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Varang Wiriyawit
Research School of Economics, ANU

Benjamin Wong
Reserve Bank of New Zealand and
Centre for Applied Macroeconomic Analysis (CAMA)

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Keywords

Structural VAR, Identification, Detrending, Bias

JEL Classification

C15, C32, C51, E37

Address for correspondence:

(E) cama.admin@anu.edu.au

Structural VARs, Deterministic and Stochastic Trends: Does Detrending Matter?

Varang Wiriyawit*   Benjamin Wong†‡

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We highlight how detrending within Structural Vector Autoregressions (SVAR) is directly linked to the shock identification. Consequences of trend misspecification are investigated using a prototypical Real Business Cycle model as the Data Generating Process. Decomposing the different sources of biases in the estimated impulse response functions, we find the biases arising directly from trend misspecification are not trivial when compared to other widely studied misspecifications. Our example also illustrates how misspecifying the trend can also distort impulse response functions of even the correctly detrended variable within the SVAR system.

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*Research School of Economics, The Australian National University. Canberra, ACT 0200, Australia. Email: varang.wiriyawit@anu.edu.au. Tel: (+61) 2 612 57405.

†Reserve Bank of New Zealand & Centre for Applied Macroeconomic Analysis, The Australian National University. 2, The Terrace, Wellington 6011, New Zealand. Email: benjamin.wong@rbnz.govt.nz. Tel: (+64) 4 471 3957.

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1 Introduction

While trends are ubiquitous in macroeconomic time series, dealing with them is often not straightforward. A subtle reality is that within Structural Vector Autoregression (SVAR) frameworks, assumptions regarding the trend play an important role in the identification of structural shocks in a system. Contributions by Pagan and Pesaran (2008) and Fisher et al. (2013) highlight how choices pertaining to the handling of trends impact upon the identification of whether shocks in a system are transitory or permanent. As an example, assuming output evolving according to a stochastic trend implies at least one structural shock within the system has a *permanent* impact on the level of output. The main contribution of the paper focuses on how a chosen detrending methodology has implications for the identification of shocks in SVARs and the estimation of structural impulse response functions. We thereafter quantify different sources of biases induced by trend misspecifications. To this end, we offer a novel decomposition of the sources of these biases.

A decision whether to difference or deterministically detrend variables cannot be a whimsical one. It is a prior stand by the researcher on how the variables respond to different identified shocks. Unfortunately, our reading of the literature suggests this is an underappreciated point.[1] Some methods of identification in an SVAR framework range from placing restrictions on contemporaneous relationships, long-run relationships, imposing directional responses of variables to particular shocks or some combination of those just mentioned. Each has implications for inferences when studying the response of the economy to different structural shocks. For instance, a common means of orthogonalising the shocks in an SVAR system is to impose short-run, often zero, restrictions on the contemporaneous response of variables to particular shocks. Suppose the orthogonalisation happens on a variable like the difference of real output, with differencing occurring prior to estimation in order to detrend the variable. Without any other restrictions, all shocks will impose a long-run impact on the level of real output. We define this type of shock with a long-run impact on the *level* of *at least one* variable in the system as a *permanent* shock.[2] While shocks having a permanent long-run impact is consistent with some shocks discussed within the macroeconomic literature (i.e. productivity shocks), they are inconsistent with a large class of shocks (e.g. demand shocks, monetary shocks, etc).

In this paper, our Monte Carlo experiments are set up as follows. We utilise standard Real Business Cycle (RBC) models in the spirit of Hansen (1985) as the Data Generating Process (DGP) to draw attention to the role of trend misspecification. The RBC model under consideration differ only with regard to the trend specification induced by the underlying

[1] As anecdotal support to this statement, a plethora of empirical studies (see e.g. Peersman and Van Robays, 2012; Cover and Mallick, 2012; Finlay and Jääskelä, 2014) first difference time series as a matter of routine or taking guidance from stationarity tests, which are known to be susceptible to low power. A more recent development in empirical macroeconomics has been the use of large datasets to extract factors for use within the FAVAR framework (see e.g. Bernanke et al., 2005). Users of such methods usually routinely first difference all their trending data as a matter of practicality, often with a failure to acknowledge the explicit link of inducing permanent shocks in all these variables.

[2] The impact of a shock, ζ, of size 1, at time t to a variable w at time $t+i$ is $\frac{\partial w_{t+i}}{\partial \zeta_t}$. A shock is transitory on w if $lim_{i\to\infty} \frac{\partial w_{t+i}}{\partial \zeta_t} = 0$. Otherwise, it is permanent. If w was first differenced, then $\lim_{i\to\infty} \frac{\partial w_{t+i}}{\partial \zeta_t} = \sum_{j=0}^{\infty} \frac{\partial \triangle w_{t+j}}{\partial \zeta_t}$. Unless a prior restriction is imposed, this sum will in general not equal to zero, implying a permanent effect on w.

technology shock process. Output specified in the DGP will then either evolve according to a stochastic or deterministic trend depending on the properties of the technology shock in the underlying DGP. Our experiments are to estimate a bi-variable SVAR using artificial data to identify a generic technology and non-technology shock. Given a DGP with some unknown trend specification, a trend assumption about output is made prior to estimation by the econometrician. The trend property of output is however incorrectly specified. Given the link between identification and trend specification, assuming the wrong trend process will also cause a misspecification with the shock identification of the SVAR. Our question is then how serious are such trend misspecifications for subsequent inference and characterise the sources of these biases.

It is worth clarifying our contribution to avoid potential confusion. The model setup will no doubt be familiar to those acquainted with a large empirical literature, starting with Galí (1999), questioning the effect of technology shocks on hours worked. Apart from the model features, this is where any similarity ends. The empirical approach in that body of work almost always assumes a stochastic trend in productivity with the fiercest debates regarding the specification of hours worked (see e.g. Christiano et al., 2003) and whether VARs are useful tools for recovering theoretical models (see e.g. Chari et al., 2008; Lindé, 2009). We have no significant contribution to the hours worked technology shock debate. Our sole interest is the role of trend misspecification in output or productivity. In particular, we also study deterministic trends, concepts which are not considered in that body of work.

A feature of our analysis is that we partition our bias into three additive components in order to present a clear trichotomy between them. We first distinguish between biases which are and are not a consequence of the trend misspecification, namely the *detrending bias* and the *non-detrending bias*. We can think of the detrending bias as consisting of two components; the *direct detrending bias* and the *indirect detrending bias*. As mentioned, there is an explicit link between detrending and shock identification, since misspecifying the trend will induce errors in the latter. Therefore, while the *direct detrending bias* is aptly described, the *indirect detrending bias* is a direct consequence of shock identification errors brought about by misspecifying the trend. The remainder *non-detrending bias* refers to the usual studied biases arising from the DSGE to VAR mapping, which are documented in various sources (see e.g. Christiano et al., 2003; Ravenna, 2007; Chari et al., 2008; Carlstrom et al., 2009). Within our conceptual framework, if the data is appropriately detrended, both components of the detrending bias disappear. Our illustrative example demonstrates that the biases induced both directly and indirectly by trend misspecification, compared to the well studied non-detrending bias, are not trivial. This emphasises incorrect assumptions about trend processes can cause considerable biases in the estimated impulse response functions, further highlighting the importance about the choice of detrending within an SVAR framework.

As an SVAR is a system of interdependent equations, the corresponding impulse response functions of a correctly detrended variable, like hours worked in our study, are also distorted if the trend in output is misspecified. That is, there can be significant spillover from trend misspecification of the trending variable to the correctly detrended variable within the system. While we caution against generalising claims on dealing with trends from our work, the illustrative example of the simple RBC model suggests trend misspecification can be the key source of bias in SVAR studies. Researchers using SVARs for empirical work should at least be mindful of the implicit assumptions within their models.

The remainder of the paper is as follows. Section 2 describes the theoretical model and the identification of the model within an SVAR setting. Section 3 explicitly links the theoretical RBC model with features of the SVAR to motivate the design of the simulation study. The simulation setup and the decomposition of biases to study the consequences of trend misspecification are then discussed in Section 4. The results are presented in Section 5 before some concluding remarks in Section 6.

2 Theoretical Model and Identification

We study an RBC model similar to that used by Hansen (1985). The parsimony of the model structure appeals with fewer identifying restrictions for the SVAR. The following subsections present the theoretical RBC model used as a DGP and a discussion of the SVAR identification.

2.1 The Theoretical Model

Under this framework, the households' problem is given by

$$\max_{\{C_t, H_t, K_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \frac{(H_t/B_t)^{1+\eta}}{(1+\eta)} \right\}$$

subject to

$$\begin{aligned} C_t + I_t &= R_t K_t + W_t H_t && (1) \\ I_t &= K_{t+1} - (1-\delta) K_t && (2) \\ \ln(B_{t+1}) &= \rho_B \ln(B_t) + \epsilon_{B,t+1} && (3) \end{aligned}$$

where $\beta \in (0,1)$ is a discount factor, $\eta > 0$ is an inverse Frisch elasticity of labour supply, $\delta \in (0,1)$ is a depreciation rate of capital, $\rho_B \in (0,1)$ is a measure of persistence and $\epsilon_{B,t+1} \sim \mathcal{N}(0, \sigma_B^2)$ is an exogenous Gaussian shock process.

Households in this economy optimise their expected discounted lifetime utility by choosing each period consumption (C_t), hours worked (H_t) and next-period capital holdings (K_{t+1}) subject to their budget constraint (1), a capital accumulation equation (2) and a stationary AR(1) exogenous process (B_t). The shock process, $\epsilon_{B,t}$, presented in Equation (3) can be interpreted as either a shock to labour supply, preference, or demand of households. In this paper, we refer to this innovation as a non-technology shock. Furthermore, as the process is stationary, the shock has a transitory impact upon variables in the system. Households earn income by supplying capital and labour services to firms. Income is either consumed or invested. Let I_t be investment, R_t be the rental rate of capital and W_t be the wage rate at period t. The First Order Conditions for households' utility maximisation are

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left\{ \left(\frac{1}{C_{t+1}} \right) (R_{t+1} + 1 - \delta) \right\} \tag{4}$$

$$\frac{1}{C_t} = \frac{H_t^{\eta}}{B_t^{1+\eta} W_t} \tag{5}$$

These necessary conditions characterise optimal decision rules for households. Equation (4) is an Euler equation for consumption stating that the marginal rate of substitution between consumption at period t and consumption at period $t+1$ equals the return of capital. Equation (5) is a labour supply equation stating that the marginal rate of substitution between consumption and leisure must equal the wage rate.

We can write the problem for firms as

$$\max_{\{K_t, H_t\}} \{Y_t - R_t K_t - W_t H_t\}$$

subject to

$$Y_t = K_t^{\alpha} (Z_t H_t)^{1-\alpha} \tag{6}$$

where $\alpha \in (0,1)$ is a capital share and Z_t is a technology innovation to productivity.

The firms maximise their profit subject to the labour-augmenting Cobb-Douglas production function. Here revenue is obtained by selling goods to households, denoted by Y_t, while costs are incurred from renting households' capital and labour services. The First Order Conditions for firms' profit maximisation are then given by

$$R_t = \alpha K_t^{\alpha-1} (Z_t H_t)^{1-\alpha} \tag{7}$$

$$W_t = (1-\alpha) Z_t^{1-\alpha} K_t^{\alpha} H_t^{-\alpha}. \tag{8}$$

Equations (7) and (8) imply that the rental rate of capital and the wage rate are set equal to the marginal productivity of an additional unit of capital and labour respectively.

Technology Shock Process

The aim of the theoretical model is to generate the aforementioned RBC model with a technology shock which can be either transitory or permanent. We therefore consider two specifications of the technology shock to achieve this. The permanent and transitory technology shock process entail a stochastic and deterministic trend process respectively. The stochastic trend specification can be represented as

$$\tilde{Z}_{t+1} = \frac{Z_{t+1}}{Z_t}$$

$$\ln(\tilde{Z}_{t+1}) = (1-\rho_z)\ln(\gamma) + \rho_z \ln(\tilde{Z}_t) + \epsilon_{z,t}$$

where γ is the average growth rate, ρ_z is persistence in the growth rate of the technology shock and $\epsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2)$ is a Gaussian shock. Under this specification, the technology innovation has a permanent impact on the level of Y_t, C_t, K_t, W_t and Z_t causing these variables to inherit unit roots. Therefore, one can obtain stationary variables with the following transformation; $\tilde{Y}_t = \frac{Y_t}{Z_t}$, $\tilde{C}_t = \frac{C_t}{Z_t}$, $\tilde{K}_{t+1} = \frac{K_{t+1}}{Z_t}$ and $\tilde{W}_t = \frac{W_t}{Z_t}$. Note that K_{t+1} is detrended by Z_t as it is determined within period t. Hereinafter, we refer to this model as RBC-rw.

On the other hand, the deterministically trending process assumes that the technology innovation grows at a constant rate γ. The process can be represented as

$$\begin{aligned}
Z_t &= \gamma^t \tilde{Z}_t \\
\ln(\tilde{Z}_{t+1}) &= \rho_z \ln(\tilde{Z}_t) + \epsilon_{z,t+1} \\
\ln(Z_t) &= t\ln(\gamma) + \rho_z \ln(\tilde{Z}_{t-1}) + \epsilon_{z,t}.
\end{aligned}$$

We obtain the stationary equilibrium condition by detrending the variables with the deterministic trend γ. The transformed variables are then given by $\tilde{Y}_t = \frac{Y_t}{\gamma^t}$, $\tilde{C}_t = \frac{C_t}{\gamma^t}$, $\tilde{K}_{t+1} = \frac{K_{t+1}}{\gamma^t}$ and $\tilde{W}_t = \frac{W_t}{\gamma^t}$. Unlike the RBC-rw specification, the technology shock considered here only has a transitory impact to all variables in the system. The variables are thus trend stationary. We refer this model as RBC-dt.

Competitive Equilibrium Definition

The competitive equilibrium is defined as follows. In a competitive equilibrium, households choose allocations of $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$ and firms will choose allocations of $\{K_t, H_t\}_{t=0}^{\infty}$ such that, given a sequence of prices $\{W_t, R_t\}_{t=0}^{\infty}$ and exogenous shocks to $\{Z_t, B_t\}_{t=0}^{\infty}$, households and firms optimise their utility and profit respectively with the market clearing such that $Y_t = C_t + I_t$.

Data Generating Process

Given this framework, we have $\tilde{X}_t = (\tilde{K}_t, \tilde{Z}_t, B_t)'$ as unobserved state variables, $\tilde{Q}_t = (\tilde{Y}_t, H_t)'$ as observable variables and $\epsilon_t = (\epsilon_{z,t}, \epsilon_{B,t})'$ as exogenous shocks where $t = 1, 2, \ldots, T$. By implementing the first-order approximation, a stable Rational Expectation Equilibrium solution to the log-linearised system of an RBC model has the following linear state-space representation,

$$\begin{aligned}
\tilde{x}_{t+1} &= R\tilde{x}_t + S\epsilon_{t+1} \\
\tilde{q}_t &= M\tilde{x}_t
\end{aligned} \tag{9}$$

where $\tilde{x}_t$ and $\tilde{q}_t$ are column vectors of log-deviation state variables and observable variables from the steady state values, R, S and M are matrices of reduced-form parameters and $\epsilon_t \epsilon_t' = \Sigma_\epsilon$ is a diagonal covariance matrix.

Given $\tilde{x}_0$, we can simulate data of output and hours worked $Q = \{Y_t, H_t\}_{t=0}^{T}$. Hours worked is always integrated of order zero, $H_t \sim I(0)$. The time series properties for output however depends on the underlying trend process. If the trend process is a deterministic trend as in the case of RBC-dt, output is trend stationary and integrated of order zero, $Y_t \sim I(0)$. On the other hand, output is integrated of order one, $Y_t \sim I(1)$, when the underlying trend process is a stochastic trend as in the case of RBC-rw. The characteristics of the structural shocks in the system also differ across model specifications. While the non-technology shock always has transitory impacts, the technology shock has transitory impact only under RBC-dt. In the alternative RBC-rw case, technology shocks have a *permanent* impact upon the *level* of output.

The values of structural parameters used in the DGPs are summarised in Table 1. Most of these parameter values are standard in the literature (see, e.g. Lindé, 2009). Some of these choices are set with respect to the objective of our study. In particular, we set the magnitude of the standard deviation of technology shocks to be twice that of the non-technology shock. It is well known that the larger the relative magnitude and persistence of any particular shock, the easier it is to recover properties of the said shock from the SVAR. (see e.g. Erceg et al., 2005; Paustian, 2007; Chari et al., 2008). We set the magnitude of these two shocks apart so as to isolate them within our analysis. It will be apparent later on that since the comparison within our Monte Carlo study is with respect to technology shocks, this is a natural choice.

2.2 Structural VAR Identification

Let $\hat{Q}_t = (\hat{Y}_t, \hat{H}_t)'$ be a column vector of (demeaned) observable variables where these "hat" variables represent time series generated from one of the DGPs described in the previous section. Define $\hat{q}_t$ as a vector of transformed observable variables, either by first differencing or linear detrending. The VAR in the transformed variable is

$$\Phi(L)\hat{q}_t = u_t \tag{10}$$

$$= A\nu_t \tag{11}$$

where $\Phi(L)$ is a lag polynomial, $I - \Phi_1 L - \Phi_2 L^2 - \ldots - \Phi_p L^p$ of finite lag order p. A is a matrix of contemporaneous impact multipliers. u_t and ν_t are the reduced-form and structural innovations with covariance matrix Σ_u and Σ_ν respectively. We define the structural shocks in this system as a technology (ν_t^T) and a non-technology (ν_t^{NT}) shock. Σ_ν is diagonal by assumption, embedding the idea that the structural shocks in the model are orthogonal. The econometrician can estimate $\Phi(L)$ consistently using least squares. However, the identification issue arises because there is one free parameter in this bi-variate system due to assuming the structural innovations are orthogonal (i.e. Σ_ν is diagonal whereas Σ_u is not).

Therefore, while the reduced-form VAR in Equation (10) can be easily estimated by least squares, the econometrician can only estimate the SVAR in Equation (11) by invoking some identifying restrictions. A common way to view this problem is the econometrician can compute impulse response functions with knowledge of both $\Phi(L)$, which is estimated by least squares, and A, which comes about after making some identifying assumptions. As the following will make clear, identifying strategies are linked to the trend assumption which the econometrician makes.

Long-Run Restrictions

Suppose the econometrician believes that output exhibits a unit root solely due to a long-run impact from a technology shock while hours worked is an I(0) process. Blanchard and Quah (1989) offer an identification strategy to impose this long-run restriction in an SVAR framework.

As output is assumed to be an I(1) process, the series enters the SVAR in first difference to achieve stationarity. To impose this restriction, first let the vector $\hat{q}_t = (\Delta\hat{y}_t, \hat{h}_t)'$, where $\hat{y}_t$ and $\hat{h}_t$ are logged output and hours worked respectively. We can rewrite the VAR in Vector Moving Average (VMA) form as follows. From Equation (11), we have

$$\hat{q}_t = \Phi^{-1}(L)A\nu_t \tag{12}$$

$$= \Psi(L)\nu_t \tag{13}$$

where $\Psi(L) = \Phi^{-1}(L)A$. Expanding (13) and substituting in the elements of the vector $\hat{q}_t$ and ν_t, we obtain

$$\begin{aligned}\begin{pmatrix}\triangle\hat{y}_t \\ \hat{h}_t\end{pmatrix} &= \Psi(L)\begin{pmatrix}\nu_t^T \\ \nu_t^{NT}\end{pmatrix} \\ &= \begin{pmatrix}\psi_{11}(L) & \psi_{12}(L) \\ \psi_{21}(L) & \psi_{22}(L)\end{pmatrix}\begin{pmatrix}\nu_t^T \\ \nu_t^{NT}\end{pmatrix} \\ &= \begin{pmatrix}\sum_{i=0}^{\infty}\psi_{11}^i\nu_t^T + \sum_{i=0}^{\infty}\psi_{12}^i\nu_t^{NT} \\ \sum_{i=0}^{\infty}\psi_{21}^i\nu_t^T + \sum_{i=0}^{\infty}\psi_{22}^i\nu_t^{NT}\end{pmatrix}.\end{aligned}$$

By assuming only the technology shock has a long-run impact on output, the required restriction is then $\psi_{12}(1) = \sum_{i=0}^{\infty}\psi_{12}^i = 0$. After imposing this restriction on $\Psi(L)$, it is straightforward to recover A.

Short-Run Restrictions

If the econometrician believes that both output and hours worked are I(0) processes, all structural shocks in the system are assumed to be transitory, with output transitory around a deterministic trend. Output is linearly detrended before estimation and thus $\hat{q}_t = (\hat{y}_t - \lambda t, \hat{h}_t)'$ where λ represents the coefficient on the deterministic trend. Regardless of the identification procedure here, shocks will always be transitory on the variables in $\hat{q}_t$, namely detrended output and hours worked. In order to invoke one identifying restriction, we place one zero restriction directly on the contemporaneous matrix, A. A natural way to identify a technology shock is to assume that a non-technology shock has no contemporaneous impact on output. This will almost by construction allow a large share of the forecast error variance to be explained by the technology shock. Our identification also directly appeals to the intellectual foundations of RBC models, where technology shocks are the dominant drivers of the business cycle.

This restriction amounts to identification with a Cholesky decomposition of the covariance matrix, ordering output first. By assumption, output will not respond contemporaneously to non-technology shocks. Given a restriction is imposed on the short-run dynamics in the model, namely the impact response to non-technology shocks, we refer to this as imposing short-run restrictions. From Equation (11), the proposed identification of this system is then given by

$$\begin{pmatrix} \hat{y}_t - \lambda t \\ \hat{h}_t \end{pmatrix} = \Phi^{-1}(L) \begin{pmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \nu_t^T \\ \nu_t^{NT} \end{pmatrix}.$$

3 Link Between RBC and SVAR

Before discussing the simulation setup, it is worth discussing the link between the theoretical RBC model, which we use as a DGP, and the SVAR identification. The Blanchard Quah identification using long-run restrictions allows for permanent shocks given that output is differenced prior to estimation. One can reconcile features of the long-run restrictions here with the RBC-rw. The non-technology shock in the theoretical RBC-rw model does not have any long-run impact on output, which is also used as the restriction to identify both shocks.

With the Cholesky decomposition, all shocks are assumed to be transitory. All the variables are thus assumed to be fluctuating either around a deterministic trend or an unconditional mean. Therefore, the features of the theoretical RBC-dt model, where all shocks are transitory, are consistent with using short-run restrictions.[3] However, there is still a misspecification with the Cholesky decomposition since output is restricted not to react contemporaneously to a non-technology shock upon impact. While this is not consistent with the theoretical RBC-dt model, we view this restriction as providing the linearly detrended SVAR with the "best" shot at matching the theoretical model, as technology shocks are the dominant shocks in the theoretical structure of the model.[4] We stress at this stage that, despite the theoretical inconsistency, the Cholesky decomposition is largely able to recover the properties of the transitory technology shock in our Monte Carlo simulation. The biases induced by the Cholesky decomposition are thus trivial in our experiments when the deterministic trend is properly specified. At the same time, the bias decomposition exercise described in the next section will make comparisons relative to correctly specifying the trend possible. Moreover, we view imposing a Cholesky decomposition for identification as a plausible strategy from an empirical perspective within the SVAR literature.

Another discrepancy is that the theoretical models under both trend specifications have a state variable, capital (K_t), which is omitted in the VAR estimation, implying an infinite order VAR is the correct specification to map the theoretical model to the SVAR (see e.g Kapetanios et al., 2007; Ravenna, 2007; Chari et al., 2008; Poskitt and Yao, 2012). A finite order VAR will thus be biased from lag length truncation. We deal the lag truncation on three dimensions. First, we estimate a VAR of a long lag order which obviates this issue to a large extent. Second, we already set the magnitude of the non-technology shock to be small relative to the technology shock in the DGPs as discussed in Section 2.1. This is with guidance from Chari et al. (2008)

[3] Note this does not in anyway imply short-run restrictions necessarily identify transitory shocks. The shocks here are transitory because of the detrending, though this impacts upon the choice to impose an identifying restriction on the short-run dynamics of the model. Another popular identification procedure is to impose impact sign restrictions on variables (e.g. Uhlig, 2005). Such restrictions do not tie down the long-run properties of the shocks. Therefore, using sign restrictions on differenced data without additional restrictions must entail these are also by construction permanent shocks.

[4] Carlstrom et al. (2009) study a similar issue by investigating the consequence of using Cholesky identification when it is not consistent with a theoretical New Keynesian DSGE model.

who show that, in a two-variable SVAR system, truncation bias can be reduced if the variance of one of the shocks in the system is large relative to the others. Third, and more importantly, we decompose the components of the biases which are a direct consequence of detrending, and the component which is not. The latter includes issues that plague SVARs even if the detrending methods are sound, and is relevant to the prior discussion on lag length truncation and the imperfect ability of the Cholesky decomposition to recover the theoretical model's impulses. Therefore, our experimental setup keeps biases which are not a consequence of detrending to a minimum and goes a step further by isolating these components for the analysis through our decomposition. This tool allows a comparison of the components which are due to trend misspecification. The decomposition is described within the next section.

4 Simulation Setup

As previously discussed, the long-run and short-run restrictions should correctly recover the correct shock properties for the RBC-rw and RBC-dt respectively. We label these cases as having the correct trend specification. The purpose of this paper is to determine the magnitude of the distortion in the estimated impulse response functions which trend misspecification in an SVAR induces. We therefore design experiments to answer the question as summarised in Table 2. Under our first trend misspecification study, output is generated I(0) by an RBC model with a deterministic trend (RBC-dt). The econometrician then wrongly first differences the series and identifies the SVAR using long-run restrictions. In the second misspecification, output is generated I(1) by an RBC model with a stochastic trend (RBC-rw). However, the econometrician wrongly detrends the series and implements short-run restrictions to identify the SVAR. Hours worked is correctly specified in levels, consistent with both DGPs. As mentioned earlier, responses to the non-technology shock are misspecified under the short-run, zero contemporaneous, restriction. The responses to this shock are thus not a fair comparison to study trend misspecification. Our analysis is therefore based on the estimated impulse response functions to a technology shock.

4.1 Bias Decomposition

In this paper, the bias due to trend misspecification is measured by a total bias, defined as the average of the difference between the estimated impulse response functions derived from a misspecified SVAR and the true ones deduced from the corresponding RBC model. For example, in Trend Misspecification 1, the total bias in the estimated impulse response functions is

$$\text{Total Bias} \quad = \quad \frac{1}{N}\sum_{i=1}^{N}\left[\widehat{IRF}^{(i)}(rw, LR) - IRF(\text{RBC-dt})\right] \tag{14}$$

where the first term is the estimated impulse response functions from an SVAR assuming a random-walk process and imposing long-run restrictions as a shock identifying strategy and the second term is true responses from an RBC model with a deterministic trend process and $N = 10,000$ is the number of simulations.

To study the consequences of trend misspecification, we quantify the size of these components by linearly decomposing the total bias in Trend Misspecification 1 expressed in Equation (14) as follows.

$$\begin{aligned}\text{Total Bias} &= \frac{1}{N}\sum_{i=1}^{N}\underbrace{\left[\widehat{IRF}^{(i)}(rw,LR)-\widehat{IRF}^{(i)}(dt,SR)\right]}_{\text{Detrending Bias}}+\\ &\quad \frac{1}{N}\sum_{i=1}^{N}\underbrace{\left[\widehat{IRF}^{(i)}(dt,SR)-IRF(\text{RBC-dt})\right]}_{\text{Non-Detrending Bias}}. \end{aligned} \tag{15}$$

The decomposition in Equation (15) distinguishes between two sources of biases. Biases as a consequence of detrending are termed *detrending bias*, while biases which are not a consequence of detrending are termed *non-detrending bias*. The non-detrending bias occurs due to the imperfect ability of the SVAR to mimic the theoretical impulses for a variety of reasons not linked to detrending. Recall these include issues ranging from lag length truncation to a degree of identification bias given the inability to perfectly pin down the matrix of impact multipliers, A. It should be clear that if the SVAR is correctly detrended, the detrending bias disappears. Hence, the non-detrending bias in Trend Misspecification 1 equals the bias incurred in Correct Trend Specification 2, when the data is correctly linearly detrended. Similar decomposition of the total bias can also be done for Trend Misspecification 2. Likewise, the non-detrending bias in this case measures the bias which occurs with Correct Trend Specification 1. The decomposition enables us to isolate the *non-detrending bias*, while also keeping it small through the experimental design. This will allow our analysis to cleanly draw out the implications of trend misspecification. If we further expand Equation (15),

$$\begin{aligned}\text{Total Bias} &= \frac{1}{N}\sum_{i=1}^{N}\underbrace{\left[\widehat{IRF}^{(i)}(rw,LR)-\widehat{IRF}^{(i)}(dt,LR)\right]}_{\text{Direct Detrending Bias}}+\\ &\quad \frac{1}{N}\sum_{i=1}^{N}\underbrace{\left[\widehat{IRF}^{(i)}(dt,LR)-\widehat{IRF}^{(i)}(dt,SR)\right]}_{\text{Indirect Detrending Bias}}+\\ &\quad \frac{1}{N}\sum_{i=1}^{N}\underbrace{\left[\widehat{IRF}^{(i)}(dt,SR)-IRF(\text{RBC-dt})\right]}_{\text{Non-Detrending Bias}}. \end{aligned}$$

The detrending bias is now further decomposed into two components, namely the *direct detrending bias* and the *indirect detrending bias*. The direct detrending bias is the average difference between the estimated impulse response functions of the correct and incorrect detrending procedure as implied by a corresponding RBC model, keeping constant the identifying restrictions assumed by the econometrician. This is isolated from the subsequent identification problem which occurs because of the wrong detrending procedure in the first step. We term this the indirect detrending bias. This bias is due to imposing incorrect identifying restrictions as a consequence of detrending and captured by the second component. This bias component is computed as the average difference between the estimated impulse response functions with different identifying restrictions, given a detrending procedure consistent to that of the corresponding RBC model.

Intuitively, from Equation (12), the responses of transformed variables $\hat{q}_t$ to structural innovations ν_t are influenced by both the estimated reduced-form coefficients $\Phi(L)$ and the matrix A. Specifically, an incorrect assumption about the trend process in output leads to trend misspecification and in turn causes estimation bias in the reduced-form coefficients, $\Phi(L)$. The bias in the coefficient estimates is also exacerbated by the nonlinear mapping involved with imposing identifying restrictions in the matrix A as it is also a function of the estimated coefficients. Furthermore, the incorrect trend assumption also leads to an error in identifying structural shocks and hence affects the way an econometrician imposes restrictions on the matrix A (i.e. imposing long-run or short-run restrictions). Another way of thinking about this bias is as follows. If detrending *did not* lead to a change in the matrix of impact multipliers of the shocks, namely A, then it is straightforward to see that the second component of the total bias would be eliminated since detrending had no marginal impact in distorting the identification of shocks. In sum, the decomposition offers a framework for thinking about how detrending within the SVAR framework induces different biases in the impulse response functions.

5 Results and Discussion

We first present the results from the baseline Monte Carlo study to first establish the role of trend misspecification. Thereafter, the quality of inferences and the consideration of alternative model set-ups is discussed.

5.1 Monte Carlo Simulation

Figure 1 depicts the simulation results under the correct trend specification while Figures 2 and 3 plot the ones under the misspecified trend. In all figures, the true responses derived from the DGPs are presented by a dashed line with circles. Red area represents estimated impulse response functions of the SVAR from 10,000 data sets of 200 observations.

Given the DGPs of interest, the dynamics of the true responses can be described as follows. Under both trend specifications, output and hours worked increase in response to a positive technology shock. The behaviour of these impulse response functions is however different depending on the underlying trend process. To recap, a transitory technology shock emerges from a deterministic trend specification while a unit root technology shock has permanent impact on the level of output. In the deterministic trend case, the impulse response function converges back to zero in the long-run due to the transitory nature of the shock. Contrast against the stochastic trend case, the impulse response function increases and reaches a new steady state level instead of converging back to zero. Hours worked, on the other hand, has a transitory response under both specifications. A stochastic trend specification reveals a hump-shape behaviour of hours worked rather than a sharp increase as per the response of output depicted under a deterministic trend specification. This is due to households' expectation of higher productivity in the future. Households are thus motivated to substitute some of their time toward current leisure and do not initially increase their labour supply as much as in the case of a deterministic trend specification. Later on, hours worked is adjusted to cope with a new long-run level of output.

Under the correct trend specification, Figures 1a and 1b present impulse response functions estimated using long-run and short-run restrictions respectively. As we can see from Figure 1a, these true responses from RBC-rw are well captured by an SVAR using long-run restrictions as the true responses lie neatly within the red area. However, there is greater sampling uncertainty about the estimated response of hours worked to a technology shock. This can be seen in the wide range of red lines which also indicates a nonzero probability of having a negative response upon impact when the true response is in fact positive. Specifically, the probability of inferring an incorrect sign upon the initial impact is 0.1793. In the case of short-run restrictions presented in Figure 1b, the estimated impulse response functions can capture the behaviour implied by RBC-dt. We note that these estimated responses have a tendency to be biased slightly upward initially and then slightly downwards. On the basis of Figures 1a and 1b, the evidence does suggest, subject to the trend being correctly specified, that the SVAR is largely able to recover the DGP impulse response functions. In other words, the experimental design has kept the non-detrending bias to a minimum. This is a good reference point for comparison as we move on to study the role of trend misspecification.

Mishandling the trend in output unsurprisingly affects the behaviour of estimated impulse response functions. Figure 2a plots the estimated impulse response functions from a misspecified SVAR using long-run restrictions along with the true responses derived from RBC-dt. As the econometrician incorrectly first differences output and imposes long-run restrictions, the estimated impulse response functions of output to a positive technology shock converges to a new steady state instead of exhibiting the transitory behaviour implied by the underlying DGP. Furthermore, on average, the estimated responses are biased upward except upon impact. The bias decomposition presented in Figure 2b suggests that the main source of bias is the direct detrending bias. The error in estimating impulse response functions is thus mainly due to mishandling the trend. Comparatively, the indirect detrending bias contributes a smaller fraction to the total bias and only distorts the short-run dynamics. In sum, biases stemming from incorrect detrending are non-trivial.

We consider our other misspecification. Wrongly imposing short-run restrictions will cause the estimated impulse response functions of output to decay over time even though the true response exhibits a permanent response as implied by RBC-rw. As the zero long-run impact of the output response to technology shock is imposed by the detrending, and then identification, procedure, we should expect a downward bias in the estimated impulse response functions. The downward bias should also get larger at longer horizons because detrending imposes a long-run effect on the system. Figure 3b shows this is indeed the case. The direct detrending bias is the dominant source of error at all horizons and is particularly noteworthy at longer horizons. Our simulations also show that the reversion of the output level to zero can be slow to take effect, perhaps reflecting the permanent shock in the underlying DGP. However, this is insufficient to prevent the obvious downward bias due to mishandling the trend in output.

Spillovers to the correctly detrended variable from trend misspecification also occurs. That is, the trend misspecification does not only affect the estimated impulse response functions of output to a technology shock. The estimated impulse response functions of hours worked to a technology shock are also distorted as the system is interdependent. In both misspecifications we consider, the shapes of the impulse response function of hours worked are fairly well preserved. Even so, they both have a tendency to exhibit upward bias in the initial periods after the technology shock. The response in Trend Misspecification 2 though, has a tendency to follow up upward biases with downward biases about 10 periods after the shock. Unlike the estimated impulse response functions of output, the dominant source of biases for hours worked differs and is generally split between the direct and indirect detrending bias. Even so, this serves to reinforce our point that mishandling the trend can lead to severely biased impulse response functions.

The results from our Monte Carlo study should be sufficient to raise warning flags. Issues like lag length truncation and identification of SVARs receive much attention given their role in allowing the researcher to interpret the data within the context of their chosen model. Even so, our Monte Carlo simulations reveal that trend misspecification is non-trivial, and could even potentially be a greater source of bias compared to these widely studied biases. While it would be tempting to conclude on the basis of Figures 2b and 3b that a misspecified SVAR with short-run restrictions provides larger distortion than a misspecified SVAR with long-run restrictions, we regard such an interpretation as premature. The illustrative example we consider is a very stylised one. Empirical reality dictates richer and larger model structures. It is an empirical question whether the patterns from our Monte Carlo study carry over. One needs to keep in mind that richer systems are inherently plagued by identification issues, which can be challenging to isolate even before considering the role of trend misspecification (see e.g. Carlstrom et al., 2009; Castelnuovo, 2012, for examples of models with typical identifying strategies). Our experiment nevertheless illustrates that trend misspecification results in non-trivial biases in the impulse response functions and these should be a relevant consideration for SVAR practitioners.

To sum up the results, our Monte Carlo experiments indicate that trend misspecification can be a key source of bias. Spillovers of biases to correctly detrended variables within the system are also non-trivial. Our results reiterate a reminder that researchers should be careful to link their detrending procedures with the identification of the structural model they have in mind.

5.2 Implications for Inference

We explore the effect that trend misspecification has on inference. Statistical inferences are often drawn based on the confidence intervals or confidence sets. Paradoxically, inferences can still be equally valid with or without the statistical bias as long as the biased estimator produces confidence intervals which still accommodate the true model parameters, producing what can be termed unbiased inference. Therefore, while we have established the possibility of trend misspecification severely biasing the estimated impulse response functions, it seems natural to investigate whether this has an effect on biasing inference. To conduct this exercise, we generate 68% confidence intervals for the impulse response functions on each run of the Monte Carlo simulation in order to investigate the coverage rate of the impulse response functions relative to the true DGP.[5] The choice of 68% is motivated as per the typical choice of VAR practitioners. Our approach of studying the coverage rates of misspecified models relative to the true DGP has strong parrallels with Christiano et al. (2003) and Wiriyawit (2014).

In a repeated sampling exercise, if the estimators are unbiased and the measure of sampling uncertainty sound, we can expect coverage rates will roughly coincide with the nominated confidence interval, 68% in our exercise (i.e. the model has good size properties). Of course, the models here are misspecified relative to the DGP derived from the RBC model on some dimension. Recall even correctly detrending the data gives rise to sources of misspecification leading to the truncation and identification biases in estimated impulse response functions as discussed earlier. This means we can only study the marginal change in the quality of inference for the same DGP moving from one with a properly specified trend to one with a misspecified trend.

Table 3 presents the coverage rates of impulse response functions at selected horizons. We first focus on the coverage rates at 10 and 20 periods after the shock. It is clear that trend misspecification deteriorates the quality of inference. For example, in the case of the RBC-dt, correctly linearly detrending and then applying the Cholesky decomposition yields coverage rates for the response of hours worked at the horizons of interest of around 30-50%. This is poor from the perspective of using 68% confidence intervals. However, the trend is misspecified by differencing output and subsequently using long-run restrictions sees coverage rates fall to zero. The coverage rate for the output response by correctly specifying a trend stationary process at 10 and 20 periods after the shock is also around 35-50%. The corresponding coverage rates for output response when incorrectly using long-run restrictions is 100%. Note that through differencing, these output responses are cumulative impulse response functions. A consequence of cumulative impulse response functions is that they are estimates of sums and therefore are susceptible to large variances. This explains the excessively high coverage rate yielding less than meaningful inferences.

The same pattern also emerges under RBC-rw as a DGP. Output and hours worked responses at 10 and 20 periods ahead are poor and have coverage rates of less than 30% when wrongly applying linear detrending instead of first differencing. This is a rapid deterioration from the hours worked response when correctly assuming a random walk specification and using long-run restrictions, whose coverage rates are between 30-45%. Once again, the coverage rates of output response using long-run restrictions at these quarters are very high, reflecting the use of cumulative impulse response functions. However, it should be sufficiently clear that misspecifying the trend results in strictly worse inference at long horizons. That is, the variance for the estimator at these horizons are either not large enough to mask deficiencies of the biased point estimate and so often yield misleading inferences. Otherwise, the variances are so large that meaningful inferences cannot be made.

[5] For each dataset generated by the DGP, we first estimate the VAR impulses. Thereafter, we take repeated joint draws from a Normal-Wishart distribution based on the OLS estimator of the reduced-form VAR parameters. We then apply the relevant detrending and identification strategy to generate the distribution of the impulse response functions. This is analogous to drawing from a Bayesian posterior with a flat prior. Since the prior contains no information, the error bands are constructed entirely from the VAR likelihood and are so interpretative from a frequentist perspective. We then count the proportion of simulations where the 68% confidence interval contains the true DGP responses.

If we turn to shorter run responses, from impact to four quarters, it is not immediately clear that trend misspecification strictly deteriorates the quality of inference. For example, in the case of having RBC-dt as a DGP, the hours worked response achieves close to the nominal coverage of 68% upon impact to the third quarter even when the trend is misspecified. Therefore, it appears that, while long-run inference is strictly improved when properly specifying the trend, this claim is ambiguous at shorter run horizons. It is worth investigating further because researchers may only be concerned with shorter run responses by arguing that long-run responses contain too much uncertainty to be interpreted in a sensible manner. For one to make valid inferences at shorter run horizons, it is prudent that contemporaneous responses, or the impact multipliers of shocks, and the more recent lag orders are estimated with sufficient precision. These are also incidentally quantities most affected by the quality of the instruments when casting the SVAR in an Instrumental Variables (IV) setting. We defer the full exposition of the link between the IV estimator and the SVAR to the appendix. In brief, the identifying strategies we consider in this paper require the VAR residuals from the output equation to be used as instruments for contemporaneous output in the hours worked equation under both identification strategies. In addition, h_{t-1} is also required as an instrument for the output equation when we implement long-run restrictions. The output equation using the short-run restrictions, on the other hand, can be estimated directly using least squares. We adopt the approach of Fry and Pagan (2005) by studying the distribution of the contemporaneous, or impact, responses to evaluate the quality of these instruments.

Figure 4 presents the distribution of the impact responses of both output and hours worked to a technology shock under both DGPs and subsequent identification strategies. In the case of RBC-rw presented in Figure 4a, the misspecified linearly detrended model is strangely able to produce an unbiased estimator of the output response to the technology shock. Confidence in this result is however misplaced because this is an artefact of the experiment setting the variance of the technology shock to be large relative to that of the non-technology shock.[6] That said, even if the IV estimator is largely unbiased for the output response under trend misspecification, the quality of instruments is poor for the hours worked equation as the distribution of the corresponding contemporaneous responses is situated far away from the true value. This explains why there is a large bias in the corresponding impulse response functions presented in Figure 3. Even if the output response is precisely pinned down on impact, this materialises into a large bias from the first quarter onwards given that there is a feedback mechanism from hours back to output after a lag. While the correctly detrended model still exhibits instruments which are slightly biased, they are better relative to the misspecified case, which experiences a sharp deterioration in the quality of instruments.

We now move to the case of RBC-dt as a DGP presented in Figure 4b. The distribution of the contemporaneous responses under trend misspecification in this case does not show a clear deterioration of the instrument quality compared to the one when the trend is properly specified. If anything, the instrument quality is poor in both cases. Therefore, estimated short-

[6] Given estimation of the output equation will omit contemporaneous hours and estimate the corresponding equation by least squares, the output response to technology is just the standard deviation of the residual. Output is largely driven by technology shocks due to the relatively larger variance of the said shock. Therefore, it is not surprising this approach can recover the variance quite well. This result effectively disappears if the variance of the non-technology shock increases. We provide this result upon request.

run responses perform, unsurprisingly, quite similarly (poor) as per their respective coverage rates. However, it is important to reiterate that, at long-run horizons, the inferences are strictly worse when the trend is misspecified.

In sum, we note that it is not at all obvious that the quality of instruments deteriorates once the trend is misspecified. In cases where it does, we can expect inference to be strictly worse at all horizons. If the instrument quality does not deteriorate, then the misspecified model may still yield inference of comparable quality relative to a properly specified trend model in the shorter run. Even so, at longer run horizons, correctly specifying the trend will always yield better inferences.

5.3 Alternative Experimental Set-up

We examine alternative experimental set-ups to explore scenarios that an econometrician may face. These settings range from setting different parameter values used in the DGPs to alternative detrending strategies the econometrician might employ.

Degree of Persistence in the Permanent Technology Shock

In contrast to the theoretical responses we have in the previous section, some empirical studies suggest that a positive permanent technology shock should lead to a fall in hours worked instead of an increase (see e.g. Galí, 2004; Francis and Ramey, 2005; Kimball et al., 2006). As shown by Lindé (2009), a reparameterisation of the model by increasing the persistence of the permanent technology shock can produce a negative response of hours worked to a technology shock. We therefore repeat the simulation exercises with RBC-rw as a DGP and increase the persistence in the permanent technology shock from 0.25 to 0.5. Other parameter values remain the same as used previously. In this subsection, the SVAR with long-run restrictions is the correct trend specification.

Figure 5 presents the result if we misspecify the trend as a deterministic trend and use the Cholesky decomposition to identify the technology shock. The output response to a technology shock is badly biased in Figure 5a, similar to that in Figure 3a. The results also reveal that none of the impulse response functions are able to capture the fall in hours worked as per the reparameterised DGP. The decomposition reveals that the predominant sources of biases is the indirect detrending bias for hours worked and a mix of both types of detrending bias for output. However, the absolute bias is much larger than in Figure 3a, the specification with less persistence in the growth rate of the technology shock. Note that there is a large degree of non-detrending bias. This suggests that even if the trend was properly specified, the total bias would still be large. This is not entirely surprising. Given a more persistent growth rate of the shock in the DGP, it is going to be the case that a higher lag order is needed to fully model the dynamics. The truncation of the lag length is thus going to be significant, and so unsurprisingly feeds into the non-detrending bias component. Even so, the source of this bias is still not as dominant as the ones induced from mishandling the trend.

Degree of Persistence in the Transitory Technology Shock

We investigate the sensitivity of a more persistent technology shock under RBC-dt. The shock persistence is increased to 0.95 relative to the lower persistence of 0.25 considered in the main Monte Carlo study. As a consequence, both output and hours worked takes at least 40 periods to revert to the steady state. We then conduct an experiment as in Trend Misspecification 1. That is, the output series is incorrectly first differenced and long-run restrictions are imposed. The bias decomposition is presented in Figure 6. The dashed line represents the total bias in Trend Misspecification 1 given the lower persistence in the transitory technology shock as a comparison. We can see that at longer horizons, the direct detrending bias is the predominant source of bias. This is similar to the case of lower persistence in the transitory technology shock except for one difference; the horizon which the direct detrending bias dominates the source of biases kicks in much later as the persistence of the technology shock increases. The total bias on average unexpectedly shrinks at shorter horizons. The reason is that, once the degree of persistence approaches 1, the deterministic trend process behaves more like a unit root process. Assuming a stochastic trend specification in this case can therefore still capture features of the underlying process, but only in the short-run. The direct detrending bias due to incorrectly first differencing eventually expands, resulting in larger total biases at longer horizons. In essence, this reveals the fact that assuming a permanent shock, and thus first differencing still results in an inability to mimic the transitory long-run behaviour of the underlying DGP even though the persistence of the technology shock has increased. The general conclusion is still similar to that shown previously. That is, detrending biases are still the main source of error when wrongly assuming a stochastic trend.

Alternative Detrending Strategies

The Hodrick-Prescott (1997) (HP) filter is a widely-used tool in empirical studies to extract the cyclical component of the series (see e.g. Leu, 2011; Kato and Miyamoto, 2013). It is therefore worth investigating the performance of the filter in estimating the true impulse response functions when the econometrician does not have adequate information regarding the underlying trend process in output. We consider a case where an econometrician employs the HP filter on the output series as a detrending methodology, and then implements short-run restrictions to identify structural shocks in the system.[7] Figure 7 plots the total bias given the DGPs of interest with different degrees of persistence in the technology process.[8] We find that the performance of the HP filter depends on the degree of persistence in the technology shock. This is not surprising. As discussed in Canova and Ferroni (2011), the relatively persistent shock process in an RBC model would produce the variability of the series at longer horizons. The HP filter however attributes the low frequency cycle as the non-cyclical component, and thus measures the true cyclical component with error. As a consequence, the higher the persistence in the technology shock, the larger the detrending bias induced by the HP filter. Another way of

[7] As we simulate quarterly data, the smoothness parameter for the HP filter is set to 1600 as Hodrick and Prescott (1997) recommend.

[8] In this exercise, the degrees of persistence used under RBC-dt are 0.25 and 0.9 for low and high persistence respectively. Under RBC-rw, 0.25 and 0.5 are set respectively for low and high persistence in the growth rate of a technology shock.

thinking about this is if there is a persistent component in the cycle, the HP filter will mistake part of this persistent cycle as a trend and systematically filters it. If one expects the persistence of technology shocks to be large empirically, we should not expect the performance of the filter to be satisfactory.

At this stage, one might suspect that estimating all series in differences may be flexible enough to consider both transitory and permanent shocks without the econometrician taking a stand. While differencing does conceptually produce permanent shocks, it is possible that empirical exercises may produce impulse response functions which are transitory. We therefore consider a common empirical strategy of just first differencing and imposing a Cholesky decomposition to identify the technology shock. Note that such an approach has no theoretical support as both shocks are permanent. We therefore generate the RBC model with two transitory shocks, RBC-dt, and then first difference output and take a Cholesky decomposition to explore whether the original transitory technology shock can be recovered. Figure 8 suggests that confidence in recovering underlying transitory shocks despite first differencing may be misplaced. In particular, none of the impulse response functions are able to recover the underlying transitory response of output. Moreover, the impulse response functions are badly biased, missing both the short and long-run properties of the underlying DGP. The intuition is similar to that of Fisher et al. (2013). By not distinguishing between permanent and transitory shocks, the identification mixes up both the transitory and permanent components. This reveals an uncomfortable reality of SVAR practitioners. Taking a stand on the underlying shock properties and modelling them as such is mandatory. As this is an identification exercise, the data cannot speak without imposing any structure. First differencing as an empirical strategy, with the intention of being agnostic about the transitory or permanent nature of shocks, is unfortunately misguided.

6 Conclusion

In this paper, we highlight that the choice of detrending is linked to SVAR identification. While identification in SVARs receive much attention given its role for sensible empirical analysis, trend misspecification remains a possible blindspot for researchers using SVARs. In an illustrative example using a Monte Carlo study, we demonstrate that the biases directly attributable to trend misspecifications can be non-trivial. While our example can be construed as being model, or even parameter, specific, this raises the empirical possibility of significant biases emanating from trend misspecification as the key source of biases in the SVAR.

We are deliberately minimalist with the SVAR structure. More work is needed before general prescriptive advice on detrending in SVARs is possible. Our approach is largely designed for the purpose of isolating the different sources of biases by keeping any potential known biases to a minimum. This serves in keeping the analysis tractable while drawing attention to the role of trend misspecification. A natural question is how important are then these sources of biases within richer and larger model environments. To this end, the decomposition of the biases which we put forward in this paper is a useful tool for addressing these questions in future research.

Appendix

A An Instrumental Variable Approach to the SVAR

In this section, we demonstrate the explicit link between the estimated SVAR and Instrumental Variables (IV). Some relevant references are Shapiro and Watson (1988) and Pagan and Pesaran (2008).

To fix ideas, we return to Equation (11),

$$\begin{aligned} \Phi(L)\hat{q}_t &= A\nu_t \\ A^{-1}\Phi(L)\hat{q}_t &= \nu_t \\ \Pi(L)\hat{q}_t &= \nu_t \end{aligned} \tag{A.1}$$

where $\Pi(L) = \Pi_0 - \Pi_1 L - \Pi_2 L^2 + \ldots - \Pi_p L^p$. ν_t has a diagonal covariance matrix Σ_ν as defined earlier. We first focus on the Cholesky decomposition. We first linear detrends the data and thus $\hat{q}_t = (\hat{y}_t - \lambda t, \hat{h}_t)'$. Define π^i_{jk} as the $(j,k)^{th}$ element of Π_i. We then expand the system and write the equations individually as

$$\hat{y}_t - \lambda t = \pi^0_{12} h_t + \sum_{i=1}^{p} \pi^i_{11}[\hat{y}_{t-i} - \lambda(t-i)] + \sum_{i=1}^{p} \pi^i_{12} h_{t-i} + \nu^T_t \tag{A.2}$$

$$h_t = \pi^0_{21}(\hat{y}_t - \lambda t) + \sum_{i=1}^{p} \pi^i_{21}[\hat{y}_{t-i} - \lambda(t-i)] + \sum_{i=1}^{p} \pi^i_{22} h_{t-i} + \nu^{NT}_t. \tag{A.3}$$

The Cholesky decomposition imposes $\pi^0_{12} = 0$. Equation (A.2) can be estimated by least squares given the restriction implies it only has terms that are predetermined (i.e. lag terms) on the righthand side. Given $(\hat{y}_t - \lambda t)$ is on the righthand side of Equation (A.3), it can be estimated using the residual, $\hat{\nu}^T_t$, as an instrument for $(\hat{y}_t - \lambda t)$. When we cast an SVAR in this framework, the implications for the instruments and identification are clear. The impact response for detrended output to a one standard deviation technology shock depends only on the OLS estimator of the standard deviation of the residual in Equation (A.2). It should be clear precisely estimating the impact response of hours worked to technology shocks depends on two quantities. First, the identification strategy needs to precisely recover the standard deviation of the technology shock through least squares. Second, the residual from the first equation is a good instrument for contemporaneous detrended output in the second equation to precisely pin down π^0_{21}.

In the case of long-run restrictions, we can expand Equation (A.1) into the following,

$$\triangle\hat{y}_t = \pi^0_{12}\triangle h_t + \sum_{i=1}^{p} \pi^i_{11}\triangle\hat{y}_{t-i} + \sum_{i=1}^{p-1}\left\{\sum_{j=0}^{i} \pi^j_{12}\right\}\triangle h_{t-i} + \left(\sum_{i}^{p} \pi^i_{12}\right) h_{t-p} + \nu^T_t \tag{A.4}$$

$$h_t = \pi^0_{21}\triangle\hat{y}_t + \pi^1_{22} h_{t-1} \sum_{i=1}^{p} \pi^i_{21}\triangle\hat{y}_{t-i} + \sum_{i=1}^{p} \pi^i_{22} h_{t-i} + \nu^{NT}_t. \tag{A.5}$$

Note Equation (A.4) is just a reparametrisation of the original equation.[9] Equation (A.5) is an expanded version of the second equation without performing any transformation. The long-run restriction imposes $\sum_i^p \pi_{12}^i = 0$ (see Fry and Pagan, 2005, for details). With the coefficient on h_{t-p} eliminated, there are p coefficients estimated on $\triangle h_{t-i}$ where $i \in [0, 1, \dots p-1]$ with p unknowns in $\pi_{12}^i, i \in [0, 1, \dots p-1]$, which will allow us to recover the original structural equation as long as Equations (A.4) can be estimated. Given contemporaneous terms on the righthand side of both equations, they both need to be estimated using IV. Equation (A.4) can be estimated using h_{t-1} to instrument for $\triangle h_t$ since it is included in the second equation, but excluded from the first. The residual of Equation (A.4) can then be used to instrument for $\triangle y_t$ in Equation (A.5). Therefore, it should be sufficiently clear that if we fail to pin down the response of output to a one standard deviation technology shock, this must be in part h_{t-1} is a poor instrument for $\triangle h_t$. Similarly, failure to pin down either the standard deviation of the technology shock and/or the technology shock series is a poor instrument for $\triangle y_t$ will mean that the hours response to technology shocks will be poorly estimated.

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[9]See Fry and Pagan (2005) for details on how to reparametrise this equation. Pagan and Pesaran (2008) show an alternate transformation using error correction ideas which will produce a similar representation.

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Table 1: List of Parameters Specified in the Model

	Structural parameters	True Value
β	Discount factor	0.99
α	Capital share	0.33
δ	Depreciation rate of capital	0.025
η	Inverse short-run labor elasticity	0
γ	Average growth rate of technology shock	1.0074
ρ_z	Persistence in technology shock	0.25
ρ_B	Persistence in non-technology shock	0.8
σ_z	Standard deviation of technology shock	0.007
σ_B	Standard deviation of non-technology shock	0.003

Table 2: Monte Carlo Simulation

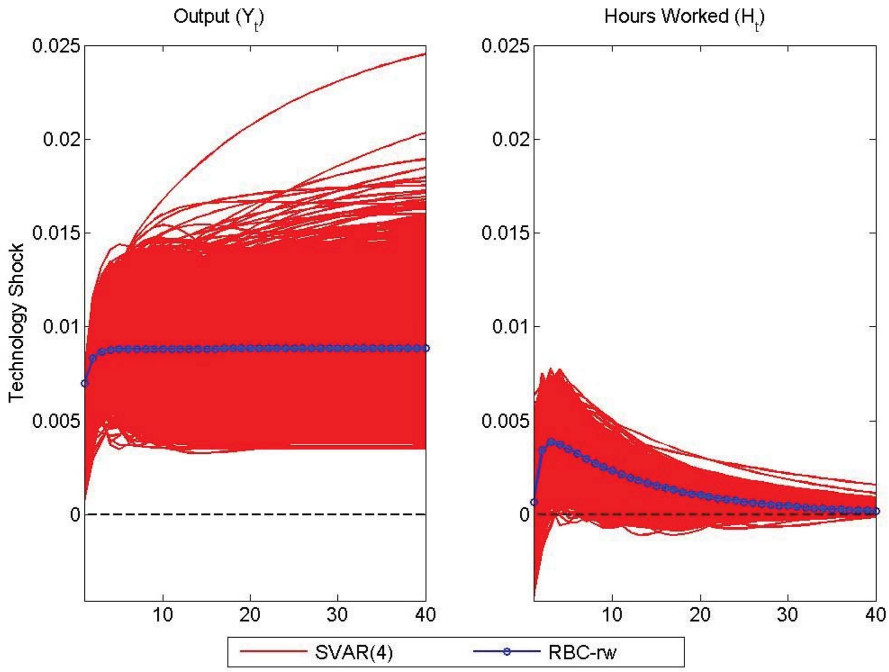
SVAR	DGP	
	RBC-rw: $Y_t \sim I(1)$	RBC-dt: $Y_t \sim I(0)$
Long-Run Restrictions	Correct Trend Specification 1	Trend Misspecification 1
Short-Run Restrictions	Trend Misspecification 2	Correct Trend Specification 2

Table 3: Coverage Rates of Impulse Response Functions to a Technology Shock

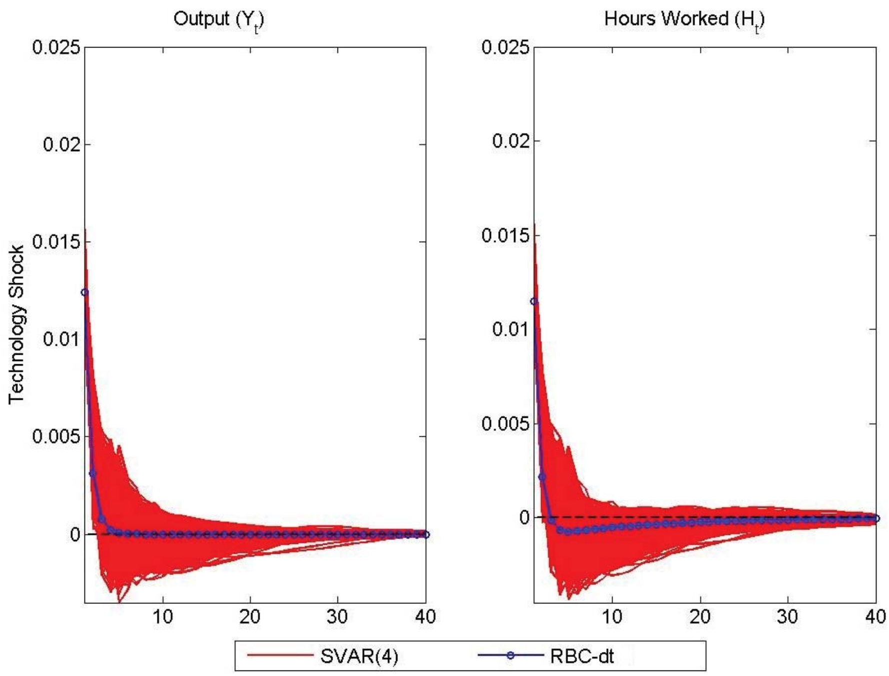
DGP		RBC-rw: $Y_t \sim I(1)$		RBC-dt: $Y_t \sim I(0)$	
SVAR		Long-Run Restrictions (C1)	Short-Run Restrictions (M2)	Short-Run Restrictions (C2)	Long-Run Restrictions (M1)
Variable	Period	Coverage Rates			
Output (Y_t)	0	0.6713	0.4879	0.2551	0.7084
	1	0.5264	0.4877	0.3017	0.8952
	2	0.6234	0.4786	0.3292	0.9599
	3	0.7431	0.4302	0.3899	0.9937
	4	0.8334	0.3572	0.3703	0.9997
	10	0.9762	0.2903	0.3805	1.0000
	20	0.9912	0.2533	0.5672	1.0000
Hours Worked (H_t)	0	0.6099	0.0000	0.0373	0.6765
	1	0.6190	0.0000	0.1824	0.7198
	2	0.6495	0.0359	0.2736	0.6136
	3	0.6442	0.1648	0.3869	0.5499
	4	0.6517	0.3486	0.4220	0.2562
	10	0.4672	0.2936	0.4824	0.0138
	20	0.3019	0.1142	0.3158	0.0000

Note: C1 and C2 are Correct Trend Specification 1 and 2 whereas M1 and M2 are Trend Misspecification 1 and 2 in our Monte Carlo exercises.

Figure 1: Impulse Response Functions to a Positive Technology Shock under Correct Trend Specification

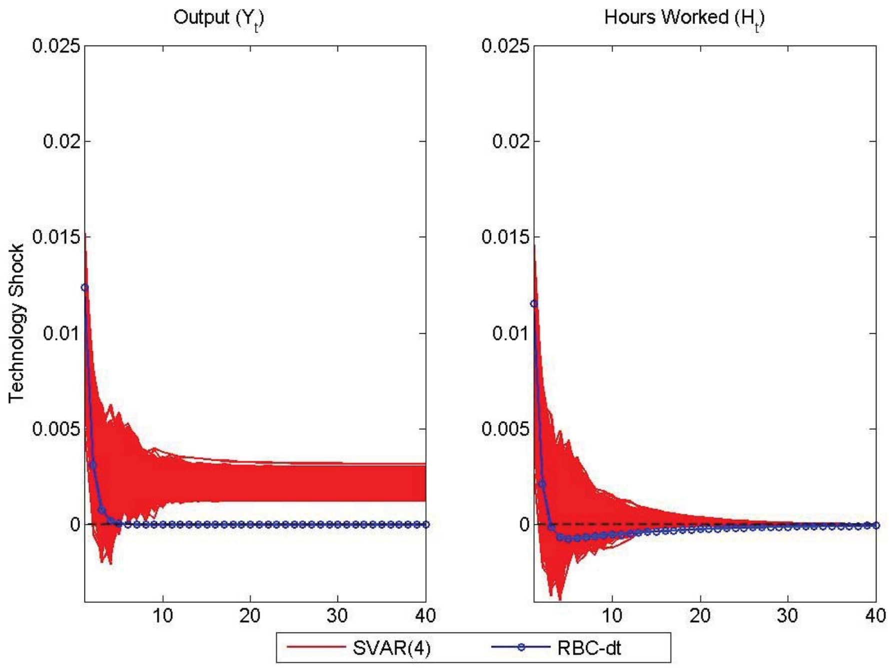


(a) Implementing Long-Run Restrictions Given RBC-rw as the DGP (Correct Trend Specification 1)

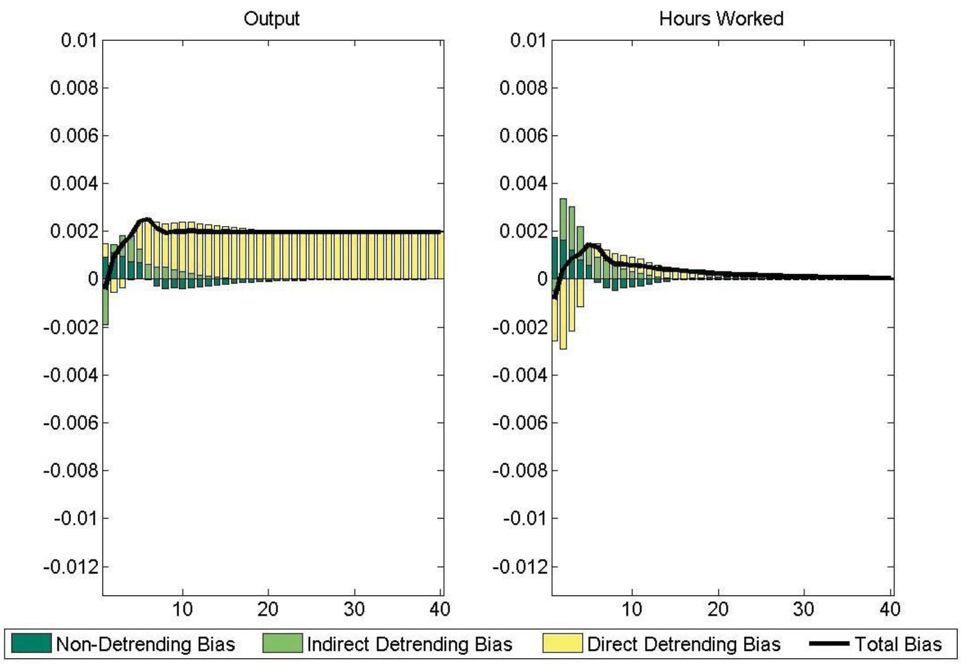


(b) Implementing Short-Run Restrictions Given RBC-dt as the DGP (Correct Trend Specification 2)

Figure 2: Monte Carlo Simulations Implementing Long-Run Restrictions Given RBC-dt as the DGP (Trend Misspecification 1)

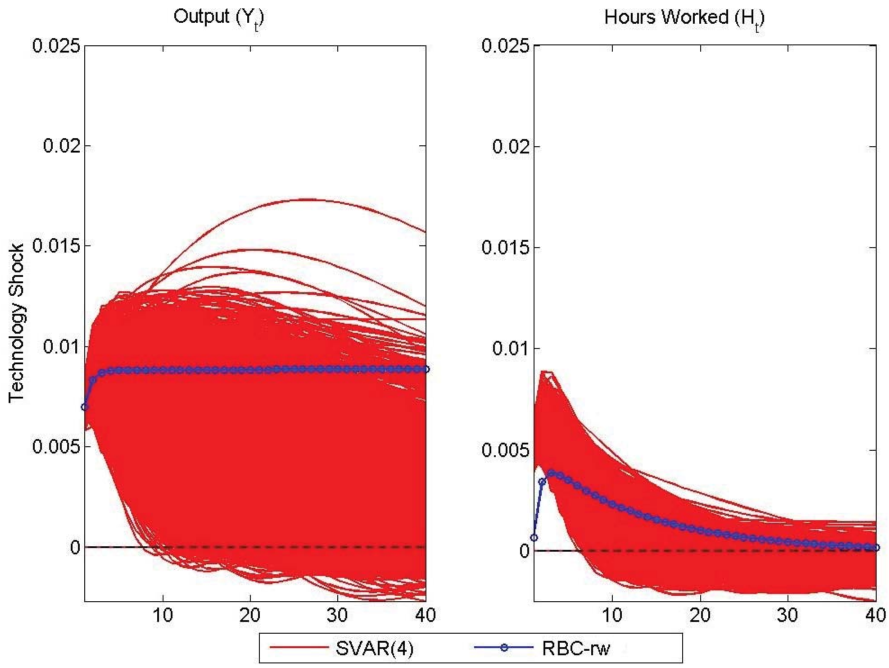


(a) Impulse Response Functions to a Positive Technology Shock

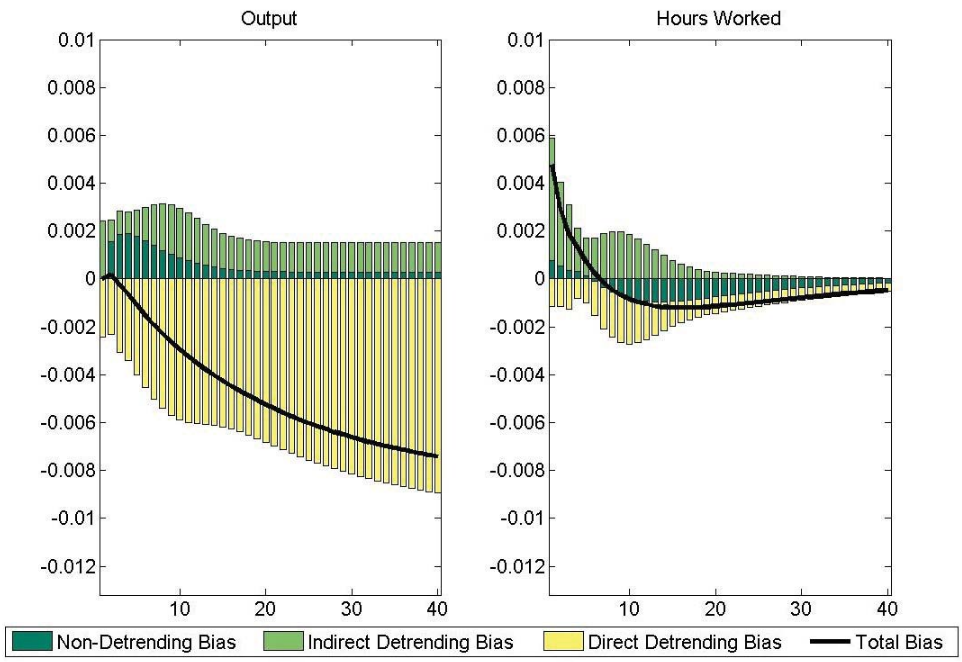


(b) Total Bias Decomposition

Figure 3: Monte Carlo Simulations Implementing Short-Run Restrictions Given RBC-rw as the DGP (Trend Misspecification 2)

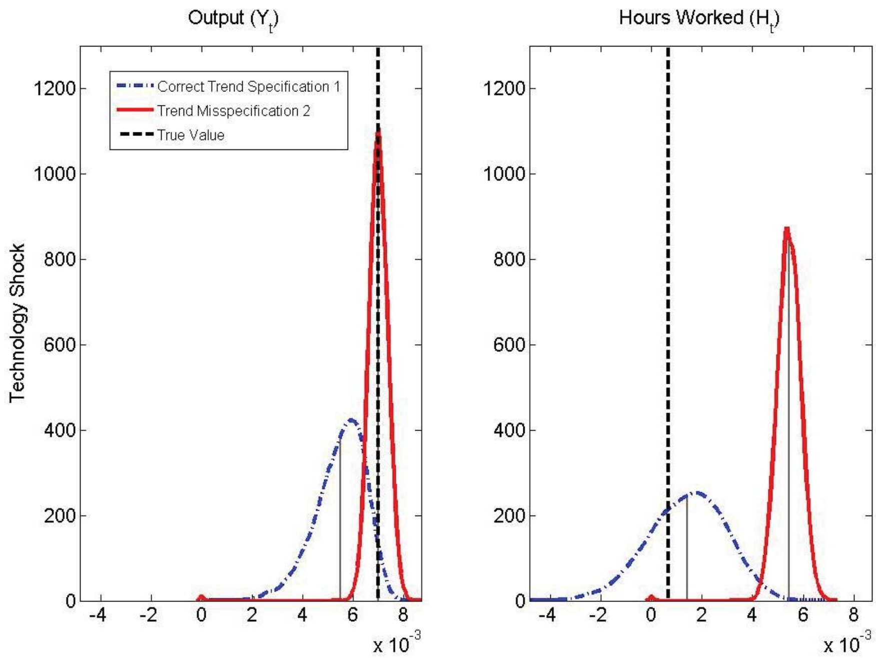


(a) Impulse Response Functions to a Positive Technology Shock

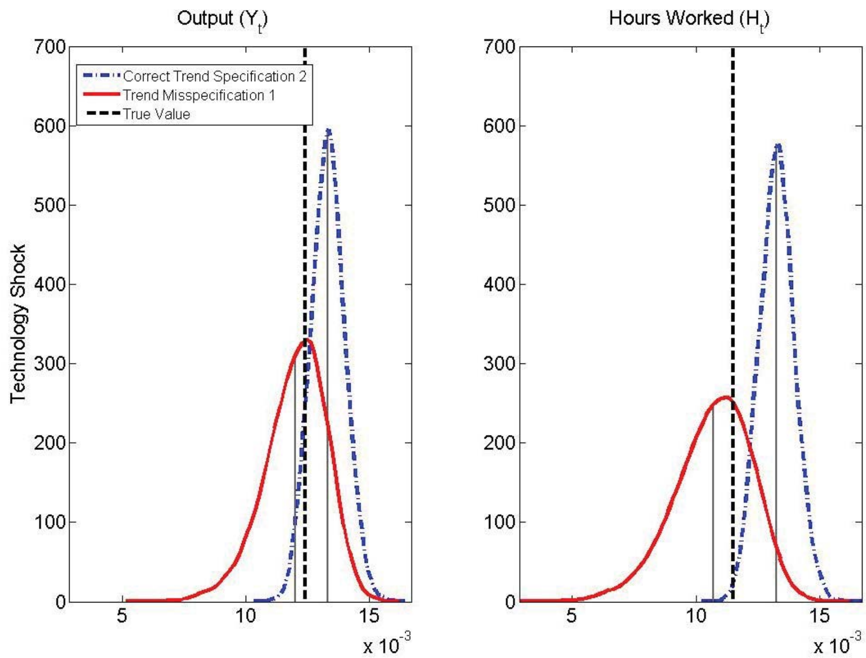


(b) Total Bias Decomposition

Figure 4: Distribution of Contemporaneous Responses

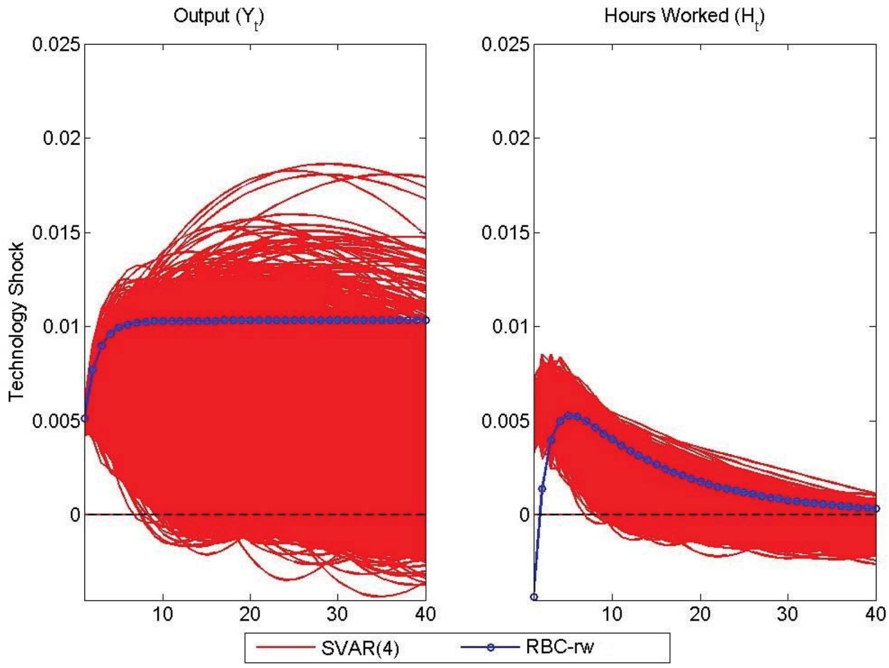


(a) Given RBC-rw as the DGP, imposing long-run restrictions is labelled as the correct trend specification while implementing short-run restrictions is considered as a trend misspecification.

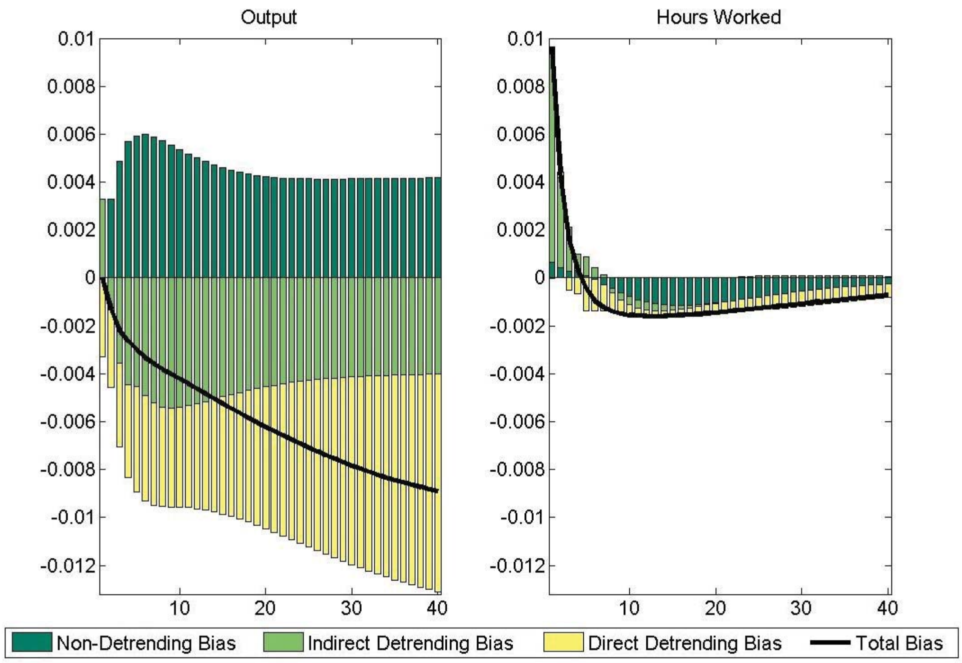


(b) Given RBC-dt as the DGP, imposing short-run restrictions is labelled as a correct trend specification while implementing long-run restrictions is considered as a trend misspecification.

Figure 5: Monte Carlo Simulations Implementing Short-Run Restrictions Given RBC-rw as the DGP (Trend Misspecification 2) where Hours Worked Responses Negatively to a Positive Technology Shock



(a) Impulse Response Functions to a Positive Technology Shock



(b) Total Bias Decomposition

Figure 6: Total Bias Decomposition Given a High Persistence in the Technology Shock with RBC-dt as the DGP (Trend Misspecification 1)

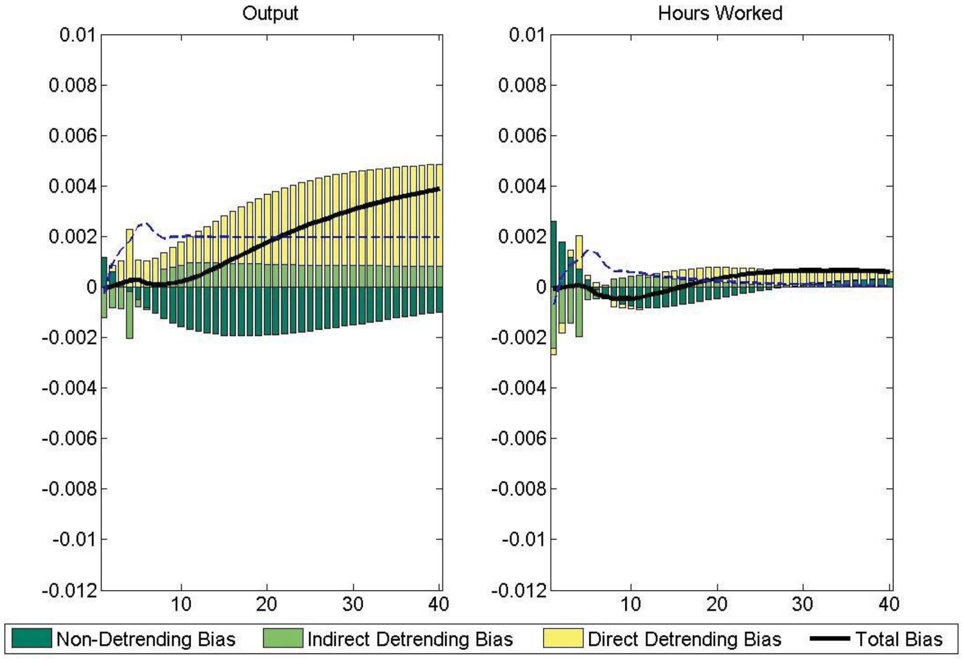


Figure 7: Total Bias Using the HP Filter and Implementing Short-Run Restrictions

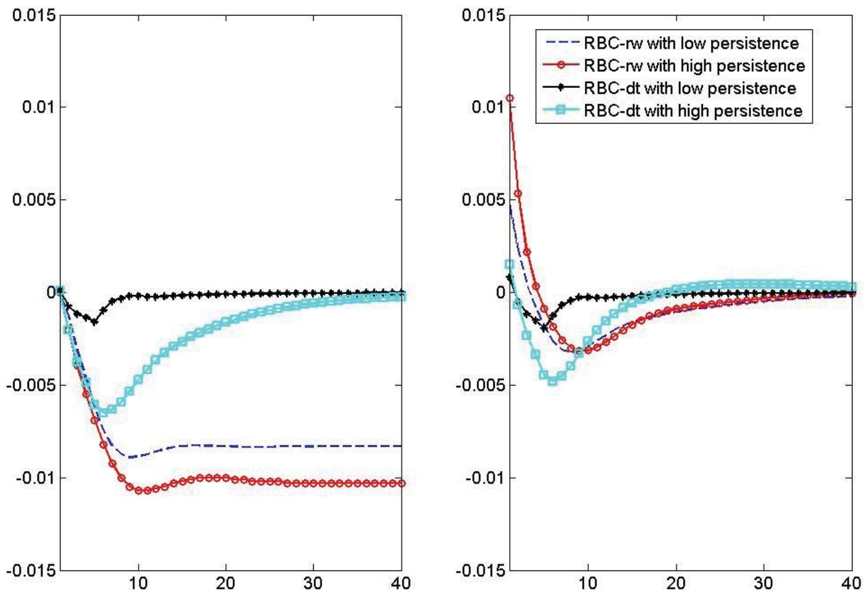


Figure 8: Monte Carlo Simulations Using First Difference and Implementing Short-Run Restrictions