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Some Consequences of Using "Measurement Error Shocks" When Estimating Time Series Models

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Abstract

In a number of time times models there are I(1) variables that appear in data sets in differenced from. This note shows that an emerging practice of assuming that observed data relates to model variables through the use of "measurement error shocks" when estimating these models can imply that there is a lack of co-integration between model and data variables, and also between data variables themselves. An analysis is provided of what the nature of the measurement error would need to be if it was desired to reproduce the same co-integration information as seen in the data. Sometimes this adjustment can be complex. It is very unlikely that measurement error can be described properly with the white noise shocks that are commonly used for measurement error.

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Some Consequences of Using "Measurement Error Shocks" When Estimating Time Series Models*

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Abstract

In a number of time times models there are I(1) variables that appear in data sets in differenced from. This note shows that an emerging practice of assuming that observed data relates to model variables through the use of "measurement error shocks" when estimating these models can imply that there is a lack of co-integration between model and data variables, and also between data variables themselves. An analysis is provided of what the nature of the measurement error would need to be if it was desired to reproduce the same co-integration information as seen in the data. Sometimes this adjustment can be complex. It is very unlikely that measurement error can be described properly with the white noise shocks that are commonly used for measurement error.

1 Introduction

Many applications of time series models use data measured as growth rates of variables such as GDP, nominal exchange rates and price levels. It is often

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now assumed that these are noisy measures of the corresponding variables in the model of interest. A recent application in Aruoba et al (2016) considered that the different measures of GDP growth could be regarded as deviating from true GDP growth and that the deviations would be described by measurement errors. Many factor models make a similar assumption. In terms of models with a greater economic emphasis, DSGE models often proceed in this way e.g. Guerron-Quintana (2010). Examples of policy models following this strategy would be the EDO model of the Federal Reserve - Chung et. al. (2010) - and the Multi-sector model of the Reserve Bank of Australia - Rees et. al. (2016). When estimation of the parameters is performed it is assumed that there is a discrepancy between model growth variables and the data on them, and these discrepancies are often described as "measurement error shocks".[1] The purpose of this note is to explore what the implication of these "measurement error shocks" is.

Section 2 shows that such shocks generally imply a lack of co-integration between the *levels* of the model variables and the corresponding data in levels, but also it implies that there is a lack of co-integration between the data variables themselves.[2] After showing this in a simple way, sections 3 and 4 turn to the question of what happens when the model implies some co-integration between the model level variables, while the data may be consistent with exactly the same number of co-integrating vectors (or perhaps more) than are implied by the model. In both cases one can make the model and data variables co-integrate by using time differences of white noise measurement error shocks as the augmenting mechanism. However, this is at the expense of using an incorrect description of what the correct measurement error shocks should be. It is rarely the case that one can treat the measurement error shocks as white noise, as is typically done in most applied studies. The exception to that occurs if there is no co-integration in the data. So using white noise shocks is making the presumption that the data lacks co-integration. Whether this was the modeller's intention is a question

[1] I have never been happy with this description. Basically what these shocks do is to measure the extent to which the model fails to track the data, and therefore Fukac and Pagan (2011) called them "tracking shocks".

[2] This is also true if the I(1) data is filtered to produce I(0) processes that are used in models e.g. as an output gap. In those cases the filtered data will be weighted averages of growth rates in variables so it is an average of growth rates in the data that would be held to deviate from model growth. Thus, the issues we describe in this note also apply when filtered data are used, although the analysis is more complex.

that always needs to be resolved.

2 Case 1: The Most Common Approach

We start with a simple situation which parallels that in Aruoba et. al. (2016). Let Δz_t be the true growth rate in GDP and Δz_{jt} $(j = 1, 2)$ being two noisy measures of it. Then we have $\Delta z_{jt} = \Delta z_t + \eta_{jt}$, where η_{jt} are said to be measurement errors. In that paper they are white noise. Now consider what this means for the relation between the level of GDP and its measures. Selecting the first data series we have

$$\begin{aligned} z_{1t} - z_t &= \sum_{k=1}^{t} (\Delta z_{1k} - \Delta z_k) \\ &= \sum_{k=1}^{t} \eta_{1k}. \end{aligned}$$

Consequently, under the assumption that measurement errors are white noise it is clear that z_{1t} and z_t must not be co-integrated i.e. the data and model variables do not co-integrate. Moreover the difference

$$\begin{aligned} z_{1t} - z_{2t} &= \sum_{k=1}^{t} (\Delta z_{1k} - \Delta z_{2k}) \\ &= \sum_{k=1}^{t} (\eta_{1k} - \eta_{2k}), \end{aligned}$$

would also be $I(1)$ and so, unless η_{1k} and η_{2k} are perfectly correlated, there would be no co-integration between the data variables z_{1t} and z_{2t}.[3]

Now it may be that the observed and model variables are not co-integrated but it would seem more satisfactory if at least one of the measured quantities did co-integrate with the true level of GDP. Therefore it doesn't seem sensible to rule it out with the chosen specification of the measurement error, and so later we look at how the latter would need to be treated. One can of course always test if z_{1t} and z_{2t} are co-integrated so that provides some check on the reasonableness of the assumptions being made about the measurement

[3] If they were perfectly correlated then basically z_{1t} and z_{2t} would be the same series.

error. For the Aruoba et al (2016) paper one model used has z_{1t} as the expenditure based series on GDP while z_{2t} is the income based series. We therefore test if there is co-integration between these two series. Using a VAR(2), Johansen's trace and eigenvalue tests are equal and of value 3.22. The 5% critical value would be 3.84, so there does seem to be co-integration between the observables, and a useful re-specification of their model would be to include some error correction terms, although whether this would have a big impact is a different issue.[4] It is interesting that the co-integrating vector seems to be (1 -.9978).

Now in the statistical model above there is only one latent (unobserved) variable and so there is no issue of co-integration between the model variables themselves. In contrast DSGE models generally have co-integration between I(1) variables in the model. To make this more concrete let there be data on three I(1) model variables - Δz^*_{Dt} = data on foreign GDP growth; Δz_{Dt} =data on domestic GDP growth and Δc_{Dt} = data on domestic consumption growth, where D =data. As well we have corresponding DSGE model variables (M = model) Δz^*_{Mt} , Δz_{Mt} and Δc_{Mt}. In the DSGE model there is a log level of technology process a_t which follows a pure random walk $a_t = a_{t-1} + \omega_t$, where ω_t are white noise innovations that have zero mean and variance σ^2. This produces unit roots in the logs of the two GDP and consumption processes, and these co-integrate with a_t. Thus the DSGE model features co-integration.

Now the assumptions often made when estimating the DSGE model with the growth rate data are that the model and data growth rate variables differ by measurement error, that is [5]

$$
\begin{aligned}
\Delta z^*_{Dt} &= \Delta z^*_{Mt} + \eta_{1t} \\
\Delta z_{Dt} &= \Delta z_{Mt} + \eta_{2t} \\
\Delta c_{Dt} &= \Delta c_{Mt} + \eta_{3t},
\end{aligned}
$$

where the η_{jt} are white noise innovations that are uncorrelated with each other. To see the relation between the data and model *level* variables assume

[4] I thank Dongho Song for sending me this data.

[5] See for example Pfeifer (2015).

initial conditions are zero so that

$$\begin{aligned} z^*_{Dt} - z^*_{Mt} &= \sum_{k=1}^{t} \eta_{1k} \\ z_{Dt} - z_{Mt} &= \sum_{k=1}^{t} \eta_{2k} \\ c_{Dt} - c_{Mt} &= \sum_{k=1}^{t} \eta_{3k}. \end{aligned}$$

Hence

$$z^*_{Dt} - z_{Dt} - (z^*_{Mt} - z_{Mt}) = \sum_{k=1}^{t} \eta_{1k} - \sum_{k=1}^{t} \eta_{2k} \tag{1}$$

Now within the model there is co-integration between z_{Mt} and z^*_{Mt} since they both co-integrate with the log level of technology a_t. Consequently, using (1),

$$\begin{aligned} z^*_{Dt} - z_{Dt} &= (z^*_{Mt} - z_{Mt}) + \sum_{k=1}^{t} \eta_{1k} - \sum_{k=1}^{t} \eta_{2k} \\ &= I(0) + \sum_{k=1}^{t} (\eta_{1k} - \eta_{2k}) \end{aligned}$$

But this must mean that $z^*_{Dt} - z_{Dt}$ is $I(1)$ unless $\sum_{k=1}^{t} (\eta_{1k} - \eta_{2k})$ is $I(0)$, which cannot happen unless $\eta_{1t} = \eta_{2t}$.We can see the same thing if we ask whether c_{Dt} and z_{Dt} co-integrate? c_{Mt} co-integrates with a_t, and so c_{Mt} and z_{Mt} co-integrate, but this is not true of $c_{D,t}$ and $c_{M,t}$.

So whenever the data is measured as growth rates in a variable z_t, and a measurement error shock is added into the observation equation, it implies there is no co-integration between the data and model variables. Moreover, if more than one I(1) variable is being treated in the same way, then this implies no co-integration between the variables in the data. This is a strong assumption and one that can be tested. Of course, if there is really no co-integration in the data then the standard method of adding on white noise measurement error shocks will be appropriate. It is interesting to note that the EDO model has measurement error shocks that are specified in this way, and so this implies a lack of co-integration between some I(1) variables. So nothing needs to be done if one is happy with that outcome. Instead the

EDO modellers introduced some extra I(1) shocks to account for the lack of co-integration seen in the data, even though this is unnecessary.

3 Case 2: Both Data and Model Have Co-integration of the Same Form

We now look at the case where both the model and data might be thought of as having the same type of co-integration. We need to be more specific about what we mean by this term and so we consider an example based on having the basic RBC model as the theoretical DSGE model and where there is a unit root in the log of technology a_t driving the model. Then this is log-linearized and model variables are expressed as deviations from a_t. Consequently the variables solved for in the model will be $\tilde{c}_t = c_t - a_t, \tilde{\imath}_t = i_t - a_t, \tilde{k}_t = k_t - a_t$ and $\tilde{y}_t = y_t - a_t$, where c_t,i_t, k_t and y_t are the logs of consumption, investment, the capital stock and output. Then $\tilde{c}_t, \tilde{\imath}_t, \tilde{k}_t$ and $\tilde{y}_t$ will be $I(0)$ and so c_t, i_t, k_t and y_t co-integrate with a_t. Alternatively, we can partially express this as y_t co-integrating with c_t, i_t and k_t because the co-integrating relations with a_t imply that $(c_t - y_t), (i_t - y_t)$ and $(k_t - y_t)$ are $I(0)$. Finally, there is a further co-integrating relation due to $(y_t - a_t)$ being $I(0)$. Thus the number of co-integrating vectors is larger in the DSGE model than in the data but those which are common i.e which relate to observable variables c_t, y_t, i_t, k_t and y_t, are of exactly the same type.

Now using the example above a_t is not an observed variable and so the DSGE model will therefore have *one more co-integrating relation than the VECM in observed data* would have. Thus, if there are n observed $I(1)$ variables z_t^D, with the corresponding model variables being z_t^M, and the DSGE model can be expressed as a VECM of the form[6]

$$\Delta z_t^M = \delta\gamma' z_{t-1}^M + \psi(z_{nt-1}^M - a_{t-1}) + e_t^M, \tag{2}$$

where γ are the common co-integrating vectors, and we have chosen to nor-

[6] If it was the case that k_t was treated as unobservable then there would be a second unobserved variable $(k_{t-1} - a_{t-1})$ to join $(y_{t-1} - a_{t-1})$. In that instance γ' would capture the common co-integrating vectors and there would be two less in the data than in the model. In Christensen et. al. (2011) an algorithm is given for converting DSGE model output into a VECM representation.

malize the extra co-integrating relation using the $n'th$ variable z_{nt}.[7] Now suppose that the data is generated by

$$\Delta z_t^D = \delta\gamma' z_{t-1}^D + e_t^D. \tag{3}$$

Then, subtracting (2) from (3), gives

$$\Delta z_t^D - \Delta z_t^M = \delta\gamma'(z_{t-1}^D - z_{t-1}^M) - \psi(z_{nt-1}^M - a_{t-1}) + e_t^D - e_t^M.$$

Defining the term $\xi_t = z_t^D - z_t^M$ this evolves as

$$\Delta\xi_t = \delta\gamma'\xi_{t-1} - \psi(z_{nt-1}^M - a_{t-1}) + e_t^D - e_t^M,$$

and this will imply that ξ_t is $I(0)$. Consequently, there will be co-integration between the levels of variables in the model and the data. Moreover, if we define $\Delta z_t^D - \Delta z_t^M = \eta_t$, it is clear that the "errors in variables shocks" η_t that reconcile data and model growth rates would need to be

$$\eta_t = \delta\gamma'\xi_{t-1} - \psi(z_{nt-1}^M - a_{t-1}) + e_t^D - e_t^M, \tag{4}$$

and it will be impossible for the vector η_t to be white noise, as is generally assumed. In fact due to the composite nature of the error term it will be a VARMA process. Notice the presence of the error correction terms in (4) and it is this that results in the co-integration.[8]

Now one might set up the shock process as in (4), but suppose we simply want to ensure co-integration between the levels of variables in the data and the model. If we make the shock reconciling growth rates η_t a white noise process, v_t, then this would mean that $\Delta(z_t^D - z_t^M) = \eta_t = v_t$, and so there would be no co-integration between model and data level variables . However, by setting $\eta_t = \Delta v_t$ we would ensure co-integration, even though the true η_t that is needed to reconcile the growth rates in the data and the model is quite different, being (4).[9]

Now in the analysis above it was assumed that the loadings δ were the same in the model as in the data. However this seems unlikely. To relax this assumption we designate the loadings from model and data as δ^M and δ^D

[7] The model shocks will be assumed to be white noise processes i.e. they are innovations, although they only need to be $I(0)$ processes for our analysis.

[8] Note that in the case where k_t was unobserved we would have $(k_{t-1} - a_{t-1})$ also driving ξ_t but because this is $I(0)$ by assumption it will not affect the result.

[9] When we have $\Delta z_t^D - \Delta z_t^M = \Delta v_t$ the solution for $z_t^D - z_t^M$ does not cumulate v_t.

respectively. Then following the same steps as the analysis above we would get

$$\eta_t = \delta^D \gamma' \xi_{t-1} + (\delta^D - \delta^M)\gamma' z_{t-1}^M - \psi(z_{nt-1}^M - a_{t-1}) + e_t^D - e_t^M$$

so the analysis above will stand because $\gamma' z_{t-1}^M$ is $I(0)$. Things would be different if the co-integrating vectors relating to observables γ' were not the same as in the model, since then the analysis shows that ξ_t would become $I(1)$. But it seems reasonable to argue that modellers would be able to discover this and make adjustments to the model to reflect that. Thus in terms of the RBC model example, if $c_t^D - y_t^D$ was $I(1)$ then it would be necessary to introduce a second independent (of a_t) common $I(1)$ latent variable into the model. Thus allowing a preference shock to be $I(1)$ would accomplish this. If this second latent $I(1)$ shock was ζ_t then we would end up with $I(0)$ variables $y_{t-1} - a_{t-1}$ and $c_{t-1} - \zeta_{t-1}$ in (4). Once again the specification of η_t would be complex.

4 Case 3: More Co-integrating Vectors in the Data than Implied by the Model

This case may arise infrequently and is most likely due to the model not being complex enough and the extra co-integrating vectors reflecting some factor not accounted for in the model. In this instance the model has the VECM structure

$$\Delta z_t^M = \delta \gamma' z_{t-1}^M + \psi(z_{nt-1}^M - a_{t-1}) + e_t^M,$$

while the VECM for the data has extra co-integrating vectors with the form

$$\Delta z_t^D = \delta \gamma' z_{t-1}^D + \alpha \beta' z_{t-1}^D + e_t^D. \tag{5}$$

Following the analysis of the previous section we would have

$$\eta_t = \Delta \xi_t = \delta \gamma' \xi_{t-1} + \alpha \beta' z_{t-1}^D - \psi(z_{nt-1}^M - a_{t-1}) + e_t^D - e_t^M. \tag{6}$$

Because $\alpha\beta' z_{t-1}^D - \psi(z_{nt-1}^M - a_{t-1}) + e_t^D - e_t^M$ is $I(0)$ then we will get co-integration again. To achieve that it is necessary to add the extra co-integrating vectors into the augmenting term η_t. Notice that once again a choice of $\eta_t = \Delta v_t$ would effect co-integration, but clearly this is a mis-specification of the actual shock needed to reconcile data and model growth rates.

5 Conclusion

The note shows that working with the traditional form of errors in variables shocks in DSGE models would fail to produce co-integration between model variables and data, and would also imply that there is no co-integration between the data variables. If there is in fact co-integration in the data, and it has either the same or more co-integrating vectors as the model, then the traditional method of assuming that model and data growth rates differ by a white noise process results in a failure of model variables to co-integrate with the data. One can produce co-integration by working with differences in a white noise process, although the correct measurement error shocks are far more complex, and involve a VARMA structure. Exactly what the consequences are of this mis-specification of the shock processes will be dependent upon the nature of the model. If one is happy to simply preserve co-integration between model and data variables using differences in white noise shocks, this would seem to be a relatively simple modification in programs that perform estimation with state space methods, such as Dynare.

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