed as important for the inequality path in the inequality literature, as discussed below.

The government debt-income ratio, \( \frac{W}{Y} \), may have affected inequality because after-tax real returns on government debt were often rendered low or negative by unexpected inflation during wars and, consequently, have driven average asset returns down. Although tax-payers may increase their saving in response to higher government debt to counter their future tax liabilities, this effect is captured by the saving terms \( s^W \) or \( s^Y \) in the regressions. The propensity to import, \( Im/Y \), before the second half of the 20th century, likely to be negatively related to inequality because imports of agricultural products tended to reduce the prices on basic food and, conversely, reduce the income of the landed class. The variance of the \( W/Y \) ratio, \( \sigma^2 \), is positively related to inequality following the predictions of the models of Kesten (1973) and Jones (2015) and standard portfolio models in which the required returns are positive functions of the riskiness of the asset. Following the life-cycle hypothesis, increasing old-age dependency may increase inequality as aging population reduces its net asset position.
As argued by Karabarbounis and Neiman (2014), the relative prices of capital, $P^{INV}/P^{CPI}$, will affect factor shares through quantity and price effects, where $P^{INV}$ is the investment price deflator. In their influential paper they show that the marked decline in the real prices of investment goods since the early 1980s across the world has been responsible for approximately half of the decline in labor’s income share. Under the assumption of an elasticity of substitution between capital and labor in excess of one, as validated by the regression analysis of Karabarbounis and Neiman (2014), a decline in real prices of capital leads to a substitution of capital for labor and, consequently, to an increasing share of income going to capital.

Top income tax rates are included as control variables following Piketty’s (2014) argument that high income earners bargain harder for income shares when income tax rates are low because they have more to gain from incremental earnings. Based on calibrations of their model, Hubmer et al. (2016) show that the marked decrease in tax progressivity since the late 1970s has been by far the most powerful force for increasing wealth inequality in the US, because a reduced tax progression spreads out the distribution of after-tax resources available for consumption and saving and increases the returns to saving; thus leading to higher wealth accumulation. Empirically, Roine et al. (2009) find that top tax rates significantly reduce income inequality. Tax rates are measured as the sum of income tax rates of the top 1% decile (statutory rates) and land tax rates because they jointly constitute taxation of high income earners. Land taxes were the dominant form of taxes on income before 1843 when income taxes were permanently introduced (low income tax rates were temporarily in place in the period 1798-1816), and land tax rates were gradually reduced and phased out after 1843 (see, for references, the online Appendix).

Institutions have often been stressed as essential determinants of inequality. As staunch advocates of the importance of institutions on economic development and inequality, Acemoglu and Robinson (2015) argue that changes in income shares are predominantly determined by institutions rather than $r$ and $g$. The Corn Laws, the restrictions on unions before the Trade Union Act in 1871, and the interest rate reducing consequences of the Glorious Revolution of 1688, as argued by North and Weingast (1989), are examples of institutional influences on asset returns. Constraints on the executive and contract intensive money are used as indicators of the quality of institutions.

Following the spirit of Polity IV, constraints on executive, $Exec$, is constructed as institutional constraints on the decision-making powers of governments, and it is operationalized by a seven-point scale, where the minimum value of 1 implies “executive authority” and the maximum value of 7 signifies “executive parity or subordination”. Changes in the index over the period from 1210 up to 1832, when it reached the maximum value and has stayed there since, mostly reflect the changing power relationship between the Crown and the Parliament. Significant institutional changes constitute
the Magna Carta of 1215, when everybody, including the king, became subject to the law, the Civil War of 1642-1649, when Parliamentary forces defeated Charles I, and the Glorious Revolution of 1688-1689, which gave Parliament supremacy over the King (see, North and Weingast, 1989, for discussion of the history of institutions in Britain).

Introduced by Clague et al. (1999), contract-intensive money, as another dimension of institutions, is proxied by \((M_2-H_0)/M_2\), where \(M_2\) is broad money supply and \(H_0\) is the monetary base. Clague et al. (1999) argue that the higher the level of contract-intensive money, the better a government enforces contracts. In economies with excellent third-party contract enforcement, credit and monetary deposits will be the preferred store of money and medium of exchange over cash money, because they are safe, efficient, in most cases pay interest, and facilitate the tracking of credit history and, thereby better enable banks to screen their borrowers. Conversely, if contracts are not enforced by the government, 1) the safety of money in financial institutions is not guaranteed; 2) repayment of loans cannot be taken for granted; and 3) lenders do not have the any security rights to mortgage assets if a borrower defaults (Clague et al., 1999). In these cases, cash will be the preferred medium of exchange over credit. Contract-intensive money affects the wealth accumulation process by simultaneously impacting \(r\) and inequality through the same channels as constraints on executive, Exec.

5.4 Results from the regressions with controls
Consider first the \(W-Y\) regressions in columns (4) and (5) in Table 5. All controls are insignificant in the regression in column (4) except for the government debt ratio and contract-intensive money. Deleting the insignificant variables yields the results in column (5). The government debt-income ratio, \((W/Y)^{GD}\), is significantly negative, which is consistent with the fact that returns on government debt have been driven down as a result of the inflationary implications of debt finance. The coefficient of contract intensive money is significantly positive, suggesting that improvements of contractual arrangements are associated with increasing wealth inequality – a result that makes sense from the perspective that excellent credit facilities and financial contracts ease asset accumulation and investment opportunities. Considering the focus variables, the coefficients of \(r\) and \(g\) remain statistically highly significant in all the \(W-Y\) regressions in which the controls are included, and their magnitudes are close to those of the baseline regressions; thus giving further support to the thesis that \(r\), \(g\) and \(s^W\) are the principal drivers of wealth inequality proxied by the \(W-Y\) ratio.

Turning to the \(S^{WCG}\)-regressions in columns (8)-(10) most of the coefficients of the controls are statistically significant determinants of \(S^{WCG}\) and \(S^W\). The coefficients of import propensity are negative and highly significant, suggesting that trade openness benefits labor income shares because British imports have traditionally competed with agricultural production and, as argued above, it has long
been in the interest of the landed class to keep profits up through restrictions on imports. The coefficients of real prices of capital, \( \frac{P^{INV}}{P^{CPI}} \), are negative and slightly significant in the \( S^W \)-regression; a result that is consistent with the hypothesis of Karabarbounis and Neiman (2014). The coefficients of top income taxes are negative; thus, giving support to Piketty’s (2014) postulate that high income earners have less incentive to bargain for higher income shares when taxes are high because of reduced expected gains from bargaining over income shares. The coefficients of \( s^Y \) are highly significantly positive, which is inconsistent with the predictions of the post-Keynesian models of Kaldor (1955) and Pasinetti (1962) in which increasing saving rates curb consumption and, consequently, reduce price-markups over marginal costs through reduced aggregate demand. However, the result is consistent with Piketty’s (2014) prediction that a saving-induced increase in the \( W-Y \) ratio increases capital’s share of income because there is no or little counter-balancing reduction in \( r \). Similarly, Kuznets (1955) argues that saving is a powerful source of inequality through asset accumulation.

The institutional quality variables, contract-intensive money and constraints on executive, have signs contrary to the hypothesis of Acemoglu and Robinson (2015). The coefficients of contract-intensive money are significantly positive in all five cases and constraints on executive are significantly positive in the factor share regressions. This does not mean that institutional improvements, in general, are adversely affecting inequality as validated by the results for import propensities and tariffs. What these results show is that institutional improvements need not be favorable to worker’s income share. Some institutional reforms favor labor and some favor capital. Although the institutions in Latin America have historically been more democratic than those in East Asia, for example, income distribution has been substantially more unequal in Latin America than East Asia (see, for data on these issues, Polity IV, and World Development Indicators). Similarly, as argued above, contract-intensive money may increase the investment opportunities of the wealthy high-income earners.

The old age dependency ratio, ‘Old Age’, is significantly positive in the \( W-Y \) estimates in column (6); however, it is insignificant in the \( S^{WCG} \)-regression in column (11), suggesting that old age dependency has an ambiguous effect on inequality (dictated by the availability of data on age distribution, the estimation period starts in 1541). Finally, the coefficients of \( r \) and \( g \) in the factor shares regressions remain highly significant and close to those of the baseline regressions, indicating that the baseline results have not been driven by important omitted variables that have simultaneously affected the dependent variables, \( r \) and \( g \).

Overall, the regressions in Table 5 show that the baseline results are robust to inclusion of variables that have been highlighted in the literature as being important for inequality. Importantly, the
coefficients of \( r \) and \( g \) remain highly significant and of the expected sign in all regressions. Moreover, the high \( R \)-squares in the regressions indicate that the models explain a large share of the variation in the \( W-Y \) ratio, \( S^{WCG} \) and \( S^W \). This is particularly true in the \( S^{WCG} \) regression in column (9) in which \( R \)-squared reaches the value of 0.77, which, combined with a very low coefficient of the lagged dependent variable, suggests that the model is very good at tracking capital-gains augmented capital shares despite the large variations in capital gains on land as well as stocks. Finally, \( r \) and \( g \) are the key-drivers in the \( S^{WCG} \) regressions. Comparing \( R \)-squared in the baseline \( S^{WCG} \) regression in column (2) in Table 4 with that of the regression in column (9) in Table 5, in which the baseline regression is extended with the significant control variables, the unadjusted \( R \)-squared improves by only 0.07 and the \( R \)-squared drops to 0.37 if \( r \) and \( g \) are excluded from the \( S^{WCG} \) regressions in column (9) in Table 5; thus reinforcing the importance of \( r \) and \( g \) as the central determinants of the evolution of inequality.

6. Concluding remarks

Piketty’s hypothesis that inequality is increasing in the \((r-g)\)-gap is highly controversial and yet little, if any, empirical research has examined the validity of the hypothesis and the extent to which \( r \) and \( g \) are significant determinants of wealth and income inequality. Constructing annual data for Britain over the period 1210-2013, this paper is a first attempt to overcome this lacuna in the literature by examining the dynamics of the \( W-Y \)-ratio and capital’s income share, \( S^{WCG} \) (where \( S^{WCG} \) includes capital gains on non-reproducibles), and their relationship to \( r \) and \( g \). It is shown that \( r \) and \( g \) have highly significant and quite persistent effects on wealth and income inequalities, as proxied by the \( W-Y \) ratio and \( S^W/S^{WCG} \). The results of the paper give three principal insights into the dynamics of inequality and the dynamics of inequality in the ultra-long run.

First, the regressions indicate that \( r \), \( g \) and to some extent saving, have been the principal drivers of inequality proxied by the \( W-Y \) ratio and \( S^{WCG} \) over the past eight centuries. Although some controls, such as institutions, top income tax rates, openness, and savings are also significant determinants of \( S^{WCG} \), \( r \) and \( g \) explain the lion’s share of the variance in \( S^{WCG} \) regression and, jointly with saving, most of the explained variance in the \( W-Y \) regressions. These results imply that an increasing wealth concentration, as signified by high \( W-Y \)-ratios, historically has not been associated with a lower \( r \) introduced by diminishing returns to wealth combined with feedback effects from the \( W-Y \) ratio on \( r \). This result remain intact if wealth is limited to non-residential reproducible capital stock, which has been a relatively stable 20% fraction of total wealth, indicating that diminishing returns to capital may have been countered by migration flows, foreign direct investment, expanding population, and Malthusian population dynamics. Under some realistic assumptions, diminishing returns to capital can be countered entirely in a Malthusian regime and, in a post-Malthusian regime, through cross-border
capital and labor flows in a small open economy. Furthermore, the analysis shows that the $W-Y$ ratio has been periodically trending upwards without reducing the gap between returns to reproducibles and growth in capital stock, and, therefore, compromising dynamic efficiency.

Second, the $(r-g)$-gap, $W/Y$ and $S^{WCG}$ have been fluctuating around a constant trend in the ultra-long run; thus, giving support for Piketty’s (2014) conjecture that the low inequality that prevailed over most of the 20th century was temporary phenomenon and that OECD economies may converge toward a higher level of inequality in the future. This supposition is supported by the analysis showing the tendency for pre- and post-industrial economies to be more unequal because potentially large capital gains on non-reproducibles are prone to aggravate wealth and income concentration. Unlike reproducibles, the increasing demand for land combined with inelastic land supply during periods of expanding population has often resulted in waves of expanding real values of non-reproducibles. The $W-Y$ and the $S^{WCG}$ elasticiticities of real capital gains are found to be significantly higher than those of dividend yields, suggesting that wealth and income inequality dynamics are heavily dependent on the source of returns. Since real capital gains on assets in steady state are approximately limited to non-reproducibles, it follows that wealth accumulation is faster in economies in which agricultural and urban land are large components of wealth, as in the pre-industrial period and today, where the expansion of the service sector in the city areas has resulted in urban property being the largest component of personal wealth. The empirical evidence give support for these effects. Furthermore, productivity spurts in pre-industrial Malthusian-trapped societies are also potential sources of inequality because the real income of the poor stays constant while land-holders, for which the income elasticity of fertility is lower than that of the poor, enjoy increasing income from the productivity advances.

The tendency for income and wealth distribution to be more unequal in pre- and post-industrial regimes stands in contrast to Kuznets’ (1955) celebrated thesis that inequality starts up low in pre-industrial societies, rises during industrialization, and declines as the economy matures, partly because wealth accumulation is more difficult in a post-industrial economy. Although referring to Europe and the US in his article, Kuznets may have had the land-abundant US economy in mind when he wrote his celebrated paper noting that agricultural and urban land is a much less important source of wealth in the US than Europe because of the relative abundance of land in the US (Piketty and Zucman, 2014).

Third, the estimates give qualified support for Piketty’s $(r-g)$-hypothesis. While support is found for Piketty’s thesis that $r$ and $g$ are the principal drivers of wealth and income inequality, the results suggest that: Piketty’s $(r-g)$-hypothesis needs to be extended to allow for saving propensities; that real capital gains on non-reproducibles have significantly stronger effects on inequality than dividend yield; and that $g$ impacts approximately twice as much on inequality as $r$, suggesting that an
$(r-2g)$-gap (perhaps further decomposing $r$) is a better approximation than the $(r-g)$-gap. Although these results do not overturn Piketty’s reasoning that an increasing $(r-g)$-gap over the rest of the 21st century will result in increasing income and wealth inequality, they nevertheless suggest that Piketty’s $(r-g)$-hypothesis needs to be extended along these dimensions. Related to this is the finding that the impact of $r$ and $g$ on the $W-Y$ ratio is much stronger for total wealth than for non-residential fixed capital. This result suggests that a broader wealth concept than the one used in standard growth models, such as the wealth concept advocated by Piketty (2014) and Piketty and Zucman (2014), is needed to analyze inequality dynamics at the macro level.

The finding that $r$, $g$, and to some extent, saving are the principal determinants of inequality and that factor shares and the $W-Y$ ratio are excellent proxies for income inequality, suggests that inequality is intrinsically a macroeconomic phenomenon and that Piketty’s models, extended in various directions, could appropriately form the basis as a general macro model of inequality. Piketty (2014) has provided a framework that challenges the long-held view of inequality as being a microeconomic phenomenon, mostly explained by individual characteristics and often divorced from the macro economy. The general nature of Piketty’s model takes it beyond the conventional macro models that seek to explain the inequality path over segmented periods by the real price of capital, biased technological progress, and institutions etc.
References


