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Cointegrated Periodically Collapsing Bubbles in the Exchange Rate of 'BRICS' Countries

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Abstract

This paper tests the occurrence of rational bubbles in the exchange rate of Brazil, Russia, India, China and South Africa (the 'BRICS' countries group) against the US dollar. We consider bubbles of the periodically recurring variety, and assume that the fundamental value follows a modified PPP relation which takes into account interest rate differentials, starting from a reference value which is endogenously determined. At each point in time the probability of collapse of the bubble to is a nonlinear logistic function of the absolute size of the bubble and is, therefore, also endogenous. The expected next period bubble size if the future regime is collapse is a linear function of the current bubble size whose parameters are also endogenously estimated. The estimation uses a maximum likelihood procedure, and the results support the model, which passes the specification tests. The hypothesis of rational expectations in the market for the forward exchange rate, which is used as a proxy of the expected exchange rate, is also tested and accepted for 3 of the 5 countries. The hypothesis of two linear regimes (rather than the non-linear regimes we use) is also tested and rejected for 3 of the 5 countries. We discuss the dynamics of the absolute bubble, and also compare the time series of the bubbles for the several countries, relative to the corresponding fundamental value. We test for unit roots in the relative bubbles and find that they are integrated, and that they pass Johansen's the cointegration test. Finally we estimate an error correction model to discuss the long term relation between the relative bubbles and the speed of adjustment of each country's relative bubble to shocks to the long term relation between them.

Keywords

Exchange rate bubbles, Cointegration test, Error correction models, BRICS

JEL Classification

C30, F31

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COINTEGRATED PERIODICALLY COLLAPSING BUBBLES IN THE EXCHANGE RATE OF 'BRICS' COUNTRIES

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1 INTRODUCTION

The increased importance of the BRICS countries group (Brazil, Russia, India, China and South Africa) in the World economy is mostly due to the current and potential size of their internal markets, and is also reflected in the growth of their share in international trade and financial flows. On the one hand, these five economies encompass three billion consumers and have an GNP of the order of US$ 16 billion, and on the other hand, they are currently the most important dynamic factor in international trade and are the destination of very significant capital flows.

A crucial variable that affects, and is affected by, this international insertion is the exchange rate of the country's currency against the other countries of the group and against the reserve currencies of the World. Here, we focus on the rate of exchange to the US dollar, and note that they have displayed a large volatility, relative to that of the reserve currencies. This may be due mostly to the volatility of the capital flows and the uncertainty it induces in the domestic economies, but speculative mechanisms may also be operating to produce it. We study this possibility by testing for the occurrence of rational bubbles in these exchange rates.

The bubble is defined (Flood and Garber (1980) and Blanchard and Watson (1982)) as the part of asset price movement that is unexplainable by the mechanisms that drive the long term asset price in the relevant theoretical model. The variables that reflect these effects are called the *fundamentals*, and the long term forecast price derived within the context of a particular model of asset price determination on the basis of the fundamentals is the fundamental price. A significant deviation between it and the actual asset price is the bubble.

Such rational bubble would, if it ever it arose, have and explosive path, as shown in Blanchard (1979) and Blanchard and Watson (1982). This points out a basic difficulty that arises in testing for the existence of rational bubbles, as pointed out by Flood and Garber (1980): the contribution of hypothetical rational bubbles to asset prices would not be directly distinguishable from the contribution of market fundamentals that the researcher cannot observe. Diba and Grossman (1984), propose an empirical strategy to deal with this problem, based on using stationarity tests for obtaining evidence against the existence of explosive rational bubbles in that case. Diba and Grossman (1988a) implements such tests for explosive rational bubbles in stock prices using a model that assumes a constant discount rate, but allows unobservable variables to affect market fundamentals, and concludes against the occurrence of explosive rational bubbles in the stock market prices he analyses. However, Diba and Grossman (1988b) have shown that the impossibility of negative rational bubbles in stock prices implies theoretically that a bubble can never restart if it ever collapses (i.e., falls to zero).

The explosive behavior of speculative bubbles implies that it would arise, grow, collapse, and definitively disappear almost surely in finite time. Since this does not appear to be an often observed behavior of asset prices, Blanchard (1979) proposed a model of periodically collapsing bubbles, whose state stochastically shifts between survival and collapse, with exogenous transition probabilities, and argued that this type of bubble can better mimic the behavior of the price of speculative assets. This idea has been further developed by Hamilton (1989) in a model that uses a Markov-switching regression to characterize the changes in the parameters of the autoregressive process, and displays nonlinear behavior when the regime shifts occur. The estimation of his model is different from that of the Kalman filter, where the estimates of the unobserved continuous state vector are produced by a linear algorithm, because the filter and smoother he proposes yield nonlinear inference about a discrete-valued unobserved state vector. Van Norden and Schaller (1993) generalized Hamilton's (1989) model and applied it to describe and analyze stock market returns, and find very strong evidence of switching behavior.

Evans (1991) expanded on Hamilton's idea by proposing a model of rational bubbles that are always positive and periodically collapse to zero, but where a later resurgence of the bubble is allowed. He also allows the endogenous determination of the probability for the occurrence of the two regimes, and shows that the tests proposed by Diba and Grossman (1988a), and others, are unable to detect that important class of rational bubbles. The point is demonstrated by constructing rational bubbles that appear to be stationary when unit root tests are applied, even though they are explosive in a relevant sense. This points out to the difficulties of testing for rational bubbles in asset price processes that allow periodically collapsing bubbles, as is our case.

Van Norden (1996) extends the Van Norden and Schaller (1993) model to test for bubbles in the exchange rate. He implicitly uses the asset market approach to exchange rate determination (see, for example, Mussa (198 4)) and views it as the relative price of the domestic assets in the two economies. The bubble is the misalignment of the exchange rate with respect to the fundamental rate, and its collapse is the correction of that "error". [3] As in the earlier model, he specifies that the probability of survival of the bubble decreases with its

[3] The use of a periodically collapsing bubble was inspired by Jarrow e Protter (2010) that, in martigale based approach, concluded that exchange rate bubbles can be negative, as opposed to asset bubbles, where this possibility does not arise.

size. Also, in his model the size of the innovation in the next period's exchange rate is a linear function of the bubble observed in the current period. To estimate the model, he log-linearizes the adjustment function, to obtain a stochastic specification with two linear regimes. It is important to note that, since the level of fundamental exchange rate in the reference year is given, its trajectory is exogenous to the model.

Maldonado et al. (2012), extend the Van Norden (1996) model considering the possibility of a non-linear relation between the innovation and the size of the bubble in the survival regime. This yields an empirical model for the innovation with non-linear regimes that allows testing the rational expectations hypothesis in the foreign exchange futures market. In addition, it allows the endogenous estimation of the fundamental value of the exchange rate at the reference date, which is arbitrarily set in Van Norden's model. The specification tests employed are those proposed by White (1982, 1987) e Hamilton (1996).

This paper tests the occurrence of regime-switching bubbles in the exchange rates of the currency of BRICS countries group to the US dollar, using the Maldonado et al. (2012) model. For the fundamental exchange rate two structural models were used: PPP and interest-rate differential modified PPP proposed by Maldonado et al. (2012). The sample period for each country was dependent on data availability, as discussed later.

The estimation of these models yield bubble estimates for these 5 exchange rate series. To be able to compare the bubble series for the different countries we define relative bubble series for each of them, dividing each bubble estimate by the contemporaneous value of the fundamental exchange rate. These relative bubble series are subjected to unit root tests, which confirm that they are integrated, and that a bubble periodically collapsing bubble is present in the original exchange rates series. We test the relative bubble series obtained from estimates of the proposed regime switching model for the presence of unit roots, instead of testing the original exchange rate series under Markov switching, as proposed by Hall, Psaradakis and Sola (1999) [4], since our approach is more consistent with the exact stochastic representation we adopt for the exchange rate process The fact that these relative bubbles series are integrated fulfills the prerequisite for testing them for for cointegration, as a way to explore the relation between the bubbles in the exchange rates of the BRICS countries. A common stochastic trend was identified, and this was interpreted as evidence that shocks to the international financial markets tend to affect in a peculiar manner the exchange rates of currencies of all countries in the BRIC group against the US dollar. An unrestricted VEC (Vector

[4] There exist other tests for rational bubbles in exchange rates in the literature that do not use a regime switching approach as, for example, Evans (1986) and Meese (1986).

Error Correction model) was fitted to this data, and provided further evidence on how deviations from long term relation between these exchange rates are corrected.

This rest of this paper is divided in 5 sections. Section 2 describes the formal specification of the time series model containing periodically collapsing bubbles proposed by Maldonado et al. (2012). Section 3 describes the empirical implementation: the data, estimation, specification tests, and tests of the rational expectations hypothesis and of Van Norden's two linear regimes specification. Section 4 discusses the dynamics of the absolute bubbles and how they can be interpreted on the basis of heuristic evidence regarding the broad movements of the exchange rate of this group of countries. Section 5 compares the bubbles for the five BRICS countries from the perspective of the bubble size relative to the fundamental value of the interest rate. It also discusses their time series characteristics, tests for the presence of a unit root and for cointegration between the countries' bubbles, and the estimation of the VEC. Finally, section 6 summarizes the main conclusions and discusses the possible future extensions.

2 THE MALDONADO ET AL. (2012) BUBBLE MODEL

Stochastic processes capable of generating asset price bubbles include the expectation of future capital gain as one determinant of current price. It is the possibility of these extraordinary gains that generate the sometimes significant departure of the market price from its structural determinants i.e. the bubble. In the simplest formulation of such models the expectation component is linear and of the first order (only involves the next period). Such a formulation, which is the basis of our model for the exchange rate, is described by the following equation:

$$Y_t = aE_t[Y_{t+1}] + f(X_t) \tag{1}$$

where Y_t is the stochastic process for the asset value, $a \in (0,1)$ is the inter-temporal discount factor (which is assumed to be constant) and $f(X_t)$ is a possibly non-linear function of all the exogenous variables (fundamental) variables that affect the price of the asset (X_t). The operator $E_t[\cdot]$ is the mathematical expectation of the argument $[\cdot]$, conditioned on all the information available at time t. It is assumed that equation (1) incorporates all the equilibrium and non-arbitrage conditions of the agents involved, and of the market for the asset.

Solving equation (1) recursively forward, and taking the limit as the terminal time tends to infinity yields:

$$Y_t = Y_t^* + b_t = \sum_{i=0}^{+\infty} a^i E_t[f(X_{t+i})] + lim_{T\to+\infty}\, a^T E_t[Y_{t+T}] \tag{2}$$

where the first term in the right hand side (r.h.s) is the fundamental value of the asset, denoted Y_t^*, which depends on the present expected value of a function of the fundamental exogenous variables. The second term in the r.h.s. of (2) is the associated bubble is b_t, which is the limit as the horizon goes to infinity of the present expected value of the terminal asset value. For this reason, any speculation regarding the future expected value of the endogenous variable must be strictly included in this term. It is clear that the bubble is just the difference between the current asset value and the fundamental value.

Shifting (2) forward in time one period, taking its expected value, and recalling that, due to the law of iterated expectations, the fundamental value also has to satisfy (1), i.e. $Y_t^* = aE_t[Y_{t+1}^*] + f(X_t)$. Therefore, it can be seen that the bubble must satisfy the following equation:

$$b_t = aE_t[b_{t+1}] \tag{3}$$

Equation (3) shows that the dynamics of the bubble is such that the expectation of its future value is the only determinant of its current value. There is no role for the exogenous structural variables in this process and the bubble, therefore, contains and isolates the speculative component of the price process.

Since (3) is a linear function, and $a \in (0,1)$, the expected bubble size next period will be strictly larger than the current bubble size - if it is not null in the current period - and will, therefore, follow an explosive path. Although the explosive path refers to the expectation of the bubble, if such solution is excluded, the only remaining solution of (1) is for the bubble to be null, which is not very satisfactory state of affairs.

Blanchard (1979) and Blanchard and Watson (1982) addressed this issue, and suggested that a Markov process be considered for the bubble dynamics: in each period the process is in either one of two states: collapse or survival. However, since it is assumed that the state of the process is not observable, only a probabilistic statement can be made about it. The probabilities in the transition matrix of the process can be assumed to be constant and exogenous, as suggested by Blanchard, but more promising models can be obtained if such probabilities are endogenous and depend on the size of the bubble. Further, in the Blanchard specification if the bubble collapses, it becomes null. It would seem more natural in that case its size would be some fraction of the former size.

Van Norden (1996) formalized these features through the two hypothesis below, which make use of the definitions. Let $\{r_t\}$ be a stochastic process that indicates which of the two states the bubble is at time t, and let $r_t = 1$ when the regime is survival (S), and $r_t = 0$ when it is collapse (C). The probability that the bubble survives in $t+1$, given the information available at time t, is $q_{t+1} = P_t[r_{t+1} = 1]$.

Hypothesis 1. There exist two functions $u : R \to R$ e $q : R \to [0,1]$ such that:

$$E_t[b_{t+1}|r_{t+1} = 0] = u(b_t) \tag{4}$$

$$q_{t+1} = q(b_t) \tag{5}$$

Equation (4) means that if, in addition to the information available at t, it is known that the bubble will collapse next time period, then the expected size of the bubble next period will be a function of its current size. Equation (5) indicates that the probability of survival of the bubble next period depends only on its current size. We will discuss the properties of these functions below, but it should be clear at this point that $u(\cdot)$ should be decreasing and that $q(\cdot)$ should be increasing in the absolute value of the bubble.

Hypothesis 2. The expectation of the fundamental asset value in the next period, given current information, is independent of the state of the bubble next period:

$$E_t[Y^*_{t+1}|r_{t+1}] = E_t[Y^*_{t+1}] \tag{6}$$

With these two hypothesis an empirical model can be specified. However, note that the bubble itself is not observable, because the fundamental value is unobservable (equation (2)). Nevertheless, equations can be derived that relate the bubble to the innovations (or surprises) of the asset price, for which a market indication can often be obtained. The innovations will be denoted R_t and are defined by:

$$R_{t+1} = Y_{t+1} - E_t[Y_{t+1}] \tag{7}$$

Using $Y_{t+1} = Y^*_{t+1} + b_{t+1}$ in (7), taking expectations, and using (3) and (4), equations that relate the innovations and to the bubble in the two regimes can be obtained:

$$E_t[R_{t+1}|r_{t+1} = 0] = E_t[R^*_{t+1}|r_{t+1} = 0] + u(b_t) - \frac{b_t}{a} \tag{8a}$$

$$E_t[R_{t+1}|r_{t+1} = 1] = E_t[R^*_{t+1}|r_{t+1} = 1] + \left(\frac{1}{q(b_t)} - 1\right)\left(\frac{b_t}{a} - u(b_t)\right) \tag{8b}$$

where $R^*_{t+1} = Y^*_{t+1} - E_t[Y^*_{t+1}]$ is the innovation in the fundamental value. However, note that under Hypothesis 2 the first term in the r.h.s. of (8a) and (8b) for the expectation of the innovation on the fundamental exchange rate is null. Therefore, the next period innovation R_{t+1} in either state depends only on the current period bubble b_t.

Maldonado et al. (2012) modified the Van Norden (1976) model in some respects. Below we summarize the changes, and direct to reader to the original paper for further details, justifications and proofs related to them.

The <u>first</u> change is the use of the following logit specification for the function $q(\cdot)$, which specifies the collapse probability:

$$q(b) = \left[1 + exp\left(\beta_{q0} + \beta_{q2} {b_t}^2\right)\right]^{-1}, \quad \beta_{q2} > 0 \tag{9}$$

It still maintains the logistic functional form, but omits a linear term in the argument of the exponential that appears in Van Norden (1996). The main benefit of not having the linear term in the argument of the exponential in (9) is that this avoids the need to log-linearize the model when estimating it, and yields a true non-linear specification. Another benefit is the reduction in the number of parameters to estimate.

The restriction on parameter β_{q2} ensures that the probability of survival decreases with the absolute size of the bubble $(dq(b)/|db| < 0)$.

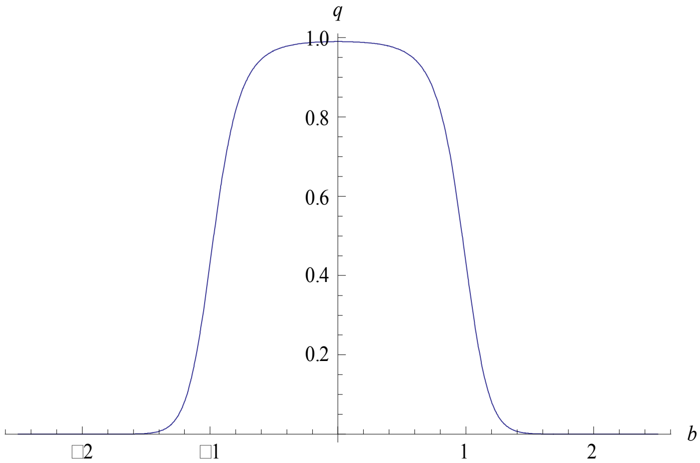

Figure 1 - Example of probability of collapse of the bubble as a function of bubble size

The <u>second</u> change proposed by Maldonado et al (2012) is the adoption of a linear specification for the $u(\cdot)$ function, defined by equation (4)). This hypothesis is motivated by the fact that the expected value of the innovation is linear with respect to the expected bubble size next period, as shown in (8a) and (8b). This helps to avoid the log-linearization that produces his two linear regimes model.

These two hypothesis yield the following empirical specification of the non-linear regime switching model:

$$R_{t+1} = \beta_{c0} + \beta_{c1} \cdot b_t + \varepsilon_{t+1}^{c} \qquad Prob = 1 - q(b_t) \tag{10a}$$

$$R_{t+1} = \beta_{s00} + (\beta_{s0} + \beta_{s1} \cdot b_t)\, exp\left(\beta_{p2} {b_t}^2\right) + \varepsilon_{t+1}^{s} \qquad Prob = q(b_t) \tag{10b}$$

where stochastic errors are defined as $\varepsilon_{t+1}^{C} = R_{t+1} - E_t[R_{t+1}|r_{t+1} = 0]$ and $\varepsilon_{t+1}^{S} = R_{t+1} - E_t[R_{t+1}|r_{t+1} = 1]$. It is assumed that the latter are random variables with Gaussian distribution: $\varepsilon_t^C \sim N(0, \sigma_C^2)$ and $\varepsilon_t^S \sim N(0, \sigma_S^2)$, respectively.

The model allows the test of several hypothesis, but here we emphasize the following:

Rational Expectations Hypothesis (REH): The expected value of the innovation next period, given the information available today, is null, i.e. $E_t[R_{t+1}] = 0$. For this to obtain it is necessary that the parameters satisfy:

$$\beta_{s0} = -\beta_{c0}\, exp(\beta_{q0}); \quad \beta_{s1} = -\beta_{c1}\, exp(\beta_{q0}); \quad \beta_{s00} = 0; \quad \beta_{p2} = \beta_{q2} \tag{11}$$

Two Linear Regimes Hypothesis (TLRH): The expected innovation in each regime is linearly related to the bubble size. This case corresponds to the Van Norden (1996) model.

$$\beta_{p2} = 0; \qquad \beta_{s00} = 0 \tag{12}$$

3 EMPIRICAL IMPLEMENTATION

The theoretical model described in the previous section refers to a (general) asset denoted $\mathrm{Y_t}$, and in this section it is applied to the spot exchange rate (S_t) of BRICS countries group (Brazil, Russia, India, China e South Africa) domestic currencies against the US dollar. It has three subsections: the description of the data base, the estimation results and the discussion of the quality of the adjustment.

3.1 DATA

We used monthly data starting in 1999:03 and ending in 2013:06 for Brazil, India and South Africa, and for China and Russia the sample started in 2005:06. For China, the earlier period was excluded because the currency was quoted with relation to a basket of currencies, instead of the dollar, and was pegged. For Russia we excluded from the sample the period where the exchange rate was managed to fluctuate within a band (dirty float). The rate used refers to that quoted for sale at the close of business in the last day of each month, available at the Bloomberg database.

Our estimation procedure relies on constructing a series for innovations to the exchange rate equal to the difference between the spot rate and its expected value. For the latter we used the value of an exchange rate forward contract of 1 month maturity, also obtained from the Bloomberg database, $F_t = E_t[S_{t+1}]$. Therefore,

$$R_{t+1} = S_{t+1} - E_t[S_{t+1}] = S_{t+1} - F_t \tag{13}$$

The fundamental asset value (Y^*) is in this case the fundamental exchange rate, denoted $\mathrm{S_t^*}$. Two models proposed in Maldonado et al. (2012), Models 1 and 2, were chosen for this implementation. They correspond respectively to the PPP model, and the interest-rate augmented PPP model, as summarized below.[5]

Model 1 - The PPP model for the fundamental exchange rate

The fundamental rate in this case is denoted and $\mathrm{S}^*_{\mathrm{t},1}$ calculated as:

$$S^*_{t,1} = k \cdot \frac{IPD_t}{IPI_t} \tag{14}$$

where IPD and IPI are the domestic and foreign price indexes, and k is a parameter whose interpretation is given below.

Model 2 - The interest-rate augmented PPP model for the fundamental exchange rate

The fundamental rate in this case is denoted $\mathrm{S}^*_{\mathrm{t},2}$ and calculated as

$$S^*_{t,2} = k \frac{(1+TJI_t)}{(1+TJD_t)} E_t \left[\frac{IPD_{t+1}}{IPI_{t+1}}\right] \tag{15}$$

Under rational expectations the expected future ratio between domestic and foreign price index in (15) is replaced by the actual ratio one period ahead, which is known at estimation time.

For both models the k parameter determines the level of the exchange rate path, since it depends on the exchange rate at $t = 1$ ($S^*_{1,1}$ or $S^*_{1,2}$ respectively).[6] It is important to note that since k is estimated in our model, the level of the path of the fundamental interest rate is endogenously determined, rather than arbitrarily assigned as in Van Norden (1996).

The domestic price index IPD used for each of the 5 countries was the PPI (producer price index), and for the international price index IPI, the PPI of the USA was used. These price indexes are monthly, referenced to the first day of the corresponding month, and were obtained from the Reuters Datastream database.

For Model 2 interest rate data is also necessary. The international interest rate (*TJI*) of all countries, except India, was calculated as the sum of the rate paid by the 4-week US Treasury bill with its EMBI+ rate.[7] For India the EMBI+ is only available after 2012, and the JACI (JP Morgan Asia Credit Index) was used instead. The domestic interest rate (*TJD*) was obtained for all countries from the interbank borrowing rate for loans of 1-month maturity. The time series

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[5] The derivation of these models in the context of the asset approach to interest rate determination can be found in Appendix 1 of Maldonado et al (2012).

[6] Note that the second subscript of the interest rate variable denotes the model to which it refers.

[7] The EMBI+ is calculated by J.P. Morgan as the average over all major issues of the difference between in the interest rate paid by a given country on sovereign debt denominated in US dollars, and the T-Bill of similar maturity.

for these variables are normalized as 1-month rates, in percents, and were also captured from Reuters, via the Datastream tool.

3.2 ESTIMATION

This section presents the results of the estimation for the BRICS countries of the model specified in (9), (10a) and (10b). Each country is treated separately, and two models are estimated for each, considering the two hypotheses presented in the last section regarding the determinants of the fundamental rate (Models 1 and 2, respectively). For each country and model, 3 equations are estimated: one unrestricted, and two restricted: one imposing rational expectations, and another imposing the occurrence of two linear regimes. These two equations allows the two corresponding hypothesis to be tested with the Likelihood Ratio test.

The results for the estimation of Models 1 and 2 are displayed in Tables 1 and 2 respectively. In each of these tables there are three versions of the respective model: the unrestricted and two restricted, as indicated in the second column of the two tables. The first restricted equation imposes rational expectations in the forward market for foreign exchange which is characterized by the restrictions (11), and is denoted RE in the sequel. The second restricted equation imposes Van Norden's two linear regimes specification, and is characterized by restrictions (12). The third column of the two tables shows the estimated value and standard deviation of the k parameter, and the next 10 columns contain the estimated values and respective standard deviations for the parameters that appear in the non-linear regime-switching regression of models (10a) and (10b). The likelihood statistic is displayed in the last column.

For all BRIC countries, except Brazil, Model 2 is the best model, as is shown by the comparison of the likelihood statistics for the unrestricted and restricted equations in the last column of the Tables 1 and 2. For Brazil, Model 1 fits the data slightly better according to that criteria, as was observed in Maldonado et al. (2012). However, the difference in the likelihood statistic of the two models for this last country is minor, as is the difference in the significant coefficients of the two models, and to simplify the discussion of the results we adopt Model 2 as the reference model for all countries.

Table 1 - Model 1 estimated parameters (standard error in parenthesis)

Country	Model	k	β_{q0}	β_{q2}	β_{S0}	β_{S1}	σ_s	β_{C0}	β_{C1}	σ_C	β_{p2}	β_{S00}	Log-likelihood
BRASIL	Unrestricted	0,81	0,29	0,88	0,14	-0,02	0,04	0,01	-0,06	0,10	0,96	-0,19	169,00
		(0,07)	(0,54)	(0,41)	(0,05)	(0,05)	(0,01)	(0,02)	(0,01)	(0,01)	(0,21)	(0,05)	
	Rational expectations	0,83	0,86	1,05	-0,02	0,07	0,01	0,01	-0,03	0,11	1,05	0,00	159,88
		(0,04)	(0,26)	(0,11)	(0,01)	(0,01)	(0,00)	(1,00)	(1,00)	(0,01)	(1,00)	-	
	Two linear regimes	0,66	-11,67	4,19	0,00	-0,03	0,09	0,88	-0,42	0,33	0,00	0,00	157,44
		(0,26)	(5,58)	(2,90)	(0,03)	(0,02)	(0,01)	(0,71)	(0,28)	(0,07)	-	-	
RUSSIA	Unrestricted	13,88	1,87	0,18	-1,74	0,14	0,43	-0,13	0,06	0,71	0,22	-0,66	-119,01
		(0,15)	(0,65)	(0,13)	(2,88)	(0,16)	(0,15)	(0,10)	(0,02)	(0,06)	(0,24)	(3,21)	
	Rational expectations	13,07	0,00	0,23	0,04	-0,04	1,90	-0,04	0,04	0,62	0,23	0,00	-123,56
		(0,70)	(0,73)	(0,15)	(0,15)	(0,04)	(0,42)	(1,00)	(1,00)	(0,06)	(1,00)	-	
	Two linear regimes	16,96	-1,94	0,03	0,10	0,01	0,47	-0,15	0,14	2,02	0,00	0,00	-124,94
		(0,92)	(0,57)	(0,02)	(0,08)	(0,02)	(0,07)	(0,67)	(0,10)	(0,34)	-	-	
INDIA	Unrestricted	66,28	-1,01	0,08	1,12	-0,10	1,43	-0,44	0,04	0,32	0,03	-1,07	-207,68
		(0,99)	(0,40)	(0,03)	(0,15)	(0,05)	(0,13)	(0,09)	(0,02)	(0,05)	(0,02)	(0,24)	
	Rational expectations	66,54	-1,04	0,09	0,15	-0,01	1,45	-0,43	0,04	0,32	0,09	0,00	-208,41
		(1,09)	(0,39)	(0,03)	(0,07)	(0,01)	(0,15)	(1,00)	(1,00)	(0,04)	(1,00)	-	
	Two linear regimes	66,07	-0,94	0,07	0,27	-0,04	1,46	-0,44	0,03	0,32	0,00	0,00	-208,90
		(1,20)	(0,44)	(0,03)	(0,21)	(0,07)	(0,14)	(0,09)	(0,02)	(0,04)	-	-	
CHINA	Unrestricted	12,31	-1,36	7,11	-0,04	-0,13	0,04	0,01	0,01	0,01	-4,62	0,02	400,83
		(0,10)	(0,49)	(2,63)	(0,03)	(0,05)	(0,00)	(0,00)	(0,00)	(0,00)	(2,88)	(0,02)	
	Rational expectations	12,39	-1,38	9,43	0,00	0,00	0,05	0,01	0,01	0,01	9,43	0,00	389,22
		(0,09)	(0,47)	(2,70)	(0,00)	(0,00)	(0,00)	(1,00)	(1,00)	(0,00)	(1,00)	-	
	Two linear regimes	12,30	-1,85	13,31	-0,02	-0,08	0,04	0,01	0,01	0,01	0,00	0,00	399,01
		(0,08)	(0,57)	(3,97)	(0,01)	(0,02)	(0,00)	(0,00)	(0,00)	(0,00)	-	-	
SOUTH AFRICA	Unrestricted	10,09	-0,75	0,15	0,25	-0,05	0,24	0,17	-0,16	0,47	0,23	-0,35	-67,89
		(0,41)	(0,55)	(0,09)	(0,19)	(0,04)	(0,03)	(0,09)	(0,04)	(0,05)	(0,07)	(0,20)	
	Rational expectations	12,94	0,31	0,39	0,17	0,11	0,12	-0,13	-0,08	0,39	0,39	0,00	-73,69
		(0,19)	(0,51)	(0,09)	(0,05)	(0,03)	(0,06)	(1,00)	(1,00)	(0,03)	(1,00)	-	
	Two linear regimes	10,47	2,07	0,22	-0,14	-0,30	0,01	0,01	-0,03	0,41	0,00	0,00	-72,99
		(0,14)	(0,35)	(0,23)	(0,03)	(0,00)	(0,00)	(0,03)	(0,03)	(0,02)	-	-	

Table 2 - Model 2 estimated parameters (standard error in parenthesis)

Country	Model	k	β_{q0}	β_{q2}	β_{S0}	β_{S1}	σ_s	β_{C0}	β_{C1}	σ_C	β_{p2}	β_{S00}	Log-likelihood
BRASIL	Unrestricted	0,80	-0,11	1,10	0,09	-0,03	0,04	0,01	-0,07	0,11	1,45	-0,13	168,95
		(0,05)	(0,48)	(0,43)	(0,05)	(0,03)	(0,01)	(0,02)	(0,01)	(0,01)	(0,39)	(0,05)	
	Rational expectations	0,81	10,82	2,67	1,67	-1,11	0,92	0,00	0,00	0,13	2,67	0,00	109,44
		(3,92)	(45,10)	(15,40)	(8,90)	(6,40)	(5,59)	(1,00)	(1,00)	(0,01)	(1,00)	-	
	Two linear regimes	0,71	-11,91	4,96	0,00	-0,03	0,09	0,91	-0,46	0,33	0,00	0,00	157,58
		(0,25)	(6,17)	(3,75)	(0,02)	(0,02)	(0,01)	(0,74)	(0,35)	(0,07)	-	-	
RUSSIA	Unrestricted	14,57	1,28	0,12	-0,92	-0,19	0,12	-0,16	0,07	0,79	0,26	1,87	-117,64
		(0,04)	(0,59)	(0,08)	(0,19)	(0,05)	(0,03)	(0,10)	(0,02)	(0,06)	(0,01)	(0,23)	
	Rational expectations	14,29	2,35	0,12	-1,77	-0,13	0,39	0,17	0,01	0,69	0,12	0,00	-121,13
		(0,17)	(0,34)	(0,02)	(0,26)	(0,15)	(0,12)	(1,00)	(1,00)	(0,05)	(1,00)	-	
	Two linear regimes	13,49	1,04	0,09	-1,57	-0,42	2,24	-0,11	0,05	0,62	0,00	0,00	-124,22
		(0,08)	(0,65)	(0,05)	(0,79)	(0,41)	(0,45)	(0,11)	(0,02)	(0,07)	-	-	
INDIA	Unrestricted	65,65	-1,45	0,10	0,08	1,06	1,44	-0,42	0,03	0,31	-0,84	0,16	-202,49
		(0,69)	(0,52)	(0,03)	(0,52)	(0,89)	(0,14)	(0,10)	(0,02)	(0,05)	(0,71)	(0,23)	
	Rational expectations	66,40	-1,39	0,12	0,10	-0,01	1,46	-0,42	0,03	0,32	0,12	0,00	-204,69
		(0,69)	(0,54)	(0,04)	(0,06)	(0,01)	(0,14)	(1,00)	(1,00)	(0,04)	(1,00)	-	
	Two linear regimes	65,88	-1,41	0,11	0,28	-0,08	1,44	-0,42	0,03	0,31	0,00	0,00	-202,79
		(0,94)	(0,49)	(0,03)	(0,21)	(0,08)	(0,13)	(0,10)	(0,02)	(0,05)	-	-	
CHINA	Unrestricted	12,32	-1,66	9,49	-0,04	-0,15	0,04	0,01	0,01	0,01	-4,90	0,02	404,87
		(0,08)	(0,58)	(3,31)	(0,03)	(0,05)	(0,00)	(0,00)	(0,00)	(0,00)	(3,20)	(0,03)	
	Rational expectations	12,33	-1,54	9,04	0,00	0,00	0,05	0,01	0,01	0,01	9,04	0,00	393,33
		(0,09)	(0,59)	(2,94)	(0,00)	(0,00)	(0,00)	(1,00)	(1,00)	(0,00)	(1,00)	-	
	Two linear regimes	12,27	-1,97	12,33	-0,01	-0,07	0,04	0,01	0,01	0,01	0,00	0,00	402,82
		(0,08)	(0,70)	(3,48)	(0,01)	(0,02)	(0,00)	(0,00)	(0,00)	(0,00)	-	-	
SOUTH AFRICA	Unrestricted	10,05	-0,63	0,12	0,21	-0,05	0,24	0,17	-0,17	0,46	0,29	-0,31	-67,14
		(0,35)	(0,59)	(0,09)	(0,13)	(0,03)	(0,03)	(0,10)	(0,04)	(0,05)	(0,06)	(0,14)	
	Rational expectations	12,96	0,61	0,38	0,21	0,15	0,09	-0,12	-0,08	0,39	0,38	0,00	-73,27
		(0,15)	(0,43)	(0,08)	(0,06)	(0,04)	(0,04)	(1,00)	(1,00)	(0,02)	(1,00)	-	
	Two linear regimes	10,78	2,37	5,34	0,59	2,48	0,00	-0,02	-0,05	0,39	0,00	0,00	-72,74
		(0,00)	(0,60)	(3,98)	(0,01)	(0,00)	(0,00)	(0,03)	(0,02)	(0,02)	-	-	

The unrestricted equations for all countries display a small coefficient of variation for the parameter k (Brazil = 7,5%, Russia = 2,7%; India = 1,5%; China = 0,6%; South Africa = 3,5%), indicating the point estimate of the fundamental exchange rate at the initial date of the sample is rather precise. Therefore, it is very unlikely that an arbitrary assignment of a value to that parameter would lead do an proper estimate of the bubble. Such as arbitrary choice is implied in the election of a certain date as the one where the actual and fundamental exchange rate coincided and there was no bubble, as is often done in calculations of PPP equilibrium rate, and was done in Van Norden (1996).

The significance of the coefficients can be examined by adopting a confidence interval of 2 standard deviations.[8] Table 3 displays, for the reference model, the insignificant coefficients for the unrestricted equation of the several countries. It shows that the fit is best for Russia, good for Brazil and China, and worse for India and South Africa, since there are, respectively, 2, 3 and 5 insignificant coefficients in these two groups of countries. The significance of the individual parameter estimates is explored below to shed some light on the nature of the fit of the model in each case.

Table 3 - The parameters of the model that are insignificant at 5%.

Brazil	Russia	India	China	South Africa
$\beta_{q0}, \beta_{S1}, \beta_{C0}$	β_{q2}, β_{C0}	$\beta_{S0}, \beta_{S1}, \beta_{C1}, \beta_{p2}, \beta_{S00}$	$\beta_{S0}, \beta_{p2}, \beta_{S00}$	$\beta_{q0}, \beta_{q2}, \beta_{S0}, \beta_{S1}, \beta_{C0}$

The intercept of the innovation equation in the collapse regime (equation 10a) is β_{C0}. It is not significant for Brazil, Russia, and South Africa. The parameters that jointly play the same role in the survival regime (equation 10b), are β_{S00} and β_{S0} . They are not significant for India and China, and for South Africa β_{S00} is significant, but β_{S0} is not. In all the five equations at least one of the intercept parameters (β_{C0}, β_{S0} and β_{S00}) is not significant, but in neither of them they are null for both regimes.

Recalling that β_{p2} is the exponent of the non-linear term of the size of the next-period innovation in the case the bubble survives (see (10b)), the fact that it may be null for the cases of India and China suggests that in those cases the two linear regimes model of Van Norden (1996) may be applicable. This is formally tested in the next section.

[8] Since this is a non-linear stochastic model, the exact distribution of the coefficient estimators is not known, so the Quasi-Maximum Likelihood method (Hamilton (1994) was used to estimate the standard errors.

The parameters of the logistic function that gives the probability of survival of the bubble($q(b)$) are β_{q0} and β_{q2} (see equation (9)). The former characterizes the intercept of that probability, i.e. the survival probability when the bubble is null, and is not significant for Brazil and South Africa. The latter (β_{q2}) indicates the speed at which the survival probability declines when the bubble increases in size (see Figure (1)), and is not significant for Russia and South Africa. This indicates that in these cases, the regime-switching periodically collapsing bubble is of the types examined in the earlier literature, where the survival probability does not depend on the size of the bubble. The estimated parameters for the restricted equations are also of interest. The likelihood statistic is employed in the next section to formally test the two hypotheses they embody, but it is useful to comment here on the heuristic assessment of their effect on the estimated value of the coefficients.

Comparison of the estimated model parameters values under RE with those in the unrestricted model indicates the nature changes produced by imposing that restriction. For Brazil, the changes in most coefficients are significant, which suggests the RE hypothesis is not supported. For Russia several coefficient estimates are not significantly affected, but β_{q0}, σ_S and β_{S0} roughly double in size, and β_{C1} which becomes non-significant. For India none of the significant parameters of the unrestricted equation is affected by imposing RE. The same is true in China, except for β_{p2}, which was negative and almost insignificant, and becomes positive and significant.[9] For South Africa, the parameter estimates which are significant in the unrestricted equation change significantly once RE is imposed. The sign of β_{q0} is reversed, but the magnitude is maintained. The estimates of β_{C1} and σ_S are reduced roughly to half, that of β_{p2} is increased by 25%, and the formerly significant β_{S00} is forced to zero.

The other restricted equation, for the two linear regimes hypothesis of Van Norden (1996), is useful to test the relevance of the non-linear specification proposed by Maldonado et al. (2012) as a generalization of the earlier specification. The two parameters whose value is restricted to be null in this specification are in the columns 12 and 13 of Tables 1 and Table 2. Recalling we have chosen Model 2 as the reference model for all countries, we restrict our attention to its parameter estimation, in Table 2.

Comparison of the value and standard deviation of the other parameters in the restricted equation with the unrestricted one reveals that they were already insignificant for India and China, so that imposing the restriction does not entail a large change in the likelihood. For the

[9] This implies that the possibility that the two linear regimes hypothesis was satisfied, which was raised above in connection with the unrestricted estimation, is not admissible once RE is imposed (see more details in the next section).

other countries, however, the coefficients of the non-linear model of Maldonado et al (2012) are significant, and it is able to achieve a larger likelihood and therefore better describe the data.

3.3 SPEFICICATION TESTS

Following Van Norden (1996), we used the White (1987) method to explore the misspecification of the error terms testing for autocorrelation, heteroscedasticity, and for Markovian state dependence in the two regimes. We also test the joint hypothesis that the errors satisfy all these hypotheses. The results of these tests are summarized in Table 4.

Table 4 - Specification tests for Model 2 [10]
(values displayed are p-values for the unrestricted model)

Test of H0: absence of effect	Note	Degrees of freedom	BRASIL	RUSSIA	INDIA	CHINA	SOUTH AFRICA
AR(1) S	(+)	1	5,5%	91%	17%	8,3%	63%
AR(1) C		1	23%	13%	**0,7%**	**0,0%**	22%
ARCH(1) S	(#)	1	51%	51%	7,7%	88,6%	78%
ARCH(1) C		1	5,2%	2,5%	19%	**0,5%**	38%
Markovian Efect	(&)	1	**0%**	7,5%	22%	2,8%	**0,9%**
Joint Test		5	**0,1%**	8,3%	1,7%	**0,0%**	11%

Notes:

(+) AR(1) C and AR(1) S are tests for the presence first order serial correlation of the residuals under the collapse and survival regimes respectively.
(#) ARCH(1) C and ARCH(1) S are tests for the presence of first order heteroscedasticity of the residuals under the collapse and survival regimes respectively.
(&) The Markovian effect tests.

Following Van Norden (1996) and Hamilton (1990) we chose a critical level of 1% for inference regarding the specification tests because our sample size is small (less than 200 observations). Therefore, for each cell in the table the null hypothesis is rejected whenever the corresponding p-value is smaller than 1% and, conversely, is no rejected whenever it is larger than that value. For example, the absence of AR(1) in the survival state for Brazil is not rejected by the data, since the corresponding value is 5,5%.

We first discuss each of the possible identification problems separately, examining each row of Table 4 in turn. The absence of first order autocorrelation of the residuals in the survival regime cannot be rejected (therefore, it is absent) for all countries. The absence of first order

[10] The values in Table 4 for Brazil are different from the ones for Model IIB in Maldonado et al (2012) because the sample here includes 28 additional data points and is, therefore, larger. With the smaller sample the results obtained before are reproduced, which verifies that the changes are due only to the additional data.

autocorrelation in the collapse regime cannot be rejected (therefore, it is absent) for Brazil, Russia and South Africa, and it is rejected (therefore, it is present) for India and China.

The absence of first order heteroscedasticity of the residuals in the survival regime cannot be rejected (therefore, it is absent) for all countries. In the collapse regime the absence of first order heteroscedasticity cannot be rejected (therefore it is absent) for all countries except China, where it is rejected (and, therefore, it is present).

The absence of Markovian state dependence is rejected (therefore, it is present) for Brazil and South Africa. For Russia, India and China it is absent.

The joint test of all 5 hypothesis indicate that from the point of view of the specification tests of the residuals, the model is rejected for Brazil and China and accepted for Russia, India and South Africa. Looking at the individual tests, it can be seen that the rejection of the model for Brazil is due to the strong rejection of the Markovian state independence, and the rejection for China is due to the presence of serial correlation and heteroscedasticity in the residuals in the collapse regime. It should be noted that the rejection of absence of serial correlation in the collapse regime for India was not strong enough to lead to a rejection of the model in the joint test. In a similar manner, the rejection of the Markovian state independence for South Africa is not strong enough to lead to a rejection of the model in the joint hypothesis test.

3.4 TESTS OF RESTRICTIONS

The Likelihood Ratio test was used to assess whether the two hypotheses raised before, the Rational Expectations (RE) hypothesis and Two Linear Regimes (TLR) hypothesis are accepted by the data on this sample. We adopted a significance level of 5% for rejection of the null i.e. that the unrestricted equation is more suitable than that with the restrictions imposed by the two hypothesis for the available data. Table 5 displays the results of the tests.

The RE hypothesis is rejected for Brazil, China and South Africa, and is not rejected for Russia and India. The TLR hypothesis is rejected for Brazil, Russia, and South Africa. Therefore, for Brazil and South Africa expectations in the futures market for the exchange rate are not formed in a rational manner, and our general non-linear model fits the data better than the more restricted 2 linear regimes model. For India we cannot reject rational expectations in the futures market for the exchange rate, but a two-linear regimes model, which is simpler than our non-linear model, may be sufficient to explain the data. Analogous interpretations can be given for Russia and China.

Table 5 - Likelihood ratio test for RE and TLR hypotheses

Country	Hypothesis	Restriction rank (n)	Log-Likelihood		Test statistic χ^2	P-value
			Unrestricted	Restricted		
BRAZIL	RE	5	168,95	109,44	119,02	**0,000%**
	TLR	2	168,95	157,58	22,74	**0,001%**
RUSSIA	RE	5	-117,64	-121,13	6,97	22,292%
	TLR	2	-117,64	-124,22	13,16	**0,139%**
INDIA	RE	5	-202,49	-204,69	4,40	49,391%
	TLR	2	-202,49	-202,79	0,60	74,215%
CHINA	RE	5	404,87	393,33	23,09	**0,032%**
	TLR	2	404,87	402,82	4,10	12,870%
SOUTH AFRICA	RE	5	-67,14	-73,27	12,27	**3,123%**
	TLR	2	-67,14	-72,74	11,21	**0,368%**

4 BUBBLE DYNAMICS

In this section we explore further the results of the estimation of the model of equations (9), (10a) and (10b), for the five BRICS countries. Figures 2 to 6 display, the time series for S_t, S_t^*, and $q(b_t)$. In the figures the actual and fundamental exchange rates (i.e. the price of 1 US$ in local currency) are labeled, respectively, "Spot" and "Fund2" (to remind the reader that our reference model for all countries is Model 2). The estimated probability of collapse of the bubble is labeled "ProbC". The exchange rates are read of the left hand axis, and the probability is read of the right hand axis. The bubble size at any point in time is the vertical difference between the actual and the fundamental exchange rate.

For Brazil (Figure 2) there is a large positive bubble in the period from March 1999 to July 2007, that reaches its peak around 2002. As been pointed out in Maldonado et al. (2012), the rapid inflation of the bubble can attributed to two major phenomena: the contagion of the Russian foreign crisis of 1998, and the domestic confidence crisis associated to the presidential election held at the end of 2002 which increased the uncertainty about future economic policy and the "terminal value" of domestic bonds. As the bubble grew, so did the probability of its collapse, which went from about 65% in 1999 to 100% in September 2002. It remained high for about 6 months until it became clear that the effects of the Russian shock in the international markets were being absorbed, and that the new leftist leaning government which was elected in the end of 2002 would not implement heterodox economic policy. During the following 5 years the bubble size slowly decreased, until it became negative in September 2007. The collapse probability also declined and stabilized at around 50%. In 2008 there was a positive bubble of short duration associated to the subprime crisis in the USA, and after that it became mostly negative until it became virtually null at the end of the sample.

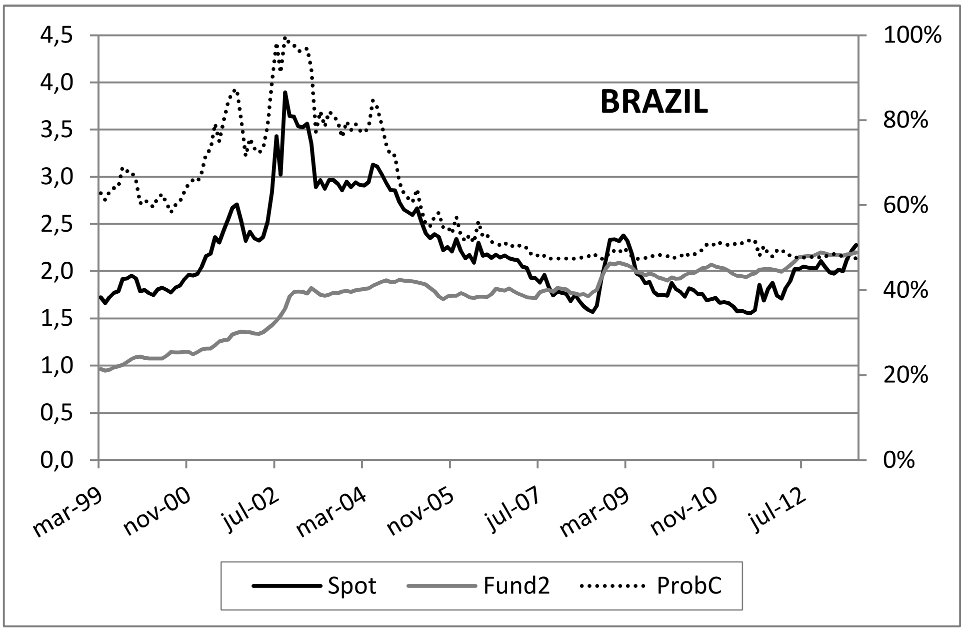

Figure 2 - Spot and Fundamental exchange rate, and collapse probability for Brazil

Prior to discussing Figure 3, for Russia, it is useful to recall the events that happened before the sample period considered here for that country. On August 17, 1998 the Russian government devalued the ruble, defaulted on domestic debt, and declared a moratorium on the payment to foreign creditors, confirming fears that had led to the collapse of the stock, bond, and currency markets in the previous week. Throughout the crisis there was a loss of international reserves of U$ 27 billion, and the domestic inflation rate increased, reaching 80% per year. However, these developments are not captured by our model because, as can be recalled from the section on the data sources for the estimation, our sample for Russia starts in 2005:6 because before that date the exchange rate was fixed or managed. Since 2005, it floats.[11]

As can be seen in Figure 3, since 2005 there have been, roughly speaking, 4 bubbles: two positive (2005 to July 2007, and September 2008 to January 2011), and two negative (August 2007 to August 2008, and February 2022 until the end of the sample). The collapse probability increased (as it should) whenever the bubble is positive, reaching 100% just before these bubbles started collapsing. As in the Brazil bubble, the collapse of the bubble that peaked in

[11] In February de 2005, the Russian central bank changed the reference of the exchange rate from the US$ to a basket of currencies composed of 10% Euros e 50% US$. The basket is now composed 45% Euros and 55% US$.

September 2008 was slow, since it only ended in December 2010, and its size toward the end of the sample is rather small.

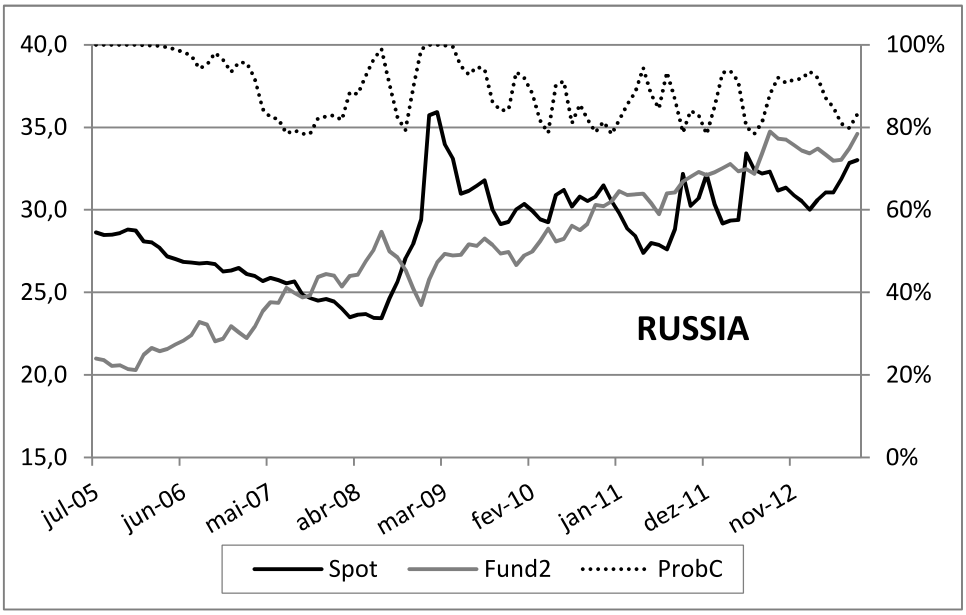

Figure 3 - Spot and Fundamental exchange rate, and collapse probability for Russia

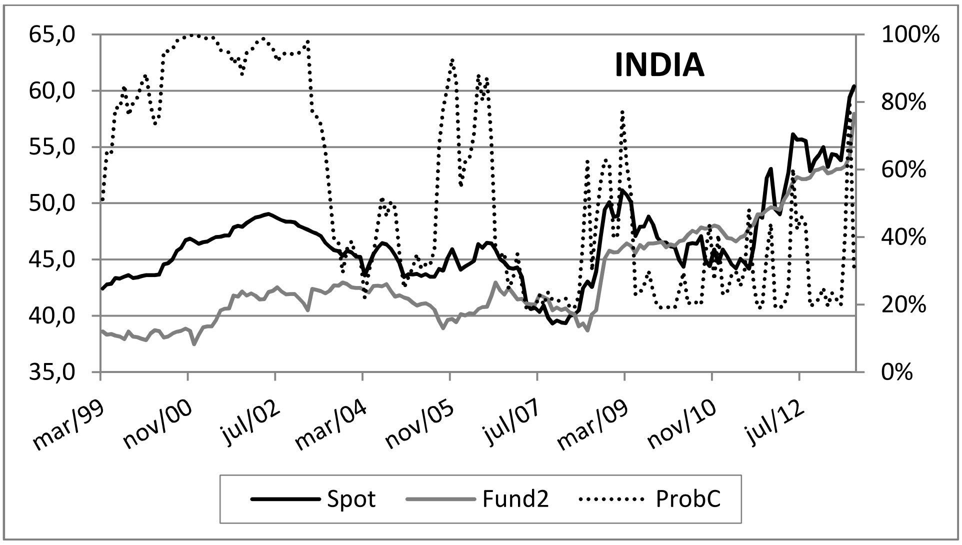

Figure 4 - Spot and Fundamental exchange rate, and collapse probability for India

Figure 4 displays the graphs for India. It shows a positive exchange rate bubble since the beginning of the sample until the beginning of 2007. While not large in relative terms, it is clearly present, and is larger at the beginning and end of that period, and smaller in the middle. The collapse probability also clearly indicates its presence, reaching close to 100% when it is larger and declining to 30% whenever it is smaller. In 2007 and after, the spot exchange rate remains very close to the fundamental rate and the collapse probability oscillates quite a bit, but remains most of the time around 30%, consistently with the absence of a bubble. This behavior of the Indian exchange rate towards the end of the sample may be due to the Liquidity Adjustment Facility (LAF) operated by the Indian Central Bank, to manage the inflow of foreign capital and avoid overvaluation of the currency whenever the international liquidity is higher.

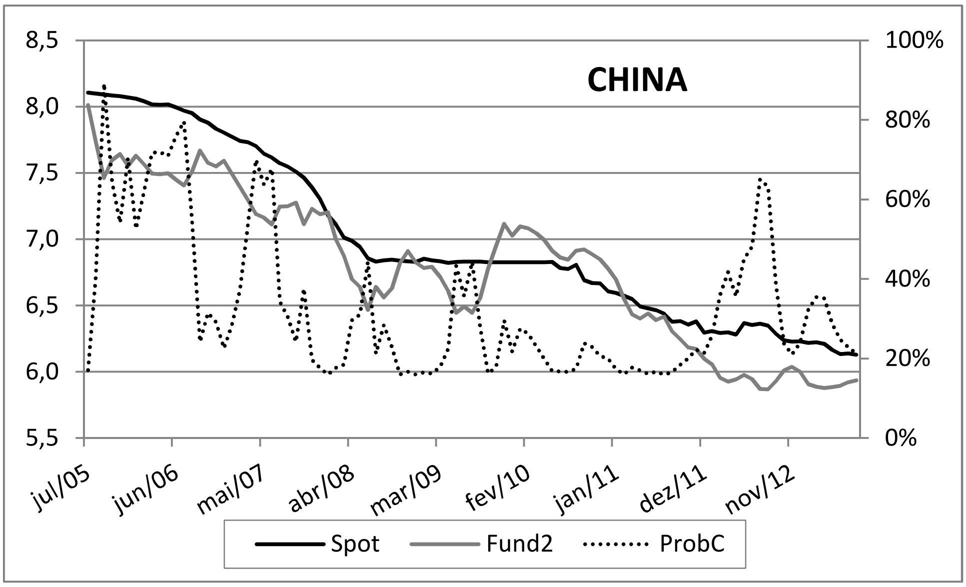

Figure 5 - Spot and Fundamental exchange rate, and collapse probability for China

Figure 5 shows the graphs for China. In this case, as for Russia, it is important to recall the events preceding the sample period. For 10 years before July 2005 the exchange rate was fixed, and for this reason our sample period starts at that date, when the Popular Bank of China (POBC) appreciated the exchange rate by 2,1%, and announced that it was abandoning the US$ as the exclusive reference and adopting in its place a basket of currencies. A similar policy was adopted by Russia at about the same time, which led us to start the sample in 2005 in that case also, as was discussed before.

Figure 5 shows that from 2005 e 2008, the currency appreciated 21% with respect to the US$, but this was the result of the appreciation of the fundamental exchange rate, and therefore it is a structural phenomenon. The spot rate followed this appreciation, since no sizeable bubble arose. The collapse probability stays below 40% for most of the time, except for brief periods in 2006 and 2012, consistently with the absence of significant bubbles.

Figure 6, for South Africa, shows that in the sample period there are two well defined positive bubbles in the exchange rate of South Africa (from march 1999 until 2003, and in 2009). The rest of the time the spot exchange rate remains very close to the fundamental rate, and the collapse probability is low, around 40%. During the two bubbles it increased, but only briefly.

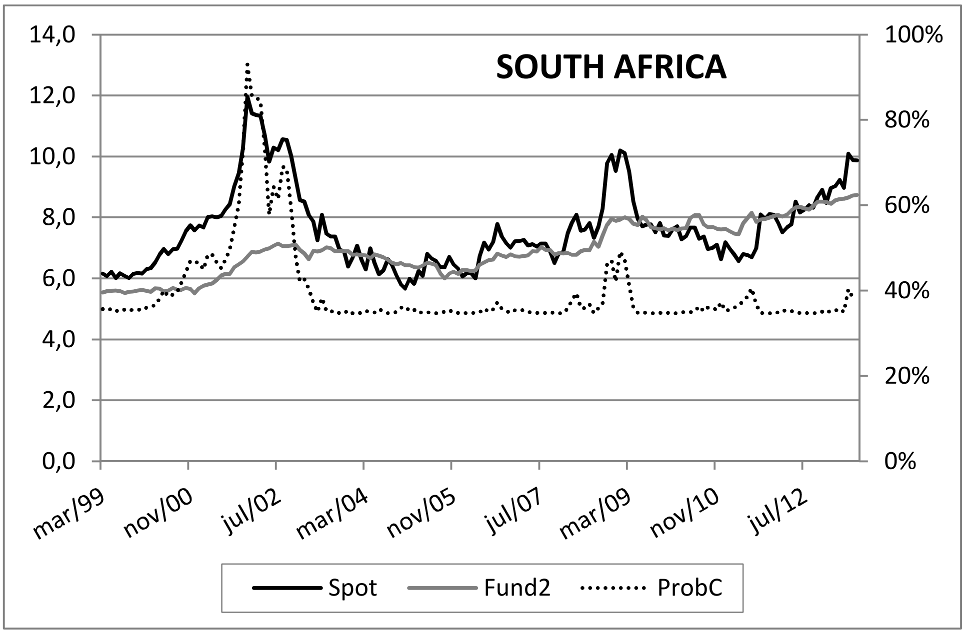

Figure 6 - Spot and Fundamental exchange rate, and collapse probability for South Africa

5 COMPARISON OF THE BUBBLES

The estimate of k discussed in section 3 allows the calculation of the path of the fundamental exchange rate (S_t^*), and the bubble can be estimated by recalling the definition $b_t = S_t - S_t^*$. To be able use the same metric to compare the size of the bubbles of different countries, a relative bubble size statistic (b_{rel}) was constructed by normalizing the bubble size with respect to the contemporaneous fundamental exchange rate:

$$b_{rel_t} = \left(\frac{S_t - S_t^*}{S_t^*}\right). \tag{16}$$

This calculation was performed for both models, but only that for the reference model (Model 2) is discussed here. The graphs of these relative bubbles are shown in Figures 7, 8 and 9 which suggest the presence of persistence in all the series, a feature that will be formally verified by testing for the existence of a unit root in these series, in the next section.

As can be seen in Figure 7, the bubbles for Brazil and Russia have similar paths, with positive peaks in 2005 and in early 2009, just after the eclosion of the subprime crisis in September 2008. The bubble for Brazil becomes negative in June 2009, but for Russia that only happens 18 months later, in December 2010. After they change sign, both bubbles remain small and roughly negative until the end of the sample.

The bubble for South Africa, in Figure 8, follows the same pattern of the Brazil and Russia bubbles after the subprime crisis, but not before that, since no significant positive bubble is present in 2005 and 2006.

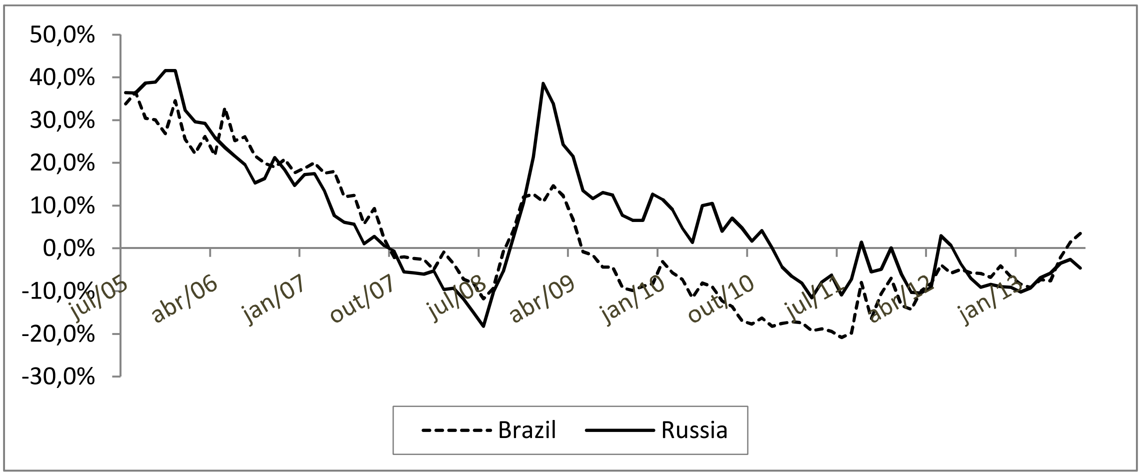

Figure 7 – Relative bubbles for Brazil and Russia – Model 2

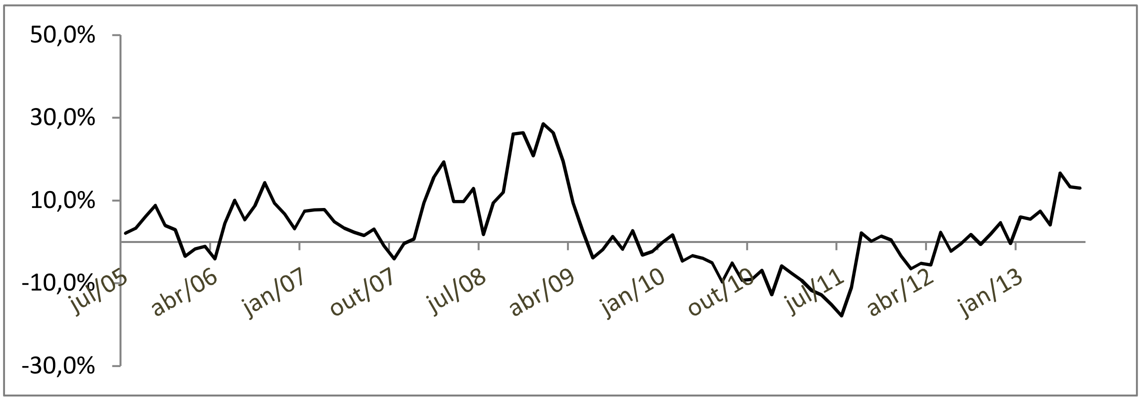

Figure 8 – Relative bubbles for South Africa – Model 2

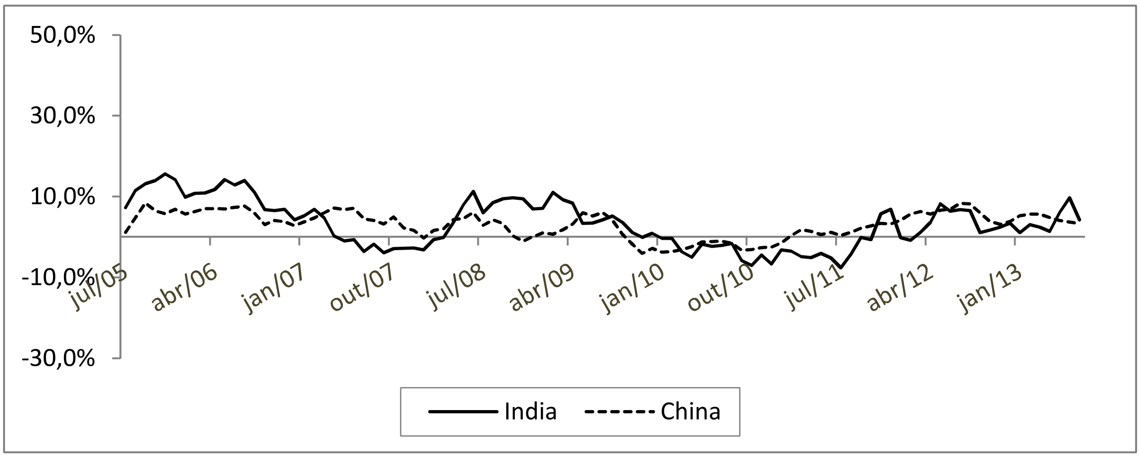

Figure 9 – Relative bubbles for India, China – Model 2

For China and India the bubbles are of markedly smaller magnitude than those for the other countries, and follow a cyclical pattern, alternating positive and negative bubbles throughout the sample, but they also display some correlation: they are positive in the beginning and end of the sample period, and are positive in the period around the subprime crisis (2008 and 2009), in the middle of the sample period. In summary, they have roughly the same dynamics of the Russia and Brazil bubbles, but with a smaller magnitude.

It is interesting to note the behavior of the bubbles during the 2008 USA sub-prime crisis. From the middle of 2007 until September 2008 there was an expansion of the bubbles, with the devaluation of the BRICS currencies with respect to the US currency. After the beginning of 2009 there is a collapse of the bubbles and the corresponding appreciation of the BRICS currencies.

A more general relation between these bubbles is also explored in the next sections. Of particular interest is the possibility that they are related in the long term, either due to the existence of common factors affecting all of them, or due to mechanisms whereby a bubble in one country's exchange rate induces the appearance of bubbles in the other BRICS countries.

5.1 UNIT ROOT TESTS

In this section we test the relative bubble series we obtained for the BRICS countries for the presence of a unit root. If the hypothesis that they are I(1) cannot be rejected, it is clear that these series contain the bubble component of the original exchange rate series, and that a cointegration test can be used to uncover any long term relation that may exist between them.

To test for a unit root we used the procedure recommended by Doldado, Jenkinson and Sosvilla-Rivero (1990) to consider systematically the possibility of the presence of an intercept and a trend in the test equation:

$$\Delta y_t = a_0 + a_1 y_{t-1} + a_2 t + \sum \beta_i \Delta y_{t-i+1} + \epsilon_t \tag{17}$$

The Augmented Dickey-Fuller (ADF) test statistics with 5% significance level was used, and the test results for out reference model (Model 2) are shown in Table 6. They show that the hypothesis that a unit root is present in the series cannot be rejected for all countries, except Russia. The deterministic terms for the intercept and trend were not significant in any case.

The Philips-Perron (PP) complements the ADF test, making a non-parametric correction to its test statistic, and is more robust with respect to unspecified autocorrelation and heteroscedasticity in the disturbance process of the test equation. For this test we did not include the intercept or trend because we had indications from the ADF test that those terms were not required. Again, the presence of a unit root cannot be rejected at the 5% level for all countries, except Russia. Therefore, the PP test confirms the results of the ADF test.

Although the presence of a unit root was rejected at the 5% significance level for Russia, we assumed that the relative bubble for that country is also integrated, because the unit root tests at 1% significance level cannot reject it. With this assumption, all relative bubble series can be taken to be I(1), and a cointegration test can be performed.

Table 6 – Unit root tests for the relative bubble of Model 2

Country	Test	Deterministic terms		Coefficient	Standard error	t-statistic	P-value
BRAZIL	ADF	With trend and intercept	a_1	-0,06	0,034	-1,881	6,2%
			a_0	0,05	0,035	1,516	13,1%
			a_2	0,00	0,000	-1,568	11,9%
		With intercept only	a_1	-0,02	0,014	-1,089	27,8%
			a_0	0,00	0,007	-0,085	93,2%
		Without trend or intercept	a_1	-0,02	0,011	-1,484	14,0%
	PP	Without trend or intercept	a_1	-0,02	0,011	-1,572	11,8%
RUSSIA	ADF	With trend and intercept	a_1	-0,11	0,041	-2,774	**0,7%**
			a_0	0,04	0,030	1,434	15,5%
			a_2	0,00	0,000	-1,417	16,0%
		With intercept only	a_1	-0,07	0,030	-2,457	**1,6%**
			a_0	0,00	0,005	0,217	82,9%
		Without trend or intercept	a_1	-0,07	0,028	-2,575	**1,2%**
	PP	Without trend or intercept	a_1	-0,07	0,028	-2,339	**2,1%**
INDIA	ADF	With trend and intercept	a_1	-0,11	0,035	-3,167	**0,2%**
			a_0	0,02	0,007	2,763	**0,6%**
			a_2	0,00	0,000	-2,472	**1,4%**
		With intercept only	a_1	-0,04	0,023	-1,951	5,3%
			a_0	0,00	0,002	1,267	20,7%
		Without trend or intercept	a_1	-0,02	0,016	-1,494	13,7%
	PP	Without trend or intercept	a_1	-0,02	0,016	-1,494	13,7%
CHINA	ADF	With trend and intercept	a_1	-0,11	0,043	-2,618	**1,0%**
			a_0	0,01	0,007	0,762	44,8%
			a_2	0,00	0,000	-0,322	74,8%
		With intercept only	a_1	-0,11	0,042	-2,629	**1,0%**
			a_0	0,00	0,002	1,686	9,5%
		Without trend or intercept	a_1	-0,06	0,031	-2,004	4,8%
	PP	Without trend or intercept	a_1	-0,05	0,032	-1,488	14,0%
SOUTH AFRICA	ADF	With trend and intercept	a_1	-0,06	0,027	-2,323	**2,1%**
			a_0	0,02	0,011	1,512	13,3%
			a_2	0,00	0,000	-1,198	23,3%
		With intercept only	a_1	-0,05	0,023	-1,990	**4,8%**
			a_0	0,00	0,005	1,005	31,6%
		Without trend or intercept	a_1	-0,03	0,020	-1,717	8,8%
	PP	Without trend or intercept	a_1	-0,03	0,020	-1,717	8,8%

Note: ADF critical values:

Significance level	With trend and intercept	With intercept only	Without trend or intercept
1%	-4.013	-3.469	-2.579
5%	-3.436	-2.878	-1.943
10%	-3.142	-2.576	-1.615

5.2 COINTEGRATION TEST AND VEC MODEL

As pointed out before, we are interested in the co-movement of the speculative rational bubbles in the exchange rate of the BRICS countries to assess if this can shed some light on the nature of the foreign exchange market crisis of these countries. For this we perform the Johansen and Juselius (1980) cointegration test, as discussed below.

We did not include a trend in the test equation, in accordance with the results of the unit root tests, which did not indicate its presence in the series. For the number of lags to be considered in the test, several criteria available in the literature were considered, and their results are shown in Table 7. The Sims (1980) criteria are based on the maximization of the likelihood statistic of the VARs with different lag lengths. The maximum lag length considered was equal to 6, due to the sample size and the remaining degrees of freedom of the VAR. The results of the Sims criteria are shown in the second column, and indicate a lag order of 6. The results of the Likelihood Ratio (LR) and the Final Forecasting Error (FFE) criteria are displayed in the third and fourth column, and indicate lag order equal to 4 and 3, respectively. The Akaike information criteria (AIC) statistic in column five also indicates a lag order of 3. The Swartz (SC) and the Hannan-Quinn (HQ) statistics are shown in columns six and seven respectively, and indicate, as is usually the case, the adoption of a smaller lag length, in this case equal to 1. In view of all these results, we have chosen a lag length equal to 3, which was indicated by the largest number of criteria, and is intermediary between the maximum and minimum lag lengths recommended by the several criteria.

Table 7 – Lag order criteria for the cointegration test – Model 2

Lag	Log L	LR	FFE	AIC	SC	HQ
0	651,7442	NA	4,62e-13	-14,21416	-14,0762	-14,1585
1	1019,547	687,1044	2,47e-16	-21,74829	**-20,92053**	**-21,41434**
2	1050,226	53,94049	2,19e-16	-21,8731	-20,35554	-21,26086
3	1079,101	47,59611	**2,04e-16**	**-21,95826**	-19,75091	-21,06773
4	1094,755	**24,08384**	2,56e-16	-21,75286	-18,85572	-20,58405
5	1114,198	27,77492	3,01e-16	-21,63072	-18,04378	-20,18361
6	**1135,261**	27,77537	3,49e-16	-21,54419	-17,26745	-19,81879

Notes: LogL: Sims criteria - maximum likelihood of VARs of different lag lengths (max. = 6)
LR: likelihood ratio test at significance level of 5% FFE: final forecasting error
AIC: Akaike SC: Schwarz HQ: Hannan-Quinn

To test for cointegration of the relative bubble size we used the Johansen test, and found that the trace and eigenvalue criteria (both) indicate the existence of only one cointegration vector, as shown in Table 8.

Table 8 - Johansen tests for cointegration rank - Model 2

Number of cointegration vectors	Trace criterion			Eigenvector criterion			
	Statistic	Critical value	P-value	Eigenvector	Statistic	Critical value	P-value
None*	91,18	76,97	0,3%	0,385	45,15	34,81	0,2%
At most 1	46,02	54,08	21,4%	0,188	19,34	28,59	46,4%

To assess the nature of error correction process we following unrestricted VEC model:

$$\Delta X_t = \mu + \alpha\beta' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \epsilon_t \quad (18)$$

where $X_t = (b_1, \ldots, b_5)$ and $j = 1, .., 5$ correspond respectively to the Brazil, Russia, India, China, and South Africa. The 5-dimensional vector β is the cointegration relation and the α is the 5-dimensional weighting vector which indicates the how the deviation from the long term relation between the variables affects each variable in X_t, i.e. how fast the errors are corrected. Larger values of α_j imply larger corrections in b_j in response to deviations of the long-term relation between the relative bubbles. The matrixes Γ_i , $i = 1, \ldots, k$ contain the weights of the autoregressive component for relative bubble changes, indexed by the number of lags i in each component ΔX_{t-i}. The maximum number of lags was set to 3, as discussed above ($k = 3$). The error component of the VEC, (ϵ_t) is assumed to be a multivariate Gaussian distribution which zero mean and a constant diagonal variance-covariance matrix. Note that while the error correction mechanism contained in the description of the stochastic process for the exchange rate is non-linear, equation (18) specifies that the error correction mechanism for the deviation of the BRICS exchange rate relative bubbles from their long term relation is linear.

The maximum likelihood procedure proposed by Johansen (1988 and 1991) was used to estimate the model in equation (18). The estimates of the cointegration and weighting vector, and the corresponding standard errors, are shown in Table 9. The cointegration vector was normalized to make the coefficient for Brazil equal to 1.

Table 9 - Estimates of the Johansen tests for cointegration rank - Model 2

	BRASIL	RUSSIA	INDIA	CHINA	SOUTH AFRICA	Constant
Cointegration coefficients	1,00000	-2,93645	7,26647	-6,21463	-2,36911	0,144735
Standard Error	-	(0,33086)	(1,24215)	(1,43992)	(0,48981)	(0,04041)
Adjustment coefficients	0,088891	0,084488	0,009770	-0,011108	0,080972	-
Standard Error	(0,017080)	(0,020490)	(0,013750)	(0,007020)	(0,026090)	-

Isolating the Brazil bubble in the cointegration equation, and rounding the coefficients, the following equation can be obtained and used to interpret the co-movements of the bubbles:

$$b_{Brazil} = 2.94 b_{Russia} - 7.27 b_{India} + 6.21 b_{China} + 2.37 b_{S.Africa} - 0.15 + \epsilon_t \quad (19)$$

Equation (19) shows that, on average and in the long term, the bubbles for Brazil, Russia, China and South Africa move in the same direction, and that the India bubble moves in the opposite direction to all of them. The relative size of the coefficients in (19) indicates that in the long-term relation between these relative bubbles, the importance of the one for Brazil is

6 to 7 times larger than that for India and China, and about 2.5 to 3 times larger than the relative bubble for Russia and South Africa. This suggests that for a perturbation of the long term equilibrium arising from a change in the overall environment affecting the BRICS exchange rates, like a shock in the foreign capital flows that accompanied the 1998 sub-prime crisis, the fluctuations in the exchange rate of Brazil are more important and elicit a more violent correction of all the exchange rates of the group than that of any of the other countries.

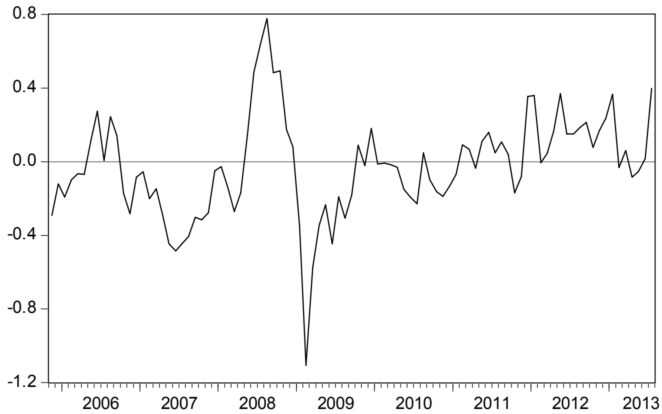

Figure 10 – Residual of the cointegration between the BRICS bubbles, for Model 2

Figure 10 displays the within sample estimated residual of the cointegration relation ($\hat{\epsilon}$), indicating that during the 2008-2009 crisis there occurred substantial deviations from the long-term equilibrium of the relative bubbles, and that the pattern of these deviations is consistent with a speculative movement whereby there is initially an overall overreaction (a positive deviation) with respect to the long term equilibrium, followed by an overcorrection (a negative deviation), and then by a "soft landing", in a pendulum-like fashion.

The estimates of the weighting vector displayed in Table 9 can also be explored to understand the economic relation between these relative bubbles. They show that the size of relative bubble of all currencies, except China, increases in period $t+1$, in response to a shock that increases the residual of the cointegration relation in period t. This is so because the coefficients in α are positive for all countries, except China. The negative coefficient for China is quite interesting, indicating that its relative bubble moves in the opposite direction to all others. This could be a consequence of a reallocation of capital flows within the BRICS group, Moreover, the change in the absolute size of relative bubble for Brazil, Russia and South Africa is of the order of 8% of the deviation in the cointegration relation, while that for India and China is of the order of 1%. This was to be expected, since it has been pointed out that the relative size of the bubbles for these two countries is of a significantly smaller magnitude than that for the other countries.

6 CONCLUSION

This paper examined the behavior of the exchange rates of the BRICS countries group (Brazil, Russia, India, China and South Africa) against the US dollar adopting an asset approach to exchange rate determination, and taking into consideration that the capital movements may lead to the occurrence of speculative rational bubbles of the periodically collapsing variety.

The strategy was as follows. We first estimate a model for each individual country and from it obtain an estimate of the bubble, defined as the difference between the actual and the fundamental exchange rate. After estimating and validating these individual models by performing appropriate specification and hypothesis testing, we conclude that the model adopted is an adequate representation of the dynamics of all those series. The heuristic discussion of the bubble series also lends support to the model. To test for interaction between the bubbles, we define *relative* bubbles, dividing the bubble obtained for each country by the corresponding fundamental exchange rate, and test them for unit roots. We find they are present, and therefore perform a cointegration test, and conclude that exactly one stochastic trend is present. This allows us to explore the nature of the interaction between the relative bubbles of different countries in the BRICS group with the help of an unrestricted VEC. Below we summarize the details of each of these steps.

For the individual countries bubble model we adopt is the one proposed in Maldonado et al (2012), which is an extension and generalization of the one proposed by Van Norden (1996). For the fundamental rate we consider both a simple PPP model (Model 1), and a modified PPP relation which takes into account interest rate differentials (Model 2), both starting from a reference value which is endogenously determined. Only China displays a fundamental rate with a negative time trend (appreciation), while all the others display a positive time trend (depreciation).

At each point in time the probability of collapse of the bubble is a non-linear logistic function of the absolute size of the bubble and is, therefore, also endogenous. The expected next period bubble size if the future regime is collapse is a linear function of the current bubble size, whose parameters are also endogenous. To obtain a workable empirical specification, we re-state the model in terms of the innovation, which is the difference between the exchange rate and its expected value in the model. This yields an empirical model in the form of a non-linear regime-switching regression for the innovation. We use the difference between the value of the forward contract of 1 month maturity and the actual value of the exchange rate as a *proxy* for the innovation. This allows us to estimate the model using a maximum likelihood procedure, considering the two alternatives for the fundamental value (Models 1 and 2 discussed above). The data is monthly, obtained from public sources, and the sample for all countries ends in 2013:06. For Brazil, India and South Africa it starts in 1999:03, while for China and Russia it only starts in 2005:06 because it was fixed before then.

The results of the estimation broadly support the periodically collapsing bubble model for this group of countries, and indicate that of the two alternatives for the fundamental rate, Model 2 is found to lead to a better fit of the model, and is adopted as the reference model for the remaining of the paper. This may be interpreted as evidence that models for the fundamental rate that include asset markets (via the interest rates) are more adequate to model speculative bubbles than the ones which do not include them explicitly. It passes the specification tests but, as expected, the quality of fit varies between the several countries. In some cases a simpler version of the model, where some coefficients are null, is as adequate as the full model. The hypothesis of rational expectations in the market for the forward exchange rate, which is used as a *proxy* of the expected exchange rate, is tested and accepted for 3 of the 5 countries at the 5% confidence level. The hypothesis of two linear regimes (rather than the non-linear regimes we use) is also tested and rejected for 3 of the 5 countries, also at the 5% level. To validate the model for each country we also discuss heuristically the dynamics of the absolute bubble, comparing the actual data with the main events that affect the exchange rate in each country.

To study the relation between the bubbles estimated as described above, we define time series of *relative* bubbles for each country, dividing the estimated bubble by the contemporaneous fundamental value. We test them for unit roots, and find that they are integrated. When submitted to Johansen's cointegration test, this set of relative bubbles is found to be cointegrated, and therefore that they have a common stochastic trend. This is an indication that the speculative component of these exchange rate series displays a long term relation. This shows the high degree of connection between the capital flows to each of these countries, and that speculation in the exchange rate market of any of them has repercussions in the exchange rate of other countries of the group. This may be an indication of the presence of the phenomenon of contagion between exchange rate markets of these countries. The instances where this is thought to have occurred are pointed out in the discussion of the corresponding time series. The similarities in the path of these relative bubbles are also pointed out in the heuristic discussion. All countries display a positive bubble in the 2008 subprime international crisis, but it collapses rather quickly (less than one year).

Finally, the nature of the interaction between the relative bubbles of different countries in the BRICS group is explored with the help of an unrestricted VEC estimated by the Johansen procedure. The coefficients of the cointegration relation allows us to show that on average and in the long term, the bubbles for Brazil, Russia, China and South Africa move in the same direction, and that the India bubble moves in the opposite direction to all of them. We find that importance in the long-term relation between these relative bubbles of Brazil is 6 to 7 times larger than that for India and China, and about 2.5 to 3 times larger than that for Russia and South Africa. The speed of adjustment of size of the relative bubbles to a shock that produces a deviation from the long term relation between them is also diverse: the change in the absolute size of relative bubble for Brazil, Russia and South Africa is of the order of 8% of the deviation in the cointegration relation, while that for India and China is of the order of only 1%. These different rates can be seen as a measure of the degree of integration of the country to the BRICS block, if it is seen as a fund within which financial capital is reallocated. The countries which are more integrated are the ones whose exchange rate adjusts faster while the smaller reaction coefficient of the others may be an indication of less integration to that block.

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