Commodity prices and labour market dynamics in small open economies

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Abstract

We investigate the connection between commodity price shocks and unemployment in advanced resource-rich small open economies from an empirical and theoretical perspective. Shocks to commodity prices are shown to influence labour market conditions primarily through the real exchange rate contrasting sharply with the transmission of technology shocks which are typically argued to affect the economy by changing labour productivity. The empirical impact of commodity price shocks is obtained from estimating a panel vector autoregression; a positive price shock is found to be expansionary for the components of GDP, causes the real exchange rate to appreciate, and improves labour market conditions. For every one percent increase in commodity prices, our estimates suggest a one basis point decline in the unemployment rate and at its peak a 0.3% increase in unfilled vacancies. We then match the impulse responses to a commodity price shock from a small open economy model with net commodity exports and search and matching frictions in the labour market to these empirical responses. As in the data, an increase in commodity prices raises consumption demand in the small open economy and induces a real appreciation. Facing higher relative prices for their goods, non-commodity producing firms post additional job vacancies, causing the number of matches between firms and workers to rise. As a result, unemployment falls, even if employment in the commodity-producing sector is negligible. For commodity price shocks, there is little difference between the standard Diamond (1982), Mortensen (1982), and Pissarides (1985) approach of modelling search and matching frictions and the alternating offer bargaining model suggested by Hall and Milgrom (2008).

Keywords

Commodity prices, search and matching unemployment.

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Abstract

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1 Introduction

Shocks to commodity prices influence labour market conditions primarily through the real exchange rate. By contrast, technology shocks affect the economy by changing labour productivity. For a small commodity producing economy, an increase in the prices of its exported commodities raises wealth and consumption demand and induces a real appreciation. Facing higher relative prices for their goods, non-commodity producing firms post additional job vacancies, causing the number of matches between firms and workers to rise. As a result, unemployment falls, even if employment in the commodity-producing sector is negligible. Documenting and analysing this hitherto unexplored link between commodity prices, the real exchange rate, and labour market conditions from both an empirical and a theoretical perspective is the key contribution of this paper.

The economies in our panel vector autoregression (PVAR) analysis include Australia, Canada, New Zealand and Norway, all of which are net commodity exporters, and importantly given the focus on labour markets, provide high quality data on unfilled vacancies, hours worked, and unemployment. Restricting attention to net exporters allows for side stepping the issue of incomplete pass-through from import prices to consumer prices faced by net commodity importers. Instead, a shock to commodity prices has a direct effect on the terms of trade and the real exchange rate of net exporters, especially if as in our sample, their net exports are small enough on the global scale not to have an effect on world commodity prices. Commodity price shocks are identified recursively as in numerous other empirical contributions that study the impact of commodity price or terms of trade shocks on small open economies. A positive price shock is found to expand the components of GDP, to cause the real exchange rate to appreciate and to improve labour market conditions. For every one percent increase in commodity prices, our estimates suggest a one basis point decline in the unemployment rate and at its peak a 0.3% increase in vacancies.

We build a small open economy model with net commodity exports that features search and matching frictions in the labour market as proposed by Diamond (1982), Mortensen (1982), and Pissarides (1985) (DMP) to obtain an economic interpretation of these empirical findings. In departure from most of the search and matching literature preferences over consumption and labour are specified as in Greenwood et al. (1988) to allow for a consumption differential between employed and unemployed agents. Wages are determined by Nash bargaining between firms and workers. To keep matters simple, all goods are traded and commodity production is fixed in our baseline model. We proceed to show that, conditional on commodity price shocks, this type of model is capable of generating data congruent labour market dynamics. Since our identification scheme easily identifies commodity price shocks in the data, the results from estimating the PVAR provide a clean yardstick against which to assess the performance of the theoretical model through impulse response function matching. This exercise yields estimates of key structural model parameters with implications for the consumption differential between employed and unemployed agents and the degree of international risk sharing through financial markets.

To achieve sufficient shock amplification for labour market tightness (the ratio of unfilled vacancies and job searchers), our formulation of the DMP model trades off between the replacement ratio, i.e., the ratio between unemployment benefits and wages, and the consumption differential between employed and unemployed agents which compensates the employed for the disutility from labour. When this consumption differential is zero, household preferences are additively separable, and a given amount of shock amplification can be achieved with
a high replacement ratio only. When fixing the replacement ratio at 40% our formulation of the DMP model requires that the consumption of the unemployed does not exceed 60% of the consumption of employed workers in order for the model to match the high volatilities of vacancies and unemployment in the data. This finding is reminiscent of the argument in Hagedorn and Manovskii (2008) to counter the criticism of the DMP framework by Shimer (2005).

Most of the search and matching literature focuses on the impact of movements in labour productivity — generally thought of as stemming from technology shocks — on the labour market in a closed economy. For shocks that impact labour productivity directly, labour market tightness and therefore unemployment and vacancies are governed by the behaviour of labour productivity; a result that withstands introducing open economy features despite (relatively minor) reactions in the real exchange rate.

The transmission of commodity price shocks contrasts sharply with this scenario. We show that labour market tightness is approximately proportional to labour productivity and an appreciation of the real exchange rate. Commodity price shocks are closely related to wealth shocks in our theoretical framework and hence a positive shock results in an immediate real appreciation and pushes up consumption demand. Facing a higher relative price for their goods, firms post additional vacancies, matches rise, and unemployment falls. Labour productivity, however, is slow to respond reflecting the pace of the expansion of the capital stock. Thus, the dynamics of labour market tightness, unemployment, and vacancies are governed by the behaviour of the real exchange rate after a commodity price shock.

Financial risk sharing plays an important role in the transmission of the commodity price shock. If the economy is characterised by financial autarky, the windfall profits from an unexpected price increase cause consumption and investment to rise sharply. Apart from missing out completely on the dynamics of the trade balance, such a model predicts a strong appreciation of the real exchange rate which in turn pushes up vacancies and lowers unemployment by more than in our empirical analysis. Thus, the countries in our sample must be able to smooth shocks through financial markets as corroborated by our impulse response function matching procedure.

To assess the sensitivity of our findings, several variations of the model are considered. First, we explore the idea in Hall and Milgrom (2008) of replacing Nash bargaining by an alternating offer bargaining game. The estimation shows that this framework can deliver a high degree of shock amplification to labour market tightness for a low replacement ratio even when employed and unemployed agents consume equal amounts. However, shock amplification depends sensitively on the split of vacancy posting costs into variable and fixed components — this is not the case in our baseline specification. Second, in principle, our framework can also capture the dynamics of a news shock about increased future commodity production analysed in Arezki et al. (2015). Finally, when augmenting the framework to allow for non-traded goods our results remain fundamentally unchanged.

2 Related literature

Our work contributes to two broad strands of the literature. The literature on the transmission of shocks in open economies and the literature on labour market dynamics in the presence of search and matching frictions. Few studies analyse commodity price shocks from the point of view of advanced commodity exporters. One notable
exception is Pieschacon (2012) in comparing the effects of oil shocks in Norway and Mexico with emphasis on
different fiscal regimes. There is, however, a large literature on the effects of oil price shocks from the perspective
of oil importers. For example, Leduc and Sill (2004) analyse the monetary policy response to oil shocks, and
Bodenstein et al. (2011) investigate the transmission channel of oil shocks in an open economy framework.\footnote{Bodenstein et al. (2011) and Bodenstein et al. (2012) discuss the literature on the impact of oil shocks on oil importing countries.}

In analysing the business cycle determinants of small open economies most contributors focus on movements
in the terms of trade instead of the narrower concept of commodity prices. For developing economies, Mendoza
(1995), Kose (2002), and Broda (2004) conclude that terms of trade shocks explain up to half the estimated
volatility in aggregate output at business cycle frequency. Reexamining the conventional view, Schmitt-Grohé
and Uribe (2015) uncover substantial heterogeneity across countries with respect to the contribution of terms of
trade shocks to business cycle fluctuations. In particular, economies that depend on commodity exports appear
to be vulnerable as limited access to financial markets and an insufficient macroeconomic policy framework
exasperate the impact of commodity price movements. An important element of these studies is the assumption
of exogenous terms of trade.

The majority of studies on small developed economies differ in this regard from those on developing economies.
The terms of trade are considered to be an endogenous variable and, similar to the closed economy literature on
the business cycle, fluctuations are viewed as the result of structural disturbances to technology and other sources.
Examples of this approach are Galí and Monacelli (2005), Justiniano and Preston (2010), and Adolfson et al.
to the terms of trade can be helpful in accounting for aggregate fluctuations in developed small open economies
just as they are in the case of developing economies.

Our paper focusses on the labour market dynamics in commodity-exporting developed small open economies.
More specifically, we investigate the empirical performance of the DMP search and matching framework conditional
on commodity price shocks. Our selection of countries provides a good laboratory for this purpose. First, the
labour markets in these countries are in general liberalised and the DMP framework captures key features of the
labour market institutions in place. Second, high quality data in employment, unemployment, and vacancies are
available. Third, despite enormous progress over the years empirical identification of structural shocks continues
to be a topic of controversy.\footnote{Since Galí (1999) its has become standard to identify technology shocks in structural VARs by imposing the restriction that only technology shocks can impact labour productivity in the long run. However, this identification approach has not remained uncriticised. Faust and Leeper (1997) argue that structural VARs with long-run restrictions perform poorly in practice given sample size limitations. Furthermore, Lippi and Reichlin (1993) discuss how a short-ordered VAR may fail to provide a good approximation of the dynamics of the variables in the VAR if the true data-generating process has a VARMA representation. For a recent analysis of these issues and further details see Erceg et al. (2005).}

Turning to the open economy offers the possibility to incorporate shocks
that are easily identified and have robust implications for the labour market. Small open economies have negligible
impact on world commodity prices; assuming a recursive identification scheme (with commodity prices ordered
first) or commodity prices to be exogenous appears to be defensible leading to possibly less controversial yardsticks

\footnote{This debate resembles the one on the hours-worked puzzle raised in Galí (1999) and its explorations by Christiano et al. (2003) and Francis and Ramey (2005).}
against which to assess the performance of theoretical models.

The search and matching framework has emerged as the leading approach for embedding labour markets into macroeconomic equilibrium models. With few exceptions, most contributions building on the DMP framework assume the economy to be closed and to be driven by (labour) productivity shocks only. In this standard formulation, Shimer (2005) points to the difficulty of the DMP framework to generate unemployment and vacancy flows that are of comparable volatility as in the U.S. data. The strong response of the real wage to labour productivity shocks dampens the incentives of firms to post new vacancies. Shimer (2005) stimulated efforts to improve the propagation of technology shocks in the DMP framework.

Early contributions to embed the standard DMP framework into a model of the business cycle are Andolfatto (1996) and Merz (1995). However, open economy models rarely feature search and matching frictions in the labour market. Hairault (2002) and Campolmi and Faia (2011) show how augmenting standard open economy model by the DMP framework impacts the transmission of shocks across countries. Christiano et al. (2011) develop a detailed small open economy DSGE model with search and matching frictions that can be employed for policy analysis. Finally, Boz et al. (2009) study search and matching frictions in a small open economy model calibrated to Mexican data. To our best knowledge, there are no published studies analysing the search and matching in the context of commodity price shocks.

3 Commodity price shocks in advanced small open economies

Among developed economies, net exports of commodities are significant only for a small set of countries. According to the IMF (2012) net commodity exports account for more than 30% of total exports in Australia, Iceland, New Zealand and Norway, and around 20% in Canada. Furthermore, net commodity exports account for 5% to 10% of GDP on average. Because of data limitations Iceland is excluded from our analysis. To quantify the impact of commodity price shocks on economic activity and labour markets we estimate structural vector autoregressive (SVAR) models.

3.1 Data description

Our dataset consists of quarterly data for Australia, Canada, New Zealand, and Norway spanning from 1994 Q3 to 2013 Q4. For each country, we include a trade-weighted real commodity price index, expressed in US dollars, except for Norway for which we use the price of Brent crude oil. Nine country-specific macroeconomic time series complete the dataset: GDP per capita, consumption per capita, investment per capita, the unemployment rate, unfilled vacancies, net exports of goods and services relative to GDP, the real effective exchange rate, the real wage deflated by consumer prices, and hours worked per capita. With the exception of the unemployment rate and net

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5 The same report shows that net commodity exports to total exports exceed 20% in South Korea, but net commodity exports account for less than 2.5% of GDP.

6 The start of the data sample in our panel VAR is dictated by the availability of quarterly vacancy data for New Zealand. We also experimented with longer time series when estimating country-specific VARs depending on data availability. Details on the data used in our analysis and additional estimation results are presented in Appendix A.
exports, the data are transformed into logs. All data, logged or otherwise, are de-trended. For the baseline VAR, we subtract a quadratic trend from the data.

To assess the importance of trade in commodities, Table 1 lists the three most important commodities for the four countries in the sample, based on net exports. Australia dominates the world trade in iron ores and concentrates, accounting for 50% of global net trade and 33% of Australia’s net exports in 2014. Whereas 30% of Canada’s net exports are accounted for by crude oil, Canada’s share in global oil trade amounts to only 5%. In New Zealand, the most important commodity is milk concentrate, accounting for 24% of net exports. New Zealand exports of milk powder account for 37% of global net trade in the commodity. Some countries in our sample may seem to have an outsized role in selected commodity markets. However, the measured share in world trade tends to overstate the importance of a small country for a given commodity since this measure does not reflect a country’s share in world production of the commodity. Countries with large domestic markets may produce and consume a significantly larger share of the commodity, but nevertheless export less. Furthermore, as we focus on a country’s role across all commodities and use a country-specific trade-weighted commodity price, each of the countries in our sample can be viewed as a price taker in the global market for commodities.

For all four countries, commodity prices have experienced high volatility over the sample as visualised by Figure 1 which plots the quarterly percentage change in four commodity price series. In comparing commodity prices to other macroeconomic variables Table 2 reveals that commodity prices have been between 5 and 21 times more volatile than GDP on average.

3.2 Estimation strategy

The effects of commodity price shocks on the labour market are estimated using a panel SVAR approach. As the relevant time series are short, but the countries in our sample experience commonalities in their economic structure, combing the data across countries can improve the quality of the coefficient estimates. Furthermore, estimation of a panel provides a single benchmark for matching the impulse response functions implied by theoretical models to their empirical counterparts.

As in Ravn et al. (2012), the baseline specification assumes that heterogeneity across countries is constant, i.e., we conduct a pooled estimation with fixed effects of a reduced form VAR:

\[
y_{i,t} = \tilde{A}_i + A(L)y_{i,t-1} + u_{i,t}.
\]

The factor \(A(L) \equiv A_0 + A_1L + A_2L^2 + ...\) denotes a lag polynomial where \(L\) is the lag operator. The vector \(u_{i,t}\) summarises the mean-zero, serially uncorrelated exogenous shocks with variance-covariance matrix \(\Sigma_u\). The lag length is set at 2 in our baseline.

The prices of the commodities traded by the countries in our sample are determined in the world markets. Commodity price shocks are identified through a recursive identification scheme. With commodity prices ordered first in the Cholesky decomposition, country-specific shocks are ruled out from affecting commodity prices contemporaneously. However, domestic developments in our sample countries can in principle feed back into the

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7 Canova et al. (2013) offers a comprehensive survey of panel VAR models used in macroeconomics.
3.3 Estimation results

Figure 2 plots the median impulse responses, denoted by the black solid lines, together with the 90% confidence intervals of the panel SVAR to a one-standard-deviation increase in commodity prices. In the baseline VAR, the data is transformed by subtracting a quadratic trend. The shock to commodity prices is both hump-shaped and persistent. Median commodity prices rise by about 8 per cent by the second quarter and have returned to steady state after 12 quarters.

Rising commodity prices lead to a boom in the commodity-exporting economies. Output, consumption and investment rise on impact. Output and investment increase gradually and peak at about 0.15% and 1%, respectively. The increase in private consumption is front loaded and reaches 0.17%. In line with the Harberger-Laursen-Metzler prediction, the net trade position improves by as much as 0.6% of GDP. The dynamic patterns of the trade balance follow closely those of the commodity price index. Accounting for the share of commodity net exports in GDP (averaging around 7.5%), we infer that the movements in the trade balance reflect primarily price rather than quantity changes suggesting a low (short-term) price elasticity of supply for commodities for these financially open economies. The measure of the real exchange rate appreciates following an increase in commodity prices, thus increasing the international purchasing power of domestic households and firms.

Turning to variables reflecting labour market conditions, we note a sustained improvement as evidenced by the fall in unemployment and the rise in unfilled vacancies. Conditions improve on impact and continue to do so beyond the rise in commodity prices. At its peak, the median response of vacancies reaches almost 2% and the unemployment rate drops by 7 basis points. CPI deflated real wages decline on impact but recover quickly, whereas hours worked rise by about 0.2% on impact and remain elevated for several periods.

As a robustness check, Figure 2 also reports the impulse responses from VARs estimated with data transformed by a linear as well as an Hodrick-Prescott filter. The shape and the magnitude of the impulse response functions appear robust to the de-trending method. The results of the panel VAR are also robust to the exclusion of individual countries from the data set. Dropping one country at a time, i.e., estimating four different VARs with only three countries, suggests that the results are not solely driven by one country in the data set.

The effects of an increase in commodity prices on commodity-exporting countries mirror those found on developed commodity-importing countries. For example, Blanchard and Galí (2007) report that a shock that raises the price of oil unexpectedly leads to a contraction in economic activity with GDP falling and unemployment rising. As in other recent studies, commodity price shocks have a significant, yet quantitatively modest effect on domestic economic activity in commodity-exporting countries. After adjusting the magnitude of the shock, Pieschacon (2012) finds that for Norway an 8% increase in the price of oil pushes up private consumption by 0.2%.

The qualitative movements and overall magnitudes of the non-labour market variables are also comparable to

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8By contrast, other empirical studies of the relationship between commodity prices (or the terms of trade) and domestic macroeconomic variables impose commodity prices (or even the terms of trade) to be exogenous. For recent examples employing this more restrictive identification assumption see Pieschacon (2012) or Schmitt-Grohé and Uribe (2015).

9These impulse responses are available from the authors.

10In Pieschacon (2012) a one standard deviation increase in the shock implies the price of oil to rise by 20% and Norwegian consumption to increase by 0.5%.
those shown in Schmitt-Grohé and Uribe (2015) for terms of trade shock (rather than commodity prices) in less developed economies. None of these studies, however, reports results for labour market variables. Medina and Naudon (2012) provide some labour market details for Chile. After an increase in mining terms of trade, vacancies expand and the unemployment rate falls by similar magnitudes as in our sample. Both the traded and the non-traded goods sector account for the increase in employment.

A complementary way of summarising the impact of commodity price shocks on commodity-exporting countries is to compute the conditional standard deviations of the variables of interest. Table 2 reports the unconditional volatilities relative to the volatility of GDP of our data; and in brackets, the volatilities conditional on commodity price shocks. We report data for individual country VARs. The conditional volatility of labour market variables is, in many cases, close to the unconditional moments in the data. The same is true for consumption. The conditional volatilities of investment, the effective real exchange rate, and net trade are somewhat larger than their unconditional counterparts. For the price of commodities, the discrepancy between the conditional and the unconditional volatilities reflects the relatively modest response of output to the changes in commodity prices.

4 Baseline model

Our results suggest that a commodity price boom is associated with a persistent fall in unemployment, and lasting increases in unfilled job vacancies, consumption and investment. To gain a deeper theoretical understanding of the economic channels at work, we build a simple model of a small open economy that exports commodities. The model features search and matching frictions in the labour market to obtain satisfying concepts of unemployment and vacancies as in the seminal contributions of Diamond (1982), Mortensen (1982), and Pissarides (1985) — the DMP framework.

The empirical analysis above provides guidance on the roles of wealth effects on the labour supply and international risk sharing. First, an increase in commodity prices raises the revenues from commodity exports. If the increase in revenues induces a strong (negative) wealth effect on the labour supply, employment, investment, and non-commodity output could contract depending on the importance of capital and labour in producing commodities in the short-term. To limit the impact of such wealth effects we either assume the labour supply to be inelastic or — as in Section 5 — we specify preferences as in Greenwood et al. (1988) to obtain an increase in the labour supply after a commodity price shock. Second, the response on the trade balance suggests that the countries in our sample are limited in their capacity to share risk in international financial markets. Under a low supply elasticity for commodities, the commodity price increase (fall) constitutes a pure wealth transfer to (from) the commodity-producing country. If financial markets were complete in the sense of Arrow and Debreu (1954), these transfers should be very small and should have a negligible impact on the domestic economy.

Apart from explicitly modelling the labour market, our model is standard. The small open economy is populated by a large number of households normalised to 1. Each household consists of a continuum of agents of measure one. In order to be employed, an agent must first be matched to a specific job at a firm. Nash bargaining between the agent and the firm determines the terms of employment. Employed agents (workers) supply labour inelastically and receive the real wage \( w_t \). Unemployed agents receive unemployment benefits in the amount of
Finally, the agents of a household share consumption risk by pooling their resources following the contributions of Andolfatto (1996) and Merz (1995). A household consumes goods (a domestically produced traded good and an imported traded good) financed through wages, unemployment benefits, firm profits, and financial assets. The only asset that trades internationally is a foreign bond. Firms accumulate capital, produce goods, and commodities. All commodities are exported. For the purpose of our subsequent discussion we refer to our baseline model as the DMP model.

4.1 Labour flows

Firms post vacancies which are filled with workers looking for jobs. The number of new matches, $m_t$, resulting from this process is described by the constant returns to scale matching function:

$$m_t = \chi u_t^\zeta v_t^{1-\zeta}. \tag{2}$$

$v_t$ is the number of vacancies and $u_t$ is the number of unemployed household members searching for a job at the beginning of the period. Newly formed matches increase the total number of employed workers immediately. Existing matches are destroyed at the exogenous rate $\rho$. As a result, employment, $n_t$, evolves according to:

$$n_t = (1 - \rho)n_{t-1} + m_t. \tag{3}$$

With the labour force being normalised to unity, $u_t$ is given by:

$$u_t = 1 - (1 - \rho)n_{t-1}. \tag{4}$$

Whereas $u_t$ is the number of unemployed workers searching for a job at the beginning of the period, the unemployment rate following standard definitions is given by:

$$\tilde{u}_t = 1 - n_t. \tag{5}$$

Finally, labour market tightness, $\theta_t$, is defined as:

$$\theta_t = \frac{v_t}{u_t} \tag{6}$$

which allows to express the matching function in terms of the job finding probability, $s_t$:

$$s_t = \frac{m_t}{u_t} = \chi \theta_t^{1-\zeta} \tag{7}$$

11 The approach of Andolfatto (1996) and Merz (1995) preserves the simplicity of the textbook DMP model with risk neutral agents but allows embedding labour market search and matching frictions into a standard business cycle framework with risk averse households. Without the construct of risk-sharing through the household, introducing risk averse agents into the DMP model complicates the analytics of the framework quickly; nonlinear numerical methods are required to obtain solutions to the model. Recent contributions that allow for risk pooling at the household level include Arseneau and Chugh (2012), Gertler and Trigari (2009), and Ravenna and Walsh (2012).

12 Endogenous separation can be introduced by adapting the framework of firm-specific productivity shocks suggested by Ramey et al. (2000).
or the job filling probability, \( q_t \):

\[
q_t = \frac{m_t}{v_t} = \chi \theta_t^{-\zeta}.
\]

4.2 Households

Households are modelled following the early contributions of Andolfatto (1996) and Merz (1995). At any point in time \( n_t \) agents of the household are employed and \( 1 - n_t \) agents are unemployed. Each household maximises the weighted utility of the employed (w) and unemployed (u) agents subject to a set of constraints.

The inter-temporal preferences of the household are given by:

\[
E_0 \sum_{t=0}^{\infty} \beta^t [n_t U(c^w_t, 1) + (1 - n_t)U(c^u_t, 0)].
\]

The period utility function \( U(c_t, h_t) \) of an agent is strictly concave and twice-continuously differentiable in consumption. The labour supply of an agent, \( h_t \), equals 1, if the agent is employed and 0 otherwise. We refrain from the common assumptions that the preferences of the individual agent over consumption feature constant relative risk aversion (CRRA) and that agents do not incur disutility from working.

Total consumption of the final consumption good by all household agents is defined as:

\[
c_t = n_t c^w_t + (1 - n_t)c^u_t.
\]

The final consumption good, \( c_t \), consists of a domestically produced good, \( c^h_t \), and an imported good, \( c^f_t \). More precisely, the final good is defined as a constant elasticity of substitution (CES) aggregate:

\[
c_t = \left[ v \left( c^h_t \right)^{\frac{\theta - 1}{\theta}} + (1 - v) \left( c^f_t \right)^{\frac{\theta - 1}{\theta}} \right]^\frac{\theta}{\theta - 1}.
\]

\( \theta \) denotes the elasticity of substitution between these two types of goods and \( v \) is the share of the domestically produced good in final consumption. The price index of the final good, \( P_t \), is chosen to be the numeraire. Consequently, all other prices are expressed relative to the home final good. For example, the relative price of domestically produced goods, \( p^h_t \), denotes the ratio \( \frac{p^h_t}{P_t} \).

The inter-temporal budget constraint of the household is defined as:

\[
n_t c^w_t + (1 - n_t)c^u_t + p^f_t b_t = w_t n_t + (1 - n_t)b^u_t + (1 + r_{t-1})p^f_t b_{t-1} + \pi_t + T_t.
\]

Households smooth consumption by trading in one-period bonds, \( b_t \), that pay out in units of the foreign intermediate good, \( p^f_t b_t \). The interest rate payable on these bonds, \( r_t \), is equal to the world interest rate adjusted for a debt elastic risk-premium. The spread (or discount) relative to the world interest rate, \( r^*_t \), depends on the debt position of the economy:

\[
1 + r_t = (1 + r^*_t) e^{-\phi h \left( \frac{p^h_t}{P_t} - b \right)}.
\]

With households owning all firms, profits from selling goods and commodities, \( \pi_t \), are passed to the household.
Additional income is derived from employment in the amount \( w_t n_t \) and unemployment benefits \( b^u(1-n_t) \). Lump-sum taxes, \( T_t \), are collected by the government to finance unemployment insurance.

The household maximises lifetime utility (9) subject to the budget constraint (12), and equations (10) and (11) by choosing \( c_t, c^w_t, c^u_t, \) and \( b_t \). The first order conditions associated with this problem can be written as:

\[
\lambda_t = U_c(c^w_t, 1) \\
\lambda_t = U_c(c^u_t, 0) \\
\frac{1}{\Gamma + r_t} = E_t \left[ \beta \frac{\lambda_{t+1} p_{t+1}^f}{\lambda_t p_t^f} \right].
\]

\( \lambda_t \) denotes the Lagrange multiplier on the household budget constraint. Equations (14) and (15) reveal that due to efficient risk pooling marginal utility is equalised across household agents irrespective of their employment status. The optimal choices for \( c_t^h \) and \( c_t^f \) are derived from minimising the costs of obtaining one unit of the aggregate consumption good subject to condition (11):

\[
c_t^h = v (p_t^h)^{-\theta} c_t \\
c_t^f = (1-v) (p_t^f)^{-\theta} c_t.
\]

The import price in terms of the final consumption good, \( p_t^f \), and the price of the domestically produced good, \( p_t^h \), are related through:

\[
1 = v \left( p_t^h \right)^{1-\theta} + (1-v) \left( p_t^f \right)^{1-\theta}.
\]

Note, that the household is not choosing the level of total employment, \( n_t \), or wages, \( w_t \). Wages are set in a bargaining game between individual workers and firms over the surplus of the match. However, the marginal value of employment to the household is a key component in determining the surplus of the match. Let \( s_t = \frac{w_t}{u_t} \) denote the probability that an unemployed agent finds a new match. Applying this definition in equation (3) yields:

\[
n_t = (1-\rho)n_{t-1} + s_t u_t = (1-\rho)(1-s_t)n_{t-1} + s_t
\]

and the marginal (monetary) value of employment to the household, \( H_t \), is shown to evolve according to:

\[
H_t = \frac{U(c_t^w, 1) - U(c_t^w, 0)}{\lambda_t} + w_t - b^u - (c_t^w - c_t^u) + (1-\rho)E_t \left[ \beta \frac{\lambda_{t+1} p_{t+1}^f}{\lambda_t p_t^f} H_{t+1}(1-s_{t+1}) \right]
\]

by applying the envelope theorem.

Expression (21) is obtained from the value function of the household and the constraints (12) and (20) as shown in Cheron and Langot (2004) and Hall and Milgrom (2008) for arbitrary time-separable preferences. Moving one household member into employment affects utility of the overall household in three ways. First, the utility of the agent changing employment status adjusts by \( U(c_t^w, 1) - U(c_t^w, 0) \). Second, total household resources rise by the
difference between wages and unemployment benefits, \( w_t - b^u \). Finally, total expenditures change by the difference between consumption provided to working and unemployed household members. Household utility increase by the product of the net increase in available resources, \( w_t - b^u - (c_t^w - c_t^u) \), and the marginal utility of wealth to the household, \( \lambda_t \). Finally, the gains from matching a household member with a firm also occur in future periods. To express the utility gain to the household in units of the final consumption good we divide by the marginal utility of wealth.

Most authors in the labour search DSGE literature assume a CRRA utility function and set the disutility from labour to zero:

\[
U(c_i^t) = \frac{c_i^{1-\sigma}}{1-\sigma}.
\]  

(22)

Under CRRA-utility, it is not only true that all household members have the same marginal utility; in fact, each agent will experience the same utility level as consumption levels do not differ by employment status. Thus, equation (21) reduces to the form commonly found in the literature:

\[
H_t = w_t - b^u + (1-\rho)E_t \left[ \frac{\beta}{\lambda_t} H_{t+1}(1-s_{t+1}) \right].
\]  

(23)

Assuming CRRA-utility in combination with efficient risk-pooling at the household level implies that the marginal value of employment to the household coincides with the value of employment in the standard DMP model with risk-neutral agents.

By contrast, we specify preferences as in Greenwood et al. (1988) (GHH), but start with the assumption that the hours worked by an employed agent are constant:

\[
U(c_i^t, h^w) = \left( c_i^t - \frac{\phi_0}{1+\phi} (h^w)^{1+\phi} \right)^{1-\sigma} \quad \frac{1}{1-\sigma}
\]  

(24)

with \( h^w = 1 \) and \( h^u = 0 \). Defining \( \Phi = \frac{\phi_0}{1+\phi} (h^w)^{1+\phi} \), equations (14), (15), (21) imply:

\[
\Phi = c_t^w - c_t^u
\]  

(25)

\[
\lambda_t = (c_t - n_t \Phi)^{-\sigma}
\]  

(26)

\[
H_t = w_t - b^u - \Phi + (1-\rho)E_t \left[ \frac{\beta}{\lambda_t} H_{t+1}(1-s_{t+1}) \right]
\]  

(27)

Under GHH preferences, all agents enjoy the same utility level, but employed agents consume more than unemployed agents. The difference in consumption levels between the employed and the unemployed, \( \Phi \), turns out to be fixed over the business cycle.
4.3 Firms

Domestic firms combine labour and capital to produce an intermediate good, $y_t^h$, with the relative price, $p_t^h$. The present discounted cash flow of these firms, $\pi_t^h$, is defined as:

$$E_0 \sum_{t=1}^{\infty} \beta^t \lambda_t \pi_t^h = E_0 \sum_{t=1}^{\infty} \beta^t \lambda_t \left( p_t^h y_t^h - w_t n_t - x_t - \kappa(v_t, v_t-1) - \bar{q}_t \right). \quad (28)$$

The real wage, $w_t$, is expressed in terms of the consumer price index. The firm’s investment into its capital stock is captured by $x_t$. In order to hire new workers, the firm needs to post vacancies. The cost function for posting a vacancy is denoted $\kappa(v_t, v_t-1)$ with $v_t$ measuring the number of vacancies. To improve the empirical fit of our model we allow the costs of posting vacancies to depend on the rate at which vacancies are posted:

$$\kappa(v_t, v_t-1) = \kappa v_t \left( \frac{v_t}{v_{t-1}} - 1 \right)^2. \quad (29)$$

Pissarides (2009) assumes that the firm has to pay a fixed cost, $\bar{\kappa}$, before the start of the bargaining process, he interprets these costs “as costs that are paid after the worker who is eventually hired arrives but before the wage bargain takes place; for example, they may be the costs of finding out about the qualities of the particular worker, of interviewing, and of negotiating with her. They are sunk before the wage bargain is concluded and the worker takes up the position, but this property is not important for volatility, because training costs that are not sunk play a similar role.” At the aggregate level $\bar{\kappa} q_t$ units of the final good are used to pay for initialising the bargaining process. We refer to $\bar{\kappa} q_t$ as the fixed component of the costs of filling a vacancy and $\kappa(v_t, v_t-1)$ as the variable component.

As with the aggregate consumption good, posting vacancies and physical investment require the use of the domestically produced good and the imported good. We assume the functional forms for these aggregate goods to be identical to the ones for aggregate consumption:

$$x_t = \left[ v_t \left( x_t^h \right)^{\frac{\alpha-1}{\alpha}} + (1 - v) \left( x_t^f \right)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}} \quad (30)$$

and similarly for vacancies. The optimal choices of producing the aggregate investment good and the payments for vacancy posting follow equations (17) to (19).

Each firm maximises its present discounted cash flow (28) subject to three constraints: its production function, the capital accumulation equation, and the evolution of employment. The Cobb-Douglas production function is defined over capital, total employment as well as total factor productivity:

$$y_t^h = a_t k_{t-1}^{\alpha} n_t^{1-\alpha}. \quad (31)$$

The capital accumulation constraint is given by:

$$k_t = (1 - \delta) k_{t-1} + \iota(x_t, x_{t-1}) \quad (32)$$
with the conventional investment adjustment cost function

\[ \iota(x_t, x_{t-1}) = x_t \left( 1 - \frac{\phi^x}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 \right). \]  

(33)

Due to the presence of search and matching frictions in the labour market, firms are also constrained by the evolution of employment. Defining the probability of filling an open vacancy as \( q_t = \frac{m_t}{v_t} \), equation (3) can be expressed as:

\[ n_t = (1 - \rho)n_{t-1} + q_t v_t. \]

(34)

The first order conditions with respect to capital and investment imply the usual restrictions:

\[ t_q = E_t \left[ \beta \lambda_{t+1} \left( \alpha \frac{p_t^h y_t^h}{k_t} + (1 - \delta)q_{t+1} \right) \right], \]

(35)

\[ 1 = t_q \frac{\partial \kappa(v_t, v_{t-1})}{\partial x_t} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} t_q \frac{\partial \kappa(v_{t+1}, v_t)}{\partial x_t} \right]. \]

(36)

where \( t_q \) denotes Tobin’s \( q \).

The first order condition for vacancies can be written as:

\[ q_t (J_t - \bar{\kappa}) = \frac{\partial \kappa(v_t, v_{t-1})}{\partial v_t} + E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \kappa(v_{t+1}, v_t)}{\partial v_t} \right]. \]

(37)

where \( J_t \) is the Lagrange multiplier on equation (34). The expected benefit from posting a vacancy, \( q_t (J_t - \bar{\kappa}) \), equals the marginal costs of posting a vacancy. Equation (37) is commonly referred to as the free entry into production condition.

As shown next, \( J_t \) measures the value that the firm assigns to an additional unit of employment. Following similar steps as for households, \( J_t \) is obtained from the firm’s value function associated with its optimisation problem. By the envelope theorem:

\[ J_t = \left( (1 - \alpha) \frac{p_t^h y_t^h}{n_t} - w_t \right) + (1 - \rho) E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1} \right]. \]

(38)

By employing one additional worker, the firm raises profits in the current period when the marginal product of labour, \( mpl_t = (1 - \alpha) \frac{p_t^h y_t^h}{n_t} \), exceeds the wage payment, \( w_t \). Furthermore, the firm receives a continuation value if the match survives.

Under standard assumption, the marginal costs of posting vacancies do not depend on past posting choices, i.e., \( \phi^v = 0 \), and there are no sunk costs of bargaining, i.e., \( \bar{\kappa} = 0 \). In this case, equations (37) and (38) reduce to the familiar system:

\[ q_t J_t = \kappa^v, \]

(39)

\[ J_t = mpl_t - w_t + (1 - \rho) E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1} \right]. \]

(40)
4.4 Wage bargaining

When a match occurs between a worker and a firm, the two negotiate over the real wage, \( w_t \). The surplus of the match is measured by \( H_t + J_t \). Assuming (efficient) Nash bargaining the solution of the bargaining game is derived from the optimisation program:

\[
\max_{w_t} H_t^{\xi} J_t^{1-\xi}
\]

subject to equations (21) and (38) which describe the evolution of the variables \( H_t \) and \( J_t \) over time. The term \( \xi \in (0, 1) \) denotes the bargaining power of the household. The power of the firm is given by \( 1 - \xi \). The surplus of the match is shared according to:

\[
J_t = \frac{1 - \xi}{\xi} H_t.
\]

4.5 Real exchange rate

We define the real exchange rate, \( rer_t \), in terms of the consumer price indices. As standard in the small open economy literature, the negligible size of the domestic country relative to the rest of the world implies that the domestic import price roughly equals the consumption real exchange rate, \( p^f_t \approx rer_t \). From equation (19) it then follows:

\[
1 = v (p^h_t)^{1-\theta} + (1-v) (rer_t)^{1-\theta}.
\]

4.6 Commodities

The commodity supply of the small open economy to the world market is assumed to be price inelastic and fixed over time.\(^\text{13}\) In addition, we abstract from the use of commodities in domestic consumption or production as for the countries in our sample the share of domestic use is minuscule relative to the overall commodity output.

Abstracting from endogenous movements in the supply of commodities focuses our work on the transmission of commodity price changes through their impact on wealth. Changes in the supply of commodities are often slow to occur. Unless sizeable excess capacity persists in the commodity-producing sector, the supply response is muted. Focusing on oil-producing Norway, Pieschacon (2012) includes oil production into a structural VAR. The estimated response of oil production after an oil price shock is small and insignificant. By contrast, the expansion in non-oil output is highly significant and about 5 to 8 times larger than the expansion in oil production depending on the horizon.

Furthermore, the direct impact of the commodity-producing sector on the labour market is likely to be small and cannot explain exclusively the economy-wide dynamics of unemployment and vacancies. For example, Australian employment in mining accounts for less than 3% of total employment although mining constitutes around 9% of GDP. Only 2% of Norwegian workers are employed in the extraction of oil and gas while the oil and gas industry accounts for 22% of Norwegian GDP.\(^\text{14}\) With commodity production being capital-intensive, employing only a small share of the domestic labour force, and being slow to respond to price shocks empirically, we deem

\(^{13}\) Even in the interwar period when commodity prices declined sharply, overall production of commodities did not contract significantly as shown in Kindleberger (1973), Chapter 4, Figure 2.

\(^{14}\) The share rise below 4% if administrative and service positions are included.
it defensible to assume that commodity price shocks primarily transmit to the remainder of the economy through their impact on transfers and thus wealth.

We denote the price of the commodities by $p^c_t$ and their supply by $y^c_t$. Profits from commodity extraction, $\pi^c_t = p^c_t y^c_t$, are distributed to the households. Commodity prices are determined in world markets and are set in foreign consumption units, $p^c_*$. The domestic price of the commodity, $p^c_t$, is related to its world price through the real exchange rate, $rer_t$:

$$p^c_t = rer_t p^c_*.$$  \hfill (44)

### 4.7 Market clearing and net trade

Demand for the domestically produced good arises from consumption, investment, filling vacancies and from abroad. Given the relative price of the domestically produced good, $p^h_t$, and aggregate consumption demand, $c_t$, the optimal consumption demand for the domestically produced good follows equation (17), i.e., $c^h_t = v\left(p^h_t\right)^{-\theta} c_t$. With similar relationships applying to the demand for the purpose of investment and covering vacancy posting costs, market clearing for the domestically produced good implies:

$$y^h_t = v\left(p^h_t\right)^{-\theta} \left(c_t + x_t + \bar{\kappa} q_t + \kappa(v_t, v_{t-1})\right) + ex^h_t.$$  \hfill (45)

Export demand from abroad is assumed to be of the form:

$$ex^h_t = v^* \left(rer_t\right)^{\theta^*} y_t^*.$$  \hfill (46)

with $y_t^*$ denoting total foreign demand for the domestic good.

Finally, the evolution of the net foreign asset position of the domestic country is obtained from the budget constraint of the household (12) and the definition of profits by goods and commodity producers. Combining these equations yields:

$$p^f_t b_t = (1 + r_{t-1}) p^f_t b_{t-1} + p^c_t y^c_t + p^h_t y^h_t - c_t - x_t - \bar{\kappa} q_t - \kappa(v_t, v_{t-1}).$$  \hfill (47)

### 5 Alternative models

To contrast the labour market dynamics after a commodity price shock in the DMP model with the dynamics derived under alternative approaches taken in the literature, we consider an equivalent model with a Walrasian labour market and the alternating offer bargaining model proposed in Hall and Milgrom (2008).

The search and matching framework is appealing not only because it is suitable for understanding the dynamics of unemployment and vacancies. In principle, the framework can also give rise to sticky real wages and volatile employment without requiring an unreasonably high labour supply elasticity. Under a Walrasian labour market, the flexible real wage adjusts instantly to induce market clearing. If the elasticity of the labour supply is set at the low values found in microeconometric studies, the real wage is too volatile a variable in comparison to the
time series data. A high elasticity is necessary to match the high variability of hours worked, together with the low variability of the real wage.\footnote{An alternative approach to overcome these tensions between theory and data and to preserve wage flexibility assumes that labour is indivisible as in Rogerson (1988).}

Shimer (2005) casts doubt whether the quantitative performance the DMP model is indeed superior to that of a model featuring a Walrasian labour market. At least simple versions of the search and matching approach have difficulty in accounting for the high volatility of labour market variables under what Shimer (2005) considers a reasonable calibration of the model — a view challenged by Mortensen and Nagypal (2007) and Hagedorn and Manovskii (2008). To reduce the volatility in the real wage and thus to raise the volatility of unemployment and vacancies Hall and Milgrom (2008) propose replacing the idea of Nash bargaining between a worker-firm pair by an alternating offer bargaining game.

5.1 Walrasian labour market

Under a Walrasian labour market, the labour supply of each agent is taken to be elastic to account for the variation in the labour input in production at the aggregate level. Household preferences over consumption and leisure follow Greenwood et al. (1988). Using preferences without a wealth effect on the labour supply are key in replicating the expansion of employment after an increase in commodity prices in our setting.

All household members are employed and have preferences:

\[
U(c_t, h_t) = \left( \frac{c_t - \phi_0}{1 + \phi} (h_t)^{1 + \phi} \right)^{1-\sigma}.
\]  

(48)

Furthermore, we follow Blanchard and Galí (2007) in introducing real wage rigidities to improve the empirical performance of this model. The real wage evolves according to:

\[
w_t = \eta w_{t-1} + (1 - \eta) mrs_t
\]

(49)

with the standard Walrasian model arising under the assumption of $\eta = 0$. The optimality conditions pertaining to the labour market are:

\[
\lambda_t = \left( c_t - \phi_0 \frac{h_t^{1+\phi}}{1 + \phi} \right)^{-\sigma}
\]

(50)

\[
mrs_t = \phi_0 h_t^\phi
\]

(51)

\[
w_t = (1 - \alpha) \frac{P^b h_t^b}{n_t}
\]

(52)

\[
h_t = n_t
\]

(53)

\[
w_t = \eta w_{t-1} + (1 - \eta) mrs_t.
\]

(54)

5.2 Alternating offer bargaining model

Under Nash bargaining, the threat points are for the worker to return to unemployment and for the firm to leave the vacancy unfilled. Hall and Milgrom (2008) suggest a noncooperative alternating offer bargaining model which
implies a change in the outside options of the negotiating parties.\textsuperscript{16} While a breakdown in the negotiations still leads to unemployment for the worker and an unfilled vacancy for the firm, the main threat is to extend bargaining rather than to terminate it. Patience determines the threat points. By breaking the tight connection between wages and outside conditions in Mortensen and Pissarides (1994), the alternating offer bargaining model implies higher volatility of unemployment than the standard DMP model for parameters that are deemed realistic by Hall and Milgrom (2008). Christiano et al. (2013) imbed the model by Hall and Milgrom (2008) into a standard monetary business cycle model and attest to it superior statistical performance based on a Bayesian procedure.

The main departure of the alternating offer bargaining model from Nash bargaining lies in the idea that a worker and a firm negotiate over a finite time span with $M^{aob}$ subperiods. The starting offer by the firm can be rejected by the worker by formulating a counteroffer. $\gamma^{aob}$ is the cost to the firm of making a counteroffer. This process continues until an agreement is reached, the time span for negotiation is over, or bargaining has broken down. The exogenous probability of a breakdown in bargaining is denoted by $\delta^{aob}$. Christiano et al. (2013) show that the surplus sharing rule in the alternating offer bargaining model can be written as:

$$J_t = \beta_1 H_t - \beta_2 \gamma^{aob} + \beta_3 (mpl_t - b^u)$$ \hspace{1cm} (55)

with $\beta_i = \alpha_{i+1}/\alpha_1$ for $i = 1, 2, 3$ and

$$\alpha_1 = 1 - \delta^{aob} + (1 - \delta^{aob})M^{aob}$$ \hspace{1cm} (56)

$$\alpha_2 = 1 - (1 - \delta^{aob})M^{aob}$$ \hspace{1cm} (57)

$$\alpha_3 = \frac{\alpha_2}{\delta^{aob}} - \alpha_1$$ \hspace{1cm} (58)

$$\alpha_4 = \frac{1 - \delta^{aob}}{2 - \delta^{aob}} \frac{\alpha_2}{M^{aob}} + 1 - \alpha_2$$ \hspace{1cm} (59)

where $mpl_t = (1 - \alpha) \frac{p_i^h y_t}{n_t}$ denotes the marginal product of labour. We start by assuming that households have CRRA preferences, i.e., consumption between employed and unemployed household members is equalised; we also discuss the implications of relaxing this assumption below. Furthermore the firm has to pay a fee $\bar{\kappa}$ to initialise the bargaining process as in the DMP model — see equation (37).

In the limit, if the number of subperiods over which bargaining occurs, $M^{aob}$, is large and the cost for the firm to make a counteroffer is low, $\gamma^{aob}$, the surplus sharing rule of the alternating offer bargaining model converges to the surplus sharing rule under Nash bargaining with $\xi = \frac{1 - \delta^{aob}}{2 - \delta^{aob}}$.

However, for a smaller value of $M^{aob}$ — Christiano et al. (2013) suggest setting $M^{aob}$ equal to 60 — the surplus sharing rules (55) can mimic the surplus sharing rule (42) only if the bargaining power of the household under Nash bargaining, $\xi$, is low and the probability of bargaining breakdown under alternating offer bargaining, $\delta^{aob}$, is high. To see this, choose $\delta^{aob}$ to satisfy the condition:

$$\beta_1 = \frac{1 - (1 - \delta^{aob})M^{aob}}{1 - \delta^{aob} + (1 - \delta^{aob})M^{aob}} = \frac{1 - \xi}{\xi}.$$ \hspace{1cm} (60)

\textsuperscript{16}The alternating offer bargaining model was introduced by Binmore et al. (1986).
Notice that $\beta_1$ is increasing in $\delta^{aob}$. Furthermore, the coefficient $\beta_3$ can be approximated as

$$\beta_3 \approx \frac{1}{2M^{aob}} + \frac{1 - \alpha_2}{\alpha_1}. \quad (61)$$

For a large value of $\delta^{aob}$ (say $> 0.1$) and $M^{aob} = 60$, $\beta_3$ will be close to zero. With the appropriate choices of $\gamma^{aob}$ and $\delta^{aob}$, the alternating offer bargaining model can be equivalent to the Nash bargaining model for arbitrary (even) values of $M^{aob}$ provided the household’s bargaining power under Nash bargaining, $\xi$, is sufficiently low.\(^{17}\)

We refer to the model with noncooperative bargaining as the AOB model in our subsequent discussion.

6 Reconciling model and VAR

Which of the labour market models is preferred by the empirical estimates provided in Section 3? To shed light on this question we estimate a number of model parameters for each model using a minimum distance strategy and assess the plausibility of the estimates.

The parameters are divided into two groups: calibrated and estimated parameters. The calibrated parameters are listed in Table 3. The first eight parameters are common to all models. The discount factor, $\beta$, implies a real interest rate of 4% per annum. The parameter $\sigma$ governs the intertemporal elasticity of substitution and is set at 1.1. The share of capital in the production function, $\alpha$, is 0.33, the depreciation rate, $\delta$, is 2.5% per quarter. All these values are standard in the literature. The elasticity of substitution between home and foreign-produced goods, $\theta$, is set at 2.0, which is within the relatively wide range of values commonly used in the literature. To assess robustness of our findings, we experiment with this parameter in Section 7.

In 2013, the goods-export to GDP ratio averages at 20% across the economies in the sample.\(^{18}\) The Harvard Atlas of Economic Complexity is used to determine the share of commodities in total exports. In 2013, the average share of commodities in net exports in our four countries was 85%, which is significantly higher than the value suggest by IMF (2012), which range from 30% to 40%, albeit over a sample dating back to 1960. To err on the side of caution, we set the share of commodity exports in total exports at 75%. The share of non-commodity exports in GDP is set at 5%. With these target values in hand, the implied value for $\nu$ is 0.8.

For the search and matching models, we set the probability that an existing match breaks up within a given quarter at 0.1, which is somewhat lower than the value of 0.15 suggested by Andolfatto (1996), but is equal to the choice in Christiano et al. (2013). We assume a replacement ratio, $r^u = \frac{h^w}{w^e}$, of 40% of wages. In the steady state, we set employment equal to 0.95, which implies a steady state unemployment rate of 5%. The implied value for the scale parameter in the matching function, $\chi$, is 0.67. Finally, we fix the share of all (expected) vacancy costs in (non-commodity) output, $\kappa_{v^i} + \bar{q}_{ss} \kappa_y^{ss}$, at 0.005.

We estimate those parameters for which there is little or no empirical evidence. Given its importance in transmitting wealth shocks, we estimate the bond holding cost parameter, $\phi^b$. To allow the models to better match the shape of the response in vacancies to the shock, the vacancy adjustment cost parameter, $\phi^v$, is also estimated, as is the investment adjustment cost parameter, $\phi^x$. For the search and matching model we also

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\(^{17}\)If preferences and the costs of initialising the bargaining process, $\bar{\kappa}$, are identical in the DMP and AOB model, the cost of making a counteroffer, $\gamma^{aob}$, needs to be zero to achieve equivalence. If preferences differ, as assumed in our analysis, $\gamma^{aob}$ will need to be a small positive number.

\(^{18}\)This ratio is taken from the OECD national accounts data.
estimate the share of unemployment in the matching function, $\zeta$. As will be highlighted in our discussion of the estimation results, for a given choice of the replacement ratio the dynamics of unemployment and vacancies are crucially influenced by our estimate of the relative consumption share of the unemployed to the employed agents, $c_u/c_w$, in the DMP model. Under the alternative of alternating offer bargaining, the probability of breakdown in bargaining, $\delta_{aob}$, is a key determinant of unemployment and vacancies given our choice of $M_{aob}$ equal to 60 as in Christiano et al. (2013). Thus, we estimate $\delta_{aob}$ in the AOB model with CRRA preferences. Finally, while we calibrate the overall costs of filling vacancies, we estimate the relative importance of the variable and the fixed component, $s^{fixed}$. Equipped with an estimate for $s^{fixed}$ we can then compute the parameters $\kappa^v$ and $\bar{\kappa}$.

Under the assumption of a Walrasian labour market, we estimate the inverse of the Frisch labour supply elasticity, $\phi$, as well as the parameter $\eta$ that introduces real wage rigidity. Appendix B provides additional details on the calibration strategy.

Given the values of the calibrated parameters — stacked in the vector $\Theta^c$ — we estimate the remaining ones — stacked in the vector $\Theta^e$ — by minimising the weighted distance between the empirical impulse response functions from the VAR in Section 3, denoted by $G$, and the impulse response function implied by one of our theoretical models, denoted by $G(\Theta^c, \Theta^e)$:

$$\hat{\Theta}^e = \arg\min_{\Theta^e} \| G - G(\Theta^c, \Theta^e) \| \Omega^{-1} \| G - G(\Theta^c, \Theta^e) \|.$$ (62)

The diagonal weighting matrix $\Omega$ is obtained from the empirical variance-covariance matrix of the estimated impulse response functions $\Psi$ by setting all off-diagonal elements in $\Psi$ to zero.\footnote{The estimate of the variance-covariance matrix $\Psi$ is obtained by means of bootstrapping in Section 3.} $\Omega$ penalises those elements of the estimated impulse responses with wide error bands. We minimise the objective (62) over the first six periods of the VAR which allows our model to closely match the initial response of the data, but leaves the subsequent dynamics of the model unrestricted. The values assigned to the calibrated parameters are provided in Table 3. The estimated values of the remaining parameters are collected in Table 4. Standard errors are constructed from the asymptotic covariance matrix of the estimator $\hat{\Theta}^e$ given by

$$[\Gamma(\Theta^e)\Omega^{-1}\Gamma(\Theta^e)]^{-1} \Gamma(\Theta^e)' [\Omega^{-1}\Psi\Omega^{-1}] \Gamma(\Theta^e) [\Gamma(\Theta^e)\Omega^{-1}\Gamma(\Theta^e)]^{-1}$$ (63)

where $\Gamma(\Theta^e) = \frac{\partial G(\Theta^e, \Theta^e)}{\partial \Theta^e}$.

### 6.1 Performance of the DMP model

The red solid lines in Figure 3 denote the fitted impulse responses of the baseline model while Table 4 reports the parameter estimates and standard errors. By minimising the objective in (62), the DMP model is able to closely replicate the behaviour of commodity prices as well as the VAR’s median impulse response for net trade.

Importantly, the DMP model matches the approximate paths of unemployment and vacancies. To account for the slight “hump” shaped path of vacancies, the estimation yields a value for the vacancy adjustment cost parameter, $\phi^v$, of 1.1, the accompanying standard errors are small. The DMP model is able to reproduce the initial decline and the following gradual decrease in unemployment, albeit compared to the data, the path of
unemployment is somewhat more persistent. The model cannot account for the initial decline or indeed the shape of the path of real wages and it is silent (by construction) on the behaviour of individual hours.

The commodity price rise transmits like a wealth shock to the economy as it induces additional transfers to households. These transfers are used to boost consumption and domestic investment and to increase savings in the form of foreign bonds. Consumption rises by 0.15% and remains elevated thereafter. The DMP model captures the initial magnitude of the increase in GDP (at constant prices) and its gradual rise, but it under predicts the response of GDP in the first 5 quarters following the commodity price shock. For investment, the model generates the same peak response as the data, but fails to capture the ‘hump’ shaped response in the first couple of periods. The persistent rise in consumption pushes up demand for both the domestic and the foreign non-commodity good, although the appreciation in the real exchange rate holds back demand for the former. In the short-run, the output expansion is driven by the increase in employment, while over time the gradual buildup of the capital stock also contributes to the modest rise in production of the domestic non-commodity good.

The shape and the magnitude of the impulse responses depend on the real interest rate movement induced by the shock and the household’s decision how to allocate the additional transfers towards savings in foreign bonds, consumption, and investment. In our framework the interest rate faced by households and firms is equal to the world interest rate adjusted by a small risk premium that depends on the country’s net foreign asset position. The elasticity of this risk premium to the net foreign asset position of the home country is estimated at 0.0085 which is in line with values commonly used in the open economy literature. Although the estimate is statistically different from zero, its value implies that the interest rate faced by households is largely exogenous. As a result, the model generates a virtually flat consumption profile following a commodity price shock. In the data as in the model, the rise in domestic consumption occurs alongside a real exchange rate appreciation suggesting that country-specific consumption risk coming from wealth shocks cannot be effectively shared via relative price movements or trade in bonds. This feature differs from the transmission of a technology shock in the open economy. Cole and Obstfeld (1991) point out that movements in the terms of trade provide a powerful source of insurance against technology shocks independently of the financial market arrangements — with the exceptions of a low trade elasticity of substitution or permanent technology shocks stressed in Corsetti et al. (2008).

The investment adjustment cost parameter, $\phi_x$, required to minimise (62) is close to but statistically different from, zero at 0.04. The share of unemployment in the matching function, $\zeta$, is highly significant and comes out at 0.72; this estimate is remarkably robust across all specifications of the search and matching models we consider. Based on a variety of econometric studies Mortensen and Nagypal (2007) consider the range from 0.3 to 0.5 plausible for the elasticity of the matching function — in our notation $1 - \zeta$. Implied by the estimated and calibrated parameters of the model is the bargaining power of households, $\xi$. It assumes the value of 0.4 which suggests that firms have a rather higher weight in the wage bargaining process than workers.

Finally, we turn towards those parameters that primarily govern the volatility of unemployment and vacancies: the share of consumption going to the unemployed, $c_u/c^w$, and the relative importance of the fixed and variable cost components in filling a vacancy, $s^{fixed}$. The data are highly informative on the share of consumption going to the unemployed. Our estimates suggest that unemployed members of a household enjoy about 60% of the

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20 For estimation purposes, we express the elasticity of the interest rate to deviations in the net foreign asset position from steady state as a percentage (multiplied by 100) in Table 4.
consumption of employed agents. With the replacement ratio set at 40% of steady state wages, there is a modest, but far from complete, requirement to reduce consumption inequality between household members. By contrast, the estimate of $s^{fixed}$ is basically zero suggesting that the fixed costs to start bargaining, $\bar{\kappa}$, are negligible. We embed the discussion of the role of these parameters in influencing labour market dynamics and of the plausibility of their estimated values into a broader analysis of the transmission mechanism following next.

6.2 Transmission in the DMP model

To structure the discussion of the transmission mechanism, Appendix C establishes an approximate relationship between labour market tightness, $\theta_t$, and the marginal product of labour, $mpl_t$, which in turn is determined by the real exchange rates and labour productivity. Abstracting from vacancy adjustment costs, the surplus sharing rule under Nash bargaining and the definitions of the marginal values of employment to the household and the firm — $H_t$ and $J_t$ — imply:

$$J_t + (1 - \xi) (b^u + \Phi) = (1 - \xi) mpl_t + (1 - \rho) E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (1 - \xi s_{t+1}) J_{t+1} \right]$$

(64)

with $J_t = \frac{\xi}{\lambda} \theta_t^c + \bar{\kappa}$ for $\phi^u = 0$. Appendix C shows in detail how equation (64) can be used to obtain an approximate decomposition of the log-deviation of labour market tightness from its steady state value, $\hat{\theta}_t$, into movements of the real exchange rate, $\hat{rer}_t$, and labour productivity, $\hat{y}_h - \hat{n}_t$:

$$\hat{\theta}_t \approx \frac{1}{\Upsilon (1 - r^u) mpl_{ss} + r^u (1 - (1 - \rho)\beta ) \left( \frac{\hat{\nu}^c}{\hat{\nu}_{ss}} + \bar{\kappa} \right) - \Phi \left( \hat{y}_h - \hat{n}_t - \frac{1 - \nu}{\nu} \hat{rer}_t \right)}.$$

(65)

The parameter $\Upsilon$ is defined in the appendix and assumes a positive value. For suitable choices of the replacement ratio $r^u$ and the consumption difference between employed and unemployed household members, $\Phi$, the second factor in equation (65) is also positive.

The role of the real exchange rate and labour market productivity in shaping the response of labour market tightness to the commodity price shock differs by time horizon. The top panel in Figure 4 plots the response of labour market tightness in the DMP model (with positive vacancy adjustment costs as estimated) together with the right hand side of equation (65) and its decomposition into movements due to changes in the real exchange rate and in labour productivity, respectively. The approximation to labour market tightness proposed in equation (65), depicted by the dashed red line, tracks the value of $\hat{\theta}_t$ derived from the estimated DMP model, depicted by the solid blue line, reasonably well in particular once the impact of the vacancy adjustment costs has worn off.

Decomposing the movements in labour market tightness shows that early on labour market conditions improve primarily due to the appreciation of the real exchange rate. Adapting the logic in Bodenstein et al. (2011), the non-commodity trade balance must go into deficit following the improvement in the commodity trade balance since the rational expectations solution imposes that the net foreign asset position is bounded away from infinity. To slow non-commodity exports, the relative price of the domestically produced good has to rise swiftly which translates into an appreciation of the real exchange rate. The magnitude of the exchange rate response is determined by the country’s ability to smooth the commodity price shock through the overall trade balance. If trade was required
to be balanced period by period (financial autarky), the non-commodity trade balance would need to go deeper
into deficit on impact which would precipitate a larger appreciation. Although the domestic good has become
more expensive, domestic demand increases for both the domestic and the foreign produced good because of the
shock’s income effect. Facing a higher relative price for their goods, firms post additional vacancies, matches rise,
and unemployment falls, which pushes up $\theta_t$. The contribution of the real exchange rate to moving labour market
tightness is depicted by the dash-dotted green line. For the first eight quarters after the shock, the real exchange
rate appreciation accounts for the bulk the change in $\theta_t$ in the DMP model.

Over time, as the real exchange rate slowly returns to its steady state value, labour market conditions remain
favourable given a lasting improvement of labour productivity. Since international risk sharing is limited in
our model, the real interest rate faced by domestic agents experiences a mild decline; the marginal product of
capital, by contrast, rises on impact as a result of the real exchange rate appreciation and the expansion in
domestic production. Thus, the wealth increase stemming from the commodity price shock is partially invested
into expanding the capital stock as the marginal product of capital would otherwise exceed the real interest
rate. As the capital stock is augmented, labour productivity rises and firms find it profitable to keep labour
demand and employment persistently above their steady state values. As the black dotted line in the top panel
of Figure 4 reveals, the improvement in labour productivity is slow to occur, but very persistent. Hence, the fall
in unemployment and the rise in vacancies continue well after commodity prices have returned to steady state.

The transmission of a commodity price shock differs significantly from the transmission of a technology shock
both with respect to its domestic and international dimension. A positive neutral technology shock raises labour
productivity on impact; but, with the price of the domestically produced good falling, the real exchange rate
depreciates persistently. The bottom panel in Figure 4 decomposes the response of labour market tightness after
a neutral technology shock into the movements due to labour productivity and due to the real exchange rate.
Labour productivity dominates in shaping the response of labour market tightness at all horizons while the real
exchange rate dampens the improvement in labour market conditions not unimportantly. Open economy aspects
are important in understanding labour market dynamics and it is not sufficient to view labour market movements
solely through the lens of changes in labour productivity as in much of the macro-labour literature referenced
here. Shocks other than to technology can have a limited impact on labour productivity, but nevertheless have a
sustained impact on the labour market through an adjustment in the real exchange rate.

Shifting the focus from the transmission of the shock to its amplification, we turn to interpreting the coefficient
in equation (65):

$$\frac{1}{Y} \frac{mpl_{ss}}{(1 - r^u)mpl_{ss} + r^u (1 - (1 - \rho)\beta) \left( \frac{\kappa}{\kappa_u} + \bar{\kappa} \right) - \Phi}.$$

Although not made explicit in our notation, the value of $Y$ depends on the replacement ratio, $r^u$, and the con-
sumption difference between employed and unemployed household members, $\Phi$. However, as argued in Appendix
C, $Y$ is not very sensitive to the values of $r^u$ and $\Phi$. In fact, for $\bar{\kappa} = 0$ as implied by our estimation of the DMP
model , $Y$ can be shown to lie in the interval $[\zeta, 1]$. Much more relevant for determining the impact of commod-
ity price induced real exchange rate and labour productivity changes is the second factor. For a given value of
the replacement ratio the denominator can be made arbitrarily small, yet positive, by making the consumption
difference between employed and unemployed household members suitably large, i.e., lowering the share $c_w^u - c_u^u$. If, however, $\Phi$ equals zero, as would be the case under CRRA preferences, a high value of the replacement ratio may be required to get sufficient amplification in $\hat{\theta}$.\footnote{If vacancy costs are calibrated to be a small number, the term $c_w^u - c_u^u + \bar{\kappa}$ will be small and the denominator can be made (arbitrarily) small by raising $r^u$ towards unity even if $\Phi = 0$. In principle, the relative importance of the fixed and variable costs components in filling vacancies and the implied value for $\kappa^u$ and $\bar{\kappa}$ can be an alternative source of amplification. However, numerically the effects are small in the DMP model. The appendix provides further elaboration on this point.}

This interplay between the parameters $r^u$ and $\Phi$ in amplifying the labour market response is not unique to the open economy context. Since Shimer (2005) has pushed the view that the textbook DMP model—in our context characterised by Nash bargaining and CRRA preferences—explains less than 10% of the volatility in U.S. unemployment and vacancies when fluctuations are driven by productivity shocks, the “correct value” of the replacement ratio, $r^u = \frac{h^{w}}{w^{ss}}$, has been the subject of lively discussion. Hagedorn and Manovskii (2008) and Hall (2008) argue that the flow value of unemployment ought to capture more than direct insurance payments to the unemployed. By appealing to the value of leisure and home production Hagedorn and Manovskii (2008) raise the flow value of unemployment to imply a replacement ratio of 95.5% and demonstrate that the DMP model can be brought in line with the data. Hall (2008) attempts to discipline the calibration and models explicitly the value of leisure.

By deviating from CRRA preferences and opting for preferences that feature complementarity between consumption and hours worked our framework explicitly incorporates a key aspect advocated in Hall (2008). As shown in equation (27), employed household members are compensated for the disutility from labour with additional consumption:

$$c_w^u - c_u^u = \left(1 - \frac{c_u^u}{c_w^u}\right) c_w^u = \frac{\phi h_w}{1 + \phi} (h_w)^{1+\phi} = \Phi. \quad (66)$$

Using U.S. data on consumption of nondurables and services from the Consumer Expenditure Survey (CE) and the Panel Study for Income Dynamics (PSID), Chodorow-Reich and Karabarbounis (2013) measure the relative consumption of an unemployed agent, $c_w^u / c_w^t$, to be 70% to 80% of an employed agent which are comparable to other estimates discussed in the literature—see Chodorow-Reich and Karabarbounis (2013) for a review. In order for our version of the DMP model to imply a satisfying match to the VAR impulse responses, the relative consumption of an unemployed agent is estimated at the lower value of 60%. Although most of the evidence in the literature is derived from U.S. data, we do not expect the ratio $c_w^u / c_w^t$ to differ dramatically for the countries in our sample. However, at least the results in Chodorow-Reich and Karabarbounis (2013) do not account for housing, health care, education, and durable goods consumption. Without pursuing the debate over the ratio $c_w^u / c_w^t$ any further, our parameter estimates imply a flow value of unemployment, $\frac{c_w^u + \bar{\kappa}}{w^{ss}}$, of 96% of steady state wages. Using different data and conditioning on commodity price shocks, we recover basically the same value for the flow value as Hagedorn and Manovskii (2008) studying unconditional U.S. data and productivity shocks.

### 6.3 Walrasian labour markets

The dash-dotted black lines in Figure 3 show the fitted impulse responses for a standard model with Walrasian labour markets. Whereas the simple RBC model is silent on key labour market variables such as unemployment and vacancies, it does assume an elastic individual labour supply that produces a data-consistent increase in hours
worked in response to a positive commodity price shock. The assumption of GHH preferences is vital to generate
the expansion in hours worked. If the wealth effect on the labour supply was not eliminated — as would be the
case under standard additive separable preferences — hours worked would fall and not rise!

The fitted impulse responses for investment and the real exchange rate are almost identical to those of the
DMP model. A key difference between the two models is found in the response of output and consumption. In
both cases, the RBC model yields a larger initial response than the DMP model. Compared to the DMP model,
there is a larger increase in employment which allows both output and consumption to increase by more.

The dashed blue lines show the fitted impulse responses of the RBC model with real wage rigidities as in
Blanchard and Galí (2007). The estimated value of $\eta$ is high at 0.84. Compared to the standard RBC model, the
model with real wage rigidities generates more volatility in output and consumption, but, as expected, slightly less
volatility in the real wage. As a result, the model generates a more volatile, and more data congruent, response
of hours worked. The dynamics of the remaining variables are very similar to the baseline DMP model or the
standard RBC model. A closer look at Table 4 reveals the reason for the relative performance of the two RBC
models. To generate a flat real wage response, the simple RBC model requires a labour supply elasticity that is
twice as high as in the RBC model with the real wage rigidity.

6.4 Alternative search and matching models

To compare the results in the DMP model with alternative formulations of the search and matching framework,
Figure 5 reports the fitted impulse responses for the alternating offer bargaining (AOB) model, depicted by the
dashed blue lines, and a version the DMP model with an elastic labour supply, depicted by the dash-dotted black
lines.

Allowing for an elastic labour supply does not alter the dynamics of the DMP model in significant ways.
Because of the endogenous response of hours worked, output, consumption and investment are somewhat more
responsive on impact. The dynamics of hours worked are similar to those generated by the RBC models analysed
in the previous section. The remaining labour market variables are not affected by the inclusion of an endogenous
labour supply. The value of the bargaining share of households, $\xi$, that is implied by the set of calibrated and
estimated parameters, turns out to be 0.3. This lower value compared to the baseline DMP model accounts for
the slightly smaller real wage response in the DMP model with elastic labour supply. The labour supply elasticity
in this more flexible version of the DMP model is ultimately determined by the difference in consumption between
employed and unemployed agents, $\Phi$. Under GHH preferences and full risk pooling the consumption difference to
compensate employed agents for their labour effort satisfies:

$$
\Phi_t = c^e_t - c^u_t = \frac{\phi_0}{1 + \phi} \left( h^w_t \right)^{1 + \phi} 
$$

at each instant in time. In periods of increased labour demand and higher hours worked, the consumption
differential increases. With $\phi_0$ being determined by other aspects of our calibration strategy, less consumption
inequality (a lower steady state value $\Phi_{ss}$) maps into a higher value of the inverse of the labour supply elasticity

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22The extension of the DMP model to include an elastic labour supply is relegated to Appendix B.3.
Although the estimated value of the inverse of the labour supply elasticity rises to 0.74 compared to 0.48 in the simple RBC model, the implied labour supply elasticity is still above unity. The elevated estimate of the labour supply elasticity goes hand in hand with the somewhat small estimate of the relative consumption of the unemployed.

As in Christiano et al. (2013), there is virtually no difference between the DMP and the AOB model. The similar dynamics trace back to virtually identical estimates of the bond holding cost parameter, φ^b, and the investment adjustment cost parameter, φ^x. Interestingly, the data prefers setting the breakdown probability of bargaining, δ^aob, as closely as possible to its lower bound in the estimation which is set at 0.001.

Despite the similar impulse responses, the AOB model differs along important dimensions from the DMP model: the bargaining setup and parameter estimates. To isolate the role of the differences in the bargaining process, we estimate two restricted versions of the AOB models. In the first version, all parameters that are common across models are set at their respective point estimates in the DMP model. The parameter M^aob is set to 60 and δ^aob is estimated. Given the values of M^aob and δ^aob, the steady state relationships pin down the value of γ^aob. The second version we consider is identical to the first one with the one exception of including the parameter \( \bar{\kappa} \) into the estimation. In the DMP model this parameter assumes the value of zero.

Figure 6 plots the implied impulse responses of unemployment and vacancies of the DMP and the AOB models and the two restricted versions of the AOB model. The first version of the model, the dashed blue lines in the figure grossly understates the responses of the labour market variables. Changing the nature of the bargaining process is not sufficient to obtain empirically plausible responses in a model with search and matching frictions. In going from the DMP model to the first version of the restricted AOB model, we also abandoned the assumption that unemployed workers consume less than employed workers by reverting to the standard assumption of a CRRA utility function for the household members and simultaneously keeping the replacement ratio at 0.4.

Instead of estimating the replacement ratio, our second version, the dash-dotted green lines, takes the alternative approach of estimating the fixed cost of bargaining, \( \bar{\kappa} \). In terms of reconciling the constraints imposed by the model’s steady state relationships and the implied dynamics, \( \bar{\kappa} \) plays a role similar to the one played by the replacement ratio.\(^{23}\) Estimation of our second version of the restricted AOB model assign the value of 0.0131 to \( \bar{\kappa} \) close to its value of 0.0095 in the unrestricted AOB model. The dynamics of unemployment and vacancies of this second version of the AOB model are hard to distinguish from those obtained under Nash bargaining.

As for the DMP model, we can derive an approximation that establishes the link between labour market tightness, the real exchange rate, and labour productivity. Appendix C treats the general case, but since the probability of a breakdown in bargaining, \( \delta^aob \), is estimated to be close to zero, we proceed differently. If \( \delta^aob \approx 0 \), the surplus sharing rule under alternating bargaining (55) reduces to:

\[
J_t \approx -\beta_2 \gamma^aob + \frac{1}{2} (mp_l - b^u)
\]

(68)

since \( \beta_1 \approx 0 \) and \( \beta_3 \approx \frac{1}{2} \). The marginal value of employment to the household, \( H_t \), is (basically) irrelevant for

\(^{23}\)In a third version of the AOB model we estimated \( \delta^aob \) and the replacement ratio. The estimate of the latter was 0.856 and \( \delta^aob \) was close to 1. In this case, the implied weight \( \beta_1 \) in the surplus sharing rule (55) approaches infinity and the AOB model resembles a version of the DMP model with a very low bargaining weight \( \xi \) for the household. See also the discussion in Section 5.2.
wage determination if $\delta^{aob}$ is small. Abstracting again from vacancy adjustment costs, we apply the first order condition with respect to vacancies (37) and log-linearise the resulting equation to obtain:

$$\hat{\theta}_t = \frac{1}{2} \frac{mplss}{\gamma} \left( \hat{y}_t^h - \bar{n}_t - \frac{1 - \nu}{\nu} \hat{r} \right).$$  \tag{69}

In this case, the amplification of shocks is unaffected by the value chosen for the replacement ratio, $\nu$.

In the AOB model, amplification of given movements in the real exchange rate and labour productivity is achieved by setting the term $\kappa^u$ suitably small. For a calibration strategy that targets the share of total vacancy filling costs in non-commodity output, $\frac{\kappa^u \mu_a + \kappa^u \mu_b}{\beta}$, choosing a small value for $\kappa^u$ may require to apportion a large share of the vacancy filling costs to its fixed component and therefore $\bar{k}$. If $\bar{k}$ is forced to be zero as in the first restricted version of the AOB model in Figure 6, the AOB model fails to deliver sufficient amplification under CRRA preferences. For the final estimates of the AOB model, the coefficient $\frac{1}{2} \frac{mplss}{\gamma} \kappa v qss$ in equation (69) assumes a value similar to the corresponding coefficient in the DMP model in equation (65).

The approximation in equation (69) also suggests that the AOB model admits high values (or at least higher than our estimate) for the probability of bargaining breakdown only if tight restrictions prevail on the parameter $\kappa^u$ such as a too low calibration target for the share of total vacancy filling costs in output, or sufficiently tight “priors” as in the Bayesian procedure applied by Christiano et al. (2013). The sensitivity of the parameter estimates to the calibration of the total vacancy posting costs relative to output is unique to the AOB model. In the DMP model a drastic reduction in these costs barely influences the parameter estimates.

7 Sensitivity analysis

The real exchange rate plays a pivotal role in the labour market response to a commodity price shock. The three model features that determine the dynamics of the real exchange rate are the degree of international risk sharing, trade openness, and the trade elasticity of substitution.

The degree of international financial risk sharing is controlled by the parameter $\phi^b$ in our model. For $\phi^b$ very close to zero, the model converges to a simple permanent income model with an exogenously fixed real interest rate. In this case, the impact of commodity price shocks is smoothed through the trade balance; except for a permanent rise in consumption, the economy remains basically unaffected. Most importantly, the real exchange rate hardly moves. By contrast, under financial autarchy, i.e., $\phi^b$ is very large, even a small increase in the net foreign asset position would lower the interest rate faced by domestic agents in international markets and redirect spending towards consumption and investment. The rise in the commodity trade balance following a commodity price increase needs to be fully offset by a deterioration in the non-commodity trade balance which requires a sharp appreciation in the real exchange rate. Domestic macroeconomic aggregates bear the full burden of the adjustment process. The top panel of Figure 7 illustrates this discussion graphically for $\phi^b = \{0.0085, 0, \infty\}$ with all other parameters set as estimated in the baseline DMP model for comparability.

The bottom panel of the figure sheds light on the role of trade openness and the trade elasticity of substitution for the response of the real exchange rate. Again, we refrain from re-estimating the model and adjust only the

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24Open economy papers utilising this framework are Schmitt-Grohé and Uribe (2003) and Aguiar and Gopinath (2007).
parameters of interest. In an even more open economy, the lower degree of home bias, i.e., $\nu = 0.7$, causing the real exchange rate to appreciate by less. The labour market response, however, depends on the product of the real exchange rate with the term $\frac{1}{1-\nu}$, which is decreasing in $\nu$. As this product barely changes, the responses of vacancies and unemployment are hardly affected. The real exchange rate appreciation is amplified when domestic and foreign goods are more complimentary under the low trade elasticity of substitution of $\theta = 0.8$. The impact of the shock on the labour market is less pronounced under the parameter estimates of the baseline model. If the model was re-estimated under the assumption of a low trade elasticity, the estimated relative consumption of the unemployed would turn out to be higher than in our baseline case to bring the labour market responses back in line with the empirical impulse response functions.

### 7.1 Commodity supply

Two potentially controversial assumptions of the baseline model affect the supply of commodities and the absence of non-traded goods. The production of commodities is assumed to be price inelastic and fixed over time. This assumption helps focus the attention on the transmission mechanism of wealth shocks, but comes at the risk of under predicting the response in GDP. If the supply of commodities reacts positively to an increase in prices, we would expect a direct effect on GDP through higher supply of commodities as well as an indirect one working via the traded goods sector. If commodity production required labour input, the effects on unemployment and vacancies would be amplified beyond the impact from the non-commodity sector. Abstracting from the supply of commodities therefore makes fitting the model to the data somewhat more challenging, but helps us to isolate the response of the non-commodity sector to commodity price shocks. Even if the production of commodities is fixed, the (net) supply of commodities to the world market can be made endogenous. If the commodity-producing country uses some of its commodity output domestically, a rise in world commodity prices may cause domestic use of the commodity to fall and (price-inelastic) commodity exports to rise. With the overall supply to the world market increased, transfers from commodity sales to the household are larger than in our baseline model. An alternative way of making the world supply elastic is through a simple storage technology. The main findings, however, remain unaffected by these changes.

Overall, the price-elasticity of supply in the short-term is regarded to be low for most commodities. Long-term considerations appear to be much more relevant in this regards as expanding production requires expensive investment over several years. From this perspective, the findings in Arezki et al. (2015) are of great interest to our work. Arezki et al. (2015) explore the empirical relationship between “news” on giant oil discoveries and macroeconomic variables for open economies in a panel VAR. Upon the breaking of the “news,” consumption rises as future permanent income has increased. Until revenues flow from the newly discovered fields roughly five years after their discovery, (oil-field) investment and consumption are financed by borrowing from abroad and the trade balance turns into deficit. GDP declines until commodity production starts whereas employment remains suppressed well beyond that point in time. Figure 8 shows that our model without investment into commodity supply can replicate these observations when the elasticity of substitution between traded goods is high with the obvious exception being investment. As in Arezki et al. (2015), the news shock occurs five years prior to production going online. Under the baseline parameterisation (solid blue lines) with a trade elasticity equal to
2, the model implies a rise in consumption and a trade balance deficit. However, the appreciation in the real exchange rate causes unemployment to fall and vacancies to rise. Although the domestically produced good has become relatively more expensive, the overall increase in consumption spending leads to higher demand for all goods given the relatively low degree of substitutability. If we follow Arezki et al. (2015) instead and assume perfect substitutability between goods (dashed red lines), domestic production can fall and with it vacancies and employment. Under somewhat less extreme assumptions on the elasticity of substitution (dash-dotted green line) or the case of a trade elasticity of 2 combined with high openness to trade (dotted black line), the decline in employment is less pronounced but still present. Thus our framework is in principle suited to imply a decline in unemployment after a price shock and an increase in unemployment after a “news” shock of higher future supply.

7.2 Non-traded goods

The effects of commodity price shocks are frequently discussed in a modelling framework that also includes non-traded goods; often in the context of the “Dutch disease” literature. In this case, the real appreciation that results from a positive commodity price shock also leads to a shift in resources from the traded goods to the non-traded goods sector.

The increase in overall spending by domestic households now concentrates on the relatively cheaper non-traded good and imports while demand for the domestically produced traded good expands little or even contracts depending on the degree of home bias and substitutability. In contrast to the model with traded goods only though, a larger share of the additional spending falls on domestically produced goods (traded and non-traded combined) thus giving a stronger impetus to investment. To reconcile model and data, the point estimate for the adjustment cost parameter in investment turned out to be significantly above our baseline estimate (around 3 instead of close to zero). As the qualitative and quantitative implications of the model with non-traded goods resemble those of the baseline model in particular with regard to the labour market, we omit a more formal treatment.

7.3 A final look at the data

Instead of embarking onto a quest for theoretical features that help bringing the model even closer to the data, we offer a final assessment of the DMP labour market framework with the help of the approximation of labour market tightness in equation (65). To this end, we use the conditional responses of labour productivity and the real exchange together with the labour market parameters estimated for the DMP model to construct a prediction for labour market tightness. Figure 9 compares this approximation (dashed red line) to the empirical impulse response of labour market tightness which is depicted by the solid blue line. After increasing the relative consumption of the unemployed to the more realistic value of 65%, the DMP framework predicts labour market

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25 We assume a nested CES framework, with non-traded and traded goods being combined to the final consumption good. Production of non-traded goods requires the input of labour and capital. The labour market is assumed to be integrated with a hiring agency negotiating wages with workers. Wages are therefore equalised across sectors. We fixed the share of non-traded goods in GDP at 70% in the steady state and assumed an elasticity of substitution between traded and non-traded goods of 0.75.

26 Under the assumption of an integrated labour market, additional model features do not fundamentally change this approximation although the dynamics of the endogenous variables may of course change.

27 We construct the responses of labour market tightness and labour productivity conditional on a commodity price shock from the empirical impulse responses in Section 3 by applying the relevant relationships obtained from the DMP model.
tightness to respond similarly to its counterpart in the data after a commodity price shock. Judged by their respective peaks the magnitudes of the responses are similar, although the timing of the responses is shifted since the approximation in (65) does not account for time leads and lags.

8 Conclusion

We analyse the effects of commodity price shocks on a set of advanced-economy commodity exporters and find an important link between the real exchange rate and labour market tightness. Our analysis has both an empirical as well as a theoretical dimension. The empirical part documents the effects of commodity price shocks on, amongst others, labour market variables and the real exchange rate using panel VARs.

The impulse responses from the VAR are then used as a yardstick against with to assess a number of small open economy models with search and matching frictions in the labour market to provide an economic interpretation. The impulse response matching exercise shows that in the data and the model, an increase in commodity prices raises consumption demand in the small open economy and induces a real appreciation. Facing higher relative prices for their goods, non-commodity producing firms post additional job vacancies, causing the number of matches between firms and workers to rise. As a result, unemployment falls, even if employment in the commodity-producing sector is negligible. The careful analysis of transmission mechanism of commodity or wealth shocks onto the labour market in search and matching models is a key contribution of this paper.

Open economy aspects are important in understanding labour market dynamics and it is not sufficient to view labour market movements solely through the lens of changes in labour productivity as in much of the macro-labour literature referenced here. Shocks other than to technology can have a limited impact on labour productivity, but nevertheless have a sustained impact on the labour market through an adjustment in the real exchange rate.
References


Note: Quarterly percentage change in the trade-weighted commodity price index, except for Norway for which we use the price of Brent crude. Data definitions are provided in Appendix A, Table 5.
Figure 2: Estimated impact of a commodity price shock

Note: The solid black lines show the median impulse response and the grey shaded area report the 90% confidence bands for the baseline VAR applying quadratic de-trending to the data. The case of linear detrending is depicted by the red-crossed lines and the blue-circled lines are featuring the case of the HP-filter.
Figure 3: DMP and RBC model versus VAR

Note: Fitted impulse responses of the DMP model (solid red), and the RBC model with GHH preferences with and without rigid real wages (dash-dotted blue and dash-dotted black, respectively) versus the baseline VAR estimates (black) and its 90% confidence bands (grey shaded area). The estimated parameter values are provided in Table 4; the values of the remaining model parameters are shown in Table 3.
Figure 4: Decomposing the response in labour market tightness

Note: Decomposition of the response in labour market tightness using the approximation provided in equation (65). The approximate response is given by the dashed red line; the contribution of the real exchange rate is depicted by the dash-dotted green line and the dotted black line shows the contribution of labour productivity. The solid blue line is the actual movement of labour market tightness generated by the DMP model. The top panel shows the movements that are induced by a commodity price shock, the bottom panel shows the case of a neutral technology shock.
Figure 5: DMP, AOB, and DMP model with elastic hours worked and versus VAR

Note: Fitted impulse responses of the DMP model (solid red), the AOB model (dashed blue), and the DMP model with elastic hours worked (dash-dotted black) versus the baseline VAR estimates (solid black) and its 90% confidence bands (grey shaded area). The estimated parameter values are provided in Table 4; the values of the remaining model parameters are shown in Table 3.
Figure 6: Restricted estimation of the AOB model

Note: Fitted impulse responses of the DMP model (solid red), the AOB model (dotted red), and the two restricted versions of the AOB model (dashed blue and dash-dotted green). AOB restricted(v1) uses the same parameters as the DMP model except for the probability of breakdown in bargaining, $\delta_{aob}$, which is estimated at $100\delta_{aob} = 0.001$. AOB restricted(v2) uses the same parameters as the DMP model, but estimates the probability of breakdown in bargaining (again at $100\delta_{aob} = 0.001$) and the relative importance of the fixed and variable cost components in posting vacancies, $s^{fixed} = 0.52$. The estimated parameter values are provided in Table 4; the values of the remaining model parameters are shown in Table 3.
Figure 7: Sensitivity to financial market arrangements, trade openness, and substitutability of goods

Note: The top panel illustrates the sensitivity of the estimated DMP model to changes in the degree of international risk sharing. Financial autarky (dash-dotted green) refers to a calibration of the baseline model where the bond holding cost parameter, $\phi^b$, is approaching infinity. The open economy PIH model (dashed blue) corresponds to a calibration of the baseline model where $\phi^b \approx 0$. The bottom panel illustrates sensitivity when the economy is more open (dashed blue) by setting $\frac{\sigma^h}{\sigma^{x_{h\to g}}_{h\to g}} = 0.15$ and when the home and foreign good are less substitutable (dash-dotted green), $\theta = 0.8$. In the estimated DMP model (red in all panels) the corresponding values are $\phi^b = 0.845$, $\frac{\sigma^h}{\sigma^{x_{h\to g}}_{h\to g}} = 0.05$, and $\theta = 2$. Domestic absorption is computed as the weighted average of consumption and investment.
Figure 8: New Shocks

Note: The figure shows the response to news about an increase in future commodity production (5 years after the news shock). The solid blue lines show the responses under the baseline parameter estimates for the DMP model. The case of perfect substitutability between traded goods is depicted by the dashed red lines; the dash-dotted green lines are derived from a model with a trade elasticity of $\theta = 7$. To obtain the dotted black lines, the baseline parameterization was changed to allow for a lower degree of home bias by setting $\frac{e}{ppi} = 0.5$. 
Note: The blue line depicts the response of labour market tightness constructed directly from the VAR estimates. The dashed red line is the predicted response of labour market tightness using equation (65) based on the empirical responses of the real exchange rate and labour market productivity.
# Table 1: Share of top three commodity exports

<table>
<thead>
<tr>
<th>Country</th>
<th>% of country net exports</th>
<th>% of world net exports</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Australia</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iron ores and concentrates</td>
<td>33</td>
<td>50</td>
</tr>
<tr>
<td>Coal; briquettes</td>
<td>18</td>
<td>35</td>
</tr>
<tr>
<td>Petroleum gases</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td><strong>Canada</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petroleum oils, crude</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>Petroleum gases</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Wheat and meslin</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td><strong>New Zealand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Milk &amp; cream concentrated</td>
<td>24</td>
<td>37</td>
</tr>
<tr>
<td>Lamb meat</td>
<td>8</td>
<td>43</td>
</tr>
<tr>
<td>Butter</td>
<td>7</td>
<td>44</td>
</tr>
<tr>
<td><strong>Norway</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petroleum oils, crude</td>
<td>43</td>
<td>3</td>
</tr>
<tr>
<td>Petroleum gases</td>
<td>24</td>
<td>8</td>
</tr>
<tr>
<td>Fish, excluding fillets</td>
<td>7</td>
<td>62</td>
</tr>
</tbody>
</table>


# Table 2: Volatilities with respect to output

<table>
<thead>
<tr>
<th>Variable</th>
<th>Australia</th>
<th>Canada</th>
<th>Norway</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unconditional moments in data</td>
<td>Conditional moments in VAR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>price of commodities</td>
<td>20.9</td>
<td>10.9</td>
<td>15.6</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>[65.9]</td>
<td>[65.7]</td>
<td>[67.7]</td>
<td>[73.1]</td>
</tr>
<tr>
<td>consumption</td>
<td>1.67</td>
<td>0.68</td>
<td>1.22</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>[1.12]</td>
<td>[1.14]</td>
<td>[1.31]</td>
<td>[1.27]</td>
</tr>
<tr>
<td>investment</td>
<td>7.05</td>
<td>3.82</td>
<td>6.01</td>
<td>5.00</td>
</tr>
<tr>
<td></td>
<td>[8.29]</td>
<td>[8.11]</td>
<td>[7.74]</td>
<td>[8.97]</td>
</tr>
<tr>
<td>unemployment rate</td>
<td>0.51</td>
<td>0.41</td>
<td>0.41</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>[0.64]</td>
<td>[0.62]</td>
<td>[0.61]</td>
<td>[0.68]</td>
</tr>
<tr>
<td>vacancies posted</td>
<td>14.1</td>
<td>10.9</td>
<td>16.9</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>[18.1]</td>
<td>[18.6]</td>
<td>[17.5]</td>
<td>[20.9]</td>
</tr>
<tr>
<td>net trade</td>
<td>1.49</td>
<td>0.87</td>
<td>2.10</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>[5.69]</td>
<td>[5.68]</td>
<td>[5.26]</td>
<td>[6.21]</td>
</tr>
<tr>
<td>real exchange rate</td>
<td>8.97</td>
<td>3.94</td>
<td>2.07</td>
<td>5.61</td>
</tr>
<tr>
<td></td>
<td>[5.65]</td>
<td>[5.93]</td>
<td>[6.97]</td>
<td>[6.33]</td>
</tr>
<tr>
<td>real wage</td>
<td>2.01</td>
<td>0.91</td>
<td>0.72</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>[0.67]</td>
<td>[0.89]</td>
<td>[0.98]</td>
<td>[0.97]</td>
</tr>
<tr>
<td>total hours worked</td>
<td>1.89</td>
<td>0.57</td>
<td>1.06</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>[1.90]</td>
<td>[1.88]</td>
<td>[1.80]</td>
<td>[2.13]</td>
</tr>
</tbody>
</table>

Notes: Unconditional and [conditional on commodity price shocks] standard deviations relative to GDP.
Table 3: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>DMP</th>
<th>DMP(l)</th>
<th>AOB</th>
<th>RBC</th>
<th>RBC(rr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
<td>(all models)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Curvature of utility function</td>
<td>1.10</td>
<td>(all models)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital</td>
<td>0.33</td>
<td>(all models)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.025</td>
<td>(all models)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>Trade elasticity</td>
<td>2.00</td>
<td>(all models)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{e^\lambda}{h^\gamma}$</td>
<td>Steady state goods export GDP ratio</td>
<td>0.05</td>
<td>(all models)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{e^\lambda}{y^\gamma}$</td>
<td>Steady state commodity export GDP ratio</td>
<td>0.15</td>
<td>(all models)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>Home-bias parameter</td>
<td>0.80</td>
<td>(all models) [implied]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n^*_{ss}$</td>
<td>Steady state employment</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$q^*_{ss}$</td>
<td>Steady state prob. of filling vacancy</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\frac{e^\lambda_{ss}+q_{ss}}{h_{ss}}$</td>
<td>Share of vacancy cost in output</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Probability that match breaks up</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$b^*/w_{ss}$</td>
<td>Replacement ratio</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Scale parameter in matching function</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Notes: DMP = baseline model, DMP(l) = baseline model with elastic labour supply, AOB = alternating offer bargaining model, RBC = Real business cycle model with Walrasian labour markets, RBC(rr) = RBC model with real wage rigidities.
Table 4: Estimated parameters

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>DMP</th>
<th>DMP(l)</th>
<th>AOB</th>
<th>RBC</th>
<th>RBC(rr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond holding cost</td>
<td>0.85</td>
<td>0.72</td>
<td>0.86</td>
<td>0.70</td>
<td>0.68</td>
</tr>
<tr>
<td>(× 100)</td>
<td>[0.004]</td>
<td>[0.004]</td>
<td>[0.004]</td>
<td>[0.004]</td>
<td>[0.004]</td>
</tr>
<tr>
<td>Cost of vacancies</td>
<td>1.10</td>
<td>0.96</td>
<td>0.84</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>[0.020]</td>
<td>[0.018]</td>
<td>[0.027]</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Investment adjustment costs</td>
<td>0.04</td>
<td>0.005</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>[0.001]</td>
</tr>
<tr>
<td>Matching function parameter</td>
<td>0.72</td>
<td>0.74</td>
<td>0.70</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>AOB parameter (× 100)</td>
<td>–</td>
<td>–</td>
<td>0.001</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Consumption share of the unemployed</td>
<td>0.60</td>
<td>0.58</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Fixed cost of bargaining</td>
<td>0.001</td>
<td>0.001</td>
<td>0.46</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>[0.03]</td>
<td>[0.03]</td>
<td>[0.03]</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Household’s bargaining weight</td>
<td>0.40</td>
<td>0.30</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>[implied]</td>
<td>[implied]</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Inverse of labour supply elasticity</td>
<td>–</td>
<td>0.74</td>
<td>–</td>
<td>0.48</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>[implied]</td>
<td>–</td>
<td>[0.003]</td>
<td>[0.01]</td>
<td></td>
</tr>
<tr>
<td>Real wage rigidity</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>[0.003]</td>
</tr>
</tbody>
</table>

Notes: DMP = baseline model, DMP(l) = baseline model with elastic labour supply, AOB = alternating offer bargaining model, RBC = Real business cycle model with Walrasian labour markets, RBC(rr) = RBC model with real wage rigidities. Numbers in brackets are standard errors.
A  Additional details for econometric analysis

A.1  Data description and definitions

Table 5 lists the raw data used in the VARs. All series are taken from Haver Analytics, and the data set is available upon request from the authors. For each country in the panel, ten time series are used, which are transformations of the raw data:

\[ peor = \ln \left( \frac{COM}{US\ CPI} \right) \]  
\[ y = \ln \left( \frac{GDP}{POP} \right) \]  
\[ c = \ln \left( \frac{C}{POP} \right) \]  
\[ i = \ln \left( \frac{I}{POP} \right) \]  
\[ nxm = \left( \frac{NXE}{GDP(L)} \right) \]  
\[ urate = \left( \frac{UNE}{LAF} \right) \]  
\[ vac = \ln(VAC) \]  
\[ fxr = \ln(REER) \]  
\[ wprr = \ln \left( \frac{WAGE}{CPI} \right) \]  
\[ thp = \ln \left( \frac{\text{Total HRS}}{POP} \right) \].
### Table 5: Data definitions

<table>
<thead>
<tr>
<th>Raw Data</th>
<th>Australia</th>
<th>Canada</th>
<th>New Zealand</th>
<th>Norway</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP(L)</td>
<td>GDP (SA, Mil.A$)</td>
<td>GDP (SA, Mil.C$)</td>
<td>GDP (SA, Mil.NZ$)</td>
<td>GDP (SA, Mil.Kroner)</td>
</tr>
<tr>
<td>UNE</td>
<td>Unemp.: All Pers. 15 and Over (SA, 1000s)</td>
<td>All Pers. 15 and Over (SA, 1000s)</td>
<td>All Pers. 15 and Over (SA, 1000s)</td>
<td>Registered Unemp. (SA, EOP, Number)</td>
</tr>
<tr>
<td>VAC</td>
<td>Unfilled Job Vacancies (SA, Number)</td>
<td>Vacancies (A. Tasci) (SA)</td>
<td>ANZ Jobs Ads (SA)</td>
<td>Unfilled Job Vacancies (SA, Number)</td>
</tr>
<tr>
<td>REER</td>
<td>Real Effective Exchange Rate</td>
<td>Real Effective Exchange Rate</td>
<td>Real Effective Exchange Rate</td>
<td>Real Effective Exchange Rate</td>
</tr>
<tr>
<td>WAGE</td>
<td>Early Est Labor Comp</td>
<td>Same as Australia</td>
<td>Same as Australia</td>
<td>Same as Australia</td>
</tr>
<tr>
<td>COM</td>
<td>RBA Commodity Prices: All Items: US$ (NSA)</td>
<td>BoC Commodity Price Index</td>
<td>ANZ World Commodity Price Index (US$, NSA)</td>
<td>Brent Crude Oil (US$ / Barrel)</td>
</tr>
<tr>
<td>CPI</td>
<td>CPI: all items excl Food &amp; Energy (NSA)</td>
<td>Consumer Price Index (NSA)</td>
<td>Consumer Price Index (NSA)</td>
<td>European Harmonized CPI (NSA)</td>
</tr>
<tr>
<td>US CPI</td>
<td>US CPI (SA)</td>
<td>US CPI (SA)</td>
<td>US CPI (SA)</td>
<td>US CPI (SA)</td>
</tr>
</tbody>
</table>

Notes: Source: All data are taken from Haver Analytics.
B Model equations

We display the equations and steady state relationships of the models presented in Sections 4 and 5.

B.1 Model equations in levels

Collecting relevant equations for numerical implementation:

- households:

\[ U_c(c_t) = \lambda_t \] (B.1)

\[ U_c(c_t^w) = U_c(c_t^u) \] (B.2)

\[ c_t = n_t c_t^w + (1 - n_t) c_t^u \] (B.3)

\[ H_t = \frac{U(c_t^w) - U(c_t^u)}{\lambda_t} + [(w_t - b^u) - (c_t^w - c_t^u)] + (1 - \rho)E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} H_{t+1}(1 - s_{t+1}) \right]. \] (B.4)

We consider a flexible formulation of individual preferences of the form

\[ U(c_i) = \left( c_i - \varphi_0 \frac{1}{1 + \varphi(h_i)} \right)^{1-\sigma} - \sigma \]

with \( h^w = 1 \) and \( h^u = 0 \), and \( \Phi = \frac{\varphi_0}{1 + \varphi} (h^w)^{1+\varphi} \) similar to Greenwood et al. (1988). By setting \( \varphi_0 = 0 \), i.e., \( \Phi = 0 \), this specification reduces to CRRA preferences. Equations (B.1) - (B.4) then imply:

1. CRRA utility:

\[ (a) \quad c_t = c_t^w = c_t^u \]

\[ (b) \quad \lambda_t = c_t^\sigma \]

\[ (c) \quad H_t = (w_t - b^u) + (1 - \rho)E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} H_{t+1}(1 - s_{t+1}) \right]. \]

2. GHH utility:

\[ (a) \quad c_t^w - c_t^u = \Phi \]

\[ (b) \quad \lambda_t = (c_t - n_t \Phi)^{-\sigma} \]

\[ (c) \quad H_t = (w_t - b^u) - \Phi + (1 - \rho)E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} H_{t+1}(1 - s_{t+1}) \right]. \]

GHH preferences consumption inequality emerges even if risk sharing is complete.

- firms:

\[ y_t^h = a_t k_{t-1}^{\alpha} n_t^{1-\alpha} \] (B.5)

\[ k_t = (1 - \delta) k_{t-1} + l(x_t, x_{t-1}) \] (B.6)

\[ q_{t+1} = E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{p_t x_t^{h+1} k_{t+1}}{k_t} + (1 - \delta) q_{t+1} \right) \right] \] (B.7)

\[ 1 = q_{t+1} \frac{\partial l(x_t, x_{t-1})}{\partial x_t} + E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial q_{t+1}}{\partial x_t} \right] \] (B.8)

\[ g_t (J_t - \bar{\kappa}) = \frac{\partial \kappa(v_t, v_{t-1})}{\partial v_t} + E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \kappa(v_{t+1}, v_t)}{\partial v_t} \right] \] (B.9)

\[ J_t = \left( 1 - \alpha \right) \frac{p_t x_t^h}{n_t} - w_t + (1 - \rho)E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1} \right] \] (B.10)

\[ ^{28} \text{Albeit using a different specification of preferences, Hall and Milgrom (2008) also assume that the marginal utility of consumption depends on hours worked, where hours worked are fixed.} \]
\[ t(x_t, x_t - 1) = \kappa^\nu x_t \left(1 - \frac{\phi^\nu}{2} \left(\frac{x_t}{x_t - 1} - 1\right)^2\right) \]  \hspace{1cm} (B.11)

\[ \frac{\partial t(x_t, x_t - 1)}{\partial x_t} = \frac{\partial t(x_t, x_t - 1)}{x_t} - \kappa^\nu \phi^\nu \left(\frac{x_t}{x_t - 1} - 1\right) \frac{x_t}{x_t - 1} \]  \hspace{1cm} (B.12)

\[ \frac{\partial t(x_t, x_t - 1)}{\partial x_{t-1}} = \kappa^\nu \phi^\nu \left(\frac{x_t}{x_{t-1}} - 1\right) \left(\frac{x_t}{x_{t-1}}\right)^2 \]  \hspace{1cm} (B.13)

\[ \kappa(v_t, v_{t-1}) = \kappa^\nu v_t \left(1 + \frac{\phi^\nu}{2} \left(\frac{v_t}{v_{t-1}} - 1\right)^2\right) \]  \hspace{1cm} (B.14)

\[ \frac{\partial \kappa(v_t, v_{t-1})}{\partial v_t} = \frac{\kappa(v_t, v_{t-1})}{v_t} + \kappa^\nu \phi^\nu \left(\frac{v_t}{v_{t-1}} - 1\right) \frac{v_t}{v_{t-1}} \]  \hspace{1cm} (B.15)

\[ \frac{\partial \kappa(v_t, v_{t-1})}{\partial v_{t-1}} = -\kappa^\nu \phi^\nu \left(\frac{v_t}{v_{t-1}} - 1\right) \left(\frac{v_t}{v_{t-1}}\right)^2 \]  \hspace{1cm} (B.16)

If \( \kappa = 0 \) and \( \phi^\nu = 0 \), the equations reduce to their standard formulations.

- trade in goods and assets:

\[ \frac{1}{1 + r_t} = E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{rer_{t+1}}{rer_t} \right] \]  \hspace{1cm} (B.17)

\[ v \left(p_t^h\right)^{-1-\theta} = 1 - (1 - v) (rer_t)^{-1-\theta} \]  \hspace{1cm} (B.18)

\[ y_t^h = v \left(p_t^h\right)^{-\theta} \left(c_t + x_t + \tilde{q}_t + \kappa(v_t, v_{t-1})\right) + ex_t^h \]  \hspace{1cm} (B.19)

\[ ex_t^h = v^* \left(\frac{rer_t}{p_t^h}\right) \theta^\nu y_t^* \]  \hspace{1cm} (B.20)

\[ tbal_t = \frac{1 - c_t + x_t + \tilde{q}_t + \kappa(v_t, v_{t-1})}{gdp_t} \]  \hspace{1cm} (B.21)

\[ \bar{b}_t = \frac{(1 + r_{t-1}) gdp_{t-1}}{gdp_t} \frac{rer_t}{rer_{t-1}} \bar{b}_{t-1} + tbal_t \]  \hspace{1cm} (B.22)

\[ 1 + r_t = (1 + r_t^*) e^{-\phi^\nu(\bar{b}_t - \bar{b})} \]  \hspace{1cm} (B.23)

\[ gdp_t = rer_t \bar{p}_t^* y_t^* + \bar{p}_t^* \bar{y}_t^* \]  \hspace{1cm} (B.24)

where \( tbal_t \) is the trade balance to GDP ratio. Net foreign assets relative to GDP, \( \bar{b}_t \), are given by \( \bar{b}_t = \frac{rer_t \bar{b}_t}{gdp_t} \).

- search and matching:

\[ m_t = \chi a_t^\nu v_t^1 - \zeta \]  \hspace{1cm} (B.25)

\[ n_t = (1 - \rho)n_{t-1} + m_t \]  \hspace{1cm} (B.26)

\[ u_t = 1 - (1 - \rho)n_{t-1} \]  \hspace{1cm} (B.27)

\[ \bar{u}_t = 1 - n_t \]  \hspace{1cm} (B.28)

\[ \theta_t = \frac{v_t}{u_t} \]  \hspace{1cm} (B.29)

\[ s_t = \chi \theta_t^1 - \zeta \]  \hspace{1cm} (B.30)

\[ q_t = \chi \theta_t^\zeta \]  \hspace{1cm} (B.31)

- exogenous variables: \( a_t, \bar{p}_t^*, y_t^*, r_t^*, \bar{y}_t^* \).

50
The final equation missing from this set is the surplus sharing rule. Under Nash bargaining, the surplus is shared according to:

\[ J_t = \frac{1 - \xi}{\xi} H_t. \]  (B.32)

Under alternating offer bargaining, we have

\[ J_t = \beta_1 H_t - \beta_2 \gamma_{aub} + \beta_3 \left( (1 - \alpha) \frac{p^h y^h}{n_t} - b^u \right) \]  (B.33)

with

\[ \alpha_1 = 1 - \delta_{aub} + (1 - \delta_{aub}) M_{aub} \]  (B.34)

\[ \alpha_2 = 1 - (1 - \delta_{aub}) M_{aub} \]  (B.35)

\[ \alpha_3 = \frac{\alpha_2 1 - \delta_{aub}}{\delta_{aub}} - \alpha_1 \]  (B.36)

\[ \alpha_4 = \frac{1 - \delta_{aub} \alpha_2}{2 - \delta_{aub} M_{aub}} + 1 - \alpha_2 \]  (B.37)

\[ \beta_1 = \alpha_2 \alpha_1 \]  (B.38)

\[ \beta_2 = \alpha_3 \alpha_1 \]  (B.39)

\[ \beta_3 = \alpha_4 \alpha_1. \]  (B.40)

### B.2 Steady state and calibration

In our calibration strategy, we fix targets for the steady state values of some of the endogenous variables: \( q_{ss}, n_{ss}, y_{c ss}, y_{h ss}, \ldots, g_{ss}, b_{wss}, \kappa_{comp} = \bar{\kappa}_{q ss}, \kappa_{ss}, \chi, v_{ss}, v_{ss}^*, b^u, \xi, \Phi. \)

In terms of parameters, we choose the values of the following ones: \( \beta, \delta, \alpha, \sigma, \xi, \kappa^x, \theta^*, \phi^x, \phi^y. \)

The remaining parameters are determined by the steady state relationships: \( \chi, v, v^*, b^u, \xi, \Phi. \) In the alternating offer model, we add \( M_{aub} \) and \( \delta_{aub} \) to the set of externally specified parameters. \( \gamma_{aub} \) is determined by the surplus sharing rule (instead of \( \xi \)). The following relationships are obtained. From the cost functions:

\[ \iota(x_{ss}, x_{ss}) = \kappa^x x_{ss} \]  (B.41)

\[ \frac{\partial \iota(x_t, x_{t-1})}{\partial x_t} \bigg|_{ss} = \kappa^x \]  (B.42)

\[ \frac{\partial \iota(x_t, x_{t-1})}{\partial x_{t-1}} \bigg|_{ss} = 0 \]  (B.43)

\[ \kappa(v_{ss}, v_{ss}) = \kappa^v v_{ss} \]  (B.44)

\[ \frac{\partial \kappa(v_t, v_{t-1})}{\partial v_t} \bigg|_{ss} = \kappa^v \]  (B.45)

\[ \frac{\partial \kappa(v_t, v_{t-1})}{\partial v_{t-1}} \bigg|_{ss} = 0, \]  (B.46)
From the labour market conditions:

\[ n_{ss} = n_{ss} \]  
\[ q_{ss} = q_{ss} \]  
\[ u_{ss} = 1 - (1 - \rho) n_{ss} \]  
\[ \bar{u}_{ss} = 1 - n_{ss} \]  
\[ m_{ss} = n_{ss} - (1 - \rho) n_{ss} \]  
\[ v_{ss} = \frac{m_{ss}}{q_{ss}} \]  
\[ s_{ss} = \frac{m_{ss}}{u_{ss}} \]  
\[ \theta_{ss} = \frac{v_{ss}}{u_{ss}} \]  
\[ \chi = \frac{m_{ss}}{u_{ss} v_{ss} \zeta} \]  

\[ \tilde{u}_{ss} = 1 - n_{ss} \]  
\[ m_{ss} = n_{ss} - (1 - \rho) n_{ss} \]  
\[ v_{ss} = \frac{m_{ss}}{q_{ss}} \]  
\[ s_{ss} = \frac{m_{ss}}{u_{ss}} \]  
\[ \theta_{ss} = \frac{v_{ss}}{u_{ss}} \]  
\[ \chi = \frac{m_{ss}}{u_{ss} v_{ss} \zeta}. \]

From the firms’ side:

\[ a_{ss} = 1 \]  
\[ t_{q_{ss}} = \frac{1}{\kappa^2} \]  
\[ y_{ss}^h = \left( \left( \frac{1}{\beta} - (1 - \delta) \right) \frac{t_{q_{ss}}}{\alpha} \right)^{-\frac{\alpha}{\gamma}} n_{ss} \]  
\[ k_{ss} = \frac{1}{\beta} \left( \beta - (1 - \delta) \right) \frac{t_{q_{ss}}}{\alpha} n_{ss} \]  
\[ x_{ss} = \frac{\delta}{\kappa^2} k_{ss} \]  
\[ \bar{\kappa} = \frac{\kappa_{q_{ss}}}{\kappa_{comp}} \frac{\kappa_{comp} y_{ss}^h}{q_{ss}} \]  
\[ \kappa^v = \frac{\kappa_{comp} y_{ss}^h - \bar{\kappa} q_{ss}}{v_{ss}} \]  
\[ J_{ss} = \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \]  
\[ w_{ss} = (1 - \alpha) \frac{y_{ss}^h}{n_{ss}} - (1 - (1 - \rho) \beta) J_{ss}. \]

From the international trade relations:

\[ r_{ss} = \frac{1}{\beta} - 1 \]  
\[ r e r_{ss} = 1 \]  
\[ p_{ss}^h = 1 \]  
\[ r_{ss}^* = r_{ss} \]  
\[ \bar{b}_{ss} = \bar{b} \]
\[ tbal_{ss} = \bar{b}_{ss} - (1 + r_{ss}) \bar{b}_{ss} \]  \hspace{1cm} (B.70)

\[ p_{ss}^c = 1 \]  \hspace{1cm} (B.71)

\[ gd_{ss} = \left( \frac{y_{ss}^c}{y_{ss}^b} + 1 \right) y_{ss}^b \]  \hspace{1cm} (B.72)

\[ v^* = \frac{c_{ss} y_{ss}^h}{y_{ss}^b} \]  \hspace{1cm} (B.73)

From the household side:

\[ c_{ss} = (1 - tbal_{ss}) gd_{ss} - (x_{ss} + \bar{r} q_{ss} + \kappa^u v_{ss}) \]  \hspace{1cm} (B.74)

\[ v = \frac{y_{ss}^b - c_{ss} y_{ss}^h}{c_{ss} + x_{ss} + \bar{r} q_{ss} + \kappa^u v_{ss}} \]  \hspace{1cm} (B.75)

\[ \lambda_{ss} = c_{ss}^{-\sigma} \left( n_{ss} \left( \frac{c_{ss}}{c_{uu}} \right) + (1 - n_{ss}) \right)^\sigma \]  \hspace{1cm} (B.76)

\[ H_{ss} = \frac{-\Phi + w_{ss} - b^u}{1 - (1 - \rho) \beta (1 - s_{ss})}. \]  \hspace{1cm} (B.77)

The bargaining sharing rule determines the bargaining weight:

\[ \xi = \frac{H_{ss}}{J_{ss} + H_{ss}}. \]  \hspace{1cm} (B.78)

An alternative approach is to fix the bargaining weight and then compute the replacement ratio. We use the bargaining sharing rule to solve for \( H_{ss} \) and then use the equation defining \( H_{ss} \) to solve for \( b^u \).

Finally, the calibration of \( \Phi \) is linked to the choice of \( \frac{c_{uu}}{c_{ss}} \):

\[ \Phi = \frac{\frac{c_{uu}}{c_{ss}} - 1}{n_{ss} \left( \frac{c_{uu}}{c_{ss}} \right) + (1 - n_{ss})} c_{ss}. \]  \hspace{1cm} (B.79)

Under alternating offer bargaining the additional restriction needed determines \( \gamma_{aob} \):

\[ \gamma_{aob} = \frac{1}{\beta_2} \left( \beta_1 H_{ss} - J_{ss} + \beta_3 \left( 1 - \alpha \right) \frac{y_{ss}^b}{n_{ss} - b^u} \right). \]  \hspace{1cm} (B.80)

**B.3 Search and matching model with elastic labour supply**

The overwhelming share of the labour search literature assumes that labour is supplied inelastically. By contrast models without search and matching frictions always model the labour supply as elastic. Here we consider the case of an elastic labour supply in the search and matching framework under the assumption of complete risk sharing among household members. If the labour supply is elastic, the marginal value of employment to the household is given by:

\[ H_t = \frac{U(c_t^w) - U(c_t^u)}{\lambda_t} + w_t h_t^w - b^u - (c_t^w - c_t^u) + (1 - \rho) E_t \left[ \frac{\beta^{\lambda_{t+1}}}{\lambda_t} H_{t+1} (1 - s_{t+1}) \right]. \]  \hspace{1cm} (B.81)
We replace the marginal value of employment to the firms by:

\[ J_t = \left( 1 - \alpha \right) \frac{p_t y_t^h}{n_t h_t^w} - w_t h_t^w + (1 - \rho) E_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} J_{t+1} \right] \]  

(B.82)

and the production function by:

\[ y_t^h = \alpha_t k_t^\alpha (n_t h_t^w)^{1-\alpha}. \]  

(B.83)

Nash bargaining occurs over wages and hours worked, yielding the conditions:

\[ J_t = 1 - \xi H_t \]  

(B.84)

\[ - \frac{U_h (c_t^w, h_t^w)}{\lambda_t} = (1 - \alpha) \frac{p_t y_t^h}{n_t h_t^w}. \]  

(B.85)

In deriving equation (B.85), we assume that the household member and the firm take \( \lambda_t \) and the marginal product of the firm \( mpd_t = (1 - \alpha) \frac{p_t y_t^h}{n_t h_t^w} \) as given in negotiating over hours worked as in Cheron and Langot (2004). Under this assumption the marginal rate of substitution between labour and consumption equals the marginal product of labour.

With GHH preferences and no labour effort by unemployed households, full risk sharing implies:

\[ \frac{c_t^w - c_t^u}{U(c_t^w) - U(c_t^u)} = \frac{\phi_0}{1 + \phi} (h_t^w)^{1+\phi} \]  

(B.86)

Consequently, the following conditions prevail:

\[ \frac{c_t^w}{c_t^u} = 1 + \left( n_t \frac{c_t^w}{c_t^u} + (1 - n_t) \right) \frac{\phi_0}{1 + \phi} (h_t^w)^{1+\phi} \]  

(B.87)

\[ \lambda_t = c_t^{-\sigma} \left( n_t \frac{c_t^w}{c_t^u} + (1 - n_t) \right)^{\sigma} \]  

(B.88)

\[ H_t = w_t h_t^w - b^u - \frac{\phi_0}{1 + \phi} (h_t^w)^{1+\phi} + (1 - \rho) E_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} H_{t+1} (1 - s_{t+1}) \right] \]  

\[ \phi_0 (h_t^w)^{\phi} = (1 - \alpha) \frac{p_t y_t^h}{n_t h_t^w}. \]  

(B.88)
C Simple analytics

C.1 DMP model

This appendix provides background on the simple analytical expressions underlying our discussion in Section 6.2. For simplicity, we abstract from vacancy adjustment costs, i.e., $\phi^v = 0$.

Noting that under Nash bargaining the surplus is shared according to $J_t = 1 - \xi H_t$, equation (21) can be written as:

$$\xi J_t = (1 - \xi) (w_t - b^u - \Phi) + \xi (1 - \rho) E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1} (1 - s_{t+1}) \right]$$

(C.1)

where $\Phi$ measures the (constant) difference in consumption between employed and unemployed household members $\Phi = c^e - c^u$. Combining the above expression with condition (38) to eliminate wages:

$$J_t + (1 - \xi) (b^u + \Phi) = (1 - \xi) mpl_t + (1 - \rho) E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \xi s_{t+1}) J_{t+1} \right].$$

(C.2)

Applying the definitions for $s_t$ and $q_t$ in equations (7) and (8), and noting that absent vacancy adjustment costs equation (37) yields the following relationship between $J_t$ and labour market tightness, $\theta_t$:

$$J_t = \kappa^v \chi \theta_t + \bar{\kappa}$$

(C.3)

equation (C.2) can be written as:

$$\frac{\kappa^v \chi}{\bar{\kappa} + \kappa} + [(1 - \xi) (b^u + \Phi) + \bar{\kappa}] = (1 - \xi) mpl_t + (1 - \rho) E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \xi \chi \theta_t \frac{1}{\theta_t}) \left( \frac{\kappa^v \chi}{\bar{\kappa} + \kappa} \right) \right].$$

(C.4)

It is immediately apparent from equation (C.4) that unemployment benefits, $b^u$, and the consumption differential, $\Phi$, enter the model in the same manner. The fixed costs of starting the bargaining process, $\bar{\kappa}$, act similarly, but given its presence in the forward-looking term we have to push the analytics a little bit further to gain a clearer image.

As the marginal product of labour is given by:

$$mpl_t = (1 - \alpha) \frac{h^h_i}{n_t}$$

(C.5)

and the relative price $p^h_i$ relates to the real exchange rate as:

$$1 = v \left( p^h_i \right)^{1-\theta} + (1 - v) (rer_t)^{1-\theta}$$

(C.6)

from equation (43), changes in commodity prices affect the marginal product of labour in the open economy through variations in the real exchange rate.
Log-linearising expression (C.4) around the steady state, delivers

\[
\begin{align*}
\left. \frac{\kappa^v}{\chi} \frac{\partial \hat{\theta}_t}{\partial t} \right|_{\text{ss}} - (1 - \rho) \beta \left( \frac{\kappa^v}{\chi} \theta_{ss} - \kappa^v \theta_{ss} - \bar{\kappa} \theta_{ss} \right) E_t \hat{\theta}_{t+1} \\
= (1 - \xi) m_{plss} \pi_{plss} + (1 - \rho) \beta \left( 1 - \chi \theta_{ss}^{-\xi} \right) \left( \frac{\kappa^v}{\chi} \theta_{ss} + \bar{\kappa} \right) E_t \left[ \hat{\lambda}_{t+1} - \hat{\lambda}_t \right].
\end{align*}
\] (C.7)

Given our strategy of keeping the steady state value of the job filling probability identical across calibrations, we make use of the fact \( q_{ss} = \chi \theta_{ss}^{-\xi} \), and we apply the definition of the marginal product of labour in our simplified environment to arrive at:

\[
\begin{align*}
\left. \frac{\kappa^v}{q_{ss}} \left( \hat{\theta}_t - E_t \hat{\theta}_{t+1} \right) \right|_{\text{ss}} + \left( \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) \left[ \xi - (1 - \rho) \beta (\xi - q_{ss} \xi \theta_{ss}) - \kappa \xi [1 - (1 - \rho) \beta (1 - q_{ss} \xi \theta_{ss})] \right] E_t \hat{\theta}_{t+1} \\
= (1 - \xi) m_{plss} \left( g^b_{ps} - \bar{\n}_t - \frac{1 - \nu}{\nu} \bar{e}_t \right) + (1 - \rho) \beta \left( \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) (1 - \xi q_{ss} \xi \theta_{ss}) E_t \left[ \hat{\lambda}_{t+1} - \hat{\lambda}_t \right].
\end{align*}
\] (C.8)

In the steady state, equation (C.4) implies:

\[
\left( \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) \left[ 1 - (1 - \rho) \beta (1 - q_{ss} \xi \theta_{ss}) \right] = (1 - \xi) \left[ m_{plss} - (b^u + \Phi) \right].
\] (C.9)

Our strategy of parameterising the DMP model implies assigning a value to the replacement ratio, \( r^u \), rather than unemployment benefits, \( b^u \). The definition of the replacement ratio and condition (38) yield the following unemployment benefits given the replacement ratio:

\[
b^u = r^u w_{ss} = r^u \left( m_{plss} - (1 - (1 - \rho) \beta) \left( \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) \right).
\] (C.10)

After substituting (C.10) into equation (C.11), we obtain the household bargaining weight, \( \xi \), as an implicit function of the replacement ratio, \( r^u \), and the consumption difference between employed and unemployed, \( \Phi \):

\[
\left( \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) \left[ 1 - (1 - \rho) \beta (1 - q_{ss} \xi \theta_{ss}) \right] = (1 - \xi) \left[ (1 - r^u) m_{plss} + r^u (1 - (1 - \rho) \beta) \left( \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) - \Phi \right]
\] (C.11)

or solved for \( \xi \):

\[
\xi = \frac{1}{1 + \frac{(1 - (1 - \rho) \beta (1 - q_{ss} \xi \theta_{ss})) \left( \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right)}{(1 - r^u) m_{plss} - (1 - (1 - \rho) \beta) \left( \frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) - \Phi}}.
\] (C.12)

\( \xi \) is decreasing in \( r^u \) and \( \Phi \).\(^2^9\)

For convenience, we define the coefficient \( \Upsilon \) as:

\[
\Upsilon = \frac{\left[ \xi - (1 - \rho) \beta (\xi - q_{ss} \xi \theta_{ss}) \right] - \kappa \xi [1 - (1 - \rho) \beta (1 - q_{ss} \xi \theta_{ss})]}{\left[ \xi - (1 - \rho) \beta (\xi - q_{ss} \xi \theta_{ss}) \right] - \kappa \xi [1 - (1 - \rho) \beta (1 - q_{ss} \xi \theta_{ss})]}
\] (C.13)

\( ^{2^9} \)It is worth pointing out that for common calibration choice of the other parameters it is much easier to obtain an implied bargaining weight around 0.5 at a moderate replacement ratio if \( \Phi > 0 \).
Only the second term in \( \Upsilon \) depends on \( \xi \); therefore \( \Upsilon \) is bounded by \( \zeta - \zeta \frac{\bar{\kappa}}{q_{ss} + \kappa} \) (for \( \xi = 0 \)) and \( 1 - \zeta \frac{\bar{\kappa}}{q_{ss} + \kappa} \).

With these expressions in hand, we can approximate the response of labour market tightness to a commodity price shock. To avoid complications from transition dynamics, we focus on the “medium run”. The commodity price shock induces highly persistent movements in the endogenous variables, thus implying little change in the endogenous variables from one period to another after the initial transition. Applying this reasoning to equation (C.7), the following approximate relationship between labour market tightness, labour productivity, and the real exchange rate emerges:

\[
\hat{y}_t \approx \frac{1}{\Upsilon} \frac{mpl_{ss}}{(1 - u_m)mpl_{ss} + (1 - (1 - \rho))\beta} \left( \frac{\epsilon_v}{q_{ss} + \kappa} \right) - \Phi \left( \hat{y}_t - \hat{n}_t - \frac{1 - \nu}{\nu} r_e \hat{r}_t \right). \tag{C.14}
\]

The impact of given changes in the real exchange rate onto labour market tightness is determined by the interplay of the parameters \( r_u \) and \( \Phi \). For now, we abstract from the coefficients \( \bar{\kappa} \) and \( \kappa_v \). Although \( \Upsilon \) is a function of \( \xi \) and therefore depends on the choices of \( r_u \) and \( \Phi \) as is apparent from equation (C.12), the role of this coefficient in delivering big amplification of the exchange rate movement is limited since \( \Upsilon \) is bounded. For \( \bar{\kappa} = 0 \), \( \Upsilon \in [\zeta, 1] \). Much more powerful in delivering amplification are therefore the parameters \( r_u \) and \( \Phi \). An increase in the replacement ratio or in the consumption gap between employed and unemployed household members reduces the denominator of the second coefficient and results in bigger movements of labour market tightness for a given movement of the real exchange rate. It is for this interplay of the parameters \( r_u \) and \( \Phi \) that our model can match the responses in unemployment and vacancies even for a moderate value of the replacement ratio of \( r_u = 0.4 \).

The choice of the parameter \( \bar{\kappa} \) also influences the transmission of the shock to labour market tightness. Our calibration strategy imposes a tight relationship with \( \kappa_v \) as we target a specific value of the total (expected) costs of vacancy posting, \( \kappa_v q_{ss} + \bar{\kappa} \), relative to non-commodity output in the steady state, \( y_{ss} \). Thus, the term \( \frac{\bar{\kappa}}{q_{ss} + \kappa} \) equals \( s_{fixed} \frac{q_{ss}}{q_{ss} + \kappa} \), where \( s_{fixed} \) is the share of vacancy posting costs that falls onto \( q_{ss} \). By increasing \( s_{fixed} \), the term \( \frac{\bar{\kappa}}{q_{ss} + \kappa} \) in (C.13) increases and implies a lower value of \( \Upsilon \). However, it can also be shown that raising \( s_{fixed} \) implies a lower value of \( \frac{\epsilon_v}{q_{ss} + \kappa} \) and a higher value of the Nash bargaining weight \( \xi \). This effect in turn, raises the value of the second term in equation (C.13) and raises the value of \( \Upsilon \). Numerically, raising \( s_{fixed} \) fails to deliver sufficient amplification.

The most effective way of raising the impact of given a change in the real exchange rate on labour market tightness is therefore to allow for preferences that are non-separable in consumption and leisure to justify a consumption gap between the employed and unemployed household members, i.e., \( \Phi > 0 \). This approach avoids having to set the replacement ratio to unrealistically high levels and is by far more effective than raising the share of the fixed costs of bargaining in the overall vacancy posting costs, \( s_{fixed} \).

30Recall, that \( u_t \) measure the pool of job-seekers at the beginning of the period and not the unemployment rate which is measured by \( \hat{u}_t \). In the medium run, the two concepts are related via \( \hat{u}_t \approx \frac{1 - (1 - \rho)\beta_{mss}}{\beta_{mss} + \rho} \hat{u}_t \).
C.2 AOB model

As in the DMP model, we can derive an equation describing the evolution of labour market tightness for the AOB model. The bargaining sharing rule implies:

\[
J_t = \frac{\beta_1 + \beta_3}{1 + \beta_1} \left( mpl_t - b^u - \frac{\beta_2}{\beta_1 + \beta_3} \gamma_{aob} \right) + (1 - \rho) E_t \left[ \frac{\beta_2}{\beta_1 + \beta_3} \left( \frac{\lambda_t}{\lambda_t} \right) \right] J_{t+1} \\
- (1 - \rho) E_t \left[ \frac{\beta_2}{\beta_1 + \beta_3} \left( mpl_{t+1} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob} \right) \right].
\] (C.15)

We proceed as under the DMP model to obtain a better understanding how labour market tightness is affected by commodity price shocks. In the steady state, equation (C.15) implies:

\[
\frac{J^\nu}{q_{ss}} + \tilde{\kappa} \left[ 1 - (1 - \rho) \beta \left( \frac{1}{1 + \beta_1 q_{ss} \theta_{ss}} \right) \right] \left( mpl_{ss} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob} \right) = \frac{\beta_1 + \beta_3}{1 + \beta_1} \left( mpl_{ss} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob} \right).
\] (C.16)

Given the values of \( \delta_{aob} \) and \( M_{aob} \), the parameters \( \beta_1, \beta_2, \beta_3 \) are determined as demonstrated in Section 5.2. Furthermore, our calibration strategy of targeting \( q_{ss} \) and \( n_{ss} \) (and therefore \( \theta_{ss} \)) implies that \( \gamma_{aob} \) is a function of \( \kappa^\nu \) and \( \tilde{\kappa} \). The parameter \( \gamma_{aob} \) is not a free parameter.

We approximate the impact of a given change in the real exchange rate on labour market tightness over the medium term as:

\[
\hat{\theta}_t = \frac{\beta_1 + \beta_3}{1 + \beta_1} \frac{\beta_2}{\beta_3} \gamma_{aob} - \Omega_2 \frac{mpl_{ss}}{1 - (1 - \rho) \beta q_{ss} \theta_{ss} (1 - \xi)} \frac{mpl_{ss} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob}}{mpl_{ss} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob}} \left( \hat{y}^h - \hat{n}_t - \frac{1 - \nu}{\nu} \hat{e}_t \right).
\] (C.17)

where

\[
\Omega_1 = \left( \frac{\kappa^\nu}{q_{ss}} + \tilde{\kappa} \right) \left[ \frac{\beta_1 + \beta_3}{1 + \beta_1} \left( mpl_{ss} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob} \right) \right] - \xi \frac{mpl_{ss} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob}}{mpl_{ss} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob}} \left( \frac{\beta_1 + \beta_3}{1 + \beta_1} \right),
\] (C.18)

\[
\Omega_2 = \frac{\kappa^\nu}{q_{ss}} + \tilde{\kappa} \left[ \frac{\beta_1 + \beta_3}{1 + \beta_1} \left( 1 - \rho \beta \left( \frac{1}{1 + \beta_1 q_{ss} \theta_{ss}} \right) \right) \right].
\] (C.19)

As under the DMP model, the impact of a given change in the real exchange rate on labour market tightness is bigger for higher values of the replacement ratio:

\[
b^u = r^u w_{ss} = r^u \left( mpl_{ss} - (1 - (1 - \rho) \beta) \left( \frac{\kappa^\nu}{q_{ss}} + \tilde{\kappa} \right) \right).
\] (C.20)

The important question is, however, to what extent can the model amplify the response of labour market tightness for a moderate value of the replacement ratio? The free parameters in the alternating offer bargaining model are: \( \delta_{aob} \), \( M_{aob} \), and \( s^{fixed} \).31

To maximize the impact of the shock, it is advisable to:

- maximize the term \( \frac{\beta_2}{\beta_3} \gamma_{aob} \) to raise \( \frac{mpl_{ss}}{mpl_{ss} - b^u - \frac{\beta_2}{\beta_3} \gamma_{aob}} \)

31 Recall that \( s^{fixed} \) measures the share of the overall vacancy posting costs apportioned to \( q_{ss} \).
• minimize $\beta_1$ to increase the term $\Omega_2 - \frac{\beta_1}{1 + \beta_1} \beta_2 \gamma_{ab}$,
• and maximize $\beta_3$ to reduce the term $\Omega_1 - (1 - \rho)\beta q_s \theta_{ss}(1 - \zeta) \frac{\beta_3}{1 + \beta_3} \left( mpl_{ss} - b^u - \frac{\beta_3}{\beta_3} \gamma_{ab} \right)$.

Setting a low probability of breakdown in bargaining, $\delta_{aob}$ achieves the second and third point. With regard to the first point, note from equation (C.16), the term $\frac{\beta_3}{\beta_3} \gamma_{ab}$ increases in the value of $s_{fixed}$ for given $M_{aob}$ and $\delta_{aob}$.

Our estimation results confirm the conjecture that $\beta_1$ is ideally set close to zero given the calibration of the total cost of vacancy posting. Across different calibrations, our point estimate for the probability $\delta_{aob}$ is always close to zero.\(^{32}\) If $\delta_{aob} \approx 0$, the bargaining equation reduces to:

$$J_t = -\beta_2 \gamma_{ab} + \beta_3 (mpl_t - b^u) \quad \text{(C.21)}$$

or in log-linearised form:

$$\hat{\theta}_t = \frac{\beta_3 mpl_{ss}}{\zeta \frac{\gamma_{ab}}{q_s}} \left( \hat{y}_t^h - \hat{n}_t - \frac{1 - \nu}{\nu} \hat{r}_t \right) . \quad \text{(C.22)}$$

In this case, the wage sharing rule is (close to) independent of the marginal value of employment to the household, $H_t$. Equation (C.22) reveals the importance of $s_{fixed}$ in the alternating offer bargaining model. For a higher value of $s_{fixed}$, we obtain that the share of total vacancy posting costs accounted for by the variable component $\kappa v_{ss}$ is smaller. The resulting value of $\kappa v$ will also be smaller. For $s_{fixed}$ sufficiently large, the model can deliver the desired amplification of the labour market tightness response of a given real exchange rate movement. Note, that restricting $s_{fixed}$ to zero very much restricts the empirical performance of the model.

It is noteworthy, that had we assumed a much lower value of the total vacancy posting costs relative to output, $\frac{\kappa v_{ss} + q_s K}{y_t^h}$, the estimated value $\delta_{aob}$ would have been further away from zero, but $s_{fixed}$ would have been close to zero. In the case of low vacancy posting costs, the implied value of $\kappa v$ might be so small — even for $s_{fixed} = 0$ — that the amplification of labour market tightness is too big relative to the data. In this case, the coefficient $\frac{\beta_3 mpl_{ss}}{\zeta \frac{\gamma_{ab}}{q_s}}$ needs to be lowered by reducing $\beta_3$, i.e., allowing for a larger value of $\delta_{aob}$. Although in this case, $H_t$ reenters the analysis and the simplified formula in equation (C.22) no longer applies directly, the intuition just laid out applies.

The sensitivity of the parameter estimates to the calibration of the total vacancy posting costs relative to output, $\frac{\kappa v_{ss} + q_s K}{y_t^h}$, is unique to the alternating offer bargaining model. In the DMP model a drastic reduction in these costs barely influences the parameter estimates; in particular the DMP model always favours $s_{fixed}$ close to zero.

\(^{32}\)Using a different procedure, Christiano et al. (2013) also estimate the probability of breakdown in bargaining to be close to zero.