



Australian  
National  
University

Crawford School of Public Policy

# CAMA

Centre for Applied Macroeconomic Analysis

---

## Asymmetry in Government Bond Returns

---

CAMA Working Paper 12/2013  
February 2013

**Ipppei Fujiwara**

Australian National University  
Centre for Applied Macroeconomic Analysis

**Lena Mareen Korber**

London School of Economics

**Daisuke Nagakura**

Keio University

### Abstract

Is there asymmetry in the distribution of government bond returns in developed countries? Can asymmetries be predicted using financial and macroeconomic variables? To answer the first question, we provide evidence for asymmetry in government bond returns in particular for short maturities. This finding has important implications for modelling and forecasting government bond returns. For example, widely used models for yield curve analysis such as the affine term structure model assume symmetrically distributed innovations. To answer the second question, we find that liquidity in government bond markets predicts the coefficient of skewness with a positive sign, meaning that the probability of a large and negative excess return is more likely in a less liquid market. In addition, a positive realized return is associated with a negative coefficient of skewness, or a small probability of a large and negative return in the future.

## **Keywords**

Government Bond Returns; Skewness; Conditional Symmetry Test

## **JEL Classification**

G10; G12; E43

## **Address for correspondence:**

(E) [cama.admin@anu.edu.au](mailto:cama.admin@anu.edu.au)

[The Centre for Applied Macroeconomic Analysis](#) in the Crawford School of Public Policy has been established to build strong links between professional macroeconomists. It provides a forum for quality macroeconomic research and discussion of policy issues between academia, government and the private sector.

**The Crawford School of Public Policy** is the Australian National University's public policy school, serving and influencing Australia, Asia and the Pacific through advanced policy research, graduate and executive education, and policy impact.

# Asymmetry in Government Bond Returns <sup>\*</sup>

Ippei Fujiwara,<sup>†</sup> Lena Mareen Körber,<sup>‡</sup> and Daisuke Nagakura<sup>§</sup>

## Abstract

Is there asymmetry in the distribution of government bond returns in developed countries? Can asymmetries be predicted using financial and macroeconomic variables? To answer the first question, we provide evidence for asymmetry in government bond returns in particular for short maturities. This finding has important implications for modelling and forecasting government bond returns. For example, widely used models for yield curve analysis such as the affine term structure model assume symmetrically distributed innovations. To answer the second question, we find that liquidity in government bond markets predicts the coefficient of skewness with a positive sign, meaning that the probability of a large and negative excess return is more likely in a less liquid market. In addition, a positive realized return is associated with a negative coefficient of skewness, or a small probability of a large and negative return in the future.

*JEL Classification:* G10; G12; E43

*Keywords:* Government Bond Returns; Skewness;  
Conditional Symmetry Test

---

<sup>\*</sup>We thank Toni Braun, Karen Lewis, Masao Ogaki, Tatsuyoshi Okimoto, participants at the Bank of Japan seminar and the 8th Conference for Applied Financial Economics for their helpful comments.

<sup>†</sup>Australian National University and CAMA. Email: [ippei.fujiwara@anu.edu.au](mailto:ippei.fujiwara@anu.edu.au)

<sup>‡</sup>London School of Economics. Email: [L.M.Koerber@lse.ac.uk](mailto:L.M.Koerber@lse.ac.uk)

<sup>§</sup>Keio University. Email: [nagakura@z7.keio.jp](mailto:nagakura@z7.keio.jp)

# 1 Introduction

In the latter half of 2010, we observed significant fluctuations in government bond yields of many developed countries. The yields gradually decreased but went up abruptly towards the end of the year. For example, the yield on a 10 year US government bond remained stable from January until April at approximately 4.0%. From the end of April to the beginning of November, the yield gradually decreased to around 2.8%. In the mid of December, the yield quickly recovered to around 3.8%. While the decrease by about 120 basis points took place over a period of six months, the increase by about 100 basis points occurred in only one month and a half. This observation suggests that ups and downs in government bond yields are asymmetric.

In this paper, we study asymmetries in government bond excess returns for five developed countries - Canada, Germany, Japan, UK and the U.S. - using different approaches based on both statistical tests and econometric models.<sup>1</sup>

First, we measure unconditional asymmetry in government bond returns using both the coefficient of skewness and the Bowley coefficient, a measure that is robust to extreme observations. Second, we investigate if there is conditional asymmetry in government bond excess returns by applying the test of Bai and Ng (2001) to our data set. Third, we analyze how quarterly asymmetry co-moves across countries and how it is related to macroeconomic and financial variables.

We find that excess returns to government bonds are negatively skewed when asymmetry is measured by the coefficient of skewness, or that there is a small probability of a large and negative excess return. In contrast, the Bowley coefficient is positive for most countries and maturities, implying that much of the asymmetries measured by the coefficient of skewness can be attributed to extreme observations. Turning to the test results, we provide evidence for conditional asymmetry in excess returns to government bonds. For bonds with a maturity of 2 years, conditional asymmetry is statistically significant in all countries but there is less evidence for asymmetry in long term bonds for the U.S. and the UK. Conditional

---

<sup>1</sup>Throughout the paper, the words asymmetry and skewness are used interchangeably.

asymmetry in government bond returns has important implications for the modelling and forecasting of government bond returns. In response to the rejection of the expectation hypothesis in the data, the affine term structure model (Vasicek (1977), Piazzesi (2010)) has become a workhorse in the analysis of the yield curve.<sup>2</sup> Affine term structure models, however, usually assume normally, and thus symmetrically distributed innovations to bond returns to make the computation of the term premium easier. This is at odds with our findings. Concerning forecasting, the presence of asymmetries calls for new models that allow for the possibility that positive and negative forecast errors are not equally likely. Additionally, asymmetry in government bond returns implies that mean and variance are not sufficient to characterize the risk in government bond returns, and has thus important implications for optimal portfolio allocation (Chunhachinda et al. (1997)). Jondreau and Rockinger (2006) document that there is an advantage of using higher moments in portfolio allocation.

Turning to the properties of quarterly asymmetry, we find that the cross-country correlation of asymmetry is increasing in the maturity of the bond. This finding implies that for longer maturities, common factors play a larger role in explaining asymmetries, while idiosyncratic factors are more important at short maturities. When asymmetry is measured robustly, cross-country correlations are smaller than when asymmetry is measured by the coefficient of skewness. Therefore, there is tail correlation in government bond excess returns. When predicting the coefficient of skewness using macroeconomic and financial variables, we find that liquidity in government bond markets predicts the coefficient of skewness with a positive sign, meaning that the probability of a large and negative excess return is more likely in a less liquid market. In addition, a positive realized return is associated with a negative coefficient of skewness, or a small probability of a large and negative return in the future. Negative skewness conditional on a positive realized excess return has

---

<sup>2</sup>Recall that the expectations hypothesis predicts that the yield on a long-term bond is the average of expected short rates. Differently put, excess bond returns or term premium are constant or nil under the expectations hypothesis. The expectation hypothesis is, however, found inconsistent with the data (e.g. Campbell and Shiller (1995), Fama and Bliss (1987), Backus, Foresi, Mozumdar, and Wu (2001), Cochrane and Piazzesi (2005)).

been documented previously by Brunnermeier et al. (2009) in the context of carry trades.

Asymmetries in equity returns have been documented in many previous studies (Peiro (2002), Premaratne and Bera (2005), Bekaert, et al. (1998), Bai and Ng (2001), Grigoletto and Lisi (2008)). These studies have analyzed data from different countries and time periods. However, while most of these studies have tested for unconditional asymmetry, our paper investigates conditional asymmetry instead. Conditional asymmetry has more significant implications for government bond markets when compared to unconditional asymmetry. As mentioned above, affine term structure models assume that the innovations are symmetrically distributed. Testing for conditional asymmetry in government bond returns is one contribution of our paper to the literature.

Skewness in equity returns has also been assessed by using models that represent it. This approach was followed by Harvey and Siddique (1999) and Ghysels et al. (2011), for example. While Harvey and Siddique (1999) propose a time series model for the coefficient of skewness, Ghysels et al. (2011) develop a quantile approach that is robust to outliers. These studies provide evidence for time-varying asymmetries in equity returns for both developed and emerging countries.

In contrast to equity returns, the literature on asymmetries in government bond returns is scarce. This is surprising since there are situations where economic behavior gives rise to asymmetric dynamics in government bond returns. Afonso et al. (2011) document that there is an asymmetry between the information contained in negative and positive events. In particular they find that the reaction of government bond spreads to downgrades in the sovereign credit rating is more pronounced when compared to upgrades. Gande and Parsley (2005) report that spillovers from rating announcements in different countries are asymmetric, too. In a paper that is similar to our work, Vähämaa (2005) relates option-implied skewness in German bond futures to monetary policy actions. He finds that there is positive skewness when the monetary policy stance is tight meaning that investors believe that a sharp increase in yields is more likely than a sharp decline. In contrast to Vähämaa (2005), our study covers a broader set of countries and relates time-varying skewness

to macroeconomic and financial variables, rather than to policy announcements.

The remainder of this paper is organized as follows. Section 2 discusses the data on excess returns to government bonds. Section 3 reports the empirical results from testing for conditional asymmetry in government bond excess returns and it discusses the properties of quarterly asymmetry. Section 4 concludes.

## 2 Government Bond Excess Returns

In this section, we explain how we calculate government bond excess returns and describe the characteristics of these returns.

### 2.1 Data

For benchmark bond yields that can easily be obtained through various media, the remaining duration is decreasing every day. In addition, the return from the coupon of a benchmark bond is unknown and it varies across different benchmark bonds. Therefore, the returns to government bonds cannot be calculated from benchmark bond yields. Instead, we need to use zero coupon yields with constant maturity in order to compute a bond return that has a coupon with constant maturity. Zero coupon yields for Canada, Germany, Japan, UK and the U.S. for the period from 1997 to 2011 are publicly available at the web site of each Central Bank.<sup>3</sup> The reported maturity on these web sites is, however, quarterly or longer. Hence, in order to calculate daily returns, which is the targeted frequency in our analysis, we

---

<sup>3</sup>For Canada, <http://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/>.  
For Germany, [http://www.bundesbank.de/Navigation/EN/Statistics/Time\\_series\\_databases/Macro\\_economic\\_time\\_series/its\\_list\\_node.html?listId=www\\_s140\\_it03c](http://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/Macro_economic_time_series/its_list_node.html?listId=www_s140_it03c).  
For Japan, <http://www.imes.boj.or.jp/research/papers/japanese/12-J-03.txt>.  
For the U.K., <http://www.bankofengland.co.uk/statistics/Pages/yieldcurve/default.aspx>.  
For the U.S., <http://www.federalreserve.gov/pubs/feds/2006>.

apply the Svensson (1994) model

$$y^{(n)} = \beta_0 + \beta_1 \frac{\tau_1 - \tau_1 \exp\left(-\frac{n}{\tau_1}\right)}{n} \quad (1)$$

$$+ \beta_2 \left[ \frac{\tau_1 - \tau_1 \exp\left(-\frac{n}{\tau_1}\right)}{n} - \exp\left(-\frac{n}{\tau_1}\right) \right] + \beta_3 \left[ \frac{\tau_2}{n} - \exp\left(-\frac{n}{\tau_2}\right) \left(1 + \frac{\tau_2}{n}\right) \right]$$

to our data, where  $n$  is the maturity and  $y^{(n)}$  is the yield at maturity  $n$ . The time dependency of the parameters  $\beta_i$  and  $\tau_i$  is suppressed. Given the data on zero coupon yields, the unknown parameters of the model are estimated by nonlinear least squares with exception of Germany and the U.S.<sup>4</sup> For these countries, the parameter estimates are reported on the web site of the Central Bank. Using the model in equation (1) with estimated parameters, we can compute zero coupon yields as well as bond prices at any maturity  $n$ .

Let  $i_{it}$  denote the risk free funding rate in country  $i$  and at day  $t$ . We approximate the risk free rate by the policy rate.<sup>5</sup> All interest rates are expressed in annualized terms. The yield spread of a  $n$ -bond in period  $t$  and country  $i$ ,  $ys_{it}^{(n)}$ , is defined as

$$ys_{it}^{(n)} = y_{it}^{(n)} - i_{it}.$$

Bond prices  $b_{it}^{(n)}$  are calculated as

$$b_{it}^{(n)} = \exp\left(-ny_{it}^{(n)}\right).$$

Then the ex-post excess return of holding a bond for one period  $x_{it}^{(n)}$  is defined as

$$x_{it}^{(n)} = \log\left(\frac{b_{i,t+1}^{(n-1)}}{b_{it}^{(n)}}\right) - i_{it},$$

where  $\log\left(b_{i,t+1}^{(n-1)}/b_{it}^{(n)}\right)$  is expressed in annualized terms by dividing it with the length of the holding period as a fraction of one year.

## 2.2 Asymmetry Measures and other Descriptive Statistics

Figure 1 shows the time series of 10 year government bond excess returns that are standardized by median and interquartile range. To ease interpretation, horizontal

<sup>4</sup>We use yields in as many maturities as possible when estimating parameters.

<sup>5</sup>In the working paper version of our paper, Fujiwara, Körber and Nagakura (2011), we used LIBOR as the funding rate. In that version, the data ended in 2007 and we therefore excluded the period of the recent financial crisis when the LIBOR rates became extremely volatile.



lines at 3 and -3 are added. If the standardized series were normally distributed, the probability of obtaining a realization that exceeds 3 in absolute value is only approximately 0.0005.<sup>6</sup> There is a significant number of values that exceed 3 in absolute value. This observation suggests that the tails in the distribution of government bond excess returns are thicker than those of a normal distribution.

An alternative method to illustrate the properties of government bond excess returns are kernel density estimates. Figure 2 reports kernel density estimates of 10 year government bond returns when compared to a normal density with equal mean and standard deviation. We find that government bond excess returns have tails that are longer and thicker when compared to the normal density. The distributions itself seem to be almost symmetric although positive and negative outliers seem to make the estimated densities slightly skewed to the right or left.

Table 1 reports descriptive statistics of daily government bond excess returns. Excess returns are positive on average and increase in the maturity of the bond. Non-zero excess returns are at odds with the expectation hypothesis. The standard deviation of the excess returns tends to increase in the maturity as well, a finding that has been documented previously by Fama (1984).

We measure asymmetry in government bond excess returns by both the coefficient of skewness and the Bowley coefficient.<sup>7</sup> While the value of coefficient skewness is sensitive to outliers, the Bowley coefficient is robust against extreme observations.<sup>8</sup> We find that the coefficient of skewness is negative for almost all countries and maturities, or that there is a small probability of a large and negative excess return.

In contrast to the coefficient of skewness, the Bowley coefficient is positive. This

---

<sup>6</sup>For normally distributed random variables, the median and mean are identical and the interquartile range equals approximately 1.35 standard deviations. Let  $Z_t$  be a random variable standardized by mean and standard deviation, and let  $\tilde{Z}_t$  be the same random variable standardized by median and interquartile range. Then,  $P(|\tilde{Z}_t| > 3) = P(|Z_t| > 1.35 \times 3) \approx 0.00005$ .

<sup>7</sup>An estimator for the Bowley coefficient is  $\hat{\zeta}_B = \frac{\hat{F}^{-1}(0.75) + \hat{F}^{-1}(0.25) - 2\hat{F}^{-1}(0.5)}{\hat{F}^{-1}(0.75) - \hat{F}^{-1}(0.25)}$ , where  $\hat{F}^{-1}(\tau) \equiv \inf\{x : \hat{F}(x) > \tau\}$  is the  $\tau$ th sample quantile,  $\hat{F}(x) \equiv \sum_{t=1}^T I(X_t \leq x)$ ,  $x \in \mathbb{R}$  is the empirical distribution function and  $I(\cdot)$  is the indicator function that is 1 if  $X_t \leq x$  and 0 otherwise. See Kim and White (2004) for other robust asymmetry measure

<sup>8</sup>The size of the different measures cannot be directly compared because the coefficient of skewness can take values on the real line, while the Bowley coefficient is bounded.

observation seems to be paradoxical at first, but it can arise in situations where the right tail of the distribution is thick (mean  $\geq$  median) but the left tail is long due to a single negative outlier (von-Hippel and Paul, 2005). This dissonance implies that skewness in bond returns are mainly driven by extreme observations. Although government bonds are usually considered as risk free assets, they are subject to a tail risk in form of a sudden drop in bond prices, similar to other risky assets such as equity returns and exchange rates.

### 3 Empirical Results

In this section, we introduce the test for conditional asymmetry of Bai and Ng (2001) and apply it to our data. Then, we calculate quarterly asymmetry measures from daily non-overlapping data on government bond excess returns and investigate their properties.

#### 3.1 Conditional Asymmetry in Government Bond Returns

To investigate if there is conditional asymmetry in the distribution of daily government bond excess returns, we apply the conditional symmetry test of Bai and Ng (2001) to our data set. Assume that a time series  $\{x_t\}_{t=1}^T$  can be represented by

$$x_t = h(\Omega_t, \beta) + \sigma(\Omega_t, \lambda)e_t, \quad (2)$$

where  $h(\Omega_t, \beta)$  is the conditional mean,  $\sigma^2(\Omega_t, \lambda)$  is the conditional variance and  $e_t$  is a disturbance term with zero mean and unit variance that is independent of  $\{x_t\}_{t=1}^t$ .  $\Omega_t$  denotes the information set at time  $t$ . The time series  $\{x_t\}_{t=1}^T$  does not necessarily have to be stationary nor i.i.d. <sup>9</sup>

In equation (2), conditional symmetry of  $\{x_t\}_{t=1}^T$  is equivalent to symmetry of  $e$  around zero, that is,  $f(e) = f(-e)$ , where  $f$  denotes the density of  $e$ . One situation that gives rise to conditional symmetry is when a time series reacts more strongly to negative announcements when compared to positive announcements. There is

---

<sup>9</sup>For our data set, we tested the i.i.d. assumption by the BDS test (Brock et al. (1996), Kanzler (1999)) and we rejected the null hypothesis that the data is i.i.d.

empirical evidence that this is the case for government bond yields (Afonso et al., 2011, Gande and Parsley, 2005).

The idea behind the test of Bai and Ng (2001) is that if  $\{x_t\}_{t=1}^T$  is conditionally symmetric, then the difference between the empirical distribution function of  $e_t$  and that of  $-e_t$  is small or  $W_T(x) = \frac{1}{\sqrt{T}} \sum_{t=1}^T [I(e_t \leq x) - I(-e_t \leq x)]$  is approximately equal to zero.

However, the disturbances  $e_t$  are unobserved and must be replaced by the normalized regression residuals  $\hat{e}_t = \frac{x_t - h(\Omega_t, \hat{\beta})}{\sigma(\hat{\Omega}_t, \hat{\lambda})}$  where  $\hat{\beta}$  and  $\hat{\lambda}$  are  $\sqrt{T}$  consistent estimators for  $\beta$  and  $\lambda$ , and  $\hat{\Omega}_t$  is the feasible information set at time  $t$ , or the information set available in practice. However, in contrast to  $W_T(x)$ , any test statistic based on  $\widehat{W}_T(x) = \frac{1}{\sqrt{T}} \sum_{t=1}^T [I(\hat{e}_t \leq x) - I(-\hat{e}_t \leq x)]$  is not asymptotically distribution free, but depends on the time series  $\{x_t\}_{t=1}^T$ . To obtain a distribution free test, Bai and Ng (2001) use a martingale transformation method. The test statistic is defined as  $CS = \max_x |S_T(x)|$  where  $S_T(x)$  is the martingale transformation of  $\widehat{W}_T(x)$ .<sup>10</sup> Under certain regularity conditions, Bai and Ng (2001) show that  $CS \rightarrow^d \max_{0 \leq x \leq 1} |B(x)|$  where  $B(x)$  is a standard Brownian motion on  $[0, 1]$ .

To apply the test of Bai and Ng (2001) in practise, we need to specify a functional form for the conditional mean  $h(\cdot)$  and the conditional variance  $\sigma^2(\cdot)$ . For our application to government bond returns, we assume a constant conditional mean. Given the small autocorrelation in returns, we expect that this specification is empirically approximately valid. We also assume that the errors are conditionally normally distributed and that the error variance has a GARCH(1,1) representation.

Panel a) in Table 2 reports the results for the test for conditional symmetry. The critical values for that test are 1.91 at a significance level of 10%, 2.21 at 5% and 2.78 at 1% (Bai and Ng, 2001) We find that conditional asymmetry is statistically significant in 11 out of 15 cases. Asymmetry tends to be more significant in 2 year bonds when compared to longer maturities, but there is also evidence for significant conditional asymmetry in 10 year bonds for Canada, Germany and Japan.<sup>11</sup>

<sup>10</sup>We refer to Bai and Ng (2001) for details.

<sup>11</sup>The results are robust to alternative specifications of the GARCH model. Test results based on a GARCH model with a time-varying conditional mean or student-t errors are reported in Tables

Previously, evidence on conditional asymmetry in equity returns as been documented by Bai and Ng (2001) and Grigoletto and Lisi (2008). Our results suggest that conditional asymmetry is not only a characteristic of equity returns, but also of government bond returns. This finding has important implications for modelling government bond returns and for portfolio choice. Models that are popular for the analysis of government bond returns usually assume symmetrically distributed errors. In addition, asymmetry in government bond returns implies that mean and variance are not sufficient to characterize the risk in government bond returns, and has thus important implications for optimal portfolio allocation (Chunhachinda et al. (1997)).

### **3.2 Properties of Time-varying Asymmetry**

In this section, we analyze how asymmetries co-move across countries and how they are related to financial and macroeconomic variables.

#### **Cross-country Co-movement of asymmetries in Government Bond Excess Returns**

Figures 3 and 4 show the time series of the coefficient of skewness and the Bowley coefficient across different maturities. Casual inspection of these time series reveals that there is a significant amount of time-variation that is decreasing in the maturity of the bond. In addition, the asymmetry measures are correlated across maturities. In particular in times of crisis, the coefficient of skewness tends to co-move across countries. For example, Panel a) of Figure 5 shows that the coefficient of skewness for 10 year bonds was strongly negative in the third quarter of 2008 in Canada, UK and the U.S. before becoming positive in the first quarter of 2009. In contrast, less cross-country co-movement is observed for the Bowley coefficient as shown in Panel b) of Figure 5.

To further investigate the cross-country co-movement of the different asymmetry measures, Tables 4 to 5 report the cross-country correlation coefficients. According to the coefficient of skewness, asymmetries in government bond excess returns are

---

A1 and A2 in the appendix.

positively correlated across countries and the correlation coefficient is significantly different from zero for most country pairs. These findings support the evidence of Ghysels et al. (2011) that common factors are important in explaining asymmetries in developed countries. In contrast, the sign of the Bowley coefficient is mixed in particular for short maturities and it is only significant between the U.S. on the one hand and Canada and Germany on the other. Because the Bowley coefficient is robust to extreme events while the coefficient of skewness is not, the observation that there is more co-movement according to the coefficient of skewness suggests that the cross-country correlation of asymmetries is driven by extreme events, such as the bankruptcy of Lehman brothers in the third quarter of 2008.

Another observation is that for both asymmetry measures, the correlation coefficient increases in the maturity of the bond. This implies that for longer maturities, common factors are even more important in driving asymmetries when compared to idiosyncratic factors.

### **Forecasting Asymmetry Using Macroeconomic and Financial Variables**

How are these quarterly asymmetry measures for government bond excess returns related to financial variables and macroeconomic conditions? In the remainder of this section, we follow Brunnermeier et al. (2009) and assess if asymmetry can be forecasted using past data on yield spreads, government bond returns, and macroeconomic conditions. In this Section, we restrict attention to bonds with a maturity of 10 years. We estimate a panel data model of the form

$$AS_{i,t+1} = \alpha_i + \beta FIN_{i,t} + \gamma MA_{i,t} + \delta R_t + \epsilon_{i,t}, \quad t = 1, \dots, T, i = 1, \dots, N \quad (3)$$

where  $AS_{i,t+1}$  is the coefficient of skewness in country  $i$  and quarter  $t + 1$ .  $FIN_{i,t}$  denotes financial variables such as the excess return or the yield spread.  $MA_{i,t}$  controls for macroeconomic variables like inflation, GDP growth, the monetary policy rate and the amount of government bonds outstanding to GDP. With exception of the amount of government bonds outstanding, all variables are taken from the International Financial Statistics that is published by the IMF. The amount of government bonds outstanding is from the International Securities Statistics of the

Bank for International Settlements. This variable has been used previously a proxy of the total liquidity in the government bond market (D'Agostino and Ehrmann (2012)).  $R_t$  is a common factor for global risk aversion that is approximated by the spread between an US corporate and a US government bond. This variable is often used in studies of government bond spreads (Attinasi et al. (2010)) and is obtained from the ECB database.  $\alpha_i$  are country fixed effects that can be correlated with the regressors in an arbitrary way. The error terms are assumed to be strongly exogenous with respect to the fixed effects  $\alpha_i$  and the regressors  $FIN_{i,t}$ ,  $MA_{i,t}$  and  $R_t$ . We estimate the model in (3) by the fixed effects estimator.

Table 6 reports the estimation results for 10 year bonds. When predicting the coefficient of skewness using macroeconomic and financial variables, we find that market liquidity at the end of the previous quarter is positively related to the coefficient of skewness. This finding suggests that a negative coefficient of skewness or small risk of a very negative return is more likely in less liquid markets. The other macroeconomic variables in contrast, are not statistically significant.

Second, the yield spread is not a significant predictor of skewness. The yield spread can be interpreted as an expected excess return when the yield curve does not shift, or the bond price does not change. Therefore, this finding implies that expected returns are not related to skewness in yield carry trades. This result is in contrast to Brunnermeier, et al. (2008). They find that there is negative skewness in the exchange rate conditional on a positive interest rate differential. Interest rate differentials can be considered as expected excess returns from carry trades when the exchange rate does not change. Positive interest rate differentials are considered as a compensation for the risk of an abrupt appreciation of low interest rate currencies, or crash risk in their terminology. The counterpart of the interest rate differential in the context of yield carry trades is the yield spread, which is defined as the difference between bond yield and funding rate. Although returns from investments in both government bonds and exchange rates are skewed, the nature of yield carry trades seems to differ from carry trades.

Third, a positive realized excess return is associated with a negative coefficient of skewness, or a small probability of a large and negative return in the future. Negative

skewness conditional on a positive realized excess return has been documented previously by Brunnermeier et al. (2009) in the context of carry trades. To interpret this finding, they show that “currency gains lead to larger speculator positions and larger future crash risk” (p. 329). However, since we do not have quarterly data on positions in government bonds, we have not tested if the same explanation is valid for government bonds.

## 4 Conclusion

In this paper, we investigated asymmetries in the distribution of government bond excess returns in developed countries. We find that there is conditional asymmetry in government bond excess returns especially if maturities are short. This finding has important implications for models that are widely used in the analysis of government bond markets, since such models usually assume symmetrically distributed errors. The evidence presented in this paper questions this assumption and calls for further research that is consistent with the evidence for asymmetry in government bond excess returns. Second, we show that liquidity in government bond markets predicts the coefficient of skewness with a positive sign, meaning that the probability of a large and negative excess return is more likely in a less liquid market. In addition, we document a negative relationship between the past excess return and the coefficient of skewness.

The analysis so far is restricted to investments in domestic government bonds. As shown by Andritzky (2012), a considerable amount of government debt is owned by non-residents. Therefore, an interesting question is to analyze the characteristics of international government bond returns. These returns are related to both domestic government bond returns and exchange rates. This question is left for future research.

**Table 1:** Descriptive statistics for daily government bond excess returns

(a) 2 year bonds					
	CN	GER	JP	UK	US
Mean	0.010	0.010	0.002	0.013	0.012
Median	0.008	0.009	0.001	0.012	0.007
Standard deviation	0.086	0.089	0.034	0.081	0.107
Bowley coefficient	0.072	0.037	0.122	0.056	0.117
Coefficient of skewness	-0.388	-0.069	-0.104	-0.058	-0.106
Kurtosis	7.041	7.044	16.653	6.495	6.846
(b) 5 year bonds					
	CN	GER	JP	UK	US
Mean	0.014	0.016	0.006	0.017	0.018
Median	0.011	0.013	0.006	0.016	0.014
Standard deviation	0.211	0.235	0.136	0.218	0.297
Bowley coefficient	0.080	0.061	0.006	0.036	0.075
Coefficient of skewness	-0.320	-0.073	-0.456	-0.019	-0.091
Kurtosis	6.202	5.393	9.514	6.019	5.906
(c) 10 year bonds					
	CN	GER	JP	UK	US
Mean	0.019	0.023	0.012	0.020	0.025
Median	0.013	0.019	0.010	0.017	0.030
Standard deviation	0.409	0.446	0.302	0.438	0.580
Bowley coefficient	0.065	0.048	0.060	0.028	0.029
Coefficient of skewness	-0.203	-0.315	-0.711	0.046	-0.042
Kurtosis	7.896	7.605	11.555	8.181	6.403

**Table 2:** Testing conditional asymmetry in daily government bond excess returns

	CN	GER	JP	UK	US
2 year bond	3.137***	2.007*	4.202***	2.217**	4.081***
5 year bond	2.889***	2.012*	1.478	1.199	2.448**
10 year bond	2.596**	2.020*	2.635**	0.822	1.340

Notes: Critical values are 1.91 (10%), 2.21 (5%) and 2.78 (1%). Significance at 10%, 5% and 1% is indicated with \*, \*\* and \*\*\*, respectively. Number of observations are 3566, 3658, 3540, 3642, 3602 for CN, GER, JP, UK and U.S.



**Table 3:** Cross-country correlations in quarterly asymmetry measures for 2 year government bond excess returns

a) Coefficient of skewness

	CN	GER	JP	UK	US
CN	1.000	0.217	0.040	0.454*	0.587*
GER	0.217	1.000	0.227	0.291*	0.331*
JP	0.040	0.227	1.000	0.050	0.165
UK	0.454*	0.291*	0.050	1.000	0.370*
US	0.587*	0.331*	0.165	0.370*	1.000

b) Bowley coefficient

	CN	GER	JP	UK	US
CN	1.000	0.054	0.125	0.045	0.334*
GER	0.054	1.000	-0.095	-0.049	-0.082
JP	0.125	-0.095	1.000	0.015	0.078
UK	0.045	-0.049	0.015	1.000	0.142
US	0.334*	-0.082	0.078	0.142	1.000

Notes: \* denotes significance at 5%.

**Table 4:** Cross-country correlations in quarterly asymmetry measures for 5 year government bond excess returns

a) Coefficient of skewness

	CN	GER	JP	UK	US
CN	1.000	0.061	0.262*	0.414*	0.727*
GER	0.061	1.000	0.472*	0.469*	0.156
JP	0.262*	0.472*	1.000	0.367*	0.355*
UK	0.414*	0.469*	0.367*	1.000	0.442
US	0.727*	0.156	0.355*	0.442*	1.000

b) Bowley coefficient

	CN	GER	JP	UK	US
CN	1.000	0.121	0.090	0.220	0.358*
GER	0.121	1.000	0.041	-0.053	0.244
JP	0.090	0.041	1.000	-0.043	-0.122
UK	0.220	-0.053	-0.043	1.000	0.180
US	0.358*	0.244	-0.122	0.180	1.000

Notes: \* denotes significance at 5%.

**Table 5:** Cross-country correlations in quarterly asymmetry measures for 10 year government bond excess returns

a) Coefficient of skewness

	CN	GER	JP	UK	US
CN	1.000	0.199	0.253	0.515*	0.710*
GER	0.199	1.000	0.460*	0.543*	0.265*
JP	0.253	0.460*	1.000	0.279*	0.226
UK	0.515*	0.543*	0.279*	1.000	0.555*
US	0.710*	0.265*	0.226	0.555*	1.000

b) Bowley coefficient

	CN	GER	JP	UK	US
CN	1.000	0.253	0.182	0.035	0.516*
GER	0.253	1.000	0.130	0.161	0.277*
JP	0.182	0.130	1.000	0.145	0.096
UK	0.035	0.161	0.145	1.000	0.041
US	0.516*	0.277*	0.096	0.041	1.000

Notes: \* denotes significance at 5%.

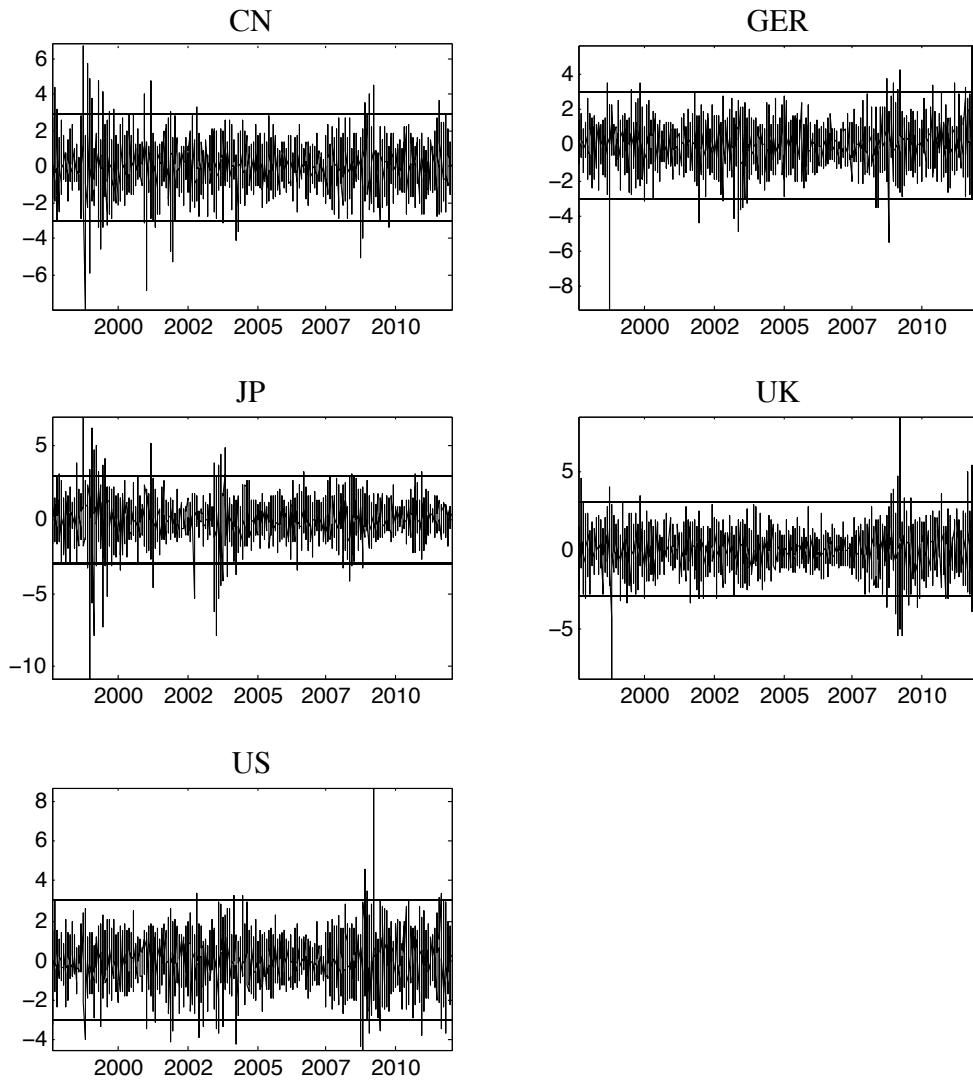
**Table 6:** Predicting the coefficient of skewness using macroeconomic and financial variables

VARIABLES	$Skew_{t+1}$	$Skew_{t+1}$	$Skew_{t+1}$	$Skew_{t+1}$	$Skew_{t+1}$
Yield spread	-0.012 (0.044)	0.038 (0.062)			0.052 (0.062)
Market liquidity/GDP		0.006* (0.003)		0.006* (0.003)	0.006* (0.003)
GDP growth		0.019 (0.023)		0.022 (0.023)	0.021 (0.023)
Inflation		-0.001 (0.039)		-0.001 (0.040)	0.000 (0.040)
Policy rate		-0.022 (0.032)		-0.004 (0.024)	-0.020 (0.031)
Risk aversion		0.000 (0.000)		0.000 (0.001)	0.000 (0.001)
Excess return			-1.405 (0.899)	-1.684* (0.963)	-1.716* (0.966)
Constant	-0.051 (0.171)	-0.523* (0.299)	-0.072** (0.035)	-0.407** (0.191)	-0.582* (0.298)
Observations	285	280	285	280	280
R-squared	0.000	0.024	0.016	0.043	0.045

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

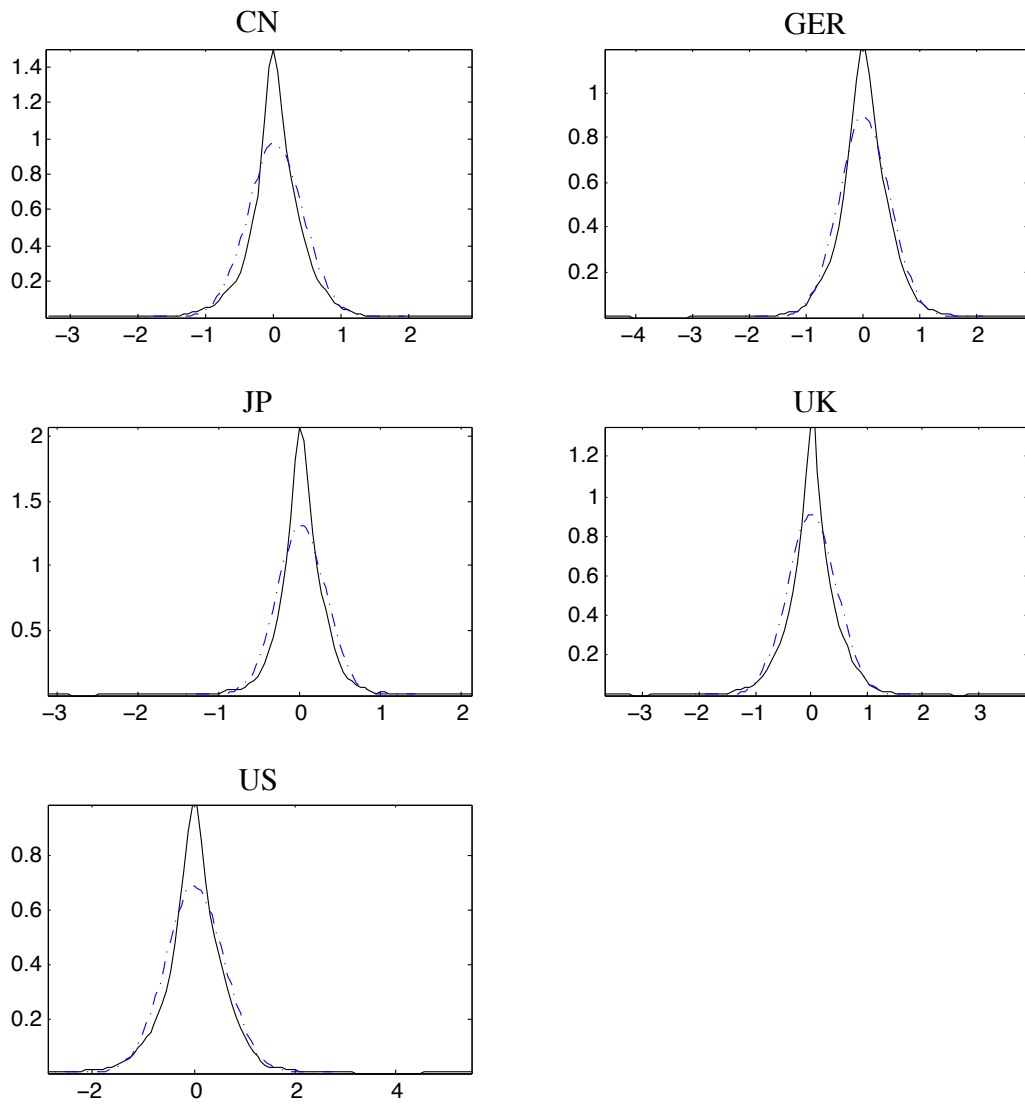
Notes: All regressions include country fixed effects. Robust standard errors are reported in in parenthesis. Frequency is quarterly. Dependent variable is the coefficient of skewness calculated from daily excess returns to 10 year government bond returns.

Figure 1: Daily 10 year government bond excess returns



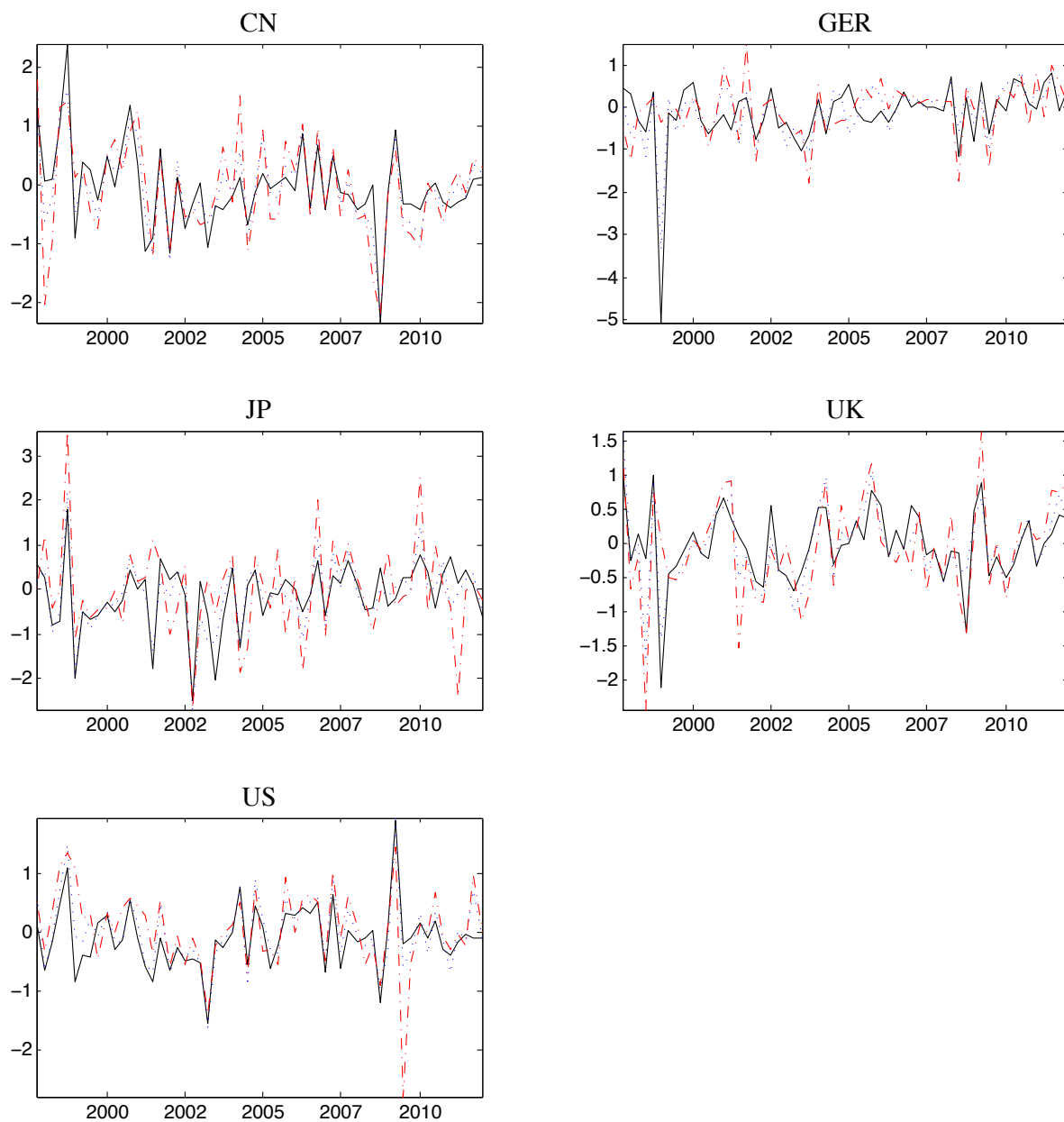
Note: Excess returns are standardized by median and interquartile range. A horizontal line at 3 and -3 is added.

**Figure 2:** Kernel density estimates of daily 10 year government bond excess returns



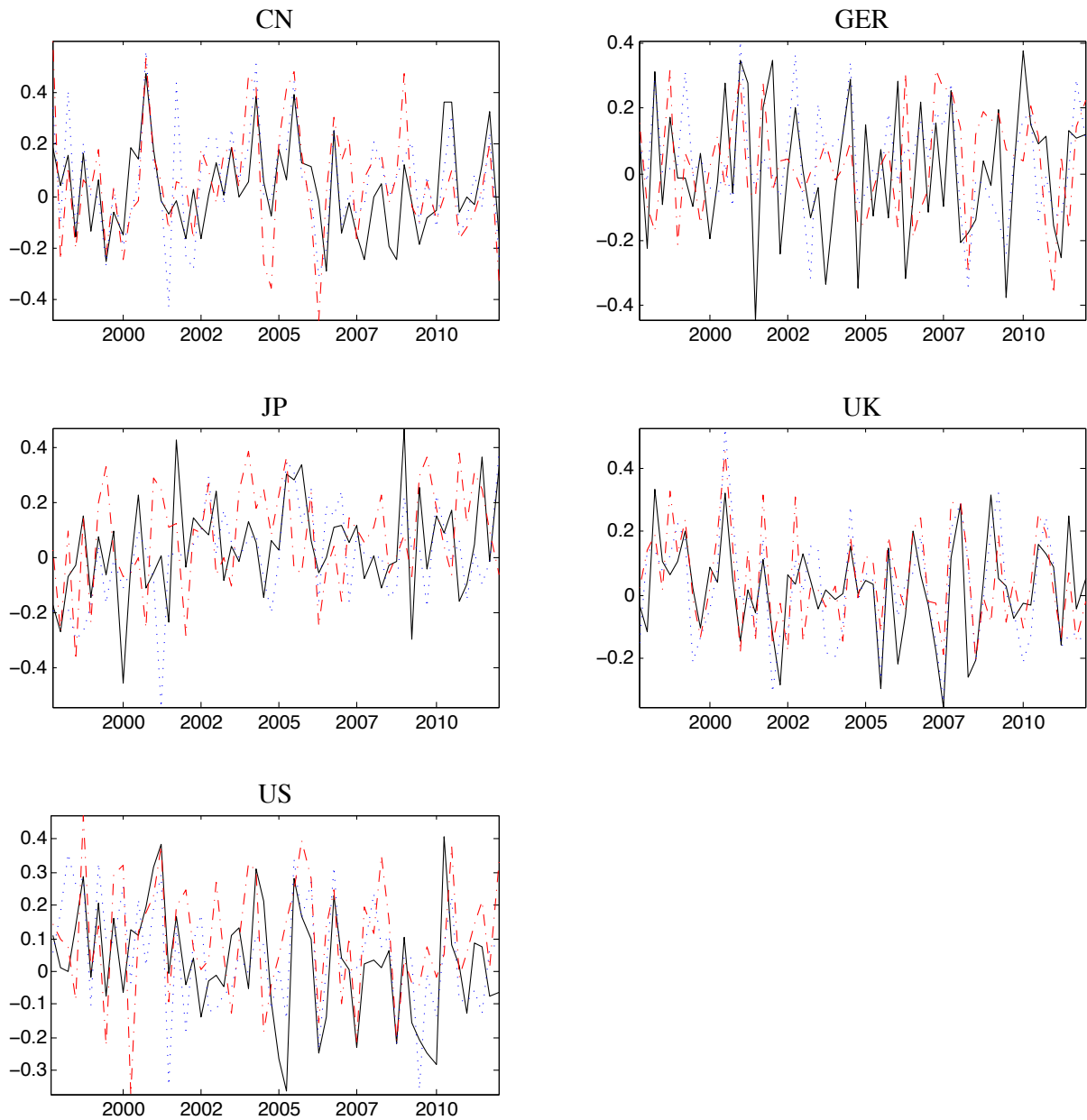
Note: A normal density with equal standard deviation and mean is superimposed in a dashed line

**Figure 3:** Quarterly coefficients of skewness of government bond excess returns



Note: 10 year bonds are shown as a solid black line, 5 year bonds as a dotted blue line and 2 year bonds as a dashed red line.

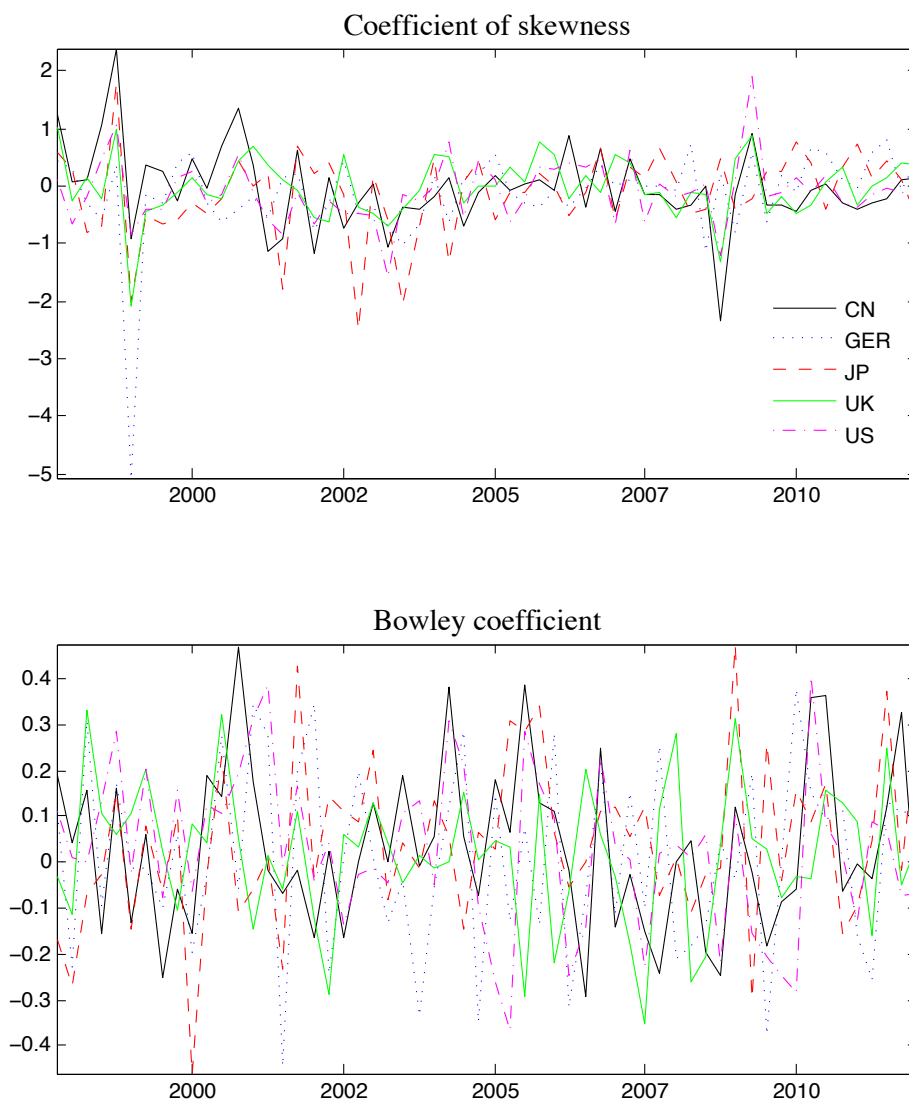
**Figure 4:** Quarterly Bowley coefficients of government bond excess returns



Note: 10 year bonds are shown as a solid black line, 5 year bonds as a dotted blue line and 2 year bonds as a dashed red line.



**Figure 5:** Quarterly asymmetry measures across countries for 10 year government bond excess returns



## Appendix

**Table A1:** Conditional asymmetry tests: Allowing for a time-varying conditional mean

	CN	GER	JP	UK	US
2 year bond	3.229***	2.020*	3.698***	1.970*	4.020***
5 year bond	2.762**	1.929*	1.393	1.189	2.460**
10 year bond	2.541**	2.147*	2.591**	0.831	1.350

Notes: The conditional mean equation includes the past excess return. Critical values are 1.91 (10%), 2.21 (5%) and 2.78 (1%). Significance at 10%, 5% and 1% is indicated with \*, \*\* and \*\*\*, respectively. Number of observations are 3566, 3658, 3540, 3642, 3602 for CN, GER, JP, UK and U.S.

**Table A2:** Conditional asymmetry tests: Assuming student-t errors

	CN	GER	JP	UK	US
2 year bond	2.924***	2.069*	3.443***	2.313**	3.494***
5 year bond	2.876**	1.930*	1.560	1.284	2.447**
10 year bond	2.465**	2.166*	2.658**	0.997	1.331

Notes: Critical values are 1.91 (10%), 2.21 (5%) and 2.78 (1%). Significance at 10%, 5% and 1% is indicated with \*, \*\* and \*\*\*, respectively. Number of observations are 3566, 3658, 3540, 3642, 3602 for CN, GER, JP, UK and U.S.

## References

- Andritzky, Jochen, 2012. Government Bonds and their Investors: What Are the Facts and Do They Matter? IMF working paper.
- Afonso, Antonio, Furceri, Davide and Pedro Gomez, 2011. Sovereign Credit Ratings and Financial Markets Linkages: Application to European Data, ECB working paper 1347.
- Attinasi, Maria-Grazia, Checherita-Westphal, Cristina and Christiane Nickel, 2009. What explains the surge in euro area sovereign spreads during the financial crisis of 2007-09? *Public Finance and Management*, 10 (4), 595-645.
- Backus, David, Silverio Foresi, Abon Mozumdar, and Liuren Wu, 2001, Predictable Changes in Yields and Forward Rates, *Journal of Financial Econometrics*, 59, 281-311.
- Bai, Jushan, and Serena Ng, 2001, A Consistent Test for Conditional Symmetry in Time Series Models, *Journal of Econometrics*, 103, 225-258.
- Bekaert, Geert, Claude Erb, Campbell Harvey and Tadas Viskanta, 1998, Distributional Characteristics of Emerging Market Returns and Asset Allocations, *The Journal of Portfolio Management* 24, 102-116.
- Brock, William, William Dechert, Jose Scheinkman, and Blake LeBaron, 1996, A Test for Independence Based on the Correlation Dimension, *Econometric Reviews* 15, 197-235.
- Brunnermeier, Markus, Stefan Nagel, and Lasse Pedersen, 2009, Carry Trades and Currency Crashes, *NBER Macroeconomics Annual 2008*, 313-347.
- Campbell, John, and Robert Shiller, 1995, Yield Spreads and Interest Rate Movements: A Bird's Eye View, *Review of Economic Studies*, 58, 495-514.
- Chunhachinda, P., Dandapani, K., Hamid, S. and A. Prakash, 1997. Portfolio Selection and Skewness: Evidence fro International Stock Markets, *Journal of Banking and Finance*, 21 (2), 143-167.

- Cochrane, John and Monika Piazzesi, 2005. Bond Risk Premia, *American Economic Review* 95, 138-160.
- Antonello, D'Agostino and Michael Ehrmann, 2012. The Pricing of G7 Sovereign Bond Spreads - the Times, they are Changing, MPRA working paper.
- Fama, Eugene, 1984, Term Premiums and Bond Rerturns, *Journal of Financial economics* 13, 529-546.
- Fama, Eugene and Robert Bliss, 1987, The Information in Long-Maturity Forward Rates, *American Economic Review* 77, 680-92.
- Fujiwara, Ippei, Lena Körber, and Daisuke Nagakura, 2011, How Much Asymmetry is There in Bond Returns and Exchange Rates? *Globalization and Monetary Policy Institute Working Paper* 93.
- Gande, A. and D. Parsley, 2005. News spillovers in the sovereign debt market, *Journal of Financial Economics*, 75, 691-734.
- Ghysels, Eric, Plazzi, Alberto and Rossen Valkanov, 2011. Conditional Skewness of Stock MArket Returns in Developed and Emerging Markets and its Economic Fundamentals, Swiss Finance Institute Research Paper No. 11-06.
- Grigoletto, Matteo and Lisi, Francesco, 2009, Looking for Skewness in Financial Time Series. *The Econometrics Journal* 12, 310-323.
- Harvey, Campbell R. and Akhtar Siddique, 1999. Autoregressive Conditional Skewness. *Journal of Financial and Quantitative Analysis*, 34 (4), 465-487.
- von Hippel, Paul T., 2005, Mean, Median, and Skew: Correcting a Textbook Rule. *Journal of Statistics Education* 13 (2).
- Jondeau, E. and M. Rockinger, 2006. Optimal Portfolio Allocation Under Higher Moments, *European Financial Management*, 12(1), 2955.
- Kanzler, Ludwig 1999, Very Fast and Correctly Sized Estimation of the BDS Statistic, DPhil Thesis, University of Oxford.

- Kim, Tae-Hwan, and Halbert White, 2004, On more Robust Estimation of Skewness and Kurtosis, *Finance Research Letters* 1, 56-73.
- Peiro, Amado, 2002, Skewness in Individual Stocks at Different Investment Horizons, *Quantitative Finance* 2, 139-146.
- Piazzesi, Monika, 2010, Affine Term Structure Models, *Handbook of Financial Econometrics* 1, 691-766.
- Premaratne, Gamini, and Anil Bera, 2005, A Test for Symmetry with Leptokurtic Financial Data, *Journal of Financial Econometrics* 3, 169-187.
- Svensson, Lars, 1994, Estimating and Interpreting Forward Rates: Sweden 1992-4, *NBER Working Paper* 4871.
- Vähämaa, Sami, 2005, Option-Implied Asymmetries in Bond Market Expectations around Monetary Policy Actions of the ECB, *Journal of Economics and Business* 57, 23-38.