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Optimal Monetary Responses to Oil Discoveries

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Abstract

This paper studies how monetary policy should respond to news about an oil discovery, using a workhorse New Keynesian model. Good news about future production can create a recession today under exchange rate pegs and a simple Taylor rule, as seen in practice. This is explained by forward-looking inflation. Recession is avoided by a Taylor rule that accommodates changes in the natural level of output, which closely approximates optimal policy. Central banks have an incentive to exploit oil revenues by appreciating the terms of trade, creating "Dutch disease" and a deflationary bias which is overcome by committing to future policy.

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First version: October 2012
This version: April 2014

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This paper studies how monetary policy should respond to news about an oil discovery, using a workhorse New Keynesian model. Good news about future production can create a recession today under exchange rate pegs and a simple Taylor rule, as seen in practice. This is explained by forward-looking inflation. Recession is avoided by a Taylor rule that accommodates changes in the natural level of output, which closely approximates optimal policy. Central banks have an incentive to exploit oil revenues by appreciating the terms of trade, creating "Dutch disease" and a deflationary bias which is overcome by committing to future policy.

Keywords: Natural resources, oil, optimal monetary policy, small open economy, news shock.

JEL Classification: E52, E62, F41, O13, Q30, Q33

1 Introduction

How should monetary policy respond to an oil discovery? This question is of broad interest, with 1.4 billion people living in the forty-seven countries economically dominated by natural resources (Baunsgaard et al., 2012). Recent technological advances in hydraulic fracturing (fracking) also means it is relevant to countries discovering shale gas and tight oil deposits, such as Australia, Canada the U.K. and the U.S. Academic literature has recently focused on medium- to long-term fiscal policy[1]. However, little has been done on how monetary policy should best respond in the short term. This poses a unique challenge because oil discoveries are a type of news shock, as there is a delay between discovery and extraction.[2] If the government does not smooth expenditure during this delay, which is typical, then monetary policy will play an important role.

This paper shows that exchange rate pegs and simple Taylor rules respond poorly to oil discoveries, but optimal policy is closely approximated with a small change to the Taylor rule. In doing so the paper makes three contributions. The first shows that oil discoveries should cause the terms of trade to appreciate twice: when households learn they are wealthier and when oil revenues are eventually spent. The second shows that good news about future oil production can create a recession today under exchange rate pegs and simple Taylor rules, because of forward-looking inflation. This happened after shale gas extraction was first demonstrated in the US. The third shows that optimal policy is closely approximated by a Taylor rule that responds to expected changes in the natural level of output. Central banks also have an incentive to exploit oil income by appreciating the terms of trade, creating "Dutch disease" and deflation, but this can be avoided if the bank can commit to future policy.

Oil discoveries will cause the terms of trade to appreciate twice. We use a workhorse New Keynesian model of a small open economy following Gali and Monacelli (2005), with rational expectations, perfect access to international capital, and oil revenues that are spent by the government when production begins. The model is tractable enough to be solved explicitly and implemented in a spreadsheet (such as Microsoft Excel). On hearing news of an oil discovery private consumption will jump, anticipating future income. When production eventually begins, the government will increase demand for domestic goods. At each stage the terms of trade must appreciate to meet the additional demand.

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[1]As surveyed by van der Ploeg and Venables (2012). Fiscal policy is important in regulating the pass-through of oil income to the economy (Pieschacon, 2012). The benchmark recommendation for oil-rich governments has been to smooth expenditure as soon as oil is discovered. In practice this means spending only the permanent income from assets that are held offshore in a sovereign wealth fund (see for example the case of Norway, Chambers et al., 2011). Expenditure smoothing has been extended to account for specific distortions a resource-exporter may face, such as bottlenecks in investment (van der Ploeg, 2012) or scarce access to capital (van der Ploeg and Venables, 2011).

[2]During this delay contracts are negotiated, permits are obtained and capital is imported or built, as seen in the data. For oil and gas discoveries, of the 400 offshore fields discovered in the UK between 1957-2011, the mean time between discovery and production was 4.5 years (DECC, 2013). For recent US shale gas the delay was 2-9 years. For mineral projects, of the 82 recently announced new projects in Western Australia at October 2012 the mean expected time to production was approximately 3 years (BREE, 2012). Of the 35 new projects that were "committed but not yet complete", only 6 percent expected to begin production within 3 months, and only 60 percent expected to begin production within 15 months.

News of an oil discovery can cause a recession under both exchange rate pegs and simple Taylor rules. Under an exchange rate peg both terms of trade appreciations must happen through domestic prices. If prices are sticky then they will be slow to adjust to the first appreciation, but respond in advance to the second, leading to an extended period of inflation. Output will initially overshoot (overemployment) and then undershoot (unemployment) its natural level before production begins. This prospect of an oil discovery-induced recession is important because 74 percent of resource-dependent economies peg their currency (Baunsgaard et al., 2012). Under a simple Taylor rule — that responds to domestic inflation and the output gap — the recession will be exacerbated. This is because a conservative central banker will tighten aggressively against forward-looking inflation (Rogoff, 1985). As inflation is driven by future income, rather than a contemporaneous shock, this response generates an even deeper recession. This highlights an important difference between contemporaneous and forward-looking inflation when conducting monetary policy.

The prospect of good news about tomorrow causing a recession today is also seen in the data. Studying news shocks is a challenge because the shock must be identified, and monetary policy must be controlled for. A useful experiment is the recent discovery of techniques to extract shale gas in the US. The date of discovery can be identified using a narrative approach (Romer and Romer, 2010). Monetary policy can be controlled for by studying how unemployment differs at a county and a national level: while shale gas revenues are important at a country level, they are not large enough to generate a monetary response nationally.[3] Figure 1.1 shows that after fracking was first successfully demonstrated, in the Barnett Formation (TX) in 1998, relative unemployment in the area rose by over 2 percent by the time production began. This was accompanied by persistently higher inflation - just as predicted by our very standard, tractable model under a currency peg.

The objective of an oil exporter's central bank will be to stabilise domestic inflation and the output gap, and to appreciate the terms of trade. The first two are common in models of symmetric small open economies (see Gali and Monacelli, 2005 and 2008). Appreciating the terms of trade exploits the asymmetry introduced by oil wealth. Doing so makes imports cheaper, but reduces non-oil exports. However, income from non-oil exports is less important for countries with oil than those without, giving the incentive to appreciate. If monetary policymakers can not commit to future policy then this central bank-induced "Dutch disease" creates a deflationary bias, which reduces welfare relative to a variety of regimes (including a currency peg) but can be overcome under commitment.[4] Thus, central bank credibility is particularly important for oil exporters, and importing credibility may explain why they are more likely than others to peg their currency.[5]

Optimal policy is closely approximated by a Taylor rule that responds to expected changes in the natural level of output, in addition to domestic inflation and the output gap. This policy ensures that both terms of trade appreciations - at discovery and at

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[3]Total natural resource rents account for less than 2 percent of US GDP (World Bank, 2013). This is like treating the county as pegging its currency to the rest of the US.

[4]Note that we abstract from any externalities in the traded sector that would make this central-bank induced "Dutch disease" sub-optimal (See Corden and Neary, 1982).

[5]74 percent of resource-dependent economies peg their exchange rate, compared to 56 percent of others. This is true for both developed (76 vs 41 percent) and developing (73 vs 66 percent) countries (Baunsgaard et al., 2012; IMF, 2008).
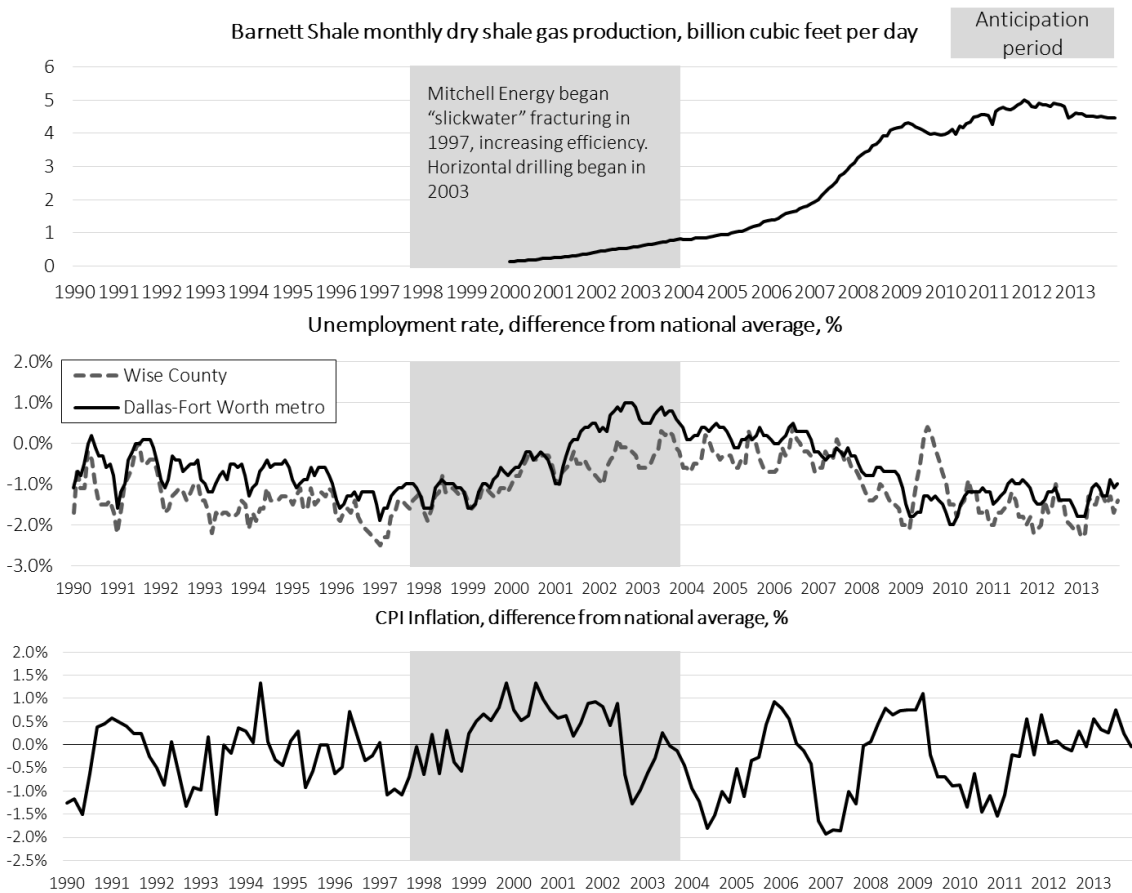
Figure 1.1: Stagflation and hydraulic fracturing in the Barnett Shale (TX). Unemployment rose in Dallas-Fort Worth by ~2 percent (relative to the national average) between the successful demonstration of fracking and large scale production. During this period inflation was also consistently above the national average.

production - happen through the nominal exchange rate rather than sticky domestic prices. The first appreciation is easily achieved with a floating currency. The second requires the bank to sharply loosen interest rates in the period before the windfall, and tighten in the period the windfall is received. This accommodates the large increase in the natural level of output, and associated fall in the natural rate of interest. It also ensures that the entire terms of trade appreciation happens through the nominal exchange rate, avoiding the need for firms to raise prices individually (and the associated distortions from price dispersion). The loosening of policy in the future does not require the central bank to be credible: such policy is also optimal for a policymaker that can not commit.

The possibility of an oil discovery creating a recession in the lead up to production has been identified before. Eastwood and Venables (1982) find that an oil discovery immediately causes the exchange rate to appreciate under the influence of expectations, in a Dornbusch-style framework. If the deflationary effect of this appreciation is not offset by increased demand, there may be a period of recession. The recession would only be reversed when oil-generated demand begins.[6] However, their analysis was performed without the benefit of today's fully-forward looking DSGE models, and assumed that households cannot borrow against future wealth. In what follows we show that such recessions can still occur because of forward-looking inflation, even if households smooth their consumption.

There is little recent work on monetary policy in oil exporters. One strand of literature has emphasized that the source of oil price shocks matters for policy in resource importers (see Kilian, 2009, Nakov and Pescatori 2010, Bodenstein et al, 2012). Another has considered the roles of monetary policy and oil prices in the US recessions of the 1970s and 80s (see Kilian and Lewis, 2011). The IMF has conducted a number of numerical analyses of oil shocks in multi-country frameworks (see for example Elekdag et al., 2007), and in specific countries such as with Ghana's 2007 discovery of oil (Dagher et al., 2010). There has been little work studying resource exporters from an optimal perspective. Romero (2008) studies how monetary policy should respond to oil price shocks in oil exporters, but abstracts from any news effects of an oil discovery.

This paper builds on the literature on optimal monetary policy in small open economies. We extend the small open economy model of Gali and Monacelli (2005, 2008) to allow for anticipated changes in income. As the model assumes perfect access to international borrowing it is best suited to oil discoveries in developed countries, like Australia, Canada, Norway, the UK and the US. Typically these models display a "divine coincidence" in which stabilising domestic inflation is equivalent to stabilising the welfare relevant output gap (Blanchard and Gali, 2007). In a closed economy, divine coincidence holds because the only distortion in the economy is sticky prices. In an open economy, divine coincidence holds only if the elasticity of substitution between home and foreign goods is one and all countries are symmetric (Monacelli, 2012). Otherwise the central bank can improve on the flexible price outcome by moving the terms of trade, as the income and substitution effects will not offset. De Paoli (2009) breaks divine coincidence by changing the elasticity of substitution between goods, introducing an incentive to stabilise

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[6]In commenting on that paper, Neary and van Wijnbergen (1984) find the results to be "relatively optimistic". By including a direct impact of oil wealth on money demand they show that the recession can continue even after government spending rises. Other papers at the time linking oil shocks to unemployment include Buiter and Purvis (1983) and van Wijnbergen (1984).

the nominal exchange rate. This paper breaks divine coincidence with asymmetric oil wealth, introducing an incentive to appreciate the terms of trade that makes credibility so important - as noted above.

The paper is also the first to study the optimal monetary response to news about future fiscal policy. News about future productivity and Pigou cycles have been the focus of the news shock literature.[7] Lorenzoni (2007) studies the optimal monetary response to productivity news shocks and finds that policymakers should announce in advance how they will react to new information. This is consistent with our findings, where forward-looking inflation is avoided because firms anticipate future monetary policy (see the forward guidance literature e.g. Woodford, 2012). News about future fiscal policy has recently been shown to be crucial in understanding the effects of government shocks (Mertens and Ravn, 2010 and 2013; Ramey, 2011). Mertens and Ravn (2011, 2012) empirically study preannounced tax cuts and show that they cause a contraction in output, investment and hours worked before they are implemented. This is also consistent with our findings, where anticipated income from the government (in our case by higher spending) causes both natural output to contract and a recession. Our paper goes further by adding nominal rigidities and considering how monetary policy should respond.

The remainder of this paper proceeds as follows. Section 2 presents a DSGE model of a small open economy with a government that receives an exogenous oil windfall and spends the revenues as they are received. Section 3 characterises the steady state and analyses dynamics around this state. Section 4 introduces the central bank's micro-founded loss function, and derives optimal monetary policy under discretion and commitment. Section 5 calibrates the model, and compares optimal policy to a flexible price benchmark and a variety of policy rules. Section 6 concludes.

2 Model

This section develops a standard, workhorse model of a small open economy with a government that receives an exogenous oil windfall. We make two additions to the standard model (see Gali and Monacelli, 2005 and 2008; Gali, 2008), to keep it as simple as possible and permit an explicit solution. The first is a government that receives an oil windfall and spends it according to a simple fiscal rule, which will be the focus of our analysis. The second is to allow relative wealth between home and foreign households to change when oil is discovered, relaxing the "divine coincidence" that characterises similar models.

2.1 Households

The representative household maximises utility subject to a per-period budget constraint,

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[7]Pigou (1927) cycles are when good news about future productivity leads to positive co-movement in consumption, investment and hours worked today. They have proved challenging to ground theoretically (see Beaudry and Portier, 2004, Den Haan and Kaltenbrunner, 2006 and Jaimovich and Rebelo, 2009). There is empirical evidence for these cycles both for (Beaudry and Portier, 2006; and Beaudry and Lucke, 2010) and against (Barksy and Sims, 2011 and 2012).

$$\max_{C_t, N_t, D_t} U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi) \ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{2.1}$$

$$s.t. \quad P_t C_t \leq W_t N_t - P_{H,t} T_t - E_t[M_{t,t+1} D_{t+1}] + D_t \tag{2.2}$$

where C_t is a domestic consumption bundle with consumer price index (CPI), P_t; G_t is the government spending bundle which is partly funded by taxes, T_t, levied in domestic currency, $P_{H,t}$; N_t is hours worked for wages, W_t; and D_{t+1} is the nominal payoff in domestic currency at time $t+1$ of the household's portfolio of perfectly internationally traded financial assets (including shares in firms), which has stochastic discount factor, $M_{t,t+1}$. Utility will depend on three key parameters: the discount factor, β, the utility weight of government spending, χ, and the elasticity of labour supply, φ. This optimisation problem yields two first order conditions, describing labour supply and the stochastic Euler equation,

$$C_t N_t^{\varphi} = (1-\chi) W_t P_t^{-1} \tag{2.3}$$

$$\beta \left(C_t / C_{t+1} \right) \left(P_t / P_{t+1} \right) = M_{t,t+1} \tag{2.4}$$

If we use lower case letters to denote the log of each variable, define the time discount rate as, $\rho \equiv \beta^{-1} - 1$, define the nominal interest rate as, $i_t = ln(E_t[M_{t,t+1}]^{-1})$, and define CPI inflation as, $\pi_t \equiv p_t - p_{t-1}$, then the first order conditions can be expressed in log-linear form[8] as,

$$w_t - p_t = c_t + \varphi n_t - \ln(1-\chi) \tag{2.5}$$

$$c_t = E_t[c_{t+1}] - (i_t - E_t[\pi_{t+1}] - \rho) \tag{2.6}$$

Total consumption is a Cobb-Douglas bundle of home (H) and foreign (F) goods, $C_t \equiv C_{H,t}^{1-\alpha} C_{F,t}^{\alpha} \cdot (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha}$. The preference for foreign goods is described by an index of openness, $\alpha \in [0,1]$. Home and foreign goods are allocated optimally to minimise, $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$, where CPI is an index of home and foreign prices, $P_t = P_{H,t}^{1-\alpha} P_{F,t}^{\alpha}$. Note that the elasticity of substitution between home and foreign goods equals one. If home and foreign consumers were also perfectly symmetric, then there would be a "divine coincidence" in the objectives of a central bank in a closed and an open economy. This happens because any movement in the terms of trade will see the increase in purchasing power over imports perfectly offset by lower demand for exports. Divine coincidence breaks down in our model because of asymmetric wealth, as discussed in Section 2.4.

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[8]The model is log-linearised around a steady state for tractability. This is appropriate because fluctuations in total resource rents in Australia, Canada, the UK and the US are less than 5 percent of GDP (World Bank, 2013). In Norway they account for 20 percent of GDP, but approximately 6 percent is released into the economy each year from the sovereign wealth fund.

Home and foreign goods are both CES bundles of individual varieties that are produced in all countries. The foreign good is a bundle of goods produced by a continuum of foreign countries, $f \in [0,1]$, with an elasticity of substitution of one. In logs this is, $c_{F,t} = \int_0^1 c_{f,t} df$, with price index, $p_{F,t} = \int_0^1 p_{f,t} df$. The consumption good produced at home and in every foreign country is a CES bundle of individual varieties, $i \in [0,1]$, $C_{j,t} \equiv \left(\int_0^1 C_{j,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ for $j = [H, f \in [0,1]]$. Each variety is produced in every country and has an elasticity of substitution of ϵ, allowing for monopolistic price setting in Section 2.5. The associated price indices are, $P_{j,t} \equiv \left(\int_0^1 P_{j,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ for $j = [H, f \in [0,1]]$.

2.2 Terms of Trade and Exchange Rates

The effective non-oil terms of trade allocates global household demand between home and foreign goods, and is defined as the price of the foreign bundle in terms of home goods. In logs this is, $s_t \equiv p_{F,t} - p_{H,t}$, where $s_t \equiv \ln S_t$ is the (log) non-oil terms of trade, and $p_{F,t}$ and $p_{H,t}$ are the (log) price indices for foreign and home goods as defined above. The effective non-oil terms of trade is an index of the bilateral non-oil terms of trade between the home country and each country f, $s_t = \int_0^1 s_{f,t} df$ where $s_{f,t} \equiv p_{f,t} - p_{H,t}$. For ease we will refer to the effective non-oil terms of trade as simply the "terms of trade" from now on.

The terms of trade links the CPI to the domestic price level. Using the (log) definitions of CPI and the terms of trade we have, $p_t = p_{H,t} + \alpha s_t$. CPI inflation can therefore be expressed as a function of domestic inflation, $\pi_t = \pi_{H,t} + \alpha \Delta s_t$.

The terms of trade is also linked to the effective nominal exchange rate. Exchange rates are defined by assuming the law of one price always holds for every variety. This implies that the price in home currency of variety i, produced in country f is (in logs), $p_{f,t}(i) = e_t^f + p_{f,t}^f(i) \ \forall i, f$, where e_t^f is the (log) bilateral nominal exchange rate between home and f, and $p_{f,t}^f(i)$ is the price of variety i, produced in country f (subscript) in currency f (superscript). The price index for all goods produced in f expressed in the home currency is, $p_{f,t} = e_t^f + p_{f,t}^f$, where $p_{f,t}^f$ is the price index of goods produced in f expressed in currency f. The price index for all goods produced in any foreign country expressed in the home currency is, $p_{F,t} = e_t + p_t^*$, where $e_t \equiv \int_0^1 e_t^f df$ is the effective nominal exchange rate and, $p_t^* \equiv \int_0^1 p_{f,t}^f df$, is the world price index. The terms of trade is thus related to the nominal exchange rate by, $s_t = e_t + p_t^* - p_{H,t}$.

Finally, the terms of trade also moves with the real exchange rate. The bilateral real exchange rate describes the CPI in country f as a proportion of the CPI at home, $q_{f,t} \equiv \ln \mathcal{Q}_{f,t} = e_t^f + p_t^f - p_t$, where p_t^f is the (log) CPI in country f. The effective real exchange rate is an index of the bilateral real exchange rates, $q_t \equiv \int_0^1 q_{f,t} df$. Linking the effective real exchange rate and the terms of trade gives, $q_t = \int_0^1 (e_t^f + p_t^f - p_t) df = e_t + p_t^* - p_t = (1-\alpha)s_t$.

2.3 Oil and Government Spending

All oil income is received by the government, which also levies lump-sum taxes and spends these revenues on public consumption and a small transfer to firms. We assume that all oil income is consumed by the government according to a simple spending rule, and we abstract from any distortionary taxes.

Government spending is summarised by its per-period budget constraint. This is given below, where F_t is the portfolio of risk-free foreign assets held at time t, T_t are lump sum taxes, τ_t is a subsidy to firms and $\varepsilon_t P_{O,t} O_t$ are oil revenues.

$$P_{H,t} G_t + \tau_t + E_t[M_{t,t+1} F_{t+1}] \leq F_t + \varepsilon_t P_{O,t} O_t + P_{H,t} T_t \tag{2.7}$$

Oil revenue in each period is, $\varepsilon_t P_{O,t} O_t$ for $t = 0, 1, \ldots$. Oil production O_t is exogenously determined, as in practice geological factors determine production in the time scales over which monetary policy is conducted. We will focus on a new oil discovery that is discovered at $t = 0$, but is extracted in the future at a constant rate for a finite period, $O_t = O_B$ for $t \in [T_A, T_B]$ and $O_t = O_A$ otherwise, where $O_B > O_A > 0$.[9] The oil price $P_{O,t}$ is expressed in units of the global price index and numeraire $P_t^* = 1$, so that fluctuations in the exchange rate will alter the relative value of oil income.

The government spends its income on public consumption and a small transfer to firms. Public consumption is a CES bundle of home-produced goods, covering categories such as health care, education and justice. We let the elasticity of substitution between goods be the same as for households, so that $G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$. The quantity of each variety is chosen optimally, yielding the government demand for each good, $G_t(i) = (P_{H,t}(i)/P_{H,t})^{-\epsilon} G_t$. The government also makes a small transfer to firms, τ_t, to offset the marginal cost distortion from monopolistic competition, following Gali and Monacelli (2005).

The amount of oil revenues spent by the government each period can be summarised by the "resource balance", RB_t. The resource balance is the net amount of resource revenues released into the government budget each period, which are expressed in units of world currency as, $RB_t = P_{O,t} O_t + F_t - E_t[M_{t,t+1} F_{t+1}]$. Substituting this into 2.7 gives, $P_{H,t} G_t + \tau_t = \varepsilon_t RB_t + T_t$, which in log-linear terms is, $\hat{g}_t = \hat{s}_t - \hat{p}_t^* + \widehat{rb}_t - \hat{t}_t$, where $t_t = -\ln(1 - \frac{T_t}{G_t})$. Foreign prices, and lump sum taxes as a share of government spending, are both assumed to be constant, so $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$. Assuming that the share, rather than the level, of lump sum taxes is constant makes the analysis more tractable, which is discussed in Appendix E.

In the main body of the paper the government is assumed to spend all oil revenues as they are received. This is not optimal, so introduces a role for the central bank. This scenario is also both practical and illustrative. In practice it characterises cases like Ecuador, Indonesia, Norway prior to 1990 and regions in Australia, Canada and the US. It also illustrates two important aspects of oil discoveries: anticipated changes in

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[9]Having non-zero oil production before and after the boom, $O_A > 0$, is necessary to avoid log-linearizing around a zero value in the steady state.

oil production, and unanticipated changes in oil prices. If we assume no initial foreign asset holdings, and that none are accumulated, $F_t = 0 \forall t$, then the resource balance is $RB_t = P_{O,t}O_t$ and (log-linear) government spending is, $\hat{g}_t = \hat{s}_t + \hat{p}_{O,t} + \hat{o}_t$. Appendix D also considers two further fiscal rules: the first-best benchmark Permanent Income rule, where expenditure is perfectly smoothed; and the Bird in Hand rule, where expenditure is a fixed proportion of sovereign wealth fund assets (as in Norway currently).

2.4   International Risk Sharing

The international risk sharing condition lets us express domestic consumption as a function of world consumption. We focus on developed economies and so assume that international markets in financial assets are partially complete. Claims on all shocks in the economy can be traded, except for claims on a nation's oil wealth. This means that an oil discovery will asymmetrically increase the wealth of home households relative to foreigners, which is the experiment we are interested in. Using this assumption and identical preferences across countries, then from equation 2.4 we have the following for every country, f,

$$E_t\left[\beta\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\right] = E_t\left[M_{t,t+1}\right] = E_t\left[\beta\left(\frac{C_t^f}{C_{t+1}^f}\right)\left(\frac{P_t^f}{P_{t+1}^f}\right)\left(\frac{\varepsilon_t^f}{\varepsilon_{t+1}^f}\right)\right] \qquad (2.8)$$

Households in each country also face a transversality condition. Everything they earn must eventually be consumed, so $\lim_{T\to\infty} M_{0,T}D_T = 0$. Summing 2.2 over an infinite horizon for both home and foreign countries, and using 2.4 (see Appendix A.1) gives the following expressions tying domestic consumption, C_t, to foreign consumption, C_t^f, adjusted for relative household wealth, Θ_t^f, and the real exchange rate, $Q_{f,t}$,

$$C_t = \Theta_t^f C_t^f \mathcal{Q}_{f,t} \qquad (2.9)$$
$$\text{where } \Theta_t^f = \frac{E_t[\sum_{s=0}^{\infty} M_{t,t+s}\left(W_{t+s}N_{t+s} - P_{H,t+s}T_{t+s}\right)] + D_t}{E_t[\sum_{s=0}^{\infty} M_{t,t+s}^f\left(W_{t+s}^f N_{t+s}^f - P_{H,t+s}^f T_{t+s}^f\right)] + D_t^f} \qquad (2.10)$$

Relative household wealth is always constant in expectation, $E_t[\Theta_{t+s}] = \Theta_t \forall s \geq 0$. Taking logs, integrating over all countries f, and using $c_t^* = \int_0^1 c_t^f df$, gives the following, where $\vartheta_t \equiv \ln \Theta_t$,

$$c_t = \vartheta_t + c_t^* + (1-\alpha)s_t \qquad (2.11)$$

A depreciation in the terms of trade ($s_t \uparrow$), means domestic goods become relatively cheap compared to foreign goods, and will boost domestic consumption if all else is equal. Relative household wealth describes the expected present value of a domestic household's future income (net taxes), relative to a household abroad. On discovering oil this will

play an important role. Domestic consumers will anticipate higher lifetime income when oil is discovered, even if it is not immediately received. Their consumption will jump accordingly. If no further surprises are received about oil income, consumption will simply evolve according to the Euler equation in 2.4.

The change in relative household wealth can be expressed in terms of the government's spending rule. Assuming households calculate their lifetime wealth under flexible prices, so $\hat{\vartheta}_t = \hat{\vartheta}_t^n$,[10] and initial asset holdings are zero, $D_0 = D_0^f = 0$, then (log) relative household wealth will be proportional to the present value of the government's resource-balance, as shown in Appendix A.1,

$$\hat{\vartheta}_0 = \frac{(1-\beta)(\gamma_G-\chi)}{(1-\chi)-(1-\gamma_G)(1-\alpha)} E_0 \left[\sum_{t=0}^{\infty} M_{0,t}\widehat{rb}_t \right]. \tag{2.12}$$

In this framework uncovered interest parity will also hold, $E_t[q_{t+1}] - q_t = r_t - r^*$. This follows from 2.6 at home and abroad, $E[\Delta c_{t+1}] = E[\Delta c_{t+1}^*] + (1-\alpha)E[\Delta s_{t+1}]$ (from equation 2.8), and the definition of the real interest rate, $r_t \equiv i_t - E[\pi_{t+1}]$.

2.5 Firms

Each domestic firm will produce a differentiated good using labour and a common technology, $Y_t(i) = A_t N_t(i)$. Oil is not included as a factor of production, to focus on the windfall effects of a resource discovery. Capital is also not included for tractability and to focus on short-term dynamics.

Firms set prices in a staggered way according to Calvo (1983). A measure of $(1 - \theta)$ randomly selected firms set new prices in each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. In a standard result, derived in Appendix A.3, the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the log-linear rule,

$$p_{H,t}^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t[mc_{t+k} + p_{H,t}] \tag{2.13}$$

where $p_{H,t}^*$ denotes the log of newly set domestic prices, and $\mu \equiv \ln\left(\frac{\epsilon}{\epsilon-1}\right)$ is the log of the optimal markup in the flexible price economy. The pricing decision is thus forward-looking, as firms recognise that the price they set will last for a random number of periods. In the flexible price limit (as $\theta \to 0$) we recover the markup rule $p_{H,t}^* = \mu + mc_t^n + p_{H,t}$, so that the marginal cost in the flexible price state is $mc_t^n = -\mu$.

---

[10]This makes the problem far more tractable, and means that short-term nominal rigidities do not have permanent effects.

3 Equilibrium Dynamics

The equilibrium of the model is characterised by standard aggregate demand (IS curve) and aggregate supply (Phillips curve) conditions, which describe dynamics around a steady state. Before we continue it is important to clarify some notation. Let us assume that before oil is discovered the economy is at the steady state (log) level of output, $y^s \equiv \ln Y^s$. Discovering oil will induce changes in the economy that cause the actual level of output, y_t, to deviate from the steady state, $\hat{y}_t \equiv y_t - y^s$. These changes will also cause the "natural" (flexible-price) level of output to deviate from the steady state, $\hat{y}_t^n \equiv y_t^n - y^s$. The difference between the actual and the natural level of output will be the "output gap", $\tilde{y}_t \equiv y_t - y_t^n$. In the analysis below we will be interested in changes to both the actual and the natural level of output, and so will be keeping track of both $\hat{y}_t$ and $\hat{y}_t^n$.

3.1 Aggregate Demand

The market clearing condition for each domestically produced good requires output to equal demand from domestic consumption, government purchases and consumption from each foreign country. In Appendix A.2 this market clearing condition is combined with the Euler equation (2.6) to give the IS curve,

$$\hat{y}_t = E_t[\hat{y}_{t+1}] - (i_t - E_t[\pi_{H,t+1}] - \rho) - (1+\varphi)E_t[\Delta \hat{y}_{t+1}^n] \tag{3.1}$$

This is the standard IS curve, as can be seen when re-written as $\tilde{y}_t = E_t[\tilde{y}_{t+1}] - (i_t - E_t[\pi_{H,t+1}] - r_t^n)$ with the natural rate of interest, $r_t^n = \rho - \varphi E_t[\Delta \hat{y}_{t+1}^n]$ (see Gali and Monacelli, 2005). An increase in the natural rate of interest is associated with an expected fall in the natural level of output, because people will consume leisure if they are wealthier tomorrow (i.e. if the natural rate of interest is high). Aggregate demand takes this simple form because of the way we have characterised government spending, $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$. We write the IS curve as in equation 3.1 in order to keep track of changes in the natural level of output, $\hat{y}_t^n$. This is derived in Appendix A.3 from marginal costs when prices are flexible, giving,

$$\hat{y}_t^n = \frac{\gamma_G}{1+\varphi}\widehat{rb}_t - \frac{\alpha + \gamma_G(1-\alpha)}{1+\varphi}\hat{\vartheta}_t + \frac{\varphi-1}{1+\varphi}\hat{a}_t \tag{3.2}$$

The natural level of output is affected by the level of government spending (the resource balance), $\widehat{rb}_t$, relative household wealth, $\hat{\vartheta}_t$ and technology, $\hat{a}_t$. When the government increases the amount of resource revenues released into the budget, it increases the natural level of output. This happens both in the current period, and in all future periods due to changing household wealth. A change in the natural level of output will affect the marginal costs of firms, which in turn affects aggregate supply. Higher relative household wealth reduces the natural level of output, as households consume leisure and higher domestic prices lead to substitution away from home goods. Higher technology increases the natural level of output by increasing labour productivity.

3.2 Aggregate Supply

Firms set prices after observing the demand schedule of households and the government. In a standard result, derived in Appendix A.3, aggregate supply can be approximated to the first order by the standard New Keynesian Phillips curve,

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(1+\varphi)(\hat{y}_t - \hat{y}_t^n) \tag{3.3}$$

where we separate the output gap into movements in the actual and natural levels of output, to keep track of both, $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$.

3.3 The Steady State

The steady state in the home country, when the home government receives oil revenues while foreign governments do not, and all countries are otherwise symmetric with $A^i = 1$, is as follows, as proved in Appendix A.4,

$$
\begin{array}{lll}
N^s = \left(\frac{1-\chi}{1-\gamma_T}\right)^{\frac{1}{1+\varphi}} & ; \quad Y^s = N^s & ; \quad \Theta^s = \frac{\alpha(1-\gamma_T)}{(1-\gamma_G-(1-\alpha)(1-\gamma_T))} \\
S^s = (N^s)^{-\varphi}(\Theta^s)^{-1} & ; \quad G^s = \gamma_G Y^s & ; \quad C^s = \Theta^s(1-\chi)(S^s)^{1-\alpha}
\end{array} \tag{3.4}
$$

where $\gamma_G \equiv G^s/Y^s$, $\gamma_T \equiv T^s/Y^s$ and $(\gamma_G - \gamma_T)$ is the steady state share of government spending financed by oil revenues, expressed as a proportion of GDP.

The steady state accounts for permanent differences in wealth due to oil income. The household wealth ratio is greater than one if government spending is partly financed by oil revenues at home but not abroad, $\Theta^s > 1$, if $\gamma_G > \gamma_T$. This will also lead to an appreciated steady state terms of trade, $S < 1$, and relatively higher home consumption, $C^s > C^*$.

4 Optimal Monetary Policy

This section explicitly derives the optimal monetary responses to an oil discovery, under both discretion and commitment. They are based on a central bank loss function that is derived from household welfare, and are compared to an exchange rate peg and two Taylor rules.

4.1 Central Bank Objective

The central bank's objective is derived from household welfare and involves stabilising domestic inflation and the output gap and appreciating the terms of trade, as stated in the following proposition.

Proposition 1. *If the government spends it oil revenues according to a fiscal rule of the form,* $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$, *then the central bank's loss function is,*

$$\mathbb{L}_0 = E_0 \sum_t^\infty \beta^t \left\{ \tfrac{1}{2}\tfrac{\epsilon}{\lambda}\pi_{H,t}^2 + \tfrac{1}{2}(1+\varphi)\hat{y}_t^2 + \alpha\,(1-\chi)\,\hat{s}_t \right\} + o(\hat{y}_t^3) + t.i.p. \qquad (4.1)$$

where t.i.p. are terms independent of policy.

Proof. See Appendix B.1. □

The incentive to stabilise domestic inflation and the output gap is consistent with standard, symmetric models of a small open economy. In a closed economy there is typically a "divine coincidence", where stabilising domestic inflation is equivalent to stabilising the output gap (see Gali, 2008). In a benchmark open economy this also holds if the effects of appreciating the terms of trade cancel out: that is, if higher purchasing power over imports is offset by lower income from exports.[11] De Paoli (2009) breaks divine coincidence by changing the elasticity of substitution between home and foreign goods. This paper breaks divine coincidence in a new way, by relaxing the symmetry in wealth across countries.

The incentive to systematically appreciate the terms of trade comes from oil wealth relaxing the symmetry between home and foreign consumers. When oil is discovered the central bank will have an incentive to appreciate the terms of trade each period ($\hat{s}_t \downarrow$), boosting domestic consumers' purchasing power abroad. The associated reduction in export income is less important because domestic consumers have an additional source of income: oil. This "appreciation bias" or "central-bank induced Dutch disease" can only be overcome if the central bank can credibly commit to future policy, as seen below. It is therefore particularly important for the central banks of resource-exporters to be credible, which may offer a reason why resource-exporters are disproportionately likely to peg their currency.

It has previously been noted that a linear term in a central bank's objective function can distort the welfare ranking of policies (Benigno and Woodford, 2005). In the main text we follow Gali (2008, Ch 5) and assume that the linear $\hat{s}_t$ term is "small" (i.e. of second order), so that it does not bias our results. The advantage of this approach is that it lets us compare policy by a discretionary and a credible central bank. In Appendix B.2.3 we re-express the objective function in quadratic terms using a second-order approximation of aggregate supply, following Benigno and Woodford (2005). This is more suitable for large oil discoveries, and considers optimal policy from a timeless perspective, but is not suitable for studying discretion. Both loss functions yield similar results.

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[11]Divine coincidence requires the small open economy to have a unit elasticity of substitution between home and foreign goods, and symmetry with the rest of the world.

4.2 Optimal Policy under Discretion

The optimal response to an oil discovery under discretion requires a sharp loosening of interest rates immediately before the windfall begins, but involves a systematic deflationary bias. Optimal policy is found by minimising the loss function subject to the Phillips curve. The loss function in equation 4.1 can be expressed in terms of the output gap using $\hat{s}_t = \hat{y}_t + t.i.p.$[12] The central bank therefore minimises the following Lagrangian,

$$
\min_{\pi_{H,t}, \hat{y}_t} \mathcal{L}_{\mathcal{D}} = E_0 \sum_t^{\infty} \beta^t \left\{ \frac{1}{2} \left[\phi_\pi \pi_{H,t}^2 + \phi_{yy} \hat{y}_t^2 \right] + \phi_y \hat{y}_t \right.
$$
$$
\left. - l_{D,t}(\lambda(1+\varphi)\hat{y}_t - \pi_{H,t}) + t.i.p. \right. \tag{4.2}
$$

where $\phi_\pi = \frac{\epsilon}{\lambda}$, $\phi_{yy} = 1 + \varphi$, $\phi_y = \alpha(1 - \chi)$ and the discretionary central bank can not choose $E_t[\pi_{H,t+1}]$. This yields paths for inflation, the output gap and interest rates, which are summarised in the following Proposition,

Proposition 2. *For a monetary authority acting with discretion and a government following an oil spending rule, the optimal paths for $\pi_{H,t}$, $\hat{y}_t$ and i_t are,*

$$
\pi_{H,t} = -a_D \lambda(1+\varphi) \left\{ \frac{\phi_y \phi_{yy}^{-1}}{1 - a_D \beta} + E_t \left[\sum_{s=0}^{\infty} (a_D \beta)^s \hat{y}_{t+s}^n \right] \right\} \tag{4.3}
$$

$$
\hat{y}_t = -\left(\lambda(1+\varphi)\phi_\pi \phi_{yy}^{-1} \right) \pi_{H,t} - \phi_y \phi_{yy}^{-1} \tag{4.4}
$$

$$
(i_t - \rho) = d_1 \pi_{H,t} - d_2 E_t[\Delta \hat{y}_{t+1}^n] + d_3 E_t \left[\sum_{s=0}^{\infty} (a_D \beta)^s \hat{y}_{t+1+s}^n \right] + d_4 \tag{4.5}
$$

where $a_D = (1 + \lambda^2(1+\varphi)^2 \phi_\pi/\phi_{yy})^{-1}$, $d_1 = \lambda(1+\varphi)\frac{\phi_\pi}{\phi_{yy}}$, $d_2 = 1 + \varphi$, $d_3 = \left(\lambda(1+\varphi)\frac{\phi_\pi}{\phi_{yy}} - 1 \right) a_D \lambda(1+\varphi)$ and $d_4 = \left(\lambda(1+\varphi)\frac{\phi_\pi}{\phi_{yy}} - 1 \right) a_D \lambda(1+\varphi) \frac{\phi_y/\phi_{yy}}{1 - a_D \beta}$.

Proof. See Appendix B.2. □

When oil is discovered a discretionary central bank should optimally respond to domestic inflation, and both current and future changes in the natural level of output. There will also be a deflationary bias, as a result of the central bank's attempts to appreciate the terms of trade through the nominal exchange rate.

Optimal discretionary policy in an oil exporter will first respond to domestic inflation, $d_1 \pi_{H,t}$. This is consistent with standard models of the small open economy (see Gali and Monacelli, 2005 and 2008). A number of interest rate paths are consistent with the optimal $\pi_{H,t}$ and $\hat{y}_t$. However, we are interested in an interest rate rule which is not only consistent with the optimal paths for $\pi_{H,t}$ and $\hat{y}_t$, but which will produce them uniquely.

---

[12]This follows from equation A.4, $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$ and $\widehat{rb}_t$, $\hat{\vartheta}_t$ are terms independent of monetary policy.

The rule in equation 4.5 will give a unique and determinate solution when $d_1 > 1$ or $\lambda(1 + \varphi)\phi_\pi > \phi_{yy}$.

The optimal monetary response to an oil windfall is to sharply loosen policy immediately before spending begins, accommodating changes in the natural rate of interest, $-d_2 E_t[\Delta \hat{y}_{t+1}^n]$. Intuitively this is worth highlighting, because this loosening ensures that the required change in relative prices from oil demand happens entirely through the nominal exchange rate. Firms anticipate this and so do not need to raise their prices in advance. It is important to note that this loosening, although it may happen in the future, does not require the central bank to be able to commit to future policy. It is also optimal for a discretionary central banker. Optimal policy also offsets the recursive effects of future demand on forward-looking inflation, $+d_3 E_t\left[\sum_{s=0}^{\infty}(a_D\beta)^s \hat{y}_{t+1+s}^n\right]$. Under standard calibrations this effect is less important, $d_2 \gg d_3$.

The final component of optimal policy is a small but systematic deflationary bias, as seen in the constant term d_4 and the first term of 4.3. This happens because the discretionary central bank will repeatedly try to appreciate the terms of trade by tightening nominal interest rates, stemming from the loss function in Proposition 1. It can be understood using uncovered interest parity, $r_t - r^* = (1 - \alpha)E_t[s_{t+1} - s_t]$. If the policymaker cannot commit to future policy it will continually raise the nominal interest rate in an attempt to raise the real interest rate ($r_t \uparrow$) and appreciate the current terms of trade ($s_t \downarrow$). This is anticipated so will be offset by deflation, leaving r_t stable.

4.3 Optimal Policy under Commitment

The optimal response to an oil discovery by a credible central bank also requires policy to loosen immediately before oil-financed demand increases. While this happens in the future it does not require commitment because it is also optimal for a discretionary central bank, as discussed above. Commitment does help to overcome the discretionary bias towards appreciating the terms of trade. For a credible central bank the Lagrangian is,

$$
\min_{\pi_{H,t}, \hat{y}_t} \mathcal{L}_\mathcal{D} = E_0 \sum_{t}^{\infty} \beta^t \left\{ \frac{1}{2}\left[\phi_\pi \pi_{H,t}^2 + \phi_{yy}\hat{y}_t^2\right] + \phi_y \hat{y}_t \right.
$$
$$
\left. -l_{c,t}(\beta\pi_{H,t+1} + \lambda(1 + \varphi)\hat{y}_t - \pi_{H,t}) + t.i.p. \right. \tag{4.6}
$$

Minimising this yields paths for the price level, the output gap and interest rates as summarised in the following Proposition,

Proposition 3. *For a monetary authority acting under commitment, and a government following an oil spending rule, the optimal paths for $\hat{p}_{H,t}$, $\hat{y}_t$ and i_t are,*

$$
\hat{p}_{H,t} = \delta\hat{p}_{H,t-1} - \lambda(1 + \varphi)\left\{ \frac{\phi_y}{\phi_{yy}}\frac{\delta}{(1 - \beta\delta)} + \delta E_t\left[\sum_{s=0}^{\infty}(\beta\delta)^s \hat{y}_{t+s}^n\right] \right\} \tag{4.7}
$$

$$
\hat{y}_t = -\left(\lambda(1 + \varphi)\phi_\pi\phi_{yy}^{-1}\right)\hat{p}_{H,t} - \phi_y\phi_{yy}^{-1} \tag{4.8}
$$

$$
(i_t - \rho) = c_1\hat{p}_{H,t} - c_2 E_t[\Delta\hat{y}_{t+1}^n] + c_3 E_t\left[\sum_{s=0}^{\infty}(\beta\delta)^s \hat{y}_{t+1+s}^n\right] + c_4 \tag{4.9}
$$

where $\delta = (1 - \sqrt{1 - 4a^2\beta})/(2a\beta)$, $a = (1 + \beta + \lambda^2(1 + \varphi)^2\phi_\pi/\phi_{yy})^{-1}$, $c_1 = (\lambda(1 + \varphi)\frac{\phi_\pi}{\phi_{yy}} - 1)(1 - \delta)$, $c_2 = 1 + \varphi$, $c_3 = (\lambda(1 + \varphi)\frac{\phi_\pi}{\phi_{yy}} - 1)\lambda(1 + \varphi)\delta$ *and* $c_4 = (\lambda(1 + \varphi)\frac{\phi_\pi}{\phi_{yy}} - 1)\lambda(1 + \varphi)\frac{\phi_\pi}{\phi_{yy}}\frac{\delta}{1-\beta\delta}$.

Proof. See Appendix B.2. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

A credible central bank in an oil exporting economy should optimally respond to the domestic price level and expected changes in the natural level of output. The small deflationary bias that exists under discretion can be overcome under commitment.

The optimal monetary rule under commitment will first respond to the domestic price level, $c_1\hat{p}_{H,t}$. This is consistent with standard small open economy literature (see Gali, 2008). The rule will give a unique and determinate solution under the same conditions as the discretionary case, when $c_1 = \left(\lambda(1 + \varphi)\phi_\pi\phi_{yy}^{-1} - 1\right)(1 - \delta) > 0$ or $\lambda(1 + \varphi)\phi_\pi > \phi_{yy}$. By committing to target the level rather than the change in prices, policy has more persistence. This allows the credible central bank to overcome the deflationary bias.

As with the discretionary case, the optimal response to an oil discovery is to sharply loosen interest rates immediately before oil revenues are spent, $-c_2E_t[\Delta\hat{y}_{t+1}^n]$. This accommodates the change in the natural rate of interest that occurs from an influx of oil wealth. It also prevents forward-looking inflation by ensuring that the necessary adjustment in the terms of trade happens entirely through the nominal exchange rate. Optimal policy also tightens slightly against anticipated demand from oil revenues in the future, to lean against firms raising prices in anticipation, $+c_3E_t\left[\sum_{s=0}^{\infty}(\beta\delta)^s\hat{y}_{t+1+s}^n\right]$.

Finally, the optimal rule under commitment overcomes (asymptotically at least) the deflationary bias under discretion. This is because the public anticipates the central bank will eventually let output return to its natural level, rather than try repeatedly to appreciate the terms of trade. This allows prices to remain stable. The term c_4 also disappears when policy is conducted from a timeless perspective (see Appendix B.2.3). Central bank credibility is therefore particularly important for resource exporters, because their additional foreign income provides an incentive to appreciate the terms of trade. Importing credibility from abroad may be a reason why so many oil exporters peg their currency.

4.4 Other Monetary Rules

In Section 5 we will compare optimal policy to three ad-hoc monetary rules: an exchange rate peg, a simple Taylor rule and an augmented Taylor rule.

The exchange rate peg is maintained using the nominal interest rate. In this model the capital account is open and international markets in financial assets are partially complete (except for claims to national oil wealth, see section 2.4). Any accumulation of foreign currency reserves by the central bank can therefore be offset by households. As a result the nominal exchange rate must be stabilised by matching the nominal interest rate at home to that abroad. The dynamics of the economy under and exchange rate peg are derived explicitly in Appendix B.3.

The simple Taylor rule will see interest rates respond to domestic inflation and the output gap, as given in equation 4.10 (see Taylor, 1993 and Woodford, 2001),

$$(i_t - \rho) = \phi_\pi \pi_{H,t} + \phi_y \hat{y}_t \tag{4.10}$$

The augmented Taylor rule will see interest rates respond to expected changes in the natural level of output, in addition to domestic inflation and the output gap as given in equation 4.11. The additional term, $E_t[\Delta \hat{y}^n_{t+1}]$, draws on similar terms in the optimal interest rate rules in equations 4.5 and 4.9. Intuitively, the rest of the economy will be responding to anticipated changes in the economy, which affect the natural rate of interest, so the monetary authority should as well. In the next section we will see that this augmented rule closely approximates optimal policy.

$$(i_t - \rho) = \phi_\pi \pi_{H,t} + \phi_y \hat{y}_t + \phi_n E_t[\Delta \hat{y}^n_{t+1}] \tag{4.11}$$

5 Results

This section illustrates the performance of different monetary regimes during an oil discovery. We focus on a news shock where oil is discovered today but begins generating revenues in the future, which are spent by the government as they are received (see section 2.3). We evaluate how each regime performs under two shocks: anticipated changes in oil production and unanticipated changes in oil prices. All the dynamics are solved in closed form (see Appendix B) and implemented in a spreadsheet (Microsoft Excel). We find that exchange rate pegs and simple Taylor rules perform poorly, but optimal policy is closely approximated if the Taylor rule also responds to expected changes in the natural level of output. Two further scenarios are illustrated in Appendix D: a Permanent Income rule where the government perfectly smooths oil expenditure; and a Bird in Hand rule where the government spends a fixed proportion of assets accumulated in a sovereign wealth fund, as is done in Norway.

5.1 Anticipated Changes in Oil Production

An oil discovery provide news about future production and so has effects in two stages: immediately when it is discovered, and in the future when it is extracted. We illustrate this by considering a range of monetary policy regimes in turn, beginning with a benchmark flexible-price case. Each stage requires a separate terms of trade appreciation as demand for home goods rises from consumers, and then the government. The latter will be anticipated by households and firms. Under an exchange rate peg (as used by 74 percent of resource-dependent countries) or a simple Taylor rule this will lead to an extended period of inflation and recession. Optimal policy avoids the recession, and is well

<div align="center">**Welfare Loss Relative to Commitment**</div>

	Comm	Flexible	TR*	TR	Peg	Disc
Welfare Loss ($\hat{c}_0$ units)	-	0.2%	0.2%	3.0%	12.1%	49.7%

Table 1: The welfare loss from an anticipated change in oil production (see figures 5.1 and 5.2), expressed as the amount of additional consumption needed at $t = 0$, $\hat{c}_0$, to replicate welfare under commitment.

approximated by a Taylor rule that also responds to expected changes in the natural level of output. These results are illustrated in Figures 5.1 and 5.2.[13]

5.1.1 Flexible Prices

If oil is discovered and the government spends all revenues as they are received then there will be two major periods of change in the economy, as illustrated in the flexible price case ("*Flex*") in Figure 5.1. The first is a jump in consumption when the discovery is announced and households immediately anticipate higher future government spending, and the associated higher wage income. Higher consumption will cause inflation and appreciate the terms of trade, reducing demand for domestically produced goods. The household will finance this consumption by borrowing from abroad, as seen in the negative change in their budget constraint.

The second major change happens when production and government spending eventually begin. This additional demand for home goods causes a second period of inflation, further appreciating the terms of trade (see "*Flex*" in Figure 5.1, at $t = 4$). Domestic inflation will affect both domestic and foreign households. Domestic households will face a tighter budget constraint, and so overall consumption will fall. They will also face a change in relative prices, and will switch from home to foreign goods. The net effect is that domestic households repay their borrowing from the first few periods. Foreign households will also respond to the appreciated terms of trade, and reduce their consumption of home goods. However, the net effect is an increase in output because the government only consumes home goods - such as education, health care and justice, while households consume a bundle of internationally produced goods.

5.1.2 Exchange Rate Peg

Under an exchange rate peg, an oil discovery will induce large fluctuations in inflation, the output gap and the terms of trade. In practice, currency pegs are used by three-quarters

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[13]Figures 5.1 and 5.2 illustrate forecasts for the mean path of each variable, rather than impulse responses. The forecasts do not return to the initial steady state, though this does not imply the model yields non-stationary responses to stochastic shocks (or alternatively unit roots in the paths of the variables). Instead, they illustrate a change in the mean level of each variable in response to an anticipated, deterministic change in oil output.
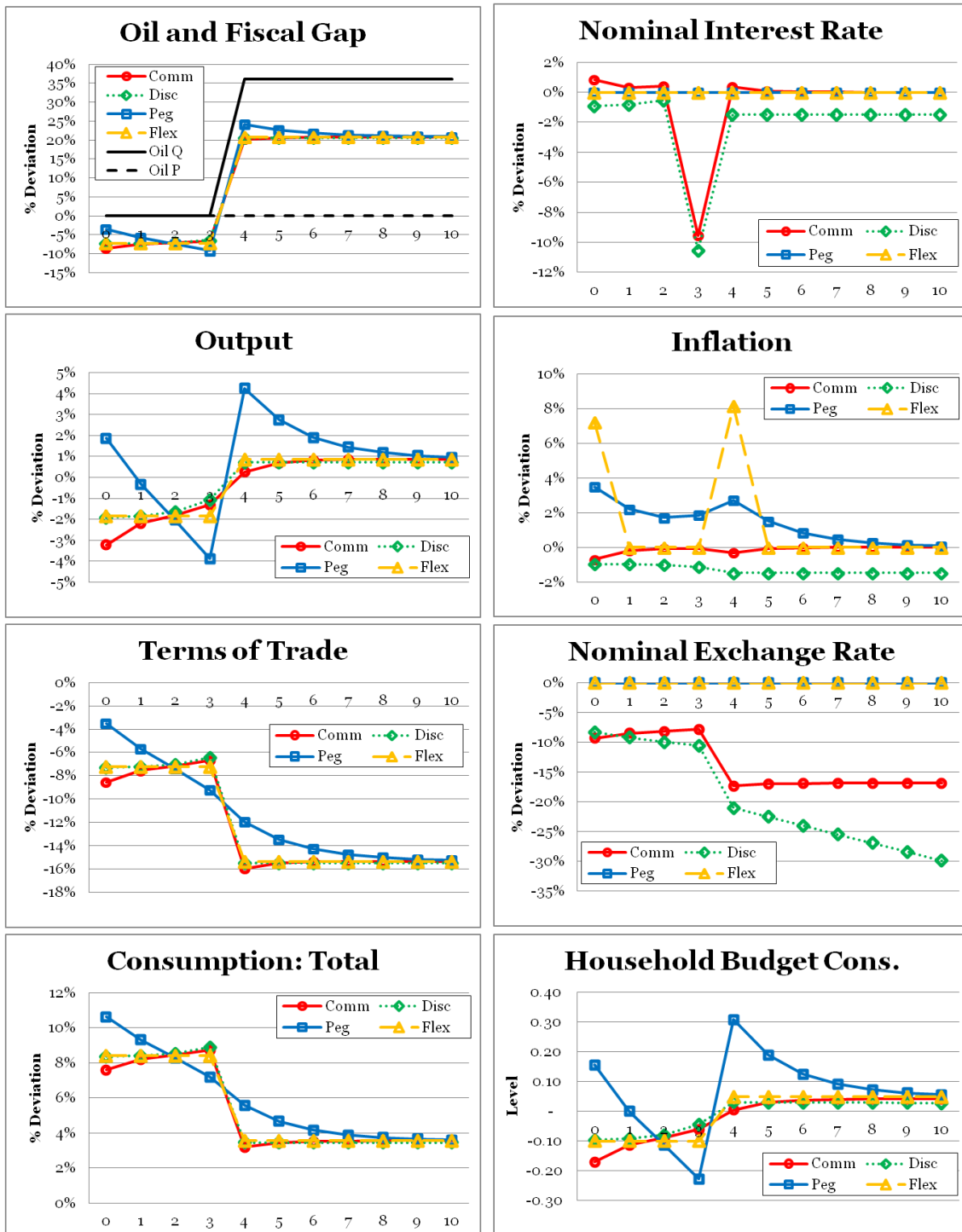
Figure 5.1: Forecast mean responses of key variables to an anticipated oil windfall, if the government spends oil revenues as they are received. Four different scenarios are illustrated: optimal commitment (Comm), optimal discretion (Disc), a pegged currency (Peg) and flexible prices (Flex).

of resource dependent economies (Baunsgaard et al., 2012). This analysis shows that a fixed exchange rate removes an important "shock absorber" for oil-exporters. This can have particularly adverse effects because of the heavy exposure to oil price shocks. The effect of an oil discovery under a peg is illustrated in the "*Peg*" case in Figure 5.1 (for derivation see Appendix B.3).

On discovering oil the terms of trade will need to appreciate for the first time. This cannot happen via the nominal exchange rate, so must happen entirely through domestic inflation. As prices are slow to adjust, output, consumption and foreign borrowing will initially overshoot their natural levels. Nominal rigidities will also cause prices to disperse between varieties of goods, leading to real distortions.

When government spending begins, the terms of trade will need to appreciate for the second time. Again, this can only happen through domestic inflation. In contrast to the appreciation when oil is discovered, this appreciation is anticipated by firms. Firms can only set prices randomly due to Calvo pricing (see Section 2.5). Any firm that has an opportunity to raise prices before government spending begins will do so. Thus, between discovering and producing oil, firms will have two incentives to raise prices. The first is to raise prices retrospectively to deal with the jump in household consumption. The second is to raise prices prospectively to anticipate future government demand. The result is an extended period of high inflation, and a rapid decline in output after the euphoria at discovery.

Output only recovers when government spending begins. It will again jump too far, as all firms will not have had an opportunity to raise prices in advance. An oil discovery under a currency peg will therefore lead to an extended period of inflation and large fluctuations in output.

5.1.3 Optimal Policy: Discretion and Commitment

Optimal policy can dampen the sharp fluctuations caused by an oil discovery under an exchange rate peg. This will involve letting the nominal exchange rate appreciate on discovery, and the key feature for our analysis: loosening policy before spending begins. There will also be an incentive to appreciate the terms of trade, leading to a deflationary bias under discretion which is overcome under commitment.

When oil is discovered, the central bank will optimally let the terms of trade appreciate via the nominal exchange rate. On discovering oil, domestic consumption will jump because of an increase in household wealth. This requires the terms of trade to appreciate. Allowing them to appreciate via the currency, rather than domestic prices, avoids the sticky price distortions in the latter. This takes care of much of the initial shock to the economy that comes from higher household wealth.

When government spending begins, the central bank will optimally loosen interest rates in the period before to delay the second terms of trade appreciation. Between oil discovery and production, forward looking firms will have an incentive to raise prices in anticipation of future demand. This will lead to distortions in the economy as prices of different varieties of goods become dispersed. The central bank can avoid this by following

a rule that will lead to a sharp loosening in policy in the period before spending begins. Loose policy in the period immediately before the windfall will cause firms to expect the currency to appreciate when the windfall begins (consistent with uncovered interest parity). Thus, firms need not raise their prices in anticipation, because they know the central bank will achieve this for them. In essence, this policy allows the central bank to take responsibility for appreciating the terms of trade from firms, using a single, flexible price rather than many sticky ones.

It is important to note that the effectiveness of loose policy in the future does not rely on the ability of the central bank to commit. Loose policy will be optimal in the period before the windfall, even for a discretionary central bank. However, the policy's effectiveness does rely on firms understanding the monetary rule that is being followed. In the model this is taken care of using rational expectations. In practice it may require communication by the central bank about the nature of the rule, and that it is optimal even for a discretionary central bank.

After government spending begins, a discretionary central bank will suffer from a deflationary bias. This stems from the incentive for the central bank to appreciate the terms of trade each period, and exploit the asymmetry in wealth between home and foreign consumers (as described in ϕ_y term in the loss function in equation 4.2). However, the resulting deflation will significantly reduce welfare relative to other regimes, including currency pegs, because of the price dispersion it creates (see Table 1). This illustrates how costly a lack of central bank credibility in a resource-exporter can be, and may explain why three-quarters of resource-dependent economies peg their exchange rate.

In contrast, a credible central bank will overcome this deflationary bias through commitment. It is seen in the initial tightening of policy in the first period. This tightening stems from the bank's incentive to further appreciate the terms of trade and exploit the asymmetry in wealth from the home country's oil. By exploiting this terms of trade externality the central bank can improve on the flexible price outcome, as seen in Table 1.

5.1.4   Monetary Rules

Optimal policy is well approximated by a particular Taylor rule. The simple Taylor rule leads to a recession between discovery and spending. However, it will closely approximate optimal policy if it also responds to expected changes in the natural level of output.

On discovering oil, the simple Taylor rule leads to a recession between discovery and production. The simple Taylor rule responds relatively aggressively to inflation in line with Rogoff's (1985) conservative central banker. It is assumed to take the form in equation 4.10 where $\phi_\pi = 1.5$ and $\phi_y = 0.5$. When oil is discovered, the terms of trade immediately start to appreciate, as forward looking firms anticipate future demand. The Taylor rule responds to this inflationary pressure by tightening interest rates relative to where they would otherwise be. This exacerbates the contraction in output, causing a deeper recession. The net effect is interest rates that are lower than they were before oil was discovered, but not low enough to avoid recession. The nominal exchange rate appreciates, compensating for lower domestic returns as dictated by uncovered interest
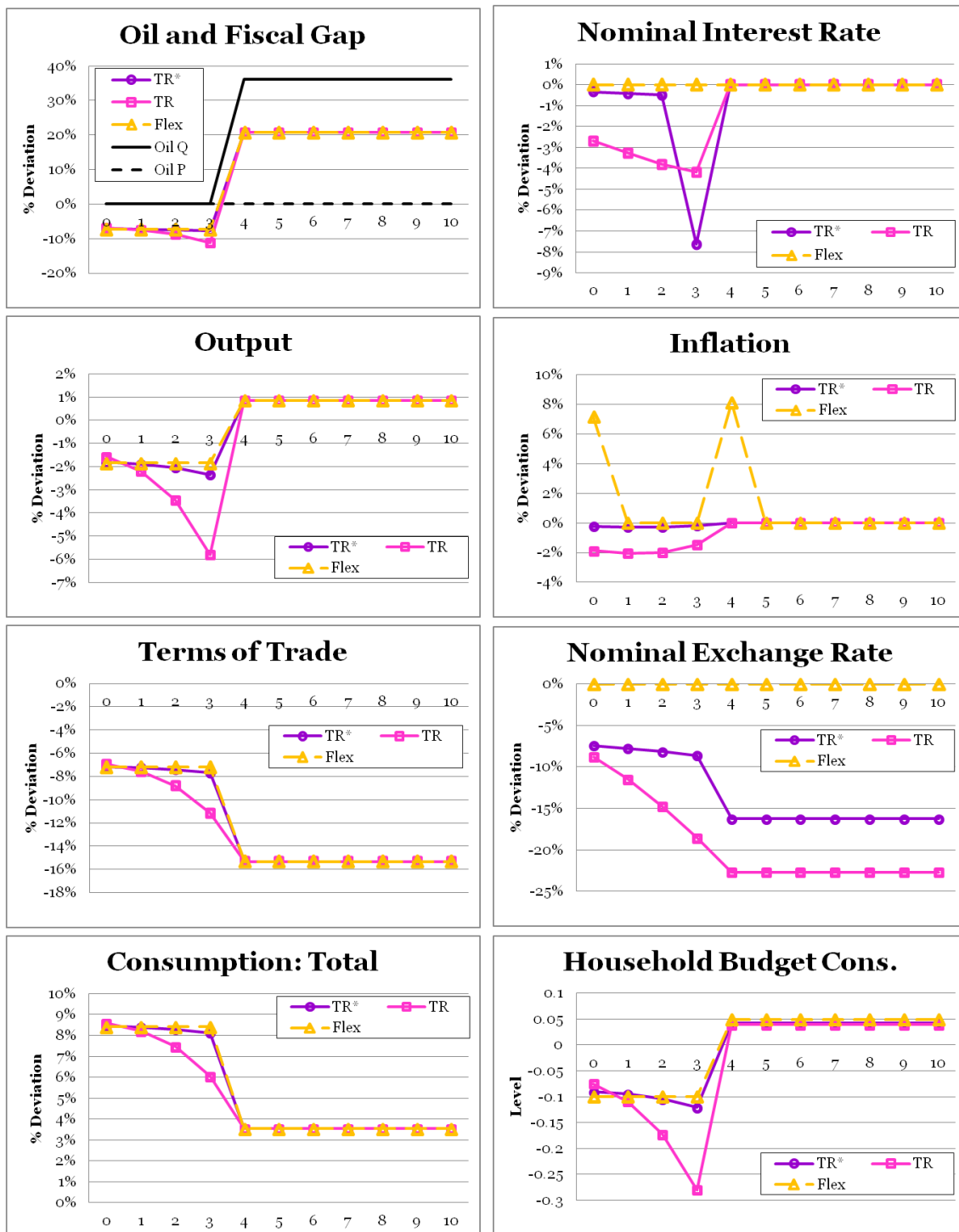
Figure 5.2: Forecast mean responses of key variables to an anticipated oil windfall, if the government spends oil revenues as they are received.. Three different scenarios are illustrated: a simple Taylor rule (TR), an optimised Taylor Rule (TR*) and flexible prices (Flex).

Standard Deviation of Key Variables

Variable	Comm	Disc	Peg	TR	TR*
$\hat{p}_{O,t}$	1.00	1.00	1.00	1.00	1.00
$\hat{g}_t$	0.75	0.76	0.92	0.81	0.78
$\pi_{H,t}$	0.01	0.01	0.07	0.04	0.01
$\hat{y}_t$	0.05	0.06	0.23	0.11	0.08
$\hat{s}_t$	0.25	0.24	0.07	0.19	0.22
$\hat{e}_t$	0.26	0.25	0	0.14	0.21
$\hat{c}_t$	0.15	0.14	0.04	0.11	0.13

Table 2: Standard deviation is calculated one period ahead, based on 10,000 Monte Carlo simulations of a N(0,1) shock to the oil price.

parity. The terms of trade also appreciates - not by firms raising prices, but rather by firms failing to lower prices as quickly as the nominal exchange rate appreciates.

The simple Taylor rule can be improved by including a term that captures the key feature of optimal policy. As discussed above, the key feature of optimal policy is the ability to loosen interest rates in the period before the windfall hits. We can approximate this by including an additional term in the Taylor rule, as given in equation 4.11. Again, we let $\phi_\pi = 1.5$ and $\phi_y = 0.5$, and choose $\phi_{yn} = -2.6$ to minimise welfare losses. By responding to expected changes in the natural level of output, which will be driven by changes in government spending, the central bank can delay the second terms of trade adjustment until government spending begins. Firms realise this and so will not raise prices in anticipation, avoiding the negative output gaps that characterise the simple Taylor rule. Welfare under this rule is as high as the flexible price case.

So far this section has focused on anticipated changes in the mean path of government spending. We now turn to the response of each monetary rule to the stochastic shock in our model - the oil price.

5.2   Unanticipated Changes in Oil Prices

If the government spends oil revenues as they are received then it will transmit stochastic oil price shocks directly to aggregate demand. In this section we compare how these shocks affect the volatility of key variables under each monetary regime. The impulse responses of each variable are illustrated in Figure 5.3, and can be interpreted as stationary fluctuations around a deterministic trend.[14] Optimal discretion will respond to both the oil price and domestic inflation, while optimal commitment responds to the oil price and the domestic price level. Key variables are significantly more volatile under a currency peg than under optimal policy.

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[14]Figure 5.1 illustrates one such deterministic trend. Removing the trend leaves the response of each variable to unanticipated, stochastic oil price shocks. These responses are stationary for all real variables.
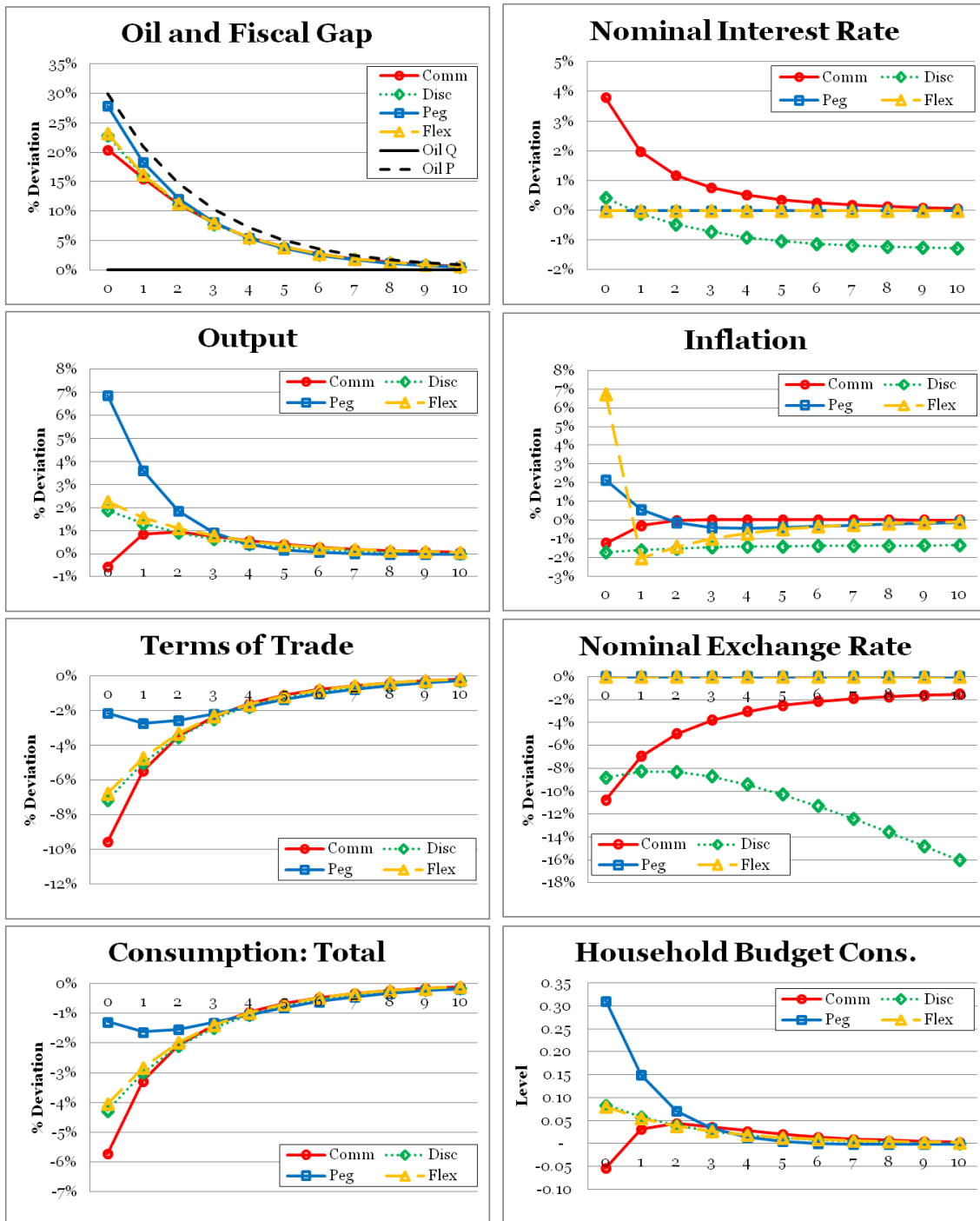
Figure 5.3: Impulse responses to a temporary oil price shock of 30% with persistence $\rho_o = 0.7$. Four different scenarios are illustrated: optimal commitment (Comm), optimal discretion (Disc), a pegged currency (Peg) and flexible prices (Flex).

Optimal policy responds to oil price shocks through their effect on prices and the natural level of output. If there is a positive shock to the oil price, interest rates will initially tighten to offset the slow response from domestic prices. If the central bank can commit then it will loosen interest rates in subsequent periods, boosting output above its flexible price level and smoothing the adjustment of the economy to the oil price shock. Discretionary policy is again affected by a deflationary bias, as discussed above.

Inflation and the output gap are significantly more volatile under a simple Taylor rule or a currency peg than under optimal policy. Table 2 illustrates the relative volatility of key variables under each monetary regime. A simple Taylor rule or a currency peg cause large fluctuations in output and inflation. In contrast, the optimal rules reduce the volatility of these key variables by letting the nominal exchange rate absorb much of the shock. The stochastic properties of these optimal policies are also well-approximated by our augmented Taylor rule ("TR^*").

6   Conclusion

This paper considers how monetary policy should optimally respond to an oil discovery. In practice there is often a significant delay between discovering oil and beginning production. Governments also typically delay spending until they receive oil revenues. Thus, oil discoveries can be thought of as a news shock about future government spending. These shocks induce changes in the economy that create a number of distortions. We have been interested in how monetary policy should manage these distortions.

The paper's first contribution is to characterise how news about an oil discovery affects a standard small open economy under rational expectations. To do this we develop a small, tractable model that permits closed form solutions and can be implemented in a spreadsheet (such as Microsoft Excel). Discovering oil will require an immediate terms of trade appreciation as private consumption jumps. If government consumption does not rise for some time, then there will also need to be a second appreciation. Firms anticipate this and will begin raising prices in advance.

The paper's second contribution is to show that forward-looking inflation can cause a recession under both an exchange-rate peg and a simple Taylor rule. Under an exchange rate peg firms anticipate the second appreciation and raise their prices in advance, causing unemployment and stagflation. Under a simple Taylor rule monetary policy will respond to this inflation by tightening, exacerbating the recession.

The paper's third contribution is to show that optimal policy is well-approximated by a simple Taylor rule that also responds to expected changes in the natural level of output. On discovering oil, optimal policy will let the currency appreciate immediately. This allows the terms of trade to appreciate without the distortions of sticky domestic prices. To prevent firms raising prices in anticipation of future demand, optimal policy will sharply loosen rates before the windfall and tighten them when it is received. This delays the terms of trade appreciation until government spending can take up the slack left by private demand. Optimal policy significantly improves welfare over standard monetary regimes according to a micro-founded loss function. The loss function illustrates that

discretionary central banks in oil exporters have an incentive to appreciate the terms of trade and exploit the asymmetry introduced by oil wealth. This raises the possibility of central-bank induced Dutch disease and creates a deflationary bias that is overcome under commitment. Thus, central-bank credibility is particularly important in oil-exporters, possibly explaining why 74 percent of oil exporters peg their currency.

Extensions to this work may consider other price setting assumptions and adding investment. Price setting plays an important role in this model, driving the initial period of recession. We have assumed Calvo pricing. An extension would investigate whether this result holds when firms set prices based on their state (such as Gertler and Leahy, 2008 and Golosov and Lucas, 2007). If there are concave menu costs then there may be an incentive to delay price rises as long as possible, overcoming distortions caused by firms raising prices in advance. Investment is also likely to have an important effect between the discovery and production of oil. Investment may reduce the necessary changes in prices, and boost employment between discovery and production.

While this analysis was conducted with an oil discovery in mind, it may also be of interest to other applications. Aid windfalls have similar characteristics to oil windfalls. If anticipating a windfall leads to recession, then it can be argued that aid grants should only be announced when government spending begins. Anticipated increases in government demand may also happen between an election and the implementation of a budget, though this would require a closer investigation of taxation.

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For Online Publication

A   Online Appendix: Model

A.1   International Risk Sharing

To derive the expressions in equations 2.9 and 2.10 we begin with the household budget constraint in equation 2.2. Summing this constraint over an infinite horizon, and using the transversality condition, $\lim_{T \to \infty} M_{0,T} D_T = 0$, gives $E_t[\sum_{s=0}^{\infty} M_{t,t+s} P_{t+s} C_{t+s}] = E_t[\sum_{s=0}^{\infty} M_{t,t+s} (W_{t+s} N_{t+s} - P_{H,t+s} T_{t+s})] + D_t$. Combining this with the Euler equation 2.4 gives,

$$P_t C_t \;=\; E_t \left[\sum_{s=0}^{\infty} M_{t,t+s} \left(W_{t+s} N_{t+s} - P_{H,t+s} T_{t+s} \right) + D_t \right] \Big/ E_t \left[\sum_{s=0}^{\infty} \beta^s \right] \qquad (A.1)$$

Combining this with a similar condition that holds in each country, f, and making use of the international risk-sharing condition in equation 2.8 gives the results in 2.9 and 2.10.

To express relative household wealth in terms of government spending, as in equation 2.12, we start with its expression at time $t = 0$

$$\Theta_{f,0} = \frac{E_0[\sum_{t=0}^{\infty} M_{0,t}(W_t N_t - P_{H,t}T_t)]}{E_0[\sum_{t=0}^{\infty} M_{0,t}^f(W_t^f N_t^f - P_{f,t}^f T_t^f)]}$$

If prices are flexible, and if the government provides a small wage subsidy to correct for monopolistic distortions, then $W_t = P_{H,t} \forall t$. Using the small open economy assumption all foreign variables remain at their steady state, and $r^* = \rho$. Thus, the denominator becomes, $E_0[\sum_0^{\infty} M_{0,t}^f(W_t^f N_t^f - P_{f,t}^f T_t^f)] = P_f^s(N_f^s - T_f^s)(1+\rho)/\rho$. In the steady state we assume that taxes are the same as the rest of the world, so $\gamma_T = \chi$. Therefore, $(N^s - T^s) = (N_f^s - T_f^s) = 1 - \chi$ and $P_H^s/P_f^s = (S_f^s)^{-1} = \Theta_f^s$. Using this, and, $\beta = (1+\rho)^{-1}$, we can log-linearise the definition of $\Theta_{f,0}$ around the steady state assuming prices are flexible,

$$\hat{\vartheta}_{f,0}^n \Theta_f^s = \frac{P_H^s(N^s - T^s)}{P_f^s(N_f^s - T_f^s)(1+\rho)/\rho} E_0\left[\sum_{t=0}^{\infty} M_{0,t}(\hat{p}_{H,t}^n + \frac{1}{1-T^s/N^s}(\hat{n}_t^n - \frac{T^s}{N^s}\hat{\mathfrak{t}}_t^n))\right]$$

$$\hat{\vartheta}_{f,0}^n = \frac{\rho}{1+\rho} E_0\left[\sum_{t=0}^{\infty} M_{0,t}(\hat{p}_{H,t}^n + \frac{1}{1-\chi}(\hat{n}_t^n - \chi\hat{\mathfrak{t}}_t^n))\right]$$

Note that taxes are $\hat{\mathfrak{t}}_t = \ln T_t - \ln T^s$, rather than $\hat{t}_t = -\ln(1 - \frac{T_t}{G_t}) + \ln(1 - \frac{T^s}{G^s})$ which we use in the text. We assume that $\hat{t}_t = 0$, so $\hat{\mathfrak{t}}_t = \hat{g}_t$. Under flexible prices it is necessary to fix a numeraire, so we let $\hat{e}_t = 0$ such that $\hat{s}_t^n = -\hat{p}_{H,t}^n$. Also, combining equations 3.2 and A.4 gives $\varphi\hat{y}_t^n = -\hat{s}_t^n - \hat{\vartheta}_t^n$. So, $\hat{p}_{H,t}^n = \varphi\hat{y}_t^n + \hat{\vartheta}_t^n$ and $\hat{g}_t^n = -\varphi\hat{y}_t^n - \hat{\vartheta}_t^n + \widehat{rb}_t$. Therefore,

$$\hat{p}_{H,t} + \frac{1}{1-\chi}(\hat{n}_t - \chi\hat{\mathfrak{t}}_t) = \varphi\hat{y}_t^n + \hat{\vartheta}_t^n + \frac{1}{1-\chi}(\hat{y}_t - \chi(-\varphi\hat{y}_t^n - \hat{\vartheta}_t^n + \widehat{rb}_t))$$

$$= \frac{1+\varphi}{1-\chi}\hat{y}_t^n + \frac{1}{1-\chi}\hat{\vartheta}_t^n - \frac{\chi}{1-\chi}\widehat{rb}_t$$

$$= \frac{1+\varphi}{1-\chi}\left\{\frac{\gamma_G}{(1+\varphi)}\widehat{rb}_t - \frac{(\alpha+\gamma_G(1-\alpha))}{(1+\varphi)}\hat{\vartheta}_t^n\right\} + \frac{1}{1-\chi}\hat{\vartheta}_t^n - \frac{\chi}{1-\chi}\widehat{rb}_t$$

$$= \frac{\gamma_G - \chi}{1-\chi}\widehat{rb}_t + \frac{(1-\gamma_G)(1-\alpha)}{1-\chi}\hat{\vartheta}_t^n$$

Substituting this into the above relationship, and using effective (rather than bilateral) relationships gives the following, where we make use of the observation that $E_t[\hat{\vartheta}_{t+s}^n] = \hat{\vartheta}_t^n \forall s$, as relative wealth incorporates all expected future income,

$$\hat{\vartheta}_0^n = \frac{\rho}{1+\rho}E_0\left[\sum_{t=0}^{\infty}M_{0,t}\left(\frac{\gamma_G-\chi}{1-\chi}\widehat{rb}_t + \frac{(1-\gamma_G)(1-\alpha)}{1-\chi}\hat{\vartheta}_t^n\right)\right]$$

$$= \frac{\rho}{1+\rho}E_0\left[\sum_{j=0}^{\infty}M_{0,t}\frac{\gamma_G-\chi}{1-\chi}\widehat{rb}_t\right] + \frac{\rho}{1+\rho}\frac{(1-\gamma_G)(1-\alpha)}{1-\chi}\frac{1+\rho}{\rho}\hat{\vartheta}_0^n$$

$$= \frac{(1-\beta)(\gamma_G-\chi)}{(1-\chi)-(1-\gamma_G)(1-\alpha)}E_0\left[\sum_{j=0}^{\infty}M_{0,t}\widehat{rb}_t\right]$$

Setting $\hat{\vartheta}_t = \hat{\vartheta}_t^n$ gives the relationship in equation 2.12. It is important to note that this is an approximation, which is subject to two types of error. The first is that the goods market equilibrium (equation A.2) is not log-linear, and so may require a higher order approximation. The second is that we approximate $-\alpha\hat{\vartheta}_t \approx \ln\left[(1-\alpha)+\alpha\Theta_t^{-1}\right] - \ln\left[(1-\alpha)+\alpha(\Theta^s)^{-1}\right]$ when approximating the goods market equilibrium. These errors can be reduced by making the jump and the distance from $\Theta^s = 1$ small.

A.2 Aggregate Demand

This appendix derives a first-order approximation of aggregate demand - the IS curve. The market clearing condition for each domestically produced good i, at each time t, requires output to equal demand from domestic consumption, government purchases and consumption from each foreign country f,

$$Y_t(i) = C_{H,t}(i) + \int_0^1 C_{H,t}^f(i)df + G_t(i)$$

$$= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon}\left\{(1-\alpha)\left(\frac{P_t C_t}{P_{H,t}}\right) + \alpha\int_0^1\left(\frac{\varepsilon_{f,t}P_t^f C_t^f}{P_{H,t}}\right)df + G_t\right\}$$

The second equality above assumes symmetric preferences across countries implying, $C_{H,t}^f(i) = \alpha\left(P_{H,t}(i)/P_{H,t}\right)^{-\epsilon}\left(\varepsilon_{f,t}P_t^f/P_{H,t}\right)C_t^f$. Aggregating across goods gives,

$$Y_t = (1-\alpha)\left(\frac{P_t C_t}{P_{H,t}}\right) + \alpha\int_0^1\left(\frac{\varepsilon_{f,t}P_t^f C_t^f}{P_{H,t}}\right)df + G_t$$

$$= S_t^{\alpha}\left[(1-\alpha)C_t + \alpha\int_0^1 \mathcal{Q}_{f,t}C_t^f df\right] + G_t$$

$$= C_t S_t^{\alpha}\left[(1-\alpha)+\alpha\Theta_t^{-1}\right] + G_t \tag{A.2}$$

Log-linearising this to the first order gives,

$$\hat{y}_t = (1-\gamma_G)\left(\hat{c}_t + \alpha\hat{s}_t - \alpha\hat{\vartheta}_t\right) + \gamma_G\hat{g}_t \tag{A.3}$$

where $\gamma_G \equiv G^s/Y^s$ and $-\alpha\hat{\vartheta}_t \approx \ln\left[(1-\alpha) + \alpha\Theta_t^{-1}\right] - \ln\left[(1-\alpha) + \alpha(\Theta^s)^{-1}\right]$ when $\Theta^s \approx 1$. This approximation makes the analysis far more tractable without a major loss in accuracy, as discussed in Appendix E. Combining this with 2.11 and $\hat{c}_t^* = 0$ gives an expression for the terms of trade,

$$\hat{s}_t = \tfrac{1}{1-\gamma_G}\hat{y}_t - \tfrac{\gamma_G}{1-\gamma_G}\hat{g}_t - (1-\alpha)\hat{\vartheta}_t \tag{A.4}$$

The IS curve in 3.1 can be found by combining 2.6; $\tilde{c}_t = (1-\alpha)\tilde{s}_t$ from 2.11; $\pi_t = \pi_{H,t} + \alpha E_t[\Delta s_{t+1}]$; $\tilde{y}_t = \tilde{s}_t$ from A.4 and $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$; and $\Delta s_t^n = -\varphi\Delta n_t^n$ from $\Delta c_t^n = (1-\alpha)\Delta s_t^n$ in 2.11, and $\Delta c_t^n = -\alpha\Delta s_t^n - \varphi\Delta n_t^n$ in 2.5, to give,

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - (i_t - E_t[\pi_{H,t+1}] - r_t^n)$$

where the natural rate of interest is, $r_t^n = \rho - \varphi E_t[\Delta\hat{y}_{t+1}^n]$.

A.3 Price-Setting and Aggregate Supply

This appendix derives the optimal price-setting rule for firms, and first- and second-order approximations of aggregate supply. The first-order approximation is the New Keynesian Phillips Curve, and the second-order one is used to incorporate higher order effects in the central bank's loss function (see Gali, 2008 Ch 3; Gali and Monacelli, 2005; and Benigno and Woodford, 2005).

Each period a measure of $(1-\theta)$ randomly selected firms choose the price, $P_{H,t}^*$ that maximises profits according to Calvo (1983),

$$\max_{P_{H,t}^*} \sum_{k=0}^{\infty} \theta^k E_t[M_{t,t+k}(P_{H,t}^* Y_{t+k} - \Psi_{t+k}(Y_{t+k}))]$$

where $\Psi_{t+k}(\cdot)$ is the nominal cost function. The first-order condition to this problem is

$$\sum_{k=0}^{\infty} \theta^k E_t[M_{t,t+k}Y_{t+k}(P_{H,t}^* - \tfrac{\epsilon}{\epsilon-1}\psi_{t+k})] = 0$$

where $\psi_t = P_{H,t}MC_t$ is the nominal marginal cost. Dividing throughout by the price level in the previous period gives the following, where $\Pi_{H,t-1,t+k} = P_{H,t+k}/P_{H,t-1}$ is gross inflation between time $t-1$ and $t+k$.

$$\sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}Y_{t+k}(\frac{P_{H,t}^*}{P_{H,t-1}} - \tfrac{\epsilon}{\epsilon-1}MC_{t+k}\Pi_{H,t-1,t+k})] = 0$$

Now, we take a second-order log-linearisation of this condition around the zero-inflation steady state. Note that this is the economy's state in the absence of an oil discovery, so inflation is zero and output is Y^s; rather than the flexible price state once oil is discovered, Y_t^n. This gives the following relationship for every k,

$$\theta^k E_t[Q_{t,t+k}Y_{t+k}(\frac{P_{H,t}^*}{P_{H,t-1}} - \frac{\epsilon}{\epsilon-1}MC_{t+k}\Pi_{H,t-1,t+k})] \approx \theta^k\beta^k Y E_t[(p_{H,t}^* - p_{H,t-1}) + \frac{1}{2}(p_{H,t}^* - p_{H,t-1})^2$$
$$-\widehat{mc}_{t+k} - \frac{1}{2}\widehat{mc}_{t+k}^2$$
$$-(p_{H,t+k} - p_{H,t-1}) - \frac{1}{2}(p_{H,t+k} - p_{H,t-1})^2$$
$$-\widehat{mc}_{t+k}(p_{H,t+k} - p_{H,t-1})] + o(\widehat{mc}_{t+k}^3)$$

Summing over all periods and using $\pi_{H,t} = (1-\theta)(p_{H,t}^* - p_{H,t-1})$ gives,

$$\frac{1}{1-\theta}\pi_{H,t} + \frac{1}{2}(\frac{1}{1-\theta}\pi_{H,t})^2 = (1-\theta\beta)\sum_{k=0}^{\infty}\theta^k\beta^k E_t[(p_{H,t+k} - p_{H,t-1}) + \frac{1}{2}(p_{H,t+k} - p_{H,t-1})^2$$
$$+\widehat{mc}_{t+k} + \frac{1}{2}\widehat{mc}_{t+k}^2 + \widehat{mc}_{t+k}(p_{H,t+k} - p_{H,t-1})] + o(\widehat{mc}_{t+k}^3)\text{(A.5)}$$

Rearranging this expression and dropping terms of order two and above gives the result in equation 2.13. Combining equation A.5 with the same expression for the next period gives,

$$V_t = \lambda[\widehat{mc}_t + \frac{1}{2}\widehat{mc}_t^2 + \frac{1}{2}c_\pi\pi_{H,t}^2] + \beta E_t[V_{t+1}] + t.i.p. + o(\widehat{mc}_t^3) \tag{A.6}$$

where $V_t = \pi_{H,t} + \frac{1}{2}(\frac{1}{1-\theta})(\pi_{H,t})^2 + \pi_{H,t}\mathcal{Z}_t$, $\mathcal{Z}_t$ is defined such that $\pi_{H,t}\mathcal{Z}_t - \beta E[\pi_{H,t+1}\mathcal{Z}_{t+1}] = -\frac{1-\theta}{\theta}\pi_{H,t}\sum_{k=0}^{\infty}\theta^k\beta^k E_t[\pi_{H,t+k} + (1-\theta\beta)\widehat{mc}_{t+k}]$, $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}$ and $c_\pi = \frac{\theta-2}{\theta\lambda}$.

First-Order Approximation: The New Keynesian Phillips Curve To the first order equation A.6 gives the standard New Keynesian Phillips curve,

$$\pi_{H,t} \approx \beta E_t[\pi_{H,t+1}] + \lambda\widehat{mc}_t \tag{A.7}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ and $\widehat{mc}_t$ is the deviation of real marginal costs in time t from their steady state level. Real marginal costs are common across domestic firms. If prices are flexible them real marginal costs are, $mc_t^n = -\mu$, which is the flexible price limit of 2.13. If prices are sticky then real marginal costs are, $mc_t = -\nu + w_t - p_{H,t} - a_t$, where $\nu \equiv -\ln(1-\tau)$ and τ is an employment subsidy that offsets the marginal cost distortion of monopolistic competition. The subsidy means that $mc_t^n - mc^s = -\mu + \nu = 0$, so $\widehat{mc}_t = \widetilde{mc}_t$.

Now we will express the Phillips curve in terms of the aggregate output gap. Aggregate domestic output is given by the index, $Y_t \equiv \left[\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)}$, similar to consumption. Aggregate employment is given by $N_t \equiv \int_0^1 N_t(i)di = \frac{Y_t Z_t}{A_t}$, where $Z_t \equiv \int_0^1 \frac{Y_t(i)}{Y_t}di = \int_0^1 \left(\frac{P_{H.,t}(i)}{P_{H,t}}\right)^{-\epsilon}$ is a measure of price dispersion. In Appendix B.1 we follow Gali and Monacelli (2008) and show that equilibrium variations in $z_t \equiv \ln Z_t$ around the perfect foresight steady state are of second order. So, up to a first-order approximation aggregate output is, $y_t = a_t + n_t$.

The deviation of real marginal costs from the steady state can be expressed as a function of domestic output, $\widehat{mc}_t = (\frac{1}{1-\gamma_G} + \varphi)\hat{y}_t - \frac{\gamma_G}{1-\gamma_G}\hat{g}_t + \alpha\hat{\vartheta}_t + (1-\varphi)\hat{a}_t$, using 2.5, the goods market equilibrium A.3, and $y_t = a_t + n_t$. A similar result holds when prices are flexible, $\widehat{mc}_t^n = (\frac{1}{1-\gamma_G} + \varphi)\hat{y}_t^n - \frac{\gamma_G}{1-\gamma_G}\hat{g}_t^n + \alpha\hat{\vartheta}_t^n + (1-\varphi)\hat{a}_t^n$. Subtracting the second from the first gives, $\widetilde{mc}_t = (\frac{1}{1-\gamma_G} + \varphi)\tilde{y}_t - \frac{\gamma_G}{1-\gamma_G}\tilde{g}_t$, as $\tilde{\vartheta}_t = \tilde{a}_t = 0$. Note that $\hat{g}_t \neq \hat{g}_t^n$ because oil revenue is received in foreign currency, so the government's purchasing power is affected by the nominal exchange rate if prices are sticky. Expressing government spending in terms of the resource balance give, $\hat{g}_t = \hat{s}_t + \widetilde{rb}_t$. Combining this with equation A.4 and $\widetilde{rb}_t = \widetilde{rb}_t^n$ gives $\tilde{g}_t = \tilde{s}_t = \tilde{y}_t$. Therefore, $\widehat{mc}_t = \widetilde{mc}_t = (1+\varphi)\tilde{y}_t$ and we have the standard New Keynesian Phillips Curve in equation 3.3, keeping track of both the actual and natural levels of output, $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$,

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(1+\varphi)(\hat{y}_t - \hat{y}_t^n).$$

The expression for $\hat{y}_t^n$ in 3.2 can be derived using $\widehat{mc}_t^n = 0$, $\hat{g}_t^n = \hat{s}_t^n + \widetilde{rb}_t$ and equation A.4,

$$\hat{y}_t^n = \frac{\gamma_G}{1+\varphi}\widetilde{rb}_t - \frac{\alpha+\gamma_G(1-\alpha)}{1+\varphi}\hat{\vartheta}_t - \frac{1-\varphi}{1+\varphi}\hat{a}_t^n$$

Second-Order Approximation Iteratively combining A.6 gives the following, which we will later substitute into the central bank loss function,

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^{t-t0}\lambda\left\{\widehat{mc}_t + \tfrac{1}{2}\widehat{mc}_t^2 + \tfrac{1}{2}c_\pi \pi_{H,t}^2\right\} + t.i.p. + o(\widehat{mc}_t^3) \qquad (A.8)$$

Now, we have $\widehat{mc}_t = \widetilde{mc}_t + \widehat{mc}_t^n = \widetilde{mc}_t = (1+\varphi)\tilde{y}_t$ as real marginal costs when prices are flexible will be the same as in the steady state, $\widehat{mc}_t^n = 0$. which is accurate to the second order.

A.4   The Steady State

This appendix defines the steady state in two cases, the symmetric case without oil, and the asymmetric case when the home government receives oil income. The symmetric

steady state is defined from the perspective of a benevolent social planner choosing the optimal levels of output, government spending and consumption. It is used as a benchmark, and to define the steady state in foreign countries. The asymmetric steady state allows the government to choose its level of spending and funding mix exogenously, deviating from this symmetric steady state.

A.4.1 Symmetric case without oil

When the home country receives no resource revenues then the steady state will be symmetric and we define it from the perspective of a benevolent social planner. As home and foreign countries are assumed to be completely symmetric we have $\Theta = S = 1$. The social planner solves the following problem,

$$
\max_{C_i^i, C_f^i, G^i, N^i} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ [(1-\chi)\ln C^i + \chi_t \ln G^i - \frac{N^{i(1+\varphi)}}{1+\varphi}] \right.
$$
$$
\left. -\lambda_1 [A^i N^i - G^i - C_i^i - \int_0^1 C_i^f df] - \lambda_2 [(C_i^i)^{1-\alpha}(C_f^i)^{\alpha} \frac{1}{(1-\alpha)\alpha} - C^i] \right\}
$$

The solution to this problem is given by, $N^i = 1$, $Y^i = A^i$, $C^i = (1-\chi)(A^i)^{1-\alpha}(\int_0^1 A^f df)^{\alpha}$, $C_i^i = (1-\alpha)(1-\chi)A^i$, $C_i^f = \alpha(1-\chi)A^i$ and $G^i = \chi A^i$. Steady state output is not affected by the share of government spending, χ, or the degree of openness, α, because of symmetry. A change in either would affect both domestic and foreign demand for domestic production, which will offset one another. The optimal allocation for the world as a whole can be found by setting $\alpha \to 0$.

A.4.2 Asymmetric case with oil

When the home government receives oil income the steady state will depend on fiscal policy, and will no longer be symmetric with the rest of the world. The steady state allocation is presented in the following lemma,

The steady state is found by simultaneously solving four equations,

$$
(1-\chi)AN^{-\varphi} = S^{\alpha}C \tag{A.9}
$$
$$
AN - T = S^{\alpha}C \tag{A.10}
$$
$$
AN = S^{\alpha}C(1 - \alpha + \alpha\Theta^{-1}) + G \tag{A.11}
$$
$$
C = S^{1-\alpha}\Theta C^* \tag{A.12}
$$

using $T = \gamma_T AN$, $G = \gamma_G AN$, and $C^* = (1 - \chi)$. The first equation comes from combining 2.3 and the steady state marginal cost condition $MC^s = W/(P_H A) = 1$. The second equation follows from the household budget constraint, 2.2, if we assume there is

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no permanent accumulation of foreign assets $D^s = 0$. The third equation is the goods market equilibrium, A.2. The fourth follows from the international risk sharing condition, 2.11. Solving simultaneously gives the results in equation 3.4.

The steady state in 3.4 will collapse to the symmetric case under certain fiscal policies. If taxes are chosen optimally, $\gamma_T = \chi$, then output will be the same as in the symmetric case, $N^s = 1$. If the government receives no oil revenues, $\gamma_G = \gamma_T$, then we have the symmetric case with $\Theta^s = S^s = 1$. We will proceed assuming $\gamma_T = \chi$ for tractability, and $\gamma_G > \gamma_T$. We will define the change in household wealth on discovering oil by comparing Θ in the steady states before and after discovery.

B Online Appendix: Monetary Policy

B.1 Central Bank Loss Function

This appendix derives a first- and second-order approximation of the central bank's loss function. The first is used in the main body of the paper and allows us to compare optimal monetary policy under discretion and commitment. However, there will be a linear term in the loss function and so it is only appropriate when the effect of the oil discovery is "small" (i.e. the linear term is of second order, Gali, 2008 Ch 5). The second is used in Appendix B.2.3 and allows us to study optimal policy under commitment from a timeless perspective. This uses a second-order approximation of aggregate supply to replace the linear term in the loss function, and is appropriate when the discovery is large (see Benigno and Woodford, 2005).

We begin with the household utility function 2.1,

$$U_0 = E_0 \sum_t^\infty \beta^t \{ (1-\chi)\ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \}$$

Let us start by taking a second-order Taylor expansion of $\ln(Y_t - G_t)$ around the steady state, using $\gamma_G = G/Y$, $(Y_t - Y)/Y \approx \hat{y}_t + \frac{1}{2}\hat{y}_t^2$ and collecting terms independent of policy ($t.i.p$),

$$
\begin{aligned}
\ln(Y_t - G_t) \approx{} & \ln(Y_s - G_s) + \frac{1}{1-G/Y}\left(\frac{Y_t - Y}{Y} - \frac{G}{Y}\frac{G_t - G}{G}\right) - \frac{1}{2}\frac{1}{(1-G/Y)^2}\left(\left(\frac{Y_t - Y}{Y}\right)^2\right. \\
& \left. + \left(\frac{G}{Y}\right)^2\left(\frac{G_t - G}{G}\right)^2 - 2\frac{YG}{Y^2}\left(\frac{Y_t - Y}{Y}\right)\left(\frac{G_t - G}{G}\right)\right) + o(\hat{y}_t^3) \quad\quad (B.1) \\
={} & \frac{1}{1-\gamma_G}(\hat{y}_t - \gamma_G\hat{g}_t) + \frac{1}{2}\frac{1}{1-\gamma_G}(\hat{y}_t^2 - \gamma_G\hat{g}_t^2) - \frac{1}{2}\frac{1}{(1-\gamma_G)^2}(\hat{y}_t^2 + \gamma_G^2\hat{g}_t^2 - 2\gamma_G\hat{y}_t\hat{g}_t) + o(\hat{y}_t^3) \\
={} & \frac{1}{1-\gamma_G}(\hat{y}_t - \gamma_G\hat{g}_t) - \frac{1}{2}\frac{\gamma_G}{(1-\gamma_G)^2}(\hat{g}_t - \hat{y}_t)^2 + o(\hat{y}_t^3) + t.i.p. \\
={} & \frac{1}{1-\gamma_G}(\tilde{y}_t - \gamma_G\tilde{g}_t) - \frac{1}{2}\frac{\gamma_G}{(1-\gamma_G)^2}\left((\tilde{g}_t - \tilde{y}_t) + (\hat{g}_t^n - \hat{y}_t^n)\right)^2 + o(\hat{y}_t^3) + t.i.p. \quad\quad (B.2)
\end{aligned}
$$

Now, to a first-order approximation $\tilde{g}_t = \tilde{s}_t = \tilde{y}_t$, from A.4. The quadratic term therefore becomes independent of policy, $(\hat{g}_t^n - \hat{y}_t^n)^2$, because any second-order or higher terms in $(\tilde{g}_t - \tilde{y}_t)$ will become third-order or higher when squared.

Substituting this second-order approximation into the goods market equilibrium in A.2 gives

$$
\begin{aligned}
\ln C_t &= \ln(Y_t - G_t) - \alpha s_t - \ln[1 - \alpha + \alpha\Theta^{-1}] \\
\tilde{c}_t &\approx \tfrac{1}{1-\gamma_G}(\tilde{y}_t - \gamma_G\tilde{g}_t) - \alpha\tilde{s}_t + o(\hat{y}_t^3) + t.i.p.
\end{aligned}
\tag{B.3}
$$

Next we take a second-order log-linearisation of the third term in the loss function 2.1,

$$
\begin{aligned}
\frac{N_t^{1+\varphi}}{1+\varphi} &= \frac{N^{1+\varphi}}{1+\varphi} + N^\varphi N\left(\frac{N_t - N}{N}\right) + \tfrac{1}{2}\varphi N^{\varphi-1}N^2\left(\frac{N_t - N}{N^2}\right)^2 + o(\hat{y}_t^3) \\
&= \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi}(\hat{n}_t + \tfrac{1}{2}\hat{n}_t^2) + \tfrac{1}{2}\varphi N^{1+\varphi}\hat{n}_t^2 + o(\hat{y}_t^3) \\
&= \hat{n}_t + \tfrac{1}{2}(1+\varphi)\hat{n}_t^2 + o(\hat{y}_t^3) + t.i.p.
\end{aligned}
\tag{B.4}
$$

Where we assume that $N = 1$ in the steady state, and $\hat{n}_t = \tilde{n}_t + \hat{n}_t^n$. To express this in terms of the output gap we use $N_t = \left(\frac{Y_t}{A}\right)\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj$ and $N_t^n = \frac{Y_t^n}{A}$. Thus, we have $\tilde{n}_t = \tilde{y}_t + z_t$ where $z_t \equiv \ln\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj$, and $\hat{n}_t^n = \hat{y}_t^n$. The following Lemma, proved in Gali and Monacelli (2008) shows that z_t is proportional to the cross-sectional variance of relative prices and thus of second order. As a result $\hat{n}_t^2 = (\tilde{y}_t + \hat{y}_t^n)^2 + o(\hat{y}_t^3)$.

Lemma 4. $z_t = \frac{\epsilon}{2}var\{p_{H,t}(j)\} + o(\|\bar{g}_t\|^3)$

Proof. See Gali and Monacelli (2008), Appendix. $\qquad\square$

The sum of the variance of relative prices can be expressed in terms of domestic inflation,

Lemma 5. $\sum_{t=0}^{\infty}\beta^t var\{p_{H,t}(j)\} = \frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t\pi_{H,t}^2$ *where* $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$

Proof. See Woodford (2001), pp22-23. $\qquad\square$

Now, in order to express welfare in terms of the output gap and inflation we will make use of the following log-linear equations, each of which is accurate to at least the second order,

$$\tilde{c}_t = (1-\alpha)\tilde{s}_t \qquad (B.5)$$

$$\widetilde{mc}_t = \tilde{c}_t + \alpha\tilde{s}_t + \varphi\tilde{n}_t \qquad (B.6)$$

$$\tilde{n}_t = \tilde{y}_t + z_t \qquad (B.7)$$

$$\tilde{c}_t = \frac{1}{1-\gamma_G}(\tilde{y}_t - \gamma_G\tilde{g}_t) - \alpha\tilde{s}_t \qquad (B.8)$$

$$\tilde{g}_t = \tilde{s}_t \qquad (B.9)$$

which follow from equations 2.11; 2.5; appendix A.3; B.3; and $\hat{g}_t = \hat{s}_t + \widehat{rb}_t$, respectively. Combining B.7, B.8, and B.9 gives $\tilde{n}_t = (1-\gamma_G)(\tilde{c}_t + \alpha\tilde{s}_t) + \gamma_G\tilde{s}_t + z_t$. Combining this with 2.1, B.4, B.5, and B.9 gives the loss function below, which is equivalent to equation 4.1 using $\tilde{s}_t = \hat{s}_t + t.i.p.$ and $\mathbb{L}_0 = -U_0$,

$$\mathbb{L}_0 = E_0\sum_t^\infty \beta^t \left\{ \tfrac{1}{2}\tfrac{\epsilon}{\lambda}\pi_{H,t}^2 + \tfrac{1}{2}(1+\varphi)(\tilde{y}_t + \hat{y}_t^n)^2 + \alpha(1-\chi)\tilde{s}_t \right\} + o(\hat{y}_t^3) + t.i.p.$$

This takes the familiar linear-quadratic form, with the exception of the linear $\tilde{s}_t$ term. This linear term can alter the welfare ranking of policies unless the fluctuations in $\tilde{s}_t$ are "small" (of second-order or above, see Benigno and Woodford, 2005). To remedy this we re-express the loss function using only quadratic terms, making use of the second-order approximation of aggregate supply in A.8. First we express the loss function in terms of marginal costs. Using B.5 to B.9 the terms of trade gap can be expressed as $\tilde{s}_t = \frac{1}{1+\varphi}(\widetilde{mc}_t - \varphi z_t)$. Substituting this into the loss function and using Lemmas 4 and 5 gives

$$\mathbb{L}_0 = E_0\sum_t^\infty \beta^t \left\{ \tfrac{1}{2}\tfrac{\epsilon}{\lambda}(1 - \tfrac{\alpha\varphi(1-\chi)}{1+\varphi})\pi_{H,t}^2 + \tfrac{1}{2}(1+\varphi)(\tilde{y}_t + \hat{y}_t^n)^2 + \tfrac{\alpha(1-\chi)}{1+\varphi}\widetilde{mc}_t \right\} + o(\hat{y}_t^3) + t.i.p.$$

Now multiplying A.8 by $\frac{\alpha(1-\chi)}{(1+\varphi)\lambda}$ and substituting into this loss function again gives,

$$\mathbb{L}_0 = E_0\sum_t^\infty \beta^t \left\{ \tfrac{1}{2}\phi_{\pi 2}\pi_{H,t}^2 + \tfrac{1}{2}\phi_{yy2}\tilde{y}_t^2 + (1+\varphi)\tilde{y}_t\hat{y}_t^n \right\} - T_0 + o(\hat{y}_t^3) + t.i.p. \quad (B.10)$$

Where $\phi_{\pi 2} = \frac{\epsilon}{\lambda}(1 - \frac{\alpha\varphi(1-\chi)}{1+\varphi}) + \frac{\alpha(1-\chi)}{(1+\varphi)}c_\pi$, $\phi_{yy2} = (1+\varphi)(1+\alpha(1-\chi))$ and $T_0 = \frac{\alpha(1-\chi)}{(1+\varphi)\lambda}V_0$ is a transitory component following Benigno and Woodford (2005). Given that the values T_0 are satisfied then this becomes predetermined and is thus independent of policy, when it is considered from a timeless perspective. This approach precludes using the loss function for studying discretionary policy (Benigno and Woodford, 2005). The loss function only consists of terms of second or higher order, and so will be accurate to the first order when differentiated, including for larger deviations of the natural level of output from its original level. Therefore, we are able to rank policies using the loss function,

$$\mathbb{L}_0 = E_0\sum_t^\infty \beta^t \left\{ \tfrac{1}{2}\phi_{\pi 2}\pi_{H,t}^2 + \tfrac{1}{2}\phi_{yy2}\tilde{y}_t^2 + (1+\varphi)\tilde{y}_t\hat{y}_t^n \right\} \qquad (B.11)$$

B.2 Optimal Monetary Policy

B.2.1 Optimal Policy under Discretion

The Lagrangian in equation 4.2 comes from substituting $\tilde{s}_t = \tilde{y}_t$ (which is accurate up to second order, see appendix A.3) into 4.1, and noting that $\tilde{y}_t = \hat{y}_t - t.i.p.$ giving,

$$
\min_{\pi_{H,t}, \hat{y}_t} \mathcal{L}_{\mathcal{D}} = E_0 \sum_t^\infty \beta^t \left\{ \frac{1}{2} \left[\phi_\pi \pi_{H,t}^2 + \phi_{yy} \hat{y}_t^2 \right] + \phi_y \hat{y}_t \right.
$$
$$
\left. - l_{D,t} (\lambda(1+\varphi)\hat{y}_t - \pi_{H,t}) + t.i.p. \right.
$$

where $\phi_\pi = \frac{\epsilon}{\lambda}$, $\phi_{yy} = (1+\varphi)$ and $\phi_y = \alpha(1-\chi)$. Minimising this yields the first order conditions,

$$
0 = \phi_\pi \pi_{H,t} + l_{D,t}
$$
$$
\lambda(1+\varphi)l_{D,t} = \phi_{yy} \hat{y}_t + \phi_y
$$

This can be simplified to,

$$
\hat{y}_t = -\frac{\lambda(1+\varphi)\phi_\pi}{\phi_{yy}} \pi_{H,t} - \frac{\phi_y}{\phi_{yy}} \tag{B.12}
$$

Substituting this path for $\hat{y}_t$ into the Phillips curve gives,

$$
\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(1+\varphi)(-\frac{\lambda(1+\varphi)\phi_\pi}{\phi_{yy}} \pi_{H,t} - \frac{\phi_y}{\phi_{yy}}) - \lambda(1+\varphi)\hat{y}_t^n
$$
$$
\pi_{H,t} = a_D \beta E[\pi_{H,t+1}] - a_D \lambda(1+\varphi)\hat{y}_t^n - a_D \lambda(1+\varphi)\frac{\phi_y}{\phi_{yy}}
$$

Where $a_D = (1 + \frac{\lambda^2(1+\varphi)^2\phi_\pi}{\phi_{yy}})^{-1}$. Iteratively substituting gives,

$$
\pi_{H,t} = \left\{ -a_D \lambda(1+\varphi)\hat{y}_t^n - a_D \lambda(1+\varphi)\frac{\phi_y}{\phi_{yy}} \right\} E\left[\sum_0^\infty (a_D\beta)^s L^{-s} \right] \tag{B.13}
$$

This can be rearranged to give,

$$
\pi_{H,t} = -a_D \lambda(1+\varphi) \left\{ \frac{\phi_y/\phi_{yy}}{1 - a_D\beta} + E\left[\sum_0^\infty (a_D\beta)^s \hat{y}_{t+s}^n \right] \right\}
$$

The central bank will systematically pursue negative inflation. This increases the value of the windfall in domestic currency. The interest rate rule can be found by combining this expression at time $t + 1$ with B.12 and the IS curve,

$$
\begin{aligned}
(i_t - \rho) &= E_t[\hat{y}_{t+1}] - \hat{y}_t + E_t[\pi_{H,t+1}] - (1 + \varphi)E_t[\Delta \hat{y}_{t+1}^n] \\
&= \frac{\lambda(1 + \varphi)\phi_\pi}{\phi_{yy}}\pi_{H,t} + \left(1 - \frac{\lambda(1 + \varphi)\phi_\pi}{\phi_{yy}}\right)E_t[\pi_{H,t+1}] - (1 + \varphi)E_t[\Delta \hat{y}_{t+1}^n] \\
&= d_1 \pi_{H,t} - d_2 E_t[\Delta \hat{y}_{t+1}^n] + d_3 E\left[\sum_0^\infty (a_D \beta)^s \hat{y}_{t+1+s}^n\right] + d_4 \qquad \text{(B.14)}
\end{aligned}
$$

where $d_1 = \lambda(1 + \varphi)\frac{\phi_\pi}{\phi_{yy}}$, $d_2 = 1 + \varphi$, $d_3 = \left(\lambda(1 + \varphi)\frac{\phi_\pi}{\phi_{yy}} - 1\right)a_D\lambda(1 + \varphi)$ and $d_4 = \left(\lambda(1 + \varphi)\frac{\phi_\pi}{\phi_{yy}} - 1\right)a_D\lambda(1 + \varphi)\frac{\phi_y/\phi_{yy}}{1 - a_D\beta}$. This rule gives a unique and determinate solution when $d_1 > 1$ or $\lambda(1 + \varphi)\phi_\pi > \phi_{yy}$.

B.2.2 Optimal Policy under Commitment

The Lagrangian in equation 4.6 comes from substituting $\tilde{s}_t = \tilde{y}_t$ (which is accurate up to second order, see appendix A.3) into 4.1 giving,

$$
\begin{aligned}
\min_{\pi_{H,t}, \hat{y}_t} \mathcal{L_D} &= E_0 \sum_t^\infty \beta^t \left\{ \frac{1}{2}\left[\phi_\pi \pi_{H,t}^2 + \phi_{yy}\hat{y}_t^2\right] + \phi_y \hat{y}_t \right. \\
&\quad \left. - l_{c,t}(\beta \pi_{H,t+1} + \lambda(1 + \varphi)\hat{y}_t - \pi_{H,t}) + t.i.p. \right.
\end{aligned}
$$

where $\phi_\pi = \frac{\epsilon}{\lambda}$, $\phi_{yy} = (1 + \varphi)$ and $\phi_y = \alpha(1 - \chi)$. Minimising this yields the first order conditions,

$$
\begin{aligned}
\phi_\pi \pi_{H,t} + l_{C,t} &= l_{C,t-1} \\
\phi_{yy}\hat{y}_t + \phi_y &= \lambda(1 + \varphi)l_{C,t} \\
\beta \pi_{H,t+1} + \lambda(1 + \varphi)\hat{y}_t - \lambda \gamma_G(\hat{p}_{o,t} + \hat{o}_t) + \omega \hat{\vartheta}_t &= \pi_{H,t}
\end{aligned}
$$

Combining the first two first order conditions yields the following result. The first relationship follows from letting $l_{C,-1} = 0$, because the Phillips curve in period -1 is not a binding constraint on the policymaker choosing the optimal plan in period 0.

$$
\begin{aligned}
\hat{y}_0 &= -\frac{\lambda(1 + \varphi)\phi_\pi}{\phi_{yy}}\pi_{H,0} - \frac{\phi_y}{\phi_{yy}} \\
\hat{y}_t &= \hat{y}_{t-1} - \frac{\lambda(1 + \varphi)\phi_\pi}{\phi_{yy}}\pi_{H,t}
\end{aligned}
$$

Iteratively combining these two relationships allows us to summarise them in a single expression in terms of the deviation of domestic prices from their period -1 level, $\hat{p}_{H,t} = p_{H,t} - p_{H,-1}$,

$$\hat{y}_t = -\frac{\lambda(1+\varphi)\phi_\pi}{\phi_{yy}}\hat{p}_{H,t} - \frac{\phi_y}{\phi_{yy}} \tag{B.15}$$

where $\phi_\pi = \frac{\epsilon}{\lambda}$, $\phi_y = \alpha(1-\chi)$ and $\phi_{yy} = \left((1+\varphi) + \frac{1-\chi}{1-\gamma_G}\right)$. To find the associated interest rate rule using the IS curve we must first find an expression for $\pi_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1}$. To do so we substitute the previous equation into the Phillips curve to give,

$$\hat{p}_{H,t} - \hat{p}_{H,t-1} = \beta E[\hat{p}_{H,t+1}] - \beta\hat{p}_{H,t} + \lambda(1+\varphi)\left(-\frac{\lambda(1+\varphi)\phi_\pi}{\phi_{yy}}\hat{p}_{H,t} - \frac{\phi_y}{\phi_{yy}}\right)$$

$$-\lambda(1+\varphi)\hat{y}_t^n$$

$$E[\hat{p}_{H,t+1}] = (a\beta)^{-1}\hat{p}_{H,t} - \beta^{-1}\hat{p}_{H,t-1} + \beta^{-1}\lambda(1+\varphi)(\hat{y}_t^n + \frac{\phi_y}{\phi_{yy}}) \tag{B.16}$$

where $a \equiv (1 + \beta + \frac{\lambda^2(1+\varphi)^2\phi_\pi}{\phi_{yy}})^{-1}, b \equiv \lambda\gamma_G, \kappa \equiv \lambda(1+\varphi)\frac{\phi_y}{\phi_{yy}}$. To find the stationary solution to this price process we rearrange and make use of the lag operator L, where $\hat{p}_{H,t-1} = L\hat{p}_{H,t}$,

$$\hat{p}_{H,t}E_t[L(\tfrac{1}{\beta} - \tfrac{1}{a\beta}L^{-1} + L^{-2})] = \beta^{-1}\lambda(1+\varphi)(\hat{y}_t^n + \frac{\phi_y}{\phi_{yy}})$$

We now factorise the quadratic expression $(\frac{1}{\beta} - \frac{1}{a\beta}L^{-1} + L^{-2}) = (L^{-1} - \delta_1)(L^{-1} - \delta_2)$. To do so let $z = L^{-1}$, so that $(\frac{1}{\beta} - \frac{1}{a\beta}z + z^2) = (z - \delta_1)(z - \delta_2)$, and $\delta_i = \left(1 \pm \sqrt{1 - 4a^2\beta}\right)/2a\beta$ for $i = 1, 2$. We assume that $\mid \delta_1 \mid < 1$ and $\mid \delta_2 \mid > 1$. Substituting this factorisation into the above expression yields,

$$\hat{p}_{H,t}(1 - \delta_1 L) = E_t\left[\left(\beta^{-1}\lambda(1+\varphi)(\hat{y}_t^n + \frac{\phi_y}{\phi_{yy}})\right)(L^{-1} - \delta_2)^{-1}\right]$$

Now, we can express the term $(L^{-1} - \delta_2)^{-1}$ as an infinite geometric series where the coefficients converge to zero because $\mid \delta_2 \mid > 1$, $(L^{-1} - \delta_2)^{-1} = -\delta_2^{-1}\left(1 + L^{-1}\delta_2^{-1} + L^{-2}\delta_2^{-2} + \dots\right)$. Substituting this into the above expression yields the following,

$$\hat{p}_{H,t}(1 - \delta_1 L) = -(\beta\delta_2)^{-1}\lambda(1+\varphi)\left\{\frac{\phi_y}{\phi_{yy}}\frac{1}{(1 - \delta_2^{-1})} + E_t\left[\sum_{s=0}^{\infty}(\delta_2^{-1}\hat{y}_{t+s}^n)\right]\right\}$$

Finally, multiplying the numerator and denominator of each term with $(1-\sqrt{1-4a^2\beta})$, and simplifying notation $\delta_1 = \delta = (1 - \sqrt{1 - 4a^2\beta})/(2a\beta)$ gives the stationary solution in equation 4.7,

$$\hat{p}_{H,t} = \delta\hat{p}_{H,t-1} - \lambda(1+\varphi)\left\{\frac{\phi_y}{\phi_{yy}}\frac{\delta}{(1-\beta\delta)} + \delta E_t\left[\sum_{s=0}^{\infty}(\beta\delta)^s\hat{y}_{t+s}^n\right]\right\} \tag{B.17}$$

An interest rate rule that brings about a unique, determinate equilibrium can be found by combining B.15, B.17 and the IS curve,

$$
\begin{aligned}
(i_t - \rho) &= E_t[\hat{y}_{t+1}] - \hat{y}_t + E_t[\pi_{H,t+1}] - (1+\varphi)E_t[\Delta\hat{y}_{t+1}^n] \\
&= (-\lambda(1+\varphi)\phi_\pi\phi_{yy}^{-1} + 1)E_t[\pi_{H,t+1}] - (1+\varphi)E_t[\Delta\hat{y}_{t+1}^n] \\
&= c_1\hat{p}_{H,t} - c_2 E_t[\Delta\hat{y}_{t+1}^n] + c_3 E_t\left[\sum_{s=0}^{\infty}(\beta\delta)^s\hat{y}_{t+1+s}^n\right] + c_4
\end{aligned}
$$

where $c_1 = (\lambda(1+\varphi)\frac{\phi_\pi}{\phi_{yy}} - 1)(1-\delta)$, $c_2 = 1+\varphi$, $c_3 = (\lambda(1+\varphi)\frac{\phi_\pi}{\phi_{yy}} - 1)\lambda(1+\varphi)\delta$ and $c_4 = (\lambda(1+\varphi)\frac{\phi_\pi}{\phi_{yy}} - 1)\lambda(1+\varphi)\frac{\phi_\pi}{\phi_{yy}}\frac{\delta}{1-\beta\delta}$. This gives a unique and determinate equilibrium when $c_1 > 0$, or $\lambda(1+\varphi)\phi_\pi > \phi_{yy}$, which is the same condition as the discretionary case.

B.2.3 Optimal Policy from a Timeless Perspective

The optimal policies considered in sections 4.2 and 4.3 are found using a loss function that includes a linear term. This is appropriate if the shocks hitting the economy are small, but can bias the welfare ranking of policies if they are large, as discussed in section 4. In this section we replace the linear term with a second order approximation of aggregate supply, yielding the loss function in equation B.11. Optimal policy from a timeless perspective (treating T_0 as transitory) can therefore be found by solving the Lagrangian,

$$
\begin{aligned}
\min_{\pi_{Ht}, \tilde{y}_t} \quad \mathcal{L}_{\mathcal{TL}} = \quad & E_0 \sum_{t}^{\infty} \beta^t \left\{\tfrac{1}{2}\phi_{\pi2}\pi_{H,t}^2 + \tfrac{1}{2}\phi_{yy2}\tilde{y}_t^2 + (1+\varphi)\tilde{y}_t\hat{y}_t^n\right\} \\
& - l_t(\beta\pi_{H,t+1} + \lambda(1+\varphi)\tilde{y}_t - \pi_{H,t})
\end{aligned} \tag{B.18}
$$

where $\phi_{\pi2} = \frac{\epsilon}{\lambda}(1 - \frac{\alpha\varphi(1-\chi)}{1+\varphi}) + \frac{\alpha(1-\chi)}{(1+\varphi)}c_\pi$ and $\phi_{yy2} = (1+\varphi)(1+\alpha(1-\chi))$. This gives the FOCs

$$
\begin{aligned}
\phi_{\pi2}\pi_{H,t} + l_t &= l_{t-1} \\
\phi_{yy2}\tilde{y}_t + (1+\varphi)\hat{y}_t^n &= \lambda(1+\varphi)l_t \\
\beta\pi_{H,t+1} + \lambda(1+\varphi)\tilde{y}_t &= \pi_{H,t}
\end{aligned}
$$

Combining the first two conditions gives the following, with the Philips curve in period -1 not being a binding constraint in period 0.

$$
\begin{aligned}
\tilde{y}_0 + \frac{(1+\varphi)}{\phi_{yy2}}\hat{y}_0^n &= -\frac{\lambda(1+\varphi)\phi_{\pi 2}}{\phi_{yy2}}\pi_{H,0} \\
\tilde{y}_t + \frac{(1+\varphi)}{\phi_{yy2}}\hat{y}_t^n &= \tilde{y}_{t-1} + \frac{(1+\varphi)}{\phi_{yy2}}\hat{y}_{t-1}^n - \frac{\lambda(1+\varphi)\phi_{\pi 2}}{\phi_{yy2}}\pi_{H,t}
\end{aligned}
$$

Iteratively combining these gives the following, where $\hat{p}_{H,t} = p_{H,t} - p_{H,-1}$,

$$
\tilde{y}_t = -\frac{\lambda(1+\varphi)\phi_{\pi 2}}{\phi_{yy2}}\hat{p}_{H,t} - \frac{(1+\varphi)}{\phi_{yy2}}\hat{y}_t^n \tag{B.19}
$$

Substituting this into the Philips curve gives

$$
\begin{aligned}
\hat{p}_{H,t} - \hat{p}_{H,t-1} &= \beta E_t[\hat{p}_{H,t+1}] - \beta\hat{p}_{H,t} + \lambda(1+\varphi)\left(-\frac{\lambda(1+\varphi)\phi_{\pi 2}}{\phi_{yy2}}\hat{p}_{H,t} - \frac{(1+\varphi)}{\phi_{yy2}}\hat{y}_t^n\right) \\
E_t[\hat{p}_{H,t+1}] &= (a\beta)^{-1}\hat{p}_{H,t} - \beta^{-1}\hat{p}_{H,t-1} + b\beta^{-1}\hat{y}_t^n
\end{aligned}
$$

where $a = \left(1 + \beta + \lambda(1+\varphi)\frac{\lambda(1+\varphi)\phi_{\pi 2}}{\phi_{yy2}}\right)^{-1}$, $b = \frac{\lambda(1+\varphi)^2}{\phi_{yy2}}$. To find the stationary solution to this price process we rearrange and make use of the lag operator L, where $\hat{p}_{H,t-1} = L\hat{p}_{H,t}$,

$$
\hat{p}_{H,t}E_t[L(\tfrac{1}{\beta} - \tfrac{1}{a\beta}L^{-1} + L^{-2})] = b\beta^{-1}\hat{y}_t^n
$$

We now factorise the quadratic expression $(\tfrac{1}{\beta} - \tfrac{1}{a\beta}L^{-1} + L^{-2}) = (L^{-1} - \delta_1)(L^{-1} - \delta_2)$. To do so let $z = L^{-1}$, so that $(\tfrac{1}{\beta} - \tfrac{1}{a\beta}z + z^2) = (z - \delta_1)(z - \delta_2)$, and $\delta_i = \left(1 \pm \sqrt{1 - 4a^2\beta}\right)/2a\beta$ for $i = 1, 2$. We assume that $|\delta_1| < 1$ and $|\delta_2| > 1$. Substituting this factorisation into the above expression yields,

$$
\hat{p}_{H,t}(1 - \delta_1 L) = E_t\left[\left(b\beta^{-1}\hat{y}_t^n\right)(L^{-1} - \delta_2)^{-1}\right]
$$

Now, we can express the term $(L^{-1} - \delta_2)^{-1}$ as an infinite geometric series where the coefficients converge to zero because $|\delta_2| > 1$, $(L^{-1} - \delta_2)^{-1} = -\delta_2^{-1}\left(1 + L^{-1}\delta_2^{-1} + L^{-2}\delta_2^{-2} + \ldots\right)$. Substituting this into the above expression yields the following,

$$
\hat{p}_{H,t}(1 - \delta_1 L) = -(\beta\delta_2)^{-1}bE_t\left[\sum_{s=0}^{\infty}(\delta_2^{-s}\hat{y}_{t+s}^n)\right]
$$

Finally, multiplying the numerator and denominator of each term with $(1-\sqrt{1-4a^2\beta})$, and simplifying notation $\delta_1 = \delta = (1-\sqrt{1-4a^2\beta})/(2a\beta)$ gives the stationary solution below,

$$\hat{p}_{H,t} = \delta\hat{p}_{H,t-1} - b\delta E_t\left[\sum_{s=0}^{\infty}(\beta\delta)^s\hat{y}_{t+s}^n\right] \tag{B.20}$$

An interest rate rule that brings about a unique, determinate equilibrium can be found by combining B.19, B.20 and the IS curve,

$$
\begin{aligned}
(i_t - \rho) &= E_t[\tilde{y}_{t+1}] - \tilde{y}_t + E_t[\pi_{H,t+1}] - \varphi E_t[\Delta\hat{y}_{t+1}^n] \\
&= \left(-\frac{\lambda(1+\varphi)\phi_{\pi2}}{\phi_{yy2}}E_t[\pi_{H,t+1}] - \frac{(1+\varphi)}{\phi_{yy2}}E_t[\Delta\hat{y}_{t+1}^n]\right) + E_t[\pi_{H,t+1}] - \varphi E_t[\Delta\hat{y}_{t+1}^n] \\
&= \left(1 - \frac{\lambda(1+\varphi)\phi_{\pi2}}{\phi_{yy2}}\right)E_t[\pi_{H,t+1}] - \left(\frac{(1+\varphi)}{\phi_{yy2}} + \varphi\right)E_t[\Delta\hat{y}_{t+1}^n] \\
&= \left(1 - \frac{\lambda(1+\varphi)\phi_{\pi2}}{\phi_{yy2}}\right)\left((\delta-1)\hat{p}_{H,t} - b\delta E_t\left[\sum_{s=0}^{\infty}(\beta\delta)^s\hat{y}_{t+1+s}^n\right]\right) - \left(\frac{(1+\varphi)}{\phi_{yy2}} + \varphi\right)E_t[\Delta\hat{y}_{t+1}^n] \\
&= c_1\hat{p}_{H,t} - c_2 E_t[\Delta\hat{y}_{t+1}^n] + c_3 E_t\left[\sum_{s=0}^{\infty}(\beta\delta)^s\hat{y}_{t+1+s}^n\right]
\end{aligned}
$$

where $c_1 = \left(\frac{\lambda(1+\varphi)\phi_{\pi2}}{\phi_{yy2}} - 1\right)(1-\delta)$, $c_2 = \frac{(1+\varphi)}{\phi_{yy2}} + \varphi$ and $c_3 = \left(\frac{\lambda(1+\varphi)\phi_{\pi2}}{\phi_{yy2}} - 1\right)b\delta$.

The difference between the optimal policy under commitment in the main text, and from a timeless perspective derived above, is illustrated in figure B.1. The scenarios differ only at the beginning of the policy experiment, because optimal policy under commitment in the main text (red) suffers from dynamic inconsistency (see McCallum and Nelson, 2004). This is overcome when policy is considered from a timeless perspective. The former is reported in the main text as it can be compared to behaviour by a discretionary central bank, allowing us to discuss the importance of central bank credibility in resource-exporters. This is not possible using the loss function in equation B.11, as noted above.

B.3   Appendix: Exchange Rate Peg

Here we derive the dynamics of the economy under a nominal exchange rate peg, $\Delta e_t = 0$. We begin by finding the implications of the nominal exchange rate peg for our key variables, $\pi_{H,t}$ and $\hat{y}_t$. To do this we follow a similar approach to Appendix 4.3.

First, taking first differences of the effective nominal exchange rate yields $\Delta s_t = \Delta e_t + \Delta p_t^* - \Delta p_{H,t} = -\pi_{H,t}$. Also, taking first differences of 2.11 and A.3 gives $\Delta\hat{s}_t = \frac{1}{1-\gamma_G}\Delta\hat{y}_t - \frac{\gamma_G}{1-\gamma_G}\Delta\hat{g}_t - (1-\alpha)\Delta\hat{\vartheta}_t$. Combining these two expressions gives,

$$\pi_{H,t} = \frac{\gamma_G}{1-\gamma_G}\Delta\hat{g}_t - \frac{1}{1-\gamma_G}\Delta\hat{y}_t + (1-\alpha)\Delta\hat{\vartheta}_t$$
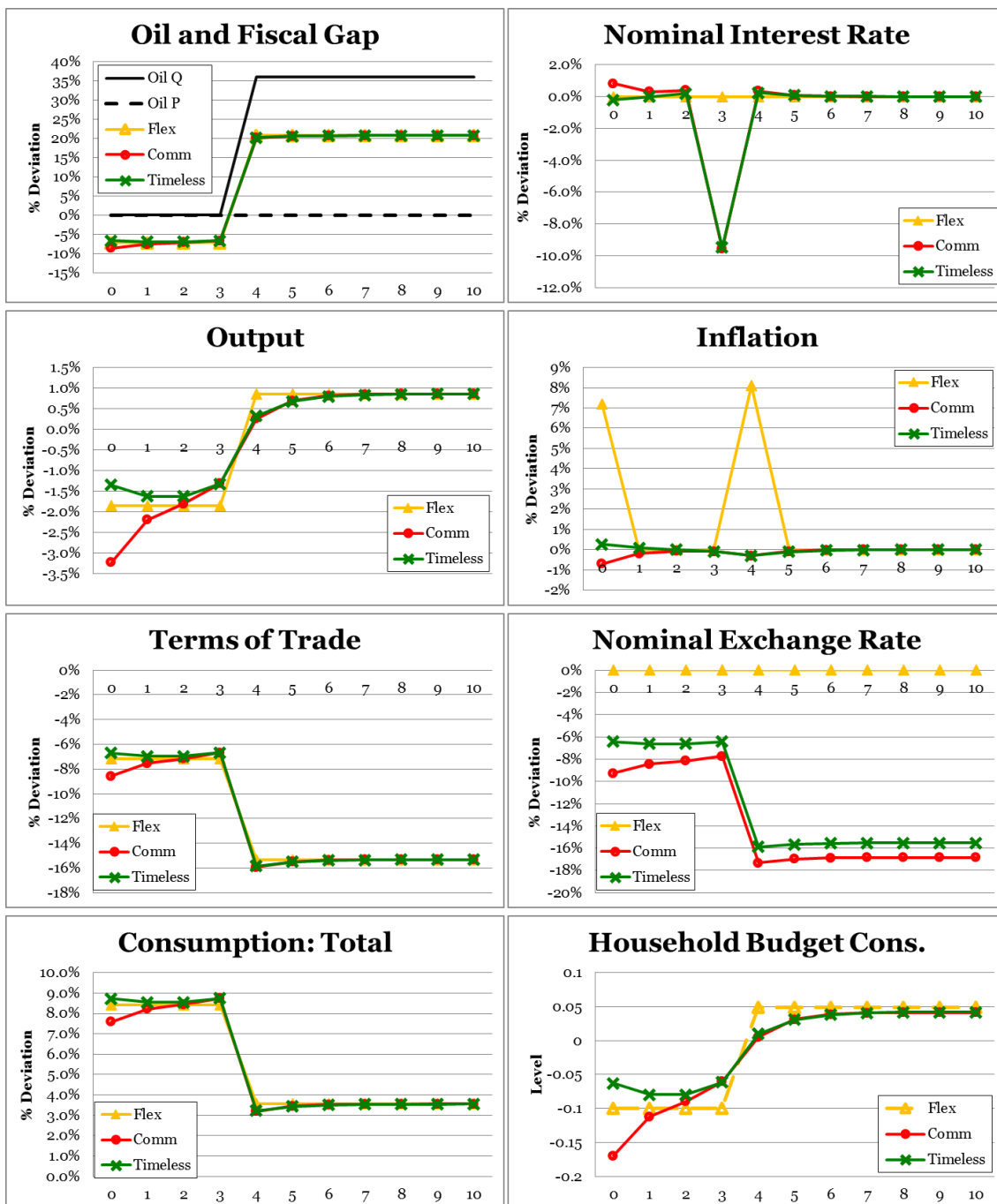
Figure B.1: Optimal policy from a timeless perspective matches the results under commitment, except in the initial periods.

Taking a similar approach to that used in the commitment case we evaluate this expression at time zero, assuming that the economy was in the steady state at all times prior to this,

$$p_{H,0} - p_{H,0-1} = \frac{\gamma_G}{1-\gamma_G}\hat{g}_0 - \frac{1}{1-\gamma_G}\hat{y}_0 + (1-\alpha)\hat{\vartheta}_0$$

Iteratively combining these two relationships allows us to express this relationship in terms of the deviation of the domestic price level, $\hat{p}_{H,t} = p_{H,t} - p_{H,-1}$,

$$\hat{p}_{H,t} = \frac{\gamma_G}{1-\gamma_G}\hat{g}_t - \frac{1}{1-\gamma_G}\hat{y}_t + (1-\alpha)\hat{\vartheta}_t$$

If we express government spending in terms of the resource balance then, $\hat{p}_{H,t} = \gamma_G\widehat{rb}_t - \hat{y}_t + (1-\gamma_G)(1-\alpha)\hat{\vartheta}_t$ or $\hat{p}_{H,t} = -\hat{y}_t + (1+\varphi)\hat{y}_t^n + \hat{\vartheta}_t$. Using this to substitute out $\hat{y}_t$ in the Phillips curve gives,

$$\hat{p}_{H,t} - c\hat{p}_{H,t-1} - c\beta E_t[\hat{p}_{H,t+1}] = c\lambda(1+\varphi)(\varphi\hat{y}_t^n + \hat{\vartheta}_t)$$

where $c = [1 + \beta + \lambda(1+\varphi)]^{-1}$. We can find the closed-form stationary solution to this linear difference equation following the method described in Appendix B.2,

$$\hat{p}_{H,t} = \delta_c\ \hat{p}_{H,t-1} + \lambda(1+\varphi)\left\{\varphi\delta_c E_t\left[\sum_{s=0}^{\infty}(\beta\delta_c)^s\hat{y}_{t+s}^n\right] + \frac{\delta_c}{1-\beta\delta_c}\hat{\vartheta}_t\right\}$$

where $\delta_c = \frac{1-\sqrt{1-4\beta c^2}}{2\beta c}$. We can use this to derive the output gap using the relationship defined above, and for the nominal interest rate by substituting our results into the IS curve. By uncovered interest parity the nominal interest rate will be constant. Uncovered interest parity states that, $(i_t - i_t^*) - E_t[\pi_{H,t+1}] = E_t[s_{t+1} - s_t]$. The definition of the terms of trade yields, $s_t = e_t + p_t^* - p_{H,t}$. So, $(i_t - i_t^*) = E_t[e_{t+1} - e_t] = 0$. While the nominal interest rate stays constant there will be relatively large fluctuations in the real interest rate. Note that while an exchange rate peg is consistent with a constant nominal interest rate, it is not uniquely associated with it. A commitment to the peg remains crucial.

B.4   Appendix: Taylor Rules

Here we derive the dynamics of the economy under a variety of Taylor rules. To do this we extend the Blanchard and Kahn (1980) method to allow for anticipated changes in the government's resource balance. As in Blanchard and Kahn this uses the widely implementable eigen-decomposition of a matrix. The method is similar to that used by Wohltmann and Winkler (2009), who use a generalized Schur matrix decomposition. As we will be interested in interest rate rules that respond to the output gap, we arrange the

Phillips curve and IS curve into a system of two equations in domestic inflation and the output gap,

$$
\begin{aligned}
E_t[\pi_{H,t+1}] &= \beta^{-1}\left(\pi_{H,t} - \lambda(1+\varphi)\tilde{y}_t\right) \\
E_t[\tilde{y}_{t+1}] &= \tilde{y}_t + (i_t - \rho) - \beta^{-1}\left(\pi_{H,t} - \lambda(1+\varphi)\tilde{y}_t\right) + \varphi E_t[\Delta \hat{y}_{t+1}^n]
\end{aligned}
$$

where $(i_t - \rho) = \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t + \phi_{yn} E_t[\Delta \hat{y}_{t+1}^n]$. This can be arranged in matrix notation, splitting the variables into control variables, $x_t = [\pi_{H,t}, \tilde{y}_t]'$, and state variables $w_t = \emptyset$ and $v_t = [\hat{y}_t^n, \hat{y}_{t+1}^n]'$,

$$
E_t\begin{bmatrix} x_{t+1} \\ w_{t+1} \end{bmatrix} = A\begin{bmatrix} x_t \\ w_t \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} v_t
$$

We decompose A into its eigenvectors, V, and eigenvalues, Λ, such that $AV = V\Lambda$. The matrix Λ is diagonal with the eigenvalues arranged in descending order along the diagonal. Replacing $A = V\Lambda V^{-1}$ and pre-multiplying by V^{-1} we get,

$$
V^{-1} E_t\begin{bmatrix} x_{t+1} \\ w_{t+1} \end{bmatrix} = \Lambda V^{-1}\begin{bmatrix} x_t \\ w_t \end{bmatrix} + V^{-1}\begin{bmatrix} B \\ 0 \end{bmatrix} v_t
$$

Let $V^{-1} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ and $\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$. For this work, where the state variables evolve independently of the controls, $Y_{21} = 0$. The matrix Λ_1 contains unstable eigenvalues (> 1) which are equal in number to the control variables, and Λ_2 contains stable eigenvalues (< 1) which are equal in number to the state variables, as imposed by Blanchard and Kahn (1980). Using $\begin{bmatrix} \tilde{x}_t \\ \tilde{w}_t \end{bmatrix} = V^{-1}\begin{bmatrix} x_t \\ w_t \end{bmatrix}$ we can describe the system in independent equations,

$$
E_t\begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{w}_{t+1} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}\begin{bmatrix} \tilde{x}_t \\ \tilde{w}_t \end{bmatrix} + \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix} v_t
$$

where $\tilde{B}_1 = Y_{11}B$ and $\tilde{B}_2 = Y_{21}B$. First taking $E_t[\tilde{x}_{t+1}] = \Lambda_1 \tilde{x}_t + \tilde{B}_1 v_t$. This can be expressed as, $(E_t[L^{-1}] - \Lambda_1)\tilde{x}_t = \tilde{B}_1 v_t$ where L^{-1} is a scalar inverse lag operator. Rearranging gives $\tilde{x}_t = (E_t[L^{-1}] - \Lambda_1)^{-1}\tilde{B}_1 v_t = -\Lambda_1^{-1}(I - E_t[L^{-1}]\Lambda_1^{-1})^{-1}\tilde{B}_1 v_t$. We can only accept stable paths for the control variables. As all the elements of Λ_1 are greater than one, the eigenvalues of Λ_1^{-1} will be less than one and the matrix geometric series, $(I - E_t[L^{-1}]\Lambda_1^{-1})^{-1} = \sum_{s=0}^{\infty}(E_t[L^{-1}]\Lambda_1^{-1})^s$ will converge. Thus,

$$
\begin{aligned}
\tilde{x}_t &= -\Lambda_1^{-1}\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s \tilde{B}_1 E_t[v_{t+s}] \\
x_t &= -(Y_{11})^{-1}Y_{12}w_t - (Y_{11})^{-1}\Lambda_1^{-1}\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s \tilde{B}_1 E_t[v_{t+s}] \qquad \text{(B.21)}
\end{aligned}
$$

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Turning now to the path of the state variables, $E_t[\tilde{w}_{t+1}] = \Lambda_2\tilde{w}_t + \tilde{B}_2 v_t$. We have $\tilde{w}_t = Y_{21}x_t + Y_{22}w_t = Kw_t - J\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1 E_t[v_{t+s}]$, where $K = -Y_{21}(Y_{11})^{-1}Y_{12} + Y_{22}$ and $J = Y_{21}(Y_{11})^{-1}\Lambda_1^{-1}$. Therefore, the path of the state variables can be described as,

$$
\begin{aligned}
E_t[\tilde{w}_{t+1}] &= \Lambda_2\tilde{w}_t + \tilde{B}_2 v_t \\
E_t[Kw_{t+1} - J\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1 E_t[v_{t+1+s}]] &= \Lambda_2(Kw_t - J\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1 E_t[v_{t+s}]) + \tilde{B}_2 v_t \\
E_t[w_{t+1}] &= K^{-1}\Lambda_2 Kw_t - K^{-1}\Lambda_2 J\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1 E_t[v_{t+s}] \\
&\quad + K^{-1}J\sum_{s=0}^{\infty}(\Lambda_1^{-1})^s\tilde{B}_1 E_t[v_{t+1+s}] \\
&\quad + K^{-1}\tilde{B}_2 v_t \quad\quad\quad\quad\quad\quad\quad (\text{B.22})
\end{aligned}
$$

The paths for the control and state variables described by B.21 and B.22 respectively are illustrated in the plots in Section 5.

C Online Appendix: Calibration

The model is calibrated in line with Gali and Monacelli (2005, 2008), allowing us to compare our results. The calibrated parameter values are summarised in Table 3. This gives a steady state and starting point for our analysis in which government spending accounts for 30 percent of output, 10 percent of which is financed by oil income and 20 percent from lump sum taxes. These taxes are set at the level that maximises welfare in an economy without oil, so we can consider all oil income to be spent on higher government spending. All steady state values are given in Table 4.

We consider scenarios in which oil's share of output increases by 20 percent, four quarters in the future. We also consider the effect of a 30 percent temporary shock to the oil price, if output remains constant.

D Online Appendix: Additional Scenarios

This appendix illustrates two additional scenarios for the way a government spends its oil revenues. The first is a base case, where the government perfectly smooths expenditure from the date oil is discovered, according to a Permanent Income rule. This requires the terms of trade to appreciate immediately, which is easily achieved with a floating exchange rate. The second scenario has the government follow a Bird in Hand rule, which involves spending a fixed proportion of assets accumulated in a sovereign wealth fund (as is done in Norway). This introduces continuous changes in the economy, as government spending rises and falls with the size of the sovereign wealth fund. An exchange rate peg or a simple Taylor rule perform poorly, but optimal policy is approximated by a Taylor rule that also responds to anticipated changes in the natural level of output, as noted in Section 5.

<div align="center">

Calibration

Parameter	Value
α	0.40
β	0.96
ρ	0.04
χ	0.20
φ	3.00
ϵ	6.00
θ	0.75
A, A^*	1.00
ρ_o	0.70

</div>

Table 3: Parameter values are calibrated following Gali and Monacelli (2005, 2008).

<div align="center">

Steady State Values

Variable	Steady State	Symmetric State
N	1.00	1.00
G	0.30	0.20
T	0.20	0.20
O	0.15	0.00
P_o	1.00	1.00
Θ	1.45	1.00
S	0.69	1.00
C	0.93	0.80

</div>

Table 4: The steady state in the cases with and without oil

D.1 Permanent Income Rule

This scenario is a base case, where the government perfectly smooths spending from the date oil is discovered. The government will borrow before the windfall begins, and save in a sovereign wealth fund during it to finance spending with the permanent income on the fund afterwards. It is the first-best policy to a first-order approximation (if the government does not face any borrowing constraints; Van der Ploeg and Venables, 2012). This is achieved by setting the resource balance in Section 2.3 to, $RB_t = r^* E_t[\sum_0^\infty M_{t,t+s} P_{O,t+s} O_{t+s}]$. There will be an immediate, permanent shock to the demand for home goods so the terms of trade must appreciate, which is easily achieved with a floating currency. Optimal monetary policy will try to improve on the flexible price outcome by appreciating the terms of trade, increasing households' buying power abroad.

An immediate jump in government spending will require the terms of trade to appreciate. This is best illustrated when prices are flexible, as seen in the "*Flex*" case in Figure D.1. To smooth spending the government will begin borrowing as soon as oil is discovered, and then save during production. The government only consumes home goods, so spending accrues as wages to domestic households. Households spend these wages on leisure and on consumption goods, both home and foreign. The extra demand for home goods will drive up their relative price, so the terms of trade must appreciate. The net
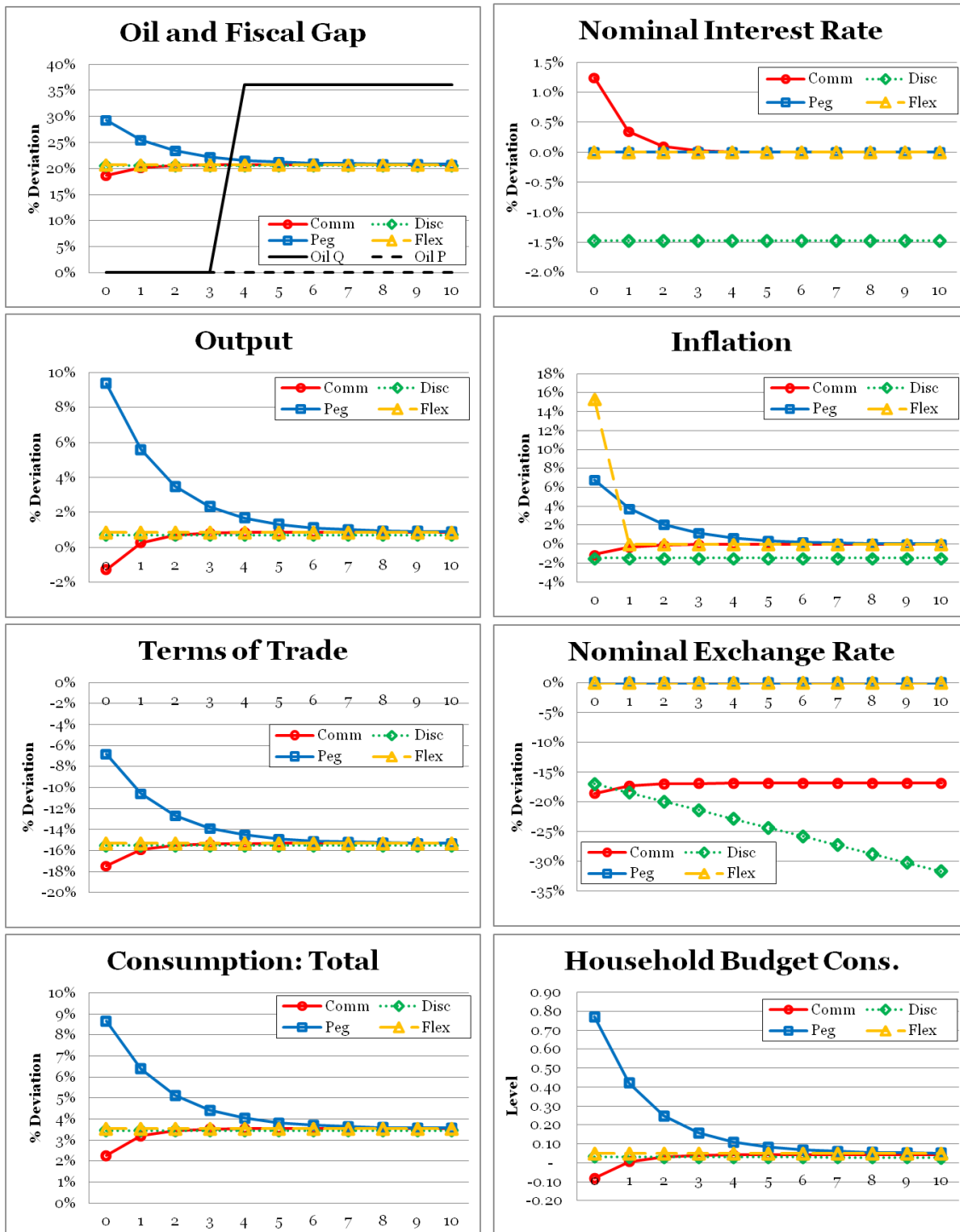
Figure D.1: Forecast mean responses of key variables to an anticipated oil windfall under the Permanent Income rule. Four different scenarios are illustrated: optimal commitment (Comm), optimal discretion (Disc), a pegged exchange rate (Peg) and flexible prices (Flex).

effect on output will depend on the elasticities of labour supply, and substitution between home and foreign goods.

Under an exchange rate peg the only way the terms of trade can appreciate is through domestic inflation. This will happen slowly, because prices are sticky. While the price of home goods is slowly rising, the consumption and production of home goods will be distorted and overshoot their natural level (see "*Peg*" in Figure D.1). These distortions only disappear once all prices have had time to adjust.

If the nominal exchange rate is floating then the flexible-price case is easily replicated. The nominal exchange rate is a single, flexible price, so can adjust far quicker than the many sticky prices set by domestic firms. The distortions from domestic inflation can therefore be avoided by letting the relative price of home and foreign goods adjust through the nominal exchange rate.

Optimal policy looks to improve on the flexible price outcome, by further appreciating the terms of trade. This additional appreciation would increase the buying power of home consumers abroad, at the cost of lost income from foreign sales of domestic goods. In standard models with a unit elasticity of substitution between home and foreign goods (Cobb-Douglas preferences), these two effects offset. However, our households also have oil wealth, so the lost income from other exports is less important. Under discretion this leads to a deflationary bias, which is overcome under commitment. Of course, it is important to note that in our simple framework there are no externalities from domestic production that would make this central bank-induced Dutch disease suboptimal.

D.2   Bird in Hand Rule

This scenario assumes that the government partially smooths its spending of a temporary windfall, according to a Bird in Hand rule. This involves the government saving all oil revenues in a sovereign wealth fund, and spending only a fixed proportion of the fund's assets each period. Since 1990 Norway has adopted a similar rule. The Bird in Hand rule is achieved by setting the resource balance in Section 2.3 to, $RB_t = \rho_{BH} F_t$, where foreign assets change according to, $E_t[M_{t,t+1} F_{t+1}] = (1 - \rho_{BH}) F_t + P_{O,t} O_t$. In log-linear terms this means $\hat{g}_t = \hat{s}_t + \hat{f}_t$. This rule will affect the economy in three phases. On discovery, consumption will immediately jump and cause the terms of trade to appreciate. During production, government spending will gradually rise which further appreciates the terms of trade and crowds out consumption. After production ends, government spending will slowly decline as foreign assets are consumed, letting the terms of trade depreciate to their new steady state. Under an exchange rate peg or a simple Taylor rule the rise and fall of government spending will create inflationary and deflationary pressure, respectively. Optimal monetary policy can overcome this, and is well approximated by our simply augmented Taylor rule.

D.2.1   Flexible Prices

If the government manages an oil windfall with a Bird in Hand rule then the economy will go through three distinct phases. It is best to illustrate these phases when prices

are flexible ("*Flex*" in Figure D.2). The first phase begins at discovery ($t = 0$ in our example), when foresighted households immediately raise their consumption on learning of their new-found wealth. As domestic households prefer home to foreign goods, the relative price of home goods (the terms of trade) must rise, meaning the terms of trade must appreciate.

The second phase happens during production ($t = 4, \ldots, 12$). Government spending will steadily rise, as it consumes a fixed share of the oil wealth accumulating in the sovereign wealth fund. The government only consumes home goods, so their relative price must rise even further. Consumers, both domestic and foreign, will substitute away from home goods during this period.

The third phase occurs once oil production ends ($t > 12$). After this time government spending will fall, as the accumulated foreign assets are run down. The relative price of home goods will duly fall as well, and household consumption will rise back to its new steady-state. Consumption in this steady state will be the same as immediately after the oil was discovered. Foresighted households will have chosen to borrow initially, before saving during the boom years, to sustain a permanently higher level of consumption once they end.

D.2.2   Exchange Rate Peg

Under an exchange rate peg, a Bird in Hand rule will lead to sustained periods of inflation and deflation. The constantly changing level of government spending requires continuous changes in the price of home goods. As these prices are sticky this will lead to large welfare losses.

On discovering oil, the terms of trade will need to appreciate. The nominal exchange rate is fixed, so this can only happen through domestic inflation. Domestic prices are sticky, so there will be an extended period of inflation between discovery and spending as firms raise prices both retrospectively and prospectively. Output will initially overshoot its natural level, as prices are slow to adjust. However, it will quickly fall as firms raise prices in anticipation of government demand.

Once oil production begins, government spending will gradually rise for the life of the oil field. The price of home goods will also have to rise. However, nominal rigidities mean that prices will consistently be below, and output above, its natural level. When the end of the boom approaches, forward-looking firms will stop raising their prices as they anticipate the decline of government demand. This causes output to rise even further beyond its natural rate, just as the boom comes to an end.

After oil production ends, government spending will decline as assets in the sovereign wealth fund are consumed. During this period of falling government spending the price of domestic goods will also need to fall. However, as prices are sticky they will remain consistently too high. As a result, during the period of decline households will respond to the sub-optimally high prices by consuming too little, causing output to be consistently below its natural level. If we assume that the negative output gap is associated with involuntary unemployment then this could have a major effect on household welfare.
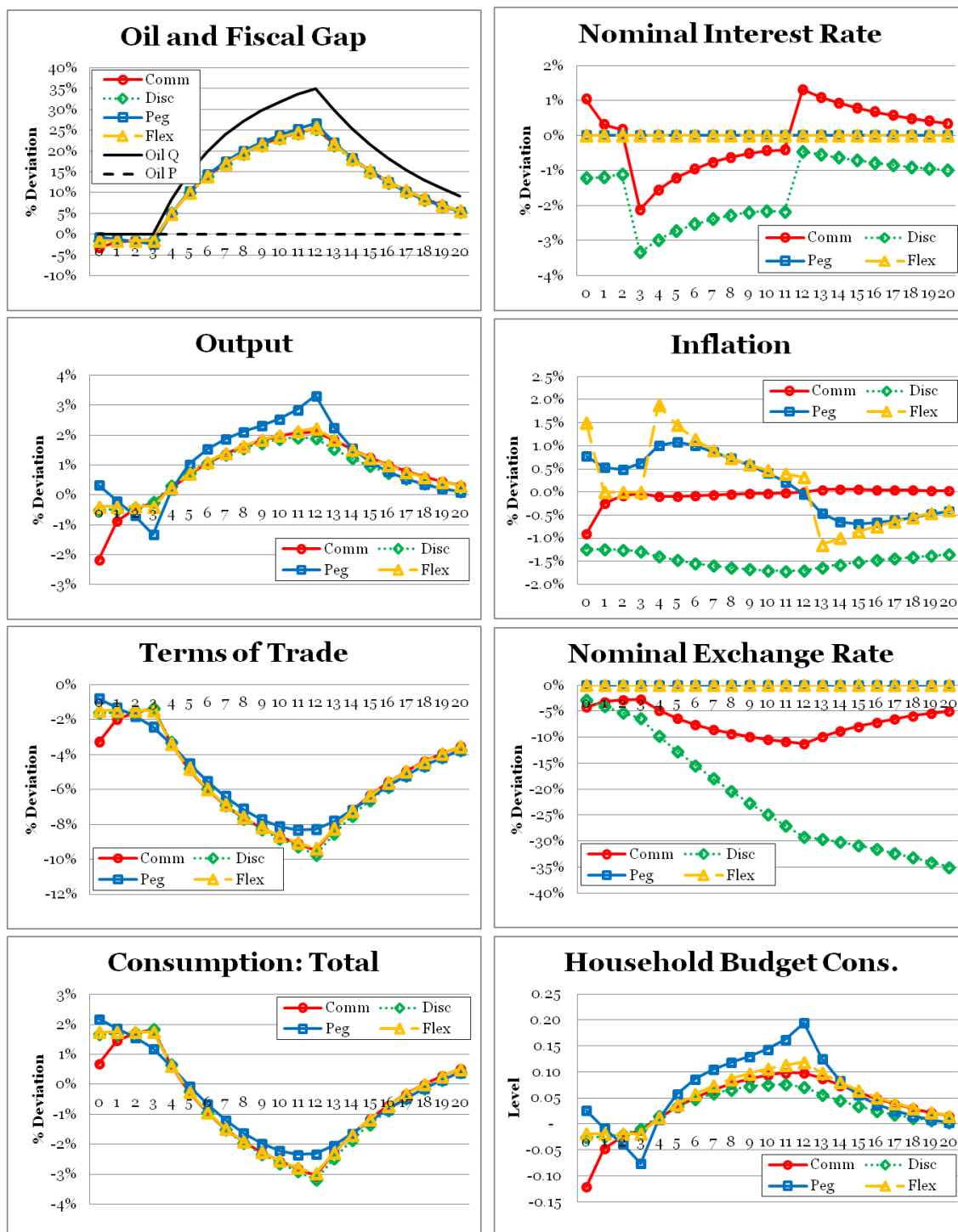
Figure D.2: Forecast mean responses of key variables to an anticipated oil windfall under the Bird in Hand rule. Four different scenarios are illustrated: optimal commitment (Comm), optimal discretion (Disc), a pegged exchange rate (Peg) and flexible prices (Flex).

D.2.3   Optimal Policy: Discretion and Commitment

Optimal policy is able to avoid the extended period of positive and negative output gaps associated with changing government spending. As with the case in the main text this involves letting the nominal exchange rate appreciate on discovering oil, and sharply loosening policy immediately before spending begins. During oil production monetary policy will slowly and continually tighten, offsetting sub-optimally low prices and preventing output from overshooting its natural level. Once production ends policy will reverse, loosening to offset the sub-optimally high prices.

When oil is discovered optimal policy will let the nominal exchange rate appreciate immediately, and loosen just before spending begins. As in the main text, households will immediately consume more on learning about their oil wealth. Home bias means that the price of home goods must appreciate relative to the price of foreign goods, which is easily achieved by a floating exchange rate. Loosening policy immediately before spending begins will remove the incentive for forward-looking firms to raise their prices, and delay the terms of trade appreciation. This loosening will be less dramatic than in the main text, as government spending only rises gradually.

During oil production, optimal monetary policy will slowly and continually tighten. Over this period government spending will rise continually, as oil revenues accumulate in the offshore sovereign wealth fund. Increasing government demand will require the price of home goods to rise, though they are sticky. To prevent output continually exceeding its natural level, due to sticky prices which are sub-optimally low, optimal monetary policy will need to tighten slightly each period.

When production ends government spending will begin to decline. In this case optimal policy will sharply tighten policy immediately before government spending falls. In the opposite case to the start of government spending, this delays the terms of trade depreciation until it is necessary - preventing output jumping too far above its natural rate. During the period of declining government spending, optimal policy will be to gradually loosen interest rates each period, offsetting sub-optimally high sticky prices. This prevents output falling below its natural level.

Once again, the broad scope of the optimal policy does not depend on the central bank's ability to commit. The policy of loosening before the windfall begins, gradually tightening during the period of rising government spending, and gradually loosening again as government spending declines is optimal under both discretion and commitment. It relies only on the rest of the economy understanding that this is optimal for the central bank. The difference between commitment and discretion comes down to the deflationary bias under discretion, as described in the main text.

D.2.4   Taylor Rules

The optimal monetary response to a Bird in Hand rule is also well approximated by the slightly augmented Taylor rule described in Section 5.1.4.
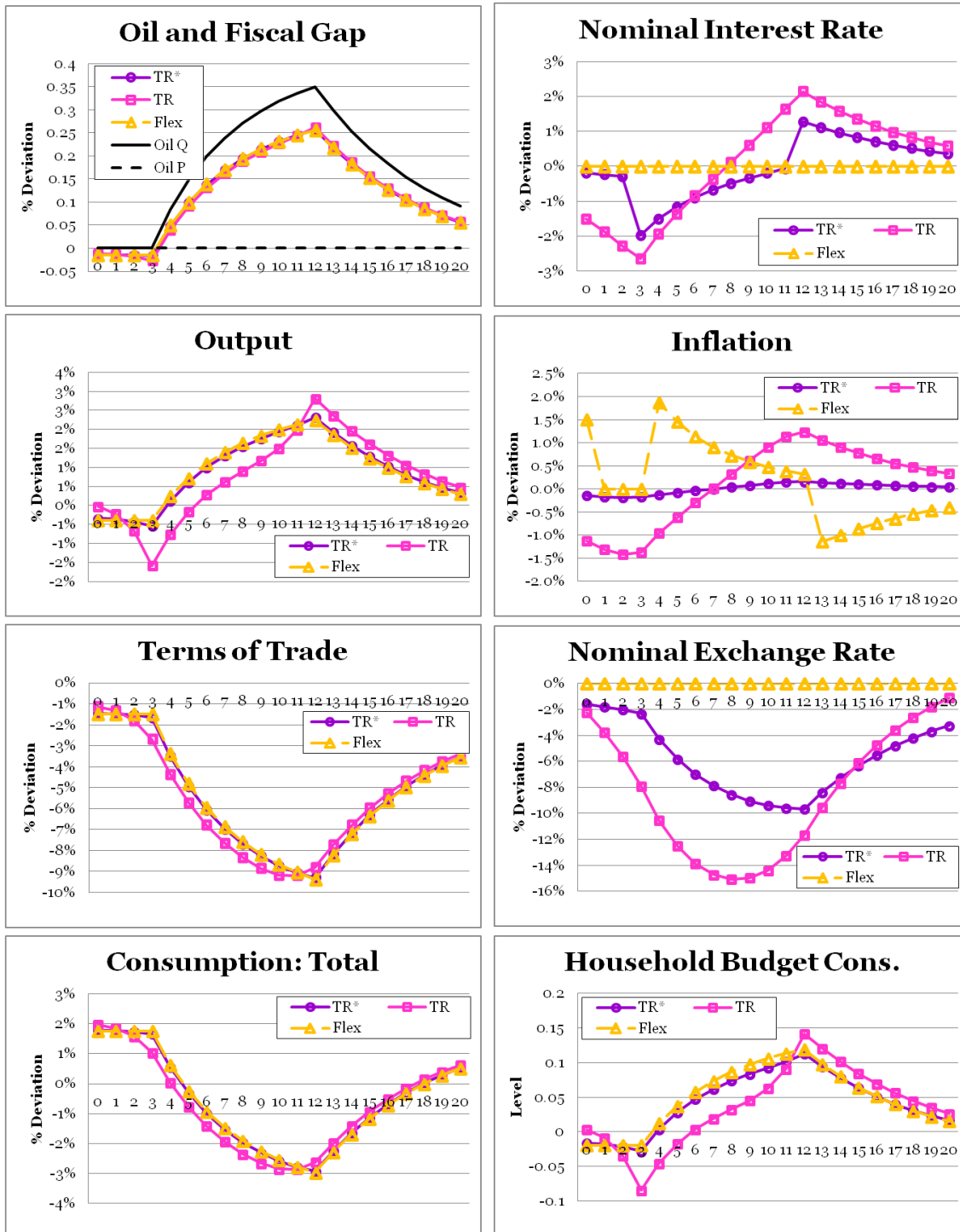
Figure D.3: Forecast mean responses of key variables to an anticipated oil windfall under the Bird in Hand rule. Three different scenarios are illustrated: a simple Taylor rule (TR), an optimised Taylor Rule (TR*) and flexible prices (Flex).

The simple Taylor rule causes large fluctuations in the output gap, if the government follows a Bird in Hand rule. Between discovery and spending there will be a similar recession to case described in the main text. Once oil production begins and the government starts consuming oil wealth, output will be consistently below its natural level. However, as the windfall comes to an end, forward looking firms and asset markets will begin to depreciate the terms of trade. Output will overshoot its natural level, and be above it as the remaining oil wealth is consumed.

In contrast, optimal policy is well approximated by a Taylor rule that responds to expected changes in the natural level of output. We again consider a Taylor rule of the form in equation 4.11, where $\phi_\pi = 1.5$ and $\phi_y = 0.5$. As in Section 5, numerically optimising the third parameter gives $\phi_{yn} = -2.6$. This gives us some confidence in the stability of the estimate, despite a large difference in the intensity and timing of the oil windfall compared to the case in the main text. Examining Figure D.3 again shows that this augmented Taylor rule closely tracks the flexible price outcome.

E   Online Appendix: Robustness

E.1   Approximating relative household wealth

In log-linearising the goods market equilibrium (equation A.2) we approximate relative household wealth. Specifically, we let $-\alpha\hat{\vartheta}_t \approx \ln(1-\alpha+\alpha\Theta_t^{-1}) - \ln(1-\alpha+\alpha\Theta_0^{-1})$. This approximation makes the analysis far more tractable and permits a closed form solution, without materially affecting our results.

To test how accurate this is we compare three different approximations, each implemented in the numerical package Dynare. The first uses a Newton-Raphson algorithm applied directly to the model in levels ("*NR*"). This is our benchmark. The second takes a first-order log-linearisation of the model around the initial steady state, but keeps track of both $\hat{\theta}_t = \ln(1-\alpha+\alpha\Theta_t^{-1}) - \ln(1-\alpha+\alpha\Theta_0^{-1})$ and $\hat{\vartheta}_t = \ln(\Theta_t) - \ln(\Theta_0)$ ("*$\theta + \vartheta$*"). The third again takes a first-order log-linearisation of the model, but approximates $-\alpha\hat{\vartheta}_t \approx \hat{\theta}_t$ as reported in the main text ("*ϑ*"). Prices are assumed to be flexible in each. As illustrated in Figure E.1, these approximations make little material difference to our analysis.

To check the accuracy of our solution method we can also compare the results from our closed-form solution to those calculated in Dynare. We find that the third case ("*ϑ*") perfectly replicates the results calculated by hand and reported in the main text.

E.2   Lump sum taxes

In log-linearising the government spending rule, $\hat{g}_t = \hat{s}_t - \hat{p}_t^* + \widehat{rb}_t - \hat{t}_t$, we make a simplifying assumption regarding lump sum taxes. We define $t_t = -\ln(1 - \frac{T_t}{G_t})$, so that the log-linearized government spending rule is exact. We also assume that $\hat{t}_t = 0$, so the ratio of lump sum taxes to government spending remains constant. Alternatively, we could have
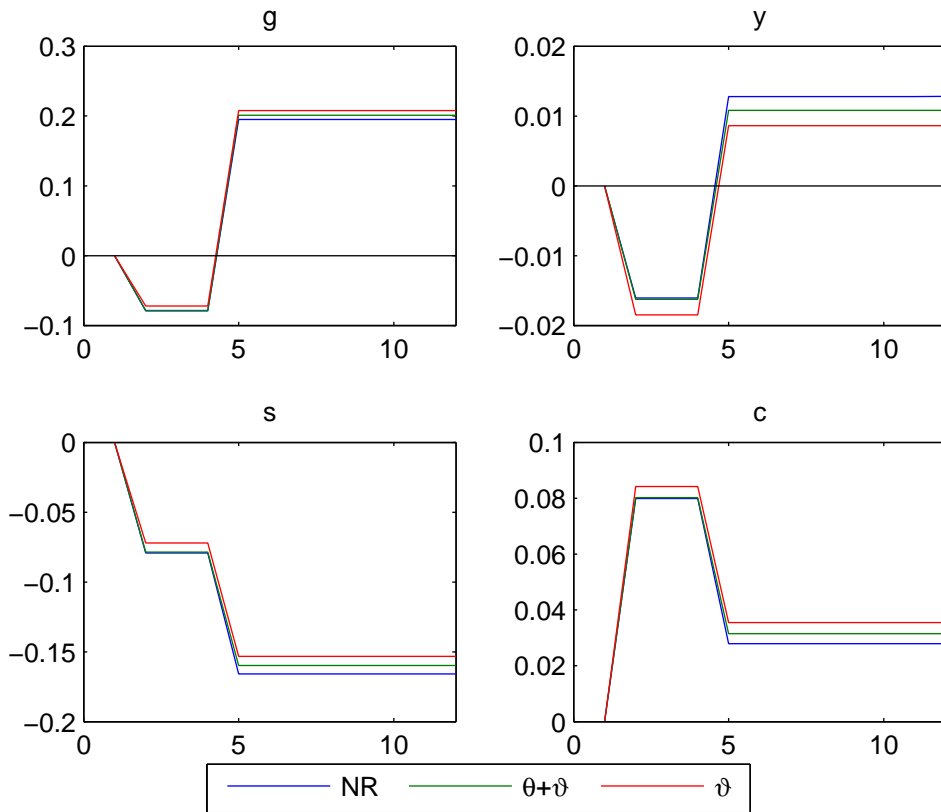
Figure E.1: Robustness check on simplifying assumptions. The spending rule in the main text is calculated under flexible prices in three cases: numerically using the Newton-Raphson method (NR), log-linearised making a distinction between θ and ϑ ($\theta + \vartheta$), and log-linearised using the approximation, $\theta \approx -\alpha\vartheta$ (ϑ). The ϑ (red) case is reported in the main body of the paper.
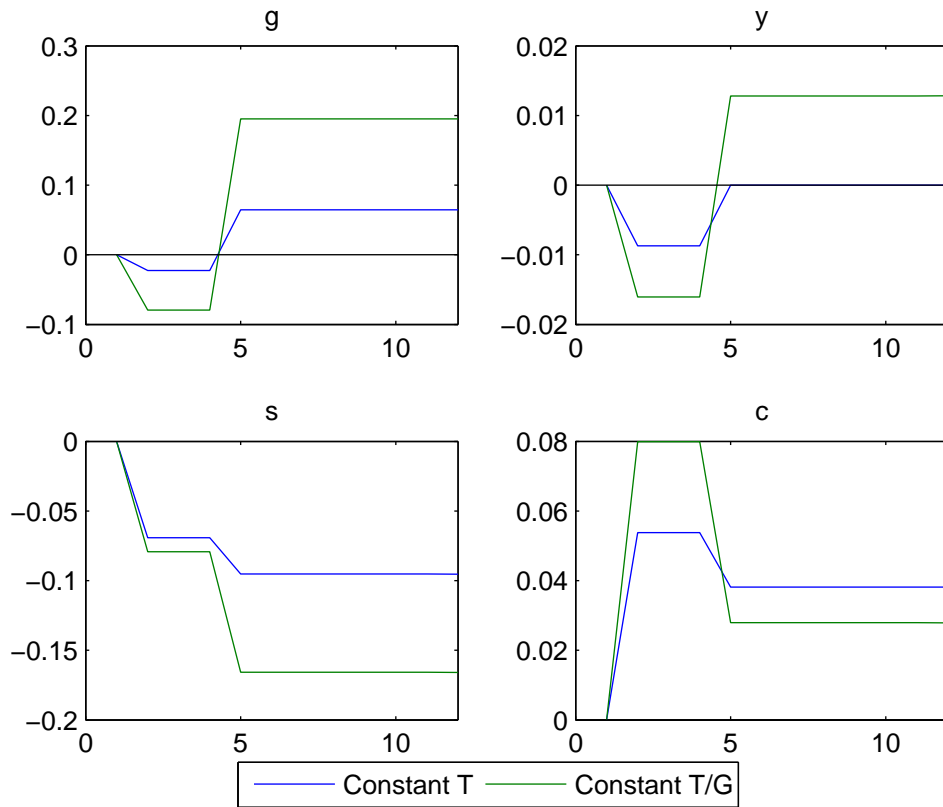
Figure E.2: Comparing different tax assumptions: constant T and constant T/G.

set the level of taxes to be constant. As illustrated in the simulations in Figure E.2, this assumption changes the magnitude of the government spending shock but the qualitative results remain the same.