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Income inequality and macroeconomic instability: a stock-flow consistent approach with heterogeneous agents

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Laura Carvalho
Department of Economics - University of Sao Paulo, Brazil

Corrado Di Guilmi
Business School - University of Technology, Sydney and
Centre for Applied Macroeconomic Analysis (CAMA), ANU

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Keywords

Stock-flow consistent model, heterogeneous agents, master equation, income inequality, financial instability

JEL Classification

C63, D31, E16, E21, E25

Address for correspondence:

(E) cama.admin@anu.edu.au

Income inequality and macroeconomic instability: a stock-flow consistent approach with heterogeneous agents*

Laura Carvalho, Corrado Di Guilmi◇

Department of Economics - University of Sao Paulo, Brazil.
Business School - University of Technology, Sydney. Australia and Centre for Applied Macroeconomic Analysis, Australian National University.

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◇Corresponding author: Corrado Di Guilmi. University of Technology, Sydney - PO Box 123, Broadway, NSW 2007, Australia. Ph.: +61295147743, Fax: +61295147711. E-mail: corrado.diguilmi@uts.edu.au.

1 Introduction

After the striking evidence presented in Piketty and Saez (2003) and extended in Piketty (2014)'s recent book, it became widely acknowledged that the degree of income inequality has not only been increasing as a whole in most advanced economies, but also that the working rich have replaced the rentiers at the top of the income distribution in the United States. In this context, several empirical and theoretical attempts have been made in recent economic literature to connect this phenomenon to the sources of the 2008 crisis and the slow recovery of aggregate demand in its aftermath.

Studies on the relationship between household consumption, debt accumulation and macroeconomic instability include Dutt (2006), Barba and Pivetti (2009), Kumhof and Ranciere (2010) and Taylor (2010). Focusing on the role of the functional distribution of income, Palley (2012) and Setterfield (2012) have examined the unstable consequences of the fall in the wage share in the US since the 1980s. When it comes to the personal – rather than functional – income distribution, empirical work by Cynamon and Fazzari (2013) has shown that the upsurge in household debt in the 2000s is largely due to an increase in demand relative to disposable income of the bottom 95% of the income distribution in the US. Setterfield and Kim (2013) developed a theoretical model linking the rise in income inequality within the top 10% of income earners to the potential instability in the household debt-to-income ratios due to the presence of emulation effects in consumption behavior.

Starting from such developments, this paper aims to build a more complete framework for the study of the relationship between both the personal and the functional distribution of income, the accumulation of household debt and the emergence of macroeconomic instabilities. It will do so by building a demand-driven stock-flow consistent (SFC) model such as developed by Tobin (1969) and Godley and Lavoie (2007), focusing on the interaction between households – workers and managers –, firms and the financial sector. By taking into account all flows of income between different sectors in the economy as well as their accumulation into financial and tangible assets, these models are able to trace the flows of credit and the accumulation of debt in the economy as a potential source of financial instability[1].

Moreover, in addition to examining aggregate macroeconomic relationships, our framework introduces heterogeneous microeconomic behavior in the household sector, which allows for the incorporation of income inequality among wage earners and its effects on consumption and borrowing patterns. For this purpose, as opposed to embedding an agent-based model into a macroeconomic SFC framework, as done for instance in the work by Godin and Kinsella (2012) and Dosi et al. (2012), the aggregation of heterogeneous agents will be performed in this study by means of an analytical methodology originally developed in statistical mechanics and recently imported into macroeconomics (see Aoki and Yoshikawa, 2006; Di Guilmi, 2008; Foley, 1994; Weidlich, 2000, among others). This modeling approach builds from the idea that, as the economy is populated by a very large number of dissimilar agents, a microfounded analytical model should look at how many agents are in a certain condition, rather than at which agents, and represent their evolution in probabilistic terms. In this paper, workers will be thus divided into two categories - that of *borrowing* and *non-borrowing* – and will be allowed to switch between them depending on their wage, financial inheritance and consumption behavior.

[1]Besides allowing for formal Minskyan analyses of debt accumulation and financial fragility (Dos Santos, 2005), SFC models have recently been used to study the macroeconomic effects of shareholder value orientation and 'financialisation' (Treeck, 2009), as well as that of household debt accumulation (Kim and Isaac, 2010).

Thanks to this approach we are able to isolate the roles of microeconomic heterogeneity and (indirect) interaction among agents. As for the former aspect, our contribution considers not only the role of the functional distribution of income but also the personal distribution and its evolution, which cannot be satisfactorily represented in an aggregative framework. Interaction of workers is represented by the "emulation" effect that drives workers to consume also according to the median level of income, rather than only to their own personal means. In this setting, policies aimed to quantify a suitable minimum level of income or to reduce the gap among wage earners can be assessed from an original perspective.

The paper is organized as follows. Section 2 presents the theoretical model, including all behavioral equations, underlying accounting identities and market equilibria. Section 3 then describes the methodology used to aggregate households in the different categories by means of a master equation. Model results are finally analyzed in Section 4.

2 The model

The economy described is composed by firms, households and a financial sector. As in the conventional neo-Kaleckian literature, prices are set as a mark-up over labor costs, investment behavior is determined independently and the degree of capacity utilization of firms adjusts to the quantity they sell. The mark-up and the functional distribution of income are assumed to (exogenously) depend on the degree of industrial concentration and the relative bargaining power of workers and capitalists.

As opposed to the model presented in Carvalho and Di Guilmi (2014), where firms were heterogeneous, in this paper they will be treated as an aggregate, and the heterogeneity will be introduced in the household sector. In particular, workers will be allowed to finance part of their desired spending by taking up debt, whenever their disposable income is insufficient. The proposed method will allow the share of *borrowing* households to affect macroeconomic dynamics.

The financial sector is considered as an aggregate: its basic role is to provide loans, hence holding debt (or bonds) as an asset, and to create money deposits endogenously as liabilities. The interest rate on loans adds a risk premium to an exogenous risk-free interest rate.

The model will be presented in discrete time, as required for simulation purposes.

2.1 The Firms

Firms prefer to finance their investment with internal resources they have previously accumulated in the form of money M^f and the flow of retained profits A^f. If these are not sufficient they issue bonds.

The investment function has the following specification:

$$i_t = \alpha\,(u_{t-1} - u^d) + \beta\, r_{t-1} + \epsilon\, h_{t-1} \qquad (1)$$

where $i_t = \frac{I_t}{pK_{t-1}}$ is the gross percentual change in capital over time (before depreciation), u_t is the actual degree of capacity utilization, u^d is firms' desired rate of capacity utilization,r_t is the profit rate and $\alpha, \beta > 0$. The variable h is the valuation ratio (Taylor, 2012), which is the ratio between the values of equity and the value of capital assets in the economy. In the present treatment we set it equal to[2]

$$h_t = \frac{Pe_t E_t}{p\,K_t} \tag{2}$$

where Pe is the stock price, p is the final good price and K the aggregate stock of capital assets.

r is defined as the ratio of profits (a constant share Π of total nominal output) to capital:

$$r_t = \frac{\Pi\,Q_t}{K_t} \tag{3}$$

All firms adopt the same Leontief-type technology with constant coefficients to produce a homogeneous good that can be used for consumption or investment. As a consequence, the demand for labor at full capacity can be residually quantified once the stock of capital is determined by investment decisions in the previous periods. The supply of labor is infinitely elastic. Accordingly the production function F gives the potential output $\bar{Q}$.

$$\bar{Q}_t = F(K_t, L_t) \tag{4}$$

with K and L representing, respectively, physical capital and productive labor. Even if excess capital may exist, the output-labor ratio is constant, so that firms are assumed not to hire excess labor [3]. Since technology exhibits fixed coefficients, it is then possible to define the potential output only as a function of capital so that

$$\bar{Q}_t = 1/\gamma\ K_t \tag{5}$$

where the inverse of the capital productivity γ is a constant parameter.

The degree of capacity utilization u is defined as the ratio of actual output Q to potential output $\bar{Q}$, being equal to one at full capacity and smaller than one with excess capacity, so that

$$u_t = \frac{Q_t}{\bar{Q}_t} = \gamma\,\frac{Q_t}{K_t} \leq 1 \tag{6}$$

Hence, with γ as a positive parameter, fluctuations in the degree of capacity utilization of the firms will track changes in the actual output-to-capital ratio[4].

Using expressions (1), (3), (6) and (2), the total level of nominal investment I_t can thus be written as:

$$I_t = (\alpha\gamma + \beta\ \Pi)pQ_{t-1} - \alpha\ \gamma pQ^d_{t-1} + \epsilon\ Pe_{t-1}E_{t-1} \tag{7}$$

where $Q^d_{t-1} = (u^d K_{t-1})/\gamma$

The law of motion for the capital stock is:

$$\Delta K = I_t - \delta K_{t-1} \tag{8}$$

where $0 < \delta < 1$ is the rate of capital depreciation.

[2] We assume that capital goods K are also priced at p.

[3] As described in Dutt (1984), the asymmetry which allows excess capital to exist, but the labor-output ratio to be fixed technologically can be justified based on the simplifying assumption that labor will not be hired if it does not contribute to production, whereas the stock of capital is determined by previous investment decisions and may be held in excess.

[4] The capital-output ratio is therefore greater than γ when there is excess capacity.

Following Kalecki (1971), firms are assumed to set the price as a mark-up on the average cost of labor, while holding excess capacity[5]

$$p = (1+\mu)\frac{w}{\xi}. \tag{9}$$

where w is the average nominal wage rate, the mark-up rate μ is taken as a parameter, and so is labor productivity $\xi = Q/L$, a constant technical coefficient obtained from the production function. The labor share of nominal output Ψ is given exogenously [6]

$$\Psi = \frac{w}{p}\frac{1}{\xi} = \frac{1}{1+\mu} \tag{10}$$

The gross profit share of aggregate output Π will then be given by

$$\Pi = 1 - \Psi = \frac{\mu}{1+\mu} \tag{11}$$

Firms' retained profits are computed as the difference between its gross profits, the net interest it pays[7] and the portion Θ of net profits (defined as gross profits minus interest payments) which is shared with firm's managers. Since the flow of gross profits is given by a constant share Π of firms' output by the mark-up rule, retained profits are given by

$$A_t = (1-\Theta)\left(\Pi p Q_t - i\ B_{f,t-1}\right) \tag{12}$$

Whenever the flow of investment desired by the firm is higher than its retained profits in the period, firms will seek external finance to cover the difference. In particular, firms will finance its investment for a share ϖ with debt and the rest by issuing new equities. Alternatively, whenever retained profits exceed desired investment, firms will pay off debt in proportion ϖ and buyback shares with the rest.

The law of motion for the accumulated stock of firms' debt B_f is then

$$\Delta B_{f,t} = \varpi\left(I_t - A_t\right) \tag{13}$$

The amount of equities evolves according to

$$\Delta E_t = (1-\varpi)\left(I_t - A_t\right)/Pe(t). \tag{14}$$

2.2 The household sector

The household sector is divided into two sub-sectors, namely that of wage earners and managers, who are profit earners. The number of workers in each period of time is given by the level of

[5]See Rowthorn (1982), Dutt (1984) and Taylor (1985) for early models of growth and distribution in this tradition.

[6]Again as in Kalecki (1971), the mark-up and the functional distribution of income are assumed to depend on structural characteristics of goods markets such as the degree of industrial concentration and the relative bargaining power of workers and capitalists. Distribution can also be endogenized in the short-run if the power of bargaining of workers is assumed to depend on the unemployment rate or the rate of capacity utilization, possibly giving rise to Goodwin (1967) type of cycles. Such predator-prey dynamics between demand and distribution are studied for instance in Skott (1989) and Barbosa-Filho and Taylor (2006), but will not be allowed for in this version of the model.

[7] Firms are assumed to pay an interest rate i on bonds B_f

employment in the economy $L = Q/\xi$, whereas the number of managers, as the number of firms, will be assumed fixed. As described in the previous subsection, workers earn wages that add up to a constant share Ψ of total nominal output pQ. Managers (profit earners) receive a share Θ out of firm's net profits. Households' disposable income Y is composed by wage or profit earnings, minus interest payments on debt D in the case of *borrowing* workers, at rate i. Managers allocate their wealth between money M and shares E. Workers keep their total wealth W in money M for simplicity, and earn no interest.

2.2.1 Managers

Disposable income for managers is given by:

$$Y_{\Theta,t} = \Theta[\Pi p Q_t - i\ B_{f,t-1}] \tag{15}$$

Consumption by managers depend positively on their disposable income and wealth, as in Godley and Lavoie (2007)).

$$C_{\Theta,t} = (1 - s_\Theta)Y_{\Theta,t} + (1 - \sigma_\Theta)W_{\Theta,t-1} \tag{16}$$

Substituting Y_Θ from expression (15):

$$C_{\Theta,t} = (1 - s_\Theta)\Theta[\Pi p Q_t - i\ B_{f,t-1}] + (1 - \sigma_\Theta)W_{\Theta,t-1} \tag{17}$$

where $s_\Theta, \sigma_\Theta < 1$ are the propensities to save out of managers' disposable income and wealth.

The wealth of managers is accumulated as an effect of savings $S_\Theta = Y_\Theta - C_\Theta$ and capital gains G on firms' shares they hold.

The law of motion for managers' wealth is then:

$$\Delta W_{\Theta,t} = S_{\Theta,t} + G_t \tag{18}$$

where savings S_Θ are defined as the difference between managers' disposable income and consumption levels $S_{\Theta,t} = Y_{\Theta,t} - C_{\Theta,t}$, and $G_{t+1} = [Pe_{t+1} - Pe_t]E_t$.

In a standard Tobinian specification, managers' demand for firms' shares is assumed to be positively dependent on the previous period's capital gain according to the following functional form:

$$\frac{Pe_t E_t}{W_{\Theta,t}} = \frac{1}{1 + exp[-\lambda_G G_{t-1}]} \tag{19}$$

with $\lambda_G > 0$.

Managers' demand for money is residually determined as

$$M_{\Theta,t} = W_{\Theta,t} - Pe_t E_t \tag{20}$$

2.2.2 Workers

A single worker is identified by the superscript j, while its state (*borrowing* or *non-borrowing*) by the subscript $z = 1, 2$. Thus when a variable is written as x_1^j, it refers to the worker j belonging to state 1; a variable with only the subscript indicates the *mean-field value* (which, in this treatment, is quantified by the average value for the units in the group). Symbols without superscript or subscript refer to aggregate variables.

The number of workers in each group is indicated by N_1 and N_2, with $N_{1,t} + N_{2,t} = L_t$. Accordingly, we can define two types of workers:

- **z = 1**: *borrowing* workers: that finance part or all their consumption with debt accumulation:

$$M^j_{t-1} + Y^j_t < C^j_t. \tag{21}$$

- **z = 2**: *non-borrowing* households: that finance all their consumption with their own resources:

$$M^j_{t-1} + Y^j_t \geq C^j_t. \tag{22}$$

where disposable income Y^j for workers in each class is defined as:

$$Y^j_{t,1} = w^j_t - i\ D^j_{t-1} \tag{23}$$

$$Y^j_{t,2} = w^j_t \tag{24}$$

The share of wages in total output Ψ is exogenous, and thus the average nominal wage rate $w = \sum_{j=1}^{L} w^j/L$ is known, but its distribution among workers is stochastic. We assume that wages w^j follow a Pareto distribution, and thus have a 'fat tail'.

Analogously, savings for each worker S^j are defined as the difference between their disposable income and consumption levels $S^j_t = Y^j_t - C^j_t$.

Each worker holds money M^j as its only asset and debt D^j as a liability. Workers' net worth then evolves according to:

$$\Delta\Omega^j_t = \Delta M^j_t - \Delta D^j = S^j_t = Y^j_t - C^j_t \tag{25}$$

Given the general constraint (25) and conditions (21) and (28), we are implicitly assuming that for *borrowing* workers, the stock of money at the end of period t will go down to zero, and the stock of debt will increase by:

$$\Delta D^j_1 = -(S^j_{t,1} + M^j_{t-1,1}) \tag{26}$$

The law of motion for money in this case is thus:

$$\Delta M^j_1 = -M^j_{t-1,1}. \tag{27}$$

Alternatively, for *non-borrowing* workers, $\Delta D^j_2 = 0$ and the law of motion for the stock of money is given by:

$$\Delta M^j_2 = S^j_{t,2}. \tag{28}$$

The total amount of money deposits (wealth) held by workers at time t then evolves according to:

$$\Delta M_w = -\sum_j M^j_{t-1,1} + \sum_j S^j_{t,2} \tag{29}$$

The total amount of debt held by workers at time t will evolve according to:

$$\Delta D_w = \Delta \left(\sum_j D^j_1 \right) = -\sum_j (S^j_{t,1} + M^j_{t-1,1}) \tag{30}$$

Finally, workers' consumption spending will also be a positive function of their income and net wealth. The two distinctive features in workers' consumption decision are:

- workers' consumption decision depends on wages 'before' interest payments, as in Setterfield and Kim (2013);
- consumption for each worker will be assumed, as in Carvalho and Rezai (2013), to also depend on the difference between their own wage w^j and the wage of the median worker $\bar{w}$.

The second feature captures empirical evidence that the consumption-to-income ratio is the higher, the lower is the household's personal income relatively to median income, or the more we move toward the bottom quantiles of personal income distribution. In other words, relatively poor consumers tend to spend a larger share of their total income due to emulation effects.

$$C^j_t = (1 - s_\Psi)w^j_t + \eta\,(\bar{w} - w^j_t) + (1 - \sigma_\Psi)\Omega^j_{t-1} \tag{31}$$

where $0 < s_\Psi, \sigma_\Psi < 1$ are the propensities to save out of workers' income and wealth, and $0 < \eta < 1$ is the sensitivity to workers' relative income, which determines the force of the relative consumption effect.

Total consumption for workers C_w is thus given by:

$$C_{w,t} = \sum_j C^j_t = (1 - s_\Psi - \eta)\Psi\, p\, Q_t + \eta\, L_t\, \bar{w} + (1 - \sigma_\Psi)(M_{w,t-1} - D_{1,t-1}) \tag{32}$$

2.3 The financial sector

Banks are considered as an aggregate sector. It gives loans to firms and workers, hence holding their debt as an asset, and creates money deposits endogenously as liabilities. It receives an interest rate i on firms' and workers' loans.

Interest receipts from worker and firm loans allow the financial intermediary to accumulate net worth. The law of motion for the financial sector is hence given by:

$$\Delta\Omega_b = i\,[D_{w,t-1} + B_{f,t-1}] \tag{33}$$

where Ω_b is the net worth and:

$$M_t = M_{w,t} + M_{f,t} + M_{\Theta,t}$$

2.4 Goods market equilibrium

Nominal output pQ is determined by aggregate consumption C and total investment I.

$$pQ_t = C_{w,t} + C_{\Theta,t} + I_t$$

After substituting I_t from expression (7), C_w from (32) and C_Θ from (17), as well as the profit share Π from expression (11) and solving for pQ we obtain:

$$pQ_t = \frac{\eta \; L_t \; \bar{w} + F_{t-1}}{1 - [(1 - s_\Psi - \eta)\Psi \; + (1 - s_\Theta)\Theta\Pi]}$$

where

$$F_t = (\alpha\gamma + \beta\Pi)pQ_t - \alpha \; pQ_t^d + (1 - \sigma_\Psi)(M_{w,t} - D_{w,t}) - (1 - s_\Theta)\Theta[i \; B_{f,t}] + (1 - \sigma_\Theta)W_{\Theta,t} + \epsilon \; Pe_t E_t$$

Further, substituting the level of employment from $L_t = Q_t/\xi$ yields:

$$Q_t^\star = \frac{F_{t-1}}{p \; [1 - (1 - s_\Psi - \eta)\Psi \; - (1 - s_\Theta)\Theta\Pi] - \frac{\eta\bar{w}}{\xi}} \tag{34}$$

2.5 Financial market equilibrium

The price of equities will equilibrate the total stock of equities supplied by firms as defined by expression (14), which as previously described will increase or decrease over time depending on the sign of $I - A$, and managers' demand for firms' equities from expression (19). Substituting (14) in (19) yields the equilibrium price of equities Pe:

$$Pe_t^\star = \frac{1}{E_{t-1}} \left[\frac{W_{\Theta,t}}{1 + exp[-\lambda_G G_{t-1}]} - (1 - \varpi)\,(I_t - A_t) \right] \tag{35}$$

Table 1 shows the balance sheets of firms, workers, managers and banks. Table 2 illustrates the social accounting matrix for our economy.

Firms		Managers		Workers		Banks	
K	B_f	M_Θ		M^j	D^j	B_f	M
	$P_e E$	$P_e E$				D_w	
	Ω_f		Ω_Θ		Ω^j		Ω_b

Table 1: Units and sectoral balance sheets

	Households	Firms		Banks	Total
		Current	Capital		
Consumption	$-(C_\Theta + C_w)$	$+C$			0
Investment		$+I$	$-\left[(\alpha\gamma + \beta\Pi)pQ_{t-1} - \alpha\, p\gamma Q^d_{t-1} + \epsilon P_{e,t-1}E_{t-1}\right]$		0
Wages	$+\Psi pQ$	$-\Psi pQ$			0
Profits	$+\Theta(\Pi pQ - i\ B_{f,t-1})$	$-(\Pi pQ - i\ B_{f,t-1}$	$+(1-\Theta)(\Pi pQ - i\ B_{f,t-1})$		0
Loan interests	$-N_1 i\ D_{1,t-1}$	$-i\ B_{f,t-1}$		$\Delta\Omega_b$	0
Change in loans	$-N_1[S_1 + M_{1,t-1}]$		$+[\bar{\varpi}(I-A)]$	$-(\Delta B + \Delta D_w)$	0
Change in deposits	$[N_1 M_{1,t-1}] - N_2 S_2 - \Delta M_\Theta$		$-(A-I)$	$+\Delta M$	0
Change in equities	$-Pe\Delta E$		$+[(1-\bar{\varpi})(I-A)]$		0
Total	0	0	0	0	0
Capital Gains	$-\Delta G$		$+\Delta P_e E$		0

Table 2: Matrix of flows.

3 Worker's dynamics

In order to provide an analytical solution that takes into account the heterogeneity of households, we shift the focus on the two types of households (borrowing and non-borrowing) and, in particular, on the density of individuals in each group. The master equation provides a valuable analytical tool to dynamically represent the evolution of these densities.

3.1 Transition probabilities

The micro-probability of a household transitioning from one state to another is determined by its capacity of satisfying conditions (21) or (28). We consider that wages are initially distributed according to a generalized Pareto distribution. Their evolution is determined by an idiosyncratic shock g, which will be assumed uniformly distributed in the interval [-0.25,0.25]. Its cumulative distribution is indicated by $F[g]$. Consequently, $w_t^j = w_{t-1}^j(1 + g_t^j)$.

For analytical tractability, we focus in the following on the mean-field variables, thus on the average values of the micro-variables for the two groups of households. Substituting $Y_{1,t} = w_{1,t-1}(1 + g_{1,t}) - i_{1,t}D_{1,t}$ and $Y_{2,t} = w_{2,t}(1 + g_{2,t})$ in the (mean-field) consumption functions we get

$$C_{1,t} = (1 - s_\psi)w_{1,t}(1 + g_{1,t}) + \eta[\bar{w}_t - w_{1,t}(1 + g_{1,t})] + (1 - \sigma_\psi)(W_{1,t-1} - D_{1,t-1}) \tag{36}$$

$$C_{2,t} = (1 - s_\psi)w_{2,t}(1 + g_{2,t}) + \eta[\bar{w}_t - w_{2,t}(1 + g_{1,t})] + (1 - \sigma_\psi)W_{2,t-1} \tag{37}$$

Accordingly, using condition (21) and considering that for workers $W^j = M^j$ a non-borrowing household becomes borrowing if

$$g_{2,t} < \frac{\eta\bar{w}_t - \sigma_\psi M_{2,t-1}}{w_{2,t}(s_\psi + \eta)} - 1 \equiv \Gamma_{2,t} \tag{38}$$

Recalling that the shocks are assumed to be uniformly distributed, using the uniform cumulative distribution function we have that

$$\zeta_t = prob(g_{2,t} < \Gamma_{2,t}) = 1 - F[\Gamma_{2,t}] = 0.5 - 2\ \Gamma_{2,t} \tag{39}$$

In the same way, using condition (28), a borrowing household becomes non-borrowing if

$$g_{1,t} \geq \frac{\eta\bar{w}_t + D_{1,t-1}(i + \sigma_\psi - 1) - \sigma_\psi M_{1,t-1}}{w_{1,t}(s_\psi + \eta)} - 1 \equiv \Gamma_{1,t} \tag{40}$$

Using the uniform cumulative distribution function we have that

$$\lambda_t = prob(g_{1,t} \geq \Gamma_{1,t}) = F[\Gamma_{1,t}] = 0.5 + 2\ \Gamma_{1,t} \tag{41}$$

These probabilities concern the transition of an individual from one state to another, thus they can be defined as micro-probabilities. In order to quantify the number of transitions from one state to the other, we need to weight these probabilities by the number of agents in each state. Consequently, the configurational transition rates (Weidlich, 2000) read as

$$\omega_{+,t} = N_{1,t}\lambda_t \tag{42}$$

$$\omega_{-,t} = N_{2,t}\zeta_t \tag{43}$$

The master equation describes the dynamics of the the probability of having N_1 household in state 1 in a given instant, assuming that the numbers of firms in the two states evolve according to a jump Markov process (Aoki, 2002; Di Guilmi, 2008). For analytical convenience we now consider time intervals that are small enough to be approximated by a continuous time reference. The master equation can be formulated as the balance equation between the aggregate transitions to and from state 1 and expressed as

$$\frac{dP(N_1,t)}{dt} = \omega_+(t)P(N_1-1)(t) + \omega_-(t)P(N_1+1)(t) + [\omega_+(t)+\omega_-(t)]\,P(N_1)(t) \tag{44}$$

The probability of having a number N_1 of borrowing household is given by the probability of transitioning from a number N_1-1 to N_1 plus the probability of transitioning from a number N_1+1 to N_1, less the probability of observing a number N_1 of borrowing households times the probability of a transition into or from the borrowing state.

3.2 Master equation's solution

The solution method for the master equation introduced by Di Guilmi (2008), developing Landini and Uberti (2008), yields a system of two equations. The first one is an ordinary differential equation which describes the time evolution of the trend of the stochastic process. The second one is a partial differential equation, known as Fokker-Planck equation, whose general solution identifies the probability distribution of the fluctuations around the drift component[8]. This solution technique splits the state variable in two components (as proposed by Aoki, 2002) according to

$$N_1(t) = N\bar{N}_1(t) + \sqrt{N}(t)v \tag{45}$$

The factor $\bar{N}_1$ is the trend and represents the deterministic component; the variable v is the spread and quantifies the stochastic noise around the trend. The solution derives an equation for each of the components. In particular, the trend evolves according to the following ODE

$$\frac{d\bar{N}_1}{d\tau} = \lambda\bar{N}_1 - (\lambda+\zeta)\bar{N}_1^2 \tag{46}$$

where $\tau = t/N$. The solution for the spread yields the Fokker-Planck equation for the noise, whose stationary distribution is given by the following Gaussian distribution

$$\theta(u) = C\exp\left(-\frac{v^2}{2\sigma^2}\right) \quad : \quad \sigma^2 = \frac{\lambda\zeta}{(\lambda+\zeta)^2} \tag{47}$$

The dynamics of n_1 can be therefore described by

$$\frac{dN_1(t)}{dt} = \lambda\bar{N}_1(t) - (\lambda+\zeta)\bar{N}_1^2(t) + \sigma\, dV(t) \tag{48}$$

where dV is a stationary Wiener increment and $\sigma\, dV$ is the stochastic fluctuation component in the proportion of households of type 1, coming from the distribution (47).

As $t\to\infty$ equation (48) can be expressed as

$$\dot{N}_1(t) = \lambda N_1(t) - (\lambda+\zeta)N_1^2(t) \tag{49}$$

which has a logistic dynamic and a steady state value of

$$N_1 = \frac{\lambda}{\lambda+\zeta} \tag{50}$$

[8]For a full detail of the solution method we refer the reader to Di Guilmi (2008), Chiarella and Di Guilmi (2011) and Landini and Uberti (2008).

3.3 Mean-field equations and stability analysis

The solution of the master equation (48) provides the first equation of the dynamical system that describes the evolution of the economy. Most importantly, the solution embodies the transition rates and, as a consequence, it is possible to express the dynamics of the macro-variables of interest as functions of households' micro-variables. By expressing equation (30) in continuous time for analytical purposes, the law of motion for the stock of household debt can be written as:

$$\dot{D} = N_1 \dot{D}_1 + \dot{N}_1 D_1 \tag{51}$$

where

$$\dot{D}_1 = (C_1(t) - Y_{1t} - M_1(t - dt)) \tag{52}$$

and from (49), (41) and (39), we have:

$$\dot{N}_1 = (0.5 + 2\,\Gamma_1(t))N_1(t) - [1 + 2\,(\Gamma_1(t) - \Gamma_2(t))]\,N_1^2(t) \tag{53}$$

Workers' debt becomes constant when $C_1(t) - Y_1(t) - M_1(t - dt) = 0$. At the steady-state where all flows and stocks are constant, we have from (27) that $\tilde{M}_1 = 0$.

From (36) and (23), we have the steady-state level of workers' debt $\tilde{D}_1$ as given by:

$$\tilde{D}_1 = \frac{1}{(1 - \sigma_\Psi) - i}\left[\eta\,\bar{w} - (s_\Psi + \eta)(1 + g_1)w_1\right] \tag{54}$$

Hence, the higher is the median wage $\bar{w}$ and the lower is the *borrowing* workers' wage w_1, the higher is the equilibrium level of household debt. This level also increases with the loan interest rate and decreases with the propensity to consume out of net wealth, which gives the negative effect of debt in consumption decisions.

The steady-state for the number of *borrowing* workers is:

$$\tilde{N}_1 = \frac{0.5 + \Gamma_1}{1 + 2(\Gamma_1 - \Gamma_2)} \tag{55}$$

From (51), the dynamics and stability of household debt depend on the dynamics of the two-dimensional system composed of $\dot{N}_1$ and $\dot{D}_1$.

$$\frac{\partial \dot{D}_1}{\partial D_1} = \frac{\partial C_1}{\partial D_1} - \frac{\partial Y_1}{\partial D_1} = i - (1 - \sigma_\Psi) \tag{56}$$

Borrowing workers' debt will only be locally stable if the interest rate charged for worker loans is low relative to the (net) wealth effect on consumption. In other words, even if the interest rate is low, a small negative effect from the stock of debt on consumption decisions may create locally unstable dynamics.

In order to analyze the joint stability of D_1 and N_1, we need to compute the Jacobian matrix of the system and evaluate it at the steady-state $\tilde{N}_1$ and $\tilde{D}_1$:

$$J_{\tilde{N}_1,\tilde{D}_1} = \begin{bmatrix} i - (1 - \sigma_\Psi) & 0 \\ 2\,\tilde{N}_1(1 + \tilde{N}_1)\frac{i - (1 - \sigma_\Psi)}{w_1(s_\psi + \eta)} & -(3.5 + g_1) \end{bmatrix}$$

Considering that the expected value of g_1 is zero, the trace is given by:

$$Tr(J) = i - (1 - \sigma_\Psi) - 3.5 = i + \sigma_\Psi - 4.5 \tag{57}$$

The trace will be positive and household debt will be unstable if and only if $i + \sigma_\psi > 4.5$.

The determinant is given by:

$$Det(J) = 3.5[(1 - \sigma_\Psi) - i] \tag{58}$$

If both N_1 and D_1 reach their steady-state values, and so do M_1 and M_2, the steady-state value for aggregate household debt D will be equal to:

$$\tilde{D} = \frac{(1 - \sigma_\Psi)[\eta\, \bar{w} + (s_\Psi + \eta)(w_2 - w_1)]}{2(1 - \sigma_\Psi - i)[w_2(s_\Psi + \eta)(1 + \sigma_\Psi) - 2\eta\bar{w}]} \tag{59}$$

Hence, the equilibrium level of aggregate debt responds positively to the interest rate and to the wage differential between *non-borrowing* and *borrowing* workers. It has an ambiguous response to the median wage $\bar{w}$, if for local stability of D_1 we assume that $1 - \sigma_\Psi - i < 0$.

4 Numerical Analysis

As specified in section 1, the model is initially simulated as agent based with full heterogeneity of households. In other words, the behavioral rules introduced in subsection 2.2.2 are applied on $L = 1,000$ potentially heterogeneous workers. At each time step the agent based model provides the mean-field values of the micro-variables. In this stage heterogeneity is reduced to two types of workers (borrowing and non-borrowing) by taking the average of the relevant variables within each group of workers. Subsequently, we identify a benchmark scenario and study the dynamics of relevant macroeconomic variables generated by shocking certain parameters. The numerical results can thus complement the analysis of the mean-field equations and steady-state solutions made in the previous section.

In the benchmark scenario the values of the parameters are $\gamma = 1; \alpha = 0.2; \beta = 0.1; \epsilon = 0.1; \delta = 0.025; u^d = 0.8; \xi = 1; \varpi = 0.05; s_\Psi = 0.05; \sigma_\Psi = 0.8; \eta = 0.3; s_\Theta = 0.57; \sigma_\Theta = 0.9; i = 0.035; \Theta = 0.5; \lambda_G = 0.2; P = 3$. Moreover, as previously described, the initial distribution of salaries is set as a Pareto, with benchmark location parameter (minimum salary) $w_{min} = 0.1$ and shape equal to 1.

Since initial salaries only suffer uniformly distributed shocks with mean zero, an increase in the location parameter w_{min} shifts the entire salary distribution to the right, and thus increases the mean salary w and the wage share Ψ of the economy, without any effect on the degree of wage inequality. In other words, the response of the economy to changes in w_{min} allows us to interpret the impact of shifts in the functional distribution of income.

The shape of the Pareto distribution, on the contrary, affects the personal (size) distribution of wages. The lower is the shape parameter, the 'fatter' is the tail of the distribution, and the higher is the degree of inequality. It is important to note, however, that changes in the shape also affect the mean and median salaries, and thus represent a shift both in the personal and the functional income distribution. In particular, the higher the shape, the lower the median salary, with the response of the mean salary being positive in our particular case.

By means of Monte Carlo simulations, we analyze the average response of aggregate demand, household debt and the degree of 'financialisation', measured as the ratio of market capitalization to GDP, $(Pe\ E)/Q$, to variations: 1. in the initial minimum wage w_{min}, capturing shifts the functional distribution of income; 2. in the shape of the Pareto distribution of wages, capturing shifts in the personal and functional distribution of income; 3. in the share of profits distributed to managers θ, and 4. in the interest rate. The average for each variable was computed from 1,000 replications of the simulation for each specific value of the parameter in exam, with each run of the simulation using a fresh set of random numbers.

Figure 1 shows the response of selected variables to an increase in the initial value of the location parameter of the wage Pareto distribution, namely the minimum wage w_{min}. As made evident by the positive response of aggregate demand Q, our benchmark scenario is that of a wage-led economy. In other words, a rise in the minimum wage, which increases the mean salary w and the wage share Ψ, has a positive effect on the economy's GDP level. Even if simulations show that the level of debt by borrowing workers D_1 decreases, aggregate debt D increases, as the total number of workers L is higher due to the rise in output and employment. This explains the observed increase in the debt-to-GDP ratio for higher minimum wages. However, the ratio between aggregate debt and workers' income, which is the appropriate measure for workers' debt sustainability, decreases substantially in the simulations. The lower profit share, which reduces managers's savings and hence their demand for firms' equities, when combined to the higher level of GDP, lead to a decrease in our proxy for 'financialisation' $(PeE)/Q$. Hence, in our benchmark scenario, a redistribution of income toward wages reduces the economy's financial fragility in general.

These results are even stronger when the shape parameter of the wage Pareto distribution is raised, as shown in Figure 2. An increase in the shape parameter, which converts itself into a reduction in workers' personal income inequality and median wage, increases aggregate demand, reduces the level of debt D_1, the debt-to-GDP and the debt-to-workers' income ratios, and also lowers the economy's degree of 'financialisation'. These results reinforce the analysis of the mean-field equations presented in subsection 3.3: a lower wage inequality, by reducing the difference between *borrowing* workers' mean wage and the median wage, decreases consumption relative to income and hence decelerate household debt accumulation. The reduction in 'financialisation' here is mostly driven by the increase in GDP, even if the wage share is also higher in some cases due to the role of the shape parameter in determining the mean wage in the Pareto distribution.

The responses of the meso and macro-variables to an interest rate rise go in the opposite direction, and complement the findings of subsection 3.3. As presented in Figure 3, the reduction in the level of aggregate demand[9] in this case, due to the lower level of firms' retained earnings and households' disposable income, comes together with a higher level of workers' debt D_1, as well as higher aggregate household debt-to-GDP and debt-to-income ratios.

Finally, Figure 4 shows the response of GDP and 'financialisation' to an increase in θ, the parameter determining the share of profits that is distributed to firms' managers. Results show an increase in aggregate demand Q, due to the positive effect on managers' consumption level. The increase in managers' income and the fall in firms' retained earnings also lead to a higher supply and demand for firms' equities, and to an overall increase in market capitalization relative to GDP.

[9]As opposed to the other simulations, where the variance of aggregate demand Q moved in the same direction as the level of Q, the higher interest rate in this case reduces aggregate demand but increases its variance, and thus the magnitude of the economy's fluctuations.

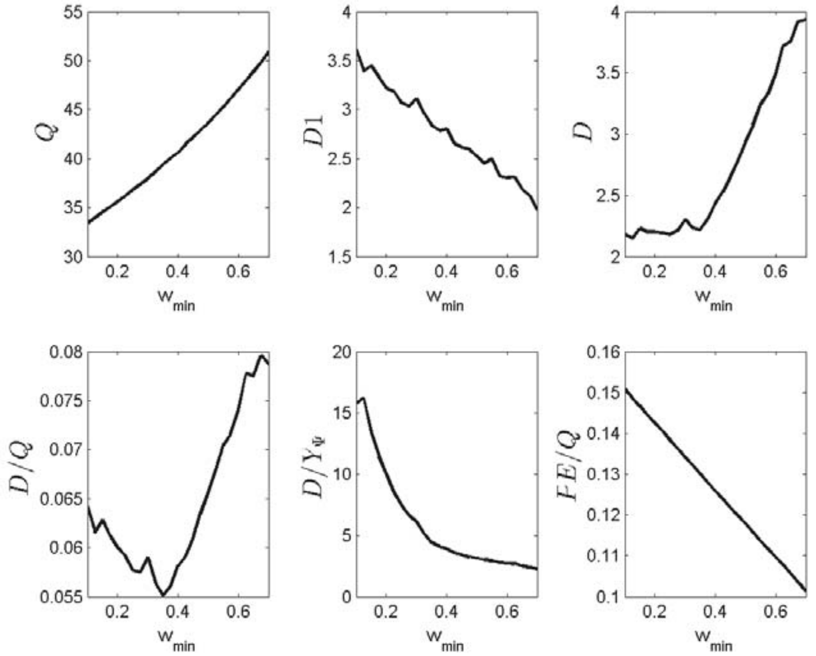

Figure 1: Average aggregate demand, household debt and 'financialisation' for different values of the minimum wage

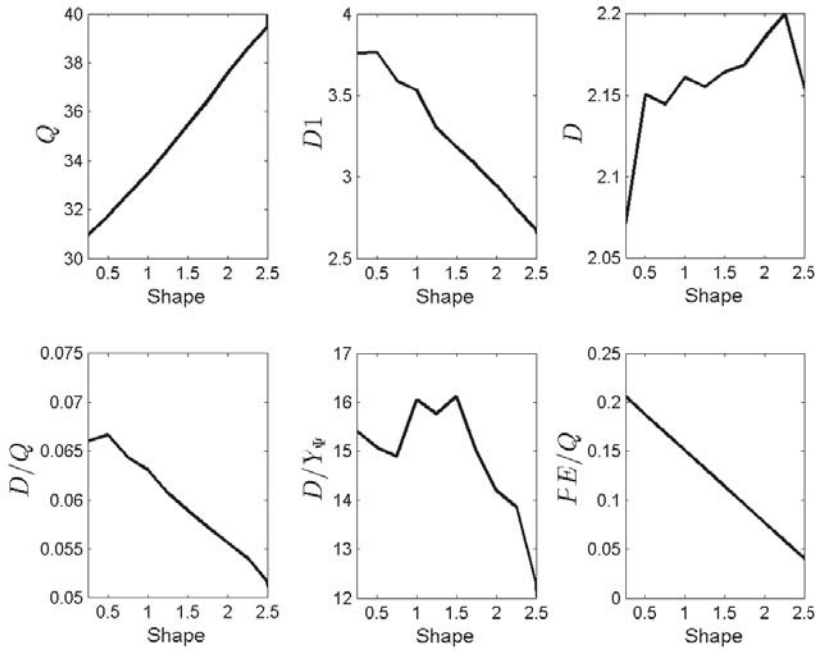

Figure 2: Average aggregate demand, household debt and 'financialisation' for different shapes of the Pareto distribution of wages

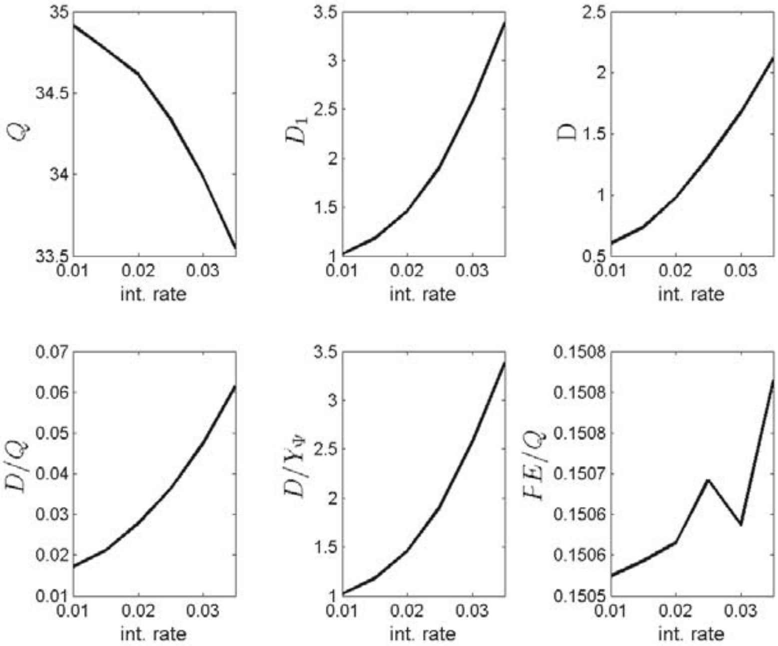

Figure 3: Average aggregate demand, household debt and 'financialisation' for different interest rate values

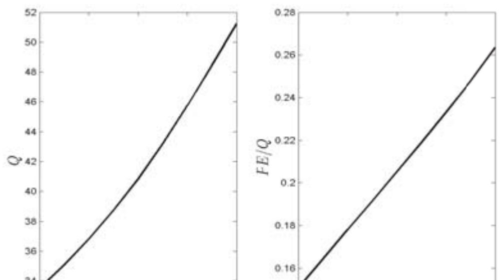

Figure 4: Average aggregate demand and 'financialisation' for different values of θ

5 Concluding remarks

After the 2007-8 crisis, an emerging literature has studied the destabilizing features of a fall in the wage share, such as observed in the United States since the 1980s, in a context of consumption-led growth fostered by household debt accumulation. The acknowledgment that this phenomenon has been accompanied in the US by an increase in inequality among wage earners, as highlighted in the findings by Piketty and Saez (2003) and Piketty (2014), is requiring an effort to incorporate the role of personal income inequality into these approaches. Indeed, as presented in Cynamon and Fazzari (2013), the upsurge in household debt in the 2000s can be largely explained by an increase in spending relative to income of the bottom 95% of the US income distribution. In this context, by introducing heterogenous microeconomic behavior among workers into a stock-flow consistent model with neo-Kaleckian features, this paper has aimed to offer a more complete framework for the study of the relationship between the personal and functional distributions of income, aggregate demand and household debt sustainability.

Besides combining functional and personal income distribution in a stock-flow consistent framework, our main contributions are twofold. First, heterogeneity among workers allows us to study the role of emulation effects in consumption behavior, and the response of aggregate demand to shifts in the size distribution of income. We do so by considering that consumption depends not only on each worker's wage, as in standard Keynesian specifications, but also on the economy's median salary. Second, even if we start by assuming full heterogeneity of workers as in standard agent-based models, we proceed our analysis by using a methodology recently imported from statistical mechanics into macroeconomics, which allows us to aggregate workers into two sub-categories – *borrowing* and *non-borrowing* – and extract their average behavior. The evolution of the share of workers in each category can thus be studied together with micro- and macroeconomic variables both numerically and analytically.

The model is composed of three sectors, namely firms, households and banks. Firms make investment decisions independently, and obtain finance from retained earnings, as well as debt accumulation and equity issuance.The household sector is further divided into two sub-classes: that of workers, whose wages add up to an exogenous share of total output, and managers, who earn a share of firms' profits. While worker's consumption behavior is heterogeneous, depending on the difference between their wage and the economy's median wage, managers are treated as an aggregate, and consume based on their income and wealth, with the latter being allocated in money and equities according to a Tobinian specification. Finally, banks provide credit endogenously to both workers and firms, and charge an constant interest rate for simplicity.

The stability analysis of household debt dynamics and its steady-state value suggest that both the functional and the personal distributions of income are crucial, together with the interest rate, in determining their degree of financial fragility and debt sustainability. These features are reinforced by a set of Montecarlo simulations, which were performed using a benchmark parameter configuration for our model.

The main numerical results are: 1. raises in the minimum wage, by shifting the wage distribution to the right and increasing the wage share, have a negative effect on average debt-to-income ratios and the degree of 'financialisation' in the economy, the latter being measured as the ratio between value of equities and GDP; 2. a reduction in wage inequality, by shifting the median income downwards, contributes even more for debt sustainability and reducing 'financialisation'; 3. the opposite results, namely higher debt ratios and higher 'financialisation' are observed when the interest rate is increased; and 4. a rise in the share of profits distributed to managers also increases the degree of 'financialisation', even when it increases aggregate demand.

In general, quantifying these effects have implications both for minimum wage and tax policies, since the latter alters the post-tax personal distribution of income. The framework can be improved and extended in several directions. Even if we start from a reasonable range, the model's parameter setting should be confronted to existing or new empirical estimations for more practical purposes. Moreover, possible extensions to the framework involve the explicit inclusion of a government sector and Central Bank for a better evaluation of the role of fiscal and monetary policies, as well as the incorporation of differential savings propensities among wage earners, which seem to appear in the data. In a further exploration, the initial distribution of wages, here set exogenously as following a Pareto, can be made endogenous based on labor market and other institutional features of the economy.

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