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Keywords

Determinacy; Generalized Taylor Principle; Sticky prices; Price indexation; Habit formation; Steady-state inflation

JEL Classification

E31, E52, E58

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Price Indexation, Habit Formation, and the Generalized Taylor Principle*

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Abstract

We prove that the Generalized Taylor Principle, under which the nominal interest rate reacts more than one-for-one to inflation in the long run, is a necessary and (under some extra mild restrictions on parameters) sufficient condition for determinacy in a sticky price model with positive steady-state inflation, interest rate smoothing in monetary policy, partial dynamic price indexation, and habit formation in consumption.

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1 Introduction

One of the most important guiding principles for practical monetary policy is the Generalized Taylor Principle, which asserts that in order to ensure price stability, the nominal interest rate needs to respond more than one-for-one to inflation in the long run. Indeed, Bullard and Mitra (2002), Woodford (2003), and Lubik and Marzo (2007) show that the Generalized Taylor Principle is a necessary and sufficient condition for a unique stable equilibrium in simple sticky price models when the central bank follows a Taylor rule, that is, a rule where the nominal interest rate responds to both inflation and output.

While these results are highly influential, most sticky price models that are taken to the data now routinely feature various propagation mechanisms such as habit formation and price indexation, following Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). To the best of our knowledge, the determinacy properties of such models have been studied only numerically.

We contribute to the literature by showing analytically that the Generalized Taylor Principle is a necessary and (under some extra mild restrictions) sufficient condition for determinacy in a more general environment than considered by previous studies. In particular, we consider a sticky price model with non-zero steady-state inflation, dynamic partial price indexation, and habit formation in consumption and in which the central bank follows a Taylor rule where the nominal interest rate is determined by its lag and partially responds to both (current) inflation and output.[1] As a by-product of our analysis, we also characterize analytically the full solution of the model when a unique equilibrium exists.[2]

We find that habit formation in consumption and interest rate smoothing in the Taylor rule does not affect the determinacy condition while dynamic partial price indexation requires monetary policy to respond to inflation and/or output more strongly to ensure determinacy. This is because dynamic partial price indexation decreases the long-run trade-off between inflation and output in the model while habit formation does not affect the long-run trade-off at all. Moreover, interest rate smoothing does not affect the determinacy condition as it does

[1]To preserve analytical tractability, we do not allow for sticky wages or investment in the model.

[2]Our paper fits generally in the literature that analyzes determinacy properties of extended versions of the prototypical sticky price model. Carlstrom, Fuerst, and Ghironi (2006) show analytically that the Taylor principle is a necessary and sufficient condition for determinacy in a two-sector model where the nominal interest rate responds only to inflation while Carlstrom and Fuerst (2004) show analytically that the Taylor principle is a necessary condition for determinacy in a one-sector model with investment where the nominal interest rate responds only to (current) inflation. Moreover, Benhabib and Eusepi (2005) show numerically that nominal interest rate responding to both inflation and output is quite effective in ensuring determinacy in models with both capital and bonds. Finally, Sveen and Weinke (2005) and (2007) show numerically that while the Taylor principle is not sufficient for determinacy in a sticky price model with investment when capital is firm-specific and where the nominal interest rate responds only to inflation, if the nominal interest rate also responds to output, determinacy is much more likely to be ensured.

not change the extent of long-run impact of interest rates on inflation.

Our results can be practically applied in likelihood-based estimation of monetary models to impose parameter restrictions that lead to determinacy or indeterminacy separately. For example, in Bhattarai, Lee, and Park (2013), we estimate a sticky price model under different combinations of monetary and fiscal policy regimes and where each regime (including one which features indeterminacy) is imposed by making use of the analytical boundary condition derived here. In particular, having an analytical boundary greatly aids in making the posterior simulation stable and helps substantially with convergence.

2 Model

The model is based on the prototypical New Keynesian set-up in Woodford (2003). The detailed exposition of the model is in the appendix. Here, we present the log-linearized equilibrium conditions and the monetary policy rule which are

$$(Y_t - \eta Y_{t-1}) = (E_t Y_{t+1} - \eta Y_t) - (1-\eta)(R_t - E_t \pi_{t+1}) + d_t, \tag{1}$$

$$(\pi_t - \gamma \pi_{t-1}) = \beta (E_t \pi_{t+1} - \gamma \pi_t) + \kappa \left[\varphi Y_t + \frac{1}{1-\eta}(Y_t - \eta Y_{t-1}) \right] + u_t, \tag{2}$$

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(\phi_\pi \pi_t + \phi_Y Y_t) + \varepsilon_{R,t}, \tag{3}$$

where Y is output, π is inflation, and R is the nominal interest rate.[3] The parameter $0 < \eta < 1$ governs habit formation, $0 < \gamma < 1$ governs dynamic price indexation, $0 < \beta < 1$ is the discount factor, $\kappa > 0$ is a composite parameter that depends inversely on the extent of price stickiness, and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. The model is therefore a generalization of the text-book, purely forward looking New Keynesian model. In particular, habit formation introduces persistence in the "IS" equation (1) while dynamic price indexation introduces persistence in the "Phillips curve" (2). Finally, the Taylor rule (3) takes a standard form with interest rate smoothing and has the smoothing parameter $0 < \rho_R < 1$ and feedback parameters $\phi_\pi \geq 0$ and $\phi_Y \geq 0$ on inflation and output, respectively.[4]

The exogenous shock d_t is a normalized preference shock and u_t is a normalized markup shock. We assume that they evolve according to an AR(1) process as follows

$$d_t = \rho_d d_{t-1} + \varepsilon_{d,t},$$

[3] Y, π, and R denote the log deviation of the variables from their respective state state value. To keep the presentation uncluttered, we do not use a hat to denote log deviations. Note that in the appendix, variables with no hats denote variables in levels, not log deviations.

[4] Clearly, when η, $\gamma = 0$ and $\rho_R = 0$, the model reduces to a completely forward-looking set-up.

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t},$$

where $\varepsilon_{d,t}$ and $\varepsilon_{u,t}$ are *i.i.d.* and have finite mean and variance. The shock $\varepsilon_{R,t}$, which is also *i.i.d.* and has finite mean and variance, captures an unanticipated deviation of monetary policy from the Taylor rule. Since stationary shocks do not matter for determinacy of the model equilibrium, we can drop d_t, u_t, and $\varepsilon_{R,t}$ from the model in most of the derivations and proofs below.

3 Results

We present our results in steps. We first show a condition on the roots of a fifth-order characteristic equation for determinacy of the model. Then we derive a necessary condition and a sufficient condition in terms of the model parameters for a unique stable equilibrium. Finally, given that this condition is met, we analytically characterize the unique solution of the model.

3.1 Condition on Characteristic Roots for Determinacy

To derive a condition for equilibrium determinacy of the model, we first collapse the three equations (1)-(3) into a single equation for Y_t and its leads and lags and then use the factorization method with the lag operator.[5] The method boils down to finding a condition about the roots of a univariate characteristic equation. It is essentially equivalent to the standard method that uses the eigenvalue decomposition, but turns out to be easier to apply in our case since we can make use of some properties of a high-order polynomial.

Because of the complicated lag structure introduced by habit formation in consumption, dynamic price indexation, and interest rate smoothing in monetary policy, we use equations in different time periods to eliminate π_t and R_t, and their leads and lags. After a series of algebraic operations, we obtain

$$\left(L^{-5} + a_4 L^{-4} + a_3 L^{-3} + a_2 L^{-2} + a_1 L^{-1} + a_0\right) E_{t-1} Y_{t-2} = E_{t-1} w_{t-1}, \tag{4}$$

where L is the lag operator,

$$w_{t-1} = \beta^{-1} (\rho_d - \rho_R) (\rho_d - \gamma) (1 - \beta \rho_d) d_{t-1} - \beta^{-1} [(1 - \rho_R) \phi_\pi + \rho_R - \rho_u] (1 - \eta) \rho_u^2 u_{t-1},$$

[5] See Sargent (1987) or Hamilton (1994) for a detailed presentation of the factorization method.

and

$$
\begin{aligned}
a_4 =& -\left[1+\beta^{-1}+(\eta+\gamma+\rho_R)+(1-\eta)\,\kappa\beta^{-1}\left(\left(\varphi+\frac{1}{1-\eta}\right)+(1-\rho_R)\,\phi_Y\kappa^{-1}\beta\right)\right]\\
a_3 =& \beta^{-1}+(\eta+\gamma+\rho_R)\left(1+\beta^{-1}\right)+(\eta\gamma+\eta\rho_R+\gamma\rho_R)\\
&+(1-\eta)\,(1-\rho_R)\,\kappa\beta^{-1}\left[\begin{array}{c}\phi_\pi\left(\varphi+\frac{1}{1-\eta}\right)+(1+\beta\gamma)\,\phi_Y\kappa^{-1}\\ +\frac{\rho_R}{1-\rho_R}\left(\varphi+\frac{1}{1-\eta}\right)+\frac{1}{1-\rho_R}\left(\frac{\eta}{1-\eta}\right)\end{array}\right],\\
a_2 =& -\left[\begin{array}{c}(\eta+\gamma+\rho_R)\,\beta^{-1}+(\eta\gamma+\eta\rho_R+\gamma\rho_R)\,(1+\beta^{-1})+\eta\gamma\rho_R\\ +(1-\eta)\,(1-\rho_R)\,\kappa\beta^{-1}\left(\phi_\pi\left(\frac{\eta}{1-\eta}\right)+\phi_Y\kappa^{-1}\gamma+\frac{\rho_R}{1-\rho_R}\left(\frac{\eta}{1-\eta}\right)\right)\end{array}\right],\\
a_1 =& \eta\gamma\beta^{-1}+\rho_R\beta^{-1}\,(\eta+\gamma+\eta\gamma+\beta\eta\gamma)\,,\\
a_0 =& -\,\eta\gamma\rho_R\beta^{-1}.
\end{aligned}
$$

Note that $a_4, a_2, a_0 < 0$ and $a_3, a_1 > 0$.[6]

Let λ_1, λ_2, λ_3, λ_4 and λ_5 be the five roots of the characteristic equation for the left hand side of (4),

$$
f(z) \equiv z^5 + a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0 = 0, \tag{5}
$$

where $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3| \leq |\lambda_4| \leq |\lambda_5|$. Then, (4) can be written as

$$
(1-\lambda_1 L)\,(1-\lambda_2 L)\,(1-\lambda_3 L)\left(L^{-1}-\lambda_4\right)\left(L^{-1}-\lambda_5\right)E_{t-1}Y_{t+1} = E_{t-1}w_{t-1}. \tag{6}
$$

The condition for (4) to have a unique stable solution for $E_{t-1}Y_{t+1}$ is therefore that $f(z) = 0$ has three roots inside the unit circle and two roots outside the unit circle

$$
|\lambda_1| \leq |\lambda_2| \leq |\lambda_3| < 1 < |\lambda_4| \leq |\lambda_5|\,. \tag{7}
$$

Under the condition (7), we can derive a unique solution for $E_{t-1}Y_{t+1}$ and use it to solve for $E_{t-1}Y_t$, which can be used in turn to solve for endogenous variables such as Y_t, π_t, and R_t. If there are more than three roots inside the unit circle, (4) is not determinate and has multiple solutions of $E_{t-1}Y_{t+1}$, which results in equilibrium indeterminacy. If there are less than three roots inside the unit circle, there does not exist any stable solution of $E_{t-1}Y_{t+1}$.

[6] The detailed derivation is presented in the appendix. Note that L^{-1} is the forward operator. The lag operator and forward operator apply to the time subscript of a variable but not on the time period in which the expectation of the same variable is taken.

3.2 Generalized Taylor Principle

We now translate the condition (7) for equilibrium determinacy into a condition with respect to the parameters of the model. We first derive a necessary condition, which turns out to be the Generalized Taylor Principle, and then show that this is also sufficient for equilibrium determinacy under a weak additional assumption.

3.2.1 Necessary Condition for Determinacy

A necessary condition for (7) is $f(1) > 0$ since $f(z)$ is a fifth-order polynomial and $f(z) < 0$ for real $z \leq 0$. Otherwise, there exist no root, two roots or four roots inside the unit circle. Note that

$$\begin{aligned} f(1) &= 1 + a_4 + a_3 + a_2 + a_1 + a_0 \\ &= (1-\eta)(1-\rho_R)\kappa\beta^{-1}(\varphi+1)\left[\phi_\pi + \frac{(1-\gamma)(1-\beta)}{\kappa(\varphi+1)}\phi_Y - 1\right], \end{aligned}$$

and therefore $f(1) > 0$ is equivalent to

$$\phi_\pi + \frac{(1-\gamma)(1-\beta)}{\kappa(\varphi+1)}\phi_Y > 1. \tag{8}$$

3.2.2 Sufficient Condition for Determinacy

It turns out that a mild restriction on parameters is needed for (8) to be sufficient for equilibrium determinacy. There exist some parameter values for which (8) is met but the model does not have a unique stable equilibrium.[7] We instead derive a sufficient condition for equilibrium determinacy and then show that this sufficient condition is only slightly stronger than (8). Using an exhaustive grid search on the parameter space, we find that the difference between the two conditions is practically unimportant.

Let $g(z) = a_3 z^3$. A stronger version of the Rouché Theorem by Glicksberg (1976) states that if the strict inequality

$$|f(z) - g(z)| < |f(z)| + |g(z)| \tag{9}$$

holds on the unit circle $C = \{z : |z| = 1\}$, and $f(z)$ and $g(z)$ have no zeros on C, then $f(z) = 0$ and $g(z) = 0$ have the same number of roots inside C.[8] Here, each root is counted

[7] These parameter values are practically not relevant as we discuss in detail later. Moreover, numerically, we find that they lead to an explosive solution.

[8] While f and g are real-valued, we extend the domain of f and g to include all the complex numbers for the proof. Note however that the roots of f and g with the extended domain are the same as the roots of f

as many times as its multiplicity.

Choose $z \in C$. Then there exists $\omega \in [0, 2\pi]$ such that $z = e^{i\omega} = \cos(\omega) + i \sin(\omega)$ and it follows that

$$\begin{aligned}
|f(z) - g(z)| &= \left|e^{i5\omega} + a_4 e^{i4\omega} + a_2 e^{i2\omega} + a_1 e^{i\omega} + a_0\right| \\
&= \left|e^{-i3\omega}\right| \left|e^{i5\omega} + a_4 e^{i4\omega} + a_2 e^{i2\omega} + a_1 e^{i\omega} + a_0\right| \\
&= \left\{ \begin{array}{c} \left[(1 + a_1)\cos(2\omega) + (a_4 + a_2)\cos(\omega) + a_0 \cos(3\omega)\right]^2 \\ + \left[(1 - a_1)\sin(2\omega) + (a_4 - a_2)\sin(\omega) - a_0 \sin(3\omega)\right]^2 \end{array} \right\}^{1/2},
\end{aligned}$$

and

$$\begin{aligned}
|f(z)| + |g(z)| &= \left|a_3 e^{i3\omega}\right| + \left|e^{i5\omega} + a_4 e^{i4\omega} + a_3 e^{i3\omega} + a_2 e^{i2\omega} + a_1 e^{i\omega} + a_0\right| \\
&= a_3 + \left|e^{-i3\omega}\right| \left|e^{i5\omega} + a_4 e^{i4\omega} + a_3 e^{i3\omega} + a_2 e^{i2\omega} + a_1 e^{i\omega} + a_0\right| \\
&= a_3 + \left\{ \begin{array}{c} \left[(1 + a_1)\cos(2\omega) + (a_4 + a_2)\cos(\omega) + a_0 \cos(3\omega) + a_3\right]^2 \\ + \left[(1 - a_1)\sin(2\omega) + (a_4 - a_2)\sin(\omega) - a_0 \sin(3\omega)\right]^2 \end{array} \right\}^{1/2}.
\end{aligned}$$

First suppose that

$$(1 - a_1)\sin(2\omega) + (a_4 - a_2)\sin(\omega) - a_0 \sin(3\omega) \neq 0.$$

Then $f(z) \neq 0$. Now define

$$h(\mu, z) \equiv \mu a_3 + \left\{ \begin{array}{c} \left[(1 + a_1)\cos(2\omega) + (a_4 + a_2)\cos(\omega) + a_0 \cos(3\omega) + \mu a_3\right]^2 \\ + \left[(1 - a_1)\sin(2\omega) + (a_4 - a_2)\sin(\omega) - a_0 \sin(3\omega)\right]^2 \end{array} \right\}^{1/2},$$

for $\mu \in [0, 1]$. Observe that

$$\frac{\partial h(\mu, z)}{\partial \mu} = \left[1 + \frac{(1 + a_1)\cos(2\omega) + (a_4 + a_2)\cos(\omega) + a_0 \cos(3\omega) + \mu a_3}{h(\mu, z) - \mu a_3}\right] a_3 > 0,$$

since $a_3 > 0$ and

$$0 < \left|\frac{(1 + a_1)\cos(2\omega) + (a_4 + a_2)\cos(\omega) + a_0 \cos(3\omega) + \mu a_3}{h(\mu, z) - \mu a_3}\right| < 1.$$

Therefore, $h(\mu, z)$ is strictly increasing in μ over $[0, 1]$, which implies that the inequality (9) holds since $h(0, z) = |f(z) - g(z)|$ and $h(1, z) = |f(z)| + |g(z)|$.

and g with the original domain.

Now suppose that

$$(1 - a_1)\sin(2\omega) + (a_4 - a_2)\sin(\omega) - a_0 \sin(3\omega) = 0. \tag{10}$$

Then we assume that

$$(1 + a_1)\cos(2\omega) + (a_4 + a_2)\cos(\omega) + a_0 \cos(3\omega) + a_3 > 0. \tag{11}$$

It follows that since $a_3 > 0$, the inequality (9) holds. Also, under this assumption, $f(z) \neq 0$.

It is obvious that $g(z)$ does not have zeros on C. Therefore, according to the stronger version of the Rouché Theorem, $f(z) = 0$ has exactly three roots inside the unit circle as $g(z) = 0$ has three roots inside the unit circle. This concludes the proof that the condition that (11) holds for any ω satisfying (10) is sufficient for equilibrium determinacy. Note that (11) does not have to hold for ω that does not satisfy (10).

Let us denote this sufficient condition by [(10) $\Rightarrow$ (11)]. Note that there is the following relationship between the conditions found so far

$$[(10) \Rightarrow (11)] \Rightarrow (7) \Rightarrow (8),$$

where (7) is the necessary and sufficient condition for equilibrium determinacy.[9]

We now summarize our main result in the following proposition.

Proposition (The Generalized Taylor Principle)**.** *Under the standard assumption on the domain of the parameters, a necessary condition for equilibrium determinacy of the model (1)-(3) is*

$$\phi_\pi + \frac{(1-\gamma)(1-\beta)}{\kappa(\varphi+1)}\phi_Y > 1. \tag{12}$$

For any $\omega \in [0, 2\pi]$ *such that*

$$(1 - a_1)\sin(2\omega) + (a_4 - a_2)\sin(\omega) - a_0 \sin(3\omega) = 0, \tag{13}$$

assume that

$$(1 + a_1)\cos(2\omega) + (a_4 + a_2)\cos(\omega) + a_0 \cos(3\omega) + a_3 > 0. \tag{14}$$

Then the condition (12) is both necessary and sufficient for equilibrium determinacy.

[9] We can derive a sufficient condition that $f(z) = 0$ has only one root or five roots inside the unit circle by defining $g(z) = a_1 z$ or $g(z) = z^5$, respectively. The sufficient conditions are mutually exclusive of each other and also with [(10) $\Rightarrow$ (11)]. A parameter value that satisfies either of these sufficient conditions is an example of the discrepancy between [(10) $\Rightarrow$ (11)] and (8). In an exhaustive grid search, we find that there are no parameter values under (8) that produce five roots inside the unit circle. See Section 3.2.4 for details.

It is easy to show that when $\eta = 0$, $\gamma = 0$, and $\rho_R = 0$, that is, when the model (1)-(3) is purely forward-looking, the sufficient condition [(13) $\Rightarrow$ (14)] is equivalent to the necessary condition (12). That is, (12) is necessary and sufficient.

In general, the sufficient condition [(13) $\Rightarrow$ (14)] is only slightly stronger than the necessary condition (12). Using an exhaustive grid search, we find that [(13) $\Rightarrow$ (14)] is practically equivalent to (12) in that those parameter values that meet (12) but not [(13) $\Rightarrow$ (14)] are not relevant. In particular, when a condition that β is greater than *either* of η, γ, or ρ_R, which is not restrictive at all, is further assumed, (12) is found to be necessary and sufficient for equilibrium determinacy.[10]

3.2.3 Economic Intuition

Economic intuition implied by the condition (12) in the proposition above is well known. Suppose that the endogenous variables are stable. Then, the Phillips curve (2) implies the following long-run relationship between inflation and output

$$dY = \frac{(1-\gamma)(1-\beta)}{\kappa(\varphi+1)} d\pi, \tag{15}$$

where $d\pi$ and dY are the sizes of a permanent change in inflation and output, respectively. Combining this with the long-run relationship implied by the Taylor rule (3) leads to

$$dR = \left[\phi_\pi + \frac{(1-\gamma)(1-\beta)}{\kappa(\varphi+1)}\phi_Y\right] d\pi, \tag{16}$$

where dR is the size of a permanent change in the nominal interest rate.[11] Note that the condition (12) is exactly given by the term in the brackets in (16) being greater than 1 and implies that when it is fulfilled, the nominal interest rate reacts to a rise in inflation by more than one-for-one in the long run. Thus, the real interest rate eventually rises when inflation rises, which works to counteract the increase in inflation and stabilizes the economy. This property is referred to as the Generalized Taylor Principle in the literature. Therefore, our main result is indeed that the Generalized Taylor Principle (12) is both necessary and sufficient for the existence of a unique stable equilibrium in our model except for some parameter values that are ruled out by the sufficient condition [(13) $\Rightarrow$ (14)].

Moreover, note that the habit formation parameter η does not directly influence condition (12) since it does not affect the long-term inflation and output gap trade-off in the model via (2). Exactly for this reason, η does not appear in (15). Our results overall generalize those

[10] We discuss the relationship between the sufficient condition [(13) $\Rightarrow$ (14)] and the necessary condition (12) and the grid search in Section 3.2.4.

[11] Note that the interest rate smoothing parameter does not appear since we consider permanent changes.

in Bullard and Mitra (2002), Woodford (2003), and Lubik and Marzo (2007), who consider a purely forward-looking New Keynesian model. When $\gamma = 0$, (12) indeed simplifies to the condition for a unique equilibrium shown in these papers. With partial dynamic price indexation, that is $0 < \gamma < 1$, our proposition shows that ceteris paribus, the Taylor rule feedback coefficients ϕ_π and/or ϕ_Y have to be larger to ensure a determinate equilibrium. This is because dynamic inflation indexation reduces the long-run trade-off between inflation and output in the model, as shown clearly by (15).[12]

3.2.4 The Sufficient Condition and the Generalized Taylor Principle

Using an exhaustive grid search on the parameter space, we find that the sufficient condition [(13) $\Rightarrow$ (14)] is practically equivalent to the Generalized Taylor Principle (12).[13] That is, for most of the parameter values, the two conditions are met simultaneously. Note that (13) can be rewritten as

$$-4a_0 \cos^2(\omega) + 2(1 - a_1)\cos(\omega) + a_4 - a_2 + a_0 = 0, \tag{17}$$

for ω such that $\sin(\omega) \neq 0$. In most of the cases, there does not exist any ω that invokes (17) since the quadratic equation in terms of $\cos(\omega)$ does not have a real root on $[0, 1]$. Even though there exists such ω solving (17), (14) is often true under (12). Also, for the values of ω such that $\sin(\omega) = 0$, (14) always holds given (12).

Next, those parameter values that satisfy (12) but not [(13) $\Rightarrow$ (14)] are not practically relevant as they are quite extreme and implausible. In particular, using the grid search we find that (12) is necessary and sufficient under an extra condition that $\beta > \eta$, γ, or ρ_R. This extra condition rules out these irrelevant and extreme parameter values.

In addition, we note that in the grid search, we do not find any parameter values that meet (12) but produce multiple stable equilibria. So we conjecture that equilibrium indeterminacy is ruled out under (12). In particular, parameter values with $\beta < \eta$, γ and ρ_R that are sometimes found to violate the sufficient condition [(13) $\Rightarrow$ (14)] while satisfying (12) generate an explosive equilibrium. Our conjecture is that such parameter values make the model dynamics too persistent, leading to an explosive solution, and that [(13) $\Rightarrow$ (14)] rules such solution out.

Lastly, using the grid search, we find [(13) $\Rightarrow$ (14)] to be necessary and sufficient for equilibrium determinacy generally. That is, any parameter values that result in equilibrium

[12] In fact, if one were to allow for complete price indexation, then the long-run trade-off between inflation and output would disappear completely, as discussed in Woodford (2003). In such a case, the condition for determinacy would simply be $\phi_\pi > 1$. Note that even with non-zero steady-state inflation, unlike in Coibion and Gorodnichenko (2011), the Generalized Taylor Principle is necessary and sufficient to ensure determinacy because we allow for partial dynamic price indexation as well as partial indexation to steady-state inflation.

[13] We provide technical details of the grid search in the appendix.

determinacy are found to satisfy [(13) $\Rightarrow$ (14)].

3.3 Model Solution

Finally we present the complete solution to the model under equilibrium determinacy.[14] We can rewrite (6) as

$$\left(L^{-1}-\lambda_4\right)\left(L^{-1}-\lambda_5\right)E_{t-1}\left[Y_{t+1}-\left(\lambda_1+\lambda_2+\lambda_3\right)Y_t+\left(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3\right)Y_{t-1}\right.$$
$$\left.-\left(\lambda_1\lambda_2\lambda_3\right)Y_{t-2}\right]=E_{t-1}w_{t-1},$$

which is solved as

$$\begin{aligned} E_{t-1}Y_{t+1}=&\left(\lambda_1+\lambda_2+\lambda_3\right)Y_t-\left(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3\right)Y_{t-1}+\left(\lambda_1\lambda_2\lambda_3\right)Y_{t-2}\\ &+\frac{\left(\rho_d-\rho_R\right)\left(\rho_d-\gamma\right)\left(1-\beta\rho_d\right)}{\beta\left(\lambda_4-\rho_d\right)\left(\lambda_5-\rho_d\right)}d_{t-1}-\frac{\left[\left(1-\rho_R\right)\phi_\pi+\rho_R-\rho_u\right]\left(1-\eta\right)\rho_u^2}{\beta\left(\lambda_4-\rho_u\right)\left(\lambda_5-\rho_u\right)}u_{t-1}. \end{aligned} \tag{18}$$

Note that $\varepsilon_{R,t-1}$ does not appear in the solution of $E_{t-1}Y_{t+1}$ since it is independent over time.

The two-step ahead expectation of output $E_{t-1}Y_{t+1}$ is uniquely determined and stable. We can further find a unique solution for $E_{t-1}Y_t$ and finally Y_t, π_t, and R_t using (18). The solution for Y_t is

$$\begin{aligned} Y_t=&\left(\tilde\Phi_{Y,0}-\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,-1}+\tilde\Phi_{Y,-1}\right)Y_{t-1}\\ &+\left(\tilde\Phi_{Y,0}-\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{\pi,-1}+\tilde\Phi_{\pi,-1}\right)\pi_{t-1}\\ &+\left(\tilde\Phi_{Y,0}-\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{R,-1}+\tilde\Phi_{R,-1}\right)R_{t-1}\\ &+\left(\tilde\Phi_{Y,0}-\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{d,0}+\tilde\Phi_{d,0}\right)d_t\\ &+\left(\tilde\Phi_{Y,0}-\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{u,0}+\tilde\Phi_{u,0}\right)u_t\\ &+\left(\tilde\Phi_{Y,0}-\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde\Phi_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{\varepsilon_R,0}+\tilde\Phi_{\varepsilon_R,0}\right)\varepsilon_{R,t}, \end{aligned}$$

where

$$\begin{aligned} \tilde\Phi_{Y,0}&=\Psi_{\pi,0}^{-1}\left[\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{Y,0}+\Psi_{Y,-1}+\Psi_{R,-1}\left(1-\rho_R\right)\phi_Y\right]+\kappa\beta^{-1}\left(\varphi+\frac{1}{1-\eta}\right),\\ \tilde\Phi_{\pi,0}&=\beta^{-1}\left(1+\beta\gamma\right)-\Psi_{\pi,0}^{-1}\left(\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{\pi,0}+\Psi_{\pi,-1}+\Psi_{R,-1}\left(1-\rho_R\right)\phi_\pi\right),\\ \tilde\Phi_{Y,-1}&=\kappa\beta^{-1}\left(\frac{\eta}{1-\eta}\right)-\Psi_{\pi,0}^{-1}\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{Y,-1}, \end{aligned}$$

[14] For the detailed derivation, see the appendix.

$$
\begin{aligned}
\tilde{\Phi}_{\pi,-1} &= -\beta^{-1}\gamma - \Psi_{\pi,0}^{-1}\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{\pi,-1}, \\
\tilde{\Phi}_{R,-1} &= -\Psi_{\pi,0}^{-1}\Psi_{R,-1}\rho_R, \\
\tilde{\Phi}_{d,0} &= -\Psi_{\pi,0}^{-1}\left(\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{d,0} + \Psi_{d,0}\rho_d\right), \\
\tilde{\Phi}_{u,0} &= -\beta^{-1} - \Psi_{\pi,0}^{-1}\left(\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{u,0} + \Psi_{u,0}\rho_u\right), \\
\tilde{\Phi}_{\varepsilon_R,0} &= -\Psi_{\pi,0}^{-1}\Psi_{R,-1},
\end{aligned}
$$

and

$$
\begin{aligned}
\Psi_{\pi,0} &= \Phi_{Y,1}^{-1}\Phi_{\pi,0} - (1-\eta)\left[(1-\rho_R)\phi_\pi - \beta^{-1}(1+\beta\gamma)\right], \\
\Psi_{Y,0} &= 1 - \Phi_{Y,1}^{-1}\Phi_{Y,0} + \eta + (1-\eta)\left[(1-\rho_R)\phi_Y + \kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right)\right], \\
\Psi_{Y,-1} &= -\eta - \Phi_{Y,1}^{-1}\Phi_{Y,-1} - \kappa\beta^{-1}\eta, \\
\Psi_{\pi,-1} &= -\Phi_{Y,1}^{-1}\Phi_{\pi,-1} + (1-\eta)\beta^{-1}\gamma, \\
\Psi_{R,-1} &= (1-\eta)\rho_R, \\
\Psi_{d,0} &= -\Phi_{Y,1}^{-1}\Phi_{d,0} - 1, \\
\Psi_{u,0} &= -\Phi_{Y,1}^{-1}\Phi_{u,0} + (1-\eta)\beta^{-1}, \\
\Psi_{\varepsilon_R,0} &= (1-\eta),
\end{aligned}
$$

and

$$
\begin{aligned}
\Phi_{Y,1} =& 1 - (\lambda_1+\lambda_2+\lambda_3) + \eta + \rho_R + (1-\eta)\left[(1-\rho_R)\phi_Y + \kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right)\right], \\
\Phi_{Y,0} =& \eta + \rho_R - (\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) + \rho_R\eta + (1-\eta)(1-\rho_R)\phi_\pi\kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right) \\
& - (1-\eta)\left[\beta^{-2}(1+\beta\gamma)\kappa\left(\varphi + \frac{1}{1-\eta}\right) - \kappa\beta^{-1}\left(\frac{\eta}{1-\eta}\right)\right] + (1-\eta)\rho_R\kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right), \\
\Phi_{\pi,0} =& (1-\eta)\beta^{-1}\left[\beta^{-1}(1+\beta\gamma)^2 - \gamma\right] - (1-\eta)\beta^{-1}(1+\beta\gamma)\left((1-\rho_R)\phi_\pi + \rho_R\right), \\
\Phi_{Y,-1} =& (\lambda_1\lambda_2\lambda_3) - \rho_R\eta - (1-\eta)(1-\rho_R)\phi_\pi\kappa\beta^{-1}\left(\frac{\eta}{1-\eta}\right) + (1-\eta)\beta^{-2}(1+\beta\gamma)\kappa\left(\frac{\eta}{1-\eta}\right) \\
& - (1-\eta)\rho_R\kappa\beta^{-1}\left(\frac{\eta}{1-\eta}\right), \\
\Phi_{\pi,-1} =& (1-\eta)(1-\rho_R)\phi_\pi\beta^{-1}\gamma - (1-\eta)\beta^{-2}\gamma(1+\beta\gamma) + (1-\eta)\rho_R\beta^{-1}\gamma, \\
\Phi_{d,0} =& (\rho_d - \rho_R)\left(1 + \frac{(\rho_d-\gamma)(1-\beta\rho_d)}{\beta(\lambda_4-\rho_d)(\lambda_5-\rho_d)}\right), \\
\Phi_{u,0} =& (1-\eta)(1-\rho_R)\beta^{-1}\phi_\pi - (1-\eta)\beta^{-1}\left[\rho_u + \beta^{-1}(1+\beta\gamma)\right] + (1-\eta)\rho_R\beta^{-1} \\
& - \frac{\left[(1-\rho_R)\phi_\pi + \rho_R - \rho_u\right](1-\eta)\rho_u^2}{\beta(\lambda_4-\rho_u)(\lambda_5-\rho_u)}.
\end{aligned}
$$

Inflation π_t is solved for as

$$\begin{aligned}\pi_t =&\Psi_{\pi,0}^{-1}\Psi_{Y,0}Y_t + \Psi_{\pi,0}^{-1}\Psi_{Y,-1}Y_{t-1} + \Psi_{\pi,0}^{-1}\Psi_{\pi,-1}\pi_{t-1} + \Psi_{\pi,0}^{-1}\Psi_{R,-1}R_{t-1}\\ &+ \Psi_{\pi,0}^{-1}\Psi_{d,0}d_t + \Psi_{\pi,0}^{-1}\Psi_{u,0}u_t + \Psi_{\pi,0}^{-1}\Psi_{\varepsilon_R,0}\varepsilon_{R,t},\end{aligned}$$

which can be further solved to remove Y_t on the right hand side. The interest rate R_t is then simply determined by the Taylor rule (3).

4 Conclusion

We show analytically that the generalized Taylor Principle, under which the nominal interest rate reacts more than one-for-one to inflation in the long-run, is a necessary and sufficient condition for determinacy in a sticky price model with non-zero steady-state inflation, partial dynamic price indexation, and habit formation in consumption.

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Appendix

A Model

A.1 Households

There is a continuum of households in the unit interval. Each household specializes in the supply of a particular type of labor. A household that supplies labor of type-j maximizes the utility function:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \delta_t \left[\log \left(C_t^j - \eta C_{t-1} \right) - \frac{\left(H_t^j \right)^{1+\varphi}}{1+\varphi} \right] \right\},$$

where C_t^j is consumption of household j, C_t is aggregate consumption, and H_t^j denotes the hours of type-j labor services. The parameters β, φ, and η are, respectively, the discount factor, the inverse of the (Frisch) elasticity of labor supply, and the degree of external habit formation, while δ_t represents an intertemporal preference shock that follows:

$$\delta_t = \delta_{t-1}^{\rho_\delta} \exp(\varepsilon_{\delta,t}).$$

Household j's flow budget constraint is:

$$P_t C_t^j + B_t^j + E_t \left[Q_{t,t+1} V_{t+1}^j \right] = W_t(j) H_t^j + V_t^j + R_{t-1} B_{t-1}^j + \Pi_t$$

where P_t is the price level, B_t^j is the amount of one-period risk-less nominal bond held by household j, R_t is the interest rate on the bond, $W_t(j)$ is the competitive nominal wage rate for type-j labor, and Π_t denotes profits of intermediate firms. In addition to the government bond, households trade at time t one-period state-contingent nominal securities V_{t+1}^j at price $Q_{t,t+1}$, and hence fully insure against idiosyncratic risk.

A.2 Firms

The final good Y_t, which is consumed by the government and households, is produced by perfectly competitive firms assembling intermediate goods, $Y_t(i)$, with the technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta_t - 1}{\theta_t}} di \right)^{\frac{\theta_t}{\theta_t - 1}}$, where θ_t denotes time-varying elasticity of substitution between intermediate goods that follows $\theta_t = \bar{\theta}^{1-\rho_\theta} \theta_{t-1}^{\rho_\theta} \exp(\varepsilon_{\theta,t})$ with the steady-state value $\bar{\theta}$. The corresponding price index for the final consumption good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta_t} di \right)^{\frac{1}{1-\theta_t}}$, where $P_t(i)$ is the price of the intermediate good i. The optimal demand for $Y_t(i)$ is given by $Y_t(i) = \left(P_t(i)/P_t \right)^{-\theta_t} Y_t$.

Monopolistically competitive firms produce intermediate goods using the production function:

$$Y_t(i) = H_t(i),$$

where $H_t(i)$ denotes the hours of type-i labor employed by firm i. We do not include a productivity shock for simplicity.

A firm resets its price optimally with probability $1 - \alpha$ every period. Firms that do not optimize adjust their price according to the simple partial dynamic indexation rule:

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\gamma}\bar{\pi}^{1-\gamma},$$

where γ measures the extent of indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$. All optimizing firms choose a common price P_t^* to maximize the present discounted value of future profits:

$$E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left[P_t^* X_{t,k} - \frac{W_{t+k}(i)}{A_{t+k}} \right] Y_{t+k}(i),$$

where

$$X_{t,k} \equiv \begin{cases} (\pi_t \pi_{t+1} \cdots \pi_{t+k-1})^{\gamma} \bar{\pi}^{(1-\gamma)k}, & k \geq 1 \\ 1, & k = 0 \end{cases} .$$

A.3 Monetary policy

The central bank sets the nominal interest rate according to a Taylor-type rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} (Y_t)^{\phi_Y} \right]^{1-\rho_R} \exp\left(\varepsilon_{R,t}\right), \tag{19}$$

which features smoothing through the dependence on the lag and systematic responses of interest rates to output and deviation of inflation from the steady-state $\bar{\pi}$. The steady-state value of R_t is $\bar{R}$ and $\varepsilon_{R,t}$ is the non-systematic monetary policy shock that is *i.i.d.*

A.4 Equilibrium

Equilibrium is characterized by the prices and quantities that satisfy the households' and firms' optimality conditions, the monetary policy rule, and the clearing conditions for the product, labor, and asset markets:

$$\int_0^1 C_t^j dj = Y_t,\ H_t(j) = H_t^j,\ \int_0^1 V_t^j dj{=}0, \text{ and} \int_0^1 B_t^j dj{=}B_t = 0.$$

Note that $C_t^j = C_t$ due to the complete market assumption.

We use approximation methods to solve for equilibrium: we obtain a first-order approximation to the equilibrium conditions around the non-stochastic steady state. The approximation leads to the equations in the text. We reparameterize the shocks so that

$$d_t = (1 - \rho_\delta)\,\delta_t,$$
$$u_t = -\kappa \frac{1}{\bar{\theta} - 1} \theta_t,$$

where $\kappa = (1 - \alpha\beta)(1 - \alpha) / [\alpha (1 + \varphi\theta)]$.

B Derivations

B.1 Derivation of the Characteristic Equation

We collapse the following three equations into a single equation with respect to Y_t and its leads and lags

$$(Y_t - \eta Y_{t-1}) = (E_t Y_{t+1} - \eta Y_t) - (1-\eta)(R_t - E_t \pi_{t+1}) + d_t, \tag{20}$$

$$(\pi_t - \gamma \pi_{t-1}) = \beta (E_t \pi_{t+1} - \gamma \pi_t) + \kappa \left[\varphi Y_t + \frac{1}{1-\eta}(Y_t - \eta Y_{t-1})\right] + u_t, \tag{21}$$

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\phi_\pi \pi_t + \phi_Y Y_t] + \varepsilon_{R,t}. \tag{22}$$

First, push (20) one period ahead, take E_t, and subtract (20) multiplied by ρ_R from it as

$$\begin{aligned}(E_t Y_{t+1} - \eta Y_t) - \rho_R (Y_t - \eta Y_{t-1}) = &(E_t Y_{t+2} - \eta E_t Y_{t+1}) - \rho_R (E_t Y_{t+1} - \eta Y_t)\\ &- (1-\eta)(E_t R_{t+1} - \rho_R R_t) + (1-\eta)(E_t \pi_{t+2} - \rho_R E_t \pi_{t+1}) + (E_t d_{t+1} - \rho_R d_t)\\ = &(E_t Y_{t+2} - \eta E_t Y_{t+1}) - \rho_R (E_t Y_{t+1} - \eta Y_t)\\ &- (1-\eta)(1-\rho_R)(\phi_\pi E_t \pi_{t+1} + \phi_Y E_t Y_{t+1})\\ &+ (1-\eta)(E_t \pi_{t+2} - \rho_R E_t \pi_{t+1}) + (\rho_d - \rho_R) d_t,\end{aligned} \tag{23}$$

where we used (22) to eliminate R_t. Now, lag (23) one period, multiply γ to it, and subtract it from the expectation of (23) given information in period $t-1$ as

$$\begin{aligned}&(E_{t-1} Y_{t+1} - \eta E_{t-1} Y_t) - \rho_R (E_{t-1} Y_t - \eta Y_{t-1}) - \gamma [(E_{t-1} Y_t - \eta Y_{t-1}) - \rho_R (Y_{t-1} - \eta Y_{t-2})]\\ = &(E_{t-1} Y_{t+2} - \eta E_{t-1} Y_{t+1}) - \rho_R (E_{t-1} Y_{t+1} - \eta E_{t-1} Y_t) - \gamma [(E_{t-1} Y_{t+1} - \eta E_{t-1} Y_t) - \rho_R (E_{t-1} Y_t - \eta Y_{t-1})]\\ &- (1-\eta)(1-\rho_R)\phi_\pi (E_{t-1}\pi_{t+1} - \gamma E_{t-1}\pi_t)\\ &- (1-\eta)(1-\rho_R)\phi_Y (E_{t-1} Y_{t+1} - \gamma E_{t-1} Y_t)\\ &+ (1-\eta)[(E_{t-1}\pi_{t+2} - \gamma E_{t-1}\pi_{t+1}) - \rho_R (E_{t-1}\pi_{t+1} - \gamma E_{t-1}\pi_t)]\\ &+ (\rho_d - \rho_R)(\rho_d - \gamma) d_{t-1}.\end{aligned} \tag{24}$$

Then push (24) one period ahead, take E_{t-1}, multiply β and subtract it from (24) to obtain

$$\begin{aligned}&(E_{t-1} Y_{t+1} - \eta E_{t-1} Y_t) - \rho_R (E_{t-1} Y_t - \eta Y_{t-1}) - \gamma [(E_{t-1} Y_t - \eta Y_{t-1}) - \rho_R (Y_{t-1} - \eta Y_{t-2})]\\ &- \beta \{(E_{t-1} Y_{t+2} - \eta E_{t-1} Y_{t+1}) - \rho_R (E_{t-1} Y_{t+1} - \eta Y_t) - \gamma [(E_{t-1} Y_{t+1} - \eta E_{t-1} Y_t) - \rho_R (E_{t-1} Y_t - \eta Y_{t-1})]\}\\ = &(E_{t-1} Y_{t+2} - \eta E_{t-1} Y_{t+1}) - \rho_R (E_{t-1} Y_{t+1} - \eta E_{t-1} Y_t) - \gamma [(E_{t-1} Y_{t+1} - \eta E_{t-1} Y_t) - \rho_R (E_{t-1} Y_t - \eta Y_{t-1})]\\ &- \beta \{(E_{t-1} Y_{t+3} - \eta E_{t-1} Y_{t+2}) - \rho_R (E_{t-1} Y_{t+2} - \eta E_{t-1} Y_{t+1}) - \gamma [(E_{t-1} Y_{t+2} - \eta E_{t-1} Y_{t+1}) - \rho_R (E_{t-1} Y_{t+1} - \eta E_{t-1} Y_t)]\}\\ &- (1-\eta)(1-\rho_R)\phi_\pi [(E_{t-1}\pi_{t+1} - \gamma E_{t-1}\pi_t) - \beta (E_{t-1}\pi_{t+2} - \gamma E_{t-1}\pi_{t+1})]\\ &- (1-\eta)(1-\rho_R)\phi_Y [(E_{t-1} Y_{t+1} - \gamma E_{t-1} Y_t) - \beta (E_{t-1} Y_{t+2} - \gamma E_{t-1} Y_{t+1})]\\ &+ (1-\eta)[(E_{t-1}\pi_{t+2} - \gamma E_{t-1}\pi_{t+1}) - \beta (E_{t-1}\pi_{t+3} - \gamma E_{t-1}\pi_{t+2})]\\ &- (1-\eta)\rho_R [(E_{t-1}\pi_{t+1} - \gamma E_{t-1}\pi_t) - \beta (E_{t-1}\pi_{t+2} - \gamma E_{t-1}\pi_{t+1})]\\ &+ (\rho_d - \rho_R)(\rho_d - \gamma)(d_{t-1} - \beta E_{t-1} d_t).\end{aligned} \tag{25}$$

But note that from (21),

$$(E_{t-1}\pi_{t+1}-\gamma E_{t-1}\pi_t)-\beta\left(E_{t-1}\pi_{t+2}-\gamma E_{t-1}\pi_{t+1}\right)=\kappa\left[\varphi E_{t-1}Y_{t+1}+\frac{1}{1-\eta}\left(E_{t-1}Y_{t+1}-\eta E_{t-1}Y_t\right)\right]+E_{t-1}u_{t+1},$$

and

$$(E_{t-1}\pi_{t+2}-\gamma E_{t-1}\pi_{t+1})-\beta\left(E_{t-1}\pi_{t+3}-\gamma E_{t-1}\pi_{t+2}\right)=\kappa\left[\varphi E_{t-1}Y_{t+2}+\frac{1}{1-\eta}\left(E_{t-1}Y_{t+2}-\eta E_{t-1}Y_{t+1}\right)\right]+E_{t-1}u_{t+2},$$

which can be plugged into (25) to eliminate π_t and its leads and lags. After arranging terms, we finally obtain

$$\left(L^{-5}+a_4L^{-4}+a_3L^{-3}+a_2L^{-2}+a_1L^{-1}+a_0\right)E_{t-1}Y_{t-2}=E_{t-1}w_{t-1},$$

where L is the lag operator,

$$w_{t-1}=\beta^{-1}\left(\rho_d-\rho_R\right)\left(\rho_d-\gamma\right)\left(1-\beta\rho_d\right)d_{t-1}-\beta^{-1}\left[\left(1-\rho_R\right)\phi_\pi+\rho_R-\rho_u\right]\left(1-\eta\right)\rho_u^2u_{t-1},$$

and

$$\begin{aligned}
a_4 =& -\left[1+\beta^{-1}+(\eta+\gamma+\rho_R)+(1-\eta)\kappa\beta^{-1}\left(\left(\varphi+\frac{1}{1-\eta}\right)+(1-\rho_R)\phi_Y\kappa^{-1}\beta\right)\right]\\
a_3 =& \beta^{-1}+(\eta+\gamma+\rho_R)\left(1+\beta^{-1}\right)+(\eta\gamma+\eta\rho_R+\gamma\rho_R)\\
&+(1-\eta)(1-\rho_R)\kappa\beta^{-1}\left[\begin{array}{c}\phi_\pi\left(\varphi+\frac{1}{1-\eta}\right)+(1+\beta\gamma)\phi_Y\kappa^{-1}\\ +\frac{\rho_R}{1-\rho_R}\left(\varphi+\frac{1}{1-\eta}\right)+\frac{1}{1-\rho_R}\left(\frac{\eta}{1-\eta}\right)\end{array}\right],\\
a_2 =& -\left[\begin{array}{c}(\eta+\gamma+\rho_R)\beta^{-1}+(\eta\gamma+\eta\rho_R+\gamma\rho_R)\left(1+\beta^{-1}\right)+\eta\gamma\rho_R\\ +(1-\eta)(1-\rho_R)\kappa\beta^{-1}\left(\phi_\pi\left(\frac{\eta}{1-\eta}\right)+\phi_Y\kappa^{-1}\gamma+\frac{\rho_R}{1-\rho_R}\left(\frac{\eta}{1-\eta}\right)\right)\end{array}\right],\\
a_1 =& \eta\gamma\beta^{-1}+\rho_R\beta^{-1}\left(\eta+\gamma+\eta\gamma+\beta\eta\gamma\right),\\
a_0 =& -\eta\gamma\rho_R.
\end{aligned}$$

B.2 Solution of (4) for Output Expectations

The expectational difference equation (4) can be written as

$$(1-\lambda_1L)(1-\lambda_2L)(1-\lambda_3L)\left(L^{-1}-\lambda_4\right)\left(L^{-1}-\lambda_5\right)E_{t-1}Y_{t+1}=E_{t-1}w_{t-1},$$

where λ_i's $(i=1,2,3,4,5)$ are the roots of the characteristic equation (4) and

$$|\lambda_1|\le|\lambda_2|\le|\lambda_3|<1<|\lambda_4|\le|\lambda_5|.$$

Note that

$$\begin{aligned}\left(L^{-1}-\lambda_4\right)E_{t-1}\left[Y_{t+1}-(\lambda_1+\lambda_2+\lambda_3)Y_t+(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)Y_{t-1}-(\lambda_1\lambda_2\lambda_3)Y_{t-2}\right]&=\left(L^{-1}-\lambda_5\right)^{-1}E_{t-1}w_{t-1}\\ &=-\lambda_5^{-1}\sum_{s=0}^{\infty}\lambda_5^{-s}E_{t-1}w_{t-1+s}.\end{aligned}$$

Since

$$
\begin{aligned}
E_{t-1}w_{t-1+s} &= \beta^{-1}\left(\rho_d-\rho_R\right)\left(\rho_d-\gamma\right)\left(1-\beta\rho_d\right)E_{t-1}d_{t-1+s}-\beta^{-1}\left[\left(1-\rho_R\right)\phi_\pi+\rho_R-\rho_u\right]\left(1-\eta\right)\rho_u^2E_{t-1}u_{t-1+s}\\
&= \beta^{-1}\left(\rho_d-\rho_R\right)\left(\rho_d-\gamma\right)\left(1-\beta\rho_d\right)\rho_d^sd_{t-1}-\beta^{-1}\left[\left(1-\rho_R\right)\phi_\pi+\rho_R-\rho_u\right]\left(1-\eta\right)\rho_u^2\rho_u^su_{t-1},
\end{aligned}
$$

for $s\geq 1$, it follows that

$$
\begin{aligned}
&\sum_{s=0}^{\infty}\lambda_5^{-s}E_{t-1}w_{t-1+s}\\
=&\beta^{-1}\left(\rho_d-\rho_R\right)\left(\rho_d-\gamma\right)\left(1-\beta\rho_d\right)\sum_{s=0}^{\infty}\left(\lambda_5^{-1}\rho_d\right)^s d_{t-1}-\beta^{-1}\left[\left(1-\rho_R\right)\phi_\pi+\rho_R-\rho_u\right]\left(1-\eta\right)\rho_u^2\sum_{s=0}^{\infty}\left(\lambda_5^{-1}\rho_u\right)^s u_{t-1}\\
=&\frac{\left(\rho_d-\rho_R\right)\left(\rho_d-\gamma\right)\left(1-\beta\rho_d\right)}{\beta\left(1-\lambda_5^{-1}\rho_d\right)}d_{t-1}-\frac{\left[\left(1-\rho_R\right)\phi_\pi+\rho_R-\rho_u\right]\left(1-\eta\right)\rho_u^2}{\beta\left(1-\lambda_5^{-1}\rho_u\right)}u_{t-1},
\end{aligned}
$$

and thus

$$
\begin{aligned}
&\left(L^{-1}-\lambda_4\right)E_{t-1}\left[Y_{t+1}-\left(\lambda_1+\lambda_2+\lambda_3\right)Y_t+\left(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3\right)Y_{t-1}-\left(\lambda_1\lambda_2\lambda_3\right)Y_{t-2}\right]\\
&=-\frac{\left(\rho_d-\rho_R\right)\left(\rho_d-\gamma\right)\left(1-\beta\rho_d\right)}{\beta\left(\lambda_5-\rho_d\right)}d_{t-1}+\frac{\left[\left(1-\rho_R\right)\phi_\pi+\rho_R-\rho_u\right]\left(1-\eta\right)\rho_u^2}{\beta\left(\lambda_5-\rho_u\right)}u_{t-1}.
\end{aligned}
$$

By inverting $\left(L^{-1}-\lambda_4\right)$ and solving the equation in the same way, we can show that

$$
\begin{aligned}
E_{t-1}Y_{t+1} =&\left(\lambda_1+\lambda_2+\lambda_3\right)E_{t-1}Y_t-\left(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3\right)Y_{t-1}+\left(\lambda_1\lambda_2\lambda_3\right)Y_{t-2} \quad (26)\\
&+\xi_dd_{t-1}+\xi_uu_{t-1}, \quad (27)
\end{aligned}
$$

where

$$
\xi_d=\frac{\left(\rho_d-\rho_R\right)\left(\rho_d-\gamma\right)\left(1-\beta\rho_d\right)}{\beta\left(\lambda_4-\rho_d\right)\left(\lambda_5-\rho_d\right)},\ \text{and}\ \xi_u=-\frac{\left[\left(1-\rho_R\right)\phi_\pi+\rho_R-\rho_u\right]\left(1-\eta\right)\rho_u^2}{\beta\left(\lambda_4-\rho_u\right)\left(\lambda_5-\rho_u\right)}.
$$

B.3 Solution to the Model

Now we use the solution of two-step ahead expected output (26) and solve the model for all the endogenous variables Y_t, π_t, and R_t. First, solve (21) for $E_t\pi_{t+1}$ and $E_t\pi_{t+2}$ as

$$
E_t\pi_{t+1}=\beta^{-1}\left(1+\beta\gamma\right)\pi_t-\beta^{-1}\gamma\pi_{t-1}-\beta^{-1}\kappa\left[\left(\varphi+\frac{1}{1-\eta}\right)Y_t-\frac{\eta}{1-\eta}Y_{t-1}\right]-u_t, \quad (28)
$$

and

$$
\begin{aligned}
E_t\pi_{t+2}=&\beta^{-1}\left(1+\beta\gamma\right)E_t\pi_{t+1}-\beta^{-1}\gamma\pi_t-\beta^{-1}\kappa\left[\left(\varphi+\frac{1}{1-\eta}\right)E_tY_{t+1}-\frac{\eta}{1-\eta}Y_t\right]-E_tu_{t+1}\\
=&\left[\beta^{-2}\left(1+\beta\gamma\right)^2-\beta^{-1}\gamma\right]\pi_t-\beta^{-2}\left(1+\beta\gamma\right)\gamma\pi_{t-1}\\
&-\beta^{-1}\kappa\left(\varphi+\frac{1}{1-\eta}\right)E_tY_{t+1}+\left[-\beta^{-2}\left(1+\beta\gamma\right)\kappa\left(\varphi+\frac{1}{1-\eta}\right)+\beta^{-1}\kappa\left(\frac{\eta}{1-\eta}\right)\right]Y_t\\
&+\beta^{-2}\left(1+\beta\gamma\right)\kappa\left(\frac{\eta}{1-\eta}\right)Y_{t-1}-\beta^{-1}\left[\rho_u+\beta^{-1}\left(1+\beta\gamma\right)\right]u_t.
\end{aligned}
$$

Plug the solution for E_tY_{t+2}, $E_t\pi_{t+1}$ and $E_t\pi_{t+2}$ into (23) to obtain

$$\Phi_{Y,1}E_tY_{t+1} = \Phi_{Y,0}Y_t + \Phi_{\pi,0}\pi_t + \Phi_{Y,-1}Y_{t-1} + \Phi_{\pi,-1}\pi_{t-1} + \Phi_{d,0}d_t + \Phi_{u,0}u_t, \tag{29}$$

where

$$\begin{aligned}
\Phi_{Y,1} =& 1 - (\lambda_1+\lambda_2+\lambda_3) + \eta + \rho_R + (1-\eta)\left[(1-\rho_R)\,\phi_Y + \kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right)\right],\\
\Phi_{Y,0} =& \eta + \rho_R - (\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) + \rho_R\eta + (1-\eta)(1-\rho_R)\,\phi_\pi\kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right)\\
&- (1-\eta)\left[\beta^{-2}(1+\beta\gamma)\,\kappa\left(\varphi + \frac{1}{1-\eta}\right) - \kappa\beta^{-1}\left(\frac{\eta}{1-\eta}\right)\right] + (1-\eta)\,\rho_R\kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right),\\
\Phi_{\pi,0} =& (1-\eta)\,\beta^{-1}\left[\beta^{-1}(1+\beta\gamma)^2 - \gamma\right] - (1-\eta)\,\beta^{-1}(1+\beta\gamma)\left((1-\rho_R)\,\phi_\pi + \rho_R\right),\\
\Phi_{Y,-1} =& (\lambda_1\lambda_2\lambda_3) - \rho_R\eta - (1-\eta)(1-\rho_R)\,\phi_\pi\kappa\beta^{-1}\left(\frac{\eta}{1-\eta}\right) + (1-\eta)\,\beta^{-2}(1+\beta\gamma)\,\kappa\left(\frac{\eta}{1-\eta}\right)\\
&- (1-\eta)\,\rho_R\kappa\beta^{-1}\left(\frac{\eta}{1-\eta}\right),\\
\Phi_{\pi,-1} =& (1-\eta)(1-\rho_R)\,\phi_\pi\beta^{-1}\gamma - (1-\eta)\,\beta^{-2}\gamma(1+\beta\gamma) + (1-\eta)\,\rho_R\beta^{-1}\gamma,\\
\Phi_{d,0} =& (\rho_d - \rho_R)\left(1 + \frac{(\rho_d-\gamma)(1-\beta\rho_d)}{\beta(\lambda_4-\rho_d)(\lambda_5-\rho_d)}\right),\\
\Phi_{u,0} =& (1-\eta)(1-\rho_R)\,\beta^{-1}\phi_\pi - (1-\eta)\,\beta^{-1}\left[\rho_u + \beta^{-1}(1+\beta\gamma)\right] + (1-\eta)\,\rho_R\beta^{-1}\\
&- \frac{\left[(1-\rho_R)\,\phi_\pi + \rho_R - \rho_u\right](1-\eta)\,\rho_u^2}{\beta(\lambda_4-\rho_u)(\lambda_5-\rho_u)}.
\end{aligned}$$

Now eliminate E_tY_{t+1} and $E_t\pi_{t+1}$ from (20) using the solution for E_tY_{t+1} in (29) and $E_t\pi_{t+1}$ in (28) and eliminate R_t using (22) to get

$$\Psi_{\pi,0}\pi_t = \Psi_{Y,0}Y_t + \Psi_{Y,-1}Y_{t-1} + \Psi_{\pi,-1}\pi_{t-1} + \Psi_{R,-1}R_{t-1} + \Psi_{d,0}d_t + \Psi_{u,0}u_t + \Psi_{\varepsilon_R,0}\varepsilon_{R,t}, \tag{30}$$

where

$$\begin{aligned}
\Psi_{\pi,0} &= \Phi_{Y,1}^{-1}\Phi_{\pi,0} - (1-\eta)\left[(1-\rho_R)\,\phi_\pi - \beta^{-1}(1+\beta\gamma)\right],\\
\Psi_{Y,0} &= 1 - \Phi_{Y,1}^{-1}\Phi_{Y,0} + \eta + (1-\eta)\left[(1-\rho_R)\,\phi_Y + \kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right)\right],\\
\Psi_{Y,-1} &= -\eta - \Phi_{Y,1}^{-1}\Phi_{Y,-1} - \kappa\beta^{-1}\eta,\\
\Psi_{\pi,-1} &= -\Phi_{Y,1}^{-1}\Phi_{\pi,-1} + (1-\eta)\,\beta^{-1}\gamma,\\
\Psi_{R,-1} &= (1-\eta)\,\rho_R,\\
\Psi_{d,0} &= -\Phi_{Y,1}^{-1}\Phi_{d,0} - 1,\\
\Psi_{u,0} &= -\Phi_{Y,1}^{-1}\Phi_{u,0} + (1-\eta)\,\beta^{-1},\\
\Psi_{\varepsilon_R,0} &= (1-\eta)\,.
\end{aligned}$$

From (30), we get another expression for $E_t\pi_{t+1}$. After substituting (29) for E_tY_{t+1} in this expression for $E_t\pi_{t+1}$, equate it with (28) to obtain

$$\tilde{\Phi}_{Y,0}Y_t = \tilde{\Phi}_{\pi,0}\pi_t + \tilde{\Phi}_{Y,-1}Y_{t-1} + \tilde{\Phi}_{\pi,-1}\pi_{t-1} + \tilde{\Phi}_{R,-1}R_{t-1} + \tilde{\Phi}_{d,0}d_t + \tilde{\Phi}_{u,0}u_t + \tilde{\Phi}_{\varepsilon_R,0}\varepsilon_{R,t},$$

where

$$\begin{aligned}
\tilde{\Phi}_{Y,0} &= \Psi_{\pi,0}^{-1}\left[\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{Y,0} + \Psi_{Y,-1} + \Psi_{R,-1}\left(1-\rho_R\right)\phi_Y\right] + \kappa\beta^{-1}\left(\varphi + \frac{1}{1-\eta}\right), \\
\tilde{\Phi}_{\pi,0} &= \beta^{-1}\left(1+\beta\gamma\right) - \Psi_{\pi,0}^{-1}\left(\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{\pi,0} + \Psi_{\pi,-1} + \Psi_{R,-1}\left(1-\rho_R\right)\phi_\pi\right), \\
\tilde{\Phi}_{Y,-1} &= \kappa\beta^{-1}\left(\frac{\eta}{1-\eta}\right) - \Psi_{\pi,0}^{-1}\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{Y,-1}, \\
\tilde{\Phi}_{\pi,-1} &= -\beta^{-1}\gamma - \Psi_{\pi,0}^{-1}\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{\pi,-1}, \\
\tilde{\Phi}_{R,-1} &= -\Psi_{\pi,0}^{-1}\Psi_{R,-1}\rho_R, \\
\tilde{\Phi}_{d,0} &= -\Psi_{\pi,0}^{-1}\left(\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{d,0} + \Psi_{d,0}\rho_d\right), \\
\tilde{\Phi}_{u,0} &= -\beta^{-1} - \Psi_{\pi,0}^{-1}\left(\Psi_{Y,0}\Phi_{Y,1}^{-1}\Phi_{u,0} + \Psi_{u,0}\rho_u\right), \\
\tilde{\Phi}_{\varepsilon_R,0} &= -\Psi_{\pi,0}^{-1}\Psi_{R,-1}.
\end{aligned}$$

Finally, using (30), we can eliminate π_t and solve for Y_t as

$$\begin{aligned}
Y_t = &\left(\tilde{\Phi}_{Y,0} - \tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,-1} + \tilde{\Phi}_{Y,-1}\right)Y_{t-1} \\
&+ \left(\tilde{\Phi}_{Y,0} - \tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{\pi,-1} + \tilde{\Phi}_{\pi,-1}\right)\pi_{t-1} \\
&+ \left(\tilde{\Phi}_{Y,0} - \tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{R,-1} + \tilde{\Phi}_{R,-1}\right)R_{t-1} \\
&+ \left(\tilde{\Phi}_{Y,0} - \tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{d,0} + \tilde{\Phi}_{d,0}\right)d_t \\
&+ \left(\tilde{\Phi}_{Y,0} - \tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{u,0} + \tilde{\Phi}_{u,0}\right)u_t \\
&+ \left(\tilde{\Phi}_{Y,0} - \tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{Y,0}\right)^{-1}\left(\tilde{\Phi}_{\pi,0}\Psi_{\pi,0}^{-1}\Psi_{\varepsilon_R,0} + \tilde{\Phi}_{\varepsilon_R,0}\right)\varepsilon_{R,t}.
\end{aligned}$$

The solution for π_t can be obtained from (30). The solution for R_t is simply determined by the Taylor rule (22).

B.4 Grid Search on the Parameter Space

We do an exhaustive grid search on the parameter space to figure out the discrepancy between the sufficient condition [(13) $\Rightarrow$ (14)] and the Generalized Taylor Principle (12). The following sets of values for each parameter are selected:

$$\begin{aligned}
\beta &\in \{0.05, 0.10, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 0.99, 0.999\}, \\
\alpha &\in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}, \\
\varphi &\in \{0.5, 1, 2, 4, 6\}, \\
\eta &\in \{0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 0.99, 0.994\}, \\
\gamma &\in \{0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 0.99, 0.995, 1\}, \\
\phi_Y &\in \{0.1, 0.2, 0.3, 0.5, 1, 2\}, \\
\rho_d &\in \{0, 0.2, 0.4, 0.6, 0.8, 0.99\}, \\
\rho_u &\in \{0, 0.2, 0.4, 0.6, 0.8, 0.99\},
\end{aligned}$$

$$\rho_R \in \{0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 0.99, 0.996\}.$$

For ϕ_π, we use the set of values

$$\phi_\pi^* + \{0.01, 0.1, 0.2, 0.3, 0.5, 1, 2, 4, 6, 9\},$$

where

$$\phi_\pi^* = 1 - \frac{(1-\gamma)(1-\beta)}{\kappa(\varphi+1)}\phi_Y$$

is the boundary value of ϕ_π for determinacy given values for the other parameters. With firm-specific labor, the Phillips curve slope parameter is computed as

$$\kappa = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\varphi\theta)},$$

where $\theta = 8$ is the steady state value of the elasticity of substitution between differentiated goods. For each parameter value, we check 1) how many roots $f(z) = 0$ has inside the unit circle; 2) whether there exists $\omega \in [0, 2\pi]$ that solves (17) and, if yes, whether (14) is met; and 3) whether the Generalized Taylor Principle (12) is satisfied.