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Mismatch Shocks and Unemployment During the Great Recession

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Abstract

We investigate the macroeconomic consequences of fluctuations in the effectiveness of the labor-market matching process with a focus on the Great Recession. We conduct our analysis in the context of an estimated medium-scale DSGE model with sticky prices and equilibrium search unemployment that features a shock to the matching efficiency (or mismatch shock). We find that this shock is not important for unemployment fluctuations in normal times. However, it plays a somewhat larger role during the Great Recession when it contributes to raise the actual unemployment rate by around 1.3 percentage points and the natural rate by around 2 percentage points. The mismatch shock is the dominant driver of the natural rate of unemployment and explains part of the recent shift of the Beveridge curve.

Keywords

Search and matching frictions, Unemployment, Natural rates.

JEL Classification

E32, C51, C52

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Mismatch Shocks and Unemployment During the Great Recession*

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1 Introduction

During the Great Recession the unemployment rate in the United States increased markedly from a value of 4.5 percent in mid-2007 to a peak of 10 percent in fall 2009. Since then the labor market has recovered slowly. Nearly three years after its peak, the unemployment rate was still above 8 percent. Some policymakers have related the persistently high rate of unemployment to an increase in sectoral and geographical mismatch between the vacant jobs that are available and the workers who are unemployed (Kocherlakota 2011, among others). This view has received some support from a series of studies that identify a decline in the effectiveness of the process by which the aggregate labor market matched vacant jobs with unemployed workers during the Great Recession (Elsby, Hobijn, and Sahin 2010; Barnichon and Figura 2014, among others). In this paper, we take a general equilibrium perspective and we estimate a medium-scale New Keynesian model with search and matching frictions in the labor market using Bayesian techniques and quarterly data for eight aggregate variables. Our goal is to measure the macroeconomic consequences of the observed decline in matching efficiency – in particular, its impact on the unemployment rate and the unemployment gap.

The spirit of our exercise is quantitative. Our model features the standard frictions and shocks that help in obtaining a good fit of the macro data (Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007). In many respects, our model is similar to Gertler, Sala, and Trigari (2008) (henceforth GST) with two main differences: (i) we introduce a shock to the efficiency of the matching function (or "mismatch shock" for short) that we identify by using data on the unemployment rate and the vacancy rate; and (ii) we use the generalized hiring cost function proposed by Yashiv (2000) which combines a pre-match and a post-match component. We discuss the two deviations from GST (2008) in turn.

Matching efficiency shocks are already present in the seminal paper by Andolfatto (1996), which interprets them as sectoral reallocation shocks of the kind emphasized by Lilien (1982).[1] These shocks can be seen as the Solow residual of the matching function and as catch-all shocks for structural changes in the labor market such as the degree of skill mismatch between jobs and workers (Sahin, Song, Topa, and Violante 2014; Herz and van Rens 2015); the importance of geographical mismatch that might have been exacerbated by house-locking effects (Sterk 2015); workers' search intensity that may have been reduced by the extended duration of unemployment

[1] More recently, DSGE models featuring matching efficiency shocks have been considered by Beauchemin and Tasci (2014), Cheremukhin and Restrepo-Echevarria (2014), Justiniano and Michelacci (2011), Krause, Lopéz-Salido, and Lubik (2008) and Lubik (2013) among others.

benefits (Fujita 2011; Nakajima 2012; Zhang 2013); firms' recruiting efforts (Davis, Faberman, and Haltiwanger 2013); and shifts in the composition of the unemployment pool, such as a rise in the share of long-term unemployed, or fluctuations in participation due to demographic factors (Barnichon and Figura 2014). If these structural factors played an important role during the Great Recession, matching efficiency shocks should emerge as a prominent driver of the surge in the unemployment rate. Our goal is to quantify their contribution.

In our model, firms' hiring costs consist of a pre-match and a post-match component. The pre-match component is the search cost of advertising vacancies, a standard ingredient of models with search and matching frictions in the labor market (Pissarides 2000). The post-match component is the cost of adjusting the hiring rate. We can think of it as capturing training costs (GST 2008). As in Yashiv (2000), we combine the two hiring costs components because the nature of hiring costs is crucial for the propagation of matching efficiency shocks. In particular, when firms do not face any pre-match costs, as in GST (2008), mismatch shocks exert no effects on the unemployment rate. In contrast, the unemployment rate fluctuates significantly in response to matching efficiency shocks when firms face pre-match hiring costs only. Therefore, the share of pre-match costs in total hiring costs is a key parameter that governs the propagation of matching efficiency shocks and that we estimate in our analysis.

We find that matching efficiency shocks propagate in our model as we estimate a low but non-negligible share of pre-match hiring costs. However, these shocks do not play an important role for business cycle fluctuations. They generate a positive conditional correlation between unemployment and vacancies while the two variables are strongly negatively correlated in the data. Nevertheless, these shocks play a somewhat larger role during and after the Great Recession when matching efficiency declines substantially and unemployment and vacancies move in the same directions for few quarters. In this episode mismatch shocks explain about 1.3 percentage points of the increase in the unemployment rate, a result that is in the ballpark of the values found by studies using alternative methodologies or more disaggregated data (Barnichon and Figura 2014, Sahin, Song, Topa, and Violante 2014). Our results suggest that the bulk of the rise in the unemployment rate during the Great Recession is driven by a series of negative demand shocks, in particular risk premium shocks and investment-specific shocks. Nevertheless, negative matching efficiency shocks contribute to weaken the recovery in the aftermath of the Great Recession and to explain the shift in the Beveridge curve.

From the perspective of a monetary policymaker, looking at the drivers of the actual unemployment rate is not sufficient. As Kocherlakota (2011) puts it, monetary policy should focus on offsetting the effects of nominal rigidities. To do so, monetary policy may aim at closing the gap between the actual and the natural rate of unemployment. A big challenge for policymakers is that the natural rate is unobservable and fluctuates over time. To address this issue, we use our estimated model to infer the path of the natural rate. We define the natural rate as the counterfactual rate of unemployment that emerges in a version of the model with flexible prices and wages, constant price mark-up, and constant bargaining power, in keeping with the previous literature (Smets and Wouters 2007; Sala, Söderström and Trigari 2008; Groshenny 2013). Even though matching efficiency shocks have limited importance for fluctuations in actual unemployment, we find that these shocks are a dominant source of variation in the natural rate. This result is due to the fact that nominal rigidities dampen the propagation of matching efficiency shocks and enhance the effects of all the other shocks. We find that the deterioration in the effectiveness of the aggregate labor market matching process during the Great Recession contributes to raising the natural rate by about 2 percentage points. Hence, negative matching efficiency shocks help close the gap between the actual and the natural rate of unemployment. The model indicates that in 2013:Q2 the natural and the actual rate almost coincide slightly below 8 percent.

Our paper is related to two strands of the literature. We contribute to the literature initiated by Lilien (1982) on the importance of reallocation shocks as a source of unemployment fluctuations. Abraham and Katz (1986) and Blanchard and Diamond (1989) look at shifts in the sectoral composition of demand and estimate a series of regressions to disentangle the importance of reallocation shocks and aggregate demand shocks. Both papers emphasize the primacy of aggregate demand shocks in producing unemployment fluctuations and find that reallocation shocks are almost irrelevant at business cycle frequencies. Our contribution to this literature is the use of an estimated dynamic stochastic general equilibrium model (DSGE), rather than a reduced-form model, with a focus on the role of the nominal rigidities and the hiring cost function.

Our paper also relates to the literature that studies the output gap derived from estimated New Keynesian models (Smets and Wouters 2007; Justiniano, Primiceri, and Tambalotti 2013). Often in this literature, the labor market is modeled only along the intensive margin (hours worked). Notable exceptions are Galí, Smets and Wouters (2011) and Sala, Söderström, and Trigari (2008). Galí, Smets, and Wouters (2011) estimate a model with unemployment and also compute a measure of the natural rate. However, in their model, unemployment is due only to the presence of sticky wages (there are no search and matching frictions) so that the natural rate fluctuates only in response to wage mark-up shocks. In our model, unemployment is due to both nominal rigidities and search and matching frictions and wage mark-up shocks play a limited role. Moreover, our measure of the natural rate fluctuates in response to several shocks. Sala, Söderström, and Trigari (2008) provide a similar model-based measure of the natural rate. Their model, however, does not feature matching efficiency shocks which are, according to our estimates, prominent drivers of the natural rate.

The paper proceeds as follows: Section 2 lays out the model. Section 3 explains our econometric strategy. Section 4 discusses the transmission mechanism of mismatch shocks. Section 5 presents our empirical results. Section 6 relates to the sensitivity analysis. Finally, Section 7 concludes.

2 Model

Our model builds upon GST (2008) and Groshenny (2013) and merges the New Keynesian model with the search and matching model of unemployment. The model incorporates the standard features introduced by Christiano, Eichenbaum, and Evans (2005) to help the model fit the postwar U.S. macro data. Moreover, as in the benchmark quantitative macroeconometric model of Smets and Wouters (2007), fluctuations are driven by multiple exogenous stochastic disturbances. GST (2008) have shown that such a model fits the macro data as accurately as the Smets and Wouters (2007) model. We extend the GST (2008) set-up by including a matching efficiency shock and by using the generalized hiring cost function.

The representative household. There is a continuum of identical households of mass one. Each household is a large family, made up of a continuum of individuals of measure one. Family members are either working or searching for a job. We assume that family members pool their income before allowing the head of the family to optimally choose per capita consumption (C_t).

The family's lifetime utility is described by

$$E_t \sum_{s=0}^{\infty} \beta^s \ln \left(C_{t+s} - hC_{t+s-1} \right), \tag{1}$$

where $0 < \beta < 1$ and $h > 0$ captures internal habit formation in consumption.

The representative family enters each period $t = 0, 1, 2, ...$, with B_{t-1} bonds and $\overline{K}_{t-1}$ units of physical capital. Bonds mature at the beginning of each period, providing B_{t-1} units of money. The representative family uses some of this money to purchase B_t new bonds at nominal cost B_t/R_t, where R_t denotes the gross nominal interest rate.

The representative household owns the stock of physical capital $\overline{K}_t$ which evolves according to

$$\overline{K}_t \leq (1-\delta)\,\overline{K}_{t-1} + \mu_t \left[1 - \pounds\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{2}$$

where δ denotes the depreciation rate. The function $\pounds$ captures the presence of adjustment costs in investment. An investment-specific technology shock μ_t affects the efficiency with which consumption goods are transformed into capital. The shock follows an autoregressive process of order one as all the other seven shocks in the model.

The household chooses the capital utilization rate, u_t, which transforms physical capital into effective capital according to $K_t = u_t\overline{K}_{t-1}$. The household faces a cost $a\,(u_t)$ of adjusting the capacity-utilization rate and rents effective capital services to firms at the nominal rate r_t^K.

Each period, N_t family members are employed. Each employee works a fixed amount of hours and earns the nominal wage W_t. The remaining $(1-N_t)$ family members are unemployed and each receives nominal unemployment benefits b_t, financed through lump-sum taxes. Unemployment benefits b_t are proportional to the nominal wage along the steady-state balanced growth path $b_t = \tau W_{ss,t}$. The fact that unemployment benefits grow along the balanced growth path ensures that unemployment remains stationary. During period t, the representative household receives total nominal factor payments $r_t^K K_t + W_t N_t + (1-N_t)\,b_t$ as well as profits D_t. The family uses these resources to purchase finished goods for both consumption and investment purposes.

The family's period t budget constraint is given by

$$\begin{aligned} P_t C_t + P_t I_t + \frac{B_t}{\epsilon_{bt} R_t} \leq {} & B_{t-1} + W_t N_t + (1-N_t)\,b_t + r_t^K u_t \overline{K}_{t-1} \\ & - P_t a\,(u_t)\,\overline{K}_{t-1} - T_t + D_t. \end{aligned} \tag{3}$$

As in Smets and Wouters (2007), the shock ϵ_{bt} drives a wedge between the central bank's instrument rate R_t and the return on assets held by the representative family. This shock captures disturbances originating in the financial markets, unlike the shock to the discount factor used in GST (2008).

The representative intermediate goods-producing firm. Each intermediate goods-producing firm $i \in [0,1]$ enters in period t with a stock of $N_{t-1}(i)$ employees. Before production starts, $\rho N_{t-1}(i)$ old jobs are destroyed. The job destruction rate ρ is constant. Those workers who have lost their jobs start searching immediately and can potentially still be hired in period t (Ravenna and Walsh 2008). Employment at firm i evolves according to $N_t(i) = (1-\rho) N_{t-1}(i) + m_t(i)$, where the flow of new hires $m_t(i)$ is given by $m_t(i) = q_t V_t(i)$. $V_t(i)$ denotes vacancies posted by firm i in period t and q_t is the aggregate probability of filling a vacancy, $q_t = \frac{m_t}{V_t}$, where $m_t = \int_0^1 m_t(i)\, di$ and $V_t = \int_0^1 V_t(i)\, di$ denote aggregate matches and vacancies respectively. This specification implies that employment is not a predetermined variable (as in GST, 2008) and delivers higher unemployment volatility. Aggregate employment $N_t = \int_0^1 N_t(i)\, di$ evolves according to

$$N_t = (1-\rho) N_{t-1} + m_t. \tag{4}$$

The matching process is described by an aggregate constant-returns-to-scale Cobb-Douglas matching function,

$$m_t = \zeta_t S_t^{\sigma} V_t^{1-\sigma}, \tag{5}$$

where S_t denotes the pool of job seekers in period t, $S_t = 1-(1-\rho) N_{t-1}$, and ζ_t is a time-varying scale parameter that captures the efficiency of the matching technology. It evolves exogenously following the autoregressive process,

$$\ln \zeta_t = (1-\rho_\zeta) \ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}, \tag{6}$$

where ζ denotes the steady-state matching efficiency and $\varepsilon_{\zeta t}$ is $i.i.d.N\left(0, \sigma_\zeta^2\right)$. Aggregate unemployment is defined by $U_t \equiv 1 - N_t$.

Firms face hiring costs $H_t(i)$ measured in terms of the finished good and given by a generalized hiring function proposed by Yashiv (2000) that combines a pre-match and a post-match component in the following way,

$$H_t(i) = \frac{\kappa}{2} \left(\frac{\phi_V V_t(i) + (1-\phi_V) m_t(i)}{N_t(i)} \right)^2 Y_t, \tag{7}$$

where κ determines to the output-share of hiring costs and $0 \leq \phi_V \leq 1$ governs the relative importance of the pre-match component. When ϕ_V is equal to 0 we are back to the model with only post-match hiring costs (GST 2008). Instead, when ϕ_V is equal to 1 we obtain a model with quadratic pre-match hiring costs (Pissarides 2000). Interestingly, the empirical literature has so far preferred a specification with post-match hiring costs, that can be interpreted as training costs.

Each period, firm i combines $N_t(i)$ homogeneous employees with $K_t(i)$ units of efficient capital to produce $Y_t(i)$ units of intermediate good i according to the constant-returns-to-scale technology described by

$$Y_t(i) = A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{1-\alpha}. \tag{8}$$

A_t is an aggregate labor-augmenting technology shock whose growth rate, $z_t \equiv A_t/A_{t-1}$, follows an exogenous process.

Intermediate goods substitute imperfectly for one another in the production function of the representative finished goods-producing firm. Hence, each intermediate goods-producing firm $i \in [0,1]$ sells its output $Y_t(i)$ in a monopolistically competitive market, setting $P_t(i)$, the price of its own product, with the commitment of satisfying the demand for good i at that price. Each intermediate goods-producing firm faces costs of adjusting its nominal price between periods, measured in terms of the finished good and given by

$$\frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 Y_t. \tag{9}$$

The term ϕ_P governs the magnitude of the price adjustment cost. The expression $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross rate of inflation in period t. The steady-state gross rate of inflation is denoted by $\pi > 1$ and coincides with the central bank's target. The parameter $0 \leq \varsigma \leq 1$ governs the importance of backward-looking behavior in price setting (Ireland 2007).

We model nominal wage rigidities as in Arseneau and Chugh (2008). Each firm faces quadratic wage-adjustment costs which are proportional to the size of its workforce and measured in terms of the finished good,

$$\frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i)\, Y_t, \tag{10}$$

where ϕ_W governs the magnitude of the wage adjustment cost. The parameter $0 \leq \varrho \leq 1$ governs the importance of backward-looking behavior in wage setting. Firms take the nominal wage as given when maximizing the discounted value of expected future profits. We use quadratic adjustment costs in prices and wages instead of staggered time-dependent contracts as in GST (2008) to simplify the model in some dimensions that are not essential for our analysis.

Wage setting. The nominal wage $W_t(i)$ is determined through surplus sharing,[2]

$$W_t(i) = \arg\max \left(\Delta_t(i)^{\eta_t} J_t(i)^{1-\eta_t} \right). \tag{11}$$

The worker's surplus, expressed in terms of final consumption goods, is given by

$$\Delta_t(i) = \frac{W_t(i)}{P_t} - \frac{b_t}{P_t} + \beta\chi E_t (1 - s_{t+1}) \frac{\Lambda_{t+1}}{\Lambda_t} \Delta_{t+1}(i), \tag{12}$$

where $\chi \equiv 1 - \rho$. Λ_t denotes the household's marginal utility of wealth and $s_t = m_t/S_t$ is the aggregate job finding rate. The firm's surplus in real terms is given by

$$\begin{aligned} J_t(i) &= \xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t \\ &+ \frac{\kappa}{N_t(i)}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 Y_t + \beta\chi E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1}(i), \end{aligned} \tag{13}$$

where $\xi_t(i)$ denotes firm i's real marginal cost. The worker's bargaining power η_t evolves exogenously and $0 < \eta < 1$ denotes the steady-state worker's bargaining power.

The representative finished goods-producing firm. During each period $t = 0, 1, 2, ...,$ the representative finished good-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0, 1]$, purchased at the nominal price $P_t(i)$, to produce Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$\left(\int_0^1 Y_t(i)^{(\theta_t - 1)/\theta_t} di \right)^{\theta_t/(\theta_t - 1)} \geq Y_t, \tag{14}$$

where $\theta > 1$ is the demand elasticity and θ_t is an exogenous process for the demand elasticity that translates in exogenous variations in the price markup.

Monetary and fiscal authorities. The central bank adjusts the short-term nominal gross

[2] In the presence of large firms with decreasing returns to labor, the solution of the bargaining problem between workers and firms should take into account intra-firm bargaining. We abstract from considering those issues since Krause and Lubik (2013) show that the effects of intra-firm bargaining on business cycle fluctuations are small.

interest rate R_t by following a Taylor-type rule similar to the one proposed by Justiniano, Primiceri and Tambalotti (2013):

$$\ln \frac{R_t}{R} = \rho_r \ln \frac{R_{t-1}}{R} + (1-\rho_r)\left(\rho_\pi \ln \frac{(P_t/P_{t-4})^{1/4}}{\pi} + \rho_y \ln \frac{(Y_t/Y_{t-4})^{1/4}}{z}\right) + \ln \epsilon_{mpt}. \tag{15}$$

The degree of interest-rate smoothing ρ_r and the reaction coefficients ρ_π and ρ_y are all positive. The monetary policy shock ϵ_{mpt} follows an exogenous process.

The government budget constraint takes the form,

$$P_t G_t + (1-N_t)\, b_t = \left(\frac{B_t}{R_t} - B_{t-1}\right) + T_t, \tag{16}$$

where T_t denotes total nominal lump-sum transfers. Public spending is an exogenous time-varying fraction of GDP, $G_t = \left(1 - \frac{1}{\epsilon_{gt}}\right) Y_t$, where ϵ_{gt} follows an exogenous process.

Details on the first order conditions, the log-linearized system, the calibration, solution and estimation of the model are provided in the Online Appendix.

3 Econometric Strategy

We calibrate 13 parameters that we report in Table 1. Since these values follow the previous literature, we focus here only on the parameters related to the labor market. The vacancy-filling rate is set equal to 0.70, which is just a normalization. Calibrated values for the steady state quarterly separation rate range in the literature from 0.05 in Krause, Lopéz-Salido, and Lubik (2008) to 0.15 in Andolfatto (1996). We use the conventional value 0.085, in line with most of the literature (Yashiv 2006). We set the elasticity of the matching function with respect to unemployment at 0.65 in the middle of the range of values estimated in five recent studies (Barnichon and Figura 2014; Justiniano and Michelacci 2011; Lubik 2013; Shimer 2005; Sedlacek 2014). The calibration of the replacement rate at 0.4 is a conservative choice based on Shimer (2005) and Yashiv (2006).

We estimate the remaining 27 parameters using Bayesian techniques. Our priors, summarized in Tables 2 and 3, are standard (Smets and Wouters 2007; GST 2008). The estimation period is 1957:Q1–2008:Q3. Following Galí, Smets and Wouters (2011) we stop our sample period in the beginning of the Great Recession to prevent our estimates from being distorted by the zero-lower bound and by other non-linearities. Nevertheless, we rely on the estimated parameters to simulate the model using data until 2013:Q2 to discuss the behavior of the aggregate variables in the recent turbulent years; that is, beyond the sample period.

The model includes as many shocks as observables. The estimation uses quarterly data on eight key macro variables. We downloaded seven series from the FREDII database maintained by the Federal Reserve Bank of St. Louis. We measure nominal consumption using data on nominal personal consumption expenditures of nondurables and services. Nominal investment corresponds to the sum of personal consumption expenditures of durables and gross private domestic investment. Nominal output is measured by nominal GDP. Per capita real GDP, consumption, and investment are obtained by dividing the nominal series by the GDP deflator and population. Real wages correspond to nominal compensation per hour in the nonfarm business sector, divided by the GDP deflator. Consistently with the model, we measure population by the labor force which is the sum of official unemployment and official employment. The unemployment rate is the official unemployment divided by the labor force. Inflation is the first difference of the log of the GDP deflator. The nominal interest rate is measured by the effective federal funds rate.

Our eighth observable variable is the vacancy rate. As in Justiniano and Michelacci (2011), data on job vacancies are used to construct the vacancy rate as the ratio of job vacancies over the sum of job vacancies and employment, consistent with the definition of job opening rate used in JOLTS. The series for job vacancies is taken from Barnichon (2010) who constructs (and updates regularly) a new vacancy index that combines the print Help-Wanted Index with the online Help-Wanted index published by the Conference Board since 2005. The series tracks closely the (rescaled) JOLTS measure of job openings that starts in December 2000. As emphasized by Shimer (2005), a shortcoming of the print Help-Wanted index is that it is subject to low frequency fluctuations that are related only tangentially to the labor market. On the one hand, the Internet may have reduced the reliance of firms on using advertising in newspapers well before 2005. On the other hand, Shimer (2005) describes how a series of newspaper consolidations and Equal Opportunity laws may have encouraged firms to rely more extensively on newspaper advertising in the first part of the sample. Therefore, to remove these secular shifts we follow Shimer (2005), Justiniano and Michelacci (2011), Davis, Faberman and Haltiwanger (2013) and we detrend the vacancy rate series using an HP filter with smoothing weight equal to 10^6.

In Tables 2 and 3 we report the outcome of our estimation exercise. Since most estimates are in line with the previous literature, we concentrate our attention on the parameters related to the labor market. The weight of the pre-match component in the convex combination ϕ_V

is estimated at 0.35 at the posterior median. Although we use an agnostic prior centered around 0.5, the data favor a large post-match component, as it has been found in the previous literature. In fact, Yashiv (2000) estimates a value of 0.3 on Israeli data using the same functional form. Sala, Söderström and Trigari (2013) and Christiano, Trabandt and Walentin (2011) use a different functional form and find a slightly lower weight on the pre-match component using US and Swedish data respectively. The posterior mode of steady state hiring costs as a percent of output is estimated at 0.25 percent. This corresponds to 4.5 percent of total wages of newly hired workers, thus inside the range between 4 and 14 percent documented by Silva and Toledo (2009). Christiano, Eichenbaum and Trabandt (2014) estimate a value of 0.5 percent.

Our model provides a rather conventional view on business cycle fluctuations over the sample period, as reported in the infinite horizon variance decomposition in Table 4. The relevant sources of output fluctuations in the model are neutral technology shocks, investment-specific technology shocks, and risk-premium shocks. Our results are consistent with GST (2008) once we take into account that the risk premium shock limits somewhat the importance of the investment specific technology shock. A less conventional implication of our model is that wage-bargaining shocks do not matter for output fluctuations. This result was already present in GST (2008) but, as far as we know, it has not been commented in the literature. Chari, Kehoe, and McGrattan (2009) have criticized the New Keynesian model for its reliance on dubiously structural shocks such as the wage-bargaining (or wage mark-up) shock. Here, we find that this criticism does not apply. Our finding suggests that search and matching frictions in the labor market, and the use of labor market variables in the estimation, absorb the explanatory power of the wage-bargaining shock. Put differently, our estimated DSGE model seems successful at endogenizing the labor wedge.

4 Inspecting the Mechanisms: the Role of Nominal Rigidities and Hiring Costs

In this section we concentrate on the macroeconomic effects of matching efficiency shocks. To set the scene we consider first a model with pre-match hiring costs only ($\phi_V = 0.99$) and flexible prices and wages (dashed lines in Figure 1). All the other parameters are set equal to their posterior mode estimates. In Figure 1 we plot impulse responses to a negative mismatch shock. When matching efficiency declines, the probability of filling a vacancy drops and hiring becomes more expensive since more vacancies have to be posted to hire a worker. In response to the increase in hiring costs, firms hire fewer workers and, given the assumption of instantaneous hiring, employment and output decline already on the impact of the shock while unemployment increases. A lower probability of filling a vacancy increases the hiring cost and that per se would lead to lower vacancy posting. There is a second effect, however. In the presence of a lower matching efficiency, forming a match is more costly and takes longer time such that being in a match becomes more valuable. As the total surplus of being in a match increases, firms tend to post more vacancies. The two effects almost compensate and the vacancy rate barely moves in this version of our model. The larger surplus of being in a match is divided between workers and firms and, as long as workers have some bargaining power, the real wage increases despite the decline in the job finding rate. Finally, higher wages and higher hiring costs lead to an increase in prices in order to maintain the real marginal cost constant, as it is optimal under flexible prices.

The solid lines in Figure 1 refer to the same model in the presence of sticky prices and wages in which the parameters related to nominal rigidities are set at their posterior mode value (as all the other parameters except ϕ_V). In this case firms cannot increase prices optimally to restore their profits impaired by the increase in costs. Prices increase less than in the flexible price case, the fall in aggregate demand is less pronounced and the contraction in hiring is more limited. This leads to higher vacancy posting. We conclude that the presence of nominal rigidities reduces the contractionary effects of the shock and generates a positive comovement between unemployment and vacancies.

In Figure 2 we vary the share of pre-match hiring costs while leaving all the other parameters at their posterior mode value. The dashed-thin lines represent the case with pre-match hiring costs only whereas the solid line refers to our baseline model with all parameters at their posterior mode value. This latter model features a large post-match component in total hiring costs ($\phi_V = 0.32$) and posting vacancies is relatively inexpensive. Firms now post more vacancies in order to avoid fluctuations in the hiring rate that are costly. Post-match hiring costs reinforce the effects generated by nominal rigidities and lead to more vacancy posting and lower effects on output and unemployment.

The dashed-bold lines in Figure 2 refer to a model with almost only post-match hiring costs ($\phi_V = 0.01$) with all the other parameters fixed at the posterior mode value: in this case posting vacancies is almost costless and firms react by posting so many more vacancies that they are able to undo the effects of the shock and output and unemployment are barely affected. In this extreme case, that corresponds to the GST (2008) model, output and unemployment are invariant to mismatch shocks. In other words, in the absence of a pre-match component a decline in matching efficiency has no effects on the macroeconomy. The search frictions are inactive and the firm has a perfect control over the hiring rate that can be constantly achieved by posting for free more or fewer vacancies. This tight link between the share of pre-match hiring costs and the propagation of mismatch shocks is the rationale for having both a pre-match and a post-match hiring cost in our model and let the data choose the appropriate weight by estimating the parameter ϕ_V.

5 Empirical Results

In this section we use our model to investigate the importance of mismatch shocks over the sample period and, more specifically, beyond the sample period during and after the Great Recession. We then provide a model based estimate of the natural rate of unemployment.

5.1 Mismatch Shocks, Unemployment and Vacancies

The estimated series for mismatch shocks is plotted in Figure 3: it reaches its minimum in the beginning of the 80s and then it starts rising around 1985 until 2002 when it peaks. The improvement in matching efficiency could reflect the firms more widely adopting information technologies (the so-called New Economy). After 2002 matching efficiency declines, with a substantial acceleration during the Great Recession, and stays at uneprecedentedly low levels in recent years. Barnichon and Figura (2014) estimate matching efficiency by regressing the job finding rate on the labor market tightness and find similar results, although they identify an even larger decline in the recent years.

Matching efficiency shocks explain only 6 percent of unemployment volatility (cf. Table 4) whereas they are almost irrelevant for output fluctuations over the sample period (1957:Q1-2008:Q3). This is not so surprising since mismatch shocks generate a large positive correlation between unemployment and vacancies whereas in the data the two series are strongly negatively correlated. A limited importance of matching efficiency shocks is consistent with fact that the data favor a substantial degree of nominal rigidities and a limited share of pre-match hiring costs and, as we have discussed in the previous section, both features tend to dampen the propagation of the shocks. Moreover, the Great Recession, a period of large fluctuations in matching efficiency, is not part of the sample period for estimation, thus maintaining low the share of variance explained by mismatch shocks.

The limited importance of mismatch shocks for business cycle fluctuations in general does not rule out that these shocks may play a relevant role in specific episodes, in particular when unemployment and vacancies move in the same direction, as in the aftermath of the Great Recession. We now make use of our estimated model to discuss the dynamics of aggregate variables over the period 2008:Q4-2013:Q2. In Figure 4 we plot the historical decomposition of the unemployment rate where the bold line represents the variable in deviation from its mean and the bars above (below) the zero-line refer to the cumulative effect of shocks that increase (decrease) unemployment in a specific quarter. Since 2009, negative mismatch shocks are responsible on average for about 1.3 percentage points of the large increase in the unemployment rate.[3] The contribution of these shocks is limited in the most acute phase of the crisis but is more relevant in the slow phase of recovery. This result is in line with other studies. Barnichon and Figura (2014) decompose movements in the Beveridge curve and conclude that without any loss in matching efficiency, unemployment would have been about 150 points lower in late 2010. Sedlacek (2014) finds results very similar to ours. Sahin, Song, Topa, and Violante (2014) confine their attention to the more narrow concept of mismatch unemployment. They combine disaggregated data to construct a mismatch index and they find that mismatch unemployment at the 2-digit industry level can account for 0.75 percentage points out of the 5.4 increase in the U.S. unemployment rate from 2006 to the Fall 2009. This result is compatible with our evidence, given that mismatch is not the only driver of matching efficiency.

From Figure 4 we see that the large increase in unemployment during the Great Recession is explained by a series of large negative demand shocks like risk-premium shocks (in particular during 2009) and investment shocks. Fiscal policy shocks have contributed materially to lower unemployment, reflecting the effects of the fiscal stimulus package implemented by the U.S. authorities in the aftermath of the crisis. Finally, we find that negative bargaining power shocks (that is, a reduction in the bargaining power of workers) have contributed to lowering the unemployment rate throughout the recent years. This finding, which is not specific to the Great Recession period, may reflect competitive pressures from abroad and threats of offshoring from the domestic market. Arseneau and Leduc (2012) show how the threat to offshore can have large effects on wages even when the actual amount of offshoring in the economy is small.

[3] Sala, Söderstöm, and Trigari (2013) conduct the same experiment in a similar model with a focus on a cross-country comparison. They find results that are in line with ours for the United States.

An alternative way to evaluate the role of mismatch shocks during the Great Recession is to consider the Beveridge curve. In Figure 5 the grey dots describe the joint evolution of unemployment and vacancies in the data over the period 2008:Q1-2013:Q2. The variables are expressed in percentage deviation from their value in 2008:Q1, when the unemployment rate was at 5 percent. The black solid line connects the counterfactual values for unemployment and vacancies obtained from our model when we turn off only matching efficiency shocks. We see that the combination of the remaining seven shocks can replicate the shape of the Beveridge curve, thus showing that standard shocks can generate shifts in the Beveridge curve by themselves. However, matching efficiency shocks are essential to match the shift from a quantitative point of view, in particular towards the end of the sample when the gap between the two loops widens. In particular, in the last quarter of our sample (2013:Q2) unemployment was around 7.5 (more than 40 percent higher than its value in 2008:Q1). The counterfactual in absence of matching efficiency shocks would predict a value of around 5.5 percent (only 10 percent higher than its value in 2008:Q1). These results are interesting in light of a recent paper by Christiano, Eichenbaum and Trabandt (2014) who show that a model similar to ours can generate a shift in the Beveridge curve and explain the data without matching efficiency shocks. Here we confirm their result but we show also that, once matching efficiency shocks are introduced, they are important to explain unemployment dynamics during the Great Recession and even more in its aftermath, unlike in other periods when they are often irrelevant. Lubik (2013) finds also that matching efficiency shocks are important to explain the Beveridge curve dynamics in a model with flexible prices and wages driven by technology and matching efficiency shocks. We show that they play a role also in the presence of nominal rigidities and several additional shocks.

5.2 Mismatch Shocks and the Natural Rate

The natural rate of unemployment is a concept often used by policy makers to compute measures of slack in the labor market. It constitutes a reference level of unemployment that emerges in the absence of monetary frictions and that moves with real forces (Friedman, 1968). The natural rate is unobservable and its estimation is a main challenge for monetary policymakers. In this section, we use our estimated medium-scale DSGE model to infer the path of the natural rate and, unlike in the previous literature, we discuss the role of mismatch shocks in its dynamics. Following Sala, Söderström, and Trigari (2008), Groshenny (2013) and the related literature on the output gap in DSGE models, we define the natural rate to be the unemployment rate that would prevail if i) prices and wages were perfectly flexible and ii) the markup of price over marginal cost and the bargaining power of workers were constant.

We adopt the standard practice of turning off the inefficient shocks to compute the natural rate.[4] Price mark-up shocks and bargaining power shocks are inefficient. The former ones affect the degree of imperfect competition in the goods market. The latter shocks induce deviations from the Hosios condition and so affect the severity of the congestion externality that characterizes the labor market in the search and matching model. This standard definition is in line with the concept of natural rate expressed in Friedman (1968), i.e. a measure of unemployment that fluctuates over time in response to shocks and that is independent from monetary factors. Moreover this definition is also shared by some monetary policymakers. For example, it is consistent with Kocherlakota (2011)'s view of the Fed's mission.[5]

In Figure 6 we plot the observed unemployment rate together with our estimate of the natural rate. If we focus on the very low frequencies, we see that the natural rate was gently trending upward until 1980, and then had been gradually decreasing, reaching a trough around 2003. Our estimate of the natural rate is rather precisely estimated, in contrast with Staiger, Stock, and Watson (1997) who argue that large confidence bands are a distinguishing feature of the natural rate. Not so surprisingly, we find that the cross-equation restrictions embedded in our estimated DSGE model provide quite a sharp identification strategy of the unobserved natural rate.

Interestingly, according to our model actual unemployment was well below the natural rate over the period 2005–2007. During the Great Recession the posterior median estimate of the natural rate rises sharply but reaches its peak as late as in the beginning of 2013. Therefore, while actual unemployment has been declining since 2010, the natural rate has been increasing until the end of our sample. How can we rationalize this diverging behavior between the actual and the natural rate? The answer is in Figure 7 where we see that large negative matching efficiency shocks (that have a more limited impact on the actual rate as shown in Figure 4) lead to an increase in the natural rate of almost 2 percentage points since the beginning of the Great

[4] The shocks affecting the natural rate are technology, investment-specific, fiscal and matching efficiency shocks. Monetary and risk premium shocks leave the natural rate unaffected because they do not propagate under flexible prices and wages.

[5] Our approach is common in the literature but is not uncontroversial. In particular, the interpretation of labor supply shocks in the New Keynesian model is the object of a recent literature (Chari, Kehoe, and McGrattan 2009; Galí, Smets, and Wouters 2011; Justiniano, Primiceri, and Tambalotti 2013) but is outside the scope of our paper. Note, however, that according to our estimates, wage bargaining shocks are almost white noise. This finding is in keeping with the interpretation of wage markup shocks as measurement errors that is favored by Justiniano, Primiceri, and Tambalotti (2013).

Recession with larger effects in recent years.

More generally, we remark that mismatch shocks are the dominant source of variation in the natural rate. Why are mismatch shocks so important for the natural rate when they have a more limited effect on the actual rate? Simply because, as already highlighted in Figure 1, mismatch shocks propagate more under flexible prices and wages. The dashed line in the fourth panel of Figure 1 represents in fact the impulse response of the natural rate of unemployment. While the natural rate reacts more than the actual rate to mismatch shocks, the opposite is true for the other shocks (neutral technology, investment-specific, and government spending shocks) that propagate little under flexible prices and wages, as shown by Shimer (2005) and in the following literature on the so-called unemployment volatility puzzle. The important role of mismatch shocks for the natural rate dynamics is a new result in the literature. The mismatch shock captures variations in structural factors (like mismatch, changes in the composition of the unemployment pool, search intensity, and demographic factors, among others) and these factors are the drivers of the natural rate in the spirit of Friedman (1968) definition.

This analysis of the natural rate of unemployment has important policy implications, at least if the Fed's focus is on offsetting the effects of nominal rigidities. According to our model, expansionary policies during the Great Recession were justified by an unemployment gap (defined as the difference between the actual rate and the natural rate) that increased from minus 1 percent to 3 percent as we see in Figure 8. All in all, our results are consistent with the view that the large increase in unemployment during the Great Recession was largely due to cyclical factors whereas structural factors have contributed only to some extent. Nevertheless, negative matching efficiency shocks play a larger role in recent years in slowing down the recovery (see Figures 4 and 5) and in closing the unemployment gap which is almost at zero at the end of our sample in 2013:Q2 (see Figure 8).

6 Sensitivity Analysis

We now evaluate the robustness of our results by considering some extensions that we summarize in Figure 9 where we plot a counterfactual historical decomposition for unemployment over the period 2008:Q1-2013Q2 in the absence of mismatch shocks. We compare these extensions to our baseline model (thin-solid line) and to the data (bold-solid line).[6]

[6] Several additional figures related to the sensitivity analysis and a more detailed discussion are reported in the Online Appendix.

Different sample period. In the first set of experiments we change the sample period used for estimation. We first extend it until 2013:Q2, to use information on the recent Beveridge curve's shift for estimation purposes, and then we limit it to the Great Moderation period (1984:Q1-2008:Q3) to rule out structural breaks' concerns. In both cases all our results are confirmed. Mismatch shocks are slightly more important when the sample period for estimation is limited to the Great Moderation period (purple-solid line).

Alternative calibration for σ. In the second set of experiments we change the calibrated value for the elasticity of the matching function to unemployment (σ) and we reestimate the model over the longest sample (1957Q1-2013Q2). We consider two values at the extremes in the range of recent estimates ($\sigma = 0.55$ and $\sigma = 0.75$). While all our results are broadly confirmed, the calibration of σ matters for the importance of mismatch shocks in generating the shift in the Beveridge curve. With σ equal to 0.55, the matching efficiency process becomes more countercyclical at business cycle frequencies and mismatch shocks are less important in explaining the recent shift in the Beveridge curve and unemployment dynamics in recent years (dotted line in Figure 9). With σ equal to 0.75, matching efficiency always declines in recessions and mismatch shocks are more important to explain unemployment dynamics (red-dashed line in Figure 9).

Alternative calibration for τ. In the third experiment we re-estimate the model using a value for the replacement rate of 0.7 instead of 0.4. High values for the workers' outside option have been often used in the recent literature since they favor a larger response of unemployment to technology shocks in models with flexible prices and wages. This is the case also in our model, as technology and investment-specific shocks have now a larger effect on the natural rate. Nevertheless, matching efficiency shocks remain the main (although not exclusive) drivers of the natural rate and their effect on the unemployment rate (orange-dashed line in Figure 9) are just slightly lower than in our baseline model.

Time-varying separation rate. In the fourth set of experiments we consider time variation in the separation rate in contrast to our baseline model where we assume a constant separation rate. While this is a convenient assumption for modeling purposes, the separation rate into unemployment is countercyclical in the data. Models with endogenous separation generate this countercyclicality in the separation rate but also a counterfactual positive correlation between unemployment and vacancies, unless search on the job is modeled explicitly, as shown by Fujita and Ramey (2012). Incorporating endogenous separation and search on the job in our set-up is a promising idea for future research that is, however, beyond the scope of the current paper. Despite not modeling endogenous separation explicitly, we nevertheless propose an extension in which the separation rate is time-varying and reacts to the state of the economy. We assume that the separation rate is negatively related to technology (ε_{zt}) and investment-specific ($\varepsilon_{\mu t}$) shocks that are the two main drivers of business cycle fluctuations in our model. It evolves according to the following specification:

$$\ln \rho_t = (1 - \rho_\rho) \ln \rho + \rho_\rho \ln \rho_{t-1} - \delta_z \varepsilon_{zt} - \delta_\mu \varepsilon_{\mu t} + \varepsilon_{\rho t}$$

where we impose in the estimation that δ_z and δ_μ have to be positive and $\varepsilon_{\rho t}$ represents a separation shock. We estimate the model with time-varying separation using data on the separation rate to unemployment from Elsby, Hobijn and Sahin (2015) as an additional observable variable.[7] In this extension we thus have nine shocks and nine observables. Mismatch shocks now coexist with separation shocks and both disturbances can potentially move unemployment and vacancies in the same direction, as discussed in Shimer (2005) among others. In this extension of our model, mismatch shocks are still important for unemployment dynamics in recent years, thus contributing to the slow recovery, but become almost irrelevant in the most acute phase of the Great Recession (blu-dashed-dotted line in Figure 9). In the Online Appendix we show that the estimated natural rate (that now is affected also by separation shocks) changes only marginally with respect to our baseline model, although investment-specific and separation shocks explain now a relevant share of its fluctuations. Furthermore, the large increase in the separation to unemployment during the Great Recession is explained mainly by negative investment-specific shocks and not by exogenous separation shocks. Finally, we re-estimate the model by considering the case of a purely exogenous separation rate (by imposing that δ_z and δ_μ are equal to zero). Not surprisingly, separation shocks become more important but mismatch shocks still play a non-negligible role in recent years (yellow-dashed-dotted line in Figure 9).

7 Conclusion

In this paper we identify a substantial decline in matching efficiency during the Great Recession and we investigate the macroeconomic consequences of this phenomenon in the context of a New

[7] Elsby, Hobijn and Sahin (2015) provide an updated series for the transition probability from employment to unemployment corrected for margin error based on Current Population Survey data. The data series starts in 1968:Q1. Therefore, the sample period in this set of experiments is 1968:Q1-2008Q3.

Keynesian model with search and matching frictions extended with matching efficiency shocks and a generalized hiring cost function. We find that the estimated decline in matching efficiency raises the actual unemployment rate by around 1.3 percentage points and the natural rate by 2 percentage points during the Great Recession. In normal times mismatch shocks are almost irrelevant for business cycle fluctuations but, nevertheless, these can play a role in periods when unemployment and vacancies comove. We find that mismatch shocks are the dominant driver of the natural rate and are thus crucial to obtain a reliable estimate of it. Ignoring mismatch shocks, as in a large part of the previous literature, is perhaps not crucial along some dimensions (Christiano, Eichenbaum and Trabandt, 2014) but is definitely problematic when considering the natural rate and the related policy implications.

Finally, we want to underscore that matching efficiency shocks have a broad interpretation. We see them as catch-all disturbances that soak up changes in various features of the aggregate labor market, not only mismatch. Like the Solow residual of the neo-classical production function, matching efficiency is likely to incorporate a non negligible endogenous component. For example, search intensity by workers and firms may play a nontrivial role, as does variable capacity utilization in the production function. Our paper is only a first step in the identification of structural factors in the labor market. More generally, we believe there is scope for future research on how to "purify" the matching function's Solow residual, as has been done for the production function (for recent advances Barnichon and Figura 2014; Borowczyk-Martins, Jolivet, and Postel-Vinay 2013; and Sedlacek 2014).

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Table 1: Calibrated Parameters

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0578
Quarterly gross growth rate	z	1.0039
Quarterly gross inflation rate	π	1.0088
Quarterly gross nominal interest rate	R	1.0139

Table 2: Priors and Posteriors of Structural Parameters

			Posteriors		
		Priors	5%	Median	95%
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.20	0.35	0.52
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.17	2.46	2.79
Habit in consump.	h	Beta (0.7,0.1)	0.59	0.65	0.69
Invest. adj. cost	ϕ_I	IGamma (5,1)	2.60	3.10	3.85
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.47	0.63	0.88
Price adjust. cost	ϕ_P	IGamma (60,10)	52.40	60.50	69.30
Wage adjust. cost	ϕ_W	IGamma (150,25)	111.50	135.90	171.60
Wage indexation	ϱ	Beta (0.5,0.2)	0.87	0.94	0.98
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.25	0.34	0.43
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.73	1.94	2.21
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.39	0.48	0.58

Table 3: Priors and Posteriors of Shock Parameters

			Posteriors		
		Priors	5%	Median	95%
Technology growth	ρ_z	Beta (0.3,0.1)	0.22	0.29	0.36
	$100\sigma_z$	IGamma (0.1,3)	1.18	1.27	1.37
Monetary policy	ρ_{mp}	Beta (0.5,0.2)	0.69	0.79	0.86
	$100\sigma_{mp}$	IGamma (0.1,3)	0.19	0.21	0.23
Investment	ρ_μ	Beta (0.5,0.2)	0.72	0.77	0.82
	$100\sigma_\mu$	IGamma (0.1,3)	5.28	6.17	7.43
Risk premium	ρ_b	Beta (0.5,0.2)	0.58	0.67	0.75
	$100\sigma_b$	IGamma (0.1,3)	0.58	0.82	1.12
Matching efficiency	ρ_ζ	Beta (0.5,0.2)	0.88	0.92	0.96
	$100\sigma_\zeta$	IGamma (0.1,3)	2.16	2.31	2.50
Price markup (rescaled)	ρ_{θ^*}	Beta (0.5,0.2)	0.90	0.93	0.96
	$100\sigma_{\theta^*}$	IGamma (0.1,3)	0.09	0.10	0.11
Bargaining power (rescaled)	ρ_{η^*}	Beta (0.5,0.2)	0.09	0.18	0.27
	$100\sigma_{\eta^*}$	IGamma (0.1,3)	130.60	161.35	204.15
Government spending	ρ_g	Beta (0.7,0.1)	0.91	0.93	0.95
	$100\sigma_g$	IGamma (0.1,3)	0.44	0.47	0.51

Table 4: Variance Decomposition (in %)

	Output	Unemp.	Vacancy	Inflation
Technology	41.2	11.8	15.4	16.8
Monetary	3.7	4.8	5.1	9.9
Investment	18.1	26.6	26.9	50.6
Matching	0.3	5.8	1.8	0.1
Risk-premium	15.6	14.5	18.0	15.5
Markup	4.6	19.8	14.0	2.6
Bargaining	2.6	12.9	9.0	1.5
Fiscal	13.9	3.8	9.9	3.0

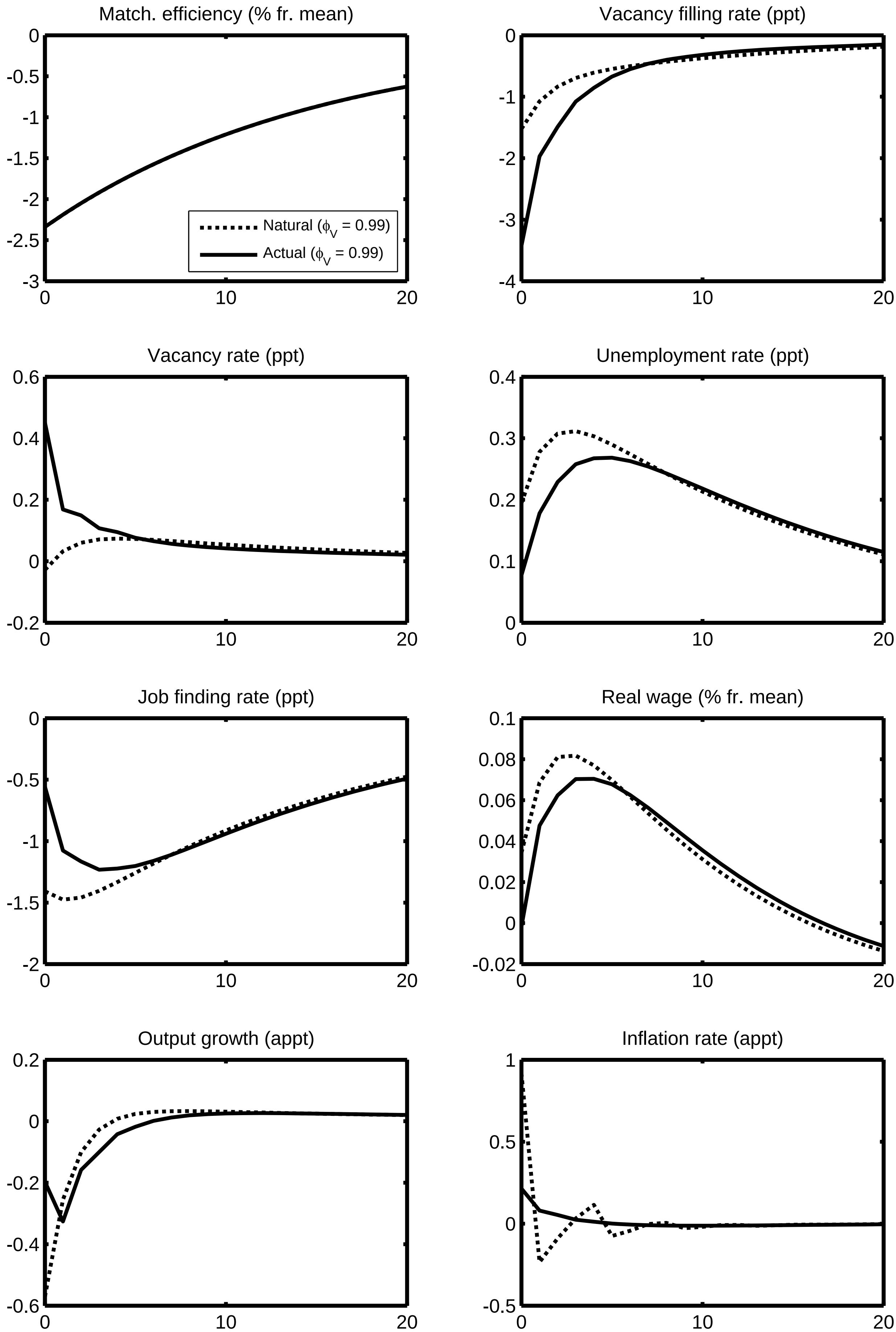


Figure 1: Impulse responses to a one-standard-deviation negative matching efficiency shock in the actual economy with nominal rigidities and in the counterfactual economy with no nominal rigidities. In both cases, the weight on pre-match hiring cost ϕ_V is set equal to 0.99. All other parameters are set equal to their posterior mode estimates.

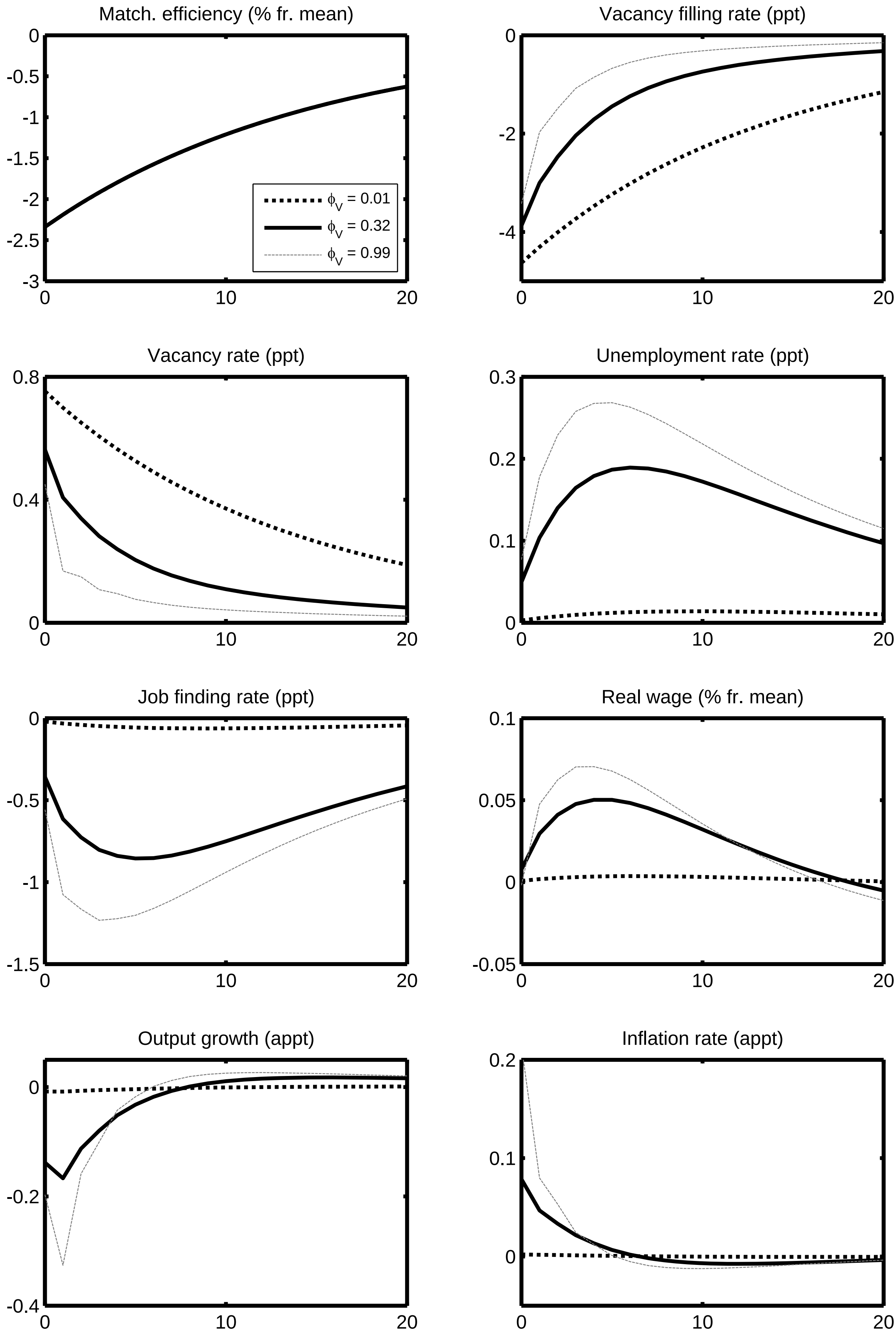


Figure 2: Impulse responses to a one-standard deviation negative matching efficiency shock, computed at the posterior mode, and under alternative calibrations of the pre-match hiring cost weight.

Figure 3: Matching Efficiency

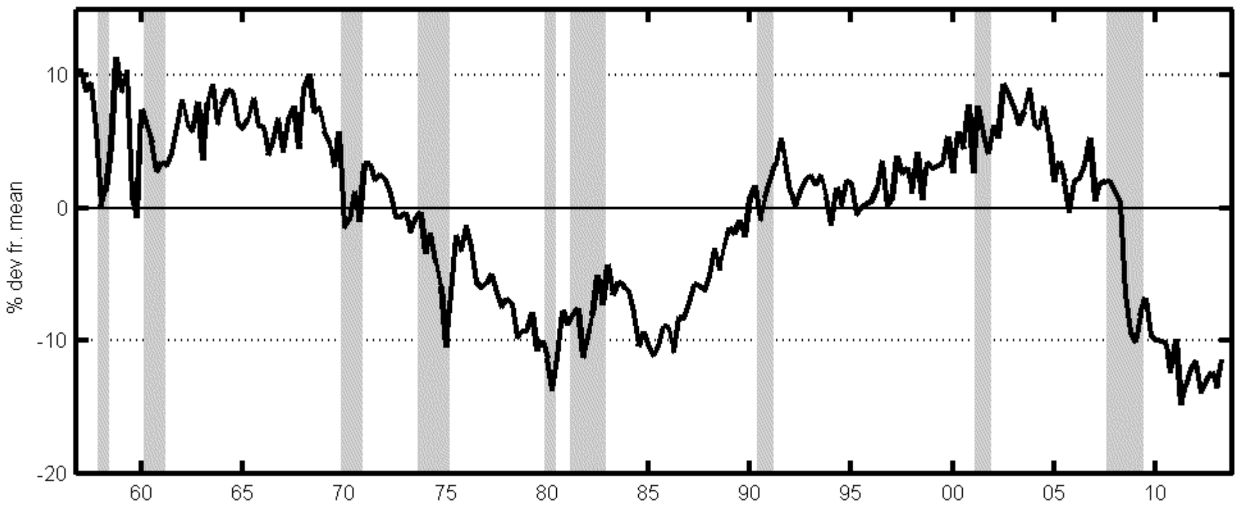


Figure 4: Historical Decomp. of Unemp. Rate (demeaned)

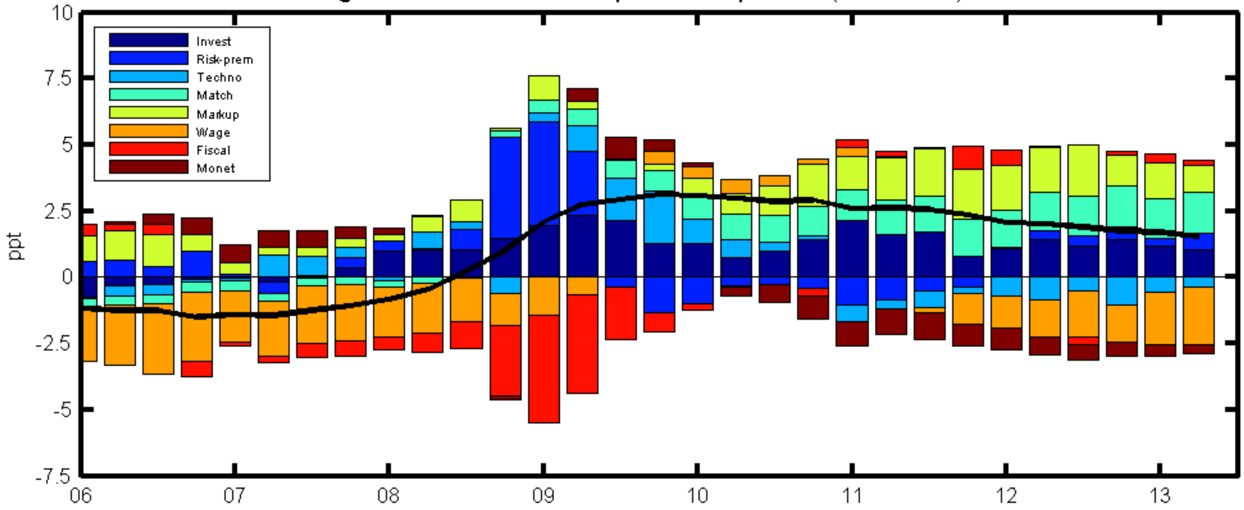


Figure 5: Beveridge Curve 2008:Q1-2013:Q2

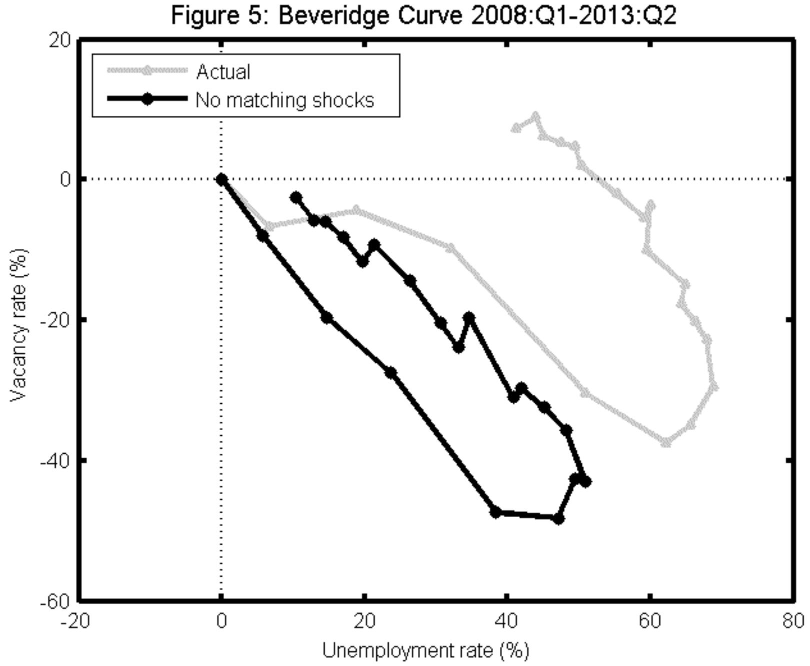


Figure 6: Unemployment Rate: Actual vs Natural

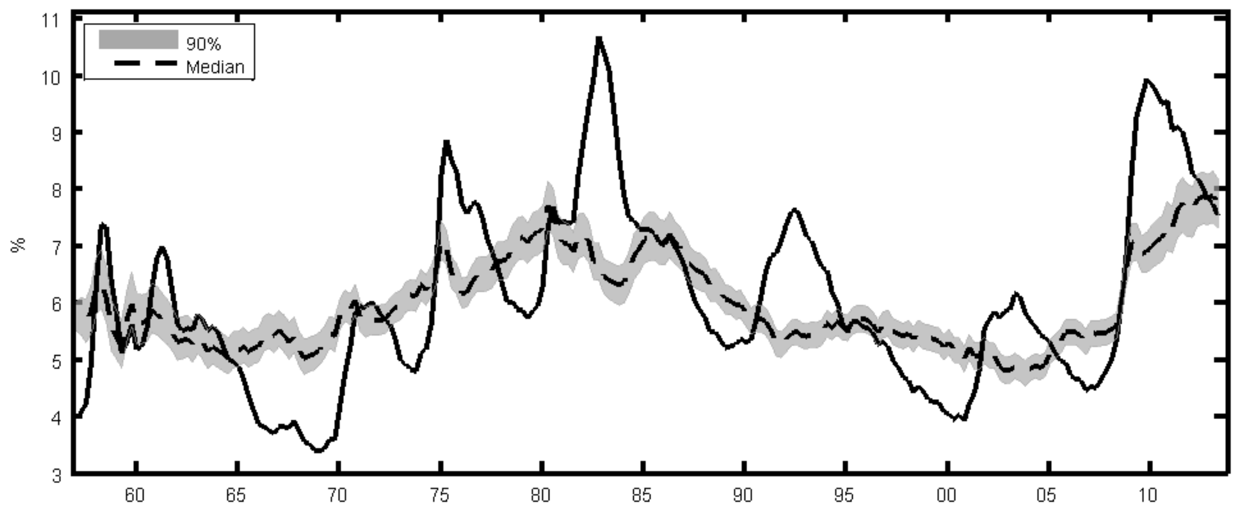


Figure 7: Historical Decomposition of the Natural Rate

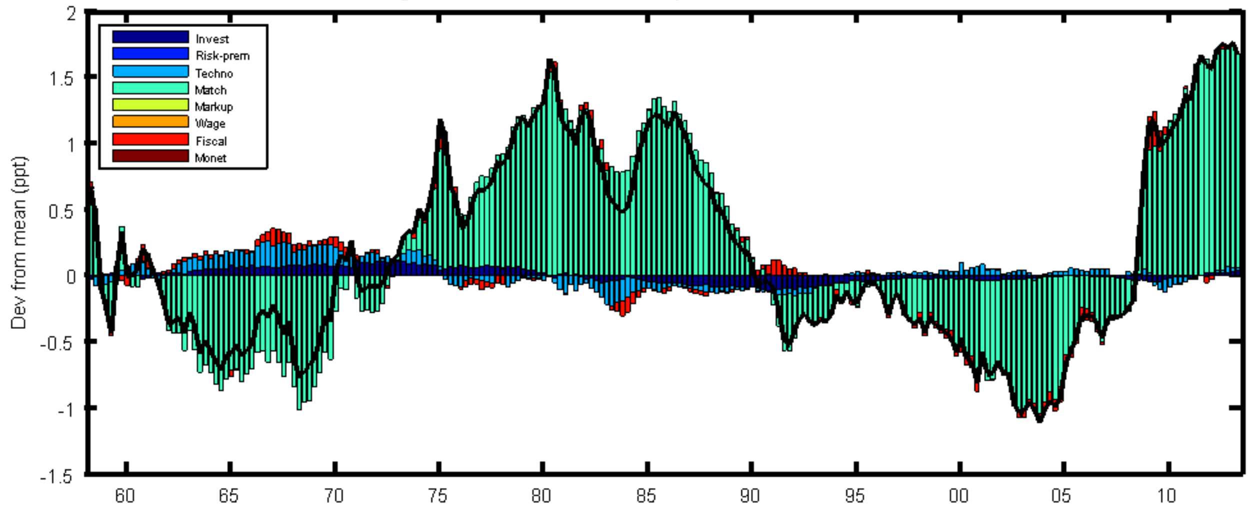


Figure 8: Unemployment Gap: Median and 90% Posterior Bands

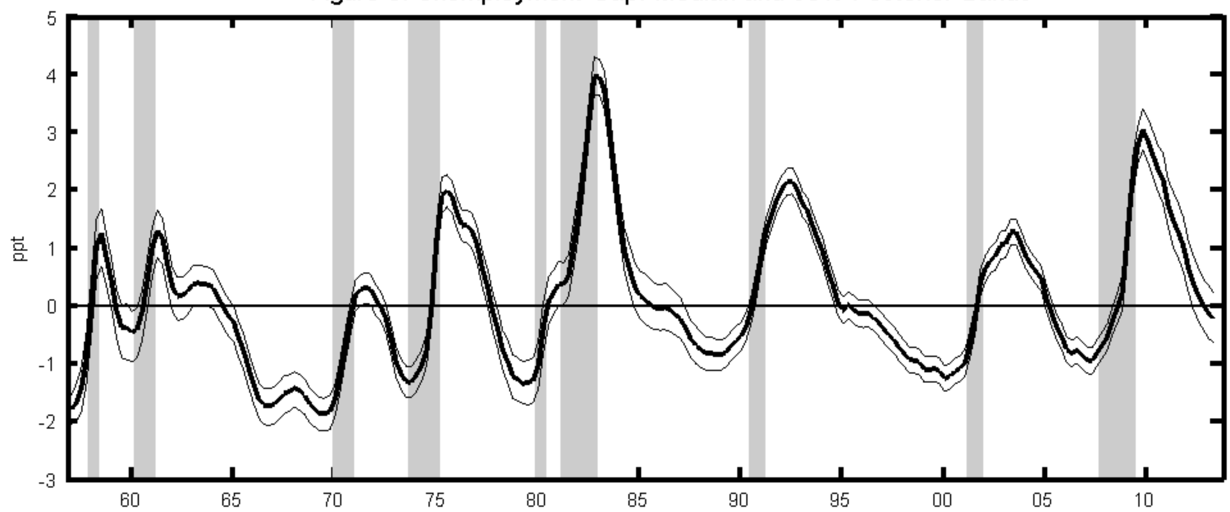
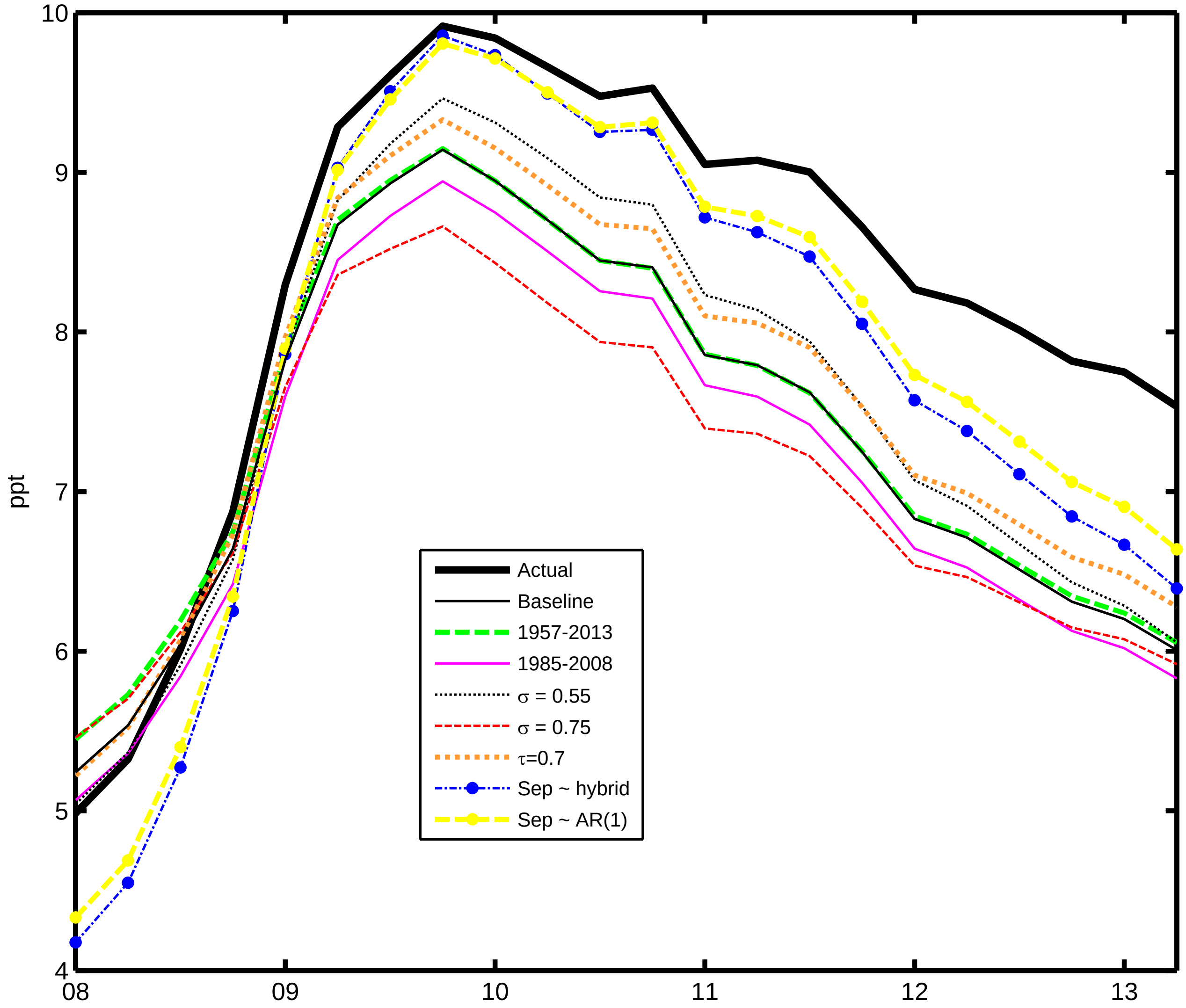


Figure 9: Actual Unemp. vs. Counterfactual (No Match Shocks)



Appendix for
"Mismatch shocks and unemployment during the Great Recession"

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May 15, 2015

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1 Roadmap

Section 2 describes the model. Section 3 deals with our empirical strategy. Section 4 provides additional results from the baseline estimation. Section 5 offers some robustness checks.

2 Model

2.1 The representative family

There is a continuum of identical households of mass one. Each household is a large family, made of a continuum of individuals of measure one. Family members are either working or searching for a job.[1] Following Merz (1995), we assume that family members pool their income before the head of the family chooses optimally per capita consumption.[2]

The representative family enters each period $t = 0, 1, 2, ...$, with B_{t-1} bonds and $\overline{K}_{t-1}$ units of physical capital. At the beginning of each period, bonds mature, providing B_{t-1} units of money. The representative family uses some of this money to purchase B_t new bonds at nominal cost B_t/R_t, where R_t denotes the gross nominal interest rate between period t and $t+1$.

The representative household owns capital and chooses the capital utilization rate, u_t, which transforms physical capital into effective capital according to

$$K_t = u_t \overline{K}_{t-1}. \tag{1}$$

The household rents $K_t(i)$ units of effective capital to intermediate-goods-producing firm $i \in [0, 1]$ at the nominal rate r_t^K. The household's choice of $K_t(i)$ must satisfy

$$K_t = \int_0^1 K_t(i)\, di. \tag{2}$$

The cost of capital utilization is $a(u_t)$ per unit of physical capital. We assume the following functional form for the function a,

$$a(u_t) = \phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t - 1)^2, \tag{3}$$

and that $u_t = 1$ in steady state.

Each period, $N_t(i)$ family members are employed at intermediate goods-producing firm $i \in [0, 1]$. Each worker employed at firm i works a fixed amount of hours and earns the nominal wage $W_t(i)$. N_t denotes aggregate employment in period t and is given by

$$N_t = \int_0^1 N_t(i)\, di. \tag{4}$$

The remaining $(1 - N_t)$ family members are unemployed and and each receives nominal unemployment benefits b_t, financed through lump-sum taxes.

During period t, the representative household receives total nominal factor payments $r_t^K K_t + W_t N_t +$

[1] The model abstracts from the labor force participation decision.

[2] The use of search and matching frictions (Pissarides 2000) in business cycle models was pioneered by Merz (1995) and Andolfatto (1996) in the real business cycle (RBC) literature. More recently, the same labor market frictions have been studied in the New Keynesian model by Blanchard and Galí (2010), Christiano, Trabandt, and Walentin (2011), Christoffel, Kuester, and Linzert (2009), Gertler, Sala and Trigari (2008), Groshenny (2009 and 2013), Krause and Lubik (2007), Krause, López-Salido, and Lubik (2008), Ravenna and Walsh (2008 and 2011), Sveen and Weinke (2009), Trigari (2009), and Walsh (2005), among many others.

$(1 - N_t)\, b_t$. In addition, the household also receives nominal profits $D_t(i)$ from each firm $i \in [0,1]$, for a total of

$$D_t = \int_0^1 D_t(i)\, di. \tag{5}$$

In each period $t = 0, 1, 2, ...$,the family uses these resources to purchase finished goods, for both consumption and investment purposes, from the representative finished goods-producing firm at the nominal price P_t. The law of motion of physical capital is

$$\overline{K}_t \leq (1-\delta)\, \overline{K}_{t-1} + \mu_t \left[1 - \pounds\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{6}$$

where δ denotes the depreciation rate. The function $\pounds$ captures the presence of adjustment costs in investment, as in Christiano, Eichenbaum and Evans (2005). We assume the following functional form for the function $\pounds$,

$$\pounds\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2, \tag{7}$$

where g_I is the steady-state growth rate of investment. Hence, along the balanced growth path, $\pounds(g_I) = \pounds'(g_I) = 0$ and $\pounds''(g_I) = \phi_I > 0$. μ_t is an investment-specific technology shock affecting the efficiency with which consumption goods are transformed into capital. The investment-specific shock follows the exogenous stationary autoregressive process

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t}, \tag{8}$$

where $\varepsilon_{\mu t}$ is $i.i.d. N\left(0, \sigma_\mu^2\right)$.

The family's budget constraint is given by

$$\begin{aligned} P_t C_t + P_t I_t + \frac{B_t}{\epsilon_{bt} R_t} \leq\ & B_{t-1} + W_t N_t + (1 - N_t)\, b_t + r_t^K u_t \overline{K}_{t-1} \\ & - P_t a(u_t)\, \overline{K}_{t-1} - T_t + D_t \end{aligned} \tag{9}$$

for all $t = 0, 1, 2, ...$ As in Smets and Wouters (2007), the shock ϵ_{bt} drives a wedge between the central bank's instrument rate R_t and the return on assets held by the representative family. As noted by De Graeve, Emiris and Wouters (2009), this disturbance works as an aggregate demand shock and generates a positive comovement between consumption and investment. The risk-premium shock ϵ_{bt} follows the autoregressive process

$$\ln \epsilon_{bt} = \rho_b \ln \epsilon_{bt-1} + \varepsilon_{bt}, \tag{10}$$

where $0 < \rho_b < 1$, and ε_{bt} is $i.i.d. N\left(0, \sigma_b^2\right)$.

The family's lifetime utility is described by

$$E_t \sum_{s=0}^{\infty} \beta^s \ln\left(C_{t+s} - hC_{t+s-1}\right) \tag{11}$$

where $0 < \beta < 1$. When $h > 0$, the model allows for habit formation in consumption and consumption responds gradually to shocks.

The head of the family chooses C_t, B_t, u_t, I_t, and $\overline{K}_t$ for each $t = 0, 1, 2, ...$ to maximize the expected lifetime utility (10) subject to the constraints (6) and (9).

The Lagrangean reads

$$E_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{c} \beta^t \ln\left(C_t - hC_{t-1}\right) + \beta^t \Lambda_t \left[\frac{B_{t-1}+W_t N_t + (1-N_t) b_t + r_t^K u_t \overline{K}_{t-1} - T_t + D_t}{P_t} - a\left(u_t\right) \overline{K}_{t-1} - C_t - I_t - \frac{B_t}{\epsilon_{bt} P_t R_t} \right] \\ +\beta^t \Upsilon_t \left[\left(1-\delta\right) \overline{K}_{t-1} + \mu_t \left(1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right) I_t - \overline{K}_t \right] \end{array} \right\} \tag{12}$$

The first order conditions for this problem are

- C_t :

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t \left(\frac{1}{C_{t+1} - hC_t} \right) \tag{13}$$

- B_t :

$$\Lambda_t = \epsilon_{bt} R_t \beta E_t \left(\Lambda_{t+1} \frac{P_t}{P_{t+1}} \right) \tag{14}$$

- u_t :

$$\left(\phi_{u1} - \phi_{u2}\right) + \phi_{u2} u_t = \widetilde{r}_t^K \tag{15}$$

where $\widetilde{r}_t^K$ denotes the real rental rate of capital $\widetilde{r}_t^K = r_t^K / P_t$.

- I_t :

$$1 = \upsilon_t \mu_t \left[1 - \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - g_I \right)^2 - \phi_I \left(\frac{I_t}{I_{t-1}} - g_I \right) \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \upsilon_{t+1} \mu_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \phi_I \left(\frac{I_{t+1}}{I_t} - g_I \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \tag{16}$$

where υ_t is the marginal Tobin's Q: the Lagrange multiplier associated with the investment adjustment constraint, Υ_t, normalized by Λ_t.

- $\overline{K}_t$:

$$\upsilon_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[\left(1-\delta\right) \upsilon_{t+1} + \widetilde{r}_{t+1}^K u_{t+1} - a\left(u_{t+1}\right) \right] \right\} \tag{17}$$

- Λ_t :

$$\frac{B_{t-1} + W_t N_t + \left(1 - N_t\right) b_t + r_t^K u_t \overline{K}_{t-1} - T_t + D_t}{P_t} - a\left(u_t\right) \overline{K}_{t-1} = C_t + I_t + \frac{B_t}{\epsilon_{bt} R_t P_t} \tag{18}$$

where Λ_t denotes the multiplier on (9) and can be interpreted as the utility to the household of an additional unit of wealth at date t.

- Υ_t :

$$\overline{K}_t = \left(1-\delta\right) \overline{K}_{t-1} + \mu_t \left[1 - \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - g_I \right)^2 \right] I_t \tag{19}$$

where Υ_t denotes the multiplier on (6) and can be interpreted as the utility to the household of an additional unit of physical capital at date t.

2.2 The representative finished goods-producing firm

During each period $t = 0, 1, 2, ...$, the representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0, 1]$, purchased at the nominal price $P_t(i)$, to manufacture Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$\left[\int_0^1 Y_t(i)^{(\theta_t-1)/\theta_t}\, di\right]^{\theta_t/(\theta_t-1)} \geq Y_t, \tag{20}$$

where θ_t translates into a random shock to the markup of price over marginal cost. This markup shock follows the autoregressive process

$$\ln \theta_t = (1-\rho_\theta)\ln\theta + \rho_\theta \ln\theta_{t-1} + \varepsilon_{\theta t}, \tag{21}$$

where $0 < \rho_\theta < 1$, $\theta > 1$, and $\varepsilon_{\theta t}$ is $i.i.d.N\left(0, \sigma_\theta^2\right)$.

Intermediate good i sells at the nominal price $P_t(i)$, while the finished good sells at the nominal price P_t. Given these prices, the finished goods-producing firm chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize its profits

$$P_t Y_t - \int_0^1 P_t(i)\, Y_t(i)\, di, \tag{22}$$

subject to the constraint (17) for each $t = 0, 1, 2, ...$ The first-order conditions for this problem are (17) with equality and

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t \tag{23}$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, ...$

Competition in the market for the finished good drives the finished goods-producing firm's profits to zero in equilibrium. This zero-profit condition determines P_t as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t}\, di\right]^{1/(1-\theta_t)} \tag{24}$$

for all $t = 0, 1, 2, ...$

2.3 The representative intermediate goods-producing firm

Each intermediate goods-producing firm $i \in [0, 1]$ enters in period t with a stock of $N_{t-1}(i)$ employees carried from the previous period. At the beginning of period t, before production starts, $\rho N_{t-1}(i)$ jobs are destroyed, where ρ is the exogenous job destruction rate. The pool of workers ρN_{t-1} who have lost their job at the beginning of period t start searching immediately and can possibly be hired in period t. The number of employees at firm i evolves according to

$$N_t(i) = (1-\rho)\, N_{t-1}(i) + m_t(i)\,. \tag{25}$$

$m_t(i)$ denotes the flow of new employees hired by firm i in period t, and is given by

$$m_t(i) = q_t V_t(i), \tag{26}$$

where $V_t(i)$ denotes vacancies posted by firm i in period t and q_t is the aggregate probability of filling a vacancy in period t. Workers hired in period t take part to period t production. Employment is therefore an *instantaneous* margin. However, each period some vacancies and job seekers remain unmatched. As a consequence, a firm-worker pair enjoys a joint surplus that motivates the existence of a long-run relationship between the two parties.

Aggregate employment $N_t = \int_0^1 N_t(i)\, di$ evolves over time according to

$$N_t = (1-\rho) N_{t-1} + m_t, \tag{27}$$

where $m_t = \int_0^1 m_t(i)\, di$ denotes aggregate matches in period t. Similarly, the aggregate vacancies is equal to $V_t = \int_0^1 V_t(i)\, di$. The pool of job seekers in period t, denoted by S_t, is given by

$$S_t = 1 - (1-\rho) N_{t-1}. \tag{28}$$

The matching process is described by the following aggregate CRS function

$$m_t = \zeta_t S_t^{\sigma} V_t^{1-\sigma}, \tag{29}$$

where ζ_t is an exogenous disturbance to the efficiency of the matching technology. We label this disturbance the mismatch shock and assume it follows the exogenous stationary stochastic process

$$\ln \zeta_t = (1-\rho_\zeta) \ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}, \tag{30}$$

where $\zeta > 0$ denotes the steady-state efficiency of the matching technology and $\varepsilon_{\zeta t}$ is $i.i.d.N\left(0, \sigma_\zeta^2\right)$. The probability q_t to fill a vacancy in period t is given by

$$q_t = \frac{m_t}{V_t} = \zeta_t \Theta_t^{-\sigma}, \tag{31}$$

where Θ denotes the tightness of the labor market $\Theta_t = V_t/S_t$. The probability s_t for a job seeker to find a job is

$$s_t = \frac{m_t}{S_t} = \zeta_t \Theta_t^{1-\sigma}. \tag{32}$$

Finally aggregate unemployment is defined by $U_t \equiv 1 - N_t$.

During each period $t = 0, 1, 2, ...$, the representative intermediate goods-producing firm combines $N_t(i)$ homogeneous employees with $K_t(i)$ units of efficient capital to produce $Y_t(i)$ units of intermediate good i according to the constant-returns-to-scale technology described by

$$Y_t(i) = A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{1-\alpha}. \tag{33}$$

A_t is an aggregate labor-augmenting technology shock whose growth rate, $z_t \equiv A_t/A_{t-1}$, follows the exogenous stationary stochastic process

$$\ln z_t = (1-\rho_z) \ln z + \rho_z \ln z_{t-1} + \varepsilon_{zt}, \tag{34}$$

where $z > 1$ denotes the steady-state growth rate of the economy and ε_{zt} is $i.i.d.N\left(0, \sigma_z^2\right)$.

The firm faces costs of hiring workers. As in Yashiv (2000 and 2006), hiring costs are a convex function of the linear combination of the number of vacancies and the number of hires. Hiring costs are measured in terms of aggregate output, and given by

$$\frac{\kappa}{2}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 Y_t, \tag{35}$$

where ϕ_V governs the magnitude of these costs.[3]

Intermediate goods substitute imperfectly for one another in the production function of the representative finished goods-producing firm. Hence, each intermediate goods-producing firm $i \in [0,1]$ sells its output $Y_t(i)$ in a monopolistically competitive market, setting $P_t(i)$, the price of its own product, with the commitment of satisfying the demand for good i at that price. Firms take the nominal wage as given when maximizing the discounted value of expected future profits.

Each intermediate goods-producing firm faces costs of adjusting its nominal price between periods (Rotemberg 1982), measured in terms of the finished good and given by

$$\frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 Y_t. \tag{36}$$

ϕ_P governs the magnitude of the price adjustment cost. $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross rate of inflation in period t. $\pi > 1$ denotes the steady-state gross rate of inflation and coincides with the central bank's target. The parameter $0 \leq \varsigma \leq 1$ governs the importance of backward-looking behavior in price setting (cf. Ireland 2007).

Following Arsenau and Chugh (2008), firms face quadratic wage-adjustment costs which are proportional to the size of their workforce and measured in terms of the finished good

$$\frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i) Y_t, \tag{37}$$

where $\phi_W \geq 0$ governs the magnitude of the wage adjustment cost. The parameter $0 \leq \varrho \leq 1$ governs the importance of backward-looking behavior in wage setting.

Adjustment costs on the hiring rate, price and wage changes make the intermediate goods-producing firm's problem dynamic. It chooses $K_t(i)$, $N_t(i)$, $V_t(i)$ and $Y_t(i)$ and $P_t(i)$ for all $t = 0, 1, 2, ...$to maximize its total market value, given by

$$E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s}\left(\frac{D_{t+s}(i)}{P_{t+s}}\right) \tag{38}$$

where $\beta^t \Lambda_t / P_t$ measures the marginal utility to the representative household of an additional dollar of profits during period t and where

$$\begin{aligned} D_t(i) = {} & P_t(i) Y_t(i) - W_t(i) N_t(i) - r_t^K K_t(i) - \frac{\kappa}{2}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 P_t Y_t \\ & - \frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 P_t Y_t - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i) P_t Y_t, \end{aligned} \tag{39}$$

[3] Hiring costs are proportional to output and thus inherit the common stochastic trend driving productivity. This specification ensures that the unemployment rate remains stationary along the balanced steady-state growth path.

subject to the constraints

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t, \tag{40}$$

$$Y_t(i) \leq K_t(i)^{\alpha}\left[A_t N_t(i)\right]^{1-\alpha}, \tag{41}$$

$$N_t(i) = \chi N_{t-1}(i) + q_t V_t(i), \tag{42}$$

where $\chi \equiv 1-\rho$ is the job survival rate.

This problem is equivalent to the one of choosing $K_t(i)$, $N_t(i)$, $V_t(i)$ and $P_t(i)$ to maximize (35), where

$$\begin{aligned}\frac{D_t(i)}{P_t} = {} & \left(\frac{P_t(i)}{P_t}\right)^{1-\theta_t} Y_t - \left(\frac{W_t(i)\,N_t(i) + r_t^K K_t(i)}{P_t}\right) - \frac{\kappa}{2}\left(\frac{\phi_V V_t(i) + (1-\phi_V)\,q_t V_t(i)}{N_t(i)}\right)^2 Y_t \\ & - \frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 Y_t - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i)\,Y_t,\end{aligned} \tag{43}$$

subject to the constraints

$$\left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t \leq K_t(i)^{\alpha}\left[A_t N_t(i)\right]^{1-\alpha}, \tag{44}$$

$$N_t(i) = \chi N_{t-1}(i) + q_t V_t(i), \tag{45}$$

for all $t = 0, 1, 2, \ldots$.

The Lagrangean reads

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[\begin{array}{c} \left(\frac{P_t(i)}{P_t}\right)^{1-\theta_t} Y_t - \left(\frac{W_t(i)N_t(i)+r_t^K K_t(i)}{P_t}\right) - \frac{\kappa}{2}\left(\frac{\phi_V V_t(i)+(1-\phi_V)q_t V_t(i)}{N_t(i)}\right)^2 Y_t \\ -\frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 Y_t - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i)\,Y_t \end{array}\right] \\ & + E_0 \sum_{t=0}^{\infty} \beta^t \Psi_t(i)\left[\chi N_{t-1}(i) + q_t V_t(i) - N_t(i)\right] + E_0 \sum_{t=0}^{\infty} \beta^t \Xi_t(i)\left[K_t(i)^{\alpha}\left(A_t N_t(i)\right)^{1-\alpha} - \left(\frac{P_t(i)}{P_t}\right)^{-\theta_t} Y_t\right]. \end{aligned} \tag{46}$$

The multiplier $\Psi_t(i)$ measures the value to firm i, expressed in utils, of an additional job in period t. The multiplier $\Xi_t(i)$ measures the value to firm i, expressed in utils, of an additional unit of output in period t. Hence, $\xi_t(i) = \Xi_t(i)/\Lambda_t$ represents firm i's real marginal cost in period t.

The first-order conditions for this problem are

- $K_t(i)$:

$$\widetilde{r}_t^K = \xi_t(i)\,\alpha K_t(i)^{\alpha-1}\left(A_t N_t(i)\right)^{1-\alpha} \tag{47}$$

- $N_t(i)$:

$$\begin{aligned}\frac{\Psi_t(i)}{\Lambda_t} = {} & \xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t \\ & + \frac{\kappa}{N_t(i)}\left[\frac{\phi_V V_t(i) + (1-\phi_V)\,q_t V_t(i)}{N_t(i)}\right]^2 Y_t + \beta\chi\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Psi_{t+1}(i)}{\Lambda_{t+1}}\end{aligned} \tag{48}$$

This condition tells that the costs and benefits of hiring an additional worker must be equal.

- $V_t(i)$:

$$\frac{\Psi_t(i)}{\Lambda_t} = \left(\frac{\phi_V + (1-\phi_V) q_t}{N_t(i)}\right)^2 \frac{\kappa Y_t V_t(i)}{q_t} \tag{49}$$

- Vacancy posting condition :

$$\begin{aligned}\left(\frac{\phi_V + (1-\phi_V) q_t}{N_t(i)}\right)^2 \frac{\kappa Y_t V_t(i)}{q_t} &= \xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t \\ &+ \frac{\kappa}{N_t(i)}\left[\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right]^2 Y_t \\ &+ \beta\chi\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\phi_V + (1-\phi_V) q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1} V_{t+1}(i)}{q_{t+1}}\end{aligned} \tag{50}$$

- $P_t(i)$:

$$\begin{aligned}(1-\theta_t)\left(\frac{P_t(i)}{P_t}\right)^{-\theta_t} &= \phi_P\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)\left(\frac{P_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)}\right) - \theta_t\xi_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-(1+\theta_t)} \\ &- \beta\phi_P E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{P_{t+1}(i)}{\pi_t^{\varsigma}\pi^{1-\varsigma}P_t(i)} - 1\right)\left(\frac{P_{t+1}(i)}{\pi_t^{\varsigma}\pi^{1-\varsigma}P_t(i)}\right)\frac{Y_{t+1}}{Y_t}\frac{P_t}{P_t(i)}\right]\end{aligned} \tag{51}$$

- $\Psi_t(i)$:

$$N_t(i) = \chi N_{t-1}(i) + q_t V_t(i) \tag{52}$$

- $\Xi_t(i)$:

$$A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{1-\alpha} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta_t} Y_t \tag{53}$$

2.4 Wage setting

Each period, intermediate-good producing firm i bargains with each of its employees individually over the nominal wage $W_t(i)$ to maximize the match surplus according to Nash bargaining,

$$W_t(i) = \arg\max\left[\Delta_t(i)^{\eta_t} J_t(i)^{1-\eta_t}\right]. \tag{54}$$

$\Delta_t(i)$ denotes the surplus of the representative worker while $J_t(i)$ denotes the surplus of the firm. Both $\Delta_t(i)$ and $J_t(i)$ are expressed in real terms. η_t denotes the worker's bargaining power which evolves exogenously according to

$$\ln\eta_t = (1-\rho_\eta)\ln\eta + \rho_\eta\ln\eta_{t-1} + \varepsilon_{\eta t}, \tag{55}$$

where $0 < \eta < 1$ and $\varepsilon_{\eta t}$ is $i.i.d.N\left(0, \sigma_\eta^2\right)$.

The family's value function is given by

$$\begin{aligned}
\Omega\left(N_t\right) &= \ln\left(C_t - hC_{t-1}\right) \\
&\quad + \Lambda_t \left[\begin{array}{c} \int_0^1 \frac{W_t(i)N_t(i)}{P_t} di + \left(1 - N_t\right)\left(\frac{b_t}{P_t}\right) \\ + \frac{B_{t-1} + r_t^K u_t \overline{K}_{t-1} - T_t + D_t}{P_t} - C_t - I_t - a\left(u_t\right)\overline{K}_{t-1} - \frac{B_t}{\epsilon_{bt} R_t P_t} \end{array} \right] \\
&\quad + \Upsilon_t \left[\left(1-\delta\right)\overline{K}_{t-1} + \mu_t \left(1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right) I_t - \overline{K}_t \right] \\
&\quad + \beta E_t \Omega\left(N_{t+1}\right). \qquad (56)
\end{aligned}$$

N_t evolves according to $N_t = \chi N_{t-1} + s_t\left(1 - \chi N_{t-1}\right)$. The family takes the job finding rate $s_t = \frac{m_t}{S_t}$ as given. To ensure that the model is consistent with balanced growth, unemployment benefits b_t are proportional to the value of the nominal wage along the balanced growth path $b_t = \tau W_{ss,t}$, where τ is the replacement ratio. Following Trigari (2009) and Ravenna and Walsh (2008), $\Delta_t\left(i\right)$ is defined as the change in the family's value function $\Omega\left(N_t\right)$ from having one additional member employed. Thus, the surplus of an employee at firm i, expressed in utils, is given by

$$\begin{aligned}
\widetilde{\Delta}_t\left(i\right) &= \frac{\partial \Omega\left(N_t\right)}{\partial N_t\left(i\right)}, \\
&= \Lambda_t\left(\frac{W_t\left(i\right)}{P_t} - \frac{b_t}{P_t}\right) + \beta E_t\left[\chi\left(1 - s_{t+1}\right)\right]\widetilde{\Delta}_{t+1}\left(i\right). \qquad (57)
\end{aligned}$$

The worker's surplus from the match, expressed in consumption goods, is given by

$$\Delta_t\left(i\right) = \frac{W_t\left(i\right)}{P_t} - \frac{b_t}{P_t} + \beta E_t\left[\chi\left(1 - s_{t+1}\right)\right]\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\Delta_{t+1}\left(i\right). \qquad (58)$$

The employer's surplus from the match, expressed in real terms, is given by $J_t\left(i\right) = \frac{\Psi_t(i)}{\Lambda_t}$

$$\begin{aligned}
J_t\left(i\right) &= \xi_t\left(i\right)\left(1-\alpha\right)\frac{Y_t\left(i\right)}{N_t\left(i\right)} - \frac{W_t\left(i\right)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t\left(i\right)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}\left(i\right)} - 1\right)^2 Y_t \\
&\quad + \frac{\kappa}{N_t\left(i\right)}\left[\frac{\phi_V V_t\left(i\right) + \left(1-\phi_V\right)q_t V_t\left(i\right)}{N_t\left(i\right)}\right]^2 Y_t + \beta\chi E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}J_{t+1}\left(i\right)\right). \qquad (59)
\end{aligned}$$

Nash bargaining over the nominal wage yields the following first-order condition

$$\eta_t J_t\left(i\right)\frac{\partial \Delta_t\left(i\right)}{\partial W_t\left(i\right)} = -\left(1-\eta_t\right)\Delta_t\left(i\right)\frac{\partial J_t\left(i\right)}{\partial W_t\left(i\right)}, \qquad (60)$$

where

$$\frac{\partial \Delta_t\left(i\right)}{\partial W_t\left(i\right)} = \frac{1}{P_t}, \qquad (61)$$

$$-\frac{\partial J_t\left(i\right)}{\partial W_t\left(i\right)} = \left\{ \begin{array}{c} \frac{1}{P_t} + \phi_W Y_t\left(\frac{1}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)}\right)\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right) \\ -\beta\chi\phi_W E_t\left[\frac{\Lambda_{t+1}Y_{t+1}}{\Lambda_t W_t(i)}\left(\frac{W_{t+1}(i)}{z\pi_t^{\varrho}\pi^{1-\varrho}W_t(i)}\right)\left(\frac{W_{t+1}(i)}{z\pi_t^{\varrho}\pi^{1-\varrho}W_t(i)} - 1\right)\right] \end{array} \right\}. \qquad (62)$$

When $\phi_W = 0$, adjusting nominal wages is costless for the firm. In that case, the effects of a marginal increase in the nominal wage on the worker's surplus and on the firm's surplus have the same magnitude (with opposite signs):

$$\text{if } \phi_W = 0, \text{ then } \frac{\partial \Delta_t (i)}{\partial W_t (i)} = -\frac{\partial J_t (i)}{\partial W_t (i)} = \frac{1}{P_t}. \tag{63}$$

In the absence of nominal wage-adjustment costs, Nash bargaining over the nominal wage implies the usual first-order condition

$$\Delta_t (i) = \left(\frac{\eta_t}{1-\eta_t} \right) J_t (i) . \tag{64}$$

Thus, as pointed out by Arsenau and Chugh (2008), Nash bargaining over the nominal wage when there are no nominal wage adjustment costs is equivalent to Nash bargaining over the real wage. The presence of nominal wage-adjustment costs (beared by the firm) affects the *effective* bargaining powers of the firm and the worker respectively. In the presence of nominal wage adjustment costs, the first-order condition from Nash bargaining is given by

$$\Delta_t (i) = \frac{\eta_t}{(1-\eta_t)} \frac{[\partial \Delta_t (i) / \partial W_t (i)]}{[-\partial J_t (i) / \partial W_t (i)]} J_t (i) , \tag{65}$$

$$\Delta_t (i) = \text{Ŋ}_{it} J_t (i) , \tag{66}$$

where we have introduced the notation

$$\text{Ŋ}_{it} \equiv \frac{\left(\frac{\eta_t}{1-\eta_t} \right) \left(\frac{\partial \Delta_t (i)}{\partial W_t (i)} \right)}{\left(-\frac{\partial J_t (i)}{\partial W_t (i)} \right)}. \tag{67}$$

Substituting the expressions of the two partial derivatives into the first-order condition, we obtain

$$\begin{aligned}
&\text{Ŋ}_{it} \left[\xi_t (i) (1-\alpha) \frac{Y_t (i)}{N_t (i)} - \frac{W_t (i)}{P_t} - \frac{\phi_W}{2} \left(\frac{W_t (i)}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} W_{t-1} (i)} - 1 \right)^2 Y_t \right] \\
&+ \text{Ŋ}_{it} \left[\frac{\kappa}{N_t (i)} \left(\frac{\phi_V V_t (i) + (1-\phi_V) q_t V_t (i)}{N_t (i)} \right)^2 Y_t \right] \\
&+ \text{Ŋ}_{it} \beta \chi E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1} (i) \right] \\
&= \frac{W_t (i)}{P_t} - \frac{b_t}{P_t} + \beta \chi E_t \left[(1-s_{t+1}) \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) \Delta_{t+1} (i) \right] ,
\end{aligned} \tag{68}$$

Using the fact that $\Delta_{t+1} (i) = \text{Ŋ}_{it+1} J_{t+1} (i)$ in the above equation, we obtain

$$\begin{aligned}
&\text{Ŋ}_{it} \left[\xi_t (i) (1-\alpha) \frac{Y_t (i)}{N_t (i)} - \frac{W_t (i)}{P_t} - \frac{\phi_W}{2} \left(\frac{W_t (i)}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} W_{t-1} (i)} - 1 \right)^2 Y_t \right] \\
&+ \text{Ŋ}_{it} \left[\frac{\kappa}{N_t (i)} \left(\frac{\phi_V V_t (i) + (1-\phi_V) q_t V_t (i)}{N_t (i)} \right)^2 Y_t \right] \\
&+ \text{Ŋ}_{it} \beta \chi E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1} (i) \right] \\
&= \frac{W_t (i)}{P_t} - \frac{b_t}{P_t} + \beta \chi E_t \left[(1-s_{t+1}) \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) \text{Ŋ}_{it+1} J_{t+1} (i) \right] ,
\end{aligned} \tag{69}$$

Now, let us recall the definition of the firm's surplus

$$J_t(i) = \frac{\Psi_t(i)}{\Lambda_t} = \left(\frac{\phi_V + (1-\phi_V) q_t}{N_t(i)}\right)^2 \frac{\kappa Y_t V_t(i)}{q_t}. \tag{70}$$

Using this expression of $J_{t+1}(i)$, the real-wage equation becomes

$$\begin{aligned}
&\frac{W_t(i)}{P_t} - \text{Ŋ}_{it}\left[\xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t\right] \\
&- \text{Ŋ}_{it}\left[\frac{\kappa}{N_t(i)}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 Y_t\right] \\
&= \text{Ŋ}_{it}\beta\chi E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\phi_V + (1-\phi_V) q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1} V_{t+1}(i)}{q_{t+1}}\right] \\
&+ \frac{b_t}{P_t} - \beta\chi E_t\left[(1-s_{t+1})\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\text{Ŋ}_{it+1}\left(\frac{\phi_V + (1-\phi_V) q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1} V_{t+1}(i)}{q_{t+1}}\right].
\end{aligned} \tag{71}$$

Finally, the equation governing the dynamics of the real wage at firm i is given by

$$\begin{aligned}
\frac{W_t(i)}{P_t} = &\left(\frac{\text{Ŋ}_{it}}{1+\text{Ŋ}_{it}}\right)\left[\begin{array}{c}
\xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t \\
+\frac{\kappa}{N_t(i)}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 Y_t \\
+\beta\chi E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{\phi_V + (1-\phi_V) q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1} V_{t+1}(i)}{q_{t+1}}
\end{array}\right] \\
&+ \frac{1}{(1+\text{Ŋ}_{it})}\left[\frac{b_t}{P_t} - \beta\chi E_t \text{Ŋ}_{it+1}(1-s_{t+1})\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{\phi_V + (1-\phi_V) q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1} V_{t+1}(i)}{q_{t+1}}\right].
\end{aligned} \tag{72}$$

2.5 Government

The central bank adjusts the short-term nominal gross interest rate R_t by following a Taylor-type rule

$$\ln\left(\frac{R_t}{R}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\left\{\rho_\pi \ln\left[\left(\frac{P_t/P_{t-4}}{\Pi}\right)^{1/4}\right] + \rho_y \ln\left[\left(\frac{Y_t/Y_{t-4}}{G_y}\right)^{1/4}\right]\right\} + \ln\epsilon_{mpt}, \tag{73}$$

where $\Pi_t = P_t/P_{t-4}$ and $G_{yt} = Y_t/Y_{t-4}$ and Π and G_y denote the steady state values of Π_t and G_{yt} respectively. The degree of interest-rate smoothing ρ_r and the reaction coefficients ρ_π, ρ_y are positive. The monetary policy shock ϵ_{mpt} follows an AR(1) process

$$\ln\epsilon_{mpt} = \rho_{mp}\ln\epsilon_{mpt-1} + \varepsilon_{mpt}, \tag{74}$$

with $0 \leq \rho_{mp} < 1$ and $\varepsilon_{mpt} \sim i.i.d.N\left(0, \sigma_{mp}^2\right)$.

The government budget constraint is of the form

$$P_t G_t + (1-N_t) b_t = \left(\frac{B_t}{R_t} - B_{t-1}\right) + T_t, \tag{75}$$

where T_t denotes total nominal lump-sum transfers. Public spending is an exogenous time-varying fraction of GDP

$$G_t = \left(1 - \frac{1}{\epsilon_{gt}}\right) Y_t, \tag{76}$$

where ϵ_{gt} evolves according to

$$\ln \epsilon_{gt} = (1-\rho_g)\ln \epsilon_g + \rho_g \ln \epsilon_{gt-1} + \varepsilon_{gt}, \tag{77}$$

with $\varepsilon_{gt} \sim i.i.d.N\left(0, \sigma_g^2\right)$.

2.6 The aggregate resource constraint

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $Y_t(i) = Y_t$, $P_t(i) = P_t$, $N_t(i) = N_t$, $V_t(i) = V_t$, $K_t(i) = K_t$ for all $i \in [0,1]$ and $t = 0,1,2,.....$ Moreover, workers are homogeneous and all workers at a given firm i receive the same nominal wage $W_t(i)$, so that $W_t(i) = W_t$ for all $i \in [0,1]$ and $t = 0,1,2,....$ The aggregate resource constraint is obtained by aggregating the household budget constraint over all intermediate sectors $i \in [0,1]$,

$$\left[\begin{array}{c} \frac{1}{\epsilon_{gt}} - \frac{\kappa}{2}\left(\frac{\phi_V V_t + (1-\phi_V) q_t V_t}{N_t}\right)^2 - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}} - 1\right)^2 - \\ \frac{\phi_W}{2}\left(\frac{W_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}} - 1\right)^2 N_t \end{array}\right] Y_t = C_t + I_t + a(u_t)\overline{K}_{t-1}. \tag{78}$$

$$\aleph_t = \frac{\phi_V V_t + (1-\phi_V) m_t}{N_t} \tag{79}$$

$$\left[\begin{array}{c} \frac{1}{\epsilon_{gt}} - \frac{\kappa}{2}\aleph_t^2 - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}} - 1\right)^2 \\ -\frac{\phi_W}{2}\left(\frac{W_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}} - 1\right)^2 N_t \end{array}\right] Y_t = C_t + I_t + \left[\phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t-1)^2\right]\overline{K}_{t-1} \tag{80}$$

2.7 The symmetric equilibrium

In a symmetric equilibrium, $Y_t(i) = Y_t$, $P_t(i) = P_t$, $N_t(i) = N_t$, $V_t(i) = V_t$, $K_t(i) = K_t$, $W_t(i) = W_t$ for all $i \in [0,1]$ and $t = 0,1,2,....$Defining the real wage $\widetilde{W}_t = W_t/P_t$, the gross rate of price inflation $\pi_t = P_t/P_{t-1}$, the system of equilibrium conditions becomes

1. Y_t

$$\left[\begin{array}{c} \frac{1}{\epsilon_{gt}} - \frac{\kappa}{2}\aleph_t^2 - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}} - 1\right)^2 \\ -\frac{\phi_W}{2}\left(\frac{W_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}} - 1\right)^2 N_t \end{array}\right] Y_t = C_t + I_t + \left[\phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t-1)^2\right]\overline{K}_{t-1}$$

2. $\aleph_t$

$$\aleph_t = \frac{\phi_V V_t + (1-\phi_V) m_t}{N_t}$$

3. m_t

$$m_t = q_t V_t$$

4. x_t

$$x_t = \frac{m_t}{N_t}$$

5. K_t

$$K_t = u_t \overline{K}_{t-1}$$

6. $\overline{K}_t$

$$\overline{K}_t = (1-\delta)\,\overline{K}_{t-1} + \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right] I_t$$

7. μ_t

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t}$$

8. ϵ_{bt}

$$\ln \epsilon_{bt} = \rho_b \ln \epsilon_{bt-1} + \varepsilon_{bt}$$

9. Λ_t

$$\Lambda_t = \beta \epsilon_{bt} R_t E_t \left(\frac{\Lambda_{t+1}}{\pi_{t+1}}\right)$$

10. C_t

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t \left(\frac{1}{C_{t+1} - hC_t}\right)$$

11. $\widetilde{r}_t^K = \frac{r_t^K}{P_t}$

$$(\phi_{u1} - \phi_{u2}) + \phi_{u2} u_t = \widetilde{r}_t^K$$

12. I_t

$$1 = \upsilon_t \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2 - \phi_I \left(\frac{I_t}{I_{t-1}} - g_I\right)\left(\frac{I_t}{I_{t-1}}\right)\right]$$
$$+ \beta E_t \upsilon_{t+1} \mu_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \phi_I \left(\frac{I_{t+1}}{I_t} - g_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

13. $\upsilon_t = \frac{\Upsilon_t}{\Lambda_t}$

$$\upsilon_t = \beta E_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left[(1-\delta)\,\upsilon_{t+1} + \widetilde{r}_{t+1}^K u_{t+1} - a\,(u_{t+1})\right]\right\}$$

14. θ_t

$$\ln \theta_t = (1-\rho_\theta)\ln\theta + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}$$

15. N_t

$$N_t = \chi N_{t-1} + q_t V_t$$

16. S_t

$$S_t = 1 - \chi N_{t-1}$$

17. U_t

$$U_t = 1 - N_t$$

18. $\Theta_t = \frac{V_t}{S_t}$

$$\Theta_t = \frac{V_t}{S_t}$$

19. q_t

$$q_t = \zeta_t \left(\frac{S_t}{V_t}\right)^{\sigma}$$
$$q_t = \zeta_t \left(\frac{V_t}{S_t}\right)^{-\sigma}$$
$$q_t = \zeta_t \Theta_t^{-\sigma}$$

20. s_t

$$s_t = \zeta_t \left(\frac{V_t}{S_t}\right)^{1-\sigma}$$
$$s_t = \zeta_t \Theta_t^{1-\sigma}$$

21. ζ_t

$$\ln \zeta_t = (1-\rho_\zeta) \ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}$$

22. V_t

$$\begin{aligned}\aleph_t^2 \frac{\kappa Y_t}{m_t} = \xi_t (1-\alpha) \frac{Y_t}{N_t} - \widetilde{W}_t - \frac{\phi_W}{2} \left(\frac{W_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} W_{t-1}} - 1\right)^2 Y_t \\ + \frac{\kappa}{N_t} \aleph_t^2 Y_t + \beta \chi \frac{\Lambda_{t+1}}{\Lambda_t} \aleph_{t+1}^2 \frac{\kappa Y_{t+1}}{m_{t+1}}\end{aligned}$$

23. u_t

$$Y_t = A_t^{1-\alpha} {K_t}^{\alpha} {N_t}^{1-\alpha}$$

24. A_t

$$z_t = \frac{A_t}{A_{t-1}}$$

25. $z_t = \frac{A_t}{A_{t-1}}$

$$\ln(z_t) = (1-\rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt}$$

26. $\xi_t = \frac{\Xi_t}{\Lambda_t}$

$$\widetilde{r}_t^K = \left(\alpha \frac{Y_t}{K_t}\right) \xi_t$$

27. π_t

$$\phi_P \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma} \pi^{1-\varsigma}} - 1\right) \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma} \pi^{1-\varsigma}}\right) = (1-\theta_t) + \theta_t \xi_t$$
$$+ \beta \phi_P E_t \left[\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \left(\frac{\pi_{t+1}}{\pi_t^{\varsigma} \pi^{1-\varsigma}} - 1\right) \left(\frac{\pi_{t+1}}{\pi_t^{\varsigma} \pi^{1-\varsigma}}\right) \left(\frac{Y_{t+1}}{Y_t}\right)\right]$$

28. $\widetilde{b}_t = \frac{b_t}{P_t}$

$$\widetilde{b}_t = \tau \widetilde{W}_{ss,t}$$

29. $\widetilde{W}_t = \frac{W_t}{P_t}$

$$\widetilde{W}_t = \left(\frac{\gimel_t}{1+\gimel_t}\right) \left[\xi_t (1-\alpha) \frac{Y_t}{N_t} - \frac{\phi_W}{2} \left(\frac{W_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} W_{t-1}} - 1\right)^2 Y_t + \frac{\kappa}{N_t} \aleph_t^2 Y_t\right]$$
$$+ \left(\frac{\gimel_t}{1+\gimel_t}\right) \left[\beta \chi E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \aleph_{t+1}^2 \frac{\kappa Y_{t+1}}{m_{t+1}}\right]$$
$$+ \frac{1}{(1+\gimel_t)} \left[\widetilde{b}_t - \beta \chi E_t \gimel_{t+1} (1-s_{t+1}) \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \aleph_{t+1}^2 \frac{\kappa Y_{t+1}}{m_{t+1}}\right]$$

30. $\gimel_t$

$$\gimel_t = \frac{\left(\frac{\eta_t}{1-\eta_t}\right)\left(\frac{\widetilde{W}_t}{Y_t}\right)}{\frac{\widetilde{W}_t}{Y_t} + \phi_W \left(\frac{W_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}} - 1\right)\left(\frac{W_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}}\right) - \beta\chi\phi_W E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{W_{t+1}}{z\pi_t^{\varrho}\pi^{1-\varrho}W_t} - 1\right)\left(\frac{W_{t+1}}{z\pi_t^{\varrho}\pi^{1-\varrho}W_t}\right)\frac{Y_{t+1}}{Y_t}\right]}$$

31. η_t

$$\ln \eta_t = (1-\rho_\eta) \ln \eta + \rho_\eta \ln \eta_{t-1} + \varepsilon_{\eta t}$$

32. R_t

$$\ln\left(\frac{R_t}{R}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\left\{\rho_\pi \ln\left[\left(\frac{P_t/P_{t-4}}{\Pi}\right)^{1/4}\right] + \rho_y \ln\left[\left(\frac{Y_t/Y_{t-4}}{G_y}\right)^{1/4}\right]\right\} + \ln \epsilon_{mpt}$$

33. ϵ_{mpt}

$$\ln \epsilon_{mpt} = \rho_{mp} \ln \epsilon_{mpt-1} + \varepsilon_{mpt}$$

34. G_t

$$G_t = \left(1 - \frac{1}{\epsilon_{gt}}\right) Y_t$$

35. ϵ_{gt}

$$\ln \epsilon_{gt} = (1-\rho_g)\ln \epsilon_g + \rho_g \ln \epsilon_{gt-1} + \varepsilon_{gt}$$

36. gy_t : Quarterly gross rate of output growth

$$gy_t = Y_t / Y_{t-1}$$

37. gc_t : Quarterly gross rate of consumption growth

$$gc_t = C_t / C_{t-1}$$

38. gi_t : Quarterly gross rate of investment growth

$$gi_t = I_t / I_{t-1}$$

39. gw_t : Quarterly gross rate of real wage growth

$$gw_t = \widetilde{W}_t / \widetilde{W}_{t-1}$$

These 39 equations determine equilibrium values for the 39 variables Y_t, K_t, $\overline{K}_t$, u_t, C_t, Λ_t, R_t, G_t, I_t, υ_t, $\widetilde{r}_t^K$, ξ_t, N_t, S_t, U_t, V_t, $\aleph_t$, m_t, x_t, Θ_t, q_t, s_t, $\widetilde{W}_t$, $\daleth_t$, $\widetilde{b}_t$, π_t, μ_t, ϵ_{bt}, A_t, z_t, ζ_t, θ_t, η_t, ϵ_{mpt}, ϵ_{gt}, gy_t, gc_t, gi_t, gw_t.

2.8 The stationary transformed economy

Output, consumption, investment, capital and the real wage share the stochastic trend induced by the unit root process of neutral technological progress. We first rewrite the model in terms of stationary variables, and then loglinearize this transformed model economy around its steady state. This approximate model can then be solved using standard methods. The following variables are stationary and need not to be transformed: u_t, R_t, $\widetilde{r}_t^K$, $\upsilon_t = \frac{\Upsilon_t}{\Lambda_t}$, ξ_t, N_t, S_t, U_t, V_t, $\aleph_t$, m_t, x_t, q_t, s_t, $\pi_t = \frac{P_t}{P_{t-1}}$, μ_t, a_t, z_t, ζ_t, θ_t, η_t, ϵ_{mpt}, ϵ_{gt} and $\daleth_t$. we define the transformed variables $y_t = Y_t/A_t$, $k_t = K_t/A_t$, $\overline{k}_t = \overline{K}_t/A_t$, $c_t = C_t/A_t$, $\lambda_t = A_t\Lambda_t$, $i_t = I_t/A_t$, $\widetilde{w}_t = \widetilde{W}_t/A_t$, $\mathbf{b}_t = \widetilde{b}_t/A_t$, $g_t = G_t/A_t$. The stationarized economy contains only 38 equations in 38 variables because the level of the non-stationary productivity shock A_t is not included.

1. $y_t = Y_t/A_t$

$$\left[\frac{1}{\epsilon_{gt}} - \frac{\kappa}{2}\aleph_t^2 - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}} - 1\right)^2 - \frac{\phi_W}{2}\left(\frac{z_t\pi_t\widetilde{w}_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}\widetilde{w}_{t-1}} - 1\right)^2 N_t\right] y_t$$
$$= c_t + i_t + \left[\phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t - 1)^2\right]\overline{k}_{t-1}\frac{1}{z_t}$$

2. $\aleph_t$

$$\aleph_t = \frac{\phi_V V_t + (1-\phi_V) m_t}{N_t}$$

3. m_t

$$m_t = q_t V_t$$

4. x_t

$$x_t = \frac{m_t}{N_t}$$

5. $k_t = K_t/A_t$

$$k_t = u_t \overline{k}_{t-1} \frac{1}{z_t}$$

6. $\overline{k}_t = \overline{K}_t/A_t$

$$\overline{k}_t = (1-\delta)\,\overline{k}_{t-1}\frac{1}{z_t} + \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{i_t}{i_{t-1}} z_t - g_I\right)^2\right] i_t$$

7. μ_t

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t}$$

8. ϵ_{bt}

$$\ln \epsilon_{bt} = \rho_b \ln \epsilon_{bt-1} + \varepsilon_{bt}$$

9. $\lambda_t = A_t \Lambda_t$

$$\lambda_t = \beta \epsilon_{bt} R_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \frac{1}{z_{t+1}}\right)$$

10. $c_t = C_t/A_t$

$$\lambda_t = \frac{z_t}{z_t c_t - h c_{t-1}} - \beta h E_t \left(\frac{1}{c_{t+1} z_{t+1} - h c_t}\right)$$

11. $\widetilde{r}_t^K = \frac{r_t^K}{P_t}$

$$(\phi_{u1} - \phi_{u2}) + \phi_{u2} u_t = \widetilde{r}_t^K$$

12. $i_t = I_t/A_t$

$$\begin{aligned} 1 = {} & \upsilon_t \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{i_t}{i_{t-1}} z_t - g_I\right)^2 - \phi_I \left(\frac{i_t}{i_{t-1}} z_t - g_I\right)\left(\frac{i_t}{i_{t-1}} z_t\right)\right] \\ & + \beta E_t \upsilon_{t+1} \mu_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{z_{t+1}} \phi_I \left(\frac{i_{t+1}}{i_t} z_{t+1} - g_I\right)\left(\frac{i_{t+1}}{i_t} z_{t+1}\right)^2 \end{aligned}$$

13. $\upsilon_t = \frac{\Upsilon_t}{\Lambda_t}$

$$\upsilon_t = \beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{z_{t+1}}\left[(1-\delta)\,\upsilon_{t+1} + \widetilde{r}_{t+1}^K u_{t+1} - \phi_{u1}(u_{t+1} - 1) - \frac{\phi_{u2}}{2}(u_{t+1} - 1)^2\right]\right\}$$

14. u_t

$$y_t = k_t^\alpha {N_t}^{1-\alpha}$$

15. $z_t = \frac{A_t}{A_{t-1}}$

$$\ln z_t = (1-\rho_z)\ln z + \rho_z \ln z_{t-1} + \varepsilon_{zt}$$

16. $\xi_t \equiv \frac{\Xi_t}{\Lambda_t}$

$$\widetilde{r}_t^K = \alpha \frac{y_t}{k_t}\xi_t$$

17. N_t

$$N_t = \chi N_{t-1} + q_t V_t$$

18. S_t

$$S_t = 1 - \chi N_{t-1}$$

19. U_t

$$U_t = 1 - N_t$$

20. $\Theta_t = \frac{V_t}{S_t}$

$$\Theta_t = \frac{V_t}{S_t}$$

21. q_t

$$q_t = \zeta_t \Theta_t^{-\sigma}$$

22. s_t

$$s_t = \zeta_t \Theta_t^{1-\sigma}$$

23. ζ_t

$$\ln \zeta_t = (1-\rho_\zeta)\ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}$$

24. V_t

$$\frac{\kappa \aleph_t^2 y_t}{m_t} = \xi_t (1-\alpha)\frac{y_t}{N_t} - \widetilde{w}_t - \frac{\phi_W}{2}\left(\frac{z_t \pi_t \widetilde{w}_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}\widetilde{w}_{t-1}} - 1\right)^2 y_t + \frac{\kappa \aleph_t^2 y_t}{N_t} + \beta\chi \frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa \aleph_{t+1}^2 y_{t+1}}{m_{t+1}}$$

25. θ_t

$$\ln \theta_t = (1-\rho_\theta)\ln \theta + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}$$

26. $\pi_t = \frac{P_t}{P_{t-1}}$

$$0 = (1-\theta_t) + \theta_t \xi_t - \phi_P \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma} \pi^{1-\varsigma}} - 1 \right) \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma} \pi^{1-\varsigma}} \right) + \beta \phi_P E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi_t^{\varsigma} \pi^{1-\varsigma}} - 1 \right) \left(\frac{\pi_{t+1}}{\pi_t^{\varsigma} \pi^{1-\varsigma}} \right) \frac{y_{t+1}}{y_t} \right]$$

27. $\widetilde{\mathbf{b}}_t = \widetilde{b}_t / A_t$

$$\widetilde{\mathbf{b}}_t = \widetilde{\mathbf{b}} = \tau \widetilde{w}$$

28. $\widetilde{w}_t = \widetilde{W}_t / A_t$

$$\widetilde{w}_t = \left(\frac{\beth_t}{1+\beth_t} \right) \left[\xi_t (1-\alpha) \frac{y_t}{N_t} - \frac{\phi_W}{2} \left(\frac{z_t \pi_t \widetilde{w}_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} \widetilde{w}_{t-1}} - 1 \right)^2 y_t + \frac{\kappa \aleph_t^2 y_t}{N_t} + \beta \chi E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{\kappa \aleph_{t+1}^2 y_{t+1}}{m_{t+1}} \right] + \frac{1}{1+\beth_t} \left[\widetilde{\mathbf{b}} - \beta \chi E_t \beth_{t+1} (1 - s_{t+1}) \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa \aleph_{t+1}^2 y_{t+1}}{m_{t+1}} \right]$$

29. $\beth_t$

$$\beth_t = \left[\left(\frac{\eta_t}{1-\eta_t} \right) \frac{\widetilde{w}_t}{y_t} \right] / \left\{ \begin{array}{c} \frac{\widetilde{w}_t}{y_t} + \phi_W \left(\frac{z_t \pi_t \widetilde{w}_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} \widetilde{w}_{t-1}} - 1 \right) \left(\frac{z_t \pi_t \widetilde{w}_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} \widetilde{w}_{t-1}} \right) \\ -\beta \chi \phi_W E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{z_{t+1} \pi_{t+1} \widetilde{w}_{t+1}}{z \pi_t^{\varrho} \pi^{1-\varrho} \widetilde{w}_t} - 1 \right) \left(\frac{z_{t+1} \pi_{t+1} \widetilde{w}_{t+1}}{z \pi_t^{\varrho} \pi^{1-\varrho} \widetilde{w}_t} \right) \frac{y_{t+1}}{y_t} \right] \end{array} \right\}$$

30. η_t

$$\ln \eta_t = (1-\rho_\eta) \ln \eta + \rho_\eta \ln \eta_{t-1} + \varepsilon_{\eta t}$$

31. R_t

$$\ln \left(\frac{R_t}{R} \right) = \rho_r \ln \left(\frac{R_{t-1}}{R} \right) + (1-\rho_r) \rho_\pi \ln \left[\frac{(\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3})^{1/4}}{\pi} \right] + (1-\rho_r) \rho_y \ln \left[\frac{(gy_t gy_{t-1} gy_{t-2} gy_{t-3})^{1/4}}{z} \right] + \ln \epsilon_{mpt}$$

32. ϵ_{mpt}

$$\ln \epsilon_{mpt} = \rho_{mp} \ln \epsilon_{mpt-1} + \varepsilon_{mpt}$$

33. $g_t = G_t / A_t$

$$g_t = \left(1 - \frac{1}{\epsilon_{gt}} \right) y_t$$

34. ϵ_{gt}

$$\ln \epsilon_{gt} = (1-\rho_g) \ln \epsilon_g + \rho_g \ln \epsilon_{gt-1} + \varepsilon_{gt}$$

35. $gy_t = Y_t/Y_{t-1}$

$$gy_t = \frac{y_t}{y_{t-1}} z_t$$

36. $gc_t = C_t/C_{t-1}$

$$gc_t = \left(\frac{c_t}{c_{t-1}}\right) z_t$$

37. $gi_t = I_t/I_{t-1}$

$$gi_t = \left(\frac{i_t}{i_{t-1}}\right) z_t$$

38. $gw_t = \widetilde{W}_t/\widetilde{W}_{t-1}$

$$gw_t = \left(\frac{\widetilde{w}_t}{\widetilde{w}_{t-1}}\right) z_t$$

2.9 The steady state of the transformed economy

In the absence of shocks, the economy converges to a steady-state growth path in which all stationary variables are constant: for all t, $y_t = y$, $k_t = k$, $\overline{k}_t = \overline{k}$, $u_t = u = 1$, $\lambda_t = \lambda$, $\upsilon_t = \upsilon$, $\xi_t = \xi$, $c_t = c$, $\widetilde{r}_t^K = \widetilde{r}^K$, $i_t = i$, $g_t = g$, $N_t = N$, $S_t = S$, $U_t = U$, $V_t = V$, $\aleph_t = \aleph$, $m_t = m$, $x_t = x$, $q_t = q$, $s_t = s$, $\gimel_t = \gimel$, $\widetilde{w}_t = \widetilde{w}$, $\widetilde{\mathbf{b}} = \widetilde{\mathbf{b}}$, $R_t = R$, $\pi_t = \pi$, $\mu_t = \mu = 1$, $\epsilon_{bt} = \epsilon_b = 1$, $z_t = z$, $\zeta_t = \zeta$, $\theta_t = \theta$, $\eta_t = \eta$, $\epsilon_{gt} = \epsilon_g$, $\epsilon_{rt} = \epsilon_r$, $g_{yt} = g_{ct} = g_{It} = g_{At} = z$. Notice that the steady-state values μ, u and ϵ_b are normalized to 1.

1. μ_t

$$\mu = 1$$

2. ϵ_{bt}

$$\epsilon_b = 1$$

3. u_t

$$u = 1$$

4. z_t

z : calibrated at sample mean of gross quarterly growth rate of per-capita real output

5. gy_t

$$gy = z$$

6. gc_t

$$gc = z$$

7. gi_t

$$gi = z$$

8. gw_t

$$gw = z$$

9. g_t

$$\frac{g}{y} = \left(1 - \frac{1}{\epsilon_g}\right) := 0.20 \text{ (calibrated)}$$

10. ϵ_{gt}

$$\frac{1}{\epsilon_g} - \frac{\kappa}{2}\aleph_t^2 = \frac{c+i}{y}$$

11. $\aleph_t$

$$\aleph = \frac{\phi_V V + (1-\phi_V)\, m}{N}$$

12. m_t

$$m = qV$$

13. x_t

$$x = \frac{m}{N}$$

14. k_t

$$zk = \overline{k}$$

15. $\overline{k}_t$

$$(z - 1 + \delta)\, \overline{k} = zi$$

16. λ_t

$$\beta = \frac{\pi z}{R}$$

17. c_t

$$\lambda c = \frac{z - \beta h}{z - h}$$

18. $\widetilde{r}_t^K$

$$\phi_{u1} = \widetilde{r}^K$$

19. i_t

$$1 = \upsilon$$

20. υ_t

$$\frac{z}{\beta} = 1 - \delta + \widetilde{r}^K$$

21. N_t

$$\rho N = qV \quad \text{where} \quad \rho \equiv 1 - \chi$$

22. S_t

$$S = 1 - \chi N$$

23. U_t

U : calibrated at sample mean of unemployment rate

24. $\Theta_t = \frac{V_t}{S_t}$

$$\Theta = \frac{V}{S}$$

25. q_t

$q = \zeta\Theta^{-\sigma} := 0.7$ (calibrated. just a normalization)

26. s_t

$$s = \zeta\Theta^{1-\sigma}$$

27. ζ_t

ζ : backed out from the steady state condition $\zeta = q\left(\frac{V}{S}\right)^{\sigma}$

28. y_t

$$y = k^\alpha N^{1-\alpha}$$

29. ξ_t

$$\widetilde{r}^K = \alpha\frac{y}{k}\xi$$

30. V_t

$$\frac{(1-\beta)\,\chi}{\rho}\kappa\aleph^2 = \xi\,(1-\alpha) - \frac{\widetilde{w}N}{y}$$

31. θ_t

$$\xi = \frac{\theta - 1}{\theta}$$

32. π_t

π : calibrated at sample mean of gross quarterly growth rate GDP deflator

33. $\widetilde{\mathbf{b}}_t$

$$\widetilde{\mathbf{b}} = \tau \widetilde{w}$$

34. $\widetilde{w}_t$

$$\begin{aligned}
\widetilde{w} &= \eta \left[(1-\alpha)\, \xi \frac{y}{N} + \left(\frac{1}{N} + \chi \beta \frac{s}{m} \right) \kappa \aleph^2 y \right] + (1-\eta)\, \widetilde{\mathbf{b}} \\
\Leftrightarrow \frac{1 - (1-\eta)\, \tau}{\eta} \frac{\widetilde{w} N}{y} &= \xi\, (1-\alpha) + \left(1 + \beta \chi \frac{s}{\rho} \right) \kappa \aleph^2
\end{aligned}$$

35. Ŋ_t

$$\text{Ŋ} = \frac{\eta}{1-\eta}$$

36. η_t

η : backed out from steady state conditions (see Table 4 below)

37. ϵ_{rt}

$$\epsilon_{mp} = 1$$

38. R_t

R : calibrated at sample mean of gross quarterly nominal rate of interest

2.10 The loglinear model with rescaled shocks

Two disturbances are normalized prior to estimation: the price-markup shock $\widehat{\theta}_t$ and the wage bargaining shock $\widehat{\eta}_t$. Rescalling these two shocks only affects the New Keynesian Phillips Curve and the equation for the evolution of the effective bargaining power.

$$\begin{aligned}
\widehat{\theta}_t^* &= \left[\frac{1}{(1+\beta\varsigma)\, \phi_P} \right] \widehat{\theta}_t \\
\widehat{\theta}_t^* &= \rho_{\theta^*} \widehat{\theta}_{t-1}^* - \varepsilon_{\theta^* t} \\
\rho_{\theta^*} &= \rho_\theta \\
\varepsilon_{\theta^* t} &\sim i.i.d.N\left(0, \sigma_{\theta^*}^2\right) \\
\sigma_{\theta^*} &= \left[\frac{1}{(1+\beta\varsigma)\, \phi_P} \right] \sigma_\theta
\end{aligned}$$

$$\widehat{\eta}_t^* = \left(\frac{1}{1-\eta}\right)\widehat{\eta}_t$$
$$\widehat{\eta}_t^* = \rho_{\eta^*}\widehat{\eta}_{t-1}^* + \varepsilon_{\eta^* t}$$
$$\rho_{\eta^*} = \rho_\eta$$
$$\varepsilon_{\eta^* t} \sim i.i.d.N\left(0,\sigma_{\eta^*}^2\right)$$
$$\sigma_{\eta^*} = \left(\frac{1}{1-\eta}\right)\sigma_\eta$$

1. y_t

$$\frac{c+i}{y}\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \phi_{u1}\frac{k}{y}\widehat{u}_t + \frac{1}{\epsilon_g}\widehat{\epsilon}_{gt} + \kappa\aleph^2\widehat{\aleph}_t$$

2. k_t

$$\widehat{k}_t = \widehat{u}_t + \widehat{\overline{k}}_{t-1} - \widehat{z}_t$$

3. $\overline{k}_t$

$$z\widehat{\overline{k}}_t = (1-\delta)\,\widehat{\overline{k}}_{t-1} - (1-\delta)\,\widehat{z}_t + (z-1+\delta)\,\widehat{\mu}_t + (z-1+\delta)\,\widehat{i}_t$$

4. λ_t

$$\widehat{\lambda}_t = \widehat{\epsilon}_{bt} + \widehat{R}_t + \widehat{\lambda}_{t+1} - \widehat{\pi}_{t+1} - \widehat{z}_{t+1}$$

5. c_t

$$\widehat{\lambda}_t = \frac{\beta hz}{(z-\beta h)(z-h)}\widehat{c}_{t+1} - \frac{z^2+\beta h^2}{(z-\beta h)(z-h)}\widehat{c}_t + \frac{hz}{(z-\beta h)(z-h)}\widehat{c}_{t-1} + \frac{\beta hz}{(z-\beta h)(z-h)}\widehat{z}_{t+1} - \frac{hz}{(z-\beta h)(z-h)}\widehat{z}_t$$

6. $\widetilde{r}_t^K$

$$\widehat{\widetilde{r}}_t^K = \left(\frac{\phi_{u2}}{\phi_{u1}}\right)\widehat{u}_t$$

7. i_t

$$\widehat{\upsilon}_t = \left[(1+\beta)\left(\phi_I z^2\right)\right]\widehat{i}_t + \left(\phi_I z^2\right)\widehat{z}_t - \left(\phi_I z^2\right)\widehat{i}_{t-1} - \widehat{\mu}_t - \left(\beta\phi_I z^2\right)\widehat{i}_{t+1} - \left(\beta\phi_I z^2\right)\widehat{z}_{t+1}$$

8. υ_t

$$\widehat{\upsilon}_t = \widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{z}_{t+1} + \left[(1-\delta)\,\beta z^{-1}\right]\widehat{\upsilon}_{t+1} + \left(\beta z^{-1}\widetilde{r}^K\right)\widehat{\widetilde{r}}_{t+1}^K$$

9. u_t

$$\widehat{y}_t = \alpha\widehat{k}_t + (1-\alpha)\,\widehat{N}_t$$

10. ξ_t

$$\widehat{\xi}_t = \widehat{\widetilde{r}}_t^K - \widehat{y}_t + \widehat{k}_t$$

11. π_t

$$\widehat{\pi}_t = \left(\frac{\varsigma}{1+\beta\varsigma}\right)\widehat{\pi}_{t-1} + \left(\frac{\beta}{1+\beta\varsigma}\right)\widehat{\pi}_{t+1} + \left(\frac{1}{1+\beta\varsigma}\right)\left(\frac{\theta-1}{\phi_P}\right)\widehat{\xi}_t - \widehat{\theta}_t^*$$

12. N_t

$$\widehat{N}_t = \chi\widehat{N}_{t-1} + (1-\chi)\,\widehat{q}_t + (1-\chi)\,\widehat{V}_t$$

13. U_t

$$\widehat{U}_t = -\left(\frac{N}{1-N}\right)\widehat{N}_t$$

14. Θ_t

$$\widehat{\Theta}_t = \widehat{V}_t + \left(\frac{\chi N}{S}\right)\widehat{N}_{t-1}$$

15. q_t

$$\widehat{q}_t = \widehat{\zeta}_t - \sigma\widehat{\Theta}_t$$

16. s_t

$$\widehat{s}_t = \widehat{\zeta}_t + (1-\sigma)\,\widehat{\Theta}_t$$

17. $\aleph_t$:

$$\widehat{\aleph}_t = \left[\frac{\phi_V V}{\phi_V V + (1-\phi_V)\,m}\right]\widehat{V}_t + \left[\frac{(1-\phi_V)\,m}{\phi_V V + (1-\phi_V)\,m}\right]\widehat{m}_t - \widehat{N}_t$$

18. m_t :

$$\widehat{m}_t = \widehat{q}_t + \widehat{V}_t$$

19. x_t :

$$\widehat{x}_t = \widehat{m}_t - \widehat{N}_t$$

20. V_t :

$$\begin{aligned}
2\frac{\chi}{\rho}\kappa\aleph^2\widehat{\aleph}_t = {} & \frac{\kappa\aleph^2}{\rho}\widehat{x}_t + (1-\alpha)\,\xi\widehat{\xi}_t + \left(\frac{\widetilde{w}N}{y} - \frac{\beta\chi}{\rho}\kappa\aleph^2\right)\left(\widehat{y}_t - \widehat{N}_t\right) \\
& - \frac{\widetilde{w}N}{y}\widehat{\widetilde{w}}_t + \frac{\beta\chi}{\rho}\kappa\aleph^2\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{y}_{t+1} - \widehat{N}_{t+1} + 2\widehat{\aleph}_{t+1} - \widehat{x}_{t+1}\right)
\end{aligned}$$

21. $\widetilde{w}_t$:

$$\frac{1}{\eta}\frac{\widetilde{w}N}{y}\widehat{\widetilde{w}}_t = (1-\alpha)\,\xi\widehat{\xi}_t + \left[(1-\alpha)\,\xi + \kappa\aleph^2\right]\left(\widehat{y}_t - \widehat{N}_t\right) + 2\kappa\aleph^2\widehat{\aleph}_t$$
$$+ \beta\chi\frac{s}{\rho}\kappa\aleph^2\left(\widehat{s}_{t+1} + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + 2\widehat{\aleph}_{t+1} + \widehat{y}_{t+1} - \widehat{m}_{t+1}\right)$$
$$- \left[\frac{\widetilde{w}N}{y} - (1-\alpha)\,\xi - \left(1 + \frac{\beta\chi}{\rho}\right)\kappa\aleph^2\right]\widehat{\gimel}_t - \beta\chi\frac{(1-s)}{\rho}\kappa\aleph^2\widehat{\gimel}_{t+1}$$

22. $\gimel_t$

$$\widehat{\gimel}_t = \widehat{\eta}_t^* + \left(\beta\chi\phi_W\frac{y}{\widetilde{w}}\right)\widehat{z}_{t+1} + \left(\beta\chi\phi_W\frac{y}{\widetilde{w}}\right)\widehat{\pi}_{t+1} + \left(\beta\chi\phi_W\frac{y}{\widetilde{w}}\right)\widehat{\widetilde{w}}_{t+1} - \left[\left(\phi_W\frac{y}{\widetilde{w}}\right)(1+\beta\chi)\right]\widehat{\widetilde{w}}_t$$
$$- \left[\left(\phi_W\frac{y}{\widetilde{w}}\right)(1+\beta\chi\varrho)\right]\widehat{\pi}_t - \left(\phi_W\frac{y}{\widetilde{w}}\right)\widehat{z}_t + \left(\phi_W\frac{y}{\widetilde{w}}\right)\widehat{\widetilde{w}}_{t-1} + \left(\phi_W\frac{y}{\widetilde{w}}\right)\varrho\widehat{\pi}_{t-1}$$

23. R_t

$$\widehat{R}_t = \rho_r\widehat{R}_{t-1} + \frac{(1-\rho_r)\,\rho_\pi}{4}\left(\widehat{\pi}_t + \widehat{\pi}_{t-1} + \widehat{\pi}_{t-2} + \widehat{\pi}_{t-3}\right)$$
$$+ \frac{(1-\rho_r)\,\rho_y}{4}\left(\widehat{gy}_t + \widehat{gy}_{t-1} + \widehat{gy}_{t-2} + \widehat{gy}_{t-3}\right) + \widehat{\epsilon}_{mpt}$$

24. $gy_t = Y_t/Y_{t-1}$

$$\widehat{gy}_t = \widehat{y}_t - \widehat{y}_{t-1} + \widehat{z}_t$$

25. $gc_t = C_t/C_{t-1}$

$$\widehat{gc}_t = \widehat{c}_t - \widehat{c}_{t-1} + \widehat{z}_t$$

26. $gi_t = I_t/I_{t-1}$

$$\widehat{gi}_t = \widehat{i}_t - \widehat{i}_{t-1} + \widehat{z}_t$$

27. $gw_t = \widetilde{W}_t/\widetilde{W}_{t-1}$

$$\widehat{gw}_t = \widehat{\widetilde{w}}_t - \widehat{\widetilde{w}}_{t-1} + \widehat{z}_t$$

28. μ_t

$$\widehat{\mu}_t = \rho_\mu\widehat{\mu}_{t-1} + \varepsilon_{\mu t}$$

29. ϵ_{bt}

$$\widehat{\epsilon}_{bt} = \rho_b\widehat{\epsilon}_{bt-1} + \varepsilon_{bt}$$

30. z_t

$$\widehat{z}_t = \rho_z\widehat{z}_{t-1} + \varepsilon_{zt}$$

31. ζ_t

$$\widehat{\zeta}_t = \rho_\zeta \widehat{\zeta}_{t-1} + \varepsilon_{\zeta t}$$

32. θ_t

$$\widehat{\theta}_t = \rho_\theta \widehat{\theta}_{t-1} + \varepsilon_{\theta t}$$

33. η_t

$$\widehat{\eta}_t = \rho_\eta \widehat{\eta}_{t-1} + \varepsilon_{\eta t}$$

34. ϵ_{gt}

$$\widehat{\epsilon}_{gt} = \rho_g \widehat{\epsilon}_{gt-1} + \varepsilon_{gt}$$

35. ϵ_{mt}

$$\widehat{\epsilon}_{mpt} = \rho_{mp} \widehat{\epsilon}_{mpt-1} + \varepsilon_{mpt}$$

2.11 Natural equilibrium: no nominal rigidities, no markup shocks and no bargaining power shocks

We compute the natural equilibrium by setting equal to zero the two parameters ϕ_P and ϕ_W that governs the degree of nominal rigidities in prices and wages respectively and by turning off the price-markup shock θ_t and the bargaining-power shock η_t.

1. c_t^p

$$\frac{c+i}{y}\widehat{y}_t^p = \frac{c}{y}\widehat{c}_t^p + \frac{i}{y}\widehat{i}_t^p + \phi_{u1}\frac{k}{y}\widehat{u}_t^p + \kappa \aleph^2 \widehat{\aleph}_t^p + \frac{1}{\epsilon_g}\widehat{\epsilon}_{gt}$$

2. k_t^p

$$\widehat{k}_t^p = \widehat{u}_t^p + \widehat{\overline{k}}_{t-1}^p - \widehat{z}_t$$

3. $\overline{k}_t^p$

$$z\widehat{\overline{k}}_t^p = (1-\delta)\,\widehat{\overline{k}}_{t-1}^p - (1-\delta)\,\widehat{z}_t + (z-1+\delta)\,\widehat{\mu}_t + (z-1+\delta)\,\widehat{i}_t^p$$

4. $\widetilde{R}_t^p$

$$\widehat{\lambda}_t^p = \widehat{\epsilon}_{bt} + \widehat{\widetilde{R}}_t^p + \widehat{\lambda}_{t+1}^p - \widehat{z}_{t+1}$$

5. λ_t^p

$$\widehat{\lambda}_t^p = \frac{\beta hz}{(z-\beta h)(z-h)}\widehat{c}_{t+1}^p - \frac{z^2+\beta h^2}{(z-\beta h)(z-h)}\widehat{c}_t^p + \frac{hz}{(z-\beta h)(z-h)}\widehat{c}_{t-1}^p + \frac{\beta hz}{(z-\beta h)(z-h)}\widehat{z}_{t+1} - \frac{hz}{(z-\beta h)(z-h)}\widehat{z}_t$$

6. u_t^p

$$\widehat{\widetilde{r}}_t^{K,p} = \left(\frac{\phi_{u2}}{\phi_{u1}}\right)\widehat{u}_t^p$$

7. i_t^p

$$\widehat{\upsilon}_t^p = \left[(1+\beta)\left(\phi_I z^2\right)\right]\widehat{i}_t^p + \left(\phi_I z^2\right)\widehat{z}_t - \left(\phi_I z^2\right)\widehat{i}_{t-1}^p - \widehat{\mu}_t - \left(\beta\phi_I z^2\right)\widehat{i}_{t+1}^p - \left(\beta\phi_I z^2\right)\widehat{z}_{t+1}$$

8. υ_t^p

$$\widehat{\upsilon}_t^p = \widehat{\lambda}_{t+1}^p - \widehat{\lambda}_t^p - \widehat{z}_{t+1} + \left[(1-\delta)\beta z^{-1}\right]\widehat{\upsilon}_{t+1}^p + \left(\beta z^{-1}\widetilde{r}^K\right)\widehat{\widetilde{r}}_{t+1}^{K,p}$$

9. y_t^p

$$\widehat{y}_t^p = \alpha\widehat{k}_t^p + (1-\alpha)\widehat{N}_t^p$$

10. $\widetilde{r}_t^{K,p}$

$$\widehat{\widetilde{r}}_t^{K,p} = \widehat{y}_t^p - \widehat{k}_t^p$$

11. N_t^p

$$\widehat{N}_t^p = \chi\widehat{N}_{t-1}^p + (1-\chi)\widehat{q}_t^p + (1-\chi)\widehat{V}_t^p$$

12. U_t^p

$$\widehat{U}_t^p = -\left(\frac{N}{1-N}\right)\widehat{N}_t^p$$

13. Θ_t^p

$$\widehat{\Theta}_t^p = \widehat{V}_t^p + \left(\frac{\chi N}{S}\right)\widehat{N}_{t-1}^p$$

14. q_t^p

$$\widehat{q}_t^p = \widehat{\zeta}_t - \sigma\widehat{\Theta}_t^p$$

15. s_t^p

$$\widehat{s}_t^p = \widehat{\zeta}_t + (1-\sigma)\widehat{\Theta}_t^p$$

16. $\aleph_t^p$

$$\widehat{\aleph}_t^p = \left[\frac{\phi_V V}{\phi_V V + (1-\phi_V)\, m}\right] \widehat{V}_t^p + \left[\frac{(1-\phi_V)\, m}{\phi_V V + (1-\phi_V)\, m}\right] \widehat{m}_t^p - \widehat{N}_t^p$$

17. m_t^p

$$\widehat{m}_t^p = \widehat{q}_t^p + \widehat{V}_t^p$$

18. x_t^p

$$\widehat{x}_t^p = \widehat{m}_t^p - \widehat{N}_t^p$$

19. V_t^p

$$\begin{aligned} 2\frac{\chi}{\rho}\kappa\aleph^2\widehat{\aleph}_t^p = {} & \frac{\kappa\aleph^2}{\rho}\widehat{x}_t^p + \left(\frac{\widetilde{w}N}{y} - \frac{\beta\chi}{\rho}\kappa\aleph^2\right)\left(\widehat{y}_t^p - \widehat{N}_t^p\right) - \frac{\widetilde{w}N}{y}\widehat{\widetilde{w}}_t^p \\ & + \frac{\beta\chi}{\rho}\kappa\aleph^2\left(\widehat{\lambda}_{t+1}^p - \widehat{\lambda}_t^p + \widehat{y}_{t+1}^p - \widehat{N}_{t+1}^p + 2\widehat{\aleph}_{t+1}^p - \widehat{x}_{t+1}^p\right) \end{aligned}$$

20. $\widetilde{w}_t^p$

$$\begin{aligned} \frac{1}{\eta}\frac{\widetilde{w}N}{y}\widehat{\widetilde{w}}_t^p = {} & \left[(1-\alpha)\,\xi + \kappa\aleph^2\right]\left(\widehat{y}_t^p - \widehat{N}_t^p\right) + 2\kappa\aleph^2\widehat{\aleph}_t^p \\ & + \beta\chi\frac{s}{\rho}\kappa\aleph^2\left(\widehat{s}_{t+1}^p + \widehat{\lambda}_{t+1}^p - \widehat{\lambda}_t^p + 2\widehat{\aleph}_{t+1}^p + \widehat{y}_{t+1}^p - \widehat{m}_{t+1}^p\right) \end{aligned}$$

3 Empirical analysis

3.1 Calibrated parameters

We calibrate 13 parameters. The steady-state values of output growth, inflation, the interest rate and the unemployment rate are set equal to their respective sample average over the period 1957Q1-2008Q3 in the baseline estimation (or, in the sensitivity analysis: 1957Q1-2013Q2 and 1985Q1-2008Q3). The value for the elasticity of the matching function with respect to unemployment is based on the recent estimates obtained by Barnichon and Figura (2014), Justiniano and Michelacci (2011), Lubik (2013), Shimer (2005) and Sedlacek (2014). The calibration of the job destruction rate is based on Yashiv (2006). The calibration of the replacement rate is a conservative value advocated by Shimer (2005). These choices avoid indeterminacy issues that are widespread in this kind of model, as shown by Kurozumi and Van Zandweghe (2010) among others. In preliminary estimation rounds, the estimate of the parameter governing the degree of indexation to past inflation was systematically driven towards zero. This phenomenon is consistent with the findings reported by Ireland (2007). It is also in line with the microevidence on price-setting behavior. Hence we calibrate that parameter to 0.01. The quarterly depreciation rate is set equal to 0.025. The capital share of output is calibrated at 0.33. The elasticity of substitution between intermediate goods is set equal to 6, implying a steady-state markup of 20 percent as in Rotemberg and Woodford (1995). The vacancy-filling rate is set equal to 0.70, which is just a normalization. The steady-state government spending/output ratio is set equal to 0.20.

Table 1: Calibrated parameters

		$57Q1-08Q3$	$57Q1-13Q2$	$85Q1-08Q3$
Capital depreciation rate	δ	0.0250	0.0250	0.0250
Capital share	α	0.33	0.33	0.33
Elasticity of substitution btw goods	θ	6	6	6
Backward-looking price setting	ς	0.01	0.01	0.01
Replacement rate	τ	0.4	0.4	0.4
Job destruction rate	ρ	0.085	0.085	0.085
Elasticity of matches to unemp.	σ	0.65	0.65	0.65
Probability to fill a vacancy within a quarter	q	0.7000	0.7000	0.7000
Exogenous spending/output ratio	g/y	0.2000	0.2000	0.2000
Unemployment rate	U	0.0578	0.0602	0.0564
Quarterly gross growth rate	z	1.0039	1.0038	1.0043
Quarterly gross inflation rate	π	1.0088	1.0083	1.0061
Quarterly gross nominal interest rate	R	1.0139	1.0128	1.0125

3.2 Bayesian estimation

Our priors are standard (Smets and Wouters 2007; Gertler, Sala and Trigari 2008). We normalize the price-markup shock and the wage-markup shock, so that these enter with a unit coefficient in the model's equations. Such procedure facilitates the identification of the standard deviations of these two disturbances.

We use the random walk Metropolis-Hasting algorithm to generate 500,000 draws from the posterior distribution. The algorithm is tuned to achieve an acceptance ratio between 25 and 30 percent. We discard the first 250,000 draws. Tables 2 and 3 summarize the priors.

Table 2: Priors of structural parameters

		Priors
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)
Hiring cost/output ratio (rescaled)	$\left(\frac{\kappa}{2}\aleph^2\right) \times 1000$	IGamma (5,1)
Habit persistence in comsump.	h	Beta (0.7,0.1)
Investment adjustment cost	ϕ_I	IGamma (5,1)
Capital utilization cost	ϕ_{u2}	IGamma (0.5,0.1)
Price adjustment cost	ϕ_P	IGamma (60,10)
Wage adjustment cost	ϕ_W	IGamma (150,25)
Backward-looking wage setting	ϱ	Beta (0.5,0.2)
Interest rate smoothing	ρ_r	Beta (0.7,0.1)
Response to inflation	ρ_π	IGamma (1.5,0.1)
Response to output growth	ρ_y	IGamma (0.5,0.1)

Table 3: Priors of shock parameters

		Priors
Technology growth	ρ_z	Beta (0.3,0.1)
	$100\sigma_z$	IGamma (0.1,3)
Monetary policy	ρ_{mp}	Beta (0.5,0.2)
	$100\sigma_{mp}$	IGamma (0.1,3)
Investment	ρ_μ	Beta (0.5,0.2)
	$100\sigma_\mu$	IGamma (0.1,3)
Risk premium	ρ_b	Beta (0.5,0.2)
	$100\sigma_b$	IGamma (0.1,3)
Matching efficiency	ρ_ζ	Beta (0.5,0.2)
	$100\sigma_\zeta$	IGamma (0.1,3)
Price markup (rescaled)	ρ_{θ^*}	Beta (0.5,0.2)
	$100\sigma_{\theta^*}$	IGamma (0.1,3)
Bargaining power (rescaled)	ρ_{η^*}	Beta (0.5,0.2)
	$100\sigma_{\eta^*}$	IGamma (0.1,3)
Government spending	ρ_g	Beta (0.7,0.1)
	$100\sigma_g$	IGamma (0.1,3)

Table 4: Parameters derived from steady-state conditions

Employment rate	N	$N = 1 - U$
Vacancy	V	$V = \frac{\rho N}{q}$
Matches	m	$m = qV$
Discount factor	β	$\beta = \frac{z\pi}{R}$
Job survival rate	χ	$\chi = 1 - \rho$
Mean of exogenous spending shock	ϵ_g	$\epsilon_g = \frac{1}{1-g/y}$
Real marginal cost	ξ	$\xi = \frac{\theta-1}{\theta}$
Quarterly net real rental rate of capital	$\widetilde{r}^K$	$\widetilde{r}^K = \frac{z}{\beta} - 1 + \delta$
Capital utilization cost first parameter	ϕ_{u1}	$\phi_{u1} = \widetilde{r}^K$
Capital/output ratio	k/y	$\frac{k}{y} = \frac{\alpha\xi}{\widetilde{r}^K}$
Investment/capital ratio	i/k	$\frac{i}{k} = z - 1 + \delta$
Investment/output ratio	i/y	$\frac{i}{y} = \frac{i}{k}\frac{k}{y}$
Consumption/output ratio	c/y	$\frac{c}{y} = \frac{1}{\epsilon_g} - \frac{\kappa}{2}\aleph^2 - \frac{i}{y}$
Pool of job seekers	S	$S = 1 - \chi N$
Matching function efficiency	ζ	$\zeta = q\left(\frac{V}{S}\right)^{\sigma}$
Job finding rate	s	$s = \zeta\left(\frac{V}{S}\right)^{1-\sigma}$
Employees' share of output	$\widetilde{w}N/y$	$\frac{\widetilde{w}N}{y} = \xi\left(1-\alpha\right) - \frac{(1-\beta)\chi}{\rho}2\left(\frac{\kappa}{2}\aleph^2\right)$
Bargaining power	η	$\eta = \frac{1-\tau}{\vartheta-\tau}$ where $\vartheta \equiv \frac{\xi(1-\alpha)+\left(1+\beta\chi\frac{s}{\rho}\right)2\left(\frac{\kappa}{2}\aleph^2\right)}{\frac{\widetilde{w}N}{y}}$
Effective bargaining power	Ŋ	$\text{Ŋ} = \frac{\eta}{1-\eta}$
Autocorrelation of (non-rescaled) markup shock	ρ_θ	$\rho_\theta = \rho_{\theta^*}$
Std dev of (non-rescaled) markup shock	σ_θ	$\sigma_\theta = \left[\left(1+\beta\varsigma\right)\phi_P\right]\sigma_{\theta^*}$
Autocorrelation of (non-rescaled) bargaining power shock	ρ_η	$\rho_\eta = \rho_{\eta^*}$
Std dev of (non-rescaled) bargaining power shock	σ_η	$\sigma_\eta = \left(1-\eta\right)\sigma_{\eta^*}$

3.3 Data transformation

X_t is the vector of observables at time t. X_t is expressed in logarithmic deviations from sample mean. X_t contains eight variables: the quarterly growth rate of output, the quarterly growth rate of consumption, the quarterly growth rate of investment, the quarterly growth rate of real wages, the vacancy rate, the unemployment rate, the quarterly inflation rate and the quarterly gross nominal interest rate

$$X_t = \begin{bmatrix} \ln(Y_t) - \ln(Y_{t-1}) - \ln(g_y) \\ \ln(C_t) - \ln(C_{t-1}) - \ln(g_c) \\ \ln(I_t) - \ln(I_{t-1}) - \ln(g_i) \\ \ln(W_t) - \ln(W_{t-1}) - \ln(g_w) \\ \ln(V_t) - \ln(V) \\ \ln(U_t) - \ln(U) \\ \ln(P_t) - \ln(P_{t-1}) - \ln(g_p) \\ \ln(R_t) - \ln(R) \end{bmatrix}.$$

Y_t is the level of real GDP per capita, C_t is the level of real consumption per capita, I_t is the level of real investment per-capita, W_t is the real wage, V_t is the ratio of the vacancy series constructed by Barnichon (2010) to the sum of the vacancy series and the number of employed people (cf. Justiniano and Michelacci, 2011), P_t is the level of the GDP deflator and R_t is the gross effective federal funds rate, expressed on a quarterly basis. Following the arguments in Shimer (2005), we are detrending the vacancy rate with an HP filter with a smoothing weight equal to 10^6 to remove the secular trend in the series (cf. also Justiniano and Michelacci 2011 and Davis, Faberman and Haltiwanger 2013).

4 Additional details on the propagation of shocks

In the main text we have concentrated our attention on the transmission mechanism for matching efficiency shocks. In this section we comment on the dynamics induced by the other shocks that are relatively standard. In Figure A0 we plot the responses of the actual and natural rates of unemployment to the six shocks that affect the natural rate. The natural rate of unemployment is defined as the counterfactual rate of unemployment that emerges in the presence of flexible prices and wages and thus corresponds to the concept of unemployment in Real Business Cycle models (Shimer 2005).

The responses of the actual rate are in line with the previous literature. Unemployment is countercyclical in response to all shocks. A partial exception is the case of the neutral technology shocks: on impact (and only on impact) an expansionary technology shock increases unemployment. This is a standard result in New Keynesian models due to the presence of nominal and real rigidities (cf. Galí 1999).

The natural rate does not react to monetary policy and risk premium shocks. It is well known that these shocks propagate only in the presence of nominal rigidities. The natural rate of unemployment reacts little also to technology and investment specific shocks. This result is also well known in the literature since Shimer (2005) and the following literature on the unemployment volatility puzzle. Notice that the nominal rigidities deliver a substantial propagation to these disturbances, thus meaning that the actual rate of unemployment is immune to the unemployment volatility puzzle. In contrast, the natural rate reacts little to technology and investment specific shocks, in line with the measures of natural rates obtained with statistical methods. In the absence of nominal rigidities, an exogenous increase in government spending leads to a very small rise in the unemployment rate. The negative wealth effect triggered by the fiscal impulse generates a fall in consumption and a rise in the real interest rate. Higher real interest rates provide firms with an incentive to raise the rate of capacity utilization, thereby substituting capital services for labor. This channel is amplified by the inelasticity of labor supply in the search and matching model.

As discussed in the main text, the matching efficiency shock has a larger effect on the natural rate than on the actual rate, unlike all the other shocks. This explains why the natural rate is driven almost exclusively by the matching efficiency shock.

In Figure A1 we plot simulated data on vacancies and unemployment conditional on each kind of disturbances. In each panel, the vertical and the horizontal axis correspond respectively to the vacancy rate and the unemployment rate, both expressed in percentage deviations from steady state. Each panel plots pseudo-data points simulated from the model calibrated at the posterior mode and drawing the i.i.d. innovations from normal distributions with mean zero and standard deviation set equal to the corresponding posterior mode estimate. We remark that only the mismatch shock generates a positive conditional correlation between unemployment and vacancies. This point is discussed in detail in the main text and is related to the presence of sticky prices and a pre-match component in total hiring costs. In the data unemployment and vacancies are strongly negatively correlated and, therefore, the other shocks have a better chance to explain aggregate dynamics. Nevertheless, mismatch shocks may play a role in periods when unemployment and vacancies move together.

In Figure A2 we plot the contribution of each shock to the Beveridge curve dynamics. The grey dots represents the dynamics induced by all the eight shocks together. The black dots show how each shock in isolation has moved the Beveridge curve over the period 2008:Q1-2013:Q2. Mismatch shocks have shifted the Beveridge curve to the right. Notice, however, that also other shocks explain part of the shift. All shocks are able to generate the loop typical of Beveridge curve dynamics in recent years and do not generate trajectories along a line. This point has been emphasized by Christiano, Eichenbaum and Trabandt (2014) in a recent paper. However, mismatch shocks are very important to match the shift to the right from a quantitative point of view and more so in recent years. Notice the large effects induced (in opposite directions) by risk premium shocks and fiscal shocks.

5 Sensitivity analysis

In this section we provide additional details on the sensitivity analysis that we conduct to investigate the robustness of our results. We modify the model along four dimensions: i) the sample period for estimation, ii) the calibration for the elasticity of the matching function to unemployment, iii) the calibration for the replacement rate, iv) the role of a time-varying separation rate. We describe each experiment in turn.

5.1 Sample period

In our baseline model the sample period used for estimation is 1957:Q1-2008:Q3. We now want to investigate the robustness of our results when we consider a longer sample (thus including the Great Recession) and a shorter sample (only the Great Moderation period).

In the first experiment we extend our sample period until 2013:Q2 to exploit the information on the recent shift of the Beveridge curve for estimation purposes. In Figures A3 to A8 we present our results for the extended sample. Matching efficiency is slightly more volatile (Figure A3) than in our baseline estimates but all in all these figures are almost identical to the ones for the baseline case.

In the second extension we focus on a shorter but more homogenous period as the Great Moderation (1985:Q1-2008:Q3). Our baseline sample period is long and may be subject to structural breaks. In contrast, the Great Moderation period is a period of relative stability that may be useful as a cross-check. In Figures A9 to A15 we present the results related to this experiment. Once again all our results on the role of matching efficiency shocks are confirmed. The only difference that we can identify with respect to the baseline case is that the relative importance of the other shocks change slightly, in particular for the risk premium shock. This point can be seen when comparing Figure A2 to Figure A12. However, even from a quantitative point of view these differences are minor. To sum up we conclude that the choice of the sample period for estimation purposes is largely inconsequential.

5.2 Alternative calibration of the matching function elasticity

A key parameter that affects directly the estimated series for matching efficiency shocks is the elasticity of the matching function to unemployment (σ).[4] In our baseline model we calibrate it at 0.65, a value in the middle of the range (0.55-0.75) found in a series of recent studies (Barnichon and Figura 2014; Justiniano and Michelacci 2011; Shimer 2005; Sedlacek 2014). These values are slightly higher than the ones advocated by Petrongolo and Pissarides (2001) and much higher than the value of 0.4 used by Blanchard and Diamond (1989). Given the importance of this parameter, we reestimate our model with σ equal to 0.55 (almost at the bottom of the Petrongolo and Pissarides' range) and with σ equal to 0.75 as in Justiniano and Michelacci (2011).

We plot the estimated series for matching efficiency shocks with σ calibrated at 0.55 in Figure A16. In our baseline case (Figure 3 in the main text) matching efficiency increases during some Recessions and declines in others. With σ equal to 0.55 matching efficiency becomes more countercyclical: it now often increases during Recessions with the clear exception of the Great Recession when we still identify a substantial decline, followed by a partial rebound and a new and even more pronounced decline. A different series for matching efficiency translates into a different estimate for the natural rate of unemployment given the prominent role of mismatch shocks in its dynamics. In Figure A17 we see that the low frequency dynamics of the

[4] In our model this parmeter should be called elasticity of the matching function to searchers since the pool of searchers is not equivalent to unempoyment.

natural rate are not affected. However, at high frequencies the correlation between the actual rate and the natural rate is now lower. The natural rate still increases during the Great Recession and keeps increasing in the aftermath as in our baseline case. Mismatch shocks are now less important to explain the shift in the Beveridge curve during the Great Recession (cf. Figure A19 and Figure 5 in the main text) but they are still crucial to explain why unemployment was so high in recent years. Mismatch shocks are still the dominant drivers of the natural rate as it can be seen in Figure A21. We conclude that our main results are confirmed but a low value of σ impacts the estimate of matching efficiency and the behavior of natural rate at high frequencies.

Not surprisingly we find the opposite results with a high value of σ. When σ is calibrated at 0.75, matching efficiency declines in almost all Recessions (thus becoming very procyclical, cf. Figure A22) and the natural rate of unemployment becomes more correlated with the actual rate at high frequencies (cf. Figure A23). Matching efficiency shocks are now crucial to explain the Beveridge curve dynamics both during the Great Recession and in its aftermath (cf. Figure A26).

5.3 Alternative calibration of the replacement rate

In our baseline model we use a conservative value for the replacement rate ($\tau = 0.4$) based on Shimer (2005). The replacement rate determines the value of the outside option for workers and is a contentious parameter in the literature. Higher values for the replacement rate, in combination with a low bargaining power for workers, have been used by Hagedorn and Manovskii (2008) among others to generate higher unemployment volatility in response to technology shocks in models with flexible prices and wages. Therefore, we may suspect that the dominant role of mismatch shocks in driving the natural rate in our baseline model may rely on a too limited propagation of the other real shocks. To investigate this issue, we set the replacement rate at 0.7 and we re-estimate the model over the same sample period. Figures A28 to A33 summarize the outcome of this experiment. All the main results described in our baseline model are confirmed under this alternative calibration. The only noticeable difference is that now mismatch shocks play a slightly lower role in the historical decomposition of the natural rate (Figure A33): now technology and investment-specific shocks propagate more under flexible prices and wages and thus play a larger role. Nevertheless, mismatch shocks are still the main drivers of the natural rate. We conclude that our results are robust to a different parameterization of the replacement rate.

5.4 Time-varying separation rate

In this last set of experiments we consider exogenous shocks to the separation rate. Hosios (1994) and Shimer (2005) among others have shown that shocks to the separation rate are also able to move unemployment and vacancies in the same direction.

Separation rate correlated with the state of the economy. In a first experiment we assume that the separation rate is negatively related to the state economy (i.e. the separation rate is low in good times) where the state of the economy is summarized by the technology and the investment-specific shocks, the two main drivers of business cycle fluctuations in our model. We assume the following specification:

$$\ln \rho_t = (1 - \rho_\rho) \ln \rho + \rho_\rho \ln \rho_{t-1} - \delta_z \varepsilon_{zt} - \delta_\mu \varepsilon_{\mu t} + \varepsilon_{\rho t}$$

where we impose in the estimation that δ_z and δ_μ have to be positive and $\varepsilon_{\rho t}$ represents an exogenous separation shock. The priors on the new parameters δ_z and δ_μ are Uniform. In this specification we extend the baseline model by including an additional shock (the separation shock) and by using an additional observable variable (the separation rate). More specifically, we use the transition probability from employment to unemployment corrected for margin error based on CPS data computed by Elsby, Hobijn and Sahin (2015). The sample period is 1968 Q1-2008Q3.

In Figures A34 to A38 we present graphically our results. In this version of our model matching efficiency increases during the Great Recession and declines only in the aftermath (Figure A34). The estimated series for the natural rate of unemployment is similar to the one derived in our baseline model (Figure A35). The increase in the separation rate during the Great Recession is mainly due to negative investment-specific shocks. Exogenous separation shocks play a role in the pre-Great Recession period and tend to lower the separation rate in recent years to compensate the effect of negative investment shocks (Figure A36). The historical decomposition for unemployment in Figure A37 reveals that matching efficiency shocks are less important in this specification of the model. Nevertheless, they still play a non-negligible role in slowing down the recovery in recent years. The natural rate of unemployment is now driven also by separation and investment-specific shocks. The role of technology and fiscal shocks is limited. The time-varying separation rate seems to be a powerful propagator for investment-specific shocks, as it can be seen also from Figure A39 where we see that they play a large role in generating the Beveridge curve dynamics observed in recent years. In contrast, exogenous separation shocks have shifted the Beveridge curve in the opposite direction.

Exogenous separation rate. In the last experiment we consider the case of a purely exogenous separation rate. The separation rate follows now the following process:

$$\ln \rho_t = (1 - \rho_\rho) \ln \rho + \rho_\rho \ln \rho_{t-1} + \varepsilon_{\rho t}$$

The results for this version of the model with nine observables and nine shocks are presented in Figures A40 to A44. Not surprisingly, exogenous separation shocks become more important in this case and are now the main drivers of the natural rate of unemployment. Nevertheless, the estimate of the natural rate is surprisingly stable across the different experiments. The decline in matching efficiency is again a feature of the post-Great Recession period when mismatch shock still contribute to slowing down the recovery and to increasing the natural rate of unemployment.

5.5 Summary

We conclude that when we change the sample period, the calibration for the elasticity of the matching function to unemployment or the calibration of the replacement rate, all our main results are confirmed. Matching efficiency shocks are not important drivers of the business cycle but they may play a role in selected periods and they are the most important driver of the natural rate. According to our analysis, they contribute substantially to explain the shift of the Beveridge curve and the weak recovery in the aftermath of the Great Recession. When we include separation shocks, the results change in some dimensions. On the one hand, mismatch shocks are not anymore the main drivers of the natural rate, although they still play a relevant role in recent years. On the other hand, the estimate of the natural rate is similar to the one obtained in our baseline model.

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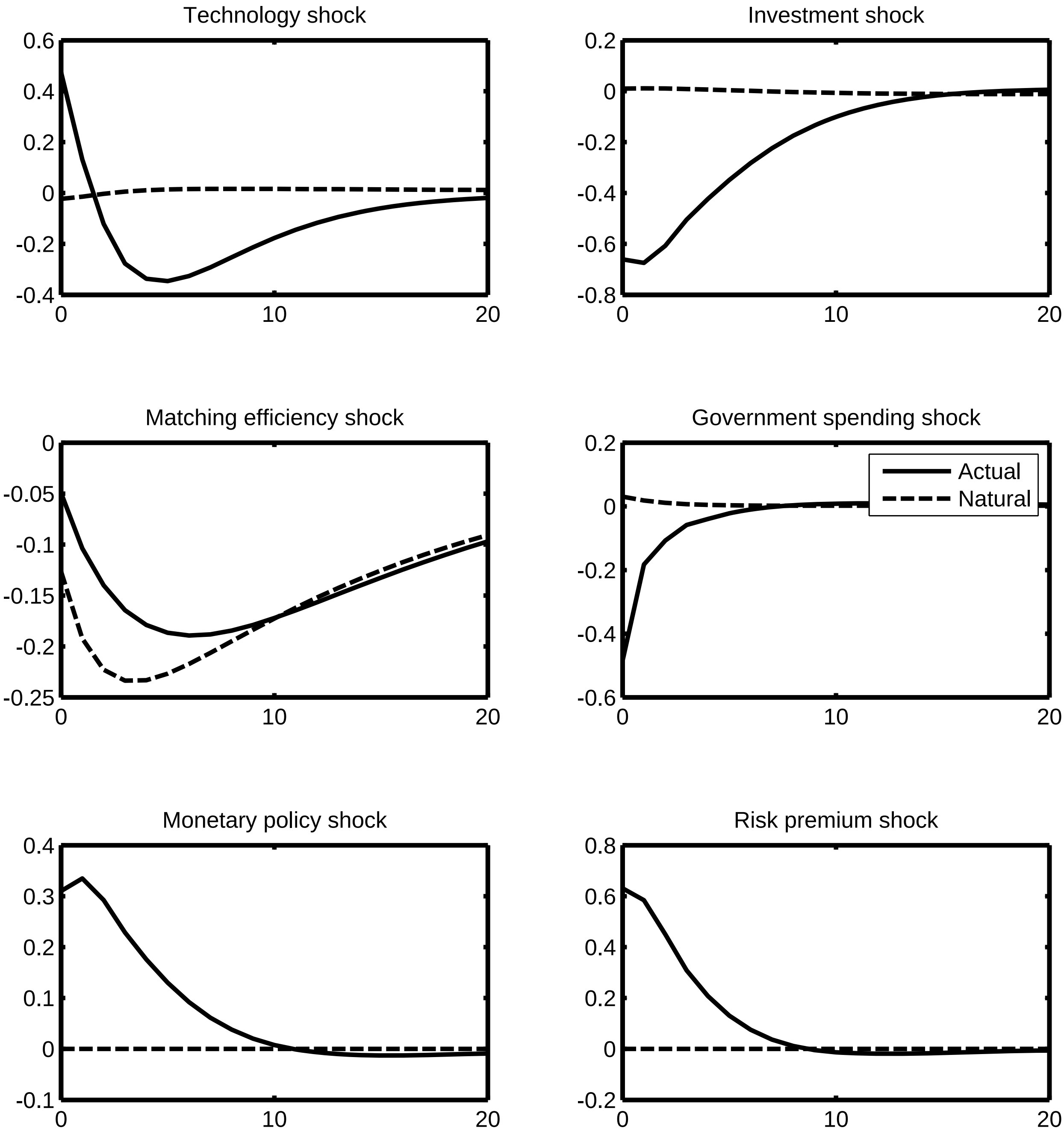


Fig. A0: Impulse responses of the actual and natural unemployment rates, expressed in percentage points. The responses are computed at the posterior mode. The size of each shock is one standard deviation.

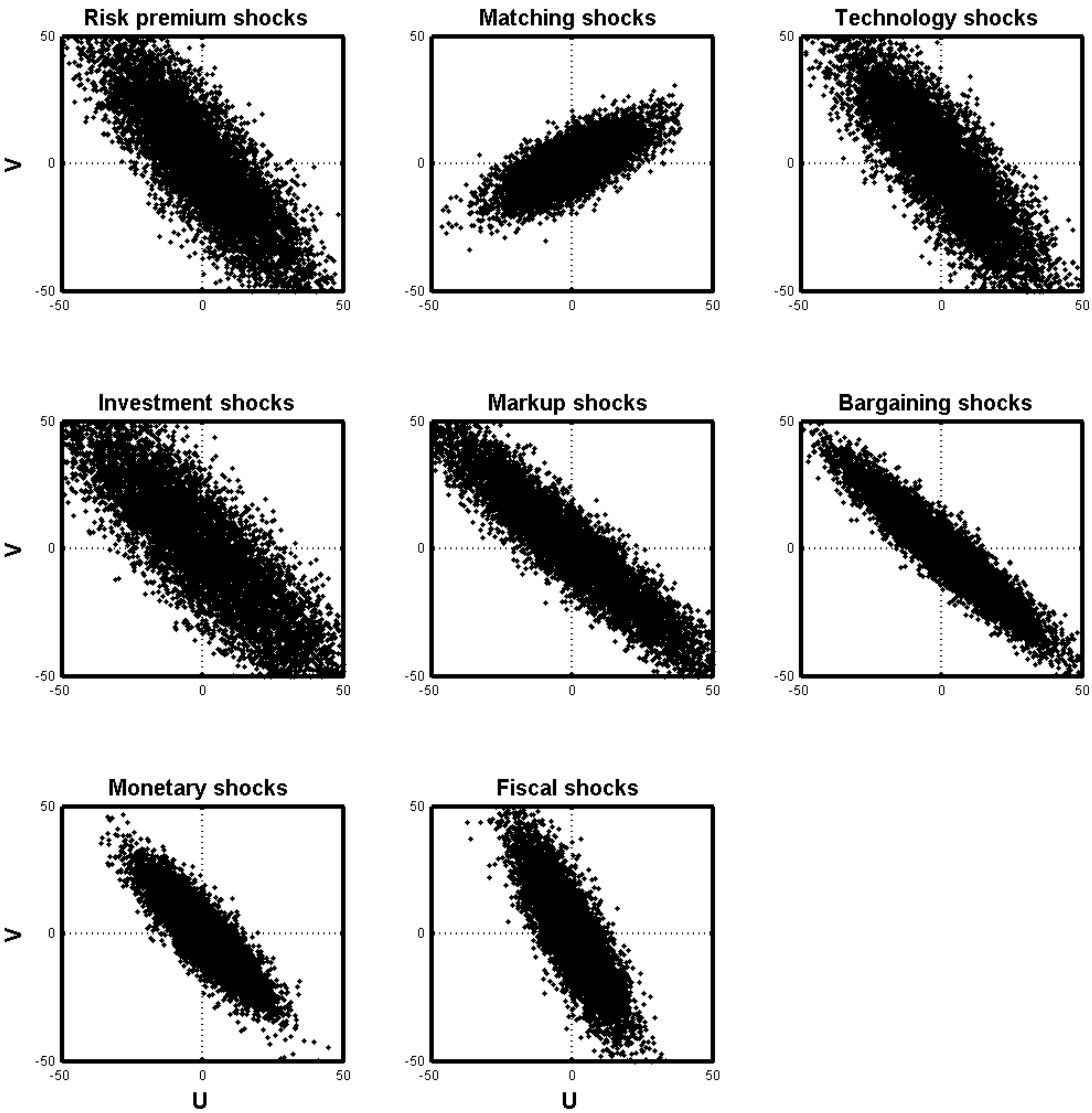


Fig A1: Simulated conditional Beveridge curves

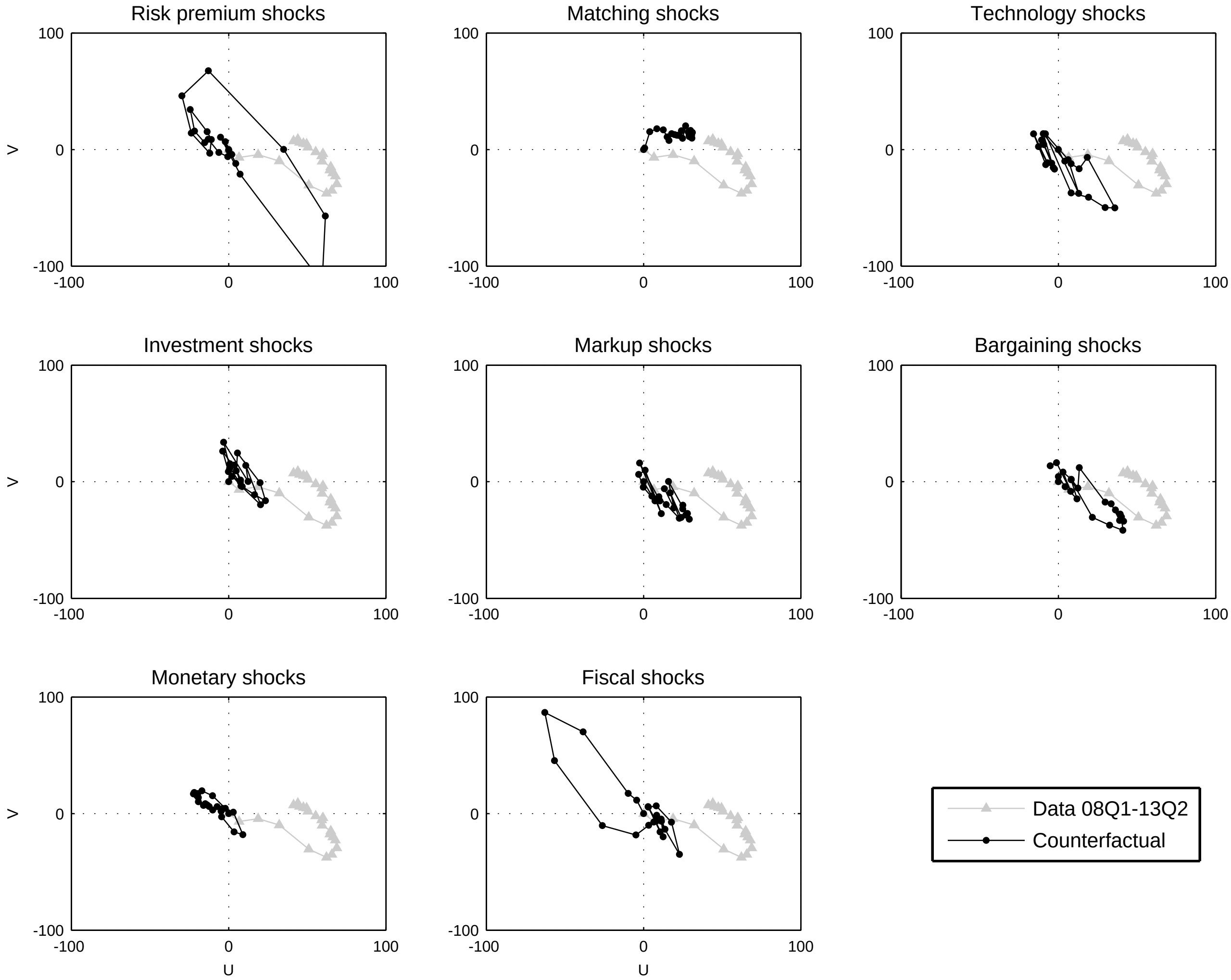


Fig A2: Contribution of each shock to the Beveridge curve 2008Q1-2013Q2 (% dev. from 2008Q1).

Robustness Check #1 - Estimation Period: 1957:Q1 - 2013:Q2

Calibrated Parameters: Check #1 [57:Q1-13:Q2]

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0602
Quarterly gross growth rate	z	1.0038
Quarterly gross inflation rate	π	1.0083
Quarterly gross nominal interest rate	R	1.0128

Priors and Posteriors: Check #1 [57:Q1-13:Q2]

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.34
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.40
Habit in consump.	h	Beta (0.7,0.1)	0.64
Invest. adj. cost	ϕ_I	IGamma (5,1)	2.90
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.63
Price adjust. cost	ϕ_P	IGamma (60,10)	60.94
Wage adjust. cost	ϕ_W	IGamma (150,25)	131.24
Wage indexation	ϱ	Beta (0.5,0.2)	0.96
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.34
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.97
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.36

Fig A3: Matching Efficiency (estim. period: 57Q1-13Q2)

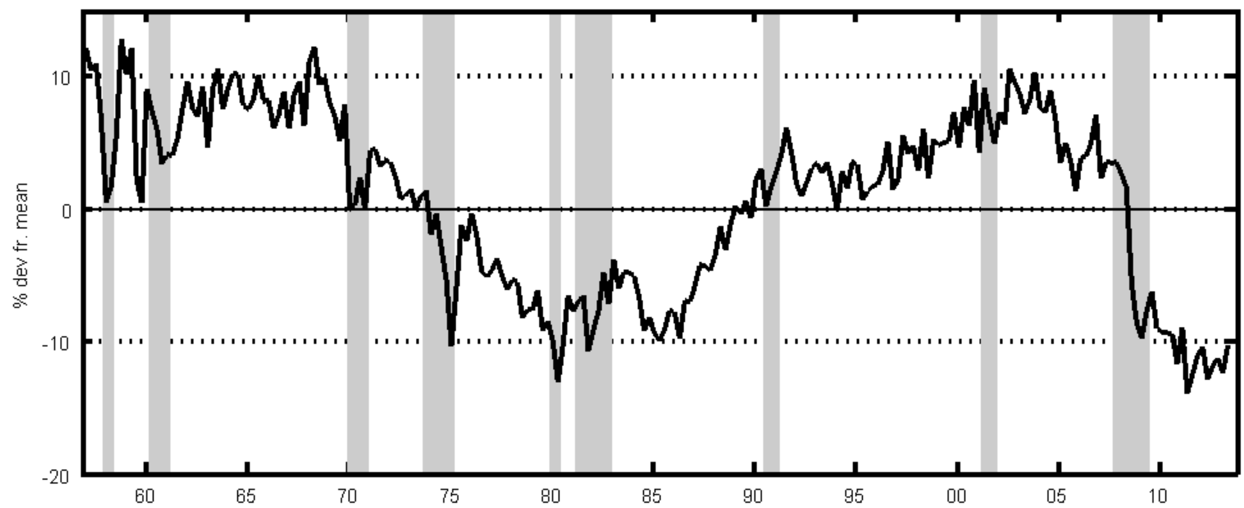


Fig A4: Unemployment Rates: Actual vs Natural (estim. period: 57Q1-13Q2)

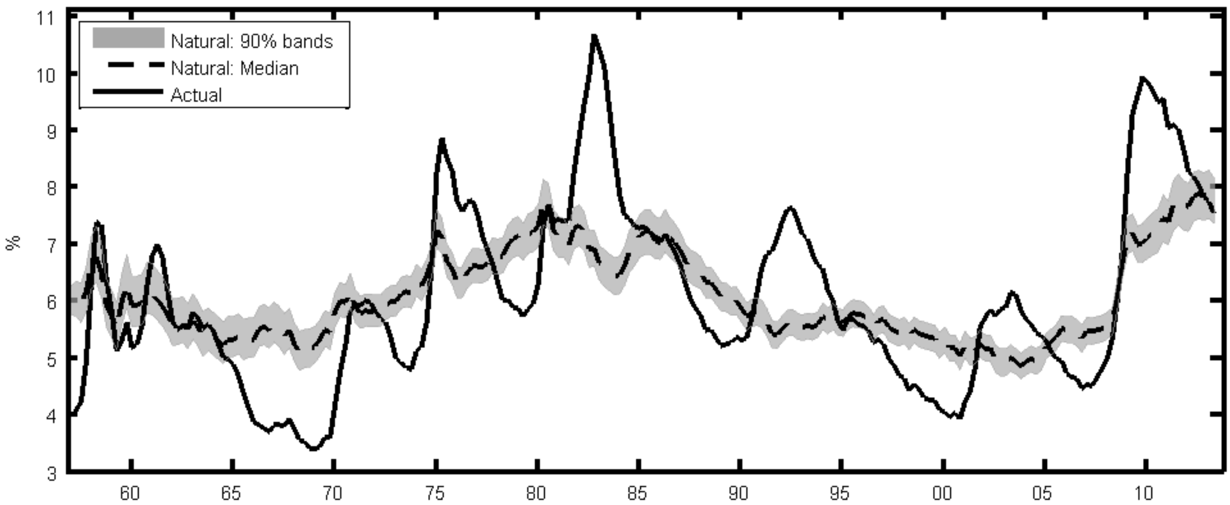


Fig A5: Unemployment Gap: Median and 90% Posterior Bands (estim. period: 57Q1-13Q2)

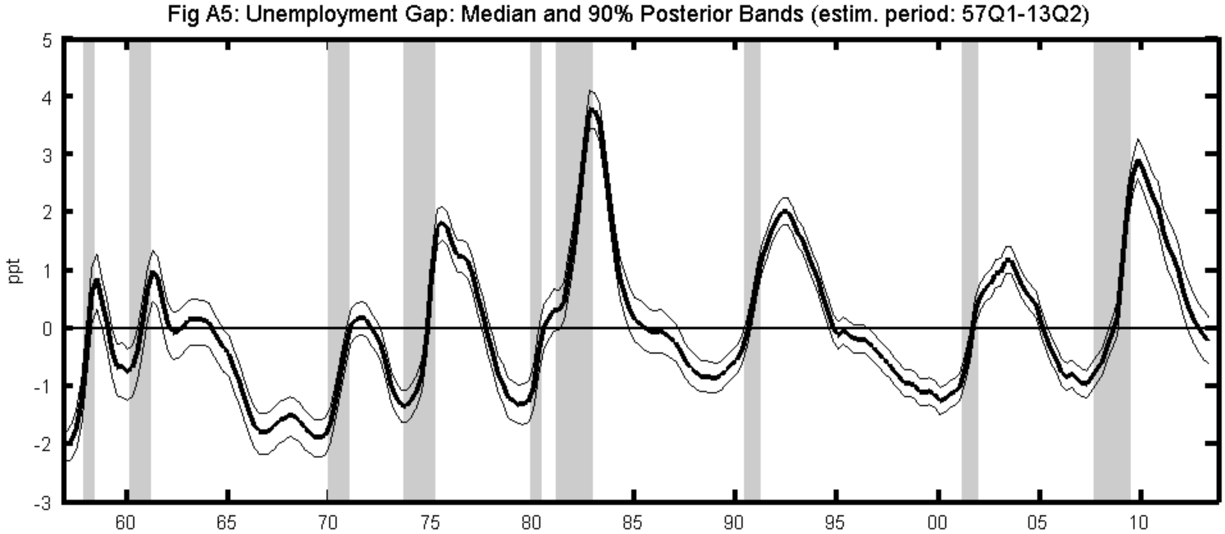


Fig A6: Beveridge Curve (estim. period: 57Q1-13Q2)

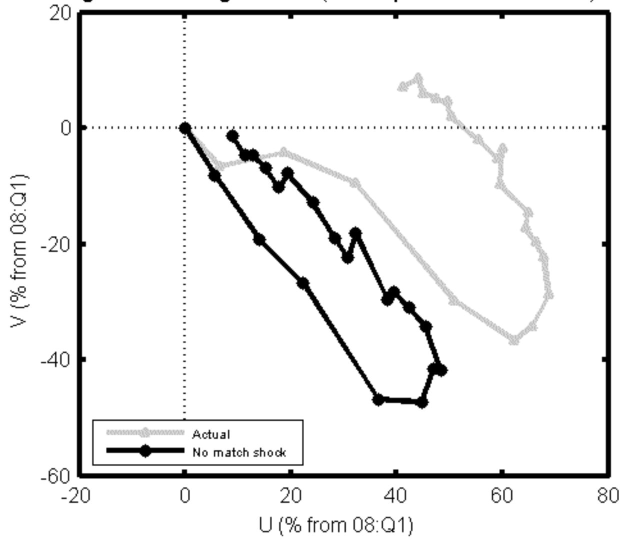


Fig A7: Historical Decomp. of Unemp. Rate (estim. period: 57Q1-13Q2)

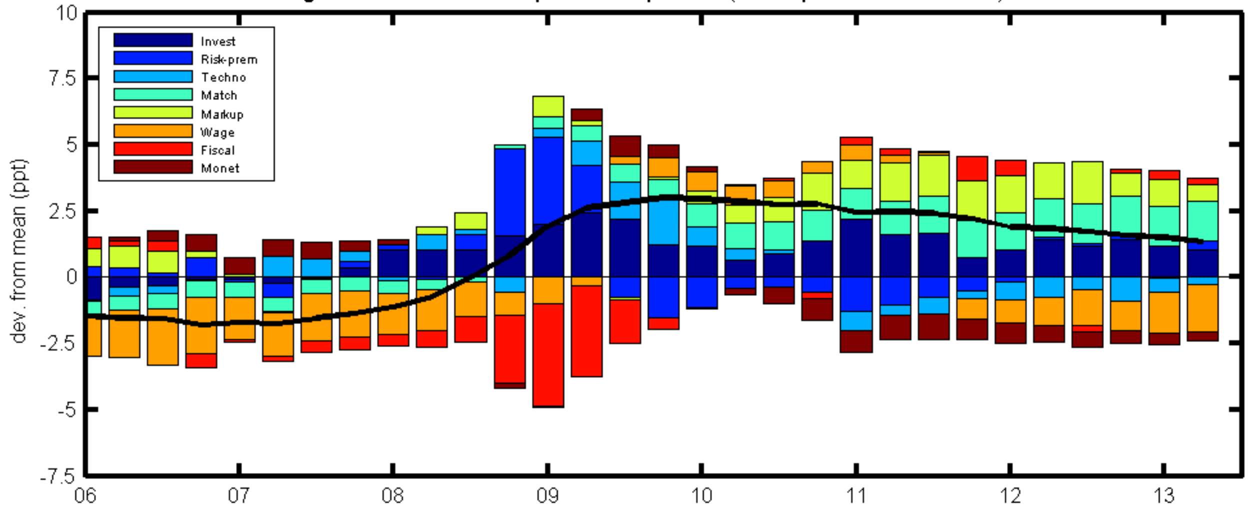
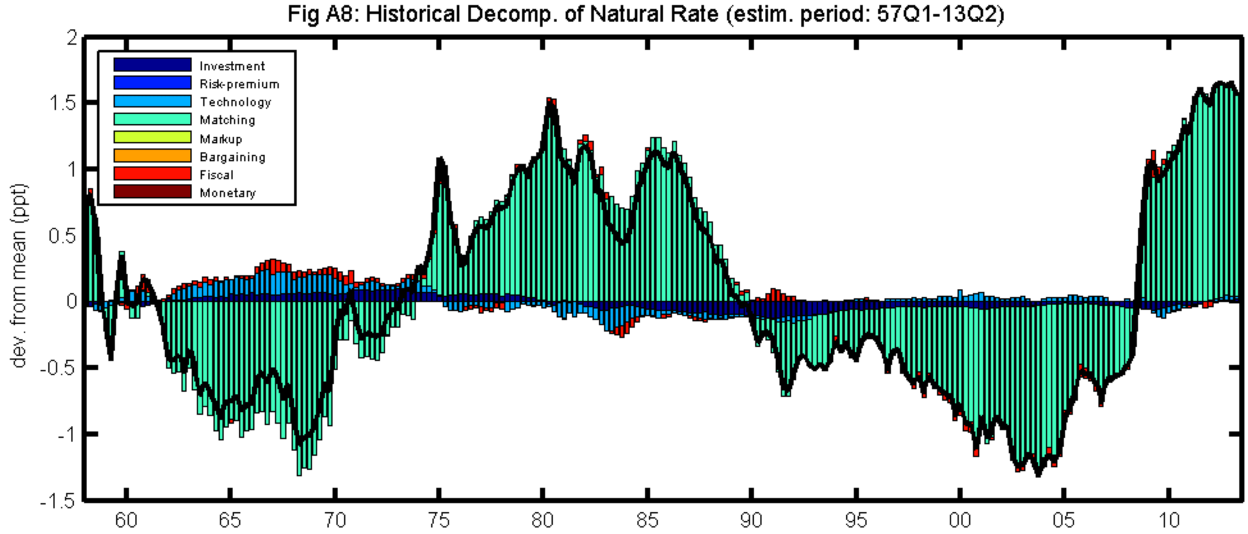


Fig A8: Historical Decomp. of Natural Rate (estim. period: 57Q1-13Q2)



Robustness Check #2 - Estimation Period: 1985:Q1 - 2008:Q3

Calibrated Parameters: Check #2 [85:Q1-08:Q3]

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0564
Quarterly gross growth rate	z	1.0043
Quarterly gross inflation rate	π	1.0061
Quarterly gross nominal interest rate	R	1.0125

Priors and Posteriors: Check #2 [85:Q1-08:Q3]

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.25
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	3.00
Habit in consump.	h	Beta (0.7,0.1)	0.54
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.7
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.54
Price adjust. cost	ϕ_P	IGamma (60,10)	81.65
Wage adjust. cost	ϕ_W	IGamma (150,25)	112.82
Wage indexation	ϱ	Beta (0.5,0.2)	0.71
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.59
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	2.33
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.41

Fig A9: Matching Efficiency (estim period: 85Q1-08Q3)

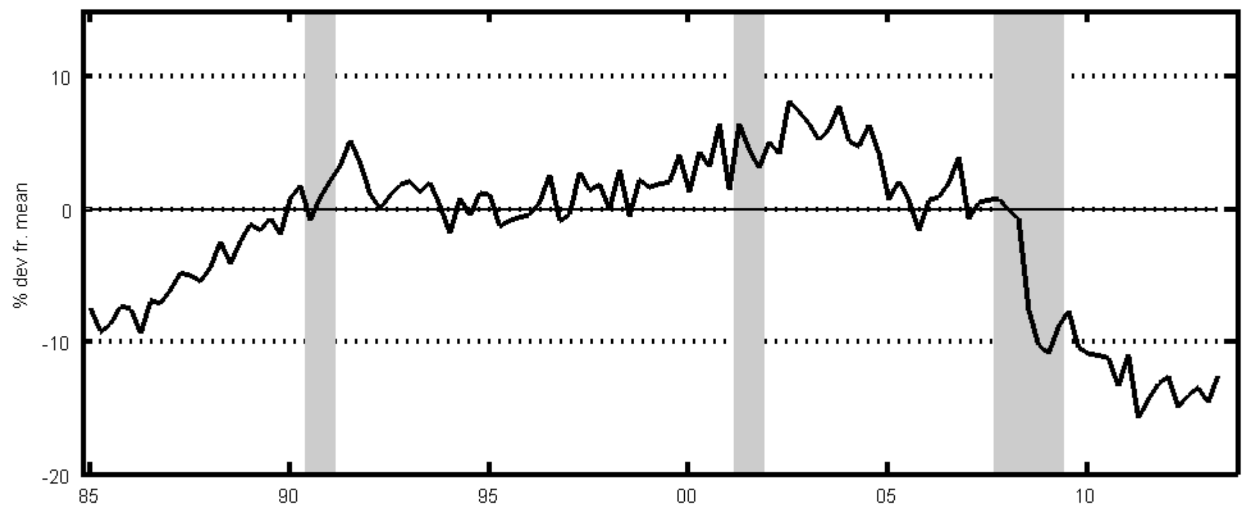


Fig A10: Unemployment Rates: Actual vs Natural (estim period: 85Q1-08Q3)

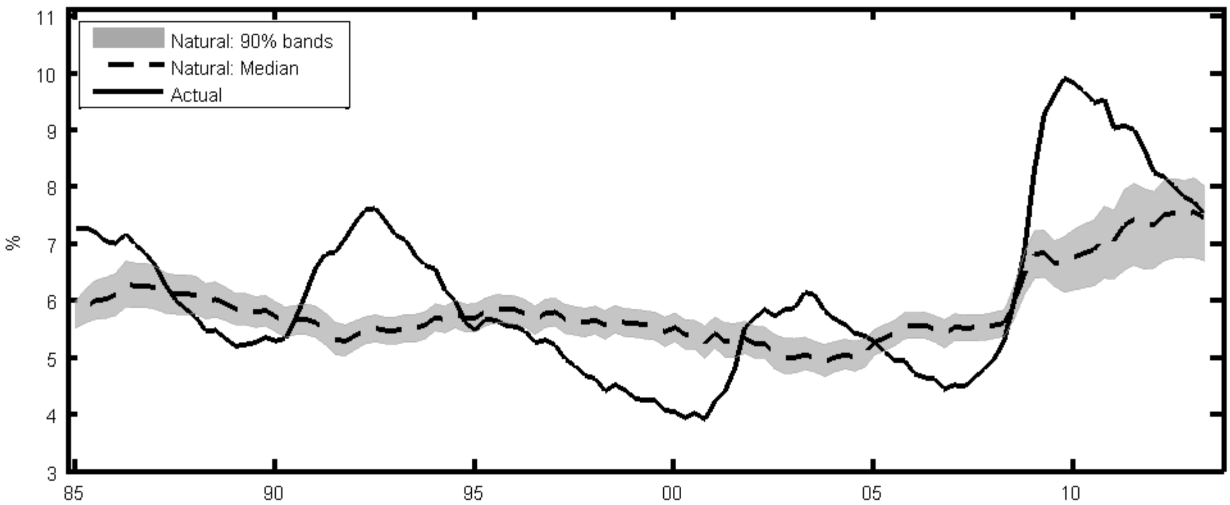
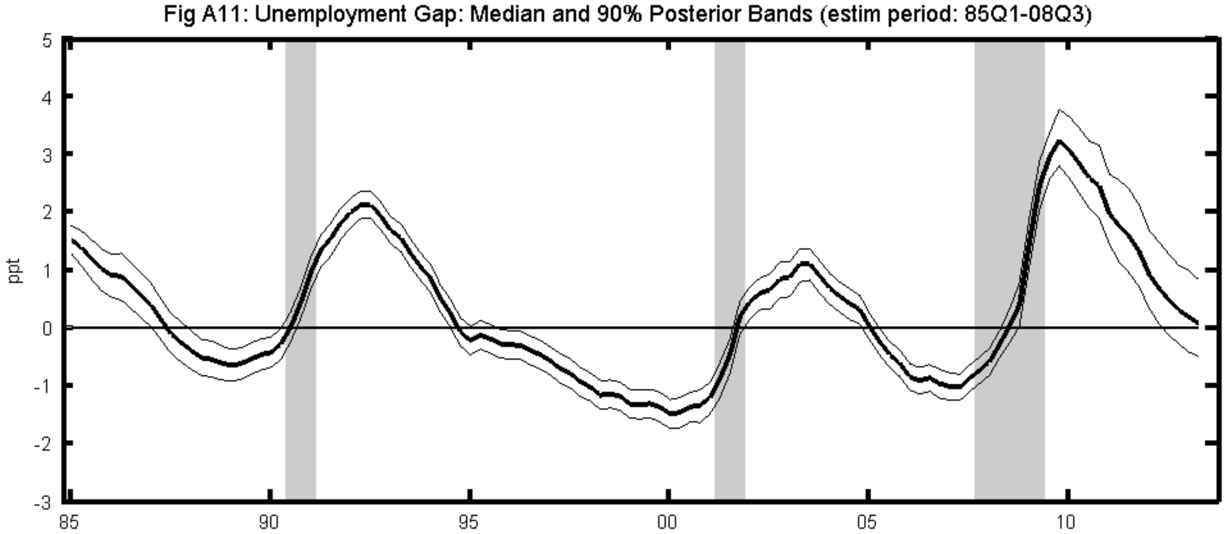


Fig A11: Unemployment Gap: Median and 90% Posterior Bands (estim period: 85Q1-08Q3)



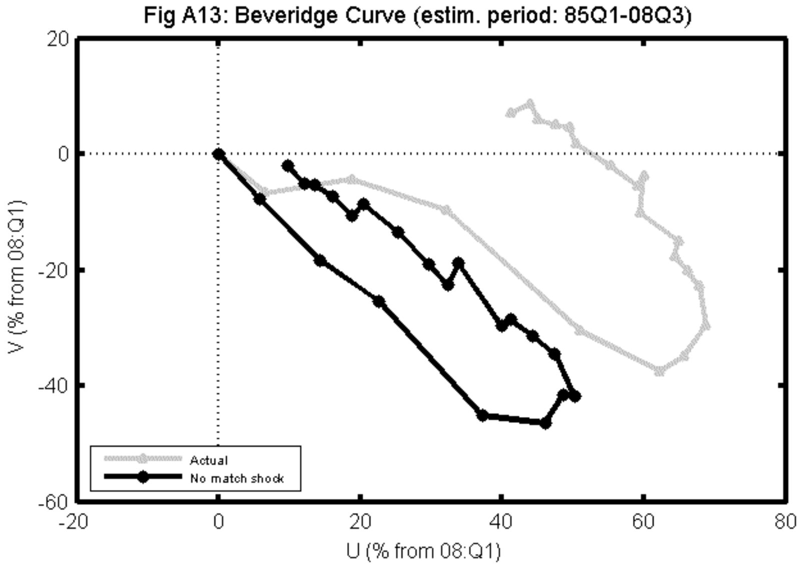


Fig A13: Beveridge Curve (estim. period: 85Q1-08Q3)
20
0
-20
-40
-60
V (% from 08:Q1)
-20
0
20
40
60
80
U (% from 08:Q1)
Actual
No match shock

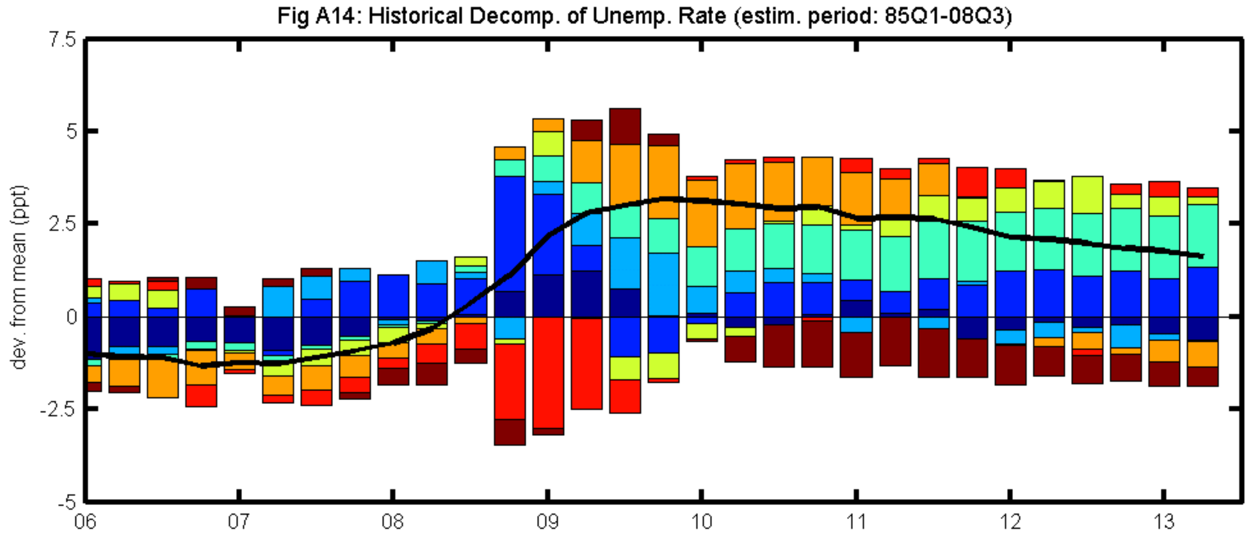


Fig A14: Historical Decomp. of Unemp. Rate (estim. period: 85Q1-08Q3)
7.5
5
2.5
0
-2.5
-5
dev. from mean (ppt)
06
07
08
09
10
11
12
13

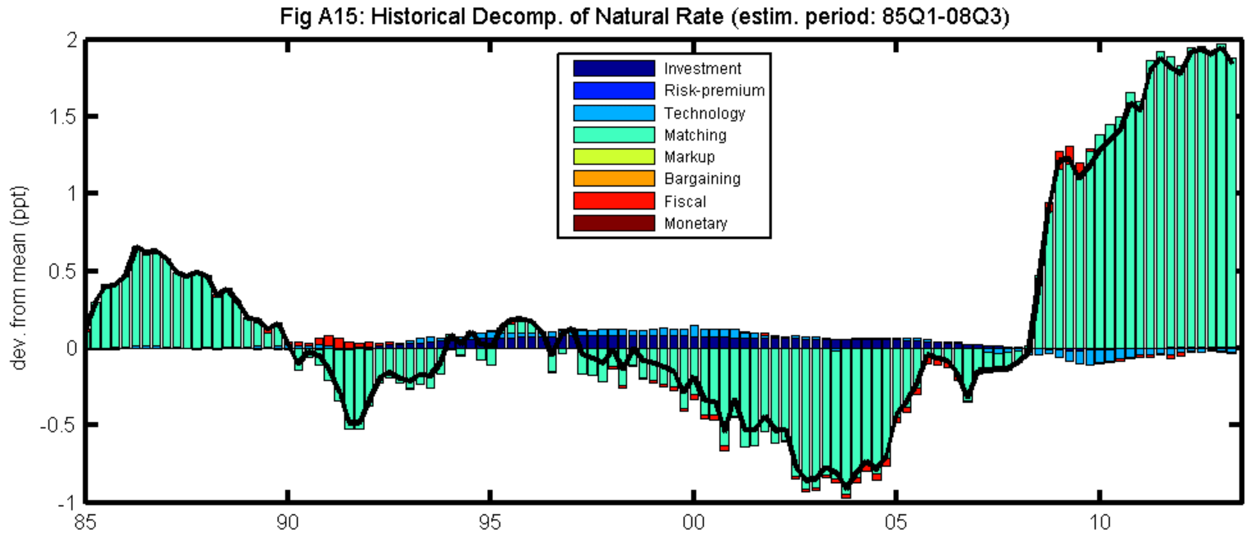


Fig A15: Historical Decomp. of Natural Rate (estim. period: 85Q1-08Q3)
2
1.5
1
0.5
0
-0.5
-1
dev. from mean (ppt)
85
90
95
00
05
10
Investment
Risk-premium
Technology
Matching
Markup
Bargaining
Fiscal
Monetary

Robustness Check #3 - $\sigma = 0.55$ (Est. Per.: 57:Q1 - 08:Q3)

Calibrated Parameters: Check #3 Low Sigma

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	$\mathbf{0.55}$
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0578
Quarterly gross growth rate	z	1.0039
Quarterly gross inflation rate	π	1.0088
Quarterly gross nominal interest rate	R	1.0139

Priors and Posteriors: Check #3 Low Sigma

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.34
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.56
Habit in consump.	h	Beta (0.7,0.1)	0.65
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.07
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.58
Price adjust. cost	ϕ_P	IGamma (60,10)	59.35
Wage adjust. cost	ϕ_W	IGamma (150,25)	141.29
Wage indexation	ϱ	Beta (0.5,0.2)	0.95
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.33
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.92
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.37

Fig A16: Matching Efficiency (σ = 0.55; estim period: 57-08Q3)

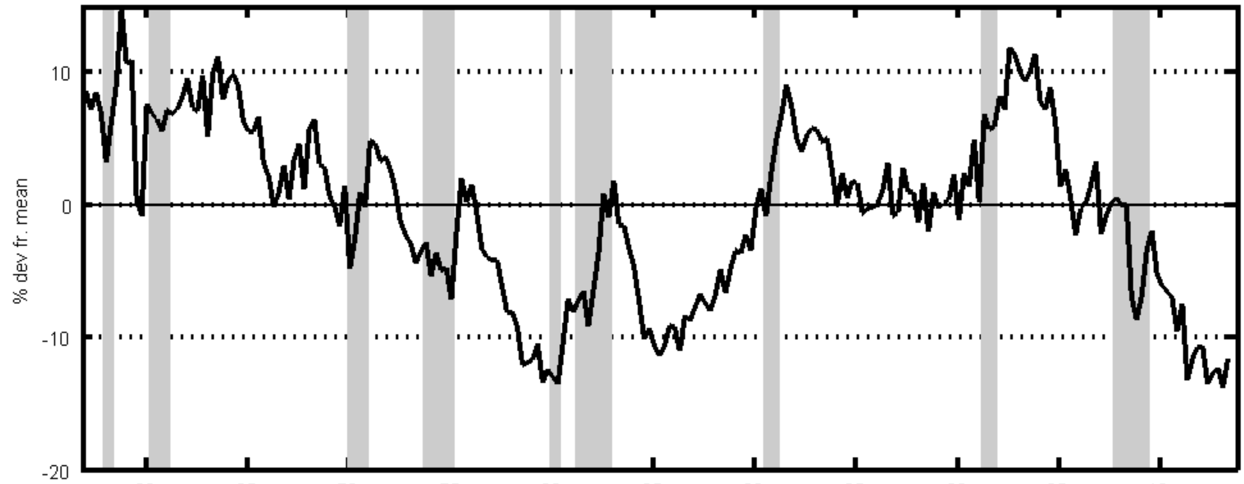


Fig A17: Unemployment Rates: Actual vs Natural (σ = 0.55; estim period: 57-08Q3)

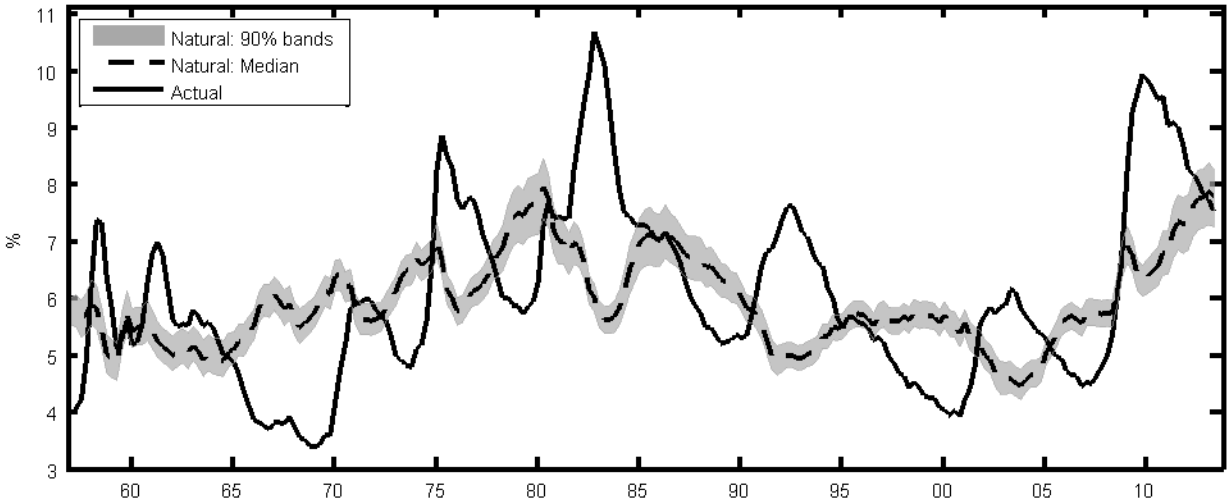


Fig A18: Unemployment Gap: Median and 90% Bands (σ = 0.55; estim period: 57-08Q3)

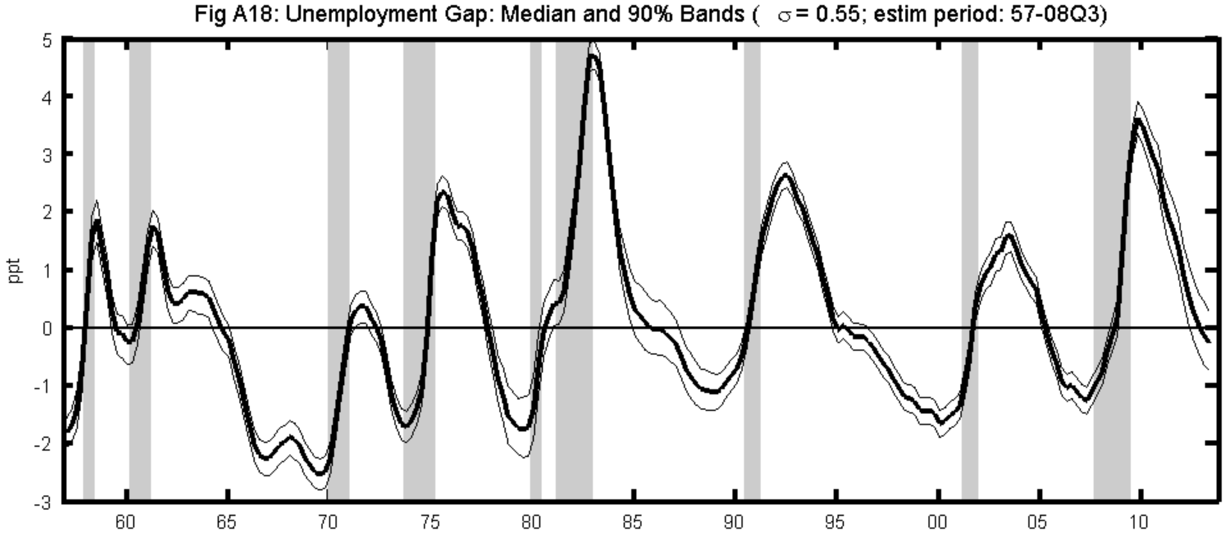


Fig A19: Beveridge Curve (σ = 0.55; estim. period: 57-08Q3)

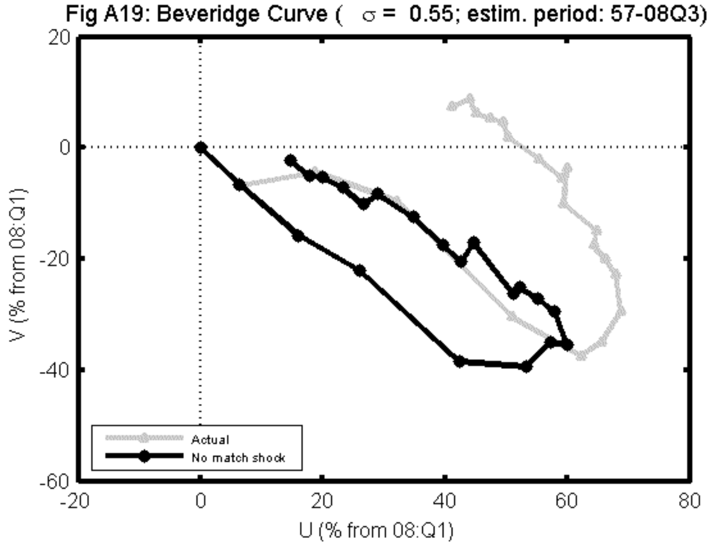


Fig A20: Historical Decomp. of Unemp. Rate (σ = 0.55; estim. period: 57-08Q3)

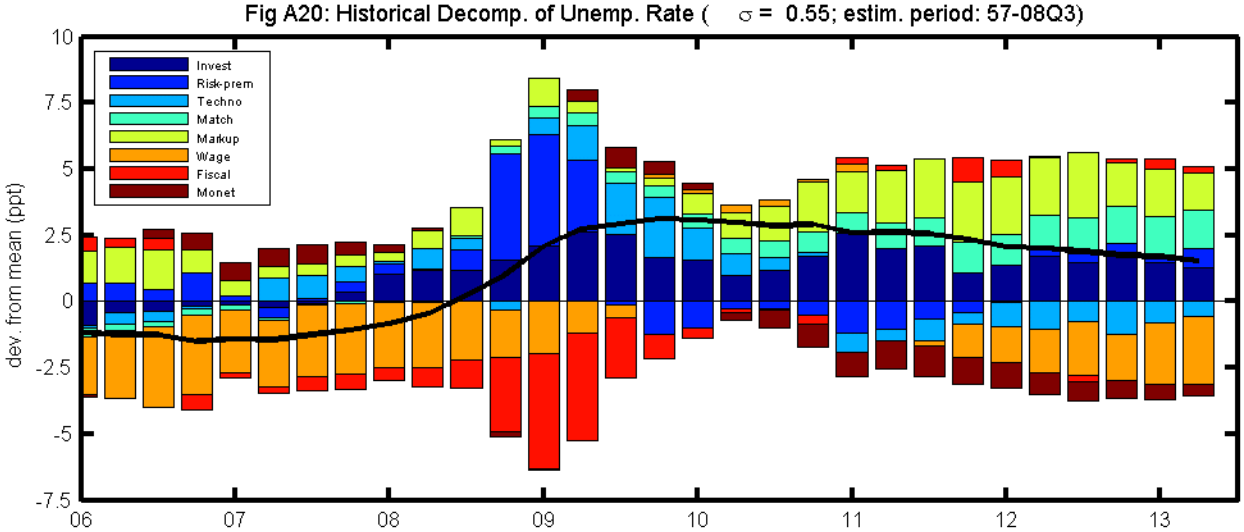
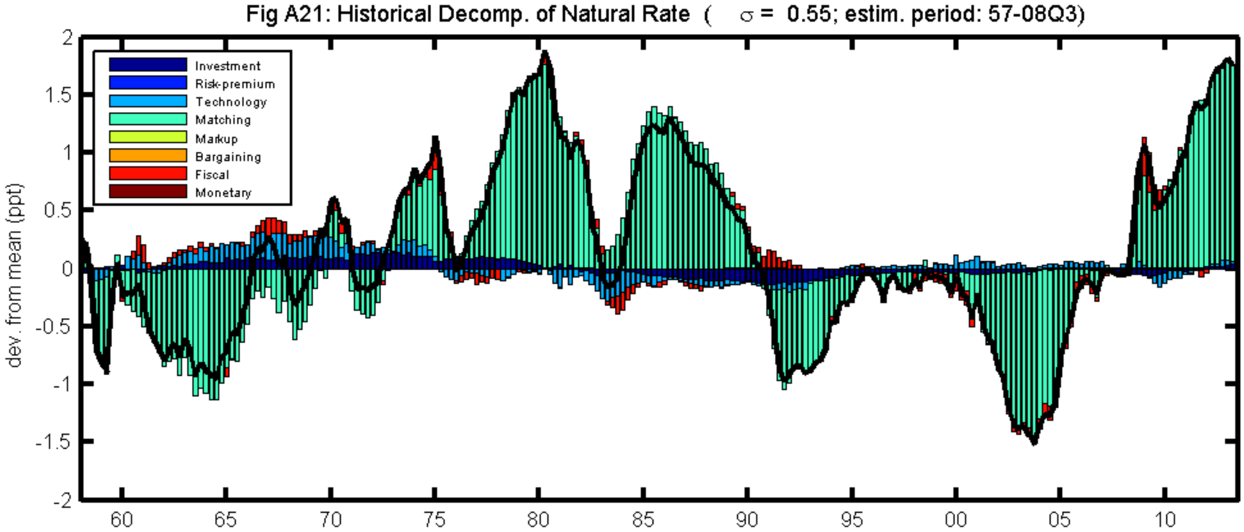


Fig A21: Historical Decomp. of Natural Rate (σ = 0.55; estim. period: 57-08Q3)



Robustness Check #4 - $\sigma = 0.75$ (Est. Per.: 57:Q1 - 08:Q3)

Calibrated Parameters: Check #4 - High Sigma

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	$\mathbf{0.75}$
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0578
Quarterly gross growth rate	z	1.0039
Quarterly gross inflation rate	π	1.0088
Quarterly gross nominal interest rate	R	1.0139

Priors and Posteriors: Check #4 - High Sigma

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.25
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.39
Habit in consump.	h	Beta (0.7,0.1)	0.64
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.03
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.60
Price adjust. cost	ϕ_P	IGamma (60,10)	62.78
Wage adjust. cost	ϕ_W	IGamma (150,25)	188.35
Wage indexation	ϱ	Beta (0.5,0.2)	0.96
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.33
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.84
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.36

Fig A22: Matching Efficiency (σ = 0.75; estim period: 57-08Q3)

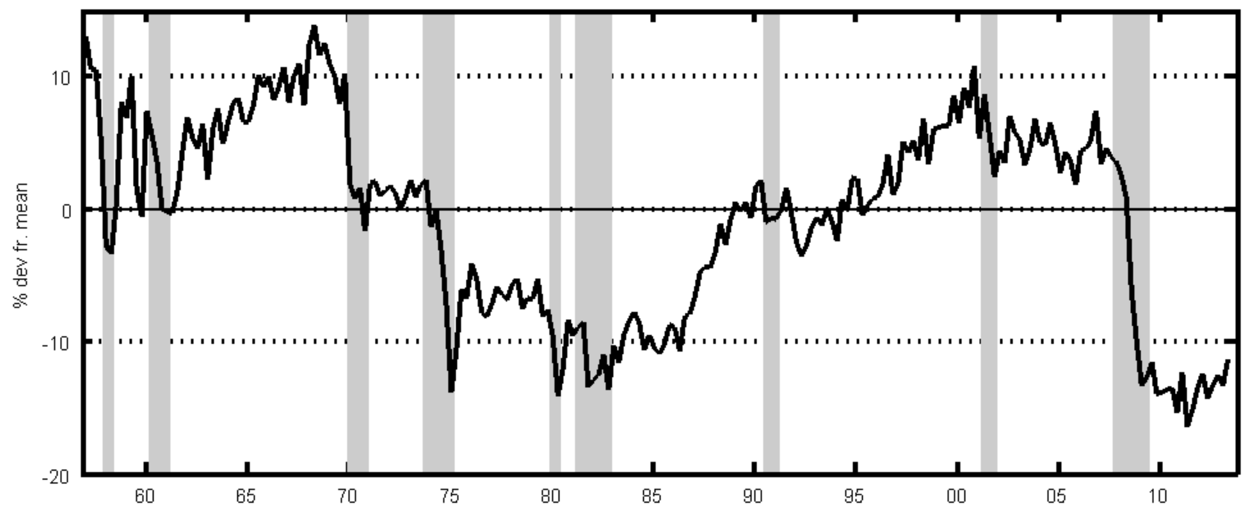


Fig A23: Unemployment Rates: Actual vs Natural (σ = 0.75; estim period: 57-08Q3)

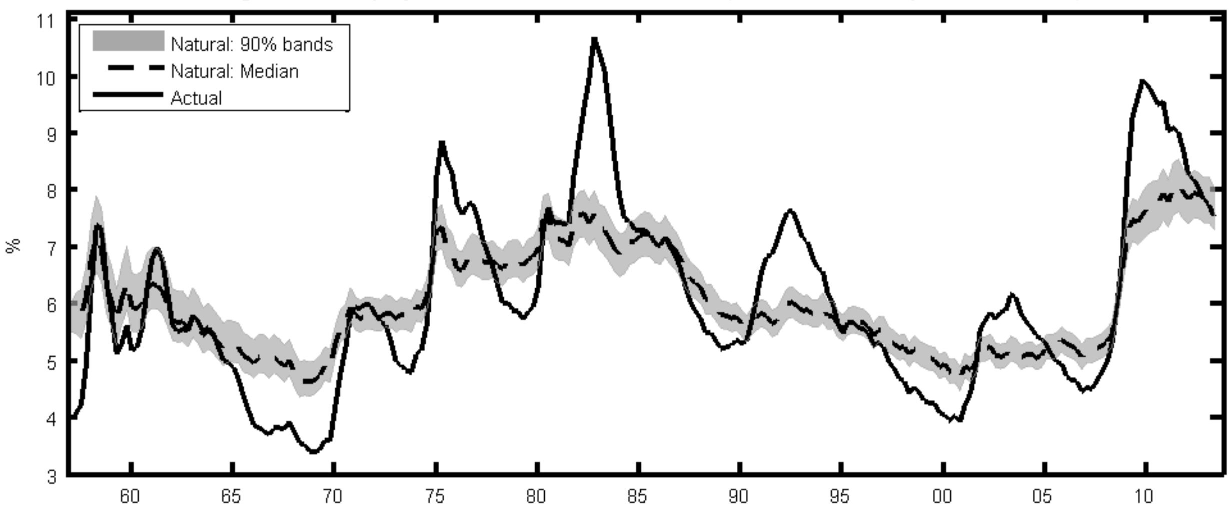


Fig A24: Unemployment Gap: Median and 90% Bands (σ = 0.75; estim period: 57-08Q3)

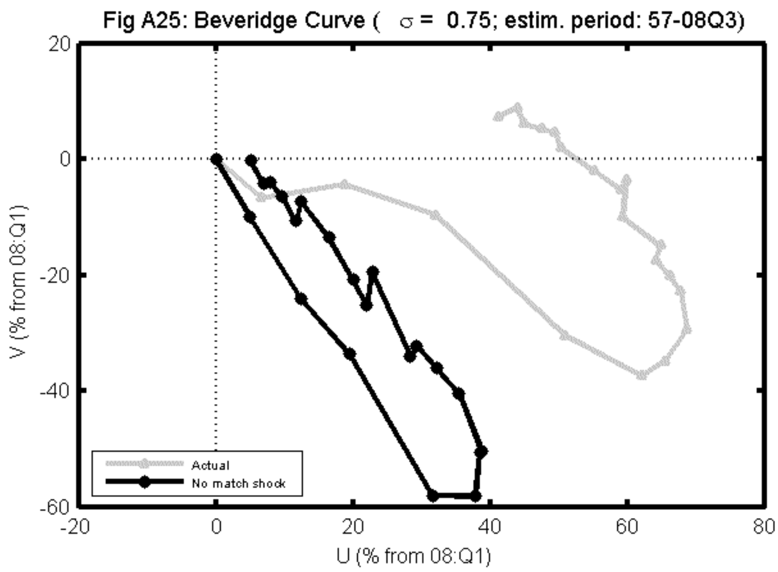


Fig A25: Beveridge Curve (σ = 0.75; estim. period: 57-08Q3)
V (% from 08:Q1)
U (% from 08:Q1)
Actual
No match shock
20
0
-20
-40
-60
-20
0
20
40
60
80

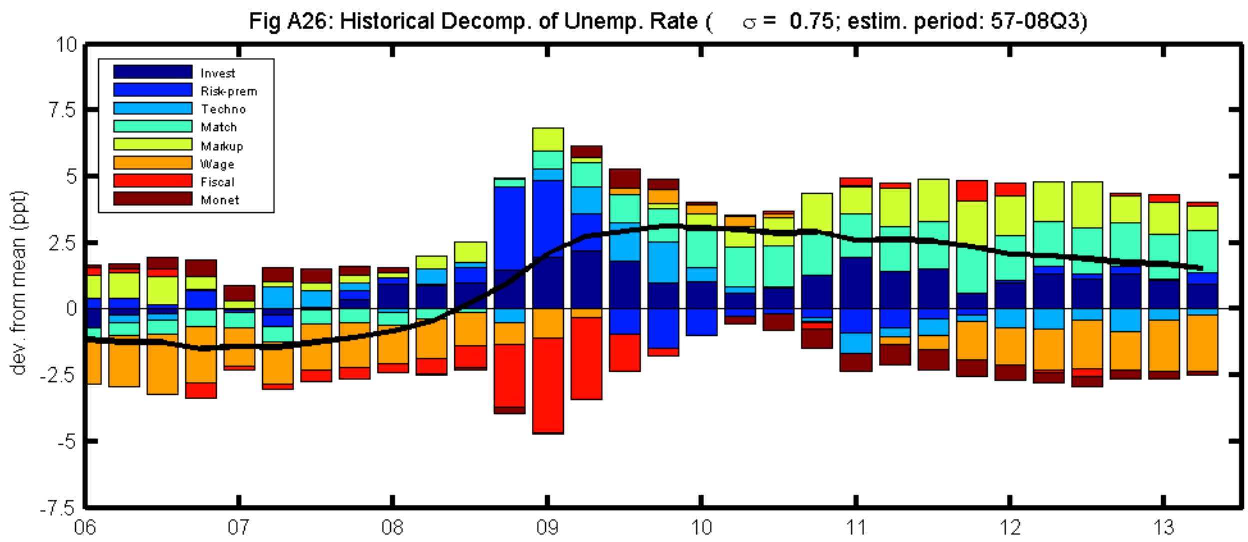


Fig A26: Historical Decomp. of Unemp. Rate (σ = 0.75; estim. period: 57-08Q3)
dev. from mean (ppt)
Invest
Risk-prem
Techno
Match
Markup
Wage
Fiscal
Monet
10
7.5
5
2.5
0
-2.5
-5
-7.5
06
07
08
09
10
11
12
13

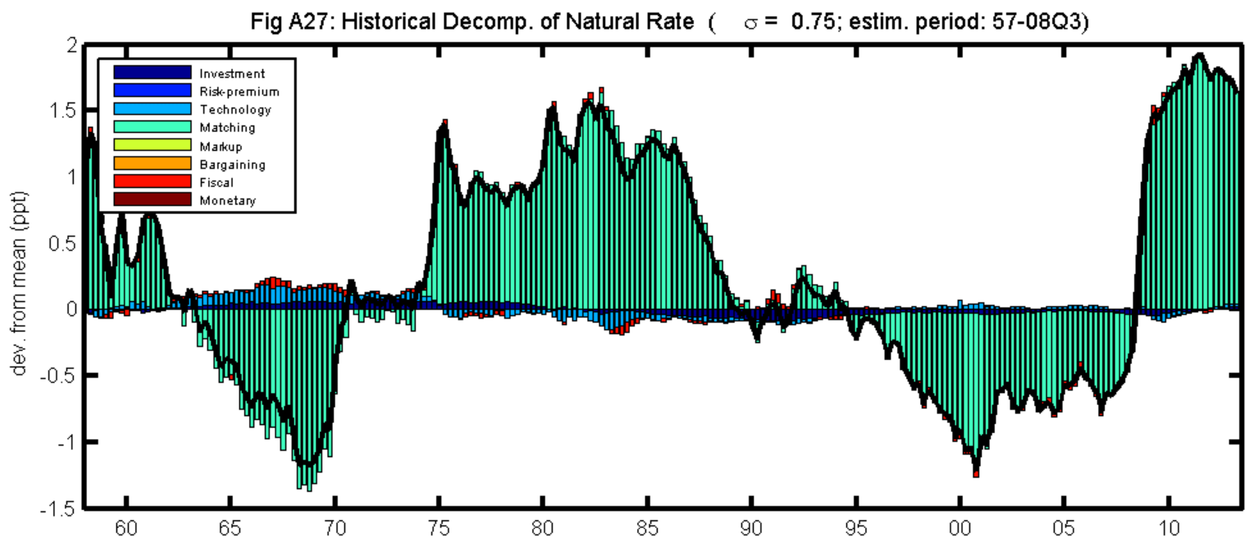


Fig A27: Historical Decomp. of Natural Rate (σ = 0.75; estim. period: 57-08Q3)
dev. from mean (ppt)
Investment
Risk-premium
Technology
Matching
Markup
Bargaining
Fiscal
Monetary
2
1.5
1
0.5
0
-0.5
-1
-1.5
60
65
70
75
80
85
90
95
00
05
10

Robustness Check #5 - $\tau = 0.70$ (Est. Per.: 57:Q1 - 08:Q3)

Calibrated Parameters

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	$\mathbf{0.70}$
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0578
Quarterly gross growth rate	z	1.0039
Quarterly gross inflation rate	π	1.0088
Quarterly gross nominal interest rate	R	1.0139

Priors and Posteriors

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.32
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.46
Habit in consump.	h	Beta (0.7,0.1)	0.65
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.06
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.59
Price adjust. cost	ϕ_P	IGamma (60,10)	62.99
Wage adjust. cost	ϕ_W	IGamma (150,25)	192.91
Wage indexation	ϱ	Beta (0.5,0.2)	0.96
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.34
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.83
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.37

Fig A28: Matching Efficiency (τ = 0.70; estim period: 57-08Q3)

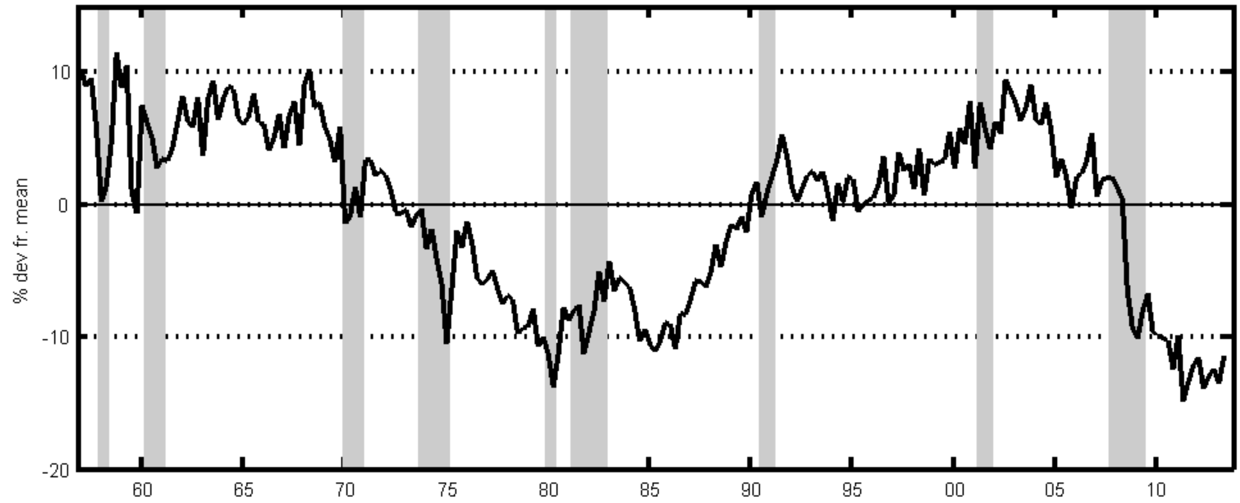


Fig A29: Unemployment Rates: Actual vs Natural (τ = 0.70; estim period: 57-08Q3)

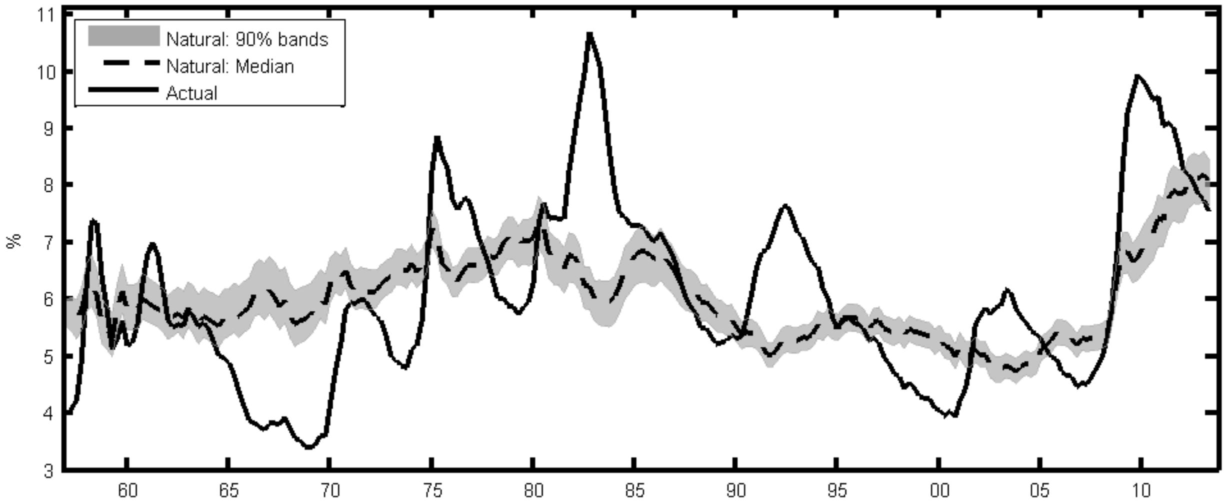


Fig A30: Unemployment Gap: Median and 90% Bands (τ = 0.70; estim period: 57-08Q3)

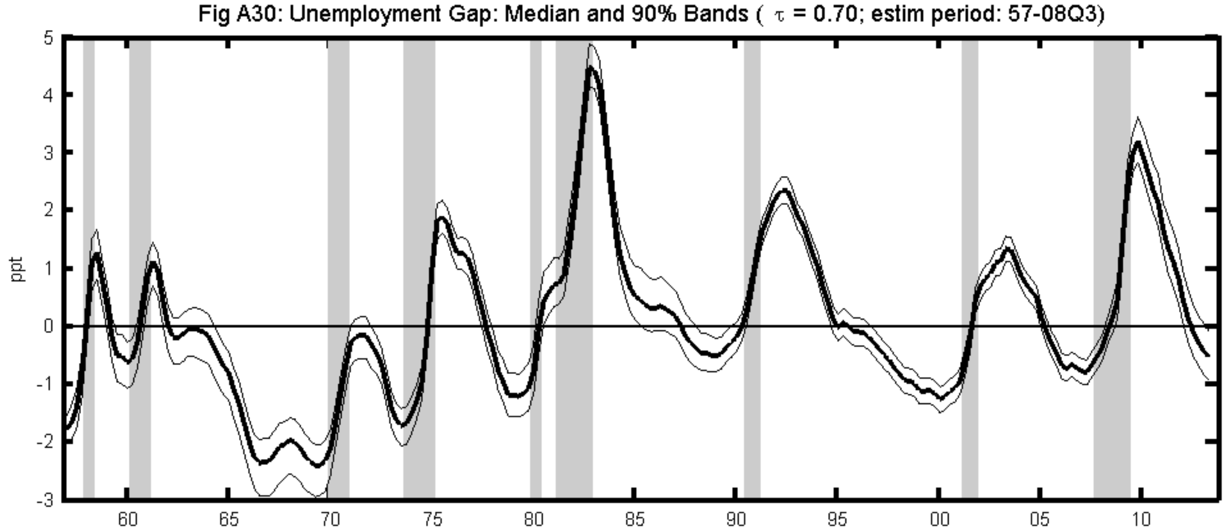


Fig A31: Beveridge Curve (τ = 0.70; estim. period: 57-08Q3)

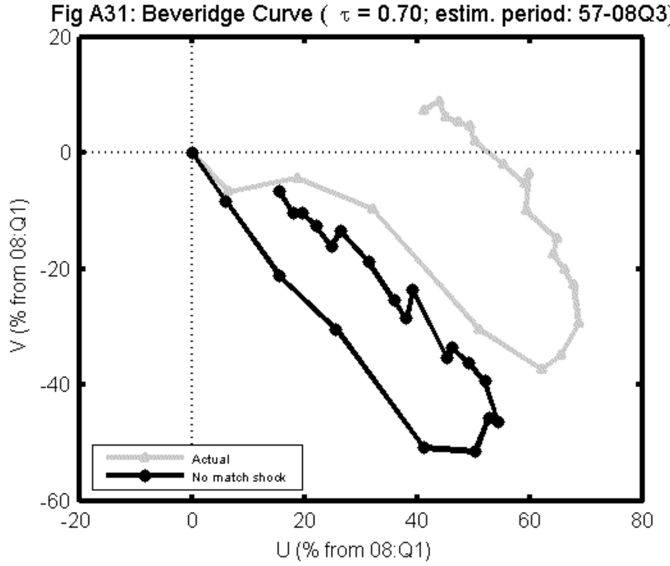


Fig A32: Historical Decomp. of Unemp. Rate (τ = 0.70; estim. period: 57-08Q3)

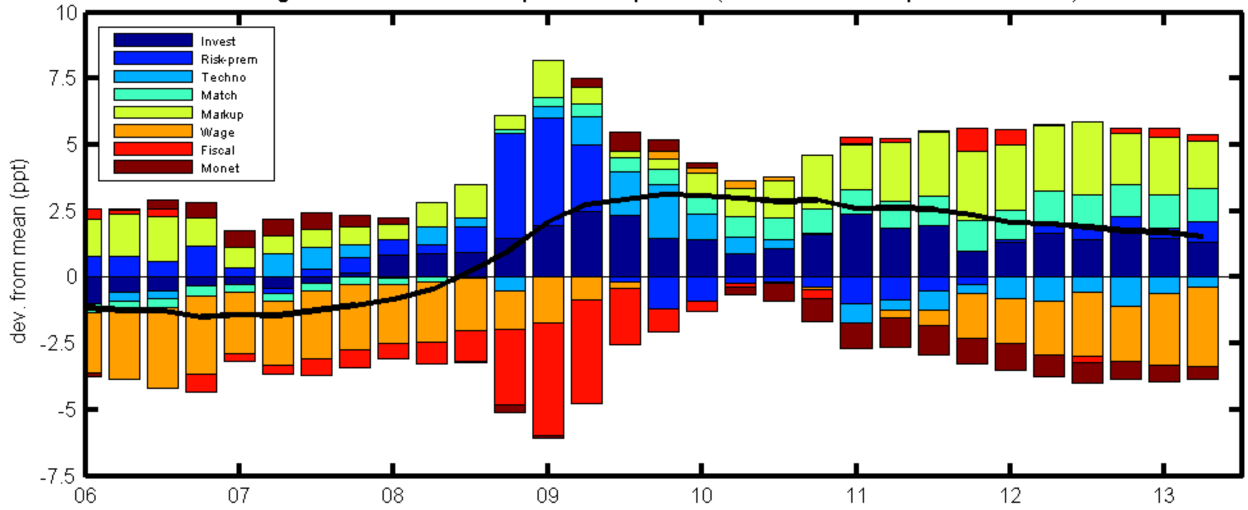
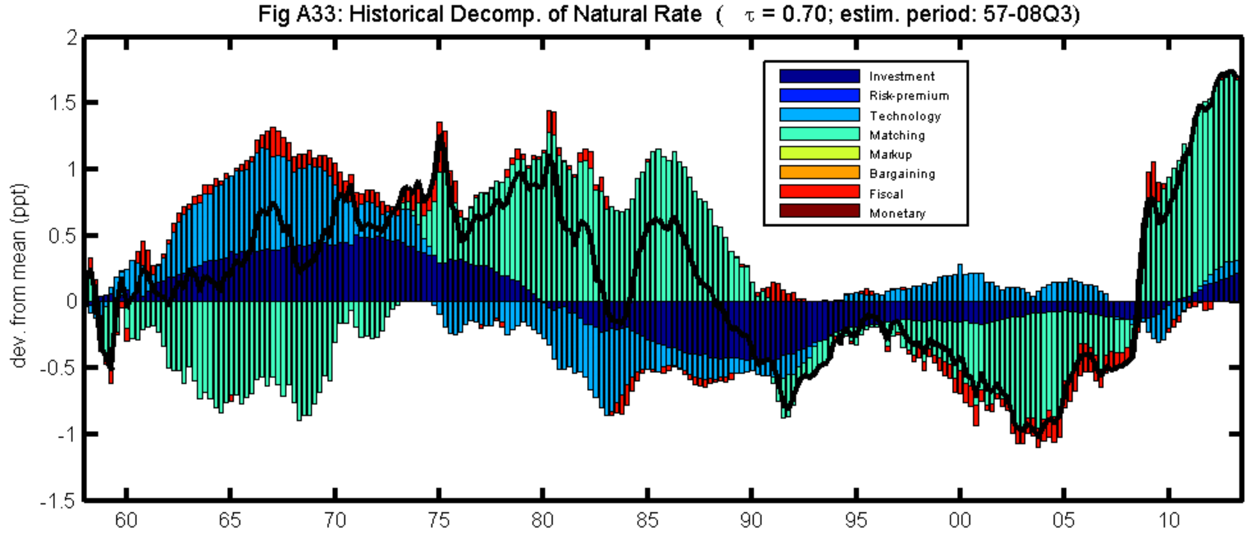


Fig A33: Historical Decomp. of Natural Rate (τ = 0.70; estim. period: 57-08Q3)



#6 - Model with 9 shocks incl. match. and separ. shock

- 9 Observables including CPS 'E-U' separation rate over 1968:Q1-2008:Q3.
- Sep. rate follows: $\ln \rho_t = (1-\rho_\rho)\ln\rho + \rho_\rho \ln\rho_{t-1} - \delta_z\varepsilon_{zt} - \delta_\mu\varepsilon_{\mu t} + \varepsilon_{\rho t}$, $\delta_z \geq 0$, $\delta_\mu \geq 0$.

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0599
Quarterly gross growth rate	z	1.0033
Quarterly gross inflation rate	π	1.0099
Quarterly gross nominal interest rate	R	1.0158

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.20
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.92
Habit in consump.	h	Beta (0.7,0.1)	0.64
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.60
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.66
Price adjust. cost	ϕ_P	IGamma (60,10)	62.96
Wage adjust. cost	ϕ_W	IGamma (150,25)	113.08
Wage indexation	ϱ	Beta (0.5,0.2)	0.92
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.28
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	2.14
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.47
Elasticity of separation wrt tech. shock	δ_z	Uniform (0,3)	1.14
Elasticity of separation wrt IST. shock	δ_μ	Uniform (0,1)	0.20

Fig. A34: Matching Efficiency and Separation Rate

Fig. A35: Unemployment Rates: Actual vs Natural

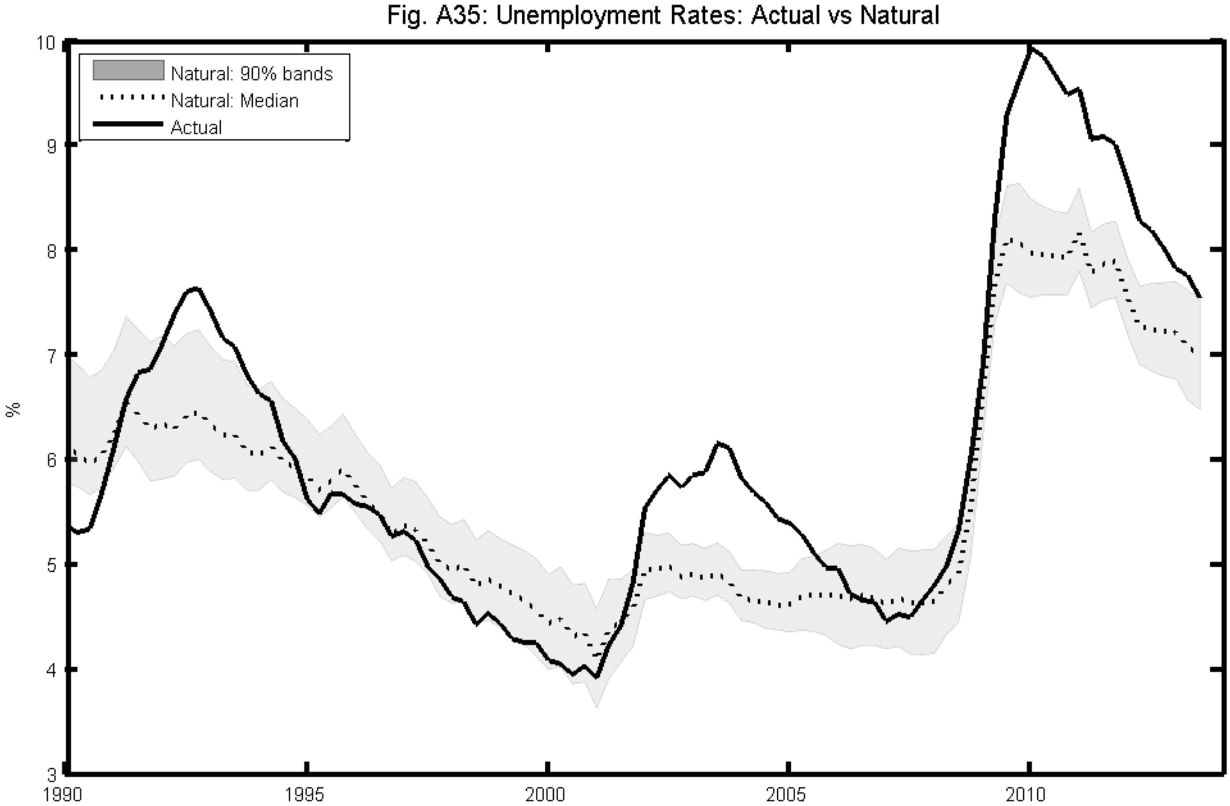


Fig A36: Historical Decomp. of Separation Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)

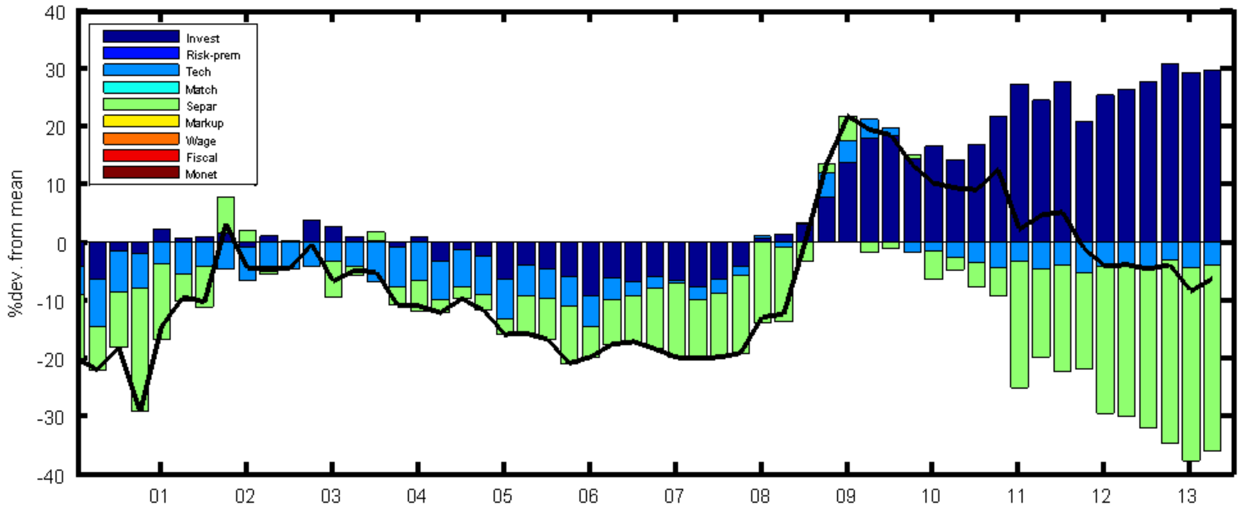


Fig A37: Historical Decomp. of Unemp. Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)

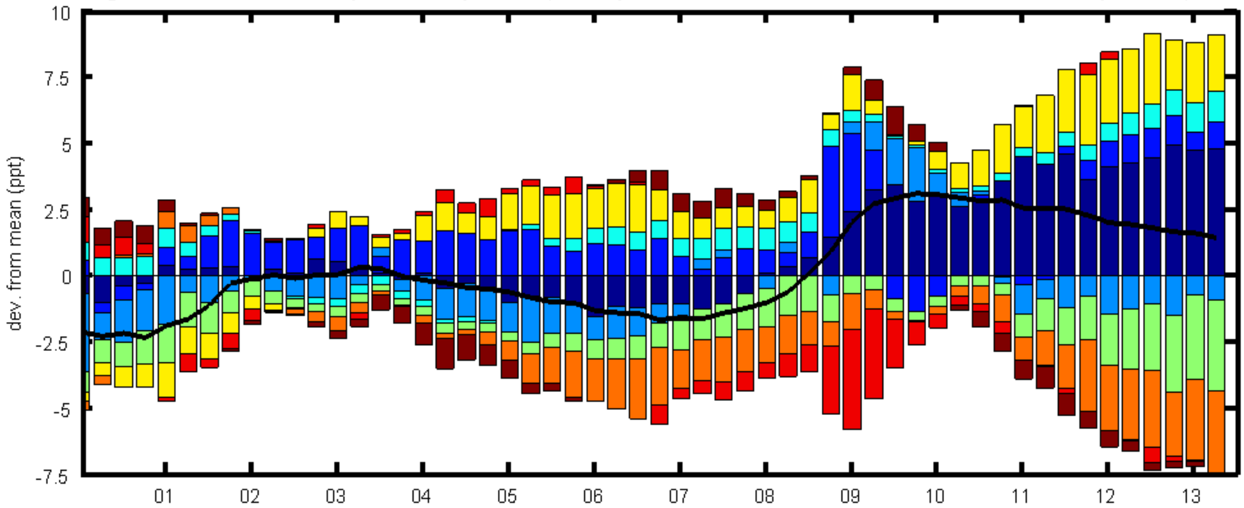
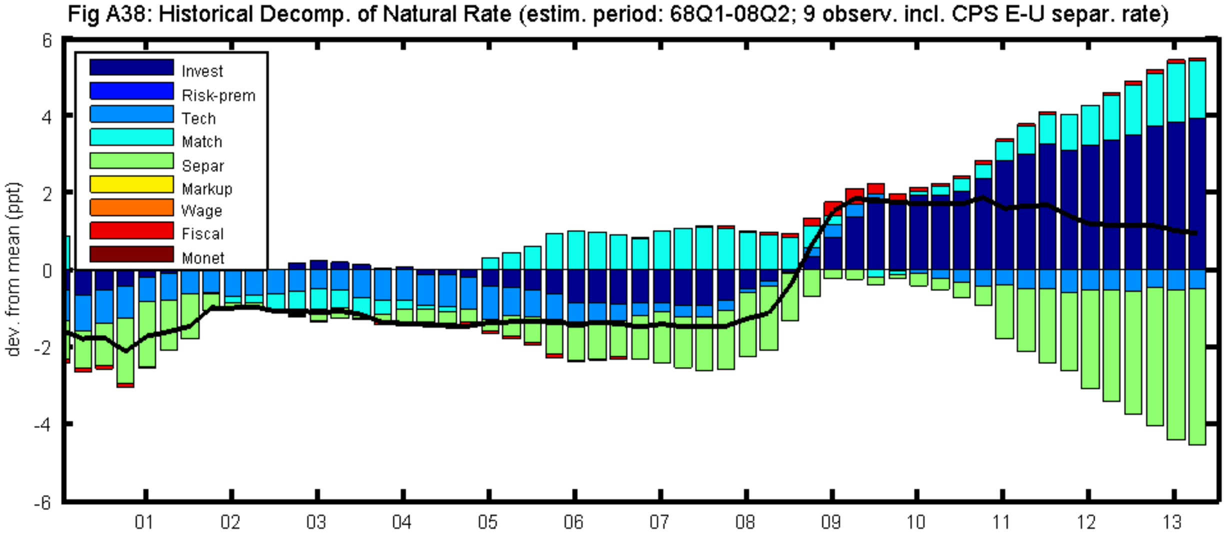


Fig A38: Historical Decomp. of Natural Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)



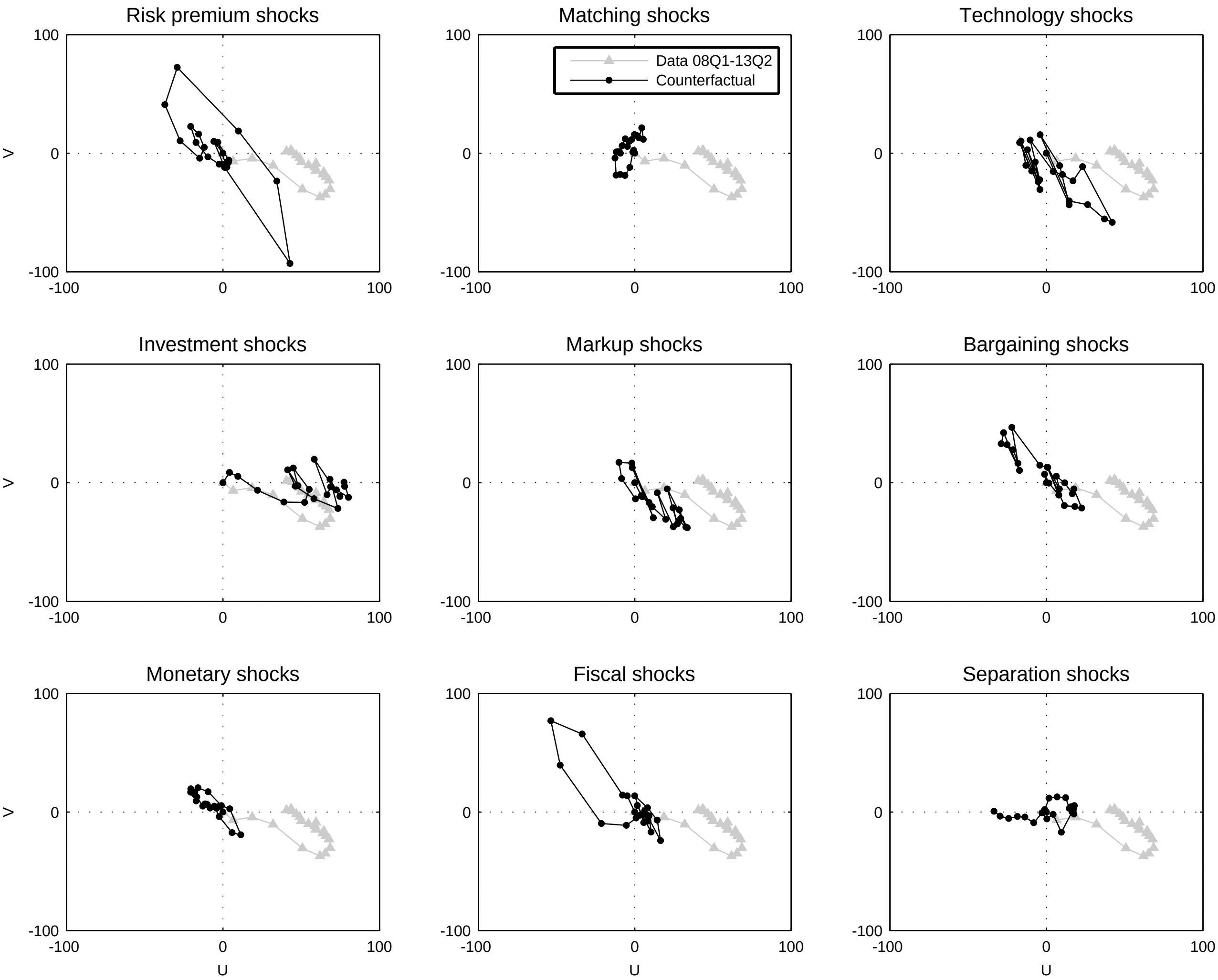


Figure A39: Conditional Beveridge curves. Model with both matching and separation shocks estimated with 9 observables over 1968:Q1 - 2008:Q3.

Check #7 - Model with 9 shocks incl. match. and separ. shock

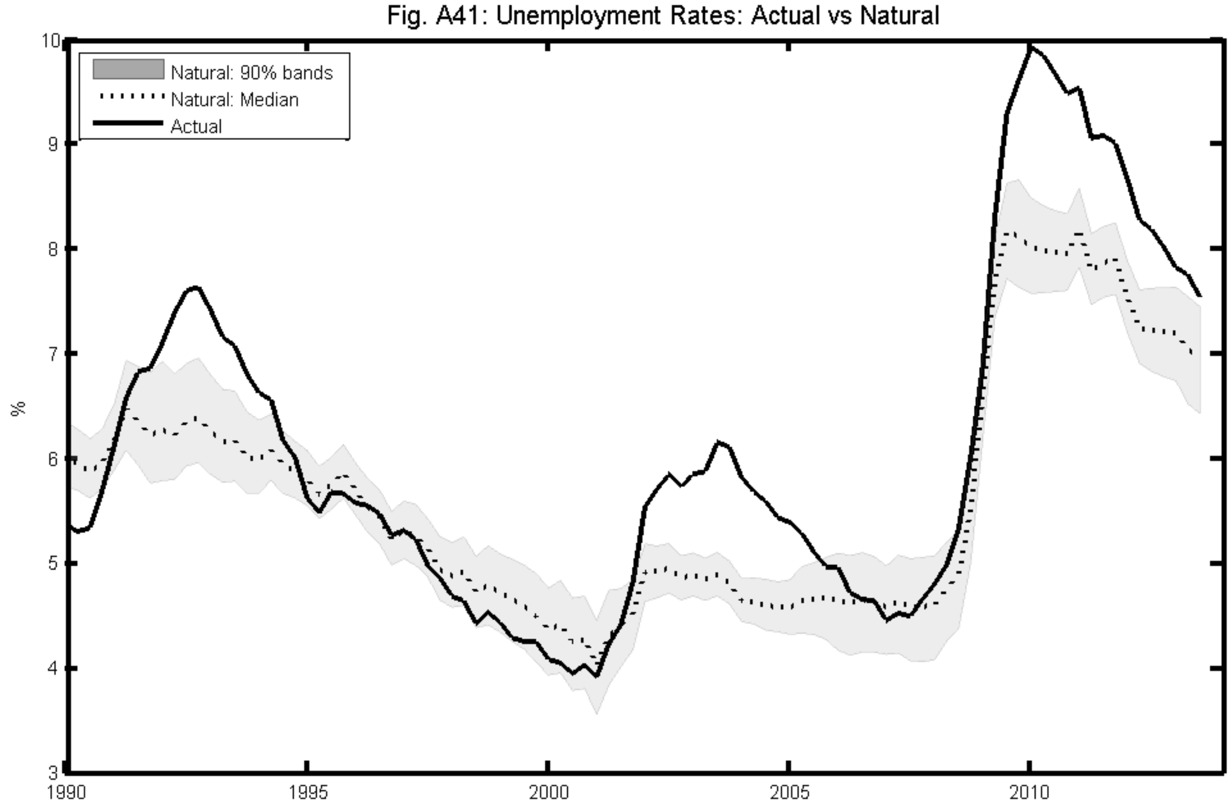
- 9 Observables including CPS 'E-U' separation rate over 1968:Q1-2008:Q3.
- Sep. rate follows: $\ln \rho_t = (1 - \rho_\rho) \ln \rho + \rho_\rho \ln \rho_{t-1} + \varepsilon_{\rho t}$.

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0599
Quarterly gross growth rate	z	1.0033
Quarterly gross inflation rate	π	1.0099
Quarterly gross nominal interest rate	R	1.0158

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.19
Hiring cost/output ratio	$1000 \frac{\kappa}{2} \aleph^2$	IGamma (5,1)	2.87
Habit in consump.	h	Beta (0.7,0.1)	0.65
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.30
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.59
Price adjust. cost	ϕ_P	IGamma (60,10)	60.53
Wage adjust. cost	ϕ_W	IGamma (150,25)	107.06
Wage indexation	ϱ	Beta (0.5,0.2)	0.95
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.26
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	2.12
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.44

Fig. A40: Matching Efficiency and Separation Rate

Fig. A41: Unemployment Rates: Actual vs Natural



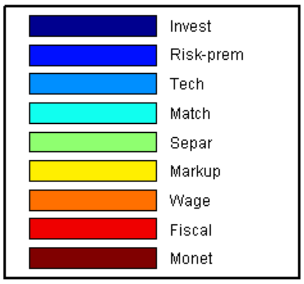


Fig A42: Beveridge Curve (estim. period: 68Q1-08Q3)

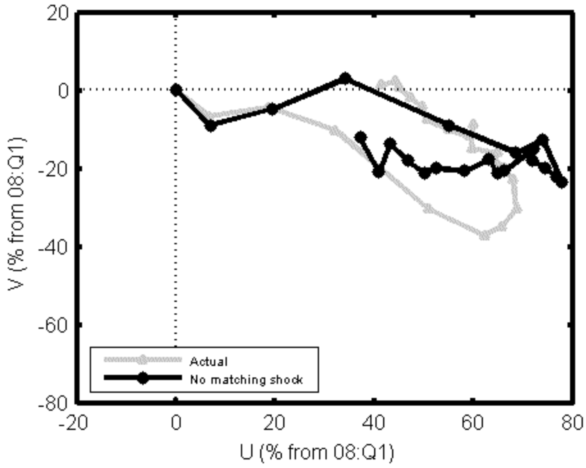


Fig A43: Historical Decomp. of Unemp. Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)

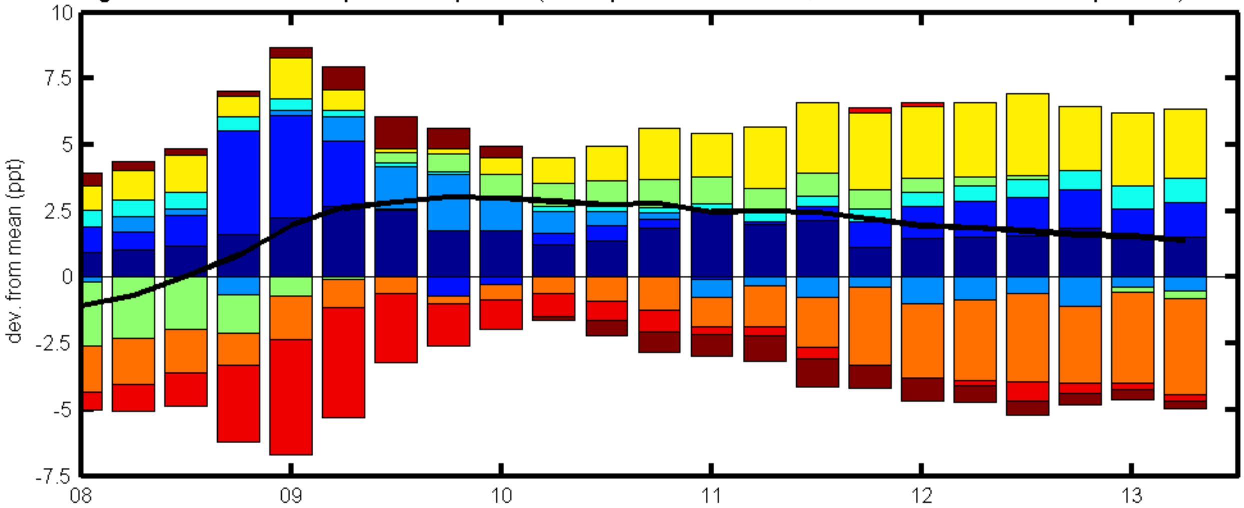


Fig A44: Historical Decomp. of Natural Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)