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State-Dependent Pricing, Firm Entry and Exit, and Non-Neutrality of Money

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(non-)uniform distribution, menu cost

JEL Classification

E31, E52

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State-Dependent Pricing, Firm Entry and Exit, and Non-Neutrality of Money

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Money is not neutral if firm entry and exit are incorporated into a menu cost model. The real effect of money increases as a firm entry and exit rate increases, and the key is non-uniform firm distribution.

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1 Introduction

Price stickiness, in general, generates a real effect of monetary policy. However, Caplin and Spulber (1987) show that money is neutral in a simple menu cost model. This result stems from endogenous changes in the extensive margin; that is, there is a change in the fraction of firms that adjust their prices. Money growth shocks increase the fraction of such firms, adjusting the aggregate price level accordingly and completely offsetting its real effect. Since their study, a number of menu cost models have been proposed to explain non-neutrality of money such as those by Caplin and Leahy (1991), Dotsey, King, and Wolman (1997), Golosov and Lucas (2007), and Midrigan (2011).

In this study, we add just one flavor to a simple menu cost model: firm entry and exit. These concepts are clearly critical for an economy. Regarding pricing, recently available microdata on prices illustrate very frequent substitutions of products, which significantly changes the implications on price stickiness (Bils and Klenow (2004), Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), Bils (2009)). By incorporating firm entry and exit, we show that money is no longer neutral. The key to this result is non-uniform firm distribution on its price. The fraction of firms that are about to change their prices is lower than that of the sum of firms that just reset their prices and those that just enter the market. This dampens the extensive margin effect, and the real effect of money increases as the firm entry and exit rates increase.

2 Model

Our model is largely borrowed from Sheshinski and Weiss (1977) and Caplin and Spulber (1987). Time is continuous. A representative household consumes and supplies labor, and firms produce goods and exit the market with a certain probability. A central bank supplies money.

In the model, firm entry and exit are assumed to be exogenous. A new firm produces exactly the same goods as an exiting firm. The reason for such a simplistic assumption is that we aim to illustrate that having firm entry and exit generates a real effect of money even in an otherwise simple menu cost model. In our companion study, Oikawa and Ueda (2014) construct a richer model where firm entry and exit are endogenous and entry induces product quality improvement using a creative destruction model by Aghion and Howitt (1992) and Grossman and Helpman (1991). This produces additional non-neutrality of money: even a deterministic change in monetary policy can have real effects.

We first investigate the steady state of the model, where there is no aggregate uncertainty.

The growth rate of nominal variables is denoted by g. We limit our attention to the case of $g > 0$.

2.1 Household

A representative household has the following preferences:

$$U_t = \int_t^{\infty} e^{-\rho t'} \left(\log C_{t'} - L_{t'} \right) dt', \tag{1}$$

$$C_t = \left[\int_0^1 c_t(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $\rho > 0$ represents the subjective discount rate; σ represents the elasticity of substitution between goods; C_t is aggregate consumption; L_t is labor supply; and $c_t(j)$ denotes the consumption of good $j \in [0, 1]$. The labor-supply optimization problem leads to

$$W_t/P_t = C_t, \tag{3}$$

where P_t and W_t represent the aggregate price and nominal wage, respectively. The price level in $t = 0$ is set as the numeraire, that is, $P = 1$.

The representative household is faced with a cash-in-advance constraint:

$$P_t C_t = M_t, \tag{4}$$

where M_t represents the aggregate price index and the money stock. From (3), we have $W_t = M_t$. Real wage, which equals the real money balance, stays constant as

$$m \equiv W_t/P_t = M_t/P_t = C_t. \tag{5}$$

2.2 Firms

Firm j, which is monopolistic in its product $j \in [0, 1]$, enters and exits the market exogenously with probability μ.

2.2.1 Firm Profits

(2) and (4) suggest that the demand for good j is given by

$$c_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\sigma} \frac{M_t}{P_t}, \tag{6}$$

where $p_t(j)$ represents the price of good j. The aggregate price level can be written as

$$P_t = \left(\int_0^1 p_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \tag{7}$$

Firms produce one unit of goods using one unit of labor. From (6), we can write the real profits of firm j as

$$\begin{aligned} &\frac{p_t(j) - W_t}{P_t} \left(\frac{p_t(j)}{P_t} \right)^{-\sigma} \frac{M_t}{P_t} \\ &= \Pi(\xi_t(j))m, \end{aligned} \tag{8}$$

where the real price $\xi_t(j)$ and the real profit function $\Pi(\xi)$ are defined using

$$\xi_t(j) \equiv \frac{p_t(j)}{P_t} \tag{9}$$

$$\Pi(\xi) \equiv (\xi - m)\, \xi^{-\sigma}. \tag{10}$$

2.2.2 Pricing under Menu Cost

Without nominal rigidity, (8) suggests that the optimal real price satisfies $\Pi'(\xi^*)$=0 where $\xi^* = \sigma m/(\sigma - 1)$.

Firms pay the menu cost when they change their prices as much as κm. When firms enter the market in period t, they set their price at p_0. They do not pay a menu cost in the initial period. The price is left unchanged, unless they pay menu cost. As shown in (9), during that period, real price ξ_t changes with the rate $-g$. In other words, these goods become cheaper as time passes. In period t_{i+1} for i=$0, 1, 2, \cdots$, they pay the menu cost and reset their price at p_{i+1}.

Considering firm exit, we can write the expected present value of firm V as

$$V = \sum_{i=0}^{\infty} \left(\int_{t_i}^{t_{i+1}} \Pi \left(p_i e^{-gt'} \right) e^{-(\rho+\mu)t'} dt' - \kappa e^{-(\rho+\mu)t_{i+1}} \right) m.$$

The first-order conditions with respect to t_i and p_i respectively yield

$$\frac{\partial V_t}{\partial t_i} = 0 = \left[\Pi \left(p_{i-1} e^{-gt_i} \right) - \Pi \left(p_i e^{-gt_i} \right) + \kappa(\rho + \mu) \right] e^{-(\rho+\mu)t_i} \tag{11}$$

$$\frac{\partial V_t}{\partial p_i} = 0 = \int_{t_i}^{t_{i+1}} \Pi' \left(p_i e^{-gt'} \right) e^{-(\rho+\mu+g)t'} dt'. \tag{12}$$

We define $S \equiv \log\left(p_i e^{-g t_i}\right)$ and $\Delta \equiv t_i - t_{i-1}$, Then, they should satisfy

$$0 = \Pi\left(e^s\right) - \Pi\left(e^S\right) + \kappa(\rho + \mu) \tag{13}$$

$$0 = \int_s^S \Pi'\left(e^z\right) e^{\frac{\rho+\mu+g}{g} z} dz \tag{14}$$

$$s \equiv \log\left(p_i e^{-g(t_i+\Delta)}\right) = S - g\Delta. \tag{15}$$

2.2.3 Firm Distribution

Because of the menu cost, firms are heterogeneous with respect to their prices. Log real prices $z \equiv \log\xi$ are distributed in the range between s and S or, after the last price change, in the range between 0 and Δ. Denote the density function of $z(t')$ using $f(z(t'))$, where $t' \in [0, \Delta]$ and $z \in [s, S]$. Because $z(t')$ changes at the rate $-g$ for $z \in [s, S]$, the density function should satisfy

$$\begin{aligned} f(z(t')) &= f(z(t' - dt'))(1 - \mu dt') \\ &= f(z(t') + g dt')(1 - \mu dt') \end{aligned}$$

for small dt' if the firm distribution is stationary. This equation implies that the density at t' should equal that of $t' - dt'$ multiplied by the survival probability of firms between $t' - dt'$ and dt', that is, $1 - \mu dt'$. This equation is transformed into $d\ \log f(z(t')) = \mu/g$. Using $1 = \int_0^\Delta f(z(t'))dt'$, we obtain the following firm density function:

$$f(z(t')) = \frac{\mu}{1 - e^{-\mu\Delta}} e^{\frac{\mu}{g}(z(t') - S)} = \frac{\mu}{1 - e^{-\mu\Delta}} e^{-\mu t'}. \tag{16}$$

2.3 The Aggregate Inflation Rate

Concerning the aggregate inflation rate, (7) is transformed into

$$\begin{aligned} g dt &= \log P_{t+dt} - \log P_t \\ &= f(z(\Delta))dt \cdot (S - s) + \mu dt \cdot S. \end{aligned}$$

This is because price changes between t and $t + dt$ can be divided into two cases. First, firms change their log real prices from s to S by paying the menu cost with probability $f(z(\Delta))dt$. Second, new firms enter the market with probability μdt. A new log real price is set at S from the average log price of zero. Thus, we have

$$g = \frac{\mu}{1 - e^{-\mu\Delta}} e^{-\mu\Delta} g\Delta + \mu S. \tag{17}$$

2.4 Real Effect of a Money Growth Shock

Thus far, we assumed no aggregate uncertainty. To consider a real effect of monetary policy shock, we now assume that a money growth shock ε^M occurs unexpectedly between periods t and $t+dt$ as

$$\log M_{t+dt} = \log M_t + gdt + \varepsilon^M dt. \tag{18}$$

Prior to period t, the economy is supposed to be at the steady state. The size of a money growth shock is sufficiently small compared to g. When a shock occurs, the real marginal cost for firms m may change, but as in Caplin and Spulber (1987), we assume that it stays unchanged. Thus, S, s, and Δ do not change. As long as the money growth shock is temporary and small, this assumption does not appears to be restrictive.

As (8) shows, the nominal marginal cost for firms increases by the growth rate of money. The surprise money growth shock thus further lowers real prices by $\varepsilon^M dt$. Therefore, the firms that reset their prices $t' \in [\Delta - dt - \varepsilon^M dt/g, \Delta]$ periods ago reset their prices between periods t and $t+dt$. The inflation rate thus becomes

$$\begin{aligned}
\pi_t dt &= \log P_{t+dt} - \log P_t \\
&= \int_{\Delta - dt - \varepsilon^M dt/g}^{\Delta} f(z(t'))dt' \cdot (S-s) + \mu dt \cdot S,
\end{aligned}$$

which yields

$$\begin{aligned}
\pi_t &= \frac{1}{dt}\int_{\Delta - dt - \varepsilon^M dt/g}^{\Delta} \frac{\mu}{1-e^{-\mu\Delta}} e^{-\mu t'} dt' \cdot (S-s) + g - \frac{\mu e^{-\mu\Delta}}{1-e^{-\mu\Delta}} g\Delta \\
&= \frac{\mu e^{-\mu\Delta}}{1-e^{-\mu\Delta}}\left(1 + \frac{\varepsilon^M}{g}\right) g\Delta + g - \frac{\mu e^{-\mu\Delta}}{1-e^{-\mu\Delta}} g\Delta \\
&= g + \frac{\mu\Delta e^{-\mu\Delta}}{1-e^{-\mu\Delta}}\varepsilon^M.
\end{aligned}$$

From (4), consumption changes as

$$\begin{aligned}
d\log C_t &= g + \varepsilon^M - \pi_t \\
&= \left(1 - \frac{\mu\Delta e^{-\mu\Delta}}{1-e^{-\mu\Delta}}\right)\varepsilon^M.
\end{aligned} \tag{19}$$

The coefficient on ε^M is positive. Therefore, this equation suggests that a positive money growth shock increases consumption. A negative growth shock decreases consumption if the size is not too large.

Proposition 1 *Money is not neutral unless the entry and exit rate μ is zero. For $\mu \ll 1$, the* *real effects of money increase as μ increases.*

The proof for the first sentence is provided previously. For the second sentence, we know that $d(\mu\Delta)/d\mu = \Delta + \mu(d\Delta/d\mu)$ and $d\Delta/d\mu$ is finite. Thus, for a sufficiently small μ, $d(\mu\Delta)/d\mu$ is positive. For $x > 0$, the function $f(x) = 1 - \frac{xe^{-x}}{1-e^{-x}}$ is positive and increasing with x. Thus, the coefficient on ε^M in (19) is increasing with μ for $\mu \ll 1$.

This proposition suggests that the money growth shock is not completely canceled out by a change in the price level. This contrasts with the result reported by Caplin and Spulber (1987). In their model, no firm entry and exit are present, and hence, firm distribution is uniform. Thus, the density of firms whose real price is close to S is the same as that of firms whose real price is close to s. In our model, because firms enter and exit, firm distribution is no longer uniform. Over time, an increasing number of firms exit, and thus, the density of firms whose real price is close to S is larger than that of firms whose real price is close to s. The money growth shock induces the latter firms to reset their prices. Because its density is relatively low, a change in the extensive margin and that in the aggregate price level are small. This generates the real effects of monetary policy.

3 Concluding Remarks

In this study, we have shown that firm entry and exit produce the real effects of monetary policy in a simple menu cost model. The key to this result is non-uniform firm distribution on its price.

Future work needs to evaluate the real effects of monetary policy quantitatively using a richer menu cost model. Moreover, firm entry and exit need to be endogenized, as Grossman and Helpman (1991), Aghion and Howitt (1992), and Oikawa and Ueda (2014) did.

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