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Working Less and Bargain Hunting More: Macro Implications of Sales during Japan's Lost Decades

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Keywords

Sales, monetary policy, lost decades, time use

JEL Classification

E3, E5

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Working Less and Bargain Hunting More:

Macro Implications of Sales during Japan's Lost Decades

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Abstract

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1 Introduction

Standard New Keynesian models have often neglected temporary sales,[1] although the frequency of sales is far higher than that of regular price changes,[2] and hence it is not necessarily guaranteed that the assumption of sticky prices holds. Ignoring this fact is justified, however, if retailers' decision to hold sales is independent of macroeconomic developments. If this is the case, temporary sales do not eliminate the real effect of monetary policy. In fact, Guimaraes and Sheedy (2011, hereafter GS) develop a dynamic stochastic general equilibrium (DSGE) model incorporating sales and show that the real effect of monetary policy remains largely unchanged. Empirical studies such as Kehoe and Midrigan (2010), Eichenbaum, Jaimovich, and Rebelo (2011), and Anderson et al. (2012) argue that retailers' decision to hold a sale is actually orthogonal to changes in macroeconomic developments.

However, in Japan, there seems to be a negative link between the frequency of temporary sales and working hours. Since the start of the 1990s, the Japanese economy has experienced what has come to be called "lost decades," a period of sluggish GDP growth and very low rates of inflation or deflation. During the lost decades, the frequency of sales increased gradually but monotonically, allowing households to purchase a greater number of goods at sale prices rather than at regular prices. At the same time, hours worked per worker followed a secular decline, reflecting both a statutory reduction in working hours and a decline in productivity growth (Hayashi and Prescott [2002]). Thus, there appears to be a negative correlation between the frequency of sales and working hours. Similarly, for the United States, Klenow and Willis (2007) and Coibion, Gorodnichenko, and Hong (2012) provide evidence showing that the frequency of temporary sales is influenced by the business cycle.[3]

The aim of the present paper is twofold. First, we examine empirically whether there is a link between the frequency of temporary sales and changes in macroeconomic developments, particularly working hours, making extensive use of Japanese supermarket scanner data. Second, we investigate whether temporary sales mitigate the real effect of monetary policy, employing a model that takes households' decisions regarding their allocation of time for work, leisure, and bargain hunting into account. To this end, we extend the GS model, in which a representative household consists of two classes of consumers, price-insensitive "loyal customers" and price-sensitive "bargain hunters," and the fraction of the two classes of consumers is fixed. In our model the fraction is endogenously determined. The more the fraction of bargain hunters

[1] See, for example, the textbooks by Woodford (2003) and Gali (2008).

[2] See, for example, Bils and Klenow (2004) and Nakamura and Steinsson (2008) for the United States and Sudo, Ueda, and Watanabe (2014) for Japan.

[3] Coibion, Gorodnichenko, and Hong (2012), however, report that the effect works in the opposite direction to that found in our paper and is therefore counter-intuitive. That is, the frequency of sales falls rather than rises when local unemployment rates rise. We discuss possible reasons for the difference in Section 2.

increases, the more the household can substitute away from relatively expensive items. At the same time, search activity for lower prices reduces the time available for work and leisure, imposing a cost on the household. These two opposing effects generate the optimal fraction of bargain hunters in the household.

Our analysis provides both new empirical and new theoretical findings. Our empirical analysis, using Japanese supermarket scanner data covering a period of two decades, reveals that the frequency of temporary sales increased during Japan's lost decades. Further, during this period, both Japanese households' price elasticity and the time households spend shopping appear to have increased. Our analysis also shows that there is a significant correlation between labor market conditions and the frequency of sales. Specifically, we find that when the time spent working decreases, the frequency of sales rises and the time spent shopping increases. The vector autoregression (VAR) analysis we conduct shows that the frequency of temporary sales and hours worked react in opposite directions to technology shocks, producing a negative correlation between the two.

Our theoretical analysis shows that the effect of a monetary policy shock on real economic activity is weaker when both temporary sales and endogenous bargain hunting are incorporated into the model, although monetary policy shocks still have real effects. For example, an expansionary monetary policy shock increases hours worked, which, in turn, increases the fraction of loyal customers (and decreases the fraction of bargain hunters). Observing this, firms lower the frequency of sales. Because the quantity of goods sold is typically higher at sales prices than at regular prices, these changes in households' and firms' behavior result in downward pressure on aggregate demand for goods. The real effect of monetary policy thus diminishes.[4] According to our simulation, the real effect of monetary policy weakens by about 40% due to endogenous bargain hunting. In contrast, the real effect of a technology shock is stronger when endogenous bargain hunting is taken into account. In our model, a positive technology shock lowers hours worked. This decreases the marginal disutility of bargain hunting, and firms raise the frequency of sales. Because the quantity of goods sold is higher at sales prices than at regular prices, aggregate production increases further. Applied to Japan, our model illustrates that the decline in hours worked helps to explain the increase in the frequency of temporary sales during the lost decades. In addition, our model implies that the fraction of loyal customers has

[4] An additional mechanism contributing to this effect is intensified strategic substitutability of sales. Suppose that all firms except for firm A raise the frequency of sales. As in GS, firm A does not have an incentive to follow others and raise the frequency of sales, because this decreases the marginal revenue from sales. In our model, an additional channel emerges. When all firms except for firm A raise the frequency of sales, the aggregate price level falls. This increases aggregate demand for goods, and in turn, aggregate demand for labor. Households supply more labor and lose time for bargain hunting. The fraction of loyal customers increases and that of bargain hunters decreases. Observing this, firm A lowers the frequency of sales. This intensified strategic substitutability of sales mitigates the real effect of monetary policy.

been shrinking and that of bargain hunters growing. In other words, households have become increasingly price sensitive.

Our paper is relevant to at least two strands of research. The first concerns the interaction between hours worked and bargain hunting on the household side. Using scanner data and time diaries to examine households' substitution between shopping and home production, Aguiar and Hurst (2007) find that older households shop the most frequently and pay the lowest prices. Lach (2007) analyzes store-level price data following the unexpected arrival of a large number of immigrants from the former Soviet Union in Israel. He finds that the immigrants have a higher price elasticity and lower search costs than the native population.[5]

The second strand of research to which our paper is related is the modeling of temporary sales. While there are several different reasons why retailers hold temporary sales, including the desire to liquidate excess stock and implicit contracts between retailers and manufacturers, the one we focus on is retailers' use of sales to maximize profits by exploiting consumer heterogeneity. Specifically the approach we take in this paper is most closely related to the ones adopted by Salop and Stiglitz (1977), Varian (1980), and Guimaraes and Sheedy (2011), an important common feature of which is that they introduce heterogeneity across consumers. Varian (1980) starts with the setting that some consumers are informed but others are not, showing that firms' optimal pricing strategy is to randomize prices to discriminate between the two types of consumers, where the lower price can be interpreted as the sale price. Guimaraes and Sheedy (2011) assume that some consumers are loyal to retailers but others are not (i.e., bargain hunters), and that they differ in terms of their price elasticity.[6] Our study is closely related to that by Guimaraes and Sheedy (2011) in this respect, but differs from it in that in our model the ratios of loyal customers and bargain hunters are endogenously determined. Our study is also related to that by Kehoe and Midrigan (2010), which has a similar research interest to ours (i.e., examining the stickiness of the aggregate price level and the real effect of monetary policy), although the way they model temporary sales is not based on consumer heterogeneity. Instead, they develop a DSGE model that incorporates not just menu costs associated with regular prices but also costs associated with the deviation of sale prices from regular prices and conclude that prices are sticky on the whole, which implies that the real effect of monetary policy remains large.

The structure of this paper is as follows. Using micro price data for Japan, Section 2

[5]See also Pashigian and Bowen (1991), Sorensen (2000), Brown and Goolsbee (2002), McKenzie and Schargrodsky (2004), and Pashigian, Peltzman, and Sun (2003).

[6]These studies are related to the literature on consumer search. How consumers search across shops and products influences how retailers set the prices of their products. Although they do not explicitly model temporary sales, studies incorporating consumer search and price setting include those by Benabou (1988), Watanabe (2008), and Coibion, Gorodnichenko, and Hong (2012).

provides evidence that bargain hunting and the frequency of temporary sales are related to macroeconomic conditions and hence are endogenously determined. In Section 3, we develop the DSGE model of temporary sales which we use to examine the implications of endogenous bargain hunting, while Section 4 presents the impulse responses of the model. Section 5 applies our model to Japan to explain the increase in the frequency of temporary sales during the lost decades. Section 6 concludes the paper.

2 Evidence for Endogenous Bargain Hunting

This section presents various types of evidence that provides the basis for our modeling strategy regarding endogenous bargain hunting. Specifically, we focus on the following. First, using household surveys for Japan, we show that working time moves in the opposite direction of shopping time. Second, using point-of-sales (POS) scanner data, we show that temporary sales have come to play an increasingly important role in retailers' business during Japan's lost decades. The frequency of sales is significantly correlated with macroeconomic indicators, especially labor market indicators. We find that when hours worked are long, the frequency of sales tends to be low.

2.1 Survey on Time Use

Let us begin by looking at the *Survey on Time Use and Leisure Activities for Japan.* The survey is conducted by the Statistics Bureau every five years. It asks around 200,000 members in 83,000 households about their daily patterns of time allocation. The questionnaire includes questions on the time used for working and shopping. These questions help us to examine the relationship between the time spent working and the time spent shopping, which is important for our model.[7]

Tables 1 and 2 show the time household members spent on shopping and working (including commuting time to work or school), respectively. The sample consists of those aged 15 and over. The numbers in the tables show the weekly average of the minutes per day. Two findings are worth highlighting. First, those not working tend to spend more time shopping than those working. Moreover, women tend to spend more time shopping than men and spend less time working. Second, time spent shopping steadily increased from 1986 to 2011. At the same time, hours worked continued to decline, although they increased slightly between 2001 and 2006. These findings provide support to our assumption that bargain hunting negatively depends on hours worked.

[7] Only aggregated data are available.

2.2 POS Data

Next, let us look at temporary sales using POS scanner data.[8] The POS data are collected by Nikkei Digital Media from retail shops around Japan. They consist of daily data covering the period from March 1, 1988 to February 28, 2013. The number of records amounts to 6 billion, where each record contains the number of units sold and the sales amount in yen for product i at shop s on date t. The cumulative number of products appearing during the sample period is 1.8 million. The data include processed food and domestic articles, but unlike the consumer price index (CPI), do not include fresh food, recreational durable goods (such as television sets or personal computers), or services (rent and utilities). Overall, the POS data cover 170 out of the 588 items in the CPI, which, according to data from the *Family Income and Expenditure Survey*, make up 17% of households' expenditure. Each product i is identified by a Japanese Article Number (JAN) code.

Our POS data have three noteworthy advantages. First, the data include quantity information as well as price information. Second, the data are at a daily frequency. This contrasts with scanner data for the United States, which are weekly. Third, the POS data span a long observation period, starting from 1988 and going up to the present, thus covering the period of the lost decades.

We aggregate the POS data using the following four steps. First, at the lowest (i.e., most detailed) JAN code level, we collect data for the variable we are interested in, such as price, for product i at shop s on date t. Second, we aggregate these data across shops using the turnover as weights to derive the weighted mean. Third, we aggregate the obtained values to the 3-digit code product level,[9] again using the turnover as weights to derive the weighted mean. Finally, we aggregate the values obtained in the previous step across 3-digit codes again using the turnover as weights to derive the weighted mean. Weights are defined by the turnover during the same month in the previous year. That is, if date t is January 1, 2012, for instance, we use the turnover in January in 2011 as weight.

2.2.1 Temporary Sales

We measure the price charged for a particular product at a particular retailer on a particular date from the POS records by dividing the turnover for product i at shop s on date t by the number of units sold of that product at that shop on that date. Recorded turnover excludes the consumption tax, which was introduced in April 1989 and raised in April 1997.

[8] See Abe and Tonogi (2010) and Sudo, Ueda, and Watanabe (2014) for earlier studies using these data.

[9] Nikkei Digital Media classifies products using 3-digit codes, where each code represents a product category such as yogurt, beer, tobacco, toothbrushes, etc.

The POS data do not include information as to whether a price is a regular price or a temporary sales price. Therefore, to isolate temporary sales prices, we follow Eichenbaum, Jaimovich, and Rebelo (2011) and define the most commonly observed price (mode price) during the three months centered on the date as the regular price of a product on that date.[10] Temporary sales are identified when the posted price differs from the regular price.

Figure 1 illustrates developments over time in four variables associated with temporary sales: (i) the frequency of sales (%), (ii) the magnitude of sales discounts (%), (iii) the ratio of quantities sold at the sale price to those at the regular price, and (iv) the ratio of turnover at the sale price to total turnover in a month (%). Variable (iii) shows the amount of products sold at sale prices relative to the amount sold at regular prices, on a certain date when a retailer chooses to hold a sale. On the other hand, variable (iv) shows how much temporary sales contribute to a retailer's total turnover in a certain month, meaning that this variable depends on the frequency of sales. In the graphs, the red dashed lines represent the original series, which are very volatile, while the black solid lines show smoothed series using the Hodrick-Prescott (HP) filter with $\lambda = 14,400$.

The figure shows that temporary sales have become increasingly important for retailers during the lost decades. The frequency of sales has risen from around 15% to 25%, indicating that recently temporary sales have been taking place once every four days. Parallel to the increase in the frequency, the magnitude of sales discounts has shrunk from around 20% to 14%. Despite these two developments, the ratio of quantities sold at sales prices to those at regular prices has been relatively stable at around 1.7. Consequently, turnover from temporary sales reached 30% of total turnover during the 2000s compared to 20% in the 1990s.

2.2.2 Price Elasticity

One of the advantages of using the POS data is that both price and quantity series are available, enabling us to investigate the relationship between the two, and in turn, the price elasticity of demand. Figure 2 shows a scatter plot for daily quantity changes (vertical axis) and the corresponding daily price changes (horizontal axis) for a particular item, namely cup noodles, of a specific type (brand, flavor, size, and so on). The slope is clearly negative. According to standard theory, which assumes that supply slopes upward and demand slopes downward, this suggests that supply shocks are more prevalent in the market for this particular item. The size of the slope can thus be interpreted as a proxy for the price elasticity of demand.

To examine how this proxy for the price elasticity of demand has changed over time, we calculate the slope of quantity changes against price changes for each product and store and

[10] They refer to this as the reference price rather than the regular price. See Sudo, Ueda, and Watanabe (2014) for a detailed discussion of the identification of sales.

then construct the weighted median of slopes across products and stores.[11] The value calculated for each year from 1989 to 2012 is depicted in Figure 3. The figure shows a clear upward trend from the mid-1990s, indicating that households have become increasingly price-sensitive in recent years. Given that bargain hunters are more price sensitive than loyal customers, the upward trend in the price elasticity suggests that the fraction of loyal customers has fallen and that of bargain hunters has risen. Thus, whereas GS in their model assume these fractions to be constant, in practice this does not seem to be the case.

2.3 The Link between the Frequency of Sales and Macroeconomic Conditions

2.3.1 Labor Market Indicators

To examine the link between the frequency of sales and macroeconomic conditions, we start by looking at labor market indicators. Figure 4 plots the frequency of sales against the unemployment rate in the upper panel and hours worked in the lower panel. The figure shows that during the 1990s, the unemployment rate increased, while hours worked decreased. These developments reflect the deterioration in the labor market in the decade following the burst of the bubble economy. In addition, the decline in hours worked also reflects the reduction in the statutory workweek length as a result of the revision of the Labor Standards Law.[12] Looking at trends in the unemployment rate and hours worked and comparing them with developments in the frequency of sales suggests that these are related.

2.3.2 Correlation between the Frequency of Sales and Macroeconomic Conditions

The relationship described in the previous subsection was for a relatively long time horizon (a decade or two). Next, we examine the link between the frequency of sales and macroeconomic conditions for a shorter time horizon, the business cycle. We isolate the cyclical component of time series between 1.5 to 8 years using the Baxter-King band pass filter and compute

[11] In order to accurately measure the price elasticity of demand, we employ the following procedure for our sample selection. First, we drop observations when demand shocks may be large, because this causes us to identify the supply curve rather than the demand curve. To this end, we only use observations in the second and fourth quadrants of the scatter plot. Second, we use monthly data and calculate the month-on-month change in quantities and prices so as to eliminate the effect of temporary sales. Price changes of less than or equal to three yen are omitted. Because we use monthly data, we have at most 12 observations a year for calculating the slope. We omit an item from the calculation of the weighted median when the number of observations falls below six. However, even when using daily data to calculate changes, we find that the price elasticity of demand has increased.

[12] The revision stipulated a gradual shortening of the workweek length from 48 to 40 hours. See Kawaguchi, Naito, and Yokoyama (2008) for an analysis of the impact of the revision on hours worked, as well as Kuroda (2010), who argues that hours worked remained essentially unchanged once demographic changes are controlled for.

the contemporaneous correlation between temporary sales and macroeconomic conditions. As indicators of macroeconomic conditions, we focus on the following seven variables: the unemployment rate, total hours worked, the number of employed persons, the index of industrial production, the monthly growth rate of the CPI, and the leading, coincident, and lagging indexes of business cycle indicators (the last three are components of the Composite Indexes compiled by the Cabinet Office). All are expressed in logarithm except for the monthly growth rate of the CPI, which is expressed in terms of the logarithm difference. In Figure 5, correlations are represented by the red circles. The blue dashed lines represent the 95% confidence interval for no correlation.[13]

The figure suggests that the frequency of sales is negatively correlated with hours worked at the 5% significance level. The frequency of sales rises when hours worked decline. Moreover, although the correlations are significant only at the 10% level, the results indicate that the frequency of sales rises when the unemployment rate rises or the lagging index worsens. All these correlations suggest that retailers raise the frequency of sales when households are less busy working, which appears plausible if such households increase time spent bargain hunting. On the other hand, no significant correlation between the frequency of sales and the CPI inflation rate, the leading index, or the coincident index can be observed.

2.3.3 Measuring Impulse Responses using a VAR Model

The above correlation analysis does not tell us anything about the direction of causality, that is, whether, for example, the change in the frequency of sales is due to a change in hours worked or vice versa. In addition, it is possible that some other factor may have caused changes in hours worked and the frequency of sales in the opposite direction. Therefore, in order to examine the causal link between the changes, we next estimate a vector autoregression (VAR) model and calculate the impulse responses to various shocks. Following Altig et al. (2011), we assume that the only shocks that affect productivity in the long run are innovations to neutral and capital-embodied technology. Monetary policy shocks have a contemporaneous impact on the central bank's policy instrument, but they do not have a contemporaneous impact on aggregate quantities and prices. We use quarterly data from 1989:1Q to 2013:1Q. We employ a total of ten variables: the relative price of investment, labor productivity, the quarterly growth rate of the CPI, hours worked multiplied by employment, the ratio of consumption to gross domestic product (GDP), the ratio of investment to GDP, the frequency of temporary sales,

[13]To take into account the possibility that the Baxter-King band pass filter may yield spurious correlations, we calculate the 95% confidence interval using Monte Carlo simulation. That is, we replicate two independent time-series variables, extract their business-cycle components using the Baxter-King band pass filter, and calculate their correlation.

the annualized short-term loan rate, the ratio of the monetary base to nominal GDP, and the quarterly growth rate of the Nikkei commodity price index. All variables are in logarithm, except for the short-term loan rate and the growth rates of the CPI and the commodity price index. While other studies include similar variables in their analysis, what is distinct here is the inclusion of the frequency of temporary sales in the VAR model.[14]

Figures 6 and 7 show the impulse responses of aggregate quantities and prices including the frequency of sales to innovations to neutral and capital-embodied technology.[15] Gray areas indicate the 95% confidence intervals. The graphs show that the frequency of temporary sales moves in the opposite direction to hours worked. Positive technology shocks, both to neutral and to capital-embodied technology, increase output and investment. In the first case, consumption increases as well. The impact on hours worked differs between the two shocks. While a neutral technology shock initially decreases hours worked before they return to their original level, a capital-embodied technology shock increases them. Meanwhile, the response of the frequency of sales also differs between the two shocks. It increases initially in response to a neutral technology shock, while it falls persistently in response to a capital-embodied shock. That is, irrespective of which type of technology shock is examined, the frequency of sales falls (rises), when hours worked rise (fall).

2.4 Difference from the United States

This section so far has provided various pieces of evidence regarding the frequency of sales and its relationship with the macroeconomic environment. The key finding is that the frequency of temporary sales moves in the opposite direction of hours worked. The likely reason is that when hours worked are lower, households have more time for bargain hunting and therefore are more price sensitive; observing this, retailers hold more frequent temporary sales. This result is in line with the findings by Aguiar and Hurst (2007) and Lach (2007), but is in stark contrast with studies on sale pricing in the United States such as Kehoe and Midrigan (2010), Eichenbaum, Jaimovich, and Rebelo (2011), and Anderson et al. (2012), who argue that retailers' decision of holding sales is orthogonal to macroeconomic circumstances. Most strikingly, Coibion, Gorodnichenko, and Hong (2012) arrive at the opposite of our result, finding that the frequency of sales falls when the unemployment rate rises.

There are at least two possible explanations for this difference. First, pricing strategies

[14] As seen in Figure 4, the frequency of temporary sales and hours worked exhibit a non-zero trend. This means that we could use changes in these variables rather than their levels in estimating the VAR model. However, we use levels because, in theory, the variables should be I(0).

[15] Appendix A shows the results when examining the economic impact of a monetary policy shock, which is found to be insignificant. We decided not to show the analysis here, since the fact that nominal interest rates were facing the zero lower bound prevents us from accurately isolating monetary policy shocks.

in Japan and the United States differ. Unlike in Japan, many retailers in the United States use an every-day-low-price strategy. The key to Coibion, Gorodnichenko, and Hong's (2012) result is the fact that it is mainly lower-priced stores such as Wal-Mart that adopt an every-day-low-price strategy, while temporary sales are conducted mainly by higher-priced stores. Given this situation, if we assume a rise in unemployment, customers are likely to switch from higher- to lower-priced stores to reduce their expenditure. Higher-priced stores give up on such price-sensitive customers and concentrate on loyal customers. They thus reduce the frequency of temporary sales. In Japan, by contrast, even lower-priced stores conduct very frequent temporary sales. As Sudo, Ueda, and Watanabe (2014) point out, prices in Japan are revised ten times more frequently than those in the United States, due to frequent temporary sales.[16] Although these differences in retailers' pricing strategies yield opposite results in Coibion, Gorodnichenko, and Hong's (2012) and our study regarding the link between unemployment and the frequency of sales, the effect of unemployment on average prices paid by consumers (effective prices) is in the same direction in both cases. That is, average prices decrease when the unemployment rate rises.

The second possible explanation is that our POS data cover a very long period, from 1988 to 2013. This is considerably longer than the typical duration of business cycles. By contrast, the U.S. price data used by Coibion, Gorodnichenko, and Hong (2012) are for a relatively short period, from 2001 to 2007.[17] The widely used Dominicks supermarket dataset is for the period from 1989 to 1997. Unless the observation period is sufficiently long relative to the duration of business cycles, it is difficult to accurately estimate the link between the frequency of sales and macroeconomic conditions. If the difference is mainly due to the second reason, our findings for Japan may hold more generally; that is, the frequency of sales decreases when hours worked rise.

3 Model

In this section, we present our DSGE model of temporary sales. We extend the model by GS so as to incorporate endogenous bargain hunting. To this end, we make two innovations.

[16] Japanese households are said to go shopping more frequently from Monday to Sunday: they walk to supermarkets nearby, purchase a small amount of products, and bring them back on foot. In contrast, U.S. households go shopping mainly on weekends by car. According to the *Survey of Consumer Behavior* by the Japanese Meat Information Service Center, the average shopping frequency in the 2000s was three to four times a week. These characteristics peculiar to Japan (or the United States) may be responsible for these differences in retailers' pricing strategies.

[17] To resolve problems due to this short period, they extend the sample size in a cross-sectional dimension using data for 50 metropolitan areas.

3.1 Setup

The Household A representative household consists of a unit mass of household members and has the following lifetime utility function:

$$U_t = \sum_{j=0}^{\infty} \beta^j \mathrm{E}_t \left[v(C_{t+j}) - Z_{t+j}^h \upsilon \left(H_{t+j} + \phi_L H \frac{(1-L_{t+j})^{\theta_L}}{(1-\lambda)^{\theta_L}} \right) \right], \tag{3.1}$$

where C_t is an aggregate composite of differentiated consumption goods that is defined below, H_t is hours worked (H is its steady state level), and $L_t \in [0, 1)$ is the fraction of household members that are chosen to be loyal customers in the household. The fraction of loyal customers L_t is endogenous, with mean λ. $1 - L_t$ is interpreted as the fraction of bargain hunters in the household. Parameter $\beta \in (0, 1)$ is the subjective discount factor, ϕ_L represents the utility weight for being a loyal customer, and $\theta_L > 0$ represents the elasticity of the fraction of loyal customers with respect to changes in hours worked. Z_t^h represents a stochastic shock to the utility weight of labor supply with unit mean, and the logarithm deviation of the stochastic shock is denoted by ε_t^h. A positive shock works to decrease labor supply and increase the fraction of loyal customers. The function $v(C_t)$ is strictly increasing and strictly concave in C_t, and $\upsilon(X_t)$ is strictly increasing and convex in X_t.

The first innovation in our model is the inclusion of the last term, that of $1 - L_{t+j}$, in equation (3.1). This suggests that the utility of the representative household increases, as the fraction of loyal customers rises (the fraction of bargain hunters falls). Bargain hunting, like supplying labor, decreases the time for leisure and hence utility.

The overall aggregator of consumption is given by

$$C_t \equiv \left[\int_T \left(\int_B c_t(\tau, b)^{\frac{\eta-1}{\eta}} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{3.2}$$

where $c_t(\tau, b)$ is the household's consumption of brand $b \in B$ of product type $\tau \in T$. GS give the example of beer and dessert as product types and Corona beer and Ben & Jerry's ice cream as brands falling into those product categories. Following GS, we assume that $\eta > \epsilon$, so that household members are more willing to substitute between different brands of a specific product type than between different product types. In the original GS model, equation (3.2) is the aggregator of consumption only for bargain hunters, and loyal customers have a different aggregator.

As the second innovation to the model, we assume that the household faces the following constraint when purchasing goods:

$$\left(\int_{\Lambda_t \in T} c_t(\tau, B)^{\frac{\epsilon-1}{\epsilon}} d\tau \right)^{\frac{\epsilon}{\epsilon-1}} \geq \bar{C}_t, \tag{3.3}$$

where

$$\bar{C}_t = \left(\frac{p_{B,t}(\tau)}{P_t} \right)^{-\epsilon} C_t^*. \tag{3.4}$$

$\Lambda_t \in T$ represents the set of product types where consumers in the household are loyal to a particular brand. The range of Λ_t increases with the fraction of loyal customers L_t, and for simplicity, we assume that the fraction of Λ_t in T equals L_t. The left-hand side of inequality (3.3) corresponds to the aggregator of consumption for loyal customers in the original GS model.

Inequality (3.3) suggests that, if the representative household assigns to a household member the role of loyal customer, the household member has to consume at least $\bar{C}_t$. At the same time, the household member has to optimize the allocation of expenditure across product types $\tau \in T$ to maximize the left-hand side of inequality (3.3), where his choice is limited to a particular brand of each product type. In other words, the loyal customer in our model is involuntarily loyal to a particular brand. Without the constraint (3.2), household members would freely choose a brand with a lower price. As for $\bar{C}_t$, equation (3.4) states that the minimum amount of consumption is decreasing in the ratio of the price index for all brands of product type τ, $p_{B,t}(\tau)$, to the average aggregate price level P_t with an elasticity of ϵ. Finally, C_t^* represents aggregate consumption spending for the representative household.

Solving the expenditure minimization problem subject to equation (3.2) as well as constraint (3.3) yields the following demand function:[18]

$$c_t(\tau, b) = \begin{cases} \left(\frac{p_t(\tau,b)}{p_{B,t}(\tau)}\right)^{-\eta} \left(\frac{p_{B,t}(\tau)}{P_t}\right)^{-\epsilon} C_t^* & \text{for the fraction of household, } 1 - L_t \\ \left(\frac{p_t(\tau,b)}{P_t}\right)^{-\epsilon} C_t^* & \text{for the fraction of household, } L_t \end{cases} \tag{3.5}$$

where $p(\tau, b)$ is the price of brand b of product type τ. This demand function has exactly the same form as that in GS, which drastically simplifies our analysis below. The first line of equation (3.5) illustrates that bargain hunters, who make up $1 - L_t$ of the household, are agents that have a higher price elasticity η. They can freely substitute from a relatively expensive brand b' to a less expensive brand b'' within type τ products. This form of demand is optimal unless constraint (3.2) binds. The second line of equation (3.5) suggests that loyal customers, who make up L_t of the household, are agents that cannot make such substitutions. They have

[18]See Appendix B. To be precise, we need to assume $c_t(\tau, b)^{-\frac{1}{\eta}} \ll c_t(\tau, b)^{-\frac{1}{\epsilon}}$. This is not a strict condition, because $\eta > \epsilon$.

a lower price elasticity $\epsilon < \eta$. As the fraction of loyal customers increases, a higher fraction of household members are bound by constraint (3.2) and obliged to consume the suboptimal amount of products.

In choosing the optimal L_t, the household faces a trade-off. On the one hand, an increase in L_t raises the household's utility. As equation (3.1) shows, the household increases time for leisure by decreasing the time spent on bargain hunting. On the other hand, the increase in L_t decreases the benefit from bargain hunting. The household decreases its utility by selecting the sub-optimal amount of demand, because the household is more strongly constrained by inequality (3.3) and forced to purchase products according to the second line of the demand function (3.5). This second effect is illustrated by the relationship between utility-related consumption C_t and spending-related consumption C_t^*. Appendix B shows that C_t depends not only on C_t^* but also on the following consumption wedge $\digamma_t$:[19]

$$C_t = \digamma_t \cdot \left(\frac{P_{B,t}}{P_t} \right)^{-\epsilon} C_t^*, \tag{3.6}$$

and that $\digamma_t \leq 1$ and $d\digamma_t / dL_t < 0$. As the household engages in more bargain hunting, L_t decreases and $\digamma_t$ increases. The household enjoys higher utility from the same amount of consumption spending C_t^*. If the household engages in bargain hunting for all product types, that is, $L_t = 0$, $\digamma_t$ becomes highest as $\digamma_t = 1$.

The household's budget constraint is given by

$$P_t C_t^* + E_t[Q_{t+1|t} A_{t+1}] \leq W_t H_t + D_t + A_t, \tag{3.7}$$

where W_t is the wage, D_t is dividends received from firms, Q_t is the asset pricing kernel, and A_t is the household's portfolio of Arrow-Debreu securities.

Additionally, following GS, we introduce Calvo-type wage stickiness into the model. That is, the household supplies differentiated labor input to firms, and wages can be adjusted at a probability of $1 - \phi_w$.

Firms An advantage of our model is that firms' behavior is described in exactly the same way as in GS. The demand function faced by firms, given by equation (3.5), is the same as that in GS. It is thus optimal for firms to randomize their prices across shopping moments from a distribution with two prices. Firms set a normal high price, $P_{N,t}$, with a frequency of $1 - s_t$ and a low sale price, $P_{S,t}$, with a frequency of s_t. Following GS, we refer to the higher price as

[19] Utility-related consumption C also depends on the price ratio P_B/P, but this fact does not influence the household's decision regarding L_t, since the household is a price taker.

the normal price instead the regular price. The only, but important, difference from GS is that firms optimize their pricing decisions by observing changes in the fraction of loyal customers, L_t.

As argued by GS, the strategic substitutability of sales plays a crucial role in firms' pricing. The more other firms hold sales, the less an individual firm will want to have a sale. Suppose that other firms always have a sale. If the individual firm stops holding a sale and sells its good at the normal price, its profit increases, since price-insensitive loyal customers tend to buy the good even at the normal price. As the opposite case, suppose that other firms do not hold sales. Because sales attract price-sensitive bargain hunters, the individual firm can increase its profit by having sales. Such strategic substitutability leads firms to randomize their prices between the high normal price and the low sale price.

Adjustments of normal prices by firms are characterized by Calvo-type price stickiness. That is, in each period, firms can reset their normal prices with a probability of $1 - \phi_p$. Sale prices can be adjusted freely.

Wholesalers produce goods using labor input:

$$Q_t = Z_t^a H_t^\alpha, \tag{3.8}$$

where α represents the elasticity of output with respect to labor input and Z_t^a represents a stochastic shock to productivity, which has a mean of one, and the logarithm deviation of which is denoted by ε_t^a.

Monetary Authority The monetary authority sets the nominal interest rate i_t based on the following monetary policy rule:

$$i_t = \rho i_{t-1} + (1 - \rho)\phi_\pi \pi_t^N + \varepsilon_t^i, \tag{3.9}$$

where π_t^N represents the change in the normal price. The official CPI excludes sales, so we assume that the monetary authority refers to the inflation rate based on the normal price. Policy parameters ρ and ϕ_π represent interest rate inertia and the size of the response to the inflation rate, respectively. ε_t^i represents a shock to monetary policy with zero mean.

Resource Constraint The resource constraint of goods is given by

$$Y_t = C_t^* + Z_t^g, \tag{3.10}$$

where Z_t^g represents a stochastic shock to government expenditure. Its mean is zero and the logarithm deviation from the steady-state Y is denoted by ε_t^g.

Exogenous Shocks We consider four types of shocks, namely, shocks to monetary policy, technology, government expenditure, and labor supply, which are respectively given by

$$\varepsilon_t^i = \eta_t^i, \tag{3.11}$$

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a, \tag{3.12}$$

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g, \tag{3.13}$$

$$\varepsilon_t^h = \rho_h \varepsilon_{t-1}^h + \eta_t^h. \tag{3.14}$$

We do not assume inertia for the monetary policy shock, since the monetary policy rule has its own inertia.

3.2 Log-Linearized Equations

Leaving detailed derivations to Appendix B, here we summarize the log-linearized equations. We denote log deviations of variables by small letters.

Sale Pricing It is optimal for a firm j to adjust its sale price, $p_{S,j,t}$, one-for-one in line with any change in its nominal marginal cost, $x_t + p_t$, that is,

$$p_{S,j,t} = x_t + p_t, \tag{3.15}$$

where the real marginal cost is denoted by x_t.

The frequency of sales is given by

$$ss_t = -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}x_t - \left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi} + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right) l_t. \tag{3.16}$$

The second term on the right-hand side, which is not in the original model by GS, indicates that as the fraction of loyal customers l_t increases, firms decrease the frequency of sales s_t. In other words, when the fraction of bargain hunters decreases, the merit of holding sales decreases, and hence firms decrease the frequency of sales. Like in GS's model, an increase in the real marginal cost x_t decreases the frequency of sales. Because the sale price responds one-for-one to changes in marginal costs, the sale price increases more than the normal price.

This decreases the relative demand for goods when they are on sales, thereby decreasing the frequency of sales.

Fraction of Loyal Customers The fraction of loyal customers l_t is given by:

$$\begin{aligned}
0 = &\left(\theta_c^{-1} + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} - 1\right) y_t - \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} w_t \\
&- \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a + \varepsilon_t^h - (\theta_c^{-1} - 1)\varepsilon_t^g \\
&- \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} B + (\theta_L - 1)\frac{\lambda}{1-\lambda} + \theta_h^{-1}\phi_L \frac{\lambda}{1-\lambda}\right) l_t \\
&+ (\theta_c^{-1} - 1)\left\{ f_t - \epsilon\left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t \right)\right\} \\
&+ \frac{\Xi}{1-\Xi}\xi_t + \frac{\eta - 1}{\eta} f_t. \qquad (3.17)
\end{aligned}$$

Two things are worth noting. First, the fraction of loyal customers l_t increases with aggregate demand, y_t, and consequently, hours worked, h_t. As hours worked increase, the disutility from bargain hunting increases, and hence the fraction of loyal customers increases.

Second, the fraction of loyal customers l_t increases with the consumption wedge, f_t. A larger consumption wedge means greater utility from a given amount of consumption spending. As the wedge grows, the benefit from bargain hunting diminishes, raising the fraction of loyal customers.

We can also show that the consumption wedge increases with the ratio of the sale price to the normal price, $\mu_t = P_{S,t}/P_{N,t}$, and decreases with the sale frequency, s_t. In other words, as the sale price converges to the normal price or sales become less frequent, prices for different brands become more homogenous and the consumption wedge grows. Specifically, the consumption wedge is given by

$$f_t = \frac{\eta}{\eta - 1}\lambda \frac{\Xi\xi_t - (1-\Xi)l_t}{\lambda\Xi + (1-\lambda)}, \qquad (3.18)$$

where

$$\begin{aligned}
\xi_t = &\frac{(\mu^{\epsilon\frac{1-\eta}{\eta}} - 1)\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}(\mu^{1-\eta} - 1)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}} s s_t \\
&+ \frac{\mu^{\epsilon\frac{1-\eta}{\eta}}\epsilon\frac{1-\eta}{\eta}\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}\mu^{1-\eta}(1-\eta)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}} s\mu_t. \qquad (3.19)
\end{aligned}$$

The ratio of the sale price to the normal price is given by

$$\mu_t = \frac{1}{1-\psi}\left(x_t + Al_t\right). \tag{3.20}$$

Phillips Curve with Sales The New Keynesian Phillips curve with sales is given by

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} \\ &+ \frac{1}{1-\psi}\left\{\kappa x_t + \psi(\Delta x_t - \beta E_t \Delta x_{t+1}) + \kappa A l_t + A(\Delta l_t - \beta E_t \Delta l_{t+1})\right\}.\end{aligned} \tag{3.21}$$

Compared with the standard New Keynesian Phillips curve, the equation has two additional terms. First, as in GS, changes in the real marginal cost, Δx_t, influence the inflation rate, π_t. This is because the overall price depends on the sale price and the sale price is proportional to the real marginal cost, as shown in equation (3.15). Second, unlike in GS, the fraction of loyal customers l_t influences the inflation rate. As the fraction of loyal customers increases, the household substitutes less from relatively expensive brands to cheaper brands. Observing this, firms lower the frequency of sales, and hence the overall price index increases.

The New Keynesian Phillips curve with respect to the normal price index is given by

$$\pi_{N,t} = \beta E_t \pi_{N,t+1} + \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p}(x_t + p_t - p_{N,t}). \tag{3.22}$$

In other words, the curve for the normal price hardly changes in the presence of sales.

The real marginal cost, x_t, is given by

$$x_t = \frac{1}{1+\gamma\delta} w_t + \frac{\gamma}{1+\gamma\delta}\left(y_t - Bl_t\right). \tag{3.23}$$

As in the standard New Keynesian model, the real marginal cost increases with the real wage, w_t.

Wage Phillips Curve The wage Phillips curve is given by

$$
\begin{aligned}
\pi_{W,t} &= \beta\pi_{W,t+1} \\
&+ \frac{(1-\phi_w)(1-\beta\phi_w)}{\phi_w}\frac{1}{1+\varsigma\theta_h^{-1}} \\
&\left[\left(\theta_c^{-1}+\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right)y_t-\left(1+\frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right)w_t\right. \\
&-\frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a+\varepsilon_t^h-\theta_c^{-1}\varepsilon_t^g \\
&-\left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}B+\theta_h^{-1}\theta_L\phi_L\frac{\lambda}{1-\lambda}\right)l_t \\
&\left.-(1-\theta_c^{-1})\left\{f_t-\epsilon\left(x_t+\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right)\right\}\right]. \qquad (3.24)
\end{aligned}
$$

where

$$\Delta w_t = \pi_{W,t}-\pi_t. \qquad (3.25)$$

An increase in the fraction of loyal customers decreases the real wage on both the labor demand and the labor supply side. On the labor demand side, when the fraction of loyal customers increases, firms lower the frequency of sales. The quantity of goods sold is generally higher when goods are on sale than when they are sold at the normal price, so if the frequency of sales decreases, total demand for such goods decreases. This reduces the supply of those goods and, in turn, labor demand and the real wage. On the labor supply side, as the fraction of loyal customers increases, the disutility from labor supply diminishes, which lowers the real wage. Moreover, the consumption wedge, f_t, worsens, which raises labor supply and lowers the real wage due to the income effect.

Euler Equation The Euler equation is given by

$$
\begin{aligned}
y_t &= E_t y_{t+1}-\theta_c(i_t-E_t\pi_{t+1})+(\varepsilon_t^g-E_t\varepsilon_{t+1}^g) \\
&+(1-\theta_c)\left\{\Delta f_{t+1}-\epsilon\left(\Delta x_{t+1}+\frac{1}{(\eta-\epsilon)(1-\lambda)}\Delta l_{t+1}\right)\right\}. \qquad (3.26)
\end{aligned}
$$

Monetary Policy Rule The monetary authority sets the nominal interest rate based on the following monetary policy rule:

$$i_t = \rho i_{t-1}+(1-\rho)\phi_\pi\pi_t^N+e_t^i. \qquad (3.27)$$

Labor Demand Production input is given by

$$h_t = \frac{1}{1+\gamma\delta}\frac{y_t - \delta w_t - Bl_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a. \tag{3.28}$$

4 Impulse Response Functions

We simulate impulse response functions (IRFs) of economic variables to four types of shock. The first shock is an accommodative shock to the monetary policy rule; the second is a positive shock to wholesalers' production technology; the third is a government expenditure shock; and the fourth is a labor supply shock.

4.1 Calibration

Most of the calibration of our model parameters is based on GS, with two exceptions. First, in our model, the central bank follows an interest rate monetary policy rule, with $\rho = 0.8$ and $\phi_\pi = 1.5$, while in GS the central bank follows a money growth rate rule. Second, we use parameters associated with sales so that they are consistent with the POS data for Japan shown in Figure 1. The average of the frequency of sales, s, is 0.18; the average ratio of quantities sold at the sale price to those at the normal prices, χ, is 1.7; and the average size of sales discounts is 17%, which means $\mu = 0.83$. Details of the setting are shown in Tables 3 and 4. As for the parameters associated with the fraction of loyal customers, we target the steady state level of the fraction of loyal customers to calibrate ϕ_L given θ_L. We set two alternative values for θ_L: 4 or 100. A lower θ_L implies a higher elasticity of the fraction of loyal customers with respect to changes in hours worked. The value of θ_L =4 is chosen to match the historical change in the frequency of sales, as shown below.

4.2 Comparison between the Standard New-Keynesian Model and the GS Model

To understand GS's results, let us begin by comparing the IRFs in the GS model and in the standard New Keynesian (NK) model. The GS model is the model discussed above without endogenous changes in the fraction of loyal customers. The standard NK model corresponds to the GS model without sales.

Figure 8 presents the IRFs of three economic variables: aggregate output y_t, the inflation rate excluding sales (i.e., changes in normal prices) $\pi_{N,t}$, and the nominal interest rate i_t. The horizontal axis shows the time up to 12 quarters after a shock. The shock we consider is an accommodative monetary policy shock of one unit. The dashed and solid lines represent the

IRFs for the GS and the standard NK model, respectively.

The left panel shows that, as argued by GS, the real effect of monetary policy in the model with sales remains large and close to that in the model without sales. The real effects are similar, because sales are strategic substitutes.

4.3 The Effects of Endogenous Bargain Hunting

In this subsection, we examine the IRFs of our model and compare them with those of the GS model. The graphs below show the IRFs of nine economic variables, with the blue solid lines representing the IRFs of the GS model. The thick solid lines and the lines with asterisks represent the IRFs of our model with endogenous changes in the fraction of loyal customers for the two different elasticity parameter values θ_L =4 and 100, respectively.

4.3.1 Monetary Policy Shock

Figure 9 presents the IRFs for the accommodative monetary policy shock. The figure indicates that temporary sales dampen the effect of such a shock on aggregate demand by around 40%. The mechanism runs as follows. In response to the monetary policy shock as a result of a lowering of the nominal interest rate, aggregate demand increases. This raises hours worked. Since the household spends more time working, its disutility from bargain hunting increases. With endogenous bargain hunting, the fraction of loyal customers (bargain hunters) increases (decreases). Observing this, firms lower the frequency of sales. Since the quantity of goods sold is higher when goods are on sale, the decrease in the frequency of sales dampens the increase in aggregate demand. The effect of monetary policy on demand is smaller the lower θ_L is.

The attenuated real effect of monetary policy is also explained by the increased strategic substitutability of sales. Suppose that all firms except for firm A raise the frequency of sales. As in GS, this reduces the incentive for firm A to raise the frequency of sales, since doing so would decrease the marginal revenue from sales. In our model, an additional channel emerges. When all firms except for firm A raise the frequency of sales, the aggregate price level falls. This increases aggregate demand for goods and, in turn, aggregate demand for labor. The household supplies more labor and loses time for bargain hunting. The fraction of loyal customers (bargain hunters) increases (decreases). Observing this, firm A lowers the frequency of sales. Such intensified strategic substitutability of sales mitigates the real effect of monetary policy.

The inflation rate excluding sales (i.e., changes in normal prices) also fluctuates less in this model than in the GS model. As explained in the previous section, an increase in the fraction of loyal customers has the effect of decreasing the real wage and the real marginal cost. Although the accommodative monetary policy shock increases hours worked and exerts upward pressure on the real marginal cost, the effect of the increase in the fraction of loyal customers dominates when θ_L is low. On the other hand, the model yields greater increases in the inflation rate including sales than the GS model. This is because the aggregate price level increases as a result of the rise in the fraction of loyal customers and the decline in the frequency of sales in response to the shock.

Finally, although our model shows that temporary sales dampen the real effect of monetary policy shocks, we should emphasize that monetary policy shocks still have real effects. Temporary sales do not fully eliminate the real effect of monetary policy shocks, despite the fact that aggregate price levels including sales look perfectly flexible.

4.3.2 Technology Shock

Figure 10 shows the IRFs for a positive technology shock. Our model yields far greater effects on aggregate demand than the GS model and brings about inflation, not deflation. In this type of sticky price model, a positive technology shock tends to decrease hours worked. This decreases (increases) the fraction of loyal customers (bargain hunters). Firms react to the shock by increasing the frequency of sales, which increases the quantity of goods, thus amplifying the increase in aggregate demand. The aggregate price level including sales falls due to the decrease in the fraction of loyal customers and the increase in the frequency of sales. In contrast, the aggregate price level excluding sales rises. This results from the increase in the real wage and in the real marginal cost brought about by the decrease in the fraction of loyal customers and the subsequent increase in the disutility of labor supply.

4.3.3 Government Expenditure Shock

Next, Figure 11 shows the IRFs for a positive government expenditure shock. As can be seen, the economic responses resemble those to an accommodative monetary policy shock. The real effect of a government expenditure shock is attenuated.

4.3.4 Labor Supply Shock

Finally, we simulate the IRFs for a shock to labor supply. The way that this shock is formulated is motivated by Hayashi and Prescott (2002). In analyzing Japan's first lost decade and incorporating the effects of the shortening of statutory work hours, they use the following utility function:

$$\log C_t - \alpha \frac{H_t}{40} E_t,$$

where H_t and E_t represent the workweek length (hours) and the fraction of household members who work. For 1990 to 1992, they take H_t as exogenous. In our model, as we showed in equation (3.1), we replace the exogenous $H_t/40$ with the labor supply shock Z_t^h and the endogenous E_t with labor supply H_t given by $\upsilon\left(H_t + \phi_L \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}}\right)$.[20]

The way that we formulate the labor supply shock is also motivated by innovations in "bargain hunting technology" made in the last two decades. Brown and Goolsbee (2002) argue that the internet lowers search costs for customers. In our model, ϕ_L in equation (3.1) can be interpreted to capture technological changes that facilitate bargain hunting in that ϕ_L presents the degree of disutility from bargain hunting. A decrease in ϕ_L reduces the disutility from bargain hunting. Thus, if we interpret the rise of the internet as an innovation in bargain hunting technology, then this innovation encourages more bargain hunting (a fall in λ).[21]

The responses to the labor supply shock are shown in Figure 12. As can be seen, this type of shock, unlike the other types of shocks considered above, results in hours worked and the fraction of loyal customers moving in opposite directions. A positive shock ε_t^h makes labor supply more costly, decreasing hours worked, h_t, which has the effect of lowering the fraction of loyal customers. At the same time, the positive labor supply shock directly raises the disutility of bargain hunting, which has the effect of increasing the fraction of loyal customers. Since the elasticity of labor supply is below one, the former effect dominates the latter and the fraction of loyal customers decreases.[22]

These effects implied by the model are inconsistent with actual observations for Japan. It therefore seems unlikely that the negative correlation between hours worked and the frequency of sales can be explained by a supply shock to labor, be it as a result of the shortening of statutory work hours or innovations in bargain hunting technology.

5 Why Did the Frequency of Sales Rise During Japan's Lost Decades?

As we saw in Figure 1, the frequency of sales, s_t, rose during Japan's lost decades. In this section, we aim to examine why the frequency of sales rose using our model and argue that the reason is the decline in hours worked coupled with the decline in employment. Moreover, we aim to calibrate the elasticity of the fraction of loyal customers with respect to changes in

[20] In our simulation, the elasticity of labor supply is 0.7. In Hayashi and Prescott (2002), it equals one.

[21] More precisely, innovations to bargain hunting technology may need to be attached to the second term of the argument of $\upsilon\left(H_t + \phi_L \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}}\right)$ rather than being attached outside the function of $\upsilon(\cdot)$ as Z_t^h. We find that even if we formulate the shock in this way, similar impulse responses are obtained.

[22] Although we do not show here, the fraction of loyal customers increases (unchanges), when the elasticity of labor supply exceeds (equals) one.

hours worked, θ_L, which plays an important role in determining the real effect of monetary policy.

5.1 Simulation Method

We conduct simulations using our model assuming that the only shocks to the economy are technology shocks. We draw the time-series path of the technology shocks so as to account for the actual movements in hours worked in Japan.[23] While we do not claim that this assumption is fully valid, the following points provide some support. First, as suggested by real business cycle theory, technology shocks are likely to be the chief driving force of business cycles. Moreover, as argued by Hayashi and Prescott (2002), a key reason for Japan's slow growth during the lost decades is the slowdown in the growth of total factor productivity, which represents the contribution of technological progress in the broadest sense.[24] Second, when the statutory workweek length was shortened, labor hoarding may have decreased. The resulting improvement in labor efficiency can be regarded as a positive technology shock.

As before, parameters associated with sales are calibrated to fit the POS data for Japan. The only parameter that we estimate is the persistence of technology shocks. We use the period from 1981:2Q to 2013:1Q as our observation period. To match the actual time-series path of hours worked with the simulated time-series path from our model, we draw the time-series path of technology shocks. Using this, we can calculate simulated time-series paths of variables associated with temporary sales, including the frequency of sales. For comparison, two models are used: the GS model and our model with endogenous changes in the fraction of loyal customers characterized by $\theta_L = 4$, which is chosen to fit the data. For simplicity, we neglect the zero lower bound on the nominal interest rate, which constrained the effectiveness of monetary policy during Japan's lost decades.

5.2 Simulation Results

Figure 13 shows simulated paths of variables associated with temporary sales, namely, the frequency of sales (%, s_t), the size of sales discounts (%, $1 - \mu_t$), the ratio of quantities sold at sale prices to those at normal prices (χ_t), and the ratio of turnover at sale prices to total turnover in a quarter (%). The figure illustrates that our model successfully explains the increase in the frequency of sales, s_t, in particular its trend. The top-left panel plots the model-based and actual sale frequencies. In terms of the direction of the trend and the size of changes, the model-based sale frequency moves very closely to the actual one. Both series

[23] As for hours worked, we use hours worked multiplied by one minus the unemployment rate to take account of total labor input.

[24] For a detailed investigation of Japan's low productivity growth during the lost decades, see Fukao (2013).

show steady increases in the frequency of sales in the 1990s and 2000s. For the 1980s, for which actual data are missing, our model suggests a slightly decreasing but stable trend in the frequency of sales. Around 2010, both series exhibit a dip. By contrast, the GS model predicts much smaller changes in the frequency of sales, which almost follows a flat line in the graph. However, our model performs as poorly as the GS model in explaining the size of sales discounts, $1 - \mu_t$, and the ratio of quantities sold at sale prices to those at normal prices, χ_t, which are drawn in the top-right and middle-left panels. The simulated values exhibit smaller fluctuations than the actual ones. In sum, these simulation results suggest that our model improves on the GS model in explaining the extensive margin of sales (the frequency of sales) but not the intensive margin (sales discounts). As for the ratio of turnover at sale prices to total turnover, our model performs well. Note that the log-linearized deviation of this variable equals $1/(1 - s + s\chi)s_t + (1 - s)/(1 - s + s\chi)\chi_t$. Although our model fails to explain χ_t, the fact that it explains s_t means that our model explains the sale turnover ratio well.

Our model also shows changes over time in the fraction of loyal customers, which is unobservable. The bottom-left panel indicates that the fraction of loyal customers remained almost constant in the 1980s. However, during the lost decades of the 1990s and 2000s, it then exhibits a downward trend. Put differently, the fraction of bargain hunters increased. Obviously, in the GS model, the fraction remains constant. The rise in the fraction of bargain hunters is consistent with the actual observations shown in Table 1 and Figure 3. Because the price elasticity of loyal customers can be assumed to be lower than that of bargain hunters, the rise in the price elasticity is consistent with the rise in the fraction of bargain hunters. In the graph, the fraction of bargain hunters started to rise in the mid-1990s, which coincides with the increase from the mid-1990s shown in Figure 3.[25]

5.3 Robustness

In the simulation above, we assumed that developments in business cycles were driven by technology shocks. However, this assumption may not necessarily be correct. Other types of shocks may better account for the actual decline in hours worked.

Therefore, to check the robustness of our results, we consider two other types of shocks: a government expenditure shock and a labor supply shock. First, the government expenditure shock, in part, captures the idea that the decline in hours worked was brought about by decreasing government expenditure and thus a decline in labor demand. Or if the statutory

[25] As for other macroeconomic variables, Appendix C reports the simulated paths of the inflation rate and the GDP growth rate. Unfortunately, the performance of our model is poor, probably because it does not incorporate several important features such as capital investment with adjustment cost and the presence of the zero lower bound.

decline in hours worked decreased firms' profits and/or households' labor income, the government may have compensated them through an increase in government spending. Furthermore, if Japan's lost decades reflect insufficient demand rather than a lack of technological innovation,[26] it would be more appropriate to focus on demand shocks than technology shocks. The government expenditure shock in our model is one way to incorporate such a demand shock, since it appears on the demand side of the goods resource constraint (3.10) and introduces fluctuations in goods demand. Based on these considerations, we conduct a simulation using government expenditure shocks. We find that the simulation results are not very different from those using technology shocks.[27]

Second, we conduct a similar exercise assuming that actual changes in hours worked are the result of labor supply shocks. Assuming a labor supply shock, the simulation results show that our model predicts movements opposite to the ones actually observed. That is, the frequency of sales falls during the lost decades, while the fraction of loyal customers rises. The reason for this can be seen from Figure 12. A decline in hours worked is accompanied by a rise in the fraction of loyal customers, which lowers the frequency of sales.

Finally, when all three shocks are incorporated into the model, we find that the simulation results are almost the same as those shown in Figure 13 when only technology shocks are considered.

6 Concluding Remarks

In this study, we first examined empirically whether there is a link between the frequency of sales and changes in macroeconomic developments. Japanese supermarket scanner data covering a period of two decades show that there is a significant negative correlation between the frequency of sales and hours worked. Moreover, our VAR model suggests that the frequency of sales and hours worked move in opposite directions in response to technology shocks, producing a negative correlation between the two. Second, we examined the real effect of monetary policy by constructing a DSGE model with temporary sales and endogenous bargain hunting. Because sale prices are frequently revised and endogenous bargain hunting increases the strategic substitutability of sales, the real effect of monetary policy shocks weakens by around 40%, although monetary policy still matters for the real side of the economy. The model also showed

[26]Sugo and Ueda (2008) estimate a sticky-price DSGE model and find that one of the main driving forces of business cycles in Japan was investment adjustment cost shocks. Bayoumi (2001) and Caballero, Hoshi, and Kashyap (2008) argue that financial shocks were the cause of Japan's lost decade(s). Although investment adjustment cost and financial shocks do not necessarily represent demand shocks, these studies suggest that Japan's business cycles are not the result of technology shocks only.

[27]Results not shown to conserve space.

that the decline in hours worked during Japan's lost decades accounts for the actual rises in the frequency of sales and implies that the fraction of price-sensitive bargain hunters has increased.

There are several potential avenues for future research. The first would be to present further qualitative and quantitative evidence for endogenous bargain hunting. In particular, more empirical research using micro data on how individuals optimize time spent bargain hunting and working is necessary. Second, our model needs to be improved to account not just for the extensive margin (the frequency of sales) but also the intensive margin (the size of sales discounts).

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Table 1: Time Spent Shopping (minutes per day)

	Men		Women	
	Working	Not working	Working	Not working
1986	6	9	27	37
1991	9	12	30	38
1996	11	15	30	39
2001	13	18	31	39
2006	14	20	31	39
2006	15	22	32	40

Source: Statistics Bureau, *Survey on Time Use and Leisure Activities*.

Table 2: Time Spent Working (including commuting time, minutes per day)

	Men		Women	
	Working	Not working	Working	Not working
1986	493	-	371	-
1991	481	-	358	-
1996	469	-	345	-
2001	456	-	324	-
2006	470	-	335	-
2011	466	-	326	-

Source: Statistics Bureau, *Survey on Time Use and Leisure Activities*.

Table 3: Model Parameters

Parameters		
β	Discount factor	0.9975
θ_c	Elasticity of consumption	0.333
θ_h	Elasticity of labor supply	0.7
ς	Elasticity btw differentiated labor	20
α	Elasticity of output to hours	0.667
γ	Elasticity of marginal cost	0.5
ϕ_p	Calvo price stickiness	0.889
ϕ_w	Calvo wage stickiness	0.889
ρ	Monetary policy rule inertia	0.8
ϕ_π	Monetary policy response to inflation	1.5

Parameters		
ρ_a	Technology shock	0.85
ρ_g	Government expenditure shock	0.85
ρ_h	Labor supply shock	0.85

Table 4: Parameters Related to Sales

Target variables		
μ	Ratio of sale prices to normal prices	0.83
χ	Ratio of sale quantity to normal quantity	1.7
s	Frequency of sales	0.18
Calibrated parameters		
ϵ	Elasticity btw product types	1.8969
η	Elasticity btw brands	8.5529
λ	Fraction of loyal customers	0.8877

Calibrated parameters		
ϕ_L	Utility weight on loyal customers	$1.3\,10^{-4}$ $\theta_L = 100$ 0.0033 $\theta_L = 4$

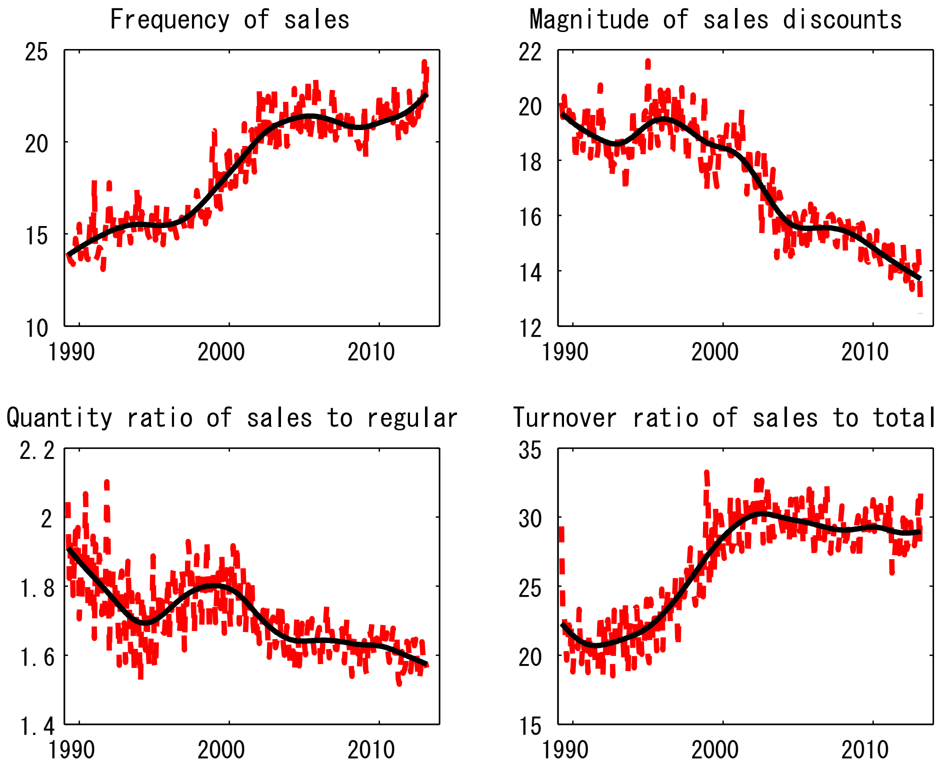

Figure 1: Variables Associated with Temporary Sales

Note: The red dashed lines represent the original series, while the black solid lines show smoothed series using the HP filter with $\lambda = 14,400$.

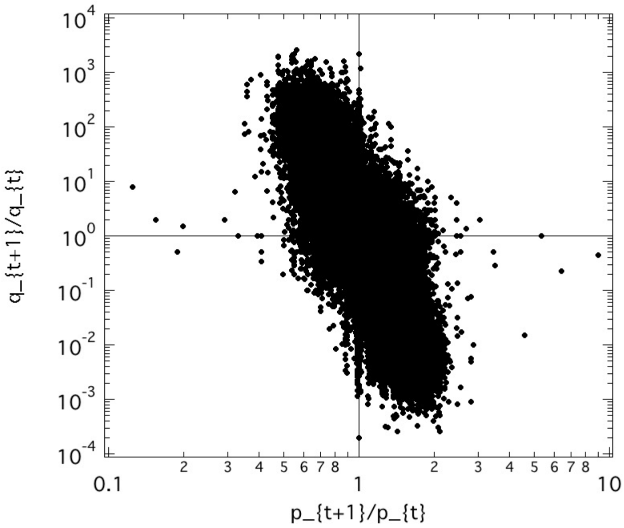

Figure 2: Price Changes vis-à-vis Quantity Changes for Cup Noodles of a Specific Type

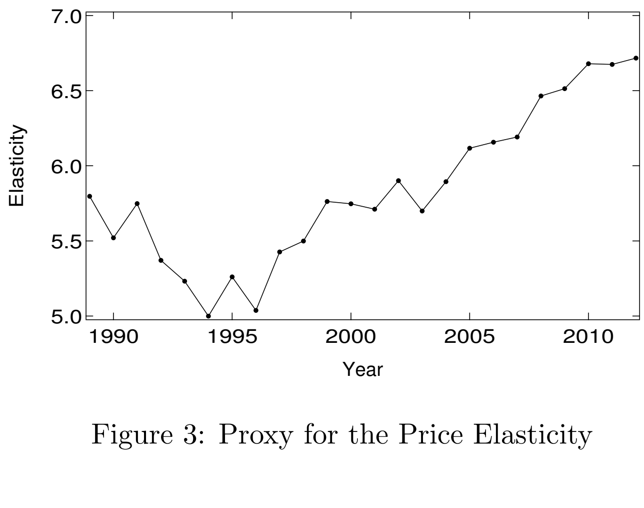

Figure 3: Proxy for the Price Elasticity

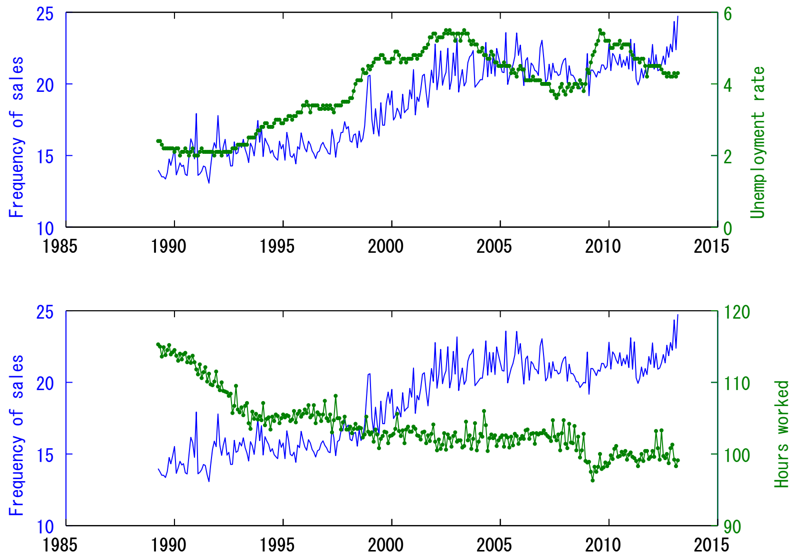

Figure 4: Frequency of Temporary Sales and Labor Market Indicators

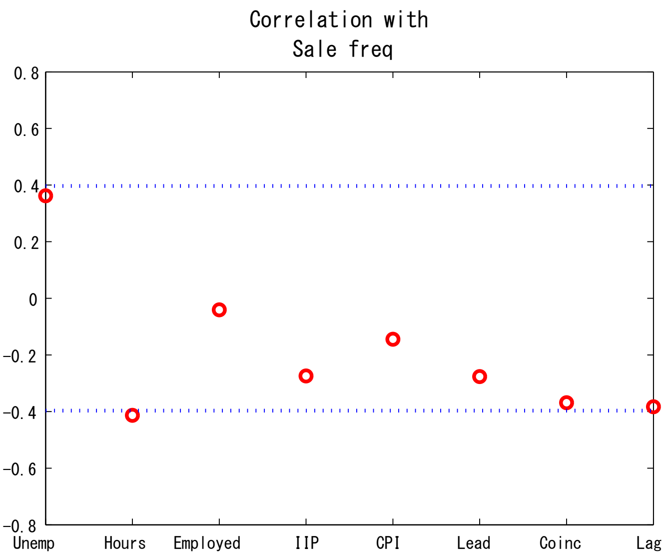

Figure 5: Correlation between Price Components and the Macroeconomy

Note: The blue dashed lines represent the 95% confidence interval for two uncorrelated band-pass filtered series calculated using Monte Carlo simulation.

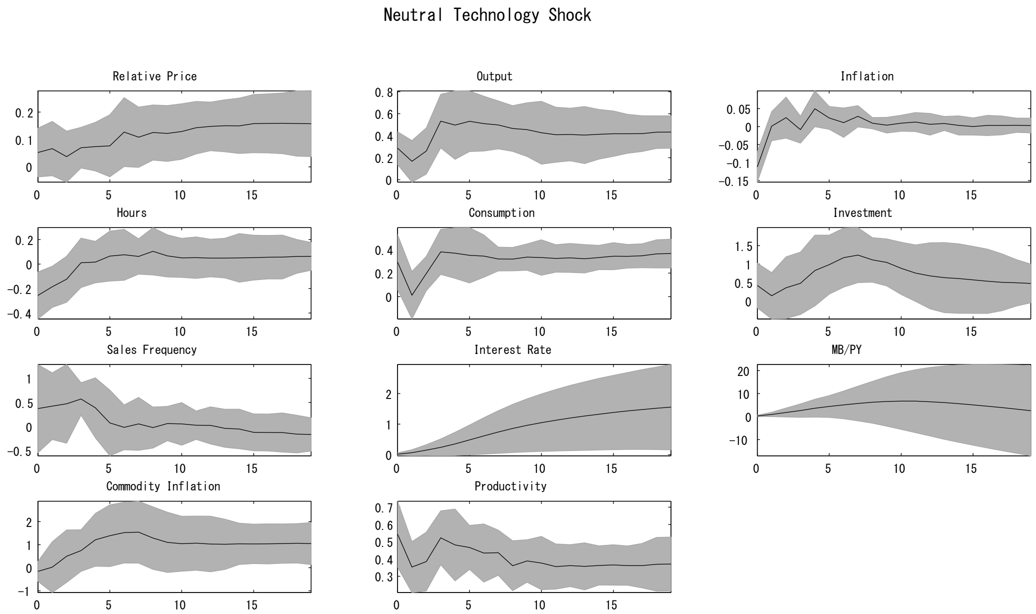

Figure 6: Response to a Neutral Technology Shock

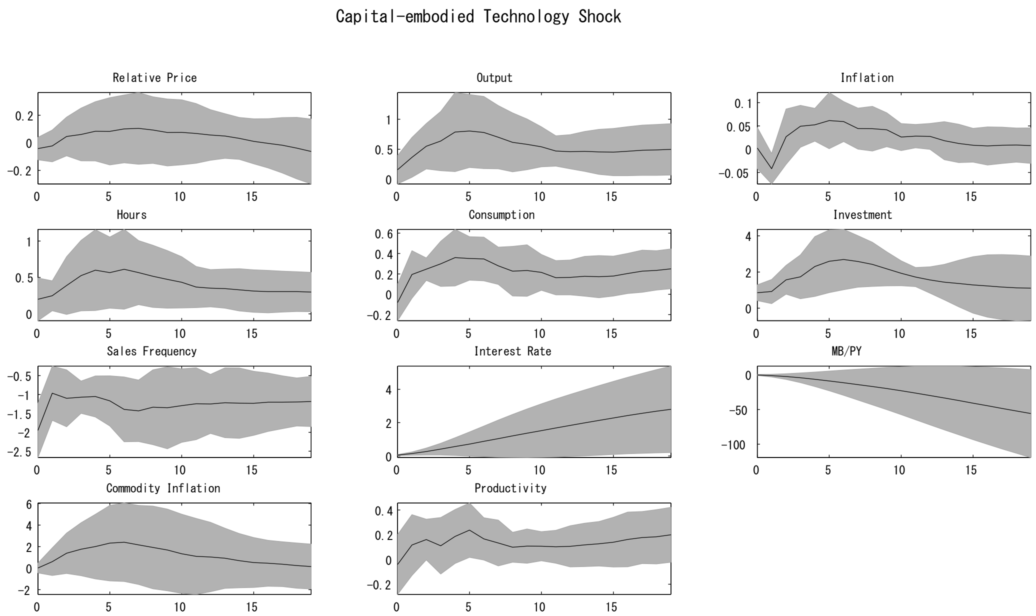

Figure 7: Response to a Capital-embodied Technology Shock

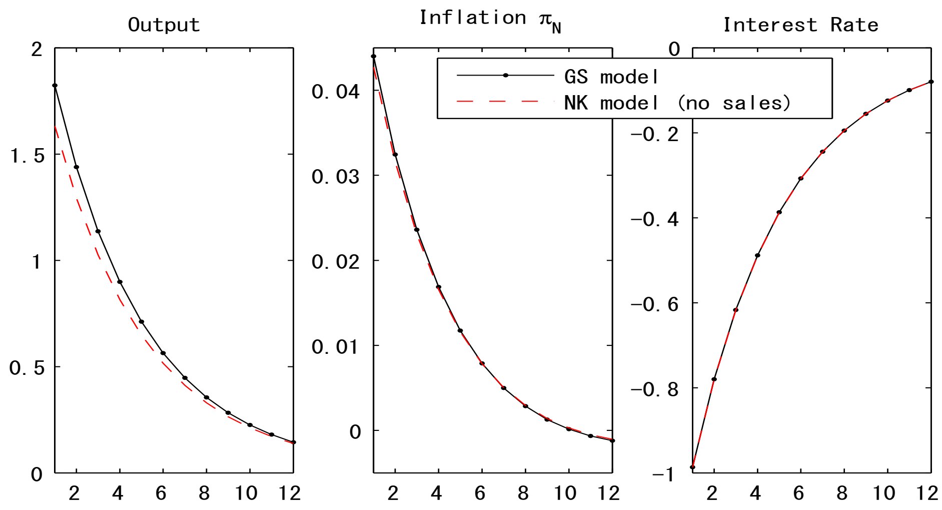

Figure 8: IRFs in the GS Model and the Standard NK Model

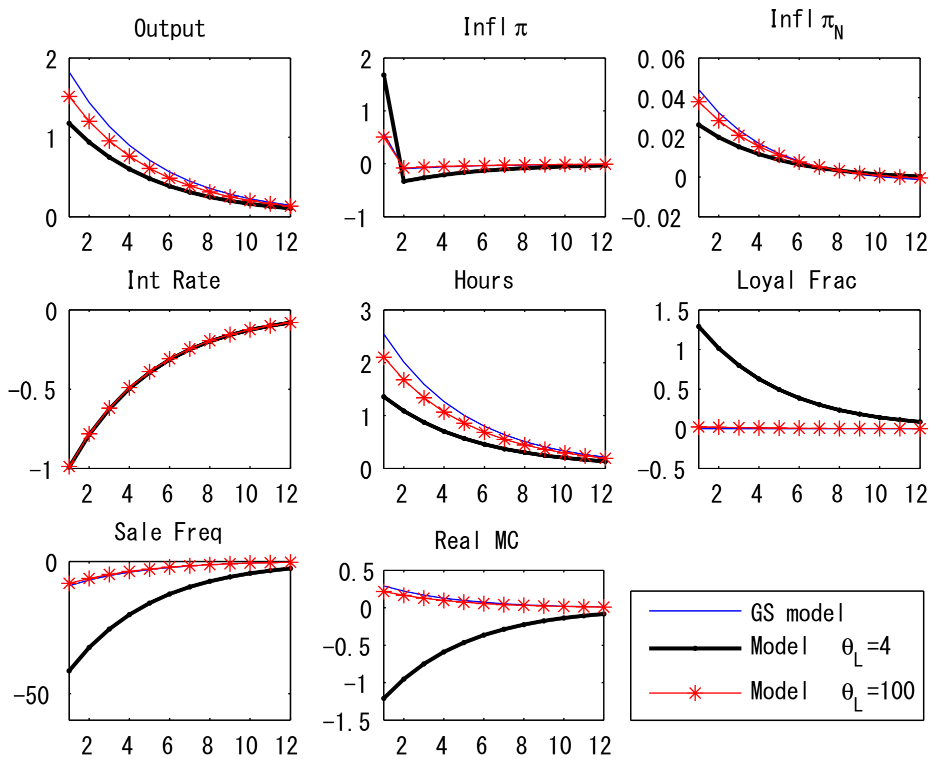

Figure 9: IRFs for an Accommodative Monetary Policy Shock

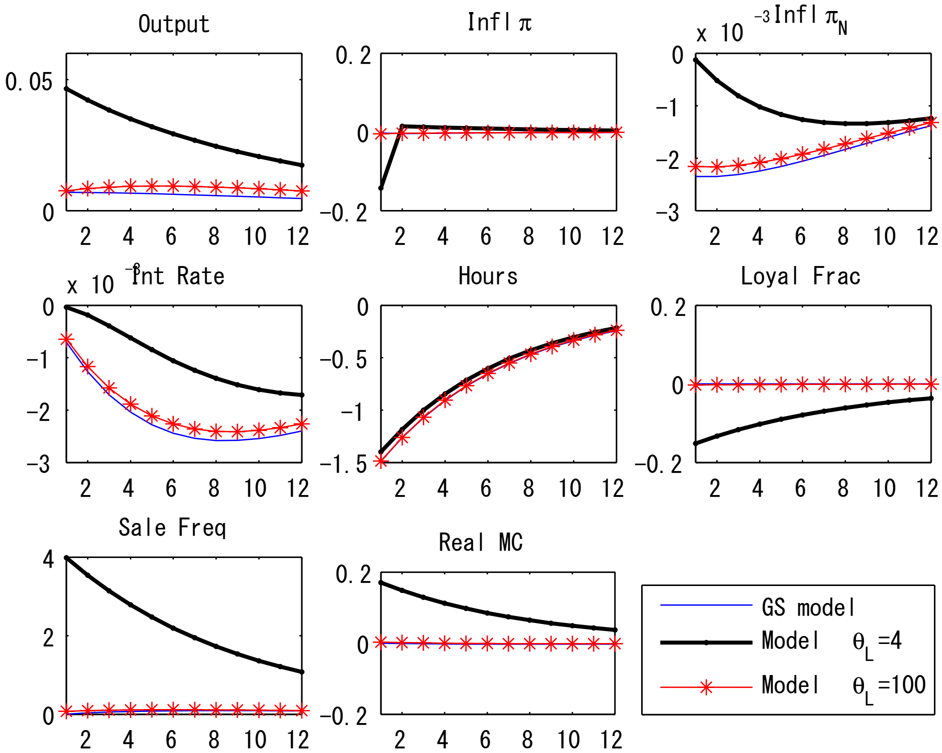

Figure 10: IRFs for a Positive Technology Shock

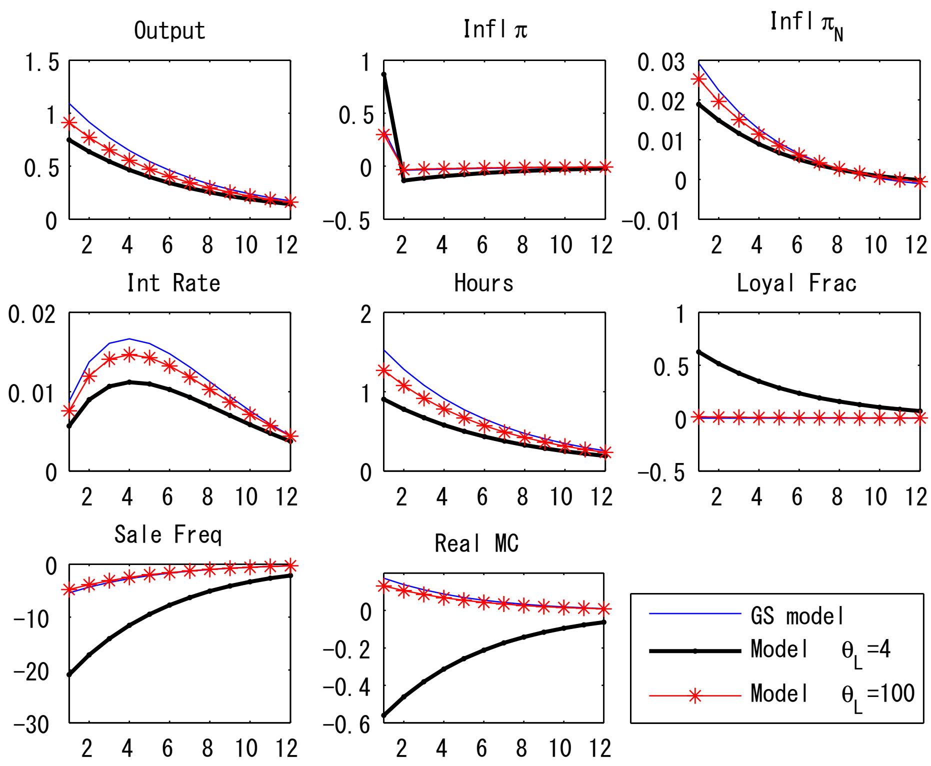

Figure 11: IRFs for a Government Expenditure Shock

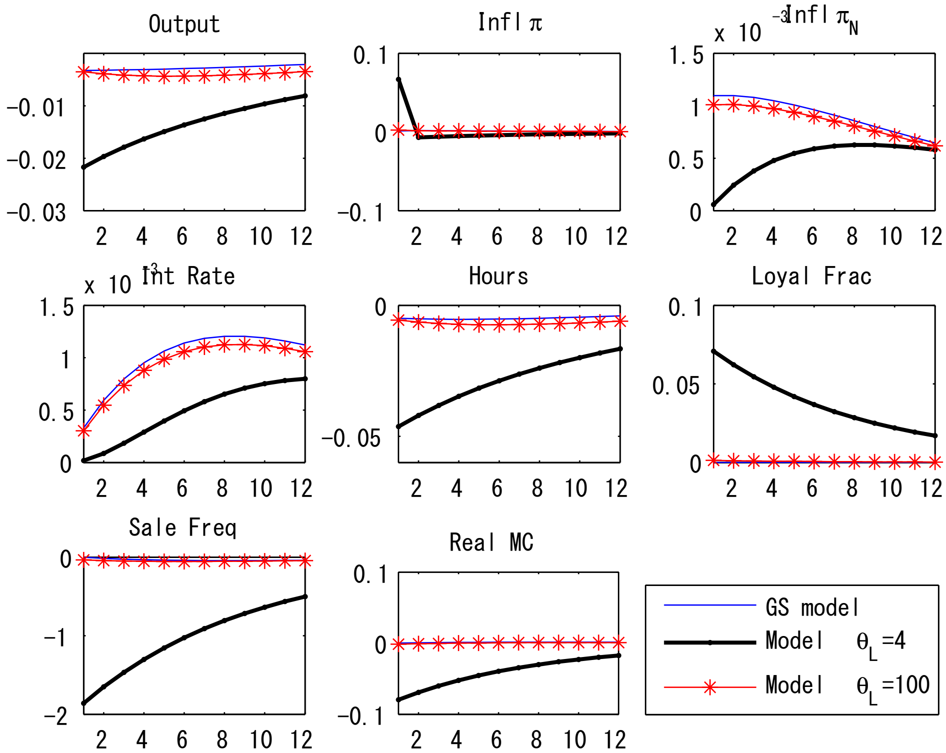

Figure 12: IRFs for a Labor Supply Shock

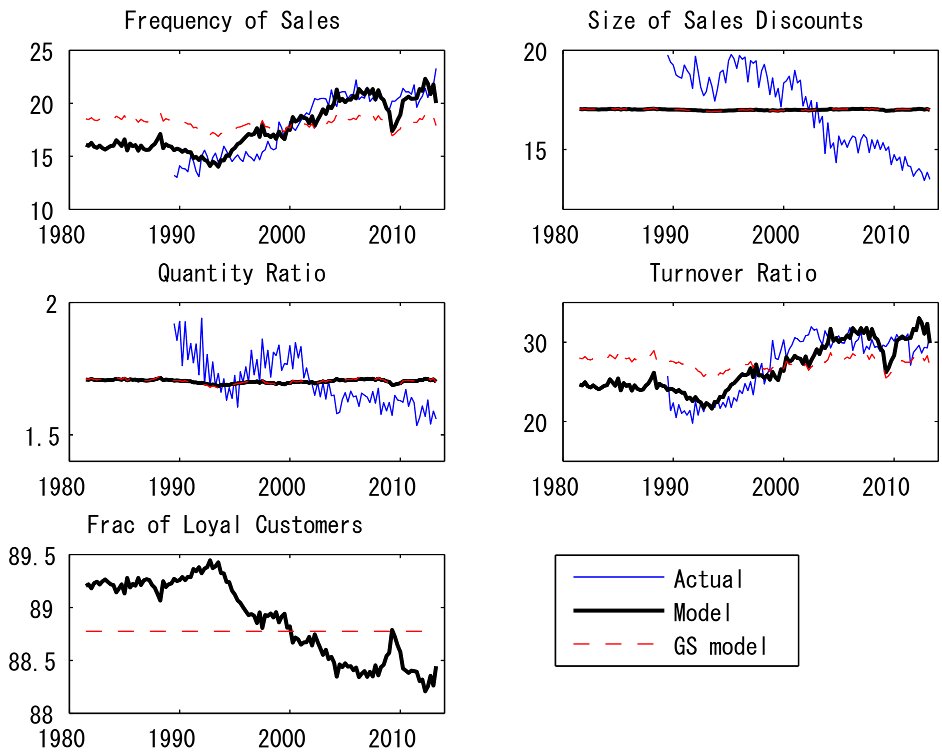

Figure 13: Model and Actual Paths of Sales-Related Variables

Appendix for "Working Less and Bargain Hunting More: Macro Implications of Sales during Japan's Lost Decades"

A Impulse Responses for a Monetary Policy Shock Using a VAR Model

In Section 2, we estimated a VAR model and presented impulse responses for two types of technology shocks. In this appendix, we show impulse responses for a monetary policy shock. To this end, we need to identify monetary policy shocks, but the zero lower bound on nominal interest rates presents a major challenge. The overnight call rate has been below 0.25% since 1995, meaning that except for the first few years, it was almost zero for most of our observation period. This makes it difficult to identify monetary policy shocks.

To overcome the challenges posed by the zero lower bound, we take various steps. The first is that we use short-term interest rates on loans as the interest rate in the VAR model.[1] The reason for using short-term interest rates on loans is that loan rates are less constrained by the zero bound because they contain a premium. Second, in addition to loan rates, we use the monetary base, since this also contains information on the stance of monetary policy. However, even using both loan rates and the monetary base, identifying monetary policy shocks remains difficult.[2] We therefore compare the impulse responses of the main macroeconomic variables using the shocks identified from the residuals of the equation corresponding to either loan rates or the monetary base and examine which responses are more consistent with those reported in previous empirical studies for the United States. We find that the monetary base is better for identifying monetary policy shocks than loan rates.

To identify monetary policy shocks, we use the following recursive restrictions. Monetary policy shocks have a contemporaneous impact on the ratio of the monetary base to nominal GDP as well as the commodity price index but do not have a contemporaneous impact on the other variables.

[1] The data are taken from the Bank of Japan.

[2] There are a number of studies that have sought to estimate a VAR model for Japan, such as Bayoumi (2001), Miyao (2002), Fujiwara (2006), and Iwata (2010), but no clear consensus has been reached so far on how to best identify monetary policy shocks.

Figure A shows the impulse responses of aggregate quantities and prices including the frequency of sales for a monetary policy shock. The gray areas indicate 95% confidence intervals. The economic impact of monetary policy is not highly significant, although the directions of changes are in line with those reported for the United States such that a reduction in the monetary base leads to an economic contraction. There is no significant change in the frequency of sales.

B Model Details

B.1 Deriving the Demand Function

In this subsection, we show that the demand function of the representative household can be derived as the outcome of an expenditure minimization problem. For simplicity, we omit the time subscript t.

Assumption 1:

$$\int_{\Lambda \in T} c(\tau, B)^{\frac{\epsilon-1}{\epsilon}} d\tau \geq \bar{C}^{\frac{\epsilon-1}{\epsilon}}, \tag{B.1}$$

where

$$\bar{C} = \left(\frac{\left(\int_{\Lambda \in T} p(\tau, B)^{1-\epsilon} d\tau\right)^{\frac{1}{1-\epsilon}}}{P} \right)^{-\epsilon} C^* \tag{B.2}$$

and Λ represents a set of product types where consumers in the household are loyal.

Assumption 2:

$$c(\tau, b)^{-\frac{1}{\eta}} \ll c(\tau, b)^{-\frac{1}{\epsilon}}. \tag{B.3}$$

Assumption 1 states that if the representative household assigns to a household member the role of loyal customer, the household member has to spend at least a certain amount of their expenditure on Λ-type products. Assumption 2 is not a strict condition, because $\eta > \epsilon$.

The expenditure minimization problem is given by

$$\min \int_T \left(\int_B p(\tau, b) c(\tau, b) db \right) d\tau, \tag{B.4}$$

subject to the overall aggregator of consumption,

$$C_t \equiv \left[\int_T \left(\int_B c_t(\tau, b)^{\frac{\eta-1}{\eta}} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}} \tag{B.5}$$

and (B.1). We will show that its solution is derived as

$$c_t(\tau,b)=\begin{cases}\left(\frac{p_t(\tau,b)}{p_{B,t}(\tau)}\right)^{-\eta}\left(\frac{p_{B,t}(\tau)}{P_t}\right)^{-\epsilon}C_t^* & \text{for the fraction of household, } 1-L_t\\ \left(\frac{p_t(\tau,b)}{P_t}\right)^{-\epsilon}C_t^* & \text{for the fraction of household, } L_t\end{cases}\tag{B.6}$$

The first-order conditions with respect to $c(\tau,b)$ are

$$\begin{aligned}0=&\ p(\tau,b)-\Phi c(\tau,b)^{-\frac{1}{\eta}}\left(\int_B c(\tau,b)^{\frac{\eta-1}{\eta}}db\right)^{\frac{\epsilon-\eta}{\epsilon(\eta-1)}}\\ &-\Psi c_t(\tau,b)^{-\frac{1}{\epsilon}}\end{aligned}\tag{B.7}$$

for $\tau\in\Lambda$ and

$$0=p(\tau,b)-\Phi c(\tau,b)^{-\frac{1}{\eta}}\left(\int_B c(\tau,b)^{\frac{\eta-1}{\eta}}db\right)^{\frac{\epsilon-\eta}{\epsilon(\eta-1)}}\tag{B.8}$$

for $\tau\in T\backslash\Lambda$, where Φ and Ψ are the Lagrange multipliers for equations (B.5) and (B.1).

By multiplying $c(\tau,b)$, taking the integral with respect to τ and b, and summing equations (B.7) and (B.8), we obtain

$$PC^*=\Phi C^{\frac{\epsilon-1}{\epsilon}}-\Psi\bar{C}^{\frac{\epsilon-1}{\epsilon}}.\tag{B.9}$$

Inserting the above Ψ into equation (B.7) and using Assumption 2 yields

$$c(\tau,b)=p(\tau,b)^{-\epsilon}\left(PC-\Phi C^{\frac{\epsilon-1}{\epsilon}}\right)^{\epsilon}\bar{C}^{1-\epsilon}.\tag{B.10}$$

Inserting this into equation (B.1) yields

$$PC-\Phi C^{\frac{\epsilon-1}{\epsilon}}=\bar{C}\left(\int_{\Lambda\in T}p(\tau,B)^{1-\epsilon}d\tau\right)^{\frac{1}{1-\epsilon}}.\tag{B.11}$$

Inserting this equation into (B.10) and using (B.2), we obtain the demand function

$$c(\tau,b)=\left(\frac{p(\tau,b)}{P}\right)^{-\epsilon}C^*\tag{B.12}$$

for $\tau\in\Lambda$.

Next, we consider $\tau\in T\backslash\Lambda$. From (B.8) and (B.11), we have

$$p(\tau,b)=\left\{PC-\int_{\Lambda\in T}p(\tau,B)^{1-\epsilon}d\tau\cdot P^{\epsilon}C^*\right\}C^{\frac{\epsilon-1}{\epsilon}}c(\tau,b)^{-\frac{1}{\eta}}\left(\int_B c(\tau,b)^{\frac{\eta-1}{\eta}}db\right)^{\frac{\epsilon-\eta}{\epsilon(\eta-1)}}.\tag{B.13}$$

Taking the power of $1-\eta$ and the integral with respect to b yields

$$P_B(\tau)^{1-\eta} = \left\{PC - \int_{\Lambda \in T} p(\tau, B)^{1-\epsilon} d\tau \cdot P^\epsilon C^*\right\}^{1-\eta} C^{\frac{\epsilon-1}{\epsilon}(1-\eta)} \left(\int_B c(\tau, b)^{\frac{\eta-1}{\eta}} db\right)^{\frac{\eta}{\epsilon}}. \tag{B.14}$$

Inserting the above $\left(\int_B c(\tau, b)^{\frac{\eta-1}{\eta}} db\right)$ into equation (B.13) yields

$$c(\tau, b) = \left(\frac{p(\tau, b)}{p_B(\tau)}\right)^{-\eta} \left(\frac{p_B(\tau)}{P}\right)^{-\epsilon} C \left\{\frac{PC - \int_{\Lambda \in T} p(\tau, B)^{1-\epsilon} d\tau \cdot P^\epsilon C^*}{PC}\right\}^\epsilon. \tag{B.15}$$

Before we simplify the right-hand side of equation (B.15), let us note that

$$\begin{aligned} PC^* &= \int_T \left(\int_B p(\tau, b) c(\tau, b) db\right) d\tau \\ &= \int_\Lambda \left(\int_B p(\tau, b) c(\tau, b) db\right) d\tau + \int_{\backslash \Lambda} \left(\int_B p(\tau, b) c(\tau, b) db\right) d\tau. \end{aligned} \tag{B.16}$$

Inserting equation (B.12) and (B.15) into this, we have

$$\begin{aligned} PC^* &= \int_\Lambda \left(\int_B p(\tau, b) c(\tau, b) db\right) d\tau + \int_{\backslash \Lambda} \left(\int_B p(\tau, b) c(\tau, b) db\right) d\tau \\ &= \int_\Lambda p(\tau, B)^{1-\epsilon} d\tau \left(\frac{1}{P}\right)^{-\epsilon} C^* \\ &+ \int_{\backslash \Lambda} \int_B p(\tau, b)^{1-\eta} db \cdot p_B(\tau)^{\eta-\epsilon} d\tau \left(\frac{1}{P}\right)^{-\epsilon} C \left\{\frac{PC - \int_{\Lambda \in T} p(\tau, B)^{1-\epsilon} d\tau \cdot P^\epsilon C^*}{PC}\right\}^\epsilon \\ &= \int_\Lambda p(\tau, B)^{1-\epsilon} d\tau \left(\frac{1}{P}\right)^{-\epsilon} C^* \\ &+ \left(P^{1-\epsilon} - \int_\Lambda p(\tau, b)^{1-\epsilon} d\tau\right) \left(\frac{1}{P}\right)^{-\epsilon} C \left\{\frac{PC - \int_{\Lambda \in T} p(\tau, B)^{1-\epsilon} d\tau \cdot P^\epsilon C^*}{PC}\right\}^\epsilon. \end{aligned}$$

Thus, we obtain

$$C^* = C \left\{\frac{PC - \int_{\Lambda \in T} p(\tau, B)^{1-\epsilon} d\tau \cdot P^\epsilon C^*}{PC}\right\}^\epsilon. \tag{B.17}$$

Inserting this into equation (B.15), we obtain the demand function

$$c(\tau, b) = \left(\frac{p(\tau, b)}{p_B(\tau)}\right)^{-\eta} \left(\frac{p_B(\tau)}{P}\right)^{-\epsilon} C^* \tag{B.18}$$

for $\tau \in T \backslash \Lambda$.

B.2 The Household

The representative household has the following lifetime utility function:

$$U_t = \sum_{j=0}^{\infty} \beta^j E_t \left[v(C_{t+j}) - Z_{t+j}^h v \left(H_{t+j} + \phi_L H \frac{(1 - L_{t+j})^{\theta_L}}{(1-\lambda)^{\theta_L}} \right) \right], \tag{B.19}$$

where

$$C_t = \left[\int_T \left(\int_B c_t(\tau, b)^{\frac{\eta-1}{\eta}} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}} \tag{B.20}$$

and Z_t^h represents a stochastic shock to the utility weight of labor supply with unit mean, and the logarithm deviation of the stochastic shock is denoted by ε_t^h. For simplicity, we do not show the time subscript t below.

As shown in Appendix B.1, given equations (B.1) and (B.3), we obtain demand functions of the same form as GS's specification [7]:

$$c(\tau, b) = \begin{cases} \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\eta} \left(\frac{p_B(\tau)}{P}\right)^{-\epsilon} C^* & \text{for the fraction of household, } 1 - L_t \\ \left(\frac{p(\tau,b)}{P}\right)^{-\epsilon} C^* & \text{for the fraction of household, } L_t \end{cases} \tag{B.21}$$

Substituting this into equation (B.20) yields

$$\begin{aligned} C &= \left[\int_T \left(\int_B c(\tau, b)^{\frac{\eta-1}{\eta}} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}} \\ &= \left[\int_T \left(\begin{array}{c} L \int_B \left(\frac{p(\tau,b)}{P}\right)^{-\epsilon \frac{\eta-1}{\eta}} C^{*\frac{\eta-1}{\eta}} db \\ +(1-L) \int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\eta \frac{\eta-1}{\eta}} \left(\frac{p_B(\tau)}{P}\right)^{-\epsilon \frac{\eta-1}{\eta}} C^{*\frac{\eta-1}{\eta}} db \end{array} \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}} \\ &= \left[\begin{array}{c} \int_T \left(L \int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\epsilon \frac{\eta-1}{\eta}} db + (1-L) \int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{1-\eta} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} \\ \left(\frac{p_B(\tau)}{P}\right)^{1-\epsilon} d\tau \end{array} \right]^{\frac{\epsilon}{\epsilon-1}} C^* \end{aligned} \tag{B.22}$$

The price index for bargain hunters, $p_B(\tau)$, is given by

$$p_B(\tau) = \left(\int_B p(\tau, b)^{1-\eta} db \right)^{\frac{1}{1-\eta}}. \tag{B.23}$$

As in equation [20] in GS, given that a fraction s of all prices are at the sale price P_S and the remaining $1-s$ are at the normal price P_N, we obtain

$$P_B = p_B(\tau) = \left(sP_S^{1-\eta} + (1-s)P_N^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{B.24}$$

Note that the above equation holds true in a flexible price model and a Rotemberg-type sticky price model, in which the normal price P_N is the same across product types τ. On the other hand, in a Calvo-type sticky price model, P_N differs across product types τ, so the above equation does not hold exactly. However, given that that equation is in a log-linearized form, it holds. As is shown in equation [E.6] in GS, P_N needs to be defined as

$$P_{N,t} = (1-\phi_p)\sum_{j=0}^{\infty}\phi_p^j R_{N,t-j}, \tag{B.25}$$

where $R_{N,t-j}$ is the new normal price set at $t-j$.

The terms in equation (B.22) can be simplified as follows:

$$\begin{aligned}\int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{1-\eta} db &= \frac{sP_S^{1-\eta} + (1-s)P_N^{1-\eta}}{P_B^{1-\eta}} \\ &= 1, \end{aligned} \tag{B.26}$$

$$\begin{aligned}\int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\epsilon\frac{\eta-1}{\eta}} db &= \frac{sP_S^{-\epsilon\frac{\eta-1}{\eta}} + (1-s)P_N^{-\epsilon\frac{\eta-1}{\eta}}}{P_B^{-\epsilon\frac{\eta-1}{\eta}}} \\ &= \frac{sP_S^{\epsilon\frac{1-\eta}{\eta}} + (1-s)P_N^{\epsilon\frac{1-\eta}{\eta}}}{\left(sP_S^{1-\eta} + (1-s)P_N^{1-\eta}\right)^{\frac{\epsilon}{\eta}}} \\ &= \frac{s\left(\frac{P_S}{P_N}\right)^{\epsilon\frac{1-\eta}{\eta}} + (1-s)}{\left(s\left(\frac{P_S}{P_N}\right)^{1-\eta} + (1-s)\right)^{\frac{\epsilon}{\eta}}}, \\ &= \frac{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)}{(s\mu^{1-\eta} + (1-s))^{\frac{\epsilon}{\eta}}}, \end{aligned} \tag{B.27}$$

where the ratio of the sale price to the nominal price is defined as

$$\mu \equiv \frac{P_S}{P_N}. \tag{B.28}$$

Equation (B.22) thus becomes

$$
\begin{aligned}
C &= \left[\left(L \frac{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)}{\left(s\mu^{1-\eta} + (1-s)\right)^{\frac{\epsilon}{\eta}}} + (1-L) \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} \left(\frac{P_B}{P}\right)^{1-\epsilon} \right]^{\frac{\epsilon}{\epsilon-1}} C^* \\
&= \left(L \frac{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)}{\left(s\mu^{1-\eta} + (1-s)\right)^{\frac{\epsilon}{\eta}}} + (1-L) \right)^{\frac{\eta}{\eta-1}} \left(\frac{P_B}{P}\right)^{-\epsilon} C^* \\
&\equiv \digamma \cdot \left(\frac{P_B}{P}\right)^{-\epsilon} C^*, \qquad \text{(B.29)}
\end{aligned}
$$

where the consumption wedge $\digamma$ is defined by

$$
\digamma = \left(L\Xi + (1-L)\right)^{\frac{\eta}{\eta-1}} \qquad \text{(B.30)}
$$

with

$$
\Xi \equiv \frac{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)}{\left(s\mu^{1-\eta} + (1-s)\right)^{\frac{\epsilon}{\eta}}}. \qquad \text{(B.31)}
$$

Suppose $\mu < 1$, $\epsilon > 1$, $\eta > 1$, and $\epsilon/\eta < 1$. We then obtain

$$
\Xi < 1, \qquad \text{(B.32)}
$$

because $f(x) = x^{\epsilon/\eta}$ is a concave increasing function, and the denominator and numerator of Ξ are the weighted average of 1 and $\mu^{1-\eta}(> 1)$. Therefore, the consumption wedge satisfies $\digamma < 1$. This means that, because the goods demand of some members of the household is suboptimal, the household's consumption decreases. The wedge decreases with L, since the first differential $d\digamma/dL$ is given by

$$
\begin{aligned}
\frac{d\digamma}{dL} &= \frac{\eta}{\eta-1} \left(L\Xi + (1-L)\right)^{\frac{\eta}{\eta-1}-1} (\Xi - 1) \\
&= -\frac{\eta}{\eta-1} \left(L\Xi + (1-L)\right)^{\frac{1}{\eta-1}} (1 - \Xi) \\
&< 0. \qquad \text{(B.33)}
\end{aligned}
$$

If all the members of the household engage in bargain hunting, that is, $L = 0$, then we have $\digamma = 1$. The household enjoys higher utility from consumption. However, the trade-off is that the household will spend more time bargain hunting, lowering utility.

The aggregate price index P satisfies

$$\begin{aligned}
PC^* &= \int_T \int_B p(\tau,b)c(\tau,b)dbd\tau \\
&= \int_T \left\{ L\int_B p(\tau,b)\left(\frac{p(\tau,b)}{P}\right)^{-\epsilon} C^* db \right. \\
&\left. +(1-L)\int_B p(\tau,b)\left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\eta}\left(\frac{p_B(\tau)}{P}\right)^{-\epsilon} C^* db \right\} d\tau,
\end{aligned} \tag{B.34}$$

which leads to

$$\begin{aligned}
P &= \left[\left(\int_T \begin{array}{c} L\int_B p(\tau,b)^{1-\epsilon}db \\ +(1-L)\int_B p(\tau,b)^{1-\eta}db\,(p_B(\tau))^{\eta-\epsilon} \end{array}\right) d\tau\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\int_T \left(L\int_B p(\tau,b)^{1-\epsilon}db + (1-L)P_B^{1-\eta}P_B^{\eta-\epsilon}\right) d\tau\right]^{\frac{1}{1-\epsilon}} \\
&= \left[L\{sP_S{}^{1-\epsilon} + (1-s)P_N^{1-\epsilon}\} + (1-L)P_B^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.
\end{aligned} \tag{B.35}$$

Substituting this P into equation (B.29), we obtain

$$\begin{aligned}
\left(\frac{P_B}{P}\right)^{-\epsilon} &= \frac{P_B^{-\epsilon}}{\left(L\{sP_S{}^{1-\epsilon} + (1-s)P_N^{1-\epsilon}\} + (1-L)P_B^{1-\epsilon}\right)^{-\frac{\epsilon}{1-\epsilon}}} \\
&= \left(\frac{L\{sP_S{}^{1-\epsilon} + (1-s)P_N^{1-\epsilon}\} + (1-L)P_B^{1-\epsilon}}{P_B^{1-\epsilon}}\right)^{\frac{\epsilon}{1-\epsilon}} \\
&= \left(L\frac{sP_S{}^{1-\epsilon} + (1-s)P_N^{1-\epsilon}}{\left(sP_S^{1-\eta} + (1-s)P_N^{1-\eta}\right)^{\frac{1-\epsilon}{1-\eta}}} + (1-L)\right)^{\frac{\epsilon}{1-\epsilon}} \\
&= \left(L\frac{s\mu^{1-\epsilon} + (1-s)}{(s\mu^{1-\eta} + (1-s))^{\frac{\epsilon-1}{\eta-1}}} + (1-L)\right)^{-\frac{\epsilon}{\epsilon-1}}.
\end{aligned} \tag{B.36}$$

The right-hand side of the equation is larger than one and decreases with $1-L$. As the fraction of bargain hunters $(1-L)$ increases, the weight of bargain price index P_B increases, and the aggregate price index P declines. Thus, the relative bargain price index to the aggregate price index increases.

The household's budget constraint is given by

$$P_tC_t^* + E_t[Q_{t+1|t}A_{t+1}] = W_tH_t + D_t + A_t. \tag{B.37}$$

The household takes P_B/P as given. The first-order conditions are written as follows: with respect to C,

$$\beta E_t\left[\frac{Z^b_{t+1}}{Z^b_t}\frac{\upsilon_C(C_{t+1})}{\upsilon_C(C_t)}\frac{P_t}{P_{t+1}}\frac{\digamma_{t+1}}{\digamma_t}\left(\frac{P_{B,t+1}/P_{B,t}}{P_{t+1}/P_t}\right)^{-\epsilon}\right] = E_t\left[Q_{t+1|t}\right] = \frac{1}{1+i_t}; \tag{B.38}$$

with respect to H,

$$\frac{Z^h_t\upsilon_H\left(H_t+\phi_L H\frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}}\right)}{\upsilon_C(C_t)} = \frac{W_t}{P_t}\digamma_t\left(\frac{P_{B,t}}{P_t}\right)^{-\epsilon}; \tag{B.39}$$

and with respect to L,

$$\theta_L\phi_L H\frac{(1-L_t)^{\theta_L-1}}{(1-\lambda)^{\theta_L}}\frac{Z^h_t\upsilon_H\left(H_t+\phi_L H\frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}}\right)}{\upsilon_C(C_t)} = -\frac{C_t}{\digamma_t}\frac{d\digamma_t}{dL_t}. \tag{B.40}$$

The last equation can be rearranged as follows:

$$\begin{aligned}
&\theta_L\phi_L H\frac{(1-L_t)^{\theta_L-1}}{(1-\lambda)^{\theta_L}}\frac{Z^h_t\upsilon_H\left(H_t+\phi_L H\frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}}\right)}{\upsilon_C(C_t)}\\
&= \frac{C_t}{\digamma_t}\frac{\eta}{\eta-1}\left(L_t\Xi_t+(1-L_t)\right)^{\frac{1}{\eta-1}}(1-\Xi_t)\\
&= \frac{\eta}{\eta-1}C_t\frac{\left(L_t\Xi_t+(1-L_t)\right)^{\frac{1}{\eta-1}}(1-\Xi_t)}{\left(L_t\Xi_t+(1-L_t)\right)^{\frac{\eta}{\eta-1}}}\\
&= \frac{\eta}{\eta-1}C_t\frac{1-\Xi_t}{L_t\Xi_t+(1-L_t)}.
\end{aligned} \tag{B.41}$$

B.3 Resource Constraint

The resource constraint is given by

$$Y_t = C^*_t + Z^g_t, \tag{B.42}$$

$$= \frac{C_t}{\digamma_t\cdot\left(\frac{P_B}{P}\right)^{-\epsilon}} + Z^g_t, \tag{B.43}$$

where Z^a_t represents a stochastic shock to productivity. Its mean is zero and the logarithmic deviation from the steady-state Y is denoted by ε^g_t. It is log-linearized as

$$c_t = y_t + f_t - \epsilon\left(p_{B,t}-p_t\right) - \varepsilon^g_t. \tag{B.44}$$

B.4 Monetary Policy

The monetary policy rule is given by

$$i_t = \rho i_{t-1} + (1-\rho)\phi_\pi \pi_t^N + e_t^i, \tag{B.45}$$

where the inflation rate for normal prices is defined by $\pi_{N,t} \equiv p_{N,t} - p_{N,t-1}$ and e_t^i represents a monetary policy shock.

B.5 Firms

This part is related to Theorem 3 in GS. Firms' problem is almost the same as that in GS, because firms face the same demand function (B.21). The share of loyal customers L_t is time-varying, but since each firm takes L_t as given, this does not change firms' optimization problem.

Regarding the demand function at the sale and normal prices, equation [22] in GS becomes

$$Q_S = (L + (1-L)v_S)(P_S/P)^{-\epsilon} Y \tag{B.46}$$

$$Q_N = (L + (1-L)v_N)(P_N/P)^{-\epsilon} Y, \tag{B.47}$$

where v is the purchase multiplier defined in equation [10] in GS:

$$v(p; P_B) = (p/P_B)^{-(\eta-\epsilon)}. \tag{B.48}$$

This represents the ratio of quantities sold at the same price to a given number of bargain hunters relative to the same number of loyal customers. Log-linearizing the above demand functions, equations [E.1a] and [E.1b] in GS become

$$q_{S,j,t} = \frac{\lambda(1-v_S)}{\lambda + (1-\lambda)v_S} l_t + \frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} v_{S,j,t} - \epsilon(p_{S,j,t} - p_t) + y_t, \tag{B.49}$$

$$q_{N,j,t} = \frac{\lambda(1-v_N)}{\lambda + (1-\lambda)v_N} l_t + \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} v_{N,j,t} - \epsilon(r_{N,t-j} - p_t) + y_t, \tag{B.50}$$

where $r_{N,t-j}$ is a normal price set j periods ago. Regarding the purchase multiplier v, we obtain an equation of the same form as equation [E.2] in GS:

$$\begin{aligned} v_{S,j,t} &= -(\eta-\epsilon)(p_{S,j,t} - p_{B,t}), \\ v_{N,j,t} &= -(\eta-\epsilon)(r_{N,t-j} - p_{B,t}). \end{aligned} \tag{B.51}$$

The above four equations yield equations that are equivalent to [E.3a] and [E.3b] in GS:

$$
\begin{aligned}
q_{S,j,t} &= \frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} l_t - \frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}(\eta-\epsilon)(p_{S,j,t}-p_{B,t}) - \epsilon(p_{S,j,t}-p_t) + y_t \\
&= \frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} l_t \\
&\quad - \frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S} p_{S,j,t} + (\eta-\epsilon)\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} p_{B,t} + \epsilon p_t + y_t, \qquad \text{(B.52)} \\
q_{N,j,t} &= \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} l_t - \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}(\eta-\epsilon)(r_{N,t-j}-p_{B,t}) - \epsilon(r_{N,t-j}-p_t) + y_t \\
&= \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} l_t \\
&\quad - \frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N} r_{N,t-j} + (\eta-\epsilon)\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} p_{B,t} + \epsilon p_t + y_t. \qquad \text{(B.53)}
\end{aligned}
$$

The optimal ratio of the sale price to the normal price is given by equation [17] in GS. GS's equation [17] can be rewritten as

$$\left\{L(\epsilon-1)+(1-L)(\eta-1)v(p;P_B)\right\}\mu(p;P_B) = L\epsilon+(1-L)\eta v(p;P_B). \qquad \text{(B.54)}$$

Log-linearization yields

$$\frac{\lambda(\epsilon-1)l_t - \lambda(\eta-1)vl_t + (1-\lambda)(\eta-1)vv_t}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v} + \mu_t = \frac{\lambda\epsilon l_t - \lambda\eta v l_t + (1-\lambda)\eta v v_t}{\lambda\epsilon+(1-\lambda)\eta v}$$

$$
\begin{aligned}
\mu_t &= \frac{\lambda\epsilon l_t - \lambda\eta v l_t + (1-\lambda)\eta v v_t}{\lambda\epsilon + (1-\lambda)\eta v} - \frac{\lambda(\epsilon-1)l_t - \lambda(\eta-1)v l_t + (1-\lambda)(\eta-1)v v_t}{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v} \\
&= \frac{\{\lambda\epsilon l_t - \lambda\eta v l_t + (1-\lambda)\eta v v_t\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}} \\
&+ \frac{\{\lambda(\epsilon-1)l_t - \lambda(\eta-1)v l_t + (1-\lambda)(\eta-1)v v_t\}\{\lambda\epsilon+(1-\lambda)\eta v\}}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}} \\
&= \frac{\{\epsilon-\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\} - \{(\epsilon-1)-(\eta-1)v\}\{\lambda\epsilon+(1-\lambda)\eta v\}}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}\lambda l_t \\
&+ \frac{\eta\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\} - (\eta-1)\{\lambda\epsilon+(1-\lambda)\eta v\}}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}(1-\lambda)v v_t \\
&= \frac{\epsilon(1-\lambda)(\eta-1)v - \eta v\lambda(\epsilon-1) - (\epsilon-1)(1-\lambda)\eta v + (\eta-1)v\lambda\epsilon}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}\lambda l_t \\
&+ \frac{\lambda(1-\lambda)(\epsilon-\eta)v}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}v_t \\
&= \frac{(1-\lambda)(\eta-\epsilon)v + v\lambda(\eta-\epsilon)}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}\lambda l_t \\
&+ \frac{\lambda(1-\lambda)(\epsilon-\eta)v}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}v_t \\
&= \frac{\lambda(\eta-\epsilon)v}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}l_t \\
&- \frac{\lambda(1-\lambda)(\eta-\epsilon)v}{\{\lambda\epsilon+(1-\lambda)\eta v\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v\}}v_t. \qquad \text{(B.55)}
\end{aligned}
$$

We define

$$
\varrho_S = \frac{\lambda(1-\lambda)(\eta-\epsilon)v_S}{\{\lambda\epsilon+(1-\lambda)\eta v_S\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S\}} \qquad \text{(B.56)}
$$

$$
\varrho_N = \frac{\lambda(1-\lambda)(\eta-\epsilon)v_N}{\{\lambda\epsilon+(1-\lambda)\eta v_N\}\{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N\}}, \qquad \text{(B.57)}
$$

and transform equations [E.4a] and [E.4b] into

$$
\mu_{S,j,t} = -\varrho_S v_{S,j,t} + \frac{1}{1-\lambda}\varrho_S l_t, \qquad \text{(B.58)}
$$

$$
\mu_{N,j,t} = -\varrho_N v_{N,j,t} + \frac{1}{1-\lambda}\varrho_N l_t. \qquad \text{(B.59)}
$$

Overall demand is given by

$$
Q_{j,t} = s_{j,t}Q_{S,j,t} + (1-s_{j,t})Q_{N,j,t},
$$

which is log-linearized as

$$
Qq_{j,t} = sQ_S(s_{j,t}+q_{S,j,t}) - sQ_N s_{j,t} + (1-s)Q_N q_{N,j,t}.
$$

Using $\chi = Q_S/Q_N$, we obtain equation [E.5] in GS:

$$(s\chi + 1 - s)q_{j,t} = s\chi(s_{j,t} + q_{S,j,t}) - ss_{j,t} + (1-s)q_{N,j,t},$$

$$q_{j,t} = \frac{\chi - 1}{s\chi + 1 - s} ss_{j,t} + \frac{s\chi}{s\chi + 1 - s} q_{S,j,t} + \frac{(1-s)}{s\chi + 1 - s} q_{N,j,t}. \tag{B.60}$$

Note that we define $s_{j,t}$ as the logarithmic deviation of the sale frequency from its steady state, while GS define it as the deviation from steady state.

We define the weighted average of variables as in equations [E.6] and [E.7] in GS:

$$s_t \equiv (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j s_{j,t},$$

$$p_{N,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j r_{N,t-j}, \quad q_{N,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j q_{N,j,t},$$

$$v_{N,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j v_{N,j,t}, \tag{B.61}$$

$$p_{S,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j p_{S,j,t}, \quad q_{S,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j q_{S,j,t},$$

$$v_{S,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j v_{S,j,t}. \tag{B.62}$$

The bargain hunters' price index $P_{B,t}$ is log-linearized as in equation [E.8] in GS:

$$p_{B,t} = \theta_B p_{S,t} + (1-\theta_B)p_{N,t} - \varphi_B ss_t, \tag{B.63}$$

where

$$\theta_B = \frac{s}{s + (1-s)\mu^{\eta-1}},$$

$$\varphi_B = \frac{1}{\eta - 1}\frac{1 - \mu^{\eta-1}}{s + (1-s)\mu^{\eta-1}}. \tag{B.64}$$

The price index for a hypothetical loyal customer $P_{L,t}$ is log-linearized as in equation [E.10] in GS:

$$p_{L,t} = \theta_L p_{S,t} + (1-\theta_L)p_{N,t} - \varphi_L ss_t, \tag{B.65}$$

where

$$\theta_L = \frac{s}{s + (1-s)\mu^{\epsilon-1}},$$
$$\varphi_L = \frac{1}{\eta - 1}\frac{1-\mu^{\epsilon-1}}{s + (1-s)\mu^{\epsilon-1}}. \tag{B.66}$$

The aggregate price level given by equation (B.35) is transformed into

$$P = \left[LP_L^{1-\epsilon} + (1-L)P_B^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{B.67}$$

In steady state, using $\overline{h} = P_B/P_L$, we have

$$1 = \lambda\left(\frac{P_L}{P}\right)^{1-\epsilon} + (1-\lambda)\left(\frac{P_B}{P}\right)^{1-\epsilon},$$
$$= \lambda\left(\frac{P_L}{P}\right)^{1-\epsilon} + (1-\lambda)\left(\overline{h}\frac{P_L}{P}\right)^{1-\epsilon}.$$

$$\left(\frac{P_L}{P}\right)^{1-\epsilon} = \frac{1}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}}, \tag{B.68}$$
$$\left(\frac{P_B}{P}\right)^{1-\epsilon} = \frac{\overline{h}^{1-\epsilon}}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}}. \tag{B.69}$$

The aggregate price level is log-linearized as

$$\begin{aligned}(1-\epsilon)P^{1-\epsilon}p_t = {} & \lambda(1-\epsilon)P_L^{1-\epsilon}p_{L,t} + (1-\lambda)(1-\epsilon)P_B^{1-\epsilon}p_{B,t} \\ & + \lambda P_L^{1-\epsilon}l_t - \lambda P_B^{1-\epsilon}l_t,\end{aligned}$$

$$
\begin{aligned}
p_t &= \lambda \left(\frac{P_L}{P}\right)^{1-\epsilon} p_{L,t} + (1-\lambda) \left(\frac{P_B}{P}\right)^{1-\epsilon} p_{B,t} \\
&\quad + \frac{\lambda}{(1-\epsilon)} \left\{ \left(\frac{P_L}{P}\right)^{1-\epsilon} - \left(\frac{P_B}{P}\right)^{1-\epsilon} \right\} l_t \\
&= \frac{\lambda}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}} p_{L,t} + \frac{(1-\lambda)\overline{h}^{1-\epsilon}}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}} p_{B,t} \\
&\quad + \frac{\lambda}{(1-\epsilon)} \frac{1-\overline{h}^{1-\epsilon}}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}} l_t \\
&= \frac{\lambda \overline{h}^{\epsilon-1}}{\lambda \overline{h}^{\epsilon-1} + (1-\lambda)} p_{L,t} + \frac{1-\lambda}{\lambda \overline{h}^{\epsilon-1} + (1-\lambda)} p_{B,t} \\
&\quad + \frac{\lambda}{(1-\epsilon)} \frac{\overline{h}^{\epsilon-1} - 1}{\lambda \overline{h}^{\epsilon-1} + (1-\lambda)} l_t.
\end{aligned}
$$

Equation [E.11] in GS thus becomes

$$
p_t = (1-\varpi) p_{L,t} + \varpi p_{B,t} - \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t, \tag{B.70}
$$

where

$$
\begin{aligned}
\varpi &\equiv \frac{1-\lambda}{\lambda \overline{h}^{\epsilon-1} + (1-\lambda)}, \\
\overline{h} &\equiv \frac{\left(s + (1-s)\mu^{\epsilon-1}\right)^{\frac{1}{\epsilon-1}}}{\left(s + (1-s)\mu^{\eta-1}\right)^{\frac{1}{\eta-1}}}.
\end{aligned} \tag{B.71}
$$

Equation [E.12] in GS becomes

$$\begin{aligned}
p_t &= (1-\varpi)p_{L,t} + \varpi p_{B,t} - \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t \\
&= (1-\varpi)(\theta_L p_{S,t} + (1-\theta_L)p_{N,t} - \varphi_L ss_t) \\
&+ \varpi(\theta_B p_{S,t} + (1-\theta_B)p_{N,t} - \varphi_B ss_t) \\
&- \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t \\
&= \{(1-\varpi)\theta_L + \varpi\theta_B\} p_{S,t} \\
&+ \{(1-\varpi)(1-\theta_L) + \varpi(1-\theta_B)\} p_{N,t} \\
&- \{(1-\varpi)\varphi_L + \varpi\varphi_B\} ss_t \\
&- \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t,
\end{aligned}$$

$$\begin{aligned}
p_t &= \theta_P p_{S,t} + (1-\theta_P)p_{N,t} - \varphi_P ss_t \\
&- \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t,
\end{aligned} \tag{B.72}$$

where

$$\begin{aligned}
\theta_P &\equiv (1-\varpi)\theta_L + \varpi\theta_B \\
\varphi_P &\equiv (1-\varpi)\varphi_L + \varpi\varphi_B.
\end{aligned} \tag{B.73}$$

The production technology is governed by constant returns to scale using labor input:

$$Q_{j,t} = Z_t^a H_{j,t}^{\alpha}, \tag{B.74}$$

where Z_t^a represents a stochastic shock to productivity, which has a mean of one, and the logarithm deviation of which is denoted by ε_t^a. Equations [E.13] and [E.14] in GS become

$$q_t = \alpha h_t + \varepsilon_t^a, \tag{B.75}$$

$$x_t = \gamma q_t + w_t. \tag{B.76}$$

Firms' profit maximizing problem yields equation [27] in GS:

$$\frac{p_{S,j,t}q_{S,j,t} - r_{N,t-j}q_{N,j,t}}{q_{S,j,t} - q_{N,j,t}} = x_{j,t} + p_t. \tag{B.77}$$

Equation [E.15] holds, as shown in the following equation:

$$(\chi - 1)(x_{j,t} + p_t) = \mu_S \chi p_{S,j,t} - \mu_N r_{N,t-j} + (\mu_S - 1)\chi(q_{S,j,t} - q_{N,j,t}). \tag{B.78}$$

Substituting equations (B.52) and (B.53), we obtain

$$\begin{aligned}
q_{S,j,t} &= \frac{\lambda(1-v_S)}{\lambda + (1-\lambda)v_S} l_t - \frac{(1-\lambda)v_S}{\lambda + (1-\lambda)v_S}(\eta - \epsilon)(p_{S,j,t} - p_{B,t}) - \epsilon(p_{S,j,t} - p_t) + y_t \\
&= \frac{\lambda(1-v_S)}{\lambda + (1-\lambda)v_S} l_t \\
&- \frac{\lambda\epsilon + (1-\lambda)\eta v_S}{\lambda + (1-\lambda)v_S} p_{S,j,t} + (\eta - \epsilon)\frac{(1-\lambda)v_S}{\lambda + (1-\lambda)v_S} p_{B,t} + \epsilon p_t + y_t, \qquad \text{(B.79)} \\
q_{N,j,t} &= \frac{\lambda(1-v_N)}{\lambda + (1-\lambda)v_N} l_t - \frac{(1-\lambda)v_N}{\lambda + (1-\lambda)v_N}(\eta - \epsilon)(r_{N,t-j} - p_{B,t}) - \epsilon(r_{N,t-j} - p_t) + y_t \\
&= \frac{\lambda(1-v_N)}{\lambda + (1-\lambda)v_N} l_t \\
&- \frac{\lambda\epsilon + (1-\lambda)\eta v_N}{\lambda + (1-\lambda)v_N} r_{N,t-j} + (\eta - \epsilon)\frac{(1-\lambda)v_N}{\lambda + (1-\lambda)v_N} p_{B,t} + \epsilon p_t + y_t. \qquad \text{(B.80)}
\end{aligned}$$

$$\begin{aligned}
(\chi - 1)(x_{j,t} + p_t) = \mu_S \chi p_{S,j,t} - \mu_N r_{N,t-j} + (\mu_S - 1)\chi \\
\cdot \Bigg\{ \left(\frac{\lambda(1-v_S)}{\lambda + (1-\lambda)v_S} - \frac{\lambda(1-v_N)}{\lambda + (1-\lambda)v_N} \right) l_t \\
- \frac{\lambda\epsilon + (1-\lambda)\eta v_S}{\lambda + (1-\lambda)v_S} p_{S,j,t} \\
+ \frac{\lambda\epsilon + (1-\lambda)\eta v_N}{\lambda + (1-\lambda)v_N} r_{N,t-j} \\
+ (\eta - \epsilon) \left(\frac{(1-\lambda)v_S}{\lambda + (1-\lambda)v_S} - \frac{(1-\lambda)v_N}{\lambda + (1-\lambda)v_N} \right) p_{B,t} \Bigg\}.
\end{aligned}$$

Hence, we have

$$
\begin{aligned}
(\chi-1)(x_{j,t}+p_t) = {} & \left\{\mu_S - (\mu_S-1)\frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S}\right\}\chi p_{S,j,t} \\
& - \left\{\mu_N - (\mu_S-1)\chi\frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N}\right\} r_{N,t-j} \\
& + (\eta-\epsilon)\left(\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} - \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}\right)(\mu_S-1)\chi p_{B,t} \\
& + \left(\frac{1-v_S}{\lambda+(1-\lambda)v_S} - \frac{1-v_N}{\lambda+(1-\lambda)v_N}\right)(\mu_S-1)\chi\lambda l_t. \qquad \text{(B.81)}
\end{aligned}
$$

Note that, under flexible prices, the optimal ratio of the sale price to the normal price is given by equation [17] in GS:

$$
\mu(p;P_B) = \frac{L\epsilon+(1-L)\eta v(p;P_B)}{L(\epsilon-1)+(1-L)(\eta-1)v(p;P_B)}. \qquad \text{(B.82)}
$$

Its steady-state value is given by

$$
\mu_S = \frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}, \qquad \text{(B.83)}
$$

$$
\mu_S - 1 = \frac{\lambda+(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}. \qquad \text{(B.84)}
$$

As for equation (B.81), the coefficient on $p_{S,l,t}$ becomes

$$
\begin{aligned}
& \left\{\mu_S - (\mu_S-1)\frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S}\right\}\chi \\
& = \left\{\mu_S - \frac{\lambda+(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S}\right\}\chi \\
& = \left\{\mu_S - \frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\right\}\chi \\
& = 0.
\end{aligned}
$$

The coefficient on $r_{N,t-l}$ similarly becomes

$$
\begin{aligned}
& \mu_N - (\mu_S-1)\chi\frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N} \\
& = \mu_N - (\mu_N-1)\chi\frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N} \\
& = 0.
\end{aligned}
$$

The coefficient on $p_{B,t}$ becomes

$$
\begin{aligned}
&(\eta-\epsilon)\left(\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}-\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}\right)(\mu_S-1)\chi \\
&=(\eta-\epsilon)\left(\begin{array}{c}\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}\frac{\lambda+(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi \\ -\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}\frac{\lambda+(1-\lambda)v_N}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\end{array}\right) \\
&=(\eta-\epsilon)\left(\frac{(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi-\frac{(1-\lambda)v_N}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\right) \\
&=\left(\frac{\lambda\epsilon+(1-\lambda)\eta v_S-\{\lambda\epsilon+(1-\lambda)\epsilon v_S\}}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi-\frac{\lambda\epsilon+(1-\lambda)\eta v_N-\{\lambda\epsilon+(1-\lambda)\epsilon v_N\}}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\right) \\
&=\mu_S\chi-(\mu_S-1)\chi\epsilon-\mu_N+(\mu_N-1)\epsilon \\
&=\mu_S\chi-\mu_N \\
&=\mu_S\frac{\mu_N-1}{\mu_S-1}-\mu_N=\frac{\mu_S(\mu_N-1)-\mu_N(\mu_S-1)}{\mu_S-1}=\frac{\mu_N-1-(\mu_S-1)}{\mu_S-1} \\
&=\chi-1.
\end{aligned}
$$

The coefficient on l_t becomes

$$
\begin{aligned}
&\left(\frac{1-v_S}{\lambda+(1-\lambda)v_S}-\frac{1-v_N}{\lambda+(1-\lambda)v_N}\right)(\mu_S-1)\chi\lambda \\
&=\frac{1-v_S}{\lambda+(1-\lambda)v_S}\frac{\lambda+(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi\lambda \\
&-\frac{1-v_N}{\lambda+(1-\lambda)v_N}\frac{\lambda+(1-\lambda)v_N}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\lambda \\
&=\frac{1-v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi\lambda \\
&-\frac{1-v_N}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\lambda \\
&=-\left\{\frac{\mu_S-\eta(\mu_S-1)}{(\eta-\epsilon)\lambda}+\frac{\mu_S-\epsilon(\mu_S-1)}{(\eta-\epsilon)(1-\lambda)}\right\}\chi\lambda \\
&+\left\{\frac{\mu_N-\eta(\mu_N-1)}{(\eta-\epsilon)\lambda}+\frac{\mu_N-\epsilon(\mu_N-1)}{(\eta-\epsilon)(1-\lambda)}\right\}\lambda \\
&=-\left\{\frac{\mu_S\chi-\eta(\mu_N-1)}{(\eta-\epsilon)\lambda}+\frac{\mu_S\chi-\epsilon(\mu_N-1)}{(\eta-\epsilon)(1-\lambda)}\right\}\lambda \\
&+\left\{\frac{\mu_N-\eta(\mu_N-1)}{(\eta-\epsilon)\lambda}+\frac{\mu_N-\epsilon(\mu_N-1)}{(\eta-\epsilon)(1-\lambda)}\right\}\lambda \\
&=-\frac{\mu_S\chi}{(\eta-\epsilon)\lambda(1-\lambda)}\lambda+-\frac{\mu_N}{(\eta-\epsilon)\lambda(1-\lambda)}\lambda \\
&=-\frac{\mu_S\chi-\mu_N}{(\eta-\epsilon)(1-\lambda)} \\
&=-\frac{\chi-1}{(\eta-\epsilon)(1-\lambda)}.
\end{aligned}
$$

Therefore, equation (B.81) is simplified as

$$\begin{aligned}(\chi-1)(x_{j,t}+p_t) &= (\chi-1)p_{B,t}-\frac{\chi-1}{(\eta-\epsilon)(1-\lambda)}l_t,\\ x_{j,t}+p_t &= p_{B,t}-\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t. \qquad \text{(B.85)}\end{aligned}$$

The right-hand side of the equation is independent of j, so all firms have the same marginal cost.

Equation [27] in GS suggests

$$p_{S,j,t}=\mu_{S,j,t}+x_{j,t}+p_t.$$

Substituting equations (B.51) and (B.58) into the above equation, we obtain the following equation:

$$\begin{aligned}p_{S,j,t} &= -\varrho_S v_{S,j,t}+\frac{1}{1-\lambda}\varrho_S l_t+x_{j,t}+p_t\\ &= \varrho_S(\eta-\epsilon)(p_{S,j,t}-p_{B,t})+\frac{1}{1-\lambda}\varrho_S l_t+x_{j,t}+p_t\\ &= \varrho_S(\eta-\epsilon)p_{S,j,t}-\varrho_S(\eta-\epsilon)\left\{x_t+p_t+\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right\}+\frac{1}{1-\lambda}\varrho_S l_t+x_t+p_t,\end{aligned}$$

and equation [E.17] in GS

$$\begin{aligned}\{1-\varrho_S(\eta-\epsilon)\}(p_{S,j,t}-x_t-p_t) &= \left\{\frac{\varrho_S}{1-\lambda}-\frac{\varrho_S(\eta-\epsilon)}{(\eta-\epsilon)(1-\lambda)}\right\}l_t\\ &= 0. \qquad \text{(B.86)}\end{aligned}$$

We thus have

$$p_{S,j,t}=x_t+p_t. \qquad \text{(B.87)}$$

Regarding normal prices, the log-linearization of the first-order condition, equation [26] in GS, becomes

$$\sum_{j=0}^{\infty}(\beta\phi_p)^j E_t\left[r_{N,t}-\mu_{N,j,t+j}-x_{t+j}-p_{t+j}\right]=0, \qquad \text{(B.88)}$$

which corresponds to equation [E.18] in GS. Using equations (B.51) and (B.59), we can express the ratio of the sale price to the normal price, $\mu_{N,j,t+j}$, as follows:

$$\begin{aligned}\mu_{N,j,t} &= -\varrho_N v_{N,j,t}+\frac{1}{1-\lambda}\varrho_N l_t\\ &= \varrho_N(\eta-\epsilon)(r_{N,t-j}-p_{B,t})+\frac{1}{1-\lambda}\varrho_N l_t.\end{aligned}$$

Equation (B.85) yields

$$\begin{aligned}\mu_{N,j,t} &= \varrho_N(\eta-\epsilon)\left(r_{N,t-j} - x_{j,t} - p_t - \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right) + \frac{1}{1-\lambda}\varrho_N l_t \\ &= \varrho_N(\eta-\epsilon)\left(r_{N,t-j} - x_{j,t} - p_t\right).\end{aligned}$$

Equation (B.88) thus becomes

$$\{1-\varrho_N(\eta-\epsilon)\}\sum_{j=0}^{\infty}(\beta\phi_p)^j\, E_t\left[r_{N,t} - x_{t+j} - p_{t+j}\right] = 0,$$

yielding equation [E.19] in GS:

$$r_{N,t} = (1-\beta\phi_p)\sum_{j=0}^{\infty}(\beta\phi_p)^j\, E_t(x_{t+j}+p_{t+j}). \tag{B.89}$$

Equations (B.72) and (B.87) mean that equation [E.20] in GS becomes

$$\begin{aligned}\varphi_P ss_t &= \theta_P p_{S,t} + (1-\theta_P)p_{N,t} - p_t - \lambda\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon-1}\right)l_t \\ &= \theta_P x_t + (1-\theta_P)(p_{N,t}-p_t) - \lambda\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon-1}\right)l_t. \end{aligned}\tag{B.90}$$

Similarly, equations (B.63), (B.85), and (B.87) mean that equation [E.21] becomes

$$\begin{aligned}\varphi_B ss_t &= \theta_B p_{S,t} + (1-\theta_B)p_{N,t} - p_{B,t} \\ &= \theta_B x_t + (1-\theta_B)p_{N,t} - x_t - p_t - \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t \\ &= (1-\theta_B)(p_{N,t}-x_t-p_t) - \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t. \end{aligned}\tag{B.91}$$

Using equations (B.90) and (B.91), we obtain

$$\begin{aligned}&\left\{(1-\theta_B)(p_{N,t}-x_t-p_t) - \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right\}/\varphi_B \\ &= \left\{\theta_P x_t + (1-\theta_P)(p_{N,t}-p_t) - \lambda\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon-1}\right)l_t\right\}/\varphi_P \\ &= \left\{x_t + (1-\theta_P)(p_{N,t}-x_t-p_t) - \lambda\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon-1}\right)l_t\right\}/\varphi_P\end{aligned}$$

$$\begin{aligned}\frac{x_t}{\varphi_P} &= \left\{\frac{1-\theta_B}{\varphi_B} - \frac{1-\theta_P}{\varphi_P}\right\}(p_{N,t} - x_t - p_t) \\ &\quad - \left\{\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B} - \frac{\lambda}{\varphi_P}\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1}\right)\right\} l_t\end{aligned}$$

$$x_t = \left\{\frac{(1-\theta_B)\varphi_P - (1-\theta_P)\varphi_B}{\varphi_B}\right\}(p_{N,t} - x_t - p_t) - Al_t.$$

where

$$A \equiv \frac{\varphi_P}{(\eta-\epsilon)(1-\lambda)\varphi_B} - \lambda\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1}\right). \tag{B.92}$$

Equation [E.22] in GS becomes

$$x_t = (1-\psi)(x_t + p_t - p_{N,t}) - Al_t, \tag{B.93}$$

where equation [E.23] is defined as

$$\begin{aligned}1-\psi &= -\frac{(1-\theta_B)\varphi_P - (1-\theta_P)\varphi_B}{\varphi_B} \\ \psi &= 1 + \frac{(1-\theta_B)\varphi_P - (1-\theta_P)\varphi_B}{\varphi_B} \\ &= \frac{(1-\theta_B)\varphi_P + \theta_P\varphi_B}{\varphi_B}.\end{aligned} \tag{B.94}$$

Rearranging equation (B.61) yields equation [E.24] in GS,

$$p_{N,t} = \phi_p p_{N,t-1} + (1-\phi_p) r_{N,t}, \tag{B.95}$$

and rearranging equation (B.89) yields equation [E.25] in GS,

$$r_{N,t} = \beta\phi_p E_t r_{N,t+1} + (1-\beta\phi_p)(x_t + p_t). \tag{B.96}$$

Multiplying the above by $(1-\phi_p)$ and substituting it into equation (B.95) yields

$$p_{N,t} = \phi_p p_{N,t-1} + (1-\phi_p)\beta\phi_p E_t r_{N,t+1} + (1-\phi_p)(1-\beta\phi_p)(x_t + p_t),$$

$$\begin{aligned}p_{N,t} - \phi_p p_{N,t-1} &= (1-\phi_p)\beta\phi_p E_t r_{N,t+1} + (1-\phi_p)(1-\beta\phi_p)(x_t + p_t) \\ &= \beta\phi_p \left\{E_t p_{N,t+1} - \phi_p p_{N,t}\right\} + (1-\phi_p)(1-\beta\phi_p)(x_t + p_t).\end{aligned}$$

Defining $\pi_{N,t} \equiv p_{N,t} - p_{N,t-1}$ and adding $(\phi_p - 1)p_{N,t}$ to both terms, we obtain equation [E.26] in GS:

$$
\begin{aligned}
\phi_p(p_{N,t} - p_{N,t-1}) &= \beta\phi_p\left\{E_t p_{N,t+1} - p_{N,t} + p_{N,t} - \phi_p p_{N,t}\right\} + (1-\phi_p)(1-\beta\phi_p)(x_t + p_t) \\
&\quad + (\phi_p - 1)p_{N,t} \\
\phi_p \pi_{N,t} &= \beta\phi_p E_t \pi_{N,t+1} + (1-\phi_p)(1-\beta\phi_p)(x_t + p_t) - (\phi_p - 1)(1-\beta\phi_p)p_{N,t} \\
\phi_p \pi_{N,t} &= \beta\phi_p E_t \pi_{N,t+1} + (1-\phi_p)(1-\beta\phi_p)(x_t + p_t - p_{N,t}) \\
\pi_{N,t} &= \beta E_t \pi_{N,t+1} + \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p}(x_t + p_t - p_{N,t}), \qquad \text{(B.97)}
\end{aligned}
$$

where we define

$$\kappa \equiv \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p}. \qquad \text{(B.98)}$$

Taking the first difference of equation (B.91) yields an equation that is equivalent to equation [E.27] in GS:

$$s\Delta s_t = -\frac{1-\theta_B}{\varphi_B}(\Delta x_t + \pi_t - \pi_{N,t}) - \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\Delta l_t. \qquad \text{(B.99)}$$

The first difference of equation (B.72) is

$$
\begin{aligned}
\pi_t &= \theta_P(p_{S,t} - p_{S,t-1}) + (1-\theta_P)\pi_{N,t} - \varphi_P s\Delta s_t \\
&\quad - \lambda\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1}\right)\Delta l_t.
\end{aligned}
$$

From $p_{S,j,t} = x_t + p_t$, we can transform the above equation into the following form:

$$
\begin{aligned}
\pi_t &= \theta_P(\Delta x_t + \pi_t) + (1-\theta_P)\pi_{N,t} - \varphi_P s\Delta s_t - \lambda\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1}\right)\Delta l_t \\
&= \pi_{N,t} + \theta_P(\Delta x_t + \pi_t - \pi_{N,t}) \\
&\quad + \varphi_P\left\{\frac{1-\theta_B}{\varphi_B}(\Delta x_t + \pi_t - \pi_{N,t}) + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\Delta l_t\right\} \\
&\quad - \lambda\left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1}\right)\Delta l_t \\
&= \pi_{N,t} + \frac{(1-\theta_B)\varphi_P + \theta_P\varphi_B}{\varphi_B}(\Delta x_t + \pi_t - \pi_{N,t}) + A\Delta l_t \\
&= \pi_{N,t} + \psi(\Delta x_t + \pi_t - \pi_{N,t}) + A\Delta l_t.
\end{aligned}
$$

We can also transform equation (B.93) into the following form:

$$x_t = (1-\psi)(x_t + p_t - p_{N,t}) - A l_t, \qquad \text{(B.100)}$$

$$\Delta x_t = (1-\psi)(\Delta x_t + \pi_t - \pi_{N,t}) - A\Delta l_t,$$

$$\pi_{N,t} = \pi_t - \frac{\psi}{1-\psi}\Delta x_t - \frac{A}{1-\psi}\Delta l_t. \tag{B.101}$$

Substituting this into equation (B.97) yields

$$\begin{aligned}
&\pi_t - \frac{\psi}{1-\psi}\Delta x_t - \frac{A}{1-\psi}\Delta l_t \\
&= \beta E_t\left\{\pi_{t+1} - \frac{\psi}{1-\psi}\Delta x_{t+1} - \frac{A}{1-\psi}\Delta l_{t+1}\right\} + \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p}(x_t + p_t - p_{N,t}).
\end{aligned}$$

Using equation (B.93), we obtain

$$\begin{aligned}
&\pi_t - \frac{\psi}{1-\psi}\Delta x_t - \frac{A}{1-\psi}\Delta l_t \\
&= \beta E_t\left\{\pi_{t+1} - \frac{\psi}{1-\psi}\Delta x_{t+1} - \frac{A}{1-\psi}\Delta l_{t+1}\right\} \\
&+ \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p}\left\{\frac{1}{1-\psi}x_t + \frac{A}{1-\psi}l_t\right\},
\end{aligned}$$

and an equation which is equivalent to equation [32] in GS:

$$\begin{aligned}
&\pi_t = \beta E_t \pi_{t+1} \\
&+ \frac{1}{1-\psi}\left\{\kappa x_t + \psi(\Delta x_t - \beta E_t \Delta x_{t+1}) + \kappa A l_t + A(\Delta l_t - \beta E_t \Delta l_{t+1})\right\}.
\end{aligned} \tag{B.102}$$

Next, we derive Lemma 4 in GS. From

$$\mu_t = p_{S,t} - p_{N,t}, \tag{B.103}$$

equation (B.93) is given by

$$\begin{aligned}
x_t &= (1-\psi)(x_t + p_t - p_{N,t}) - A l_t, \\
&= (1-\psi)(p_{S,t} - p_{N,t}) - A l_t \\
&= (1-\psi)\mu_t - A l_t.
\end{aligned}$$

This yields an equation which is equivalent to equation [E.37] in GS:

$$\mu_t = \frac{1}{1-\psi}(x_t + A l_t). \tag{B.104}$$

From equations (B.49) to (B.51), we have

$$q_{S,t} = \frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} l_t + \frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} \{-(\eta-\epsilon)(p_{S,t}-p_{B,t})\} - \epsilon(p_{S,t}-p_t) + y_t,$$

$$q_{N,t} = \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} l_t + \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} \{-(\eta-\epsilon)(p_{N,t}-p_{B,t})\} - \epsilon(p_{N,t}-p_t) + y_t.$$

Using equations (B.85) and (B.87), we have

$$\begin{aligned} q_{S,t} &= \frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} l_t + \frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} (\eta-\epsilon) \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t - \epsilon x_t + y_t \\ &= l_t - \epsilon x_t + y_t \end{aligned} \tag{B.105}$$

$$\begin{aligned} q_{N,t} &= \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} l_t - \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} (\eta-\epsilon) \left(p_{N,t} - p_{S,t} - \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t \right) \\ &\quad - \epsilon(p_{N,t}-p_t) + y_t \\ &= l_t - \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} (\eta-\epsilon) \left(p_{N,t}-p_{S,t}\right) - \epsilon(p_{N,t}-p_{S,t}+p_{S,t}-p_t) + y_t \\ &= l_t - \frac{(\eta-\epsilon)(1-\lambda)v_N + \epsilon\{\lambda+(1-\lambda)v_N\}}{\lambda+(1-\lambda)v_N} \left(p_{N,t}-p_{S,t}\right) - \epsilon x_t + y_t \\ &= l_t + \frac{\epsilon\lambda + \eta(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} \mu_t - \epsilon x_t + y_t. \end{aligned} \tag{B.106}$$

Thus, the quantity ratio becomes

$$\begin{aligned} \chi_t &= q_{S,t} - q_{N,t} \\ &= -\varsigma_N \mu_t, \end{aligned} \tag{B.107}$$

where

$$\varsigma_N = \frac{\epsilon\lambda + \eta(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}. \tag{B.108}$$

From equation (B.104), we can express the quantity ratio as follows:

$$\chi_t = -\frac{\varsigma_N}{1-\psi} \left(x_t + A l_t\right). \tag{B.109}$$

The ratio of turnover at the sale price to total turnover is defined as

$$\frac{s_t Q_{S,t}}{(1-s_t)Q_{N,t} + s_t Q_{S,t}}, \tag{B.110}$$

which is log-linearized as

$$\frac{1}{s\chi+1-s}s_t+\frac{1-s}{s\chi+1-s}\chi_t. \tag{B.111}$$

Equation (B.91) is transformed into an equation which is equivalent to [E.39] in GS:

$$\begin{aligned}
ss_t &= \frac{1-\theta_B}{\varphi_B}(p_{N,t}-x_t-p_t)-\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}l_t\\
&= \frac{1-\theta_B}{\varphi_B}(p_{N,t}-p_{S,t})-\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}l_t\\
&= -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}(x_t+Al_t)-\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}l_t\\
&= -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}x_t-\left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi}+\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right)l_t.
\end{aligned} \tag{B.112}$$

Let

$$\Delta_t \equiv y_t - q_t. \tag{B.113}$$

Using equation (B.60), total output becomes

$$q_t=\frac{\chi-1}{s\chi+1-s}ss_t+\frac{s\chi}{s\chi+1-s}q_{S,t}+\frac{(1-s)}{s\chi+1-s}q_{N,t}.$$

Equations (B.105) and (B.106) yield

$$\begin{aligned}
q_t &= \frac{\chi-1}{s\chi+1-s}ss_t+\frac{s\chi}{s\chi+1-s}(l_t-\epsilon x_t+y_t)\\
&\quad+\frac{(1-s)}{s\chi+1-s}\left\{l_t+\frac{\epsilon\lambda+\eta(1-\lambda)\upsilon_N}{\lambda+(1-\lambda)\upsilon_N}\mu_t-\epsilon x_t+y_t\right\}\\
&= \frac{\chi-1}{s\chi+1-s}ss_t+l_t-\epsilon x_t+y_t\\
&\quad+\frac{1-s}{s\chi+1-s}\frac{\varsigma_N}{1-\psi}(x_t+Al_t).
\end{aligned}$$

Using equation (B.112), we obtain

$$\begin{aligned}
q_t &= \frac{\chi-1}{s\chi+1-s}\left\{-\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}x_t-\left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi}+\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right)l_t\right\}\\
&\quad+l_t-\epsilon x_t+y_t+\frac{1-s}{s\chi+1-s}\frac{\varsigma_N}{1-\psi}(x_t+Al_t)\\
&= y_t-\delta x_t-Bl_t,
\end{aligned}$$

where equation [E.40] in GS becomes

$$\begin{aligned}\delta &\equiv \epsilon + \frac{\chi-1}{s\chi+1-s}\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi} - \frac{1-s}{s\chi+1-s}\frac{\varsigma_N}{1-\psi} \\ &= \epsilon + \frac{1}{s\chi+1-s}\frac{1}{1-\psi}\left(\frac{(\chi-1)(1-\theta_B)}{\varphi_B} - (1-s)\varsigma_N\right),\end{aligned} \tag{B.114}$$

$$\begin{aligned}B &\equiv -1 + \frac{\chi-1}{s\chi+1-s}\left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi} + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right) \\ &\quad - \frac{1-s}{s\chi+1-s}\frac{\varsigma_N}{1-\psi}A \\ &= -1 + \frac{1}{s\chi+1-s}\frac{1}{1-\psi}\left(\frac{(\chi-1)(1-\theta_B)}{\varphi_B} - (1-s)\varsigma_N\right)A \\ &\quad + \frac{\chi-1}{s\chi+1-s}\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B} \\ &= -1 + (\delta-\epsilon)A + \frac{\chi-1}{s\chi+1-s}\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}.\end{aligned} \tag{B.115}$$

Thus, we have

$$\begin{aligned}\Delta_t &\equiv y_t - q_t \\ &= \delta x_t + Bl_t.\end{aligned} \tag{B.116}$$

From equation (B.76), we can express the real marginal cost as follows:

$$\begin{aligned}x_t &= \gamma q_t + w_t \\ &= \gamma(y_t - \Delta_t) + w_t \\ &= \gamma(y_t - \delta x_t - Bl_t) + w_t.\end{aligned} \tag{B.117}$$

Equation [A.9b] in GS becomes

$$x_t = \frac{1}{1+\gamma\delta}w_t + \frac{\gamma}{1+\gamma\delta}\left(y_t - Bl_t\right). \tag{B.118}$$

From equation (B.76), we can express the quantity of products sold as follows:

$$\begin{aligned} q_t &= y_t - \Delta_t \\ &= y_t - \delta x_t - Bl_t \\ &= y_t - \delta\left(\frac{1}{1+\gamma\delta}w_t + \frac{\gamma}{1+\gamma\delta}(y_t - Bl_t)\right) - Bl_t \\ &= \frac{1}{1+\gamma\delta}y_t - \frac{\delta}{1+\gamma\delta}w_t - \frac{1}{1+\gamma\delta}Bl_t. \end{aligned} \tag{B.119}$$

B.6 Log-Linearization of the Equations for the Household's Actions

Using equations (B.30) and (B.31), we can derive the log-linearized consumption wedge F_t as follows:

$$\begin{aligned} f_t &= \frac{\eta}{\eta-1}\frac{\lambda\Xi(l_t+\xi_t) - \lambda l_t}{\lambda\Xi + (1-\lambda)} \\ &= \frac{\eta}{\eta-1}\lambda\frac{\Xi\xi_t - (1-\Xi)l_t}{\lambda\Xi + (1-\lambda)}, \end{aligned} \tag{B.120}$$

where

$$\begin{aligned} \xi_t &= \frac{s\mu^{\epsilon\frac{1-\eta}{\eta}}(s_t + \epsilon\frac{1-\eta}{\eta}\mu_t) - ss_t}{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)} - \frac{\epsilon}{\eta}\frac{s\mu^{1-\eta}(s_t + (1-\eta)\mu_t) - ss_t}{s\mu^{1-\eta} + (1-s)} \\ &= \frac{s(\mu^{\epsilon\frac{1-\eta}{\eta}} - 1)s_t + s\mu^{\epsilon\frac{1-\eta}{\eta}}\epsilon\frac{1-\eta}{\eta}\mu_t}{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)} - \frac{\epsilon}{\eta}\frac{s(\mu^{1-\eta}-1)s_t + s\mu^{1-\eta}(1-\eta)\mu_t}{s\mu^{1-\eta} + (1-s)} \\ &= \frac{(\mu^{\epsilon\frac{1-\eta}{\eta}} - 1)\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}(\mu^{1-\eta}-1)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}}ss_t \\ &+ \frac{\mu^{\epsilon\frac{1-\eta}{\eta}}\epsilon\frac{1-\eta}{\eta}\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}\mu^{1-\eta}(1-\eta)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}}s\mu_t \end{aligned} \tag{B.121}$$

Using equation (B.44), we transform equation (B.38) into

$$\begin{aligned} 0 &= \frac{v_{CC}}{v_C}C(E_tc_{t+1} - c_t) + (i_t - E_t\pi_{t+1}) \\ &+ E_t(f_{t+1} - f_t) - \epsilon E_t\left[(p_{B,t+1} - p_{t+1}) - (p_{B,t} - p_t)\right] \\ &= -\theta_c^{-1}\left\{E_t(y_{t+1} + f_{t+1} - \epsilon(p_{B,t+1} - p_{t+1}) - \varepsilon^g_{t+1}) - (y_t + f_t - \epsilon(p_{B,t} - p_t) - \varepsilon^g_t)\right\} \\ &+ (i_t - E_t\pi_{t+1}) + E_t(f_{t+1} - f_t) - \epsilon E_t\left[(p_{B,t+1} - p_{t+1}) - (p_{B,t} - p_t)\right]. \end{aligned}$$

Equation (B.85) yields

$$0 = \theta_c^{-1} \left\{ \begin{array}{c} E_t(y_{t+1} + f_{t+1} - \epsilon \left[x_{t+1} + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_{t+1}\right] - \varepsilon_{t+1}^g) \\ -(y_t + f_t - \epsilon \left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t - \varepsilon_t^g\right]) \end{array} \right\}$$
$$- (i_t - E_t\pi_{t+1}) - E_t(f_{t+1} - f_t)$$
$$+ \epsilon E_t \left[x_{t+1} \frac{1}{(\eta-\epsilon)(1-\lambda)} l_{t+1} - x_t - \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t\right]$$

and

$$\begin{aligned} y_t = {} & E_t y_{t+1} - \theta_c(i_t - E_t\pi_{t+1}) + (\varepsilon_t^g - E_t\varepsilon_{t+1}^g) \\ & + (1-\theta_c)\left\{\Delta f_{t+1} - \epsilon\left(\Delta x_{t+1} + \frac{1}{(\eta-\epsilon)(1-\lambda)}\Delta l_{t+1}\right)\right\}. \end{aligned} \tag{B.122}$$

Equation (B.39) becomes

$$\begin{aligned} \varepsilon_t^h + & \frac{\upsilon_{HH}}{\upsilon_H}\left\{H h_t - \theta_L \phi_L H \frac{(1-\lambda)^{\theta_L - 1}}{(1-\lambda)^{\theta_L}} \lambda l_t\right\} - \frac{\upsilon_{CC}}{\upsilon_C} C c_t \\ & = w_t + f_t - \epsilon(p_{B,t} - p_t). \end{aligned} \tag{B.123}$$

The second term on the left-hand side of the equation implies that, all other things being equal, h_t is positively correlated with l_t. A decline in hours worked involves a decrease in the fraction of loyal customers. From equations (B.44) and (B.85), we can rewrite this equation as follows:

$$\begin{aligned} \varepsilon_t^h + & \theta_h^{-1}\left(h_t - \theta_L \phi_L H (1-\lambda)^{-1} \frac{\lambda}{H} l_t\right) \\ & + \theta_c^{-1}(y_t + f_t - \epsilon\left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t\right] - \varepsilon_t^g) \\ & = w_t + f_t - \epsilon\left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t\right]. \end{aligned} \tag{B.124}$$

From equations (B.75) and (B.119), we can express hours worked as follows:

$$\begin{aligned} h_t & = \frac{q_t - \varepsilon_t^a}{\alpha} \\ & = \frac{1}{1+\gamma\delta} \frac{y_t - \delta w_t - B l_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a. \end{aligned} \tag{B.125}$$

Substituting equation (B.125) into (B.124) yields

$$\begin{aligned}
\varepsilon_t^h + \theta_h^{-1} & \left(\frac{1}{1+\gamma\delta} \frac{y_t - \delta w_t - B l_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a \right) \\
& - \theta_h^{-1}\theta_L \phi_L (1-\lambda)^{-1}\lambda l_t \\
& + \theta_c^{-1}(y_t + f_t - \epsilon \left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t \right] - \varepsilon_t^g) \\
& = w_t + f_t - \epsilon \left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t \right],
\end{aligned}$$

$$\begin{aligned}
0 = & \left(\theta_c^{-1} + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} \right) y_t - \left(1 + \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} \right) w_t \\
& - \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a + \varepsilon_t^h - \theta_c^{-1}\varepsilon_t^g \\
& - \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} B + \theta_h^{-1}\theta_L\phi_L(1-\lambda)^{-1}\lambda \right) l_t \\
& - (1-\theta_c^{-1}) \left\{ f_t - \epsilon \left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t \right) \right\}. \qquad \text{(B.126)}
\end{aligned}$$

In the presence of wage stickiness, the right-hand side of the equation deviates from zero and the following equation can be derived:

$$\begin{aligned}
\pi_{W,t} = & \beta \pi_{W,t+1} \\
& + \frac{(1-\phi_w)(1-\beta\phi_w)}{\phi_w} \frac{1}{1+\varsigma\theta_h^{-1}} \\
& \left[\left(\theta_c^{-1} + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} \right) y_t - \left(1 + \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} \right) w_t \right. \\
& - \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a + \varepsilon_t^h - \theta_c^{-1}\varepsilon_t^g \\
& - \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} B + \theta_h^{-1}\theta_L\phi_L(1-\lambda)^{-1}\lambda \right) l_t \\
& \left. -(1-\theta_c^{-1}) \left\{ f_t - \epsilon \left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t \right) \right\} \right]. \qquad \text{(B.127)}
\end{aligned}$$

Finally, equation (B.41) becomes

$$
\begin{aligned}
& -(\theta_L - 1)\frac{\lambda}{1-\lambda} l_t \\
& + \frac{\upsilon_{HH}}{\upsilon_H}(Hh_t - \theta_L \phi_L H(1-\lambda)^{-1}\lambda l_t) - \frac{\upsilon_{CC}}{\upsilon_C} C c_t + \varepsilon_t^h \\
& = c_t - \frac{\Xi}{1-\Xi}\xi_t - \frac{\lambda \Xi(l_t + \xi_t) - \lambda l_t}{\lambda \Xi + (1-\lambda)},
\end{aligned}
$$

$$
\begin{aligned}
& -(\theta_L - 1)\frac{\lambda}{1-\lambda} l_t \\
& + \theta_h^{-1} h_t - \theta_h^{-1}\phi_L(1-\lambda)^{-1}\lambda l_t + \theta_c^{-1} c_t + \varepsilon_t^h \\
& = c_t - \frac{\Xi}{1-\Xi}\xi_t - \frac{\eta - 1}{\eta} f_t,
\end{aligned}
$$

$$
\begin{aligned}
& -(\theta_L - 1)\frac{\lambda}{1-\lambda} l_t \\
& + \theta_h^{-1}\left(\frac{1}{1+\gamma\delta}\frac{y_t - \delta w_t - B l_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a - \varepsilon_t^h\right) + \varepsilon_t^h \\
& - \theta_h^{-1}\phi_L(1-\lambda)^{-1}\lambda l_t \\
& + (\theta_c^{-1} - 1)\left(y_t + f_t - \epsilon\left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t\right] - \varepsilon_t^g\right) \\
& = -\frac{\Xi}{1-\Xi}\xi_t - \frac{\eta - 1}{\eta} f_t.
\end{aligned}
$$

$$
\begin{aligned}
0 = & \left(\theta_c^{-1} + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} - 1\right) y_t - \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} w_t \\
& - \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a + \varepsilon_t^h - (\theta_c^{-1} - 1)\varepsilon_t^g \\
& - \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} B + (\theta_L - 1)\frac{\lambda}{1-\lambda} + \theta_h^{-1}\phi_L \frac{\lambda}{1-\lambda}\right) l_t \\
& + (\theta_c^{-1} - 1)\left\{f_t - \epsilon\left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t\right)\right\} \\
& + \frac{\Xi}{1-\Xi}\xi_t + \frac{\eta - 1}{\eta} f_t.
\end{aligned}
\tag{B.128}
$$

B.7 Calibration from Steady-State Conditions

To calibrate the parameter associated with the fraction of loyal customers ϕ_L given θ_L, we examine the steady state conditions.

From equations (B.39) and (B.41)

$$\frac{\upsilon_H(H_t + \phi_L H \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}})}{\upsilon_C(C_t)} = \frac{W_t}{P_t} \digamma_t \left(\frac{P_{B,t}}{P_t}\right)^{-\epsilon},$$

$$\begin{aligned}&\theta_L \phi_L H (1-L_t)^{-1} \frac{\upsilon_H(H_t + \phi_L H \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}})}{\upsilon_C(C_t)} \\ &= \frac{\eta}{\eta-1} C_t \frac{1-\Xi}{L_t \Xi + (1-L_t)},\end{aligned}$$

we can obtain the following steady state condition:

$$\begin{aligned}&\theta_L \phi_L H (1-\lambda)^{-1} \frac{W}{P} \digamma \left(\frac{P_B}{P}\right)^{-\epsilon} \\ &= \frac{\eta}{\eta-1} C \frac{1-\Xi}{\lambda \Xi + (1-\lambda)}.\end{aligned}$$

Substituting equation (B.43), that is, $Y = C/\left(\digamma \cdot \left(\frac{P_B}{P}\right)^{-\epsilon}\right)$ into the above equation, we obtain

$$\begin{aligned}&\theta_L \phi_L (1-\lambda)^{-1} \frac{WHQ}{PQY} \\ &= \frac{\eta}{\eta-1} \frac{1-\Xi}{\lambda \Xi + (1-\lambda)}.\end{aligned} \tag{B.129}$$

As for the right-hand side of the equation, Ξ is given by equation (B.31):

$$\Xi = \frac{s\mu^{\epsilon \frac{1-\eta}{\eta}} + (1-s)}{(s\mu^{1-\eta} + (1-s))^{\frac{\epsilon}{\eta}}}. \tag{B.130}$$

As for the left-side of the equation, we can obtain WH/PQ using the fact that from equation [26] in GS firms' optimal normal price satisfies

$$\frac{P_N}{P} = \frac{\epsilon}{\epsilon-1} \frac{X}{P} = \frac{\epsilon}{\epsilon-1} \frac{WH}{\alpha PQ}. \tag{B.131}$$

From the definition of the price index, we can rewrite P_N/P as follows:

$$\begin{aligned}
\frac{P_N}{P} &= \frac{P_N}{\{\lambda P_L^{1-\epsilon} + (1-\lambda)P_B^{1-\epsilon}\}^{\frac{1}{1-\epsilon}}} \\
&= \frac{P_N/P_L}{\left\{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}} \\
&= \frac{1}{\left\{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}} \frac{P_N}{\{sP_S^{1-\epsilon} + (1-s)P_N^{1-\epsilon}\}^{\frac{1}{1-\epsilon}}} \\
&= \frac{1}{\left\{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}} \frac{1}{\{s\mu^{1-\epsilon} + 1 - s\}^{\frac{1}{1-\epsilon}}}.
\end{aligned} \tag{B.132}$$

The relationship between the sum of output of products, Q, and aggregate demand for them, Y, is given by

$$\begin{aligned}
Q &= sQ_S + (1-s)Q_N \\
&= s(\lambda + (1-\lambda)v_S)\left(\frac{P_S}{P}\right)^{-\epsilon} Y \\
&+ (1-s)(\lambda + (1-\lambda)v_N)\left(\frac{P_N}{P}\right)^{-\epsilon} Y,
\end{aligned}$$

where

$$\begin{aligned}
v_S = \left(\frac{P_S}{P_B}\right)^{-(\eta-\epsilon)} &= \left(\frac{P_S}{\overline{h}P_L}\right)^{-(\eta-\epsilon)} \\
&= \left(\frac{1}{\overline{h}} \frac{\mu}{\{s\mu^{1-\epsilon} + 1 - s\}^{\frac{1}{1-\epsilon}}}\right)^{-(\eta-\epsilon)}
\end{aligned}$$

$$\begin{aligned}
v_N = \left(\frac{P_N}{P_B}\right)^{-(\eta-\epsilon)} &= \left(\frac{P_N}{\overline{h}P_L}\right)^{-(\eta-\epsilon)} \\
&= \left(\frac{1}{\overline{h}} \frac{1}{\{s\mu^{1-\epsilon} + 1 - s\}^{\frac{1}{1-\epsilon}}}\right)^{-(\eta-\epsilon)}.
\end{aligned}$$

Therefore, we obtain

$$
\begin{aligned}
\frac{Q}{Y} &= s(\lambda+(1-\lambda)v_S)\left(\frac{P_S}{P}\right)^{-\epsilon} \\
&+(1-s)(\lambda+(1-\lambda)v_N)\left(\frac{P_N}{P}\right)^{-\epsilon} \\
&= s\left\{\lambda+(1-\lambda)\left(\frac{1}{\overline{h}}\frac{\mu}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right)^{-(\eta-\epsilon)}\right\} \\
&\cdot\left\{\frac{1}{\left\{\lambda+(1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}}\frac{\mu}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right\}^{-\epsilon} \\
&+(1-s)\left\{\lambda+(1-\lambda)\left(\frac{1}{\overline{h}}\frac{1}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right)^{-(\eta-\epsilon)}\right\} \\
&\cdot\left\{\frac{1}{\left\{\lambda+(1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}}\frac{1}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right\}^{-\epsilon}.
\end{aligned} \tag{B.133}
$$

Equation (B.129) together with equations (B.130), (B.131), (B.132), and (B.133) yields the steady state conditions to calibrate ϕ_L given θ_L.

B.8 Summary of the Model

Equation [A.9a] in GS becomes equation (B.102):

$$
\begin{aligned}
\pi_t &= \beta E_t\pi_{t+1} \\
&+\frac{1}{1-\psi}\left\{\kappa x_t+\psi(\Delta x_t-\beta E_t\Delta x_{t+1})+\kappa A l_t+A(\Delta l_t-\beta E_t\Delta l_{t+1})\right\}.
\end{aligned} \tag{B.134}
$$

The equivalent to equation [A.9b] in GS is given by equation (B.118):

$$
x_t=\frac{1}{1+\gamma\delta}w_t+\frac{\gamma}{1+\gamma\delta}\left(y_t-Bl_t\right). \tag{B.135}
$$

Equation [A.9c] in GS becomes equation (B.127):

$$\begin{aligned}\pi_{W,t} &= \beta\pi_{W,t+1} \\ &+ \frac{(1-\phi_w)(1-\beta\phi_w)}{\phi_w}\frac{1}{1+\varsigma\theta_h^{-1}} \\ &\left[\left(\theta_c^{-1} + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right)y_t - \left(1 + \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right)w_t\right. \\ &- \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a + \varepsilon_t^h - \theta_c^{-1}\varepsilon_t^g \\ &- \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}B + \theta_h^{-1}\theta_L\phi_L\frac{\lambda}{1-\lambda}\right)l_t \\ &\left.-(1-\theta_c^{-1})\left\{f_t - \epsilon\left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right)\right\}\right].\end{aligned} \tag{B.136}$$

Equation [A.9d] in GS holds:

$$\Delta w_t = \pi_{W,t} - \pi_t. \tag{B.137}$$

Equation [A.9e] in GS becomes equation (B.122):

$$\begin{aligned}y_t &= E_t y_{t+1} - \theta_c(i_t - E_t\pi_{t+1}) + (\varepsilon_t^g - E_t\varepsilon_{t+1}^g) \\ &+ (1-\theta_c)\left\{\Delta f_{t+1} - \epsilon\left(\Delta x_{t+1} + \frac{1}{(\eta-\epsilon)(1-\lambda)}\Delta l_{t+1}\right)\right\}.\end{aligned} \tag{B.138}$$

The monetary policy rule is given by

$$i_t = \rho i_{t-1} + (1-\rho)\phi_\pi\pi_t^N + e_t^i. \tag{B.139}$$

The fraction of loyal customers is given by equation (B.128):

$$\begin{aligned}0 &= \left(\theta_c^{-1} + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} - 1\right)y_t - \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}w_t \\ &- \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a + \varepsilon_t^h - (\theta_c^{-1} - 1)\varepsilon_t^g \\ &- \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}B + (\theta_L - 1)\frac{\lambda}{1-\lambda} + \theta_h^{-1}\phi_L\frac{\lambda}{1-\lambda}\right)l_t \\ &+ (\theta_c^{-1} - 1)\left\{f_t - \epsilon\left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right)\right\} \\ &+ \frac{\Xi}{1-\Xi}\xi_t + \frac{\eta-1}{\eta}f_t.\end{aligned} \tag{B.140}$$

The consumption wedge is given by equation (B.120) with (B.121):

$$f_t = \frac{\eta}{\eta - 1}\lambda\frac{\Xi\xi_t - (1-\Xi)l_t}{\lambda\Xi + (1-\lambda)}, \tag{B.141}$$

where

$$\begin{aligned}\xi_t = &\frac{(\mu^{\epsilon\frac{1-\eta}{\eta}} - 1)\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}(\mu^{1-\eta} - 1)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}} ss_t \\ &+ \frac{\mu^{\epsilon\frac{1-\eta}{\eta}}\epsilon\frac{1-\eta}{\eta}\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}\mu^{1-\eta}(1-\eta)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}} s\mu_t. \end{aligned} \tag{B.142}$$

The ratio of the sale price to the normal price is given by equation (B.104):

$$\mu_t = \frac{1}{1-\psi}\left(x_t + Al_t\right). \tag{B.143}$$

The frequency of sales is given by equation (B.112):

$$ss_t = -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}x_t - \left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi} + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right)l_t. \tag{B.144}$$

Production input is given by equation (B.125):

$$h_t = \frac{1}{1+\gamma\delta}\frac{y_t - \delta w_t - Bl_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a. \tag{B.145}$$

The Phillips curve for the normal price index is given by equation (B.101):

$$\pi_{N,t} = \pi_t - \frac{\psi}{1-\psi}\Delta x_t - \frac{A}{1-\psi}\Delta l_t. \tag{B.146}$$

C Other Simulated Variables

The aim of our study was not to construct a DSGE model that improves the goodness of fit of macroeconomic variables such as the inflation rate or GDP growth by incorporating temporary sales and endogenous bargain hunting. Rather, the focus of our study was to model the frequency of temporary sales using the actual path of hours worked. Nevertheless, it is important to examine how our model performs in explaining changes in inflation and GDP growth. Unfortunately, it performs very poorly. Figure B shows that the paths of the inflation rate and the GDP growth rate simulated by our model are very different from the actual paths. This poor performance suggests that we need to construct a richer model by incorporating many features not included here, such as capital investment with adjustment cost and the zero lower bound on nominal interest rates.

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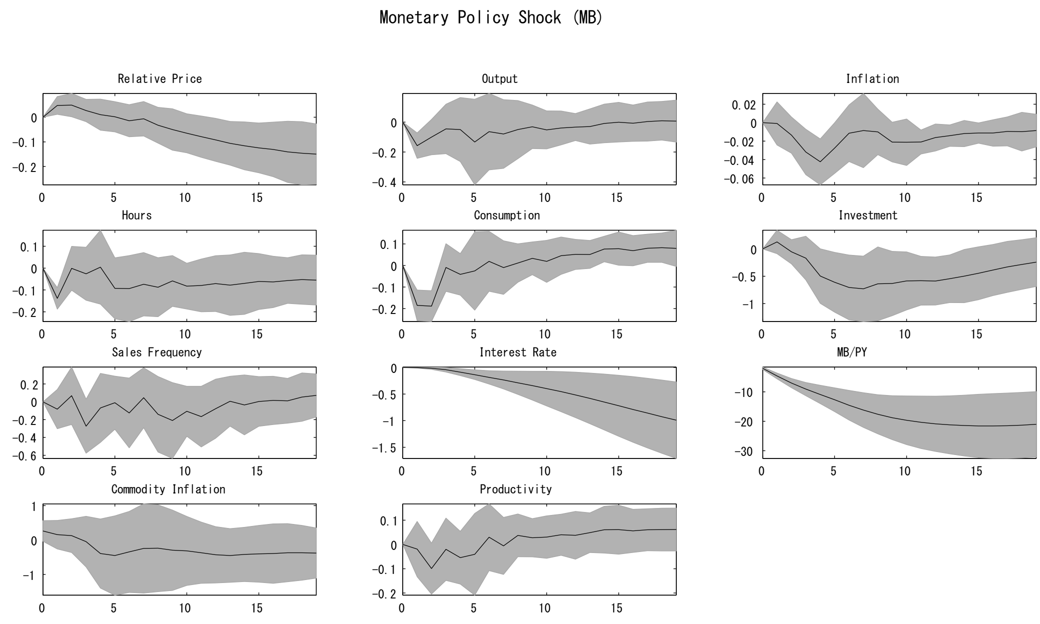

Figure A: Response for a Monetary Policy Shock through the Monetary Base

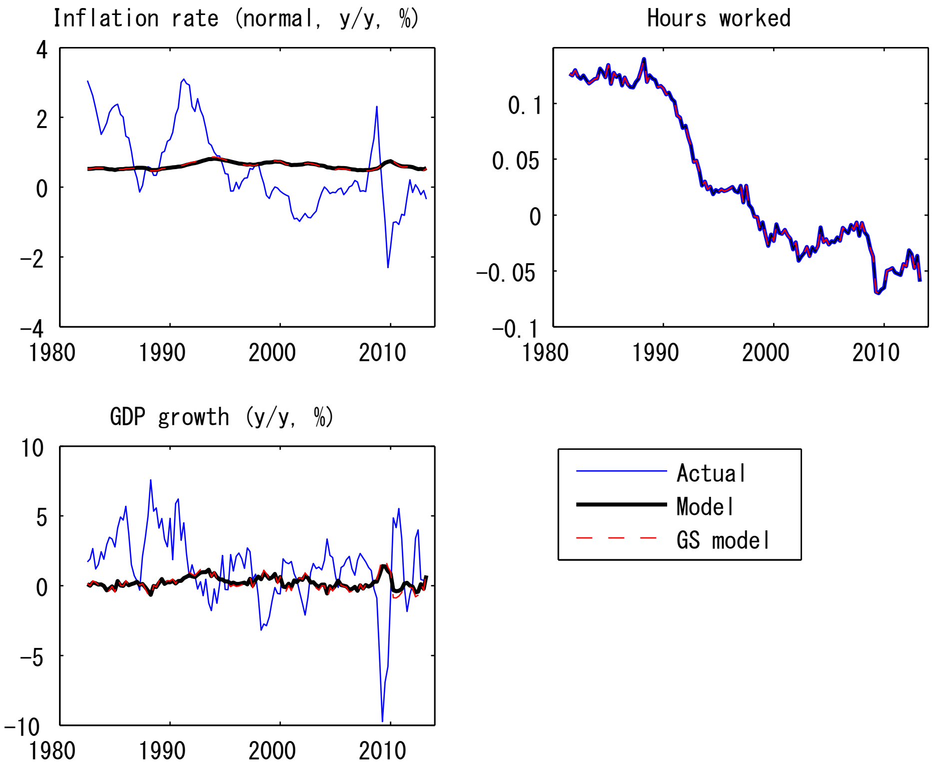

Figure B: Simulated and Actual Paths of Macroeconomic Variables