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Extremal Dependence and Contagion

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Abstract

A new test for financial market contagion based on changes in extremal dependence defined as co-kurtosis and co-volatility is developed to identify the propagation mechanism of shocks across international financial markets. The proposed approach captures changes in various aspects of the asset return relationships such as cross-market mean and skewness (co-kurtosis) as well as cross-market volatilities (co-volatility). In an empirical application involving the global financial crisis of 2008-09, the results show that significant contagion effects are widespread from the US banking sector to global equity markets and banking sectors through either the co-kurtosis or the co-volatility channel, reinforcing that higher order moments matter during crises.

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JEL Classification

C12, F30, G11, G21

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Extremal Dependence and Contagion

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A new test for financial market contagion based on changes in extremal dependence defined as co-kurtosis and co-volatility is developed to identify the propagation mechanism of shocks across international financial markets. The proposed approach captures changes in various aspects of the asset return relationships such as cross-market mean and skewness (co-kurtosis) as well as cross-market volatilities (co-volatility). In an empirical application involving the global financial crisis of 2008-09, the results show that significant contagion effects are widespread from the US banking sector to global equity markets and banking sectors through either the co-kurtosis or the co-volatility channel, reinforcing that higher order moments matter during crises.

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1 Introduction

Recent global financial crises have reminded investors that asset returns are driven by asymmetric and fat-tailed distributions, suggesting once more (Kraus and Litzenberger, 1976; Friend and Westerfield, 1980; Engle et al., 1990; Harvey and Siddique, 2000) that the Gaussian distribution underlying the traditional mean-variance framework is not appropriate for modelling asset returns. In addition to being concerned with the distribution of asset returns on a univariate basis, market dependence through the contagious effects of financial crises is also of great concern to policy makers and financial market participants because of the global consequences for monetary policy, risk management and asset pricing.

Crisis episodes are characterized by asset returns falling in value and volatility increasing compared to non-crisis periods. Under the traditional mean-variance framework, these statistics are consistent with a investors realizing a higher level of excess returns in a non-crisis period and a higher level of risk in a crisis period (Sharpe, 1964; Lintner, 1965; Black, 1972). However, it is evident that higher order moments also change. For example, asymmetry in the returns distribution measured by skewness generated by the preferences of investors are subject to change in different regimes. This asymmetry can be attributed to two types of effects. The first effect is the volatility skew and smile. Volatility skew is common in financial markets where stock returns appear to be negatively correlated with return volatility (Black, 1976; Bekaert and Wu, 2000). The bias phenomenon known as the volatility smile is often observed in financial markets during a crisis period where the volatility-return co-movement exhibits an upward relation, resulting in positive skewness in the crisis period (Shleifer and Vishny, 1997; Conrad et al., 2013). The second effect is the risk return trade-off in a utility maximizing model of portfolio choice between expected excess returns and higher order moments. Lower asset returns occurring in a financial crisis are realized by a risk-averse investor in conjunction with positive skewness of returns (Fry et al. 2010).

Asset returns also typically yield leptokurtic behavior and kurtosis rises during crisis periods. The relatively lower kurtosis commonly displayed in non-crisis periods is documented theoretically by Brunnermeier and Pedersen (2008). They find that speculators invest in securities with a positive average return and negative skewness, giving rise to the low value of kurtosis. However, extreme events result in speculators investing in securities with a negative average return and negative skewness, thus increasing kurtosis risk but providing a good hedge during the crisis period.[1]

Dependence between markets is measured by the co-moments of asset returns and changes in these relationships are also often observed in financial data. These co-moment changes are the basis of different types of tests for contagion including the correlation test of Forbes and Rigobon (2002) and the coskewness test of Fry et al. (2010) and Fry-McKibbin et al. (2013). In the simplest case, changes in linear dependence (or correlation), is commonly used to measure financial contagion where it is expected that correlation between markets should increase in a crisis period in the presence of contagion.[2] However, it is also suggested that linear co-movement does not fully capture market information since it is measured by the equal weight of small and large returns (Embrechts et al., 2003). Moreover, correlation is a linear measure of dependence that can be applied only when investors display mean-variance preferences, or in other words, when returns follow a normal distribution.

To counter this limitation, the coskewness class of contagion tests was developed to captures changes in the asymmetry of the joint probability distribution of asset returns in a crisis period compared to a non-crisis period. Co-skewness measures the relationship between the volatility in market i and the mean of the asset returns in market j. The shift from negative co-skewness to positive co-skewness during the financial crisis comes from the two potential explanations of asymmetry above, applied to a bi-variate setting.

This paper extends current measures for detecting contagion by examining higher order elements of asset return distributions frequently overlooked despite the inadequacy of the Gaussian distribution in describing asset returns during crisis episodes. Two new contagion tests are derived, defined broadly as a significant change in extremal dependence between two markets in a non-crisis and a crisis period. Extremal dependence is measured by co-kurtosis (the relationship between the asset return in market i and return skewness in market j) and co-volatility (the relationship between the return volatility of markets i and j). To illustrate this new approach, we apply the tests to equity markets and banking sectors during the global financial crisis of 2008-09.

The extremal dependence measures derived in this paper capture more co-movements

[1] Brunnermeier and Pedersen (2008) show that the funding constraint influences not only the price level but also the skewness of the price distribution. In extreme events, the security price is below the market fundamental price, resulting in negative returns. Holding the security leads to losses as speculators face funding constraints, inducing negative skewness in the price distribution

[2] Contagion is often defined as a significant incresase in correlation between two markets during the crisis period after controlling for market fundamentals (Forbes and Rigobon, 2002).

than the linear dependence measures in the worst events and is similar to Garcia and Tsafack (2011) who illustrate that the tail dependence coefficient can be treated as the probability of the worst event occurring in one market given that the worst event occurs in another market. The theories of Brunnermeier and Pedersen (2008) and Fry et. al (2010) touched on above imply different signs of higher order moments. It is possible that either sign could eventuate in a crisis period as investors optimize under incomplete information (Vaugirard, 2007; Gorton and Metrick, 2009) and liquidity constraints (Cifuentes et al., 2005; Allen and Gale, 2000), particularly when the source crisis country is also considered a safe haven country as is the case for the US here (Vayanos, 2004). In this paper, contagion is considered to be a two sided test to allow for the range possibilities as it is not possible to tell *a priori* in which direction the signs will change.

The results of the tests show that significant contagion effects are widespread from the US banking sector to global equity markets and from the US banking sector to global banking sectors during the crisis period. More evidence of contagion is found through extremal dependence than through asymmetric dependence, indicating the extremal dependence tests capture more co-movements during crises than the asymmetric dependence tests in extreme events. In terms of the size of the tests, the results of simulation experiments show that the co-volatility change test presents a good approximation of the finite sample distribution even when comparing a relatively long non-crisis sample period with a relatively short crisis sample period. The tests for changes in co-kurtosis tend to be biased compared to the asymmetric distribution hence critical values require adjusting.

The remaining sections of this paper are organized as follows. Section 2 shows the moment and co-moment of asset returns in equity and banking markets to illustrate the characteristics actually observed in a non-crisis period compared to a crisis period. Two series are then selected to illustrate the presence of the volatility skew in a non-crisis period and the volatility smile in a crisis period in Section 3. It is found that there is evidence of these types of data in both the asymmetric and extremal dependence relationships. Section 4 presents the traditional CAPM model with the addition of higher order moments and co-moments in understanding the risk-return trade-off between the expected excess return and higher order moments and co-moments. Section 5 describes the properties of a bivariate generalized exponential family of distributions with asymmetry and leptokurtosis, which provides the framework for developing tests of co-kurtosis and co-volatility and then tests of contagion based on changes in extremal dependence in Section 6. These tests are applied in Section 7 to investigate channels of contagion in equity markets and the banking sector during the financial crisis of 2008-09. The results show that the extremal dependence tests capture more market co-movements than the asymmetric dependence tests in extreme events. Section 8 concludes.

2 Higher Order Co-moments in a Crisis

To place the relevance of the contagion tests and especially the extremal dependence tests in the context of the observed data, this section examines the descriptive statistics of moments and co-moments of the data used in the empirical application of Section 7 and also presents anecdotal evidence on the volatility skew and smile concepts.

The data is composed of the daily banking equity price indices and aggregate equity indices of eight countries selected from Asia, Europe, Latin America and North America, expressed in US dollars.[3] Daily percentage equity returns of each market i are calculated as

$$R_{i,t} = 100\left(ln(P_{i,t}) - ln(P_{i,t-1})\right),$$

where $P_{i,t}$ is the equity index in market i at time t and $R_{i,t}$ is the percentage return of the equity in market i at time t. The sample period starts on April 1, 2005 and ends on August 31, 2009. It is divided into two periods, defined from April 1, 2005 to February 29, 2008 (the non-crisis period), a total of $T_x = 760$ observations, and from March 3, 2008 to August 31, 2009 (the crisis period), a total of $T_y = 391$ observations.[4] All returns are plotted in Figure

[3] The purpose of the data set is illustrative only and covers Hong Kong and Korea, France, Germany and the UK, Chile and Mexico, and the US. Many other combinations of data could be considered, but to keep the analysis tractable a small subset of possibilities was chosen.

The equity indices are collected from Datastream. The mneumonics are: Hong Kong - Hang Seng price index (HNGKNGI); Korea - Korea Se Composite price index (KORCOMP); Chile - General price index (IGPAGEN); Mexico - Mexico Ipc Bolsa price index (MXIPC35); France - CAC 40 price index (FRCAC40); Germany - MDAX Frankfurt price index (MDAXIDX); the UK - FTSE100 price index (FTSE100); and the US - Dow Jones Industrials (DJINDUS). The banking equity indices are collected from Bloomberg. The mneumonics are: Hong Kong - FTSE China A 600 Banks (XA81); Korea - Korea Banking Index (KOSPBANK); Chile - MSCI Chile Banks (MXCL0BK); Mexico - MSCI Mexico Banks (MXMX0BK); Germany - MSCI Germany Banks (MXDE0BK); the UK - FTSE 350 Banking Index (F3BANK); and the US - PHLX KBW Bank Sector Index (BKX).

[4] The starting date of the crisis is chosen on March 3, 2008 since one of the major US financial institutions, Bear Stearns, was rescued by the Fed using emergency funding on that day due to large losses. The end of August 2009 is generally considered as an ending date of the crisis in financial markets as economic activity in

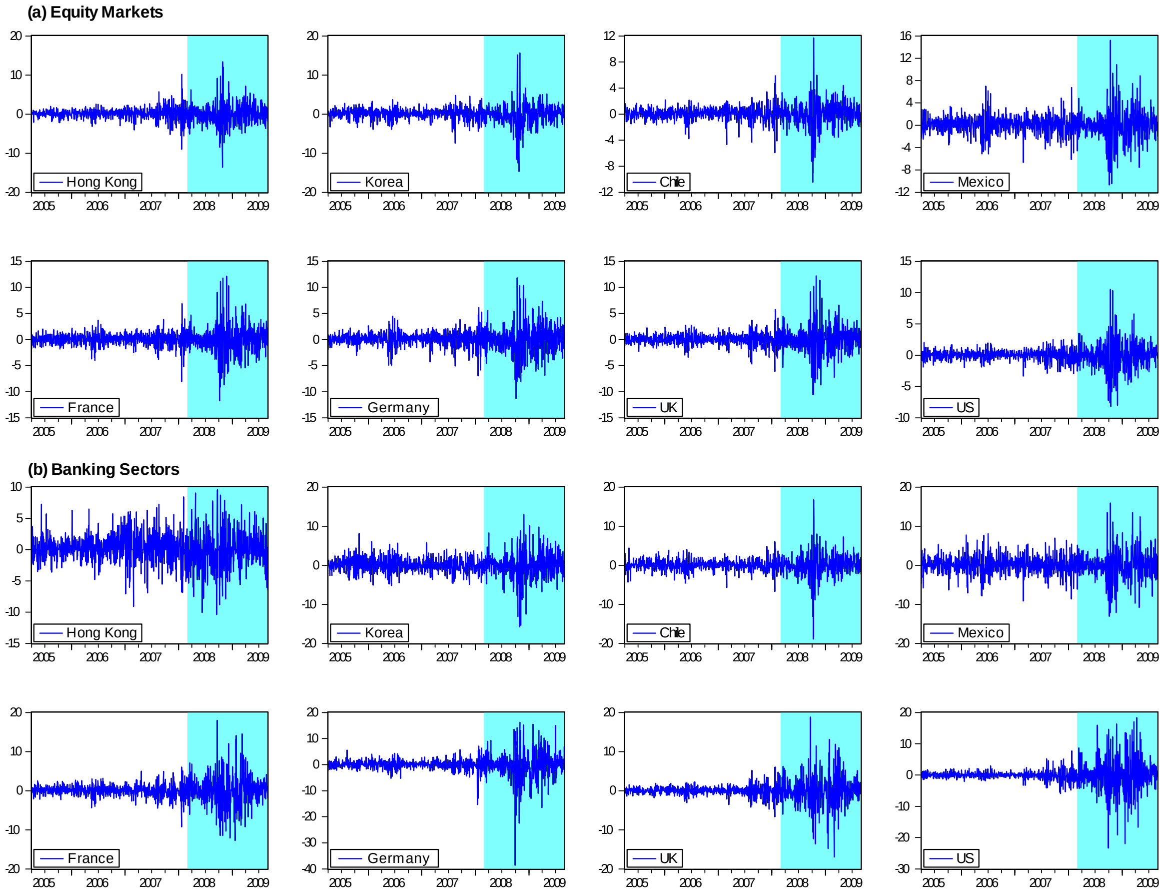

Figure 1: Daily percentage returns of equity markets and equity returns of banking sectors for eight countries (Hong Kong, Korea, Chile, Mexico, France, Germany, the UK and the US) during January 1, 2005 to August 31, 2009. The shaded areas refer to the global crisis starting from March 3, 2008 to August 31, 2009.

1. The figure illustrates that the volatility of equity returns changes dramatically in equity markets and banking sectors globally during the financial crisis of 2008-09.

Tables 1 to 3 report the moment and co-moment statistics of the dataset. Several features characteristic of crises are highlighted in this table and are compared to a period in which there is no crisis. Table 1 reports descriptive statistics of the own-moments of the mean, standard deviation, skewness and kurtosis for each return series in both equity markets and banking sectors for each period.

The first and second moments in the table show that average daily returns of the equity

the US bounced back (Moore and Palumbo, 2010). This paper does not consider the period of the European sovereign debt crisis, and we also acknowledge jitters in international banking markets such as the issues with Northern Rock as early as 2007.

indices decrease and volatility increases across the board in the crisis period as expected.[5] These statistics are consistent with a risk-averse investor realizing a higher level of excess returns in the non-crisis period in conjunction with a higher level of risk across the two regimes (Sharpe, 1964; Lintner, 1965; Black, 1972). Inspection of the third and fourth moments of the returns shows that it is not only the first and second moments that change. Non-normality with asymmetry and fat tails is a major characteristic of returns in the equity markets and banking sectors. The third moment of skewness is generally negative in the non-crisis period but switches sign to be positive in the crisis period for most markets. Asset returns also yield leptokurtic behavior and kurtosis rises during the crisis period, illustrating that the return distributions are far from normal.

Tables 2 and 3 reports statistics of the higher order co-moments of the equity and banking returns with the US banking returns during the non-crisis and crisis periods. Each co-moment captures different features of the joint asset return relationship including: i) linear dependence; ii) asymmetric dependence; and iii) extremal dependence. The tables show that in line with the theories touched on earlier, the three types of market dependence change in the crisis period.

Correlation between the US banking sector and all other returns increase apart from the correlation of the US banking sector with the US equity market indicating foremost the nature of the crisis as being one of a banking issue, rather than of the aggregate equity markets in the US (Table 2).

Turning to the asymmetric dependence statistics, an increase in the magnitude of the value of co-skewness$_{12}$ indicates that the market volatilities for the countries in the sample are more strongly related to the returns in the US banking sector than previously. Negative co-skewness$_{12}$ values suggest that an asset invested in the US banking sector should achieve a relatively higher return when other markets are less volatile. Positive co-skewness$_{12}$ is desirable from the point of view of the US investor as higher returns during the volatile period provide a good hedge. That is, an asset pays higher returns to the US banking sector as other equity markets become more volatile. The logic is reversed for the co-skewness$_{21}$ case. The co-skewness$_{12}$ statistics in the equity markets show a fall in value in most cases, while for the banking sector all coefficients are higher. A similar property is presented in Table 2 where co-skewness$_{21}$ rises or moves towards positive values in almost all markets

[5] Volatility in the banking sectors display much higher values than that in the equity markets.

during the crisis period.

Table 3 presents statistics on the extremal (fourth order) co-moments of the returns for the two periods. As the table shows, co-kurtosis$_{13}$ and co-kurtosis$_{31}$ are positive during both periods for all countries apart from Hong Kong and Korea where both versions are negative during the non-crisis period for the banking sector and for Korea for the equity sector. Co-kurtosis$_{13}$ ranges from -1.728 (Korea) to 9.386 (US) during the non-crisis period and from 0.061 (Hong Kong) to 5.563 (US) during the crisis period. Co-kurtosis increases in the crisis period in most cases, indicating that the joint distribution of returns has a sharper peak and longer and fatter tails during the crisis period. The higher values of co-kurtosis$_{13}$ in the crisis period implies returns in the US banking sector are low with a positive skewness of returns in the other markets, thus increasing contagion risk in the crisis period.

Similarly, co-kurtosis$_{31}$ is higher in the crisis period, suggesting that the distribution of returns in the US banking sector exhibits negative skewness as the returns in the other markets are low during the crisis period, again increasing risk. The co-volatility$_{22}$ statistics, which measures the correlation between volatilities, are all positive in both periods. Interestingly, the co-volatility relationship does not change as systematically as the other statistics. Sometimes the statistic takes a higher value than in the non-crisis period and other times it does not. Most interesting is that the co-volatility statistics for the US are lower for both the equity and banking sectors in the crisis period.

3 Volatility Skew and Smile

This section provides evidence on the volatility skew and smile effect in higher order co-moments, while the next section explores the risk-return trade-off effect in a portfolio choice model accounting for higher order co-moments, motivating the development of the test statistics for contagion through extremal dependence.

Figure 2 illustrates an example of the volatility skew effect in non-crisis data and the smile effect evident in crisis data using the example of the Korean and US banking equity data used in the empirical section. Panels A and C of the figure illustrate co-movements between the equity returns and return volatility in the Korean and US equity markets. The figures show that the data exhibits volatility skew, a negative relation (left skew) in the non-crisis period for both combinations of asymmetry. This result suggests that an asset invested in the equity market in the US achieves high returns when the Korean banking sector is less volatile. However, the crash period can result in speculators investing in securities with a negative average return during the volatile periods. This phenomenon is highlighted in Panel D of Figure 2 where the volatility-return co-movements in the crisis period display an upward relation (right skew), indicating positive co-skewness and increasing risk in the crisis period. Asset returns in financial markets can hence show a smile when considering the non-crisis and crisis periods together, as the return volatility plotted against the asset return turns from downward sloping to upward sloping (Panels C and D).

There is also evidence of the smile effect through the co-kurtosis or co-volatility channel. There is a negative relationship between asset returns and positive skewness of returns observed in financial markets in the non-crisis periods, leading to low co-kurtosis risk in the pre-crisis period. As Panels A and C of Figure 3 shows, there is a slight negative correlation between the asset returns and return skewness in the Korean and the US banking sectors in the non-crisis period. This result implies a lower co-kurtosis between the two markets and indicates that an asset that has lower co-kurtosis achieves high returns when another market returns exhibit negative skewness. The financial shocks lead to speculator losses (negative returns) as speculators hit funding constraints and remain holding securities with negative skewness, thus increasing co-kurtosis risk (Brunnermeier et al., 2008). As shown in Panels B and D of Figure 3, there is a positive correlation between equity returns and return skewness in the equity markets of Korean and the US banking sector during the crisis period, indicating the smile effect through co-kurtosis channel that appears in the financial markets during the crash period.

Similarly, changes in co-movements between return volatilities turning from a negative relationship to a positive one in the crisis period reflects the smile effect through the co-volatility channel. Figure 4 shows that co-volatility between the Korean and the US banking sectors is much higher in the crisis period compared with the non-crisis period.

4 Portfolio Choice

In standard theory of portfolio choice, investors achieve the optimal asset allocation by maximizing the expected value of a utility function subject to the variance of the portfolio in the mean-variance framework (Sharpe, 1964, Lintner, 1965, Black, 1972). The descriptive sta-

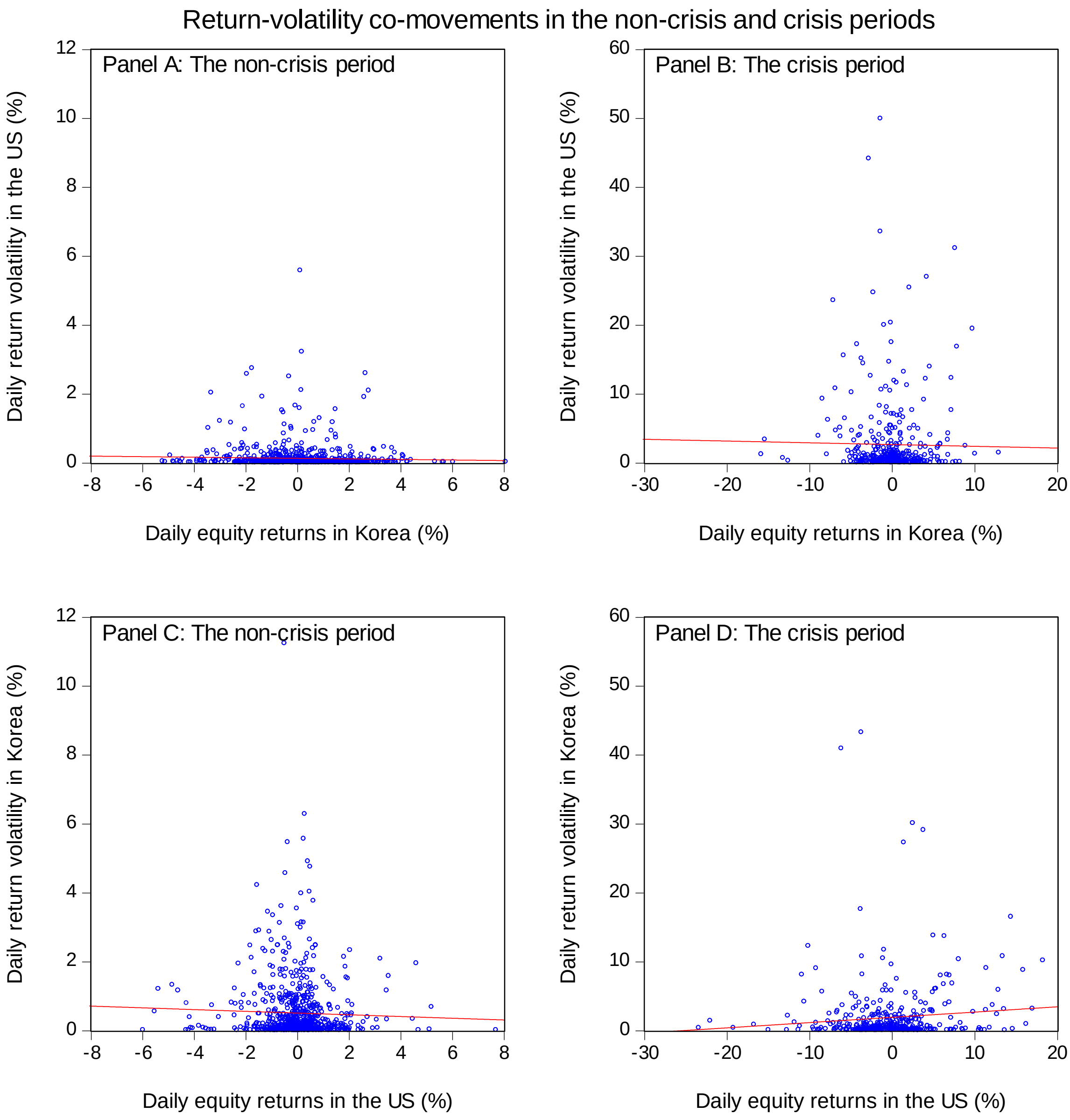

Figure 2: Asymmetric co-movements between the equity returns and return volatility for Korea and the US in the non-crisis period (Panels A and C) and the crisis period (Panels B and D). Circles denote data points and solid lines represent the linear regression line. The data is composed of the daily percentage returns on Korean banking sector and daily percentage returns of banking sector in the US equity index. The non-crisis period is from April 1, 2005 to February 29, 2008. The crisis period is from March 3, 2008 to August 31, 2009.

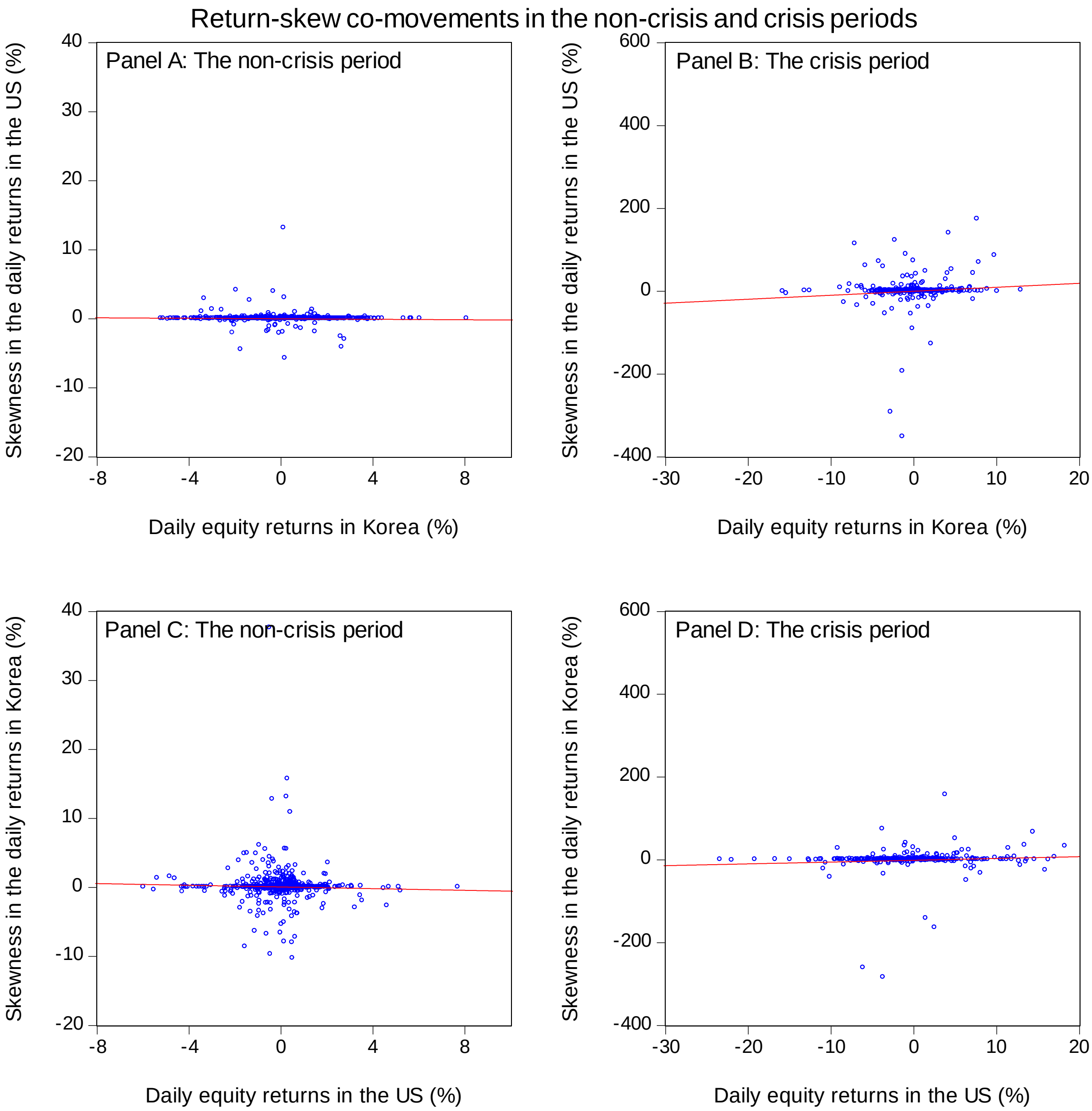

Figure 3: Asymmetric co-movements between equity returns and skewness of returns for Korea and the US in the non-crisis period (Panels A and C) and the crisis period (Panels B and D). Circles denote data points and solid lines represent the linear regression line. The data is composed of the daily percentage returns in the Korean banking sector and daily percentage returns of banking sector in the US equity index. The non-crisis period is from April 1, 2005 to February 29, 2008. The crisis period is from March 3, 2008 to August 31, 2009.

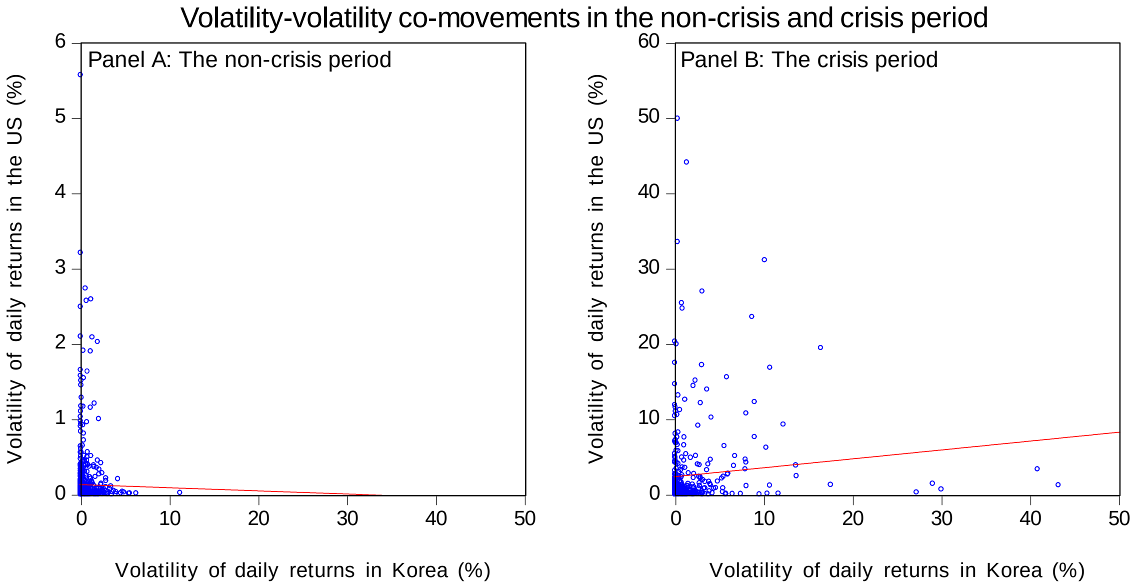

Figure 4: Co-movements between volatility and volatility for Korea and the US in the non-crisis period (Panel A) and the crisis period (Panel B). Circles denote data points and solid lines represent the linear regression line. The data is composed of the daily percentage returns in Korean banking sector and daily percentage returns of banking sector in the US equity index. The non-crisis period is from April 1, 2005 to February 29, 2008. The crisis period is from March 3, 2008 to August 31, 2009.

tistics in Tables 1 to 3 emphasize the importance of modelling asymmetric and tail risks. To take these asymmetries into account, a traditional CAPM model augmented with higher order moments and co-moments is built to illustrate the risk return trade-off between expected excess returns and the higher order terms.

4.1 Higher Order Moments and Portfolio Choice

Consider the standard utility function of an investor allocating their portfolio across N risky assets to maximize their end of period wealth W

$$\underset{\alpha_o,\alpha_1,\ldots,\alpha_N}{Max} \quad E\left[U\left(W\right)\right] \ , \tag{1}$$

given the budget constraint:

$$\alpha_0 + \sum_{i=1}^{N} \alpha_i = 1.$$

The fraction of wealth allocated to the risk-free asset is α_0, and the fraction of wealth allocated to the i^{th} risky asset is α_i. The end of period wealth is

$$W = \alpha_0\left(1+R_f\right) + \sum_{i=1}^{N} \alpha_i\left(1+R_i\right), \tag{2}$$

where R_f and R_i are the rates of return on the risk-free and risky assets respectively.

Assume the distribution of returns of the investor's portfolio of risky assets to be asymmetric and fat-tailed. In this case, the investor's utility function is constructed to include higher order moments since the quadratic mean and variance do not completely determine the distribution. Scott and Horvath (1980) show that investor preferences for higher moments are essential for portfolio selection. The expected utility of the return on investment W of the investor in this case is given by an infinite-order Taylor series expansion around expected wealth:

$$E\left[U\left(W\right)\right] = \sum_{q=0}^{\infty} \frac{U^{(q)}\left(\overline{W}\right)}{q!} E\left[\left(W-\overline{W}\right)^q\right], \tag{3}$$

where the expected value of the end of period wealth is $\overline{W} = E\left(W\right)$ and the q-th derivative of U is $U^{(q)}$.

Equation (3) can be decomposed into the investor's risk preferences $U^{(q)}\left(\overline{W}\right)$ and the q moments of the distribution $E\left[\left(W-\overline{W}\right)^{q}\right]$. Scott and Horvath (1980) show that under the assumption of positive marginal utility of wealth, a utility function exhibits decreasing absolute risk aversion at all wealth levels with strict consistency for moment preferences of $U^{(q)}\left(\overline{W}\right)>0$ if q is odd and $U^{(q)}\left(\overline{W}\right)<0$ if q is even.

Consider equation (3) for the case of the moments $q=1$ to $q=4$. The expected utility function can be written as

$$\begin{aligned} E\left[U\left(W\right)\right] &= U\left(\overline{W}\right)+U^{'}\left(\overline{W}\right)E\left[\left(W-\overline{W}\right)\right]+\frac{1}{2}U^{''}\left(\overline{W}\right)E\left[\left(W-\overline{W}\right)^{2}\right]+ \\ & \frac{1}{3!}U^{'''}\left(\overline{W}\right)E\left[\left(W-\overline{W}\right)^{3}\right]+\frac{1}{4!}U^{(4)}\left(\overline{W}\right)E\left[\left(W-\overline{W}\right)^{4}\right]+ \\ & o(W). \end{aligned} \tag{4}$$

where $o(W)$ is the Taylor remainder. The expected return, variance, skewness and kurtosis for the end of period return, R_p, are denoted by

$$\begin{aligned} \mu_p &= E\left[R_p\right] \\ \sigma_p^2 &= E\left[\left(R_p-\mu_p\right)^2\right]=E\left[\left(W-\overline{W}\right)^2\right] \\ s_p^3 &= E\left[\left(R_p-\mu_p\right)^3\right]=E\left[\left(W-\overline{W}\right)^3\right] \\ k_p^4 &= E\left[\left(R_p-\mu_p\right)^4\right]=E\left[\left(W-\overline{W}\right)^4\right], \end{aligned} \tag{5}$$

where μ_p is the expected return on the portfolio defined as

$$\mu_p=E\left[\alpha_0 R_f\right]+E\left[\sum_{i=1}^{N}\alpha_i R_i\right]=\sum_{i=0}^{N}\alpha_i\mu_i, \tag{6}$$

where α_i is the weight of the i^{th} asset in the portfolio with $\alpha_i\geq 0$ and $\sum_{i=0}^{N}\alpha_i=1$.

The investor's equilibrium condition is solved by taking the first order conditions of the Lagrange of the utility function of wealth subject to the budget constraint as shown in Appendix A.1. The expected excess return on each risky asset over the risk free rate is given by

$$E(R_i)-R_f=\left(\frac{\partial E\left[U\left(W\right)\right]}{\partial\sigma_p^2}\right)\sigma_p^2+\left(\frac{\partial E\left[U\left(W\right)\right]}{\partial s_p^3}\right)s_P^3+\left(\frac{\partial E\left[U\left(W\right)\right]}{\partial k_p^4}\right)k_P^4, \tag{7}$$

where

$$\left(\frac{\partial E[U(W)]}{\partial \sigma_p^2}\right) = -\frac{\left(\frac{\partial(\frac{1}{2}U''(\overline{W}))}{\partial \alpha_i}\right)}{\frac{\partial U(\overline{W})}{\partial \overline{W}}}, \quad \left(\frac{\partial E[U(W)]}{\partial s_p^3}\right) = -\frac{\left(\frac{\partial(\frac{1}{3!}U'''(\overline{W}))}{\partial \alpha_i}\right)}{\frac{\partial U(\overline{W})}{\partial \overline{W}}}, \quad \left(\frac{\partial E[U(W)]}{\partial k_p^4}\right) = -\frac{\left(\frac{\partial(\frac{1}{4!}U^{(4)}(\overline{W}))}{\partial \alpha_i}\right)}{\frac{\partial U(\overline{W})}{\partial \overline{W}}}. \tag{8}$$

Equation (7) shows that the expected excess return on each risky asset contains two elements: i) the portfolio risk premium involving higher order moments of volatility (σ_p^2), skewness (s_p^3) and kurtosis (k_p^4); and ii) measures of the investor's risk preferences for volatility $\left(\frac{\partial E[U(W)]}{\partial \sigma_p^2}\right)$, skewness $\left(\frac{\partial E[U(W)]}{\partial s_p^3}\right)$ and kurtosis $\left(\frac{\partial E[U(W)]}{\partial k_p^4}\right)$.

4.2 Higher Order Co-moments and Portfolio Choice

If an investor invests in two risky assets, $N = 2$, then the variance, skewness and kurtosis of the end of period returns can be decomposed into

$$\begin{aligned} \sigma_p^2 &= E\left[\left(R_p - \mu_p\right)^2\right] \\ &= E\left[\left(\sum_{i=1}^{N=2} \alpha_i \left(R_i - \mu_i\right)\right)^2\right] \\ &= \alpha_1^2 E\left[\left(R_1 - \mu_1\right)^2\right] + \alpha_2^2 E\left[\left(R_2 - \mu_2\right)^2\right] + 2\alpha_1\alpha_2 E\left[\left(R_1 - \mu_1\right)\left(R_2 - \mu_2\right)\right], \end{aligned} \tag{9}$$

$$\begin{aligned} s_P^3 &= E\left[\left(\sum_{i=1}^{N=2} \alpha_i \left(R_i - \mu_i\right)\right)^3\right] \\ &= \alpha_1^3 E\left[\left(R_1 - \mu_1\right)^3\right] + \alpha_2^3 E\left[\left(R_2 - \mu_2\right)^3\right] + \\ &\quad 3\alpha_1^2\alpha_2 E\left[\left(R_1 - \mu_1\right)^2\left(R_2 - \mu_2\right)\right] + 3\alpha_1\alpha_2^2 E\left[\left(R_1 - \mu_1\right)\left(R_2 - \mu_2\right)^2\right], \end{aligned} \tag{10}$$

$$\begin{aligned} k_P^4 &= E\left[\left(\sum_{i=1}^{N=2} \alpha_i \left(R_i - \mu_i\right)\right)^4\right] \\ &= \alpha_1^4 E\left[\left(R_1 - \mu_1\right)^4\right] + \alpha_2^4 E\left[\left(R_2 - \mu_2\right)^4\right] + 4\alpha_1^3\alpha_2 E\left[\left(R_1 - \mu_1\right)^3\left(R_2 - \mu_2\right)\right] \\ &\quad +4\alpha_1\alpha_2^3 E\left[\left(R_1 - \mu_1\right)\left(R_2 - \mu_2\right)^3\right] + 6\alpha_1^2\alpha_2^2 E\left[\left(R_1 - \mu_1\right)^2\left(R_2 - \mu_2\right)^2\right], \end{aligned} \tag{11}$$

respectively.

Substituting equations (9) to (11) into equation (7) gives the expected excess return for asset i in terms of the second, third and fourth order moments and co-moments. The co-moments include the correlation, co-skewness, co-kurtosis and co-volatility as shown below:

$$\begin{aligned} E(R_i) - R_f \;=\;& \theta_1 E\left[(R_1-\mu_1)^2\right] + \theta_2 E\left[(R_2-\mu_2)^2\right] + \theta_3 E\left[(R_1-\mu_1)(R_2-\mu_2)\right] \\ & + \theta_4 E\left[(R_1-\mu_1)^3\right] + \theta_5 E\left[(R_2-\mu_2)^3\right] + \theta_6 E\left[(R_1-\mu_1)^2(R_2-\mu_2)\right] \\ & + \theta_7 E\left[(R_1-\mu_1)(R_2-\mu_2)^2\right] + \theta_8 E\left[(R_1-\mu_1)^4\right] + \theta_9 E\left[(R_2-\mu_2)^4\right] \\ & + \theta_{10} E\left[(R_1-\mu_1)^3(R_2-\mu_2)\right] + \theta_{11} E\left[(R_1-\mu_1)(R_2-\mu_2)^3\right] \\ & + \theta_{12} E\left[(R_1-\mu_1)^2(R_2-\mu_2)^2\right], \end{aligned} \tag{12}$$

where

$$\begin{array}{lll} \theta_1 = \alpha_1^2\left(\frac{\partial E[U(W)]}{\partial \sigma_p^2}\right), & \theta_5 = \alpha_2^3\left(\frac{\partial E[U(W)]}{\partial s_P^3}\right), & \theta_9 = \alpha_2^4\left(\frac{\partial E[U(W)]}{\partial k_P^4}\right), \\ \theta_2 = \alpha_2^2\left(\frac{\partial E[U(W)]}{\partial \sigma_p^2}\right), & \theta_6 = 3\alpha_1^2\alpha_2\left(\frac{\partial E[U(W)]}{\partial s_P^3}\right), & \theta_{10} = 4\alpha_1^3\alpha_2\left(\frac{\partial E[U(W)]}{\partial k_P^4}\right), \\ \theta_3 = 2\alpha_1\alpha_2\left(\frac{\partial E[U(W)]}{\partial \sigma_p^2}\right), & \theta_7 = 3\alpha_1\alpha_2^2\left(\frac{\partial E[U(W)]}{\partial s_P^3}\right), & \theta_{11} = 4\alpha_1\alpha_2^3\left(\frac{\partial E[U(W)]}{\partial k_P^4}\right), \\ \theta_4 = \alpha_1^3\left(\frac{\partial E[U(W)]}{\partial s_P^3}\right), & \theta_8 = \alpha_1^4\left(\frac{\partial E[U(W)]}{\partial k_P^4}\right), & \theta_{12} = 6\alpha_1^2\alpha_2^2\left(\frac{\partial E[U(W)]}{\partial k_P^4}\right). \end{array} \tag{13}$$

Equations (12) and (13) are valid for all i.

Rewriting the expression in equation (12) for asset 1 gives:

$$\begin{aligned} E(R_1) - R_f \;=\;& \theta_1 E\left[(R_1-\mu_1)^2\right] + \theta_2 E\left[(R_2-\mu_2)^2\right] + \theta_3 E\left[(R_1-\mu_1)(R_2-\mu_2)\right] \\ & + \theta_4 E\left[(R_1-\mu_1)^3\right] + \theta_5 E\left[(R_2-\mu_2)^3\right] + \theta_6 E\left[(R_1-\mu_1)^2(R_2-\mu_2)\right] \\ & + \theta_7 E\left[(R_1-\mu_1)(R_2-\mu_2)^2\right] + \theta_8 E\left[(R_1-\mu_1)^4\right] + \theta_9 E\left[(R_2-\mu_2)^4\right] \\ & + \theta_{10} E\left[(R_1-\mu_1)^3(R_2-\mu_2)\right] + \theta_{11} E\left[(R_1-\mu_1)(R_2-\mu_2)^3\right] \\ & + \theta_{12} E\left[(R_1-\mu_1)^2(R_2-\mu_2)^2\right]. \end{aligned} \tag{14}$$

This equation decomposes the expected excess return for the risky asset 1 in terms of risk prices and risk quantities. The risk prices (the θ_i in equation (13)) are expressed in terms of the various risk aversion measures arising from the utility function of the investor ($\left(\frac{\partial E[U(W)]}{\partial \sigma_p^2}\right)$, $\left(\frac{\partial E[U(W)]}{\partial s_P^3}\right)$ and $\left(\frac{\partial E[U(W)]}{\partial k_P^4}\right)$) and the shares of the asset in the portfolio

(α_i). The risk quantities contain the second moment terms of the variance and covariance, the third moment terms of skewness ($E\left[(R_1-\mu_1)^3\right]$ and $E\left[(R_2-\mu_2)^3\right]$) and co-skewness ($E\left[(R_1-\mu_1)^2(R_2-\mu_2)\right]$ and $E\left[(R_1-\mu_1)(R_2-\mu_2)^2\right]$), as well as the fourth moment terms of kurtosis ($E\left[(R_1-\mu_1)^4\right]$ and $E\left[(R_2-\mu_2)^4\right]$), co-kurtosis ($E\left[(R_1-\mu_1)^3(R_2-\mu_2)\right]$ and $E\left[(R_1-\mu_1)(R_2-\mu_2)^3\right]$) and co-volatility ($E\left[(R_1-\mu_1)^2(R_2-\mu_2)^2\right]$).

4.3 Effects of Higher Order Moments on Risky Assets

To illustrate the properties of the expected excess return for risky asset 1 in equation (14) under the scenario of higher order co-moments, the risk-return trade-off surfaces between the expected excess return, the variance, skewness and kurtosis, are presented in Figure 5. The figure is generated by simulating equation (14) where the parameters governing the simulation are chosen as

$$\begin{aligned} \theta_1 &= 0.5, \theta_2 = 0.7, \theta_3 = 2.0, \theta_4 = \theta_5 = \theta_6 = \theta_7 = -1.5, \\ \theta_8 &= \theta_9 = 4, \theta_{10} = \theta_{11} = \theta_{12} = 1.5, \end{aligned} \tag{15}$$

with the values of co-moments given by

$$\begin{aligned} E\left[(R_1-\mu_1)(R_2-\mu_2)\right] &= 0.80, E\left[(R_1-\mu_1)^2(R_2-\mu_2)\right] = 0.00, \\ E\left[(R_1-\mu_1)(R_2-\mu_2)^2\right] &= 0.00, E\left[(R_1-\mu_1)(R_2-\mu_2)^3\right] = 1.47, \\ E\left[(R_1-\mu_1)^3(R_2-\mu_2)\right] &= 1.46, E\left[(R_1-\mu_1)^2(R_2-\mu_2)^2\right] = 2.20. \end{aligned}$$

The parameters $\theta_4, \ldots, \theta_7$ are restricted to have a negative sign due to the assumption that a risk-averse investor has a utility function with decreasing absolute risk aversion, while the other parameters $\theta_8, \ldots, \theta_{12}$ are set up to have a positive sign due to the utility function exhibiting decreasing absolute prudence (Scott and Horvath, 1980).

Panel A of Figure 5 presents the mean-skewness-kurtosis surface for a case where there is no volatility ($E\left[(R_1-\mu_1)^2\right]=0$ and $E\left[(R_2-\mu_2)^2\right]=0$) and panel B for a case where there is volatility ($E\left[(R_1-\mu_1)^2\right]=2$ and $E\left[(R_2-\mu_2)^2\right]=2$). Table 4 summarizes the simulation parameters for the case of no volatility (Panel A of Figure 5) and the case of volatility (Panel B of Figure 5). These parameters are chosen from the data on which the

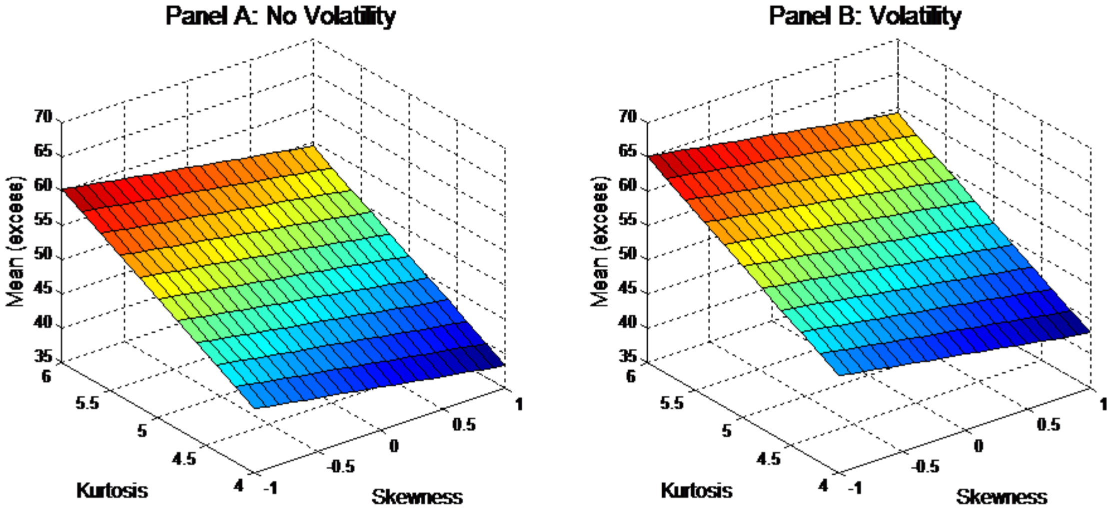

Figure 5: Mean, volatility, skewness and kurtosis trade-offs. Panel A presents a mean-skewness-kurtosis surface for the case of no volatility ($E\left[(R_1-\mu_1)^2\right] = 0$ and $E\left[(R_2-\mu_2)^2\right]=0$). Panel B shows a mean-skewness-kurtosis surface for the case of volatility ($E\left[(R_1-\mu_1)^2\right]=2$ and $E\left[(R_2-\mu_2)^2\right]=2$). Table 4 summarizes the parameters of the moments and co-moments chosen to equation 14 which generates these surfaces.

empirical results are based in Section 7.

Given any level of volatility in Panels A and B of Figure 5, there is a positive relationship between the expected excess return and kurtosis while the relationship between skewness and kurtosis is negative. Comparing panel A with panel B of Figure 5, an investor needs to be compensated for higher risk (volatility) with higher expected excess returns. Given any level of kurtosis, there is a negative trade-off between the expected excess return and skewness. These findings suggest that an investor requires a higher expected excess return for taking more volatility and kurtosis risks; whereas, an investor also realizes a lower expected excess return for the benefit of positive skewness. Fang and Lai (1997) and Hwang and Satchell (1999) also show that expected excess return for the risky asset is associated not only with volatility but also with skewness and kurtosis. Moreover, evidence supports that a four-moment CAPM is able to price the cross-moments of returns much better than the traditional CAPM (Dittmar, 1999).

5 Fourth Order Co-moments Statistics

The previous sections emphasize the impacts of a financial crisis on the risk-return trade-off between expected asset returns and higher order moments and co-moments in understanding the effects of contagion on risk during financial crises. To develop a new approach of testing for contagion through changes in extremal dependence, test statistics of the fourth order co-moments are developed in this section using the framework of a non-normal multivariate returns distribution by setting co-moment restrictions.

5.1 A Bivariate Generalized Exponential Family

To develop the statistics of higher order co-moments, a non-normal multivariate returns distribution is specified in order to model the distribution with asymmetry and leptokurtosis. Following the work of Fry et al. (2010), the bivariate generalized exponential family of the distribution through the first to the fourth order moments and co-moments is used for developing the statistics of co-moments and it is represented as[6]

$$\begin{aligned} f(r) &= \exp(h-\eta) \qquad (16) \\ &= \exp\left(\sum_{i=1}^{12}\theta_i g_i(r)-\eta\right) \\ &= \exp\big(\theta_1 r_1^2+\theta_2 r_2^2+\theta_3 r_1 r_2+\theta_4 r_1 r_2^2+\theta_5 r_1^2 r_2+\theta_6 r_1^2 r_2^2 \\ &\quad +\theta_7 r_1^1 r_2^3+\theta_8 r_1^3 r_2^1+\theta_9 r_1^3+\theta_{10} r_2^3+\theta_{11} r_1^4+\theta_{12} r_2^4-\eta\big), \end{aligned}$$

where h is a function of parameters θ_i and $g_i(r)$. The choices for $g_i(r)$ are polynomials and cross-products in the elements of r which are assumed to follow an independent bivariate normal distribution, and η is a normalizing constant denoted as

$$\begin{aligned} \eta = \ln\int\int \exp\big(&\theta_1 r_1^2+\theta_2 r_2^2+\theta_3 r_1 r_2+\theta_4 r_1 r_2^2+\theta_5 r_1^2 r_2+\theta_6 r_1^2 r_2^2 \qquad (17) \\ &+\theta_7 r_1^1 r_2^3+\theta_8 r_1^3 r_2^1+\theta_9 r_1^3+\theta_{10} r_2^3+\theta_{11} r_1^4+\theta_{12} r_2^4\big)\, dr_1 dr_2. \end{aligned}$$

In terms of equation (16), the parameters θ_1 and θ_2 control the variances of assets 1 and

[6] The univariate class of generalized exponential family of distributions is developed by Cobb et al. (1983) and Lye and Martin (1993).

2 respectively. The parameter θ_3 controls the degree of linear dependence in the relationship between assets 1 and 2 (r_1 and r_2), referring to a correlation coefficient. The parameters θ_4 and θ_5 measure the asymmetry of the probability distribution of the two assets, capturing dependent links between the first moment of asset 1 (r_1) and the second moment of asset 2 (r_2^2) and between the second moment of asset 1 (r_1^2) and the first moment of asset 2 (r_2). These parameters represent the co-skewness coefficients. The coefficients of co-volatility and co-kurtosis (the fourth co-moments) measure the extent to which observations tend to have relatively large frequencies around the centre and in the tails of the joint distribution. These parameters (θ_6, θ_7 and θ_8) capture the interaction between the second moment of two assets 1 and 2 (r_1^2 and r_2^2), between the first moment and third moment of assets 1 and 2 (r_1^1 and r_2^3) and between the third moment and the first moment of assets 1 and 2 (r_1^3 and r_2^1). The parameter θ_6 denotes the co-volatility coefficient and the parameters θ_7 and θ_8 denote the co-kurtosis coefficients. The parameters θ_9 and θ_{10} as well as θ_{11} and θ_{12} control the skewness (r_1^3 and r_2^3) and kurtosis (r_1^4 and r_2^4) for assets 1 and 2 respectively.

5.2 Co-kurtosis and Co-volatility Test Statistics

In this paper, the Lagrange multiplier test is adopted to develop the statistics of co-kurtosis and co-volatility as the bivariate generalized exponential family of the distribution in equation (16) is nested in the bivariate normal distribution by setting the restrictions $\theta_4 = \ldots = \theta_{12} = 0$.

Consider a sample of size T from the bivariate generalized exponential family of the distribution with a finite number K of unknown parameters $\theta = (\theta_1, ..., \theta_K)'$ summarizing the moments of a log likelihood function $\ln L_t(\theta) = h - \eta$ in equation (16) where $h = \sum_{i=1}^{K} \theta_i g_i(r)$ and η is the normalizing constant respectively. The hypothesis to be tested is specified as

$$H_0 : \theta_1 = \cdots = \theta_p = 0; p \leq K. \tag{18}$$

Let $\widehat{\theta}$ be the maximum likelihood estimator of θ. The Lagrange multiplier test statistic is given by

$$LM = q\left(\widehat{\theta}\right)' I\left(\widehat{\theta}\right)^{-1} q\left(\widehat{\theta}\right), \tag{19}$$

which is asymptotically distributed as the chi-squared with p degrees of freedom

$$LM \xrightarrow{d} \chi_p^2. \tag{20}$$

Here, $q\left(\widehat{\theta}\right)$ is the score function evaluated at $\widehat{\theta}$ given by

$$q\left(\widehat{\theta}\right) = \left(\frac{\partial \ln L_t(\theta)}{\partial \theta}\right)_{\theta=\widehat{\theta}}, \tag{21}$$

and $I\left(\widehat{\theta}\right)$ is the asymptotic information matrix evaluated at $\widehat{\theta}$, that is

$$I\left(\widehat{\theta}\right) = T\left(E\left[\frac{\partial h}{\partial \theta}\frac{\partial h}{\partial \theta'}\right]_{\theta=\widehat{\theta}} - E\left[\frac{\partial h}{\partial \theta}\right]_{\theta=\widehat{\theta}} E\left[\frac{\partial h}{\partial \theta'}\right]_{\theta=\widehat{\theta}}\right). \tag{22}$$

The proof of the asymptotic information matrix is shown in Appendix A.2.

Consider the restricted model, the bivariate generalized normal distribution with co-volatility, that is

$$\begin{aligned} f(r_{1,t}, r_{2,t}) &= \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\ &\quad \left. +\theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - \eta\right], \end{aligned} \tag{23}$$

where

$$\begin{aligned} \eta &= \ln \iint \exp\left[\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\ &\quad \left. +\theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2\right] dr_1 dr_2. \end{aligned} \tag{24}$$

A test of the restriction for the hypothesis of normality is given by

$$H_0 : \theta_6 = 0, \tag{25}$$

which constitutes a test of co-volatility.

If the expression of the fourth order co-moment term, $\theta_6 \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2$, in equations (23) and (24) is replaced with the expression of $\theta_7 \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1 \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3$ or $\theta_8 \left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^3 \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^1$, these are for a test of normality. A test of the restriction for the hypothesis of normality is set up as

$$H_0 : \theta_p = 0, \;\; p = 7, 8,$$

which constitutes a test of the first form of co-kurtosis ($\theta_7 = 0$) and a test of the second form of co-kurtosis ($\theta_8 = 0$) respectively. Under the null hypothesis, the maximum likelihood estimators of the unknown parameters under the restricted model in equation (23) are

$$\widehat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t};\widehat{\sigma}_i^2 = \frac{1}{T}\sum_{t=1}^{T}\left(r_{i,t}-\widehat{\mu}_i\right)^2;\widehat{\rho} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right);i=1,2. \tag{26}$$

The Lagrange multiplier statistics for co-volatility (LM_1) and for co-kurtosis (LM_2 and LM_3) are used to test for extremal dependence and are shown as

$$\begin{aligned} LM_1 &= \frac{1}{T\left(4\widehat{\rho}^4 + 16\widehat{\rho}^2 + 4\right)}\left[\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 - T\left(1+2\widehat{\rho}^2\right)\right]^2 \\ LM_2 &= \frac{1}{T\left(18\widehat{\rho}^2 + 6\right)}\left[\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^1\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^3 - T\left(3\widehat{\rho}\right)\right]^2 \\ LM_3 &= \frac{1}{T\left(18\widehat{\rho}^2 + 6\right)}\left[\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^3\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^1 - T\left(3\widehat{\rho}\right)\right]^2. \end{aligned} \tag{27}$$

The derivations for the test statistics of co-volatility and co-kurtosis are shown in Appendix A.3.

Under the null hypothesis in equation (18), LM_1, LM_2 and LM_3 are distributed asymptotically as χ_1^2.

6 Higher Order Co-moment Contagion Tests

In this section, two types of contagion tests are developed based on a change in the non-linear dependence between two assets, which is motivated by the documentation of asymmetry and fat tails in asset markets in earlier studies by Harvey and Siddique (2000) and Dittmar (2002), and by the more recent findings of the effects of shocks on the behavior of asset returns by Jondeau and Rockinger (2009). The first is the asymmetric dependence tests developed by Fry et al. (2010). The second is the extremal dependence tests, which are proposed in this paper. Extremal dependence captures the dependence between an extreme event in one market and a similar event in another market and it is used to derive a new test of financial contagion in this paper.

In deriving the contagion tests, the following notation is used. Let x and y denote the non-crisis and crisis periods, respectively. T_x and T_y are the sample sizes of the non-crisis and crisis periods respectively. $T = T_x + T_y$ is the sample size of the full period. Then, the sample correlation coefficient during the non-crisis period (low-volatility) is ρ_x and during the crisis period (high-volatility) is ρ_y. Let i denote the source crisis asset market and j denote the recipient market of contagion. $\widehat{\mu}_{xi}$, $\widehat{\mu}_{xj}$, $\widehat{\mu}_{yi}$ and $\widehat{\mu}_{yj}$ are the sample means of the asset returns for markets i and j during the non-crisis and crisis periods and $\widehat{\sigma}_{xi}$, $\widehat{\sigma}_{xj}$, $\widehat{\sigma}_{yi}$ and $\widehat{\sigma}_{yj}$ are the sample standard deviations of the asset returns for markets i and j during the non-crisis and crisis period, respectively.

6.1 Asymmetric Dependence Tests

The aim of the asymmetric dependence tests of contagion by Fry et al. (2010) is to identify whether there is a statistically significant change in co-skewness between the non-crisis and crisis period after controlling for the market fundamentals. There are two forms of this tests, which depend on whether the source country coincides with asset returns (CS_{12}) or asset volatility (CS_{21}).

The first type is to test for contagion where the shocks transmit from the returns of a source market i to the volatility of asset returns of a recipient market j. The second type is to test for contagion where the shocks spread from the volatility of asset returns of a source market i to the asset returns of a recipient market j given as

$$CS_{12}(i \rightarrow j; r_i^1, r_j^2) = \left(\frac{\widehat{\psi}_y\left(r_i^1, r_j^2\right) - \widehat{\psi}_x\left(r_i^1, r_j^2\right)}{\sqrt{\frac{4\widehat{v}_{y|x_i}^2+2}{T_y} + \frac{4\widehat{\rho}_x^2+2}{T_x}}} \right)^2 \tag{28}$$

$$CS_{21}(i \rightarrow j; r_i^2, r_j^1) = \left(\frac{\widehat{\psi}_y\left(r_i^2, r_j^1\right) - \widehat{\psi}_x\left(r_i^2, r_j^1\right)}{\sqrt{\frac{4\widehat{v}_{y|x_i}^2+2}{T_y} + \frac{4\widehat{\rho}_x^2+2}{T_x}}} \right)^2, \tag{29}$$

where

$$\begin{aligned} \widehat{\psi}_y(r_i^m, r_j^n) &= \frac{1}{T_y}\sum_{t=1}^{T_y}\left(\frac{y_{i,t}-\widehat{\mu}_{yi}}{\widehat{\sigma}_{yi}}\right)^m \left(\frac{y_{j,t}-\widehat{\mu}_{yj}}{\widehat{\sigma}_{yj}}\right)^n \\ \widehat{\psi}_x(r_i^m, r_j^n) &= \frac{1}{T_x}\sum_{t=1}^{T_x}\left(\frac{x_{i,t}-\widehat{\mu}_{xi}}{\widehat{\sigma}_{xi}}\right)^m \left(\frac{x_{j,t}-\widehat{\mu}_{xj}}{\widehat{\sigma}_{xj}}\right)^n, \end{aligned}$$

and

$$\widehat{v}_{y|x_i} = \frac{\widehat{\rho}_y}{\sqrt{1 + \left(\frac{s_{y,i}^2 - s_{x,i}^2}{s_{x,i}^2}\right)\left(1-\widehat{\rho}_y^2\right)}}, \tag{30}$$

represents the adjusted correlation coefficient proposed by Forbes and Rigobon (2002). Forbes and Rigobon (2002) argue that estimation of cross-market correlation coefficients is biased due to heteroscedasticity in market returns. The adjusted correlation coefficient is scaled by a non-linear function of the percentage change in volatility of the source market returns $\left(\left(s_{y,i}^2 - s_{x,i}^2\right)/s_{x,i}^2\right)$ to solve this problem.

To test that there is a significant change in co-skewness between the non-crisis and crisis period, the null and alternative hypotheses are

$$\begin{aligned} H_0 &: \psi_y(r_i^m, r_j^n) = \psi_x(r_i^m, r_j^n) \\ H_1 &: \psi_y(r_i^m, r_j^n) \neq \psi_x(r_i^m, r_j^n). \end{aligned}$$

Under the null hypothesis of no contagion, tests of contagion based on changes in co-skewness are asymptotically distributed as

$$CS_{12}, CS_{21}(i \to j) \xrightarrow{d} \chi_1^2.$$

6.2 Extremal Dependence Tests

Co-kurtosis and co-volatility are utilized here for measuring extremal dependence. The idea of the extremal dependence tests is to identify whether there is a significant change in co-kurtosis or co-volatility between a non-crisis and a crisis period.

Three types of extremal dependence tests are specified depending on whether asset returns in the source market are expressed in terms of returns and cubed returns in computing co-kurtosis and in terms of return volatility in computing co-volatility. The first type of statistic CK_{13} is to detect the shocks emanating from the asset returns of a source market i to the cubed returns (akin to skewness) of an asset in a recipient market j. The second type of statistic CK_{31} is to measure the shocks transmitting from the cubed returns of asset in a source market i to the returns of asset in a recipient market j given as

$$CK_{13}(i \quad \to \quad j; r_i^1, r_j^3) = \left(\frac{\widehat{\xi}_y \left(r_i^1, r_j^3\right) - \widehat{\xi}_x \left(r_i^1, r_j^3\right)}{\sqrt{\frac{18\widehat{v}_{y|x_i}^2 + 6}{T_y} + \frac{18\widehat{\rho}_x^2 + 6}{T_x}}} \right)^2 \tag{31}$$

$$CK_{31}(i \quad \to \quad j; r_i^3, r_j^1) = \left(\frac{\widehat{\xi}_y \left(r_i^3, r_j^1\right) - \widehat{\xi}_x \left(r_i^3, r_j^1\right)}{\sqrt{\frac{18\widehat{v}_{y|x_i}^2 + 6}{T_y} + \frac{18\widehat{\rho}_x^2 + 6}{T_x}}} \right)^2, \tag{32}$$

where

$$\widehat{\xi}_y(r_i^m, r_j^n) = \frac{1}{T_y} \sum_{t=1}^{T_y} \left(\frac{y_{i,t} - \widehat{\mu}_{yi}}{\widehat{\sigma}_{yi}} \right)^m \left(\frac{y_{j,t} - \widehat{\mu}_{yj}}{\widehat{\sigma}_{yj}} \right)^n - \left(3\widehat{v}_{y|x_i}\right)$$

$$\widehat{\xi}_x(r_i^m, r_j^n) = \frac{1}{T_x} \sum_{t=1}^{Tx} \left(\frac{x_{i,t} - \widehat{\mu}_{xi}}{\widehat{\sigma}_{xi}} \right)^m \left(\frac{x_{j,t} - \widehat{\mu}_{xj}}{\widehat{\sigma}_{xj}} \right)^n - \left(3\widehat{\rho}_x\right),$$

and

$$\widehat{v}_{y|x_i} = \frac{\widehat{\rho}_y}{\sqrt{1+\left(\frac{s_{y,i}^2-s_{x,i}^2}{s_{x,i}^2}\right)\left(1-\widehat{\rho}_y^2\right)}}.$$

The third type of statistic CV_{22} is to detect the shocks transmitting from the volatility of asset returns in a source market i to the volatility of asset returns in a recipient market j. The statistic of co-volatility can be represented as

$$CV_{22}(i \to j; r_i^2, r_j^2) = \left(\frac{\widehat{\xi}_y\left(r_i^2, r_j^2\right) - \widehat{\xi}_x\left(r_i^2, r_j^2\right)}{\sqrt{\frac{4\widehat{v}_{y|x_i}^4+16\widehat{v}_{y|x_i}^2+4}{T_y} + \frac{4\widehat{\rho}_x^4+16\widehat{\rho}_x^2+4}{T_x}}}\right)^2, \tag{33}$$

where

$$\begin{aligned}
\widehat{\xi}_y(r_i^2, r_j^2) &= \frac{1}{T_y}\sum_{t=1}^{T_y}\left(\frac{y_{i,t}-\widehat{\mu}_{yi}}{\widehat{\sigma}_{yi}}\right)^2\left(\frac{y_{j,t}-\widehat{\mu}_{yj}}{\widehat{\sigma}_{yj}}\right)^2 - \left(1+2\widehat{v}_{y|x_i}^2\right) \\
\widehat{\xi}_x(r_i^2, r_j^2) &= \frac{1}{T_x}\sum_{t=1}^{T_x}\left(\frac{x_{i,t}-\widehat{\mu}_{xi}}{\widehat{\sigma}_{xi}}\right)^2\left(\frac{x_{j,t}-\widehat{\mu}_{xj}}{\widehat{\sigma}_{xj}}\right)^2 - \left(1+2\widehat{\rho}_x^2\right).
\end{aligned}$$

The co-volatility change test of contagion proposed in this paper is quite similar to the approach of the volatility spillovers across markets introduced by Diebolz and Yilmaz (2009). They construct indices of return spillovers between markets, and volatility spillovers between markets.

To test that there is a significant change in co-kurtosis or co-volatility between the non-crisis period and the crisis period, the null and alternative hypotheses are

$$\begin{aligned}
H_0 &: \xi_y(r_i^m, r_j^n) = \xi_x(r_i^m, r_j^n) \\
H_1 &: \xi_y(r_i^m, r_j^n) \neq \xi_x(r_i^m, r_j^n).
\end{aligned}$$

Under the null hypothesis of no contagion, tests of contagion based on changes in co-kurtosis or co-volatility are asymptotically distributed as

$$CK_{13}, CK_{31}, CV_{22}(i \to j) \xrightarrow{d} \chi_1^2.$$

6.3 Finite Sample Properties

The properties of the contagion test statistics are illustrated in terms of the relatively large sample period of the non-crisis but the relatively short sample period of the crisis which is a characteristic of most financial market crises. To investigate this issue, the finite sample distribution properties of the contagion test statistics under the null hypothesis of no contagion are conducted through simulation.

To generate the asset returns in the simulation, the parameters are chosen for the following non-crisis and crisis variance-covariance matrices of returns in two equity markets i and j as

$$V_x = \begin{bmatrix} 0.557 & 0.143 \\ 0.143 & 2.660 \end{bmatrix}, \ V_y = \begin{bmatrix} 29.195 & 14.350 \\ 14.350 & 16.430 \end{bmatrix}. \tag{34}$$

The parameters chosen are based on the data used in the empirical applications shown in section 7. For the non-crisis period matrix of the returns (V_x), the parameters for the variance of equity returns in two markets, 0.557 and 2.660, are chosen respectively based on the sample variances of equity returns in the US banking sector and the average of the seven selected recipient markets. The parameter for the covariance, 0.143, is set in terms of average sample covariances of equity returns between the US banking sector and the selected markets during the non-crisis period. The parameters for the crisis variance-covariance matrix are specified using the same concept as that used to set up the parameters of the non-crisis variance-covariance matrix. The variance-covariance matrices of V_x and V_y show that the increase in volatility of returns over the two periods is 5145% for the source market and 517% for the recipient market, respectively. The correlation coefficients during the non-crisis and crisis periods yield 0.118 (ρ_x) and 0.655 (ρ_y). Furthermore, the adjusted correlation coefficient shown in equation (30) is $v_{y|x_i} = 0.119$.

The returns are randomly generated to follow a bivariate normal distribution with zero mean and variances given by equation (34) in computing the contagion tests. The contagion statistics of co-skewness (CS_{12}, CS_{21}), co-kurtosis (CK_{13}, CK_{31}) and co-volatility (CV_{22}) are conducted by Monte Carlo simulation using $10,000$ replications given the sample sizes of the non-crisis period ($T_x = 760$) and of the crisis period ($T_y = 391$). The contagion statistics of CS_{12}, CS_{21}, CK_{13}, CK_{31} and CV_{22} as well as the statistic of the asymptotic distribution (χ_1^2) for three values of the significance level (α) are presented in Table 5.

The results show that the statistics for contagion based on changes in co-skewness (CS_{12} and CS_{21}) and co-volatility (CV_{22}) are good approximations of the finite sample distribution for the three values of the significance level (α). These statistics, CS_{12}, CS_{21} and CV_{22}, tend to follow a chi-square distribution with one degree of freedom (χ_1^2). However, the test statistics for contagion based on changes in co-kurtosis (CK_{13} and CK_{31}) tend to be biased compared with the asymptotic distribution (χ_1^2). The critical values require adjusting as shown in Table 5 for the latter statistics.[7]

7 Empirical Application

The financial crisis of 2008-09 originated in the US interbank market following the collapse of Lehman Brothers and escalated into a global phenomenon. It was triggered by a complex interaction between valuation in the housing market and liquidity problems in the US banking system. The US banking sector is considered to be a potentially essential channel for contagion risk due to the high volume of transactions through the different segments of the financial sector both locally and across countries. In particular, the US banking sector played a decisive role in the development of the financial crises as financial intermediaries faced a potential refinancing problem in the maturity mismatch between loans and assets (Brunnermeier, 2009). Such mismatching resulted in a liquidity shortage for investments banks, ultimately leading to the failure of Lehman Brothers occurring in mid-September, 2008. Subsequently, the insolvency of the US shadow banks rapidly developed and spread across the entire financial system, leading to a number of European bank failures and slumping global stock markets.

The tests of contagion developed in Section 6 are applied to identify transmission channels through changes in asymmetric and extremal dependences during the financial crisis of 2008-09 for the data set described in Section 2. An extensive set of linkages for financial contagion in this paper are investigated by testing: i) contagion linkages between banking sectors of different countries; and ii) contagion linkages between banking sectors and equity markets within countries.[8]

[7] If the non-crisis period and crisis period sample sizes are the same, this bias does not occur in the CK_{13} and CK_{31} statistics.

[8] In computing the statistics of contagion testing, a Vector Autoregressive (VAR) model is first specified and estimated not only to control for market fundamentals (country-specific and cross market relationships that always exist) but also to handle with the problems of serial correlation in the data set (Forbes and

The first set of linkages is to detect contagion through the transmission of shocks among banks. Cross-border contagion between banks is studied by Gropp and Moerman (2004) and Gropp et al. (2010). They use non-parametric tests of banks' changes in distances-to-default to test for contagion between two banks and measure contagion as the changes in the tail dependence of joint distribution (equivalent to co-exceedances) during the crisis period. It is worth emphasizing that tail (extremal) dependence is a key component for risk management as it is interpreted as the probability of the worst event occurring in one market given that the worst event occurs in another market (Garcia and Tsafack, 2011). Second, contagion linkages between the banking sectors and equity markets are presented as the characteristics of the crisis are disorder in a range of financial markets, not just in the source market.

The next sub-sections present the empirical results of the contagion tests based on changes in asymmetric dependence (co-skewness) and extremal dependence (co-kurtosis and co-volatility) during the global financial crisis of 2008-09 using the crisis sourced in the US banking sector as the source of the crisis. Under the null hypothesis of no contagion, the critical values for the contagion tests are generated from the simulation study of Section 6.3 and shown in Table 5.

7.1 Contagion Channel through Asymmetric Dependence

Table 6 presents the empirical results for the asymmetric dependence tests of contagion during the crisis of 2008-09 with the source market specified to be the US banking sector. Inspection of this table reveals that significant evidence of contagion is found within banking sectors, but less contagion effects are shown between the US banking sector and the equity markets through the co-skewness channel. In particular, the majority of the banking sectors, especially these located in the European region are affected by at least one of the contagion

Rigobon, 2002). The model specification is given by

$$\begin{aligned} Z_t &= \phi(L)Z_t + \eta_t \\ Z_t &= \{x_t, y_t\}'. \end{aligned} \tag{35}$$

where Z_t is a transposed vector of returns across a set of equity markets and banking sectors during the non-crisis (x_t) and crisis (y_t) periods; $\phi(L)$ is a vector of lags and η_t is a vector of the residual terms.

Instead of using daily returns, two-days rolling average returns (Z_t) are utilized to deal with the fact that equity markets are open in different time zones (Forbes and Rigobon, 2002). Five lags of $\phi(L)$ are set in the VAR based on the criteria of the sequential modified log-likelihood ratio test statistic (LR) and Akaike information (AI). The residuals estimated from the VAR are used in computing the co-skewness contagion statistics in (28) and (29), the co-kurtosis contagion statistics in (31) and (32) as well as the co-volatility contagion statistic in (33).

channels through asymmetric dependence during the financial crisis. The key channels detected by the asymmetric dependence tests are from the returns in the US banking sector to the volatility of the banking sectors in Korea, France, Germany, the UK and Chile and also from the volatility of the US banking sector to the returns of the banking sectors in Hong Kong, Korea and the UK. This result indicates that the US banking sector is of systemic importance through the asymmetric dependence channel.[9]

Panel A of Table 6 shows there are no significant and widespread cross-border contagious linkages from the US banking sector to equity markets in France, Germany, the UK, Mexico and the US through the co-skewness channel. This result is in sharp contrast to the result that the equity markets in Hong Kong, Korea and Chile are exposed to contagion risk from the US banking sector through the co-skewness channel.

7.2 Contagion Channel through Extremal Dependence

The results of the extremal dependence tests presented in Table 7 show that significant contagion effects are pervasive from the US banking sector to the selected equity markets (Panel A) and to the selected banking sectors (Panel B) in four regions including Asia, Europe, Latin America and North America during the global crisis of 2008-09. Among these regions, the European region is much exposed to contagion risks from the US banking sector since both equity markets and banking sectors in the European region have experienced dramatic increases in extremal dependence (i.e. co-kurtosis and co-volatility) with the US banking sector in the crisis period compared with the non-crisis period. The smallest value of the test statistic is 55.82 for the contagion linkages between the US banking sector and German equity markets and 79.54 for the contagion linkages between the US banking sector and the German banking sectors through extremal dependence, which is much higher than the critical value at the 1% significant level. The results are consistent with the fact that after the end of the crisis as considered in this paper, the European region faced its own financial crisis so called “European sovereign debt crisis” due to the problem of refinancing government debt.

Among the four regions, the evidence of contagion through the co-volatility channel is weak to both Asia and Latin America during the global financial crisis of 2008-09. In

[9] Gropp and Moerman (2004) define the term “systemic importance” in terms of the banking sector which tends to have a net contagious influence on other banks.

particular, there is no evidence of contagion between the US banking sector and equity markets as well as the banking sectors in Hong Kong and Chile through the co-volatility channel. However, the evidence of contagion is strong to both the Asian and Latin American regions through the two forms of the co-kurtosis channels. The key channels operating are from the returns of the US banking sector to the return skewness of both Asian and Latin American equity markets, and from the return skewness of the US banking sector to the equity market returns in both Asian and Latin American except for Hong Kong. As for the North American region, the US equity market is affected by the global financial crisis of 2008-09 through either one of the co-kurtosis channel or the co-volatility channel.

Comparing Table 6 with Table 7, more contagion channels are found through extremal dependence than through asymmetric dependence during the financial crisis of 2008-09. In particular, all four regions of Asia, Europe, Latin America and North America are affected by the financial shocks originating from the US banking sector through extremal dependence between 2008 and 2009. The finding suggests that in extreme events, the extremal dependence test are more important in capturing markets linkages than the asymmetric dependence linkages.

8 Conclusions

This paper proposes a class of new tests of financial contagion based on changes in extremal dependence (co-kurtosis and co-volatility) during a financial crisis. Extremal dependence is one of non-linear dependence, enabling the capture of the probability of the worst event in one market given the worst event that appears in another market. Extremal dependence structure is subject to change in different regimes and attributed to two types of effects. The first is the risk return trade-off effect between the expected excess returns and higher order moments. The higher kurtosis observed in a financial crisis is realized by a risk-averse investor in conjunction with positive expected excess returns, leading to a sudden decrease in current asset returns. The risk return trade-off effect is explored based on the traditional CAPM model with higher order moments and co-moments where the expected excess return on risky asset is expressed in terms of risk prices and risk quantities. Risk prices are comprised of risk preferences determined by the marginal utility function and the share of the asset in the portfolio. Risk quantities are determined by any moment and co-moment preferences, with special emphasis on the co-kurtosis and co-volatility for modelling financial contagion.

The second is the smile effect through the co-kurtosis or co-volatility channel, which is observed in financial markets where the return-skew and the volatility-volatility co-moments exhibit a negative relation in the non-crisis period but switch to a positive relation in the crisis period, resulting in higher co-kurtosis and co-volatility risks in the crash period.

In deriving a new test of contagion through the changes in extremal dependence, the bivariate generalized exponential distribution is specified to allow for higher order moments and co-moments. The Lagrange multiplier test is adopted to develop the statistics of co-kurtosis and co-volatility, which gives the framework for developing a new test of contagion based on changes in extremal dependence.

This new approach is applied to test for financial contagion in equity markets and banking sectors during the financial crisis of 2008-09. The results of the tests show significant contagion effects are pervasive from the US banking sector to global equity markets and from the US banking sector to the global banking sector based on changes in extremal dependence. The extremal dependence tests capture more market co-movements than the asymmetric dependence tests developed by Fry et al. (2010) in extreme events. Simulation studies show the statistic for contagion based on changes in co-volatility presents a good approximation of the finite sample distribution regarding the relatively large sample period of the non-crisis but the relatively short sample period of the crisis.

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A Appendix - Derivation of Test Statistics

A.1 An Optimum Problem

Consider an investor allocating their portfolio across N risky assets to maximize the expected utility function of end of period wealth

$$\underset{\alpha_o,\alpha_1,\ldots,\alpha_N}{Max} \quad E\left[U\left(W\right)\right] \ , \tag{36}$$

where

$$W = \alpha_0\left(1+R_f\right)+\sum_{i=1}^{N}\alpha_i\left(1+R_i\right),$$

subject to the budget constraint

$$\alpha_0+\sum_{i=1}^{N}\alpha_i = 1. \tag{37}$$

To maximize an investor's portfolio problem described above, a Lagrange function is written as

$$L = E\left[U\left(W\right)\right]+\lambda\left(1-\alpha_0+\sum_{i=1}^{N}\alpha_i\right). \tag{38}$$

In solving this problem by taking the two partial derivatives of the Lagrange function, two pairs of simultaneous linear equations are written as

$$\frac{\partial L}{\partial \alpha_0} = \frac{\partial U\left(\overline{W}\right)}{\partial \overline{W}}R_f-\lambda = 0, \tag{39}$$

and

$$\begin{aligned}\frac{\partial L}{\partial \alpha_i} &= \frac{\partial U\left(\overline{W}\right)}{\partial \overline{W}}E\left(R_i\right)+\frac{\partial(\frac{1}{2}U''\left(\overline{W}\right))}{\partial \alpha_i}\sigma_p^2 \\ &+\frac{\partial(\frac{1}{3!}U'''\left(\overline{W}\right))}{\partial \alpha_i}s_p^3+\frac{\partial(\frac{1}{4!}U^{(4)}\left(\overline{W}\right))}{\partial \alpha_i}k_p^4-\lambda = 0.\end{aligned} \tag{40}$$

Rearranging equations (39) and (40), the investor's equilibrium condition is expressed as

$$E\left(R_i\right)-R_f = \left(\frac{\partial E\left[U\left(W\right)\right]}{\partial \sigma_p^2}\right)\sigma_p^2+\left(\frac{\partial E\left[U\left(W\right)\right]}{\partial s_p^3}\right)s_p^3+\left(\frac{\partial E\left[U\left(W\right)\right]}{\partial k_p^4}\right)k_P^4, \tag{41}$$

where

$$\left(\frac{\partial E\left[U\left(W\right)\right]}{\partial \sigma_p^2}\right) = -\frac{\frac{\partial\left(\frac{1}{2}U''\left(\overline{W}\right)\right)}{\partial \alpha_i}}{\frac{\partial U\left(\overline{W}\right)}{\partial \overline{W}}} \tag{42}$$

$$\left(\frac{\partial E\left[U\left(W\right)\right]}{\partial s_p^3}\right) = -\frac{\frac{\partial\left(\frac{1}{3!}U'''\left(\overline{W}\right)\right)}{\partial \alpha_i}}{\frac{\partial U\left(\overline{W}\right)}{\partial \overline{W}}}$$

$$\left(\frac{\partial E\left[U\left(W\right)\right]}{\partial k_p^4}\right) = -\frac{\frac{\partial\left(\frac{1}{4!}U^{(4)}\left(\overline{W}\right)\right)}{\partial \alpha_i}}{\frac{\partial U\left(\overline{W}\right)}{\partial \overline{W}}}.$$

A.2 Proof of the Asymptotic Information Matrix

The log of the likelihood function at time t in equation (16) is

$$\ln L_t(\theta) = h - \eta. \tag{43}$$

The first and second derivatives are given respectively by

$$\frac{\partial \ln L_t(\theta)}{\partial \theta} = \frac{\partial h}{\partial \theta} - \frac{\partial \eta}{\partial \theta}, \tag{44}$$

and

$$\frac{\partial^2 \ln L_t(\theta)}{\partial \theta \partial \theta'} = \frac{\partial^2 h}{\partial \theta \partial \theta'} - \frac{\partial^2 \eta}{\partial \theta \partial \theta'}. \tag{45}$$

The information matrix at time t is

$$\begin{aligned} I_t\left(\theta\right) &= -E\left[\frac{\partial^2 \ln L_t(\theta)}{\partial \theta \partial \theta'}\right] \\ &= \frac{\partial^2 \eta}{\partial \theta \partial \theta'} - E\left[\frac{\partial^2 h}{\partial \theta \partial \theta'}\right]. \end{aligned} \tag{46}$$

Differentiating a first and second time gives

$$\frac{\partial \eta}{\partial \theta} = \frac{\int \left(\frac{\partial h}{\partial \theta}\right) \exp\left(h\right) dr}{\int \exp\left(h\right) dr} = E\left[\frac{\partial h}{\partial \theta}\right]. \tag{47}$$

$$\frac{\partial^2 \eta}{\partial\theta\partial\theta'} = \frac{\left(\int \left(\frac{\partial^2 h}{\partial\theta\partial\theta'}\right) \exp(h)\, dr + \int \left(\frac{\partial h}{\partial\theta}\right)\left(\frac{\partial h}{\partial\theta'}\right) \exp(h)\, dr\right)\left(\int \exp(h)\, dr\right)}{\left(\int \exp(h)\, dr\right)^2} - \frac{\left(\int \left(\frac{\partial h}{\partial\theta}\right) \exp(h)\, dr\right)\left(\int \left(\frac{\partial h}{\partial\theta}\right) \exp(h)\, dr\right)}{\left(\int \exp(h)\, dr\right)^2}$$
$$= E\left[\frac{\partial^2 h}{\partial\theta\partial\theta'}\right] + E\left[\frac{\partial h}{\partial\theta}\frac{\partial h}{\partial\theta'}\right] - E\left[\frac{\partial h}{\partial\theta}\right] E\left[\frac{\partial h}{\partial\theta'}\right]. \tag{48}$$

Substituting equation (48) into the information matrix at time t in equation (46) yields

$$I_t(\theta) = E\left[\frac{\partial^2 h}{\partial\theta\partial\theta'}\right] + E\left[\frac{\partial h}{\partial\theta}\frac{\partial h}{\partial\theta'}\right] - E\left[\frac{\partial h}{\partial\theta}\right] E\left[\frac{\partial h}{\partial\theta'}\right] - E\left[\frac{\partial^2 h}{\partial\theta\partial\theta'}\right]$$
$$= E\left[\frac{\partial h}{\partial\theta}\frac{\partial h}{\partial\theta'}\right] - E\left[\frac{\partial h}{\partial\theta}\right] E\left[\frac{\partial h}{\partial\theta'}\right]. \tag{49}$$

Finally, $I\left(\widehat{\theta}\right)$ is the asymptotic information matrix evaluated at $\widehat{\theta}$, that is

$$I\left(\widehat{\theta}\right) = \sum_t I_t\left(\widehat{\theta}\right)$$
$$= T\left(E\left[\frac{\partial h}{\partial\theta}\frac{\partial h}{\partial\theta'}\right]_{\theta=\widehat{\theta}} - E\left[\frac{\partial h}{\partial\theta}\right]_{\theta=\widehat{\theta}} E\left[\frac{\partial h}{\partial\theta'}\right]_{\theta=\widehat{\theta}}\right). \tag{50}$$

A.3 Derivation of the Test Statistics

A.3.1 Statistics of Co-volatility

Consider the following bivariate generalized normal distribution with co-volatility

$$f(r_{1,t}, r_{2,t}) = \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) + \theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - \eta\right] \tag{51}$$
$$= \exp[h-\eta],$$

where

$$\eta = \ln\iint \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) + \theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2\right] dr_1 dr_2 \tag{52}$$
$$= \ln\iint \exp[h]\, dr_1 dr_2,$$

and

$$
\begin{aligned}
h &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2+\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\
&\quad +\theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2. \qquad (53)
\end{aligned}
$$

A test of the restriction for normality is

$$H_0 : \theta_6 = 0. \qquad (54)$$

Under the null hypothesis, the maximum likelihood estimators of the unknown parameters are

$$\widehat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t};\ \widehat{\sigma}_i^2 = \frac{1}{T}\sum_{t=1}^{T}\left(r_{i,t}-\widehat{\mu}_i\right)^2;\ \widehat{\rho} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right);\ i=1,2. \qquad (55)$$

Let the parameters of equation (51) be

$$\theta = \left\{\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho, \theta_6\right\}. \qquad (56)$$

By taking the log function in equation (51), the log likelihood function at time t is given by

$$
\begin{aligned}
\ln L_t(\theta) &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2+\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\
&\quad +\theta_6\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2-\eta \qquad (57) \\
&= h-\eta,
\end{aligned}
$$

where h and η are given by equations (53) and (52).

Taking the expectations for the first and second derivatives of the log likelihood function ($\ln L_t(\theta)$) in equation (53) under the null hypothesis of the bivariate normality ($\theta_6 = 0$) is

$$
\begin{aligned}
E\left(\frac{\partial h}{\partial \mu_1}\right) &= 0 \\
E\left(\frac{\partial h}{\partial \mu_2}\right) &= 0 \\
E\left(\frac{\partial h}{\partial \sigma_1^2}\right) &= \frac{1}{2}\frac{1}{\sigma_1^2} \\
E\left(\frac{\partial h}{\partial \sigma_2^2}\right) &= \frac{1}{2}\frac{1}{\sigma_2^2} \\
E\left(\frac{\partial h}{\partial \rho}\right) &= \frac{-\rho}{1-\rho^2} \\
E\left(\frac{\partial h}{\partial \theta_6}\right) &= 1+2\rho^2,
\end{aligned}
$$

$$
\begin{aligned}
E\left[\left(\frac{\partial h}{\partial \mu_1}\right)^2\right] &= \frac{1}{1-\rho^2}\frac{1}{\sigma_1^2} \\
E\left[\left(\frac{\partial h}{\partial \mu_2}\right)^2\right] &= \frac{1}{1-\rho^2}\frac{1}{\sigma_2^2} \\
E\left[\left(\frac{\partial h}{\partial \sigma_1^2}\right)^2\right] &= -\frac{1}{4}\frac{1}{1-\rho^2}\left(\frac{1}{\sigma_1^2}\right)^2\left(2\rho^2-3\right) \\
E\left[\left(\frac{\partial h}{\partial \sigma_2^2}\right)^2\right] &= -\frac{1}{4}\frac{1}{1-\rho^2}\left(\frac{1}{\sigma_2^2}\right)^2\left(2\rho^2-3\right) \\
E\left[\left(\frac{\partial h}{\partial \rho}\right)^2\right] &= \left(\frac{1}{1-\rho^2}\right)^4\left(2\rho^6-3\rho^4+1\right) \\
E\left[\left(\frac{\partial h}{\partial \theta_6}\right)^2\right] &= 9+72\rho^2+24\rho^4,
\end{aligned}
$$

$$\begin{aligned}
E\left[\left(\frac{\partial h}{\partial \mu_1}\right)\left(\frac{\partial h}{\partial \mu_2}\right)\right] &= -\frac{1}{\sigma_1}\frac{1}{\sigma_2}\left(\frac{\rho}{1-\rho^2}\right) \\
E\left[\left(\frac{\partial h}{\partial \mu_1}\right)\left(\frac{\partial h}{\partial \sigma_1^2}\right)\right] &= 0 \\
E\left[\left(\frac{\partial h}{\partial \mu_1}\right)\left(\frac{\partial h}{\partial \sigma_2^2}\right)\right] &= 0 \\
E\left[\left(\frac{\partial h}{\partial \mu_1}\right)\left(\frac{\partial h}{\partial \rho}\right)\right] &= 0 \\
E\left[\left(\frac{\partial h}{\partial \mu_1}\right)\left(\frac{\partial h}{\partial \theta_6}\right)\right] &= 0,
\end{aligned}$$

$$\begin{aligned}
E\left[\left(\frac{\partial h}{\partial \mu_2}\right)\left(\frac{\partial h}{\partial \sigma_1^2}\right)\right] &= 0 \\
E\left[\left(\frac{\partial h}{\partial \mu_2}\right)\left(\frac{\partial h}{\partial \sigma_2^2}\right)\right] &= 0 \\
E\left[\left(\frac{\partial h}{\partial \mu_2}\right)\left(\frac{\partial h}{\partial \rho}\right)\right] &= 0 \\
E\left[\left(\frac{\partial h}{\partial \mu_2}\right)\left(\frac{\partial h}{\partial \theta_6}\right)\right] &= 0,
\end{aligned}$$

$$\begin{aligned}
E\left[\left(\frac{\partial h}{\partial \sigma_1^2}\right)\left(\frac{\partial h}{\partial \sigma_2^2}\right)\right] &= -\frac{1}{4}\frac{1}{\sigma_1^2}\frac{1}{\sigma_2^2}(2\rho^2-1)\left(\frac{1}{1-\rho^2}\right) \\
E\left[\left(\frac{\partial h}{\partial \sigma_1^2}\right)\left(\frac{\partial h}{\partial \rho}\right)\right] &= -\rho\frac{1}{\sigma_1^2}\left(\frac{1}{1-\rho^2}\right) \\
E\left[\left(\frac{\partial h}{\partial \sigma_1^2}\right)\left(\frac{\partial h}{\partial \theta_6}\right)\right] &= \frac{3}{2}\frac{1}{\sigma_1^2}\left(2\rho^2+1\right),
\end{aligned}$$

$$\begin{aligned}
E\left[\left(\frac{\partial h}{\partial \sigma_2^2}\right)\left(\frac{\partial h}{\partial \rho}\right)\right] &= -\rho\frac{1}{\sigma_2^2}\left(\frac{1}{1-\rho^2}\right) \\
E\left[\left(\frac{\partial h}{\partial \sigma_2^2}\right)\left(\frac{\partial h}{\partial \theta_6}\right)\right] &= \frac{3}{2}\frac{1}{\sigma_2^2}(2\rho^2+1) \\
E\left[\left(\frac{\partial h}{\partial \rho}\right)\left(\frac{\partial h}{\partial \theta_6}\right)\right] &= 3(-2\rho^3+\rho)\left(\frac{1}{1-\rho^2}\right),
\end{aligned}$$

where

$$\begin{aligned}
\mu_i &= E\left[r_{i,t}\right] \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^2\right] &= 1 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)\right] &= \rho, i \neq j \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^2\right] &= 0 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^3\right] &= 0 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^4\right] &= 0 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^3\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^4\right] &= 0 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^3\right] &= 0 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^4\right] &= 3 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^2\right] &= 1+2\rho^2 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^3\right] &= 3\rho \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^4\right] &= 3+12\rho^2 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^3\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^3\right] &= 9\rho+6\rho^3 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^4\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^4\right] &= 9+72\rho^2+24\rho^4.
\end{aligned}$$

The elements of the information matrix at time t, evaluated under the null are

$$\begin{aligned}
I_{1,1,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \mu_1}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right]E\left[\frac{\partial h}{\partial \mu_1}\right] \\
&= \left(\frac{1}{1-\rho^2}\right)\left(\frac{1}{\sigma_1^2}\right)
\end{aligned}$$

$$
\begin{aligned}
I_{1,2,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \mu_2}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right] E\left[\frac{\partial h}{\partial \mu_2}\right] \\
&= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho}{\sigma_1\sigma_2}\right)
\end{aligned}
$$

$$
\begin{aligned}
I_{1,3,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \sigma_1^2}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right] E\left[\frac{\partial h}{\partial \sigma_1^2}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{1,4,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \sigma_2^2}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right] E\left[\frac{\partial h}{\partial \sigma_2^2}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{1,5,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \rho}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right] E\left[\frac{\partial h}{\partial \rho}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{1,6,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right] E\left[\frac{\partial h}{\partial \theta_6}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{2,2,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \mu_2}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right] E\left[\frac{\partial h}{\partial \mu_2}\right] \\
&= \left(\frac{1}{1-\rho^2}\right)\left(\frac{1}{\sigma_2^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
I_{2,3,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \sigma_1^2}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right] E\left[\frac{\partial h}{\partial \sigma_1^2}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{2,4,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \sigma_2^2}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right] E\left[\frac{\partial h}{\partial \sigma_2^2}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{2,5,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \rho}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right] E\left[\frac{\partial h}{\partial \rho}\right] \\
&= 0
\end{aligned}
$$

$$I_{2,6,t} = E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right] E\left[\frac{\partial h}{\partial \theta_6}\right]$$
$$= 0$$

$$I_{3,3,t} = E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \sigma_1^2}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \sigma_1^2}\right]$$
$$= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho^2+2}{4\sigma_1^4}\right)$$

$$I_{3,4,t} = E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \sigma_2^2}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \sigma_2^2}\right]$$
$$= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho^2}{4\sigma_1^2\sigma_2^2}\right)$$

$$I_{3,5,t} = E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \rho}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \rho}\right]$$
$$= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho}{2\sigma_1^2}\right)$$

$$I_{3,6,t} = E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \theta_6}\right]$$
$$= \frac{(2\rho^2+1)}{\sigma_1^2}$$

$$I_{4,4,t} = E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \sigma_2^2}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right] E\left[\frac{\partial h}{\partial \sigma_2^2}\right]$$
$$= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho^2+2}{4\sigma_2^4}\right)$$

$$I_{4,5,t} = E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \rho}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right] E\left[\frac{\partial h}{\partial \rho}\right]$$
$$= \left(\frac{1}{1-\rho^2}\right)\left(\frac{-\rho}{2\sigma_2^2}\right)$$

$$I_{4,6,t} = E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right] E\left[\frac{\partial h}{\partial \theta_6}\right]$$
$$= \frac{(2\rho^2+1)}{\sigma_2^2}$$

$$I_{5,5,t} = E\left[\frac{\partial h}{\partial \rho}\frac{\partial h}{\partial \rho}\right] - E\left[\frac{\partial h}{\partial \rho}\right] E\left[\frac{\partial h}{\partial \rho}\right] = \left(\frac{1}{1-\rho^2}\right)\left(\frac{\rho^2+1}{1-\rho^2}\right)$$

$$I_{5,6,t} = E\left[\frac{\partial h}{\partial \rho}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \rho}\right] E\left[\frac{\partial h}{\partial \theta_6}\right] = 4\rho$$

$$I_{6,6,t} = E\left[\frac{\partial h}{\partial \theta_6}\frac{\partial h}{\partial \theta_6}\right] - E\left[\frac{\partial h}{\partial \theta_6}\right] E\left[\frac{\partial h}{\partial \theta_6}\right] = (8+68\rho^2+20\rho^4).$$

The information matrix under the null hypothesis of the bivariate normality ($\theta_6 = 0$) is

$$I\left(\widehat{\theta}\right) = T\left(E\left[\frac{\partial h}{\partial \theta}\frac{\partial h}{\partial \theta'}\right]_{\theta_6=0} - E\left[\frac{\partial h}{\partial \theta}\right]_{\theta_6=0} E\left[\frac{\partial h}{\partial \theta'}\right]_{\theta_6=0}\right) \tag{58}$$

$$= \left(\frac{T}{1-\widehat{\rho}^2}\right) \times \begin{bmatrix} \frac{1}{\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & 0 & 0 & 0 & 0 \\ \frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & \frac{1}{\widehat{\sigma}_2^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-\widehat{\rho}^2+2}{4\widehat{\sigma}_1^4} & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{(2\widehat{\rho}^2+1)(1-\widehat{\rho}^2)}{\widehat{\sigma}_1^2} \\ 0 & 0 & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}^2+2}{4\widehat{\sigma}_2^4} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{(2\widehat{\rho}^2+1)(1-\widehat{\rho}^2)}{\widehat{\sigma}_2^2} \\ 0 & 0 & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{\widehat{\rho}^2+1}{1-\widehat{\rho}^2} & 4\widehat{\rho}(1-\widehat{\rho}^2) \\ 0 & 0 & \frac{(2\widehat{\rho}^2+1)(1-\widehat{\rho}^2)}{\widehat{\sigma}_1^2} & \frac{(2\widehat{\rho}^2+1)(1-\widehat{\rho}^2)}{\widehat{\sigma}_2^2} & 4\widehat{\rho}(1-\widehat{\rho}^2) & (8+68\widehat{\rho}^2+20\widehat{\rho}^4)(1-\widehat{\rho}^2) \end{bmatrix},$$

so that

$$I\left(\widehat{\theta}\right)^{-1} = \left(\frac{1-\widehat{\rho}^2}{T}\right) \times \begin{bmatrix} \frac{1}{\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & 0 & 0 & 0 & 0 \\ \frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & \frac{1}{\widehat{\sigma}_2^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-\widehat{\rho}^2+2}{4\widehat{\sigma}_1^4} & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{(2\widehat{\rho}^2+1)(1-\widehat{\rho}^2)}{\widehat{\sigma}_1^2} \\ 0 & 0 & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}^2+2}{4\widehat{\sigma}_2^4} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{(2\widehat{\rho}^2+1)(1-\widehat{\rho}^2)}{\widehat{\sigma}_2^2} \\ 0 & 0 & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{\widehat{\rho}^2+1}{1-\widehat{\rho}^2} & 4\widehat{\rho}(1-\widehat{\rho}^2) \\ 0 & 0 & \frac{(2\widehat{\rho}^2+1)(1-\widehat{\rho}^2)}{\widehat{\sigma}_1^2} & \frac{(2\widehat{\rho}^2+1)(1-\widehat{\rho}^2)}{\widehat{\sigma}_2^2} & 4\widehat{\rho}(1-\widehat{\rho}^2) & (8+68\widehat{\rho}^2+20\widehat{\rho}^4)(1-\widehat{\rho}^2) \end{bmatrix}^{-1}. \tag{59}$$

Evaluating the gradient for θ_6 under the null hypothesis gives

$$\begin{aligned}\frac{\partial \ln L_t(\theta)}{\partial \theta}|_{\theta_6=0} &= \sum_{t=1}^{T}\left(\frac{\partial h}{\partial \theta_6}\right) - T\left(\frac{\partial \eta}{\partial \theta_6}\right) \\ &= \sum_{t=1}^{T}\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - T\left(1+2\rho^2\right).\end{aligned} \tag{60}$$

By taking the first derivatives of the log likelihood function, the score function under H_0 is given as

$$\begin{aligned}q\left(\widehat{\theta}\right) &= \frac{\partial \ln L_t(\theta)}{\partial \theta}|_{\theta_6=0} \\ &= \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & \sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 - T\left(1+2\widehat{\rho}^2\right)\end{array}\right]'.\end{aligned} \tag{61}$$

Substituting equations (59) and (61) into the Lagrange multiplier statistic in equation (22) gives

$$\begin{aligned}LM_1 &= q\left(\widehat{\theta}\right)' I\left(\widehat{\theta}\right)^{-1} q\left(\widehat{\theta}\right) \\ &= \frac{1}{T\left(4\widehat{\rho}^4+16\widehat{\rho}^2+4\right)}\left[\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 - T\left(1+2\widehat{\rho}^2\right)\right]^2 \\ &= \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{1,t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2,t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 - \left(1+2\widehat{\rho}^2\right)}{\sqrt{\frac{\left(4\widehat{\rho}^4+16\widehat{\rho}^2+4\right)}{T}}}\right)^2.\end{aligned} \tag{62}$$

A.3.2 A Statistic of Co-kurtosis

Consider the following bivariate generalized normal distribution with co-kurtosis given as

$$\begin{aligned}f(r_{1,t}, r_{2,t}) &= \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\ &\quad \left. +\theta_7\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3 - \eta\right] \qquad (63) \\ &= \exp\left[h-\eta\right],\end{aligned}$$

where

$$
\begin{aligned}
\eta &= \ln \iint \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right)\right. \\
&\quad \left. +\theta_7\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3\right] dr_1 dr_2 \qquad (64) \\
&= \ln \iint \exp[h]\, dr_1 dr_2,
\end{aligned}
$$

and

$$
\begin{aligned}
h &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\
&\quad +\theta_7\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3. \qquad (65)
\end{aligned}
$$

A test of the restriction for normality is

$$H_0 : \theta_7 = 0.$$

Let the parameters of equation (65) be

$$\theta = \left\{\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho, \theta_7\right\}. \qquad (66)$$

By taking the log function of equation (63), the log likelihood function at time t is given by

$$
\begin{aligned}
\ln L_t(\theta) &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)\right) \\
&\quad +\theta_7\left(\frac{r_{1,t}-\mu_1}{\sigma_1}\right)^1\left(\frac{r_{2,t}-\mu_2}{\sigma_2}\right)^3 - \eta \qquad (67) \\
&= h - \eta,
\end{aligned}
$$

where h and η are given by equations (65) and (64).

Taking the expectations for the first and second derivatives of the log likelihood function in equation (67) under the null hypothesis of the bivariate normality ($\theta_7 = 0$) is

$$
\begin{aligned}
E\left(\frac{\partial h}{\partial \theta_7}\right) &= 3\rho \\
E\left[\left(\frac{\partial h}{\partial \theta_7}\right)^2\right] &= 15 + 90\rho^2 \\
E\left[\left(\frac{\partial h}{\partial \mu_1}\right)\left(\frac{\partial h}{\partial \theta_7}\right)\right] &= 0 \\
E\left[\left(\frac{\partial h}{\partial \mu_2}\right)\left(\frac{\partial h}{\partial \theta_7}\right)\right] &= 0 \\
E\left[\left(\frac{\partial h}{\partial \sigma_1^2}\right)\left(\frac{\partial h}{\partial \theta_7}\right)\right] &= \frac{3\rho}{\sigma_1^2} \\
E\left[\left(\frac{\partial h}{\partial \sigma_2^2}\right)\left(\frac{\partial h}{\partial \theta_7}\right)\right] &= \frac{6\rho}{\sigma_2^2} \\
E\left[\left(\frac{\partial h}{\partial \rho}\right)\left(\frac{\partial h}{\partial \theta_7}\right)\right] &= 3(1-2\rho^2)\left(\frac{1}{1-\rho^2}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^6\right] &= 15 \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^5\right] &= 15\rho, i \neq j \\
E\left[\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{j,t}-\mu_j}{\sigma_j}\right)^6\right] &= 15 + 90\rho^2, i \neq j.
\end{aligned}
$$

The elements of the information matrix at observation t, evaluated under the null are

$$
\begin{aligned}
I_{1,6,t} &= E\left[\frac{\partial h}{\partial \mu_1}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \mu_1}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{2,6,t} &= E\left[\frac{\partial h}{\partial \mu_2}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \mu_2}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
I_{3,6,t} &= E\left[\frac{\partial h}{\partial \sigma_1^2}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \sigma_1^2}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= \left(\frac{3\rho}{2\sigma_1^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
I_{4,6,t} &= E\left[\frac{\partial h}{\partial \sigma_2^2}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \sigma_2^2}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= \left(\frac{9\rho}{2\sigma_2^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
I_{5,6,t} &= E\left[\frac{\partial h}{\partial \rho}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \rho}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= 3
\end{aligned}
$$

$$
\begin{aligned}
I_{6,6,t} &= E\left[\frac{\partial h}{\partial \theta_7}\frac{\partial h}{\partial \theta_7}\right] - E\left[\frac{\partial h}{\partial \theta_7}\right] E\left[\frac{\partial h}{\partial \theta_7}\right] \\
&= (15+81\rho^2).
\end{aligned}
$$

The information matrix under the null hypothesis of the bivariate normality ($\theta_7 = 0$) is given as

$$
\begin{aligned}
I\left(\widehat{\theta}\right) &= T\left(E\left[\frac{\partial h}{\partial \theta}\frac{\partial h}{\partial \theta'}\right]_{\theta_7=0} - E\left[\frac{\partial h}{\partial \theta}\right]_{\theta_7=0} E\left[\frac{\partial h}{\partial \theta'}\right]_{\theta_7=0}\right) \qquad (68)\\
&= \left(\frac{T}{1-\widehat{\rho}^2}\right)\begin{bmatrix}
\frac{1}{\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & 0 & 0 & 0 & 0 \\
\frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & \frac{1}{\widehat{\sigma}_2^2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-\widehat{\rho}^2+2}{4\widehat{\sigma}_1^4} & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{3\widehat{\rho}(1-\widehat{\rho}^2)}{2\widehat{\sigma}_1^2} \\
0 & 0 & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}^2+2}{4\widehat{\sigma}_2^4} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{9\widehat{\rho}(1-\widehat{\rho}^2)}{2\widehat{\sigma}_2^2} \\
0 & 0 & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{\widehat{\rho}^2+1}{1-\widehat{\rho}^2} & 3(1-\widehat{\rho}^2) \\
0 & 0 & \frac{3\widehat{\rho}(1-\widehat{\rho}^2)}{2\widehat{\sigma}_1^2} & \frac{9\widehat{\rho}(1-\widehat{\rho}^2)}{2\widehat{\sigma}_2^2} & 3(1-\widehat{\rho}^2) & (15+81\widehat{\rho}^2)(1-\widehat{\rho}^2)
\end{bmatrix},
\end{aligned}
$$

so that

$$
I\left(\widehat{\theta}\right)^{-1} = \left(\frac{1-\widehat{\rho}^2}{T}\right)\begin{bmatrix}
\frac{1}{\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & 0 & 0 & 0 & 0 \\
\frac{-\widehat{\rho}}{\widehat{\sigma}_1\widehat{\sigma}_2} & \frac{1}{\widehat{\sigma}_2^2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-\widehat{\rho}^2+2}{4\widehat{\sigma}_1^4} & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{3\widehat{\rho}(1-\widehat{\rho}^2)}{2\widehat{\sigma}_1^2} \\
0 & 0 & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_1^2\widehat{\sigma}_2^2} & \frac{-\widehat{\rho}^2+2}{4\widehat{\sigma}_2^4} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{9\widehat{\rho}(1-\widehat{\rho}^2)}{2\widehat{\sigma}_2^2} \\
0 & 0 & \frac{-\widehat{\rho}}{2\widehat{\sigma}_1^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_2^2} & \frac{\widehat{\rho}^2+1}{1-\widehat{\rho}^2} & 3(1-\widehat{\rho}^2) \\
0 & 0 & \frac{3\widehat{\rho}(1-\widehat{\rho}^2)}{2\widehat{\sigma}_1^2} & \frac{9\widehat{\rho}(1-\widehat{\rho}^2)}{2\widehat{\sigma}_2^2} & 3(1-\widehat{\rho}^2) & (15+81\widehat{\rho}^2)(1-\widehat{\rho}^2)
\end{bmatrix}. \qquad (69)
$$

Evaluating the gradient for θ_7 under the null hypothesis gives

$$\begin{aligned} \frac{\partial \ln L_t(\theta)}{\partial \theta_7}|_{\theta_7=0} &= \sum_{t=1}^{T} \left(\frac{\partial h}{\partial \theta_7} \right) - T \left(\frac{\partial \eta}{\partial \theta_7} \right) \\ &= \sum_{t=1}^{T} \left(\frac{r_{1,t} - \mu_1}{\sigma_1} \right)^1 \left(\frac{r_{2,t} - \mu_2}{\sigma_2} \right)^3 - T\left(3\rho\right). \end{aligned} \tag{70}$$

By taking the first derivatives of the log likelihood function, the score function under H_0 is given as

$$\begin{aligned} q\left(\widehat{\theta}\right) &= \frac{\partial \ln L_t(\theta)}{\partial \theta}|_{\theta_7=0} \\ &= \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & \sum_{t=1}^{T} \left(\frac{r_{1,t} - \widehat{\mu}_1}{\widehat{\sigma}_1} \right)^1 \left(\frac{r_{2,t} - \widehat{\mu}_2}{\widehat{\sigma}_2} \right)^3 - T\left(3\widehat{\rho}\right) \end{array} \right]'. \end{aligned} \tag{71}$$

Substituting equations (69) and (71) into the Lagrange multiplier statistic in equation (22) gives

$$\begin{aligned} LM &= q\left(\widehat{\theta}\right)' I\left(\widehat{\theta}\right)^{-1} q\left(\widehat{\theta}\right) \\ &= \frac{1}{T\left(18\widehat{\rho}^2 + 6\right)} \left[\sum_{t=1}^{T} \left(\frac{r_{1,t} - \widehat{\mu}_1}{\widehat{\sigma}_1} \right)^1 \left(\frac{r_{2,t} - \widehat{\mu}_2}{\widehat{\sigma}_2} \right)^3 - T\left(3\widehat{\rho}\right) \right]^2 \\ &= \left(\frac{\frac{1}{T}\sum_{t=1}^{T} \left(\frac{r_{1,t} - \widehat{\mu}_1}{\widehat{\sigma}_1} \right)^1 \left(\frac{r_{2,t} - \widehat{\mu}_2}{\widehat{\sigma}_2} \right)^3 - \left(3\widehat{\rho}\right)}{\sqrt{\frac{\left(18\widehat{\rho}^2 + 6\right)}{T}}} \right)^2. \end{aligned} \tag{72}$$

Table 1: Summary moments of equity returns for eight equity markets and banking sectors during the non-crisis (NC) and crisis (C) periods. The non-crisis period is from April 1, 2005 to February 29, 2008 and the crisis period is from March 3, 2008 to August 31, 2009.

Country	Moment							
	Mean		Std.Dev		Skewness		kurtosis	
	NC	C	NC	C	NC	C	NC	C
(A) Equity markets								
Asia								
Hong Kong	0.078	-0.053	0.886	1.951	-0.107	0.218	11.446	6.917
Korea	0.084	-0.091	0.929	1.843	-0.676	-0.082	5.672	11.011
Europe								
France	0.042	-0.084	0.839	1.872	-0.528	0.251	8.641	7.100
Germany	0.081	-0.092	0.938	2.005	-0.526	0.087	6.584	5.505
UK	0.031	-0.097	0.780	1.819	-0.312	0.079	6.715	7.082
Latin America								
Chile	0.078	-0.016	0.713	1.275	-0.470	-0.404	8.319	10.430
Mexico	0.113	-0.063	1.122	1.920	-0.230	0.198	5.453	7.043
North America								
US	0.022	-0.065	0.557	1.498	-0.514	0.166	4.937	6.728
(B) Banking sectors								
Asia								
Hong Kong	0.219	-0.119	1.542	2.206	0.005	-0.008	4.781	4.200
Korea	0.046	0.001	1.289	2.321	0.170	-0.379	4.102	6.522
Europe								
France	-0.003	-0.050	1.016	2.792	-0.400	0.264	6.838	5.446
Germany	0.024	-0.312	1.173	3.584	-1.389	-1.010	15.664	10.701
UK	-0.018	-0.118	0.807	2.651	-0.032	0.051	7.888	6.562
Latin America								
Chile	0.050	-0.016	0.974	1.819	-0.181	-0.425	5.368	12.854
Mexico	0.118	-0.051	1.624	2.518	-0.170	0.097	4.716	5.971
North America								
US	-0.020	-0.141	0.785	3.796	-0.063	0.000	9.386	5.563

Table 2: Linear and asymmetric co-moments of equity returns for eight equity markets and banking sectors with the US banking sector during the non-crisis (NC) and crisis (C) periods. The non-crisis period is from April 1, 2005 to February 29, 2008 and the crisis period is from March 3, 2008 to August 31, 2009.

Country	Linear		Asymmetric			
	Correlation		Co-skewness_{12}		Co-skewness_{21}	
	NC	C	NC	C	NC	C
(A) Equity markets						
Asia						
Hong Kong	-0.020	0.166	0.476	0.223	0.345	0.085
Korea	-0.082	0.173	0.193	0.228	-0.202	0.031
Europe						
France	0.266	0.404	0.155	0.012	-0.495	-0.105
Germany	0.230	0.404	0.058	-0.103	-0.417	-0.177
UK	0.262	0.380	0.214	-0.064	-0.397	-0.191
Latin America						
Chile	0.309	0.376	0.045	-0.165	-0.440	-0.066
Mexico	0.492	0.585	-0.023	0.025	-0.158	-0.036
North America						
US	0.799	0.772	-0.360	-0.116	-0.291	-0.122
(B) Banking sectors						
Asia						
Hong Kong	-0.026	-0.001	0.006	0.023	-0.059	0.123
Korea	-0.041	0.129	-0.059	0.214	-0.100	-0.028
Europe						
France	0.378	0.452	0.055	0.117	-0.421	-0.095
Germany	0.328	0.402	-0.275	-0.580	-0.509	-0.339
UK	0.384	0.458	0.059	0.095	-0.288	-0.013
Latin America						
Chile	0.325	0.382	0.142	0.284	-0.276	-0.005
Mexico	0.290	0.483	-0.109	0.069	-0.223	-0.051
North America						
US	1.000	1.000	-0.063	0.000	-0.063	0.000

Notes: The statistics are measured in terms of:-
Correlation: returns of market j and returns of the US bank.
Co-skewness_{12}: returns of market j and squared returns of the US bank.
Co-skewness_{21}: squared returns of market j and returns of the US bank.

Table 3: Extremal co-moments of equity returns for eight equity markets and banking sectors during the non-crisis (NC) and crisis (C) periods. The non-crisis period is from April 1, 2005 to February 29, 2008 and the crisis period is from March 3, 2008 to August 31, 2009.

Country	Extremal					
	Co-kurtosis$_{13}$		Co-kurtosis$_{31}$		Co-volatility$_{22}$	
	NC	C	NC	C	NC	C
(A) Equity markets						
Asia						
Hong Kong	1.436	1.392	1.271	1.063	4.853	1.494
Korea	-1.728	1.792	-1.186	0.682	2.202	1.344
Europe						
France	0.868	2.178	0.296	2.457	2.922	2.307
Germany	1.289	1.967	0.699	2.544	2.398	2.263
UK	0.887	2.144	0.840	2.482	2.651	2.494
Latin America						
Chile	2.189	2.467	1.759	2.317	2.966	2.207
Mexico	2.670	3.222	3.677	3.169	2.830	2.818
North America						
US	4.491	4.031	6.283	3.967	5.045	3.549
(B) Banking sectors						
Asia						
Hong Kong	-0.401	0.061	-0.426	-0.085	1.788	1.038
Korea	-0.200	0.867	-0.604	0.731	0.957	1.479
Europe						
France	1.834	2.315	1.584	2.689	2.707	2.668
Germany	4.402	5.334	1.757	3.092	3.476	4.147
UK	2.628	2.518	2.820	2.571	3.042	2.392
Latin America						
Chile	1.511	1.295	1.787	2.248	2.181	2.330
Mexico	1.403	2.363	1.638	2.556	1.745	2.249
North America						
US	9.386	5.563	9.386	5.563	9.386	5.563

Notes: The statistics are measured in terms of:-
Co-kurtosis$_{13}$: returns of market j and cubed returns of the US bank.
Co-kurtosis$_{31}$: cubed returns of market j and returns of the US bank.
Co-volatility$_{22}$: squared returns of market j and squared returns of the US bank.

Table 4: Simulation parameters of the moment and co-moment terms in equation (14). These parameters produce Figure 5.

Parameters	No volatility (Fig 2.1 Panel A)	Volatility (Fig 2.1 Panel B)
θ_1	0.50	0.50
θ_2	0.70	0.70
θ_3	2.00	2.00
θ_4	-1.50	-1.50
θ_5	-1.50	-1.50
θ_6	-1.50	-1.50
θ_7	-1.50	-1.50
θ_8	4.00	4.00
θ_9	4.00	4.00
θ_{10}	1.50	1.50
θ_{11}	1.50	1.50
θ_{12}	1.50	1.50
$E[(R_1-\mu_1)]^2$	0.00	2.00
$E[(R_2-\mu_2)]^2$	0.00	2.00
$E[(R_1-\mu_1)(R_2-\mu_2)]$	0.80	0.80
$E[(R_1-\mu_1)]^3$	$[-1,1]$	$[-1,1]$
$E[(R_2-\mu_2)]^3$	$[-1,1]$	$[-1,1]$
$E[(R_1-\mu_1)(R_2-\mu_2)^2]$	0.00	0.00
$E[(R_1-\mu_1)^2(R_2-\mu_2)]$	0.00	0.00
$E[(R_1-\mu_1)]^4$	$[4,9]$	$[4,9]$
$E[(R_2-\mu_2)]^4$	$[4,9]$	$[4,9]$
$E[(R_1-\mu_1)(R_2-\mu_2)^3]$	1.47	1.47
$E[(R_1-\mu_1)^3(R_2-\mu_2)]$	1.46	1.46
$E[(R_1-\mu_1)^2(R_2-\mu_2)^2]$	2.20	2.20

Table 5: Critical values for the test statistics of contagion based on changes in asymmetric dependence (CS_{12}, CS_{21}) and extremal dependence (CK_{13}, CK_{31} and CV_{22}) and for the Chi-squared statistic (χ_1^2) with one degree of freedom. The size of the non-crisis period is $T_x = 760$ and the size of the crisis period is $T_y = 391$. The number of replications in the Monte Carlo simulations is $10,000$.

	Significance level		
Test statistics	$\alpha = 0.025$	$\alpha = 0.05$	$\alpha = 0.1$
CS_{12}	5.09	3.94	2.79
CS_{21}	4.98	3.84	2.73
CK_{13}	9.74	7.34	5.15
CK_{31}	9.63	7.37	5.18
CV_{22}	5.13	3.78	2.63
χ_1^2	5.02	3.84	2.71

Table 6: Test statistics for contagion based on changes in asymmetric dependence during the global financial crisis of 2008-09.

	(A) Equity markets		(B) Banking sectors	
Country	$CS_{12}^{(a)}$	$CS_{21}^{(b)}$	$CS_{12}^{(a)}$	$CS_{21}^{(b)}$
Asia				
Hong Kong	0.03	9.01**	0.14	8.08**
Korea	0.40	3.17*	12.94**	4.06**
Europe				
France	0.18	0.21	7.78**	0.97
Germany	0.30	0.18	5.68**	1.33
UK	2.08	1.19	17.63**	4.13**
Latin America				
Chile	14.91**	1.83	3.42*	2.53
Mexico	0.19	0.64	0.13	0.01
North America				
US	0.04	0.04	n.a.	n.a.

Notes: (a) CS_{12}: Co-skewness contagion test measured in terms of the returns of the US banking sector and squared returns of market j. (b) CS_{21}: Co-skewness contagion test measured in terms of the squared returns of the US banking sector and returns of market j. The critical values of CS_{12} and CS_{21} are presented in Table 5. * denotes significance at the 10% level and ** significance at the 5% level.

Table 7: Test statistics for contagion based on changes in extremal dependence during the global financial crisis of 2008-09.

	(A) Equity markets			(B) Banking sectors		
Country	$CK_{13}^{(a)}$	$CK_{31}^{(b)}$	$CV_{22}^{(c)}$	$CK_{13}^{(a)}$	$CK_{13}^{(b)}$	$CV_{22}^{(c)}$
Asia						
Hong Kong	90.68**	210.36**	0.87	1.42	23.73**	2.42
Korea	137.86**	170.76**	14.66**	11.88**	107.80**	9.83**
Europe						
France	111.21**	155.68**	86.34**	218.07**	116.85**	157.81**
Germany	83.51**	151.50**	55.82**	351.51**	79.54**	167.91**
UK	151.14**	100.17**	88.87**	246.91**	83.32**	128.01**
Latin America						
Chile	25.56**	7.93**	2.62	62.29**	9.71**	1.31
Mexico	46.40**	21.23**	20.45**	140.49**	104.35**	95.01**
North America						
US	28.42**	2.08	5.77**	n.a.	n.a.	n.a.

Notes: (a) CK_{13} : Co-kurtosis contagion test measured in terms of the returns of the US banking sector and cubed returns of market j. (b) CK_{31} : Co-kurtosis contagion test measured in terms of the cubed returns of the US banking sector and returns of market j. (c) CV_{22} : Co-volatility contagion test measured in terms of the squared returns of the US banking sector and squared returns of market j. The critical values of CK_{13}, CK_{31} and CV_{22} are presented in Table 5. ** denotes the significance of contagion at the 5% level.