Testing for Explosive Behavior in Relative Price Measures: Implications for Inflation Expectations*

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Abstract

We show that U.S. consumer inflation expectations behave differently when food and energy prices rise and fall sharply relative to other prices for the 1982-2010 period. Using the recently proposed test of Phillips et al. (2011a), we identify three periods where the headline price index of personal consumption expenditures (PCE) moves explosively relative to the core PCE. During upward explosive periods, consumers are more forward-looking in that they rely less on past inflation in forming inflation expectations as compared with non-explosive and downward explosive periods. Both current and past unemployment and interest rates can also improve consumer inflation expectation predictions when headline PCE deviates explosively upward from core PCE. The changes in inflation expectations are also found to be more important than the relative volatile periods implied by a Markov-switching model. Taken together, our results indicate that the upward explosive behavior of food and energy prices should be taken into consideration when designing policies that aim to anchor inflation expectations.

Keywords: Explosive behavior, core inflation, relative measure, inflation expectations.
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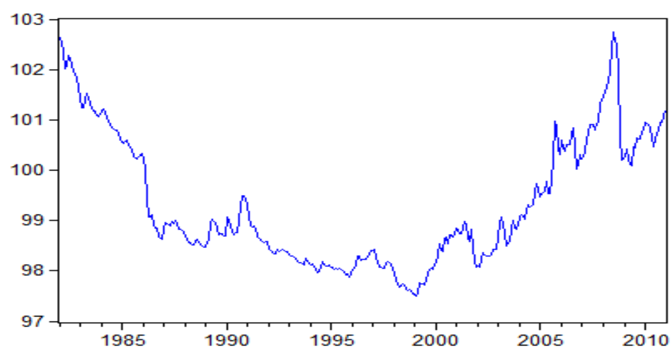
1 Introduction

Many central bankers focus on measures of core inflation which exclude certain items that face volatile price movements, notably food and energy.[1] Consumers confront individual prices rather than price indices, and might interpret large changes in energy or food items as signals of emerging inflation, changing their expectations about its future path.[2] However, quantifying the divergences between energy and food prices and the prices of other goods which can lead consumers to change their inflation expectations is difficult. This presents a policy challenge for central bankers because the impact of monetary policy depends on these inflation expectations being "anchored".

In this paper we examine the impact of explosive (large and sudden) deviations between energy and food prices and other prices on consumer inflation expectations.[3] To identify the periods where headline PCE has deviated from core PCE in an explosive manner we use the recently proposed test of Phillips et al. (2011a). We then compare the behavior and formation of consumer inflation expectations during these explosive periods with other periods using the methods of Mankiw et al. (2004). We also compare the behavior and formation of consumer inflation expectations within the explosive periods themselves. As an alternative to the regime classification, our final exercise is to use a Markov-switching model in analysing these inflation expectations. Our work is motivated by the recent and important debate over the appropriateness of using core or headline measures of inflation expectations in conducting monetary policy, as discussed by Thornton (2007), Bodenstein et al. (2008), Thornton (2011), and Bullard (2011).

This debate has recently been revived as food and energy prices have sharply increased relative to the prices of other goods. The prices of food and energy have recently shown substantial deviations from headline price indices. Figure 1 highlights this fact by plotting the PCE index deflated by the core PCE index. As we can see, differences between the headline and core price measures have been observed over several decades and are not just a transitory phenomenon.

Figure 1: Headline PCE (deflated by core PCE) from January 1982 to December 2010



This divergence in price indices may create some difficulty when trying to anchor consumer inflation expectations. Bullard (2011) has argued that relative price movements can have an important impact on the public's inflation expectations. Commodities such as oil and food are particularly important in

[1] The Federal Reserve, for instance, closely monitors the rate of growth of the core personal consumption expenditure (PCE) deflator. Since February 2000 in the Monetary Policy Report to the Congress, the Federal Reserve Board reports the projections of Federal Open Market Committee participants regarding core PCE inflation, rather than headline inflation which include food and energy prices.

[2] Within this spirit, Trehan (2011) argues that households are more sensitive to changes in commodity prices and tend to respond by revising their inflation expectations by more than historical relationships warrant.

[3] Throughout the paper we refer to *explosiveness* as the statistical property of a time series whose characteristic equation has a root inside the unit circle.

this case. Consumers frequently observe food prices. Moreover, oil prices impact the production and distribution costs of a broad range of different goods and services, affecting many other prices. Thus it is plausible that a sharp or prolonged divergence of food and energy prices from other prices in the economy may pose some challenges when trying to anchor inflation expectations. Bernanke (2007) has argued that when inflation expectations become unanchored the stance of monetary policy may not be as effective.

Central bankers tend to focus more on core inflation, the narrower measure, because it can help prevent them from responding too strongly to transitory movements in inflation. Measures of core inflation attempt to strip out or smooth volatile changes in prices to distinguish the inflation signal from transitory noise.[4] The underlying assumption of this policy measure is that differences in food and energy prices will not be prolonged, and so will not affect the general price level in the medium-term. However, it is unclear how central bankers should react when there are sharp differences between core and non-core measures of inflation as seen in Figure 1.

To some extent this depends on the how quickly such price increases might feed through to the headline price level, the expected duration of the price spikes, and the impact on consumer inflation expectations. We have nothing here to say about the first or second point, although we do assess the actual duration of such price spikes. With respect to the impact on inflation expectations, it is reasonable to assume that with quickly rising food and energy prices, households will be paying less attention to core inflation when forming their inflation expectations. If nothing else, focusing on a narrow definition of the price level can complicate the conduct of monetary policy in this case if household inflation expectations are not anchored.

We assess if and how consumer inflation expectations change when food and energy prices deviate *sharply* from other prices in the economy. We define *sharp deviations* as periods of explosive deviation between core and headline price measures for the period 1982-2010. To identify these explosive periods, we apply the recently proposed test of Phillips et al. (2011a), PSY hereafter, which generalises the test of Phillips et al. (2011b), PWY hereafter. The test of PSY is capable of locating locally (temporarily) explosive behavior within the sample period. The locating strategy utilises information up to the current period and can be used as a warning mechanism for the existence of explosive behavior.[5] Our results indicate that the headline measure of PCE deviates from core PCE in an explosive manner on three occasions in our sample.

We also isolate the components of the headline measure which are responsible for this behavior. Results indicate that two of the explosive periods may be due to energy supply shocks. From March 2008 to September 2008, both food and energy are behind this rise, as each individual index also shows patterns of explosive movements. Inflation expectations, as measured by surveys, also rise during this period. Additionally, it seems that large energy price movements drive the explosive differences in headline versus core PCE. All three periods of explosive behavior in the headline series correspond to periods of explosive behavior in the energy series. This is not true of the food index, as there is a period of explosive behavior in 2001-2002 that is not represented in the headline measure.

We then assess the impact of periods with sharp differences on consumer inflation expectations using the methods of Mankiw et al. (2004). First, during non-explosive periods we are unable to reject the hypothesis that inflation expectations are formed by adaptive expectations. But the hypothesis of adaptive expectations is decisively rejected during explosive periods. This indicates that these sharp deviations may be important in determining how inflation expectations evolve. Second, our results also show that during the periods of *upward* explosive behavior consumers are more forward-looking, as they rely less on past inflation in forming their expectations. Consumers also utilise both current and past unemployment and interest rates in forming inflation expectations during those periods.

[4] See Motley (1997) and Mehra and Sawhney (2010) for more on this topic.

[5] The PSY test was applied to historical stock market data, and identified many periods of explosive behavior between 1871 and 2010 (Phillips et al., 2011a).

Finally, we consider the impact of volatilities in the relative measure between headline and core inflation on consumers' inflation expectations using a Markov-switching model and the methods of Mankiw et al. (2004). Our findings suggest that explosive deviations between headline and core inflation are more important than the relative volatile periods implied by the Markov-switching model. Taken together, our results indicate that the explosive behavior of food and energy prices should be taken into consideration when evaluating policies that rely on consumer inflation expectations, particularly monetary policy.

2 Testing for Explosive Deviations

PSY show that the sup augmented Dickey-Fuller (SADF) test of PWY may fail to reveal the existence of explosive behavior when there are multiple episodes of this behavior within the same sample period. The generalised sup ADF (GSADF) test of PSY, which was proposed to address this difficulty, significantly improves discriminatory power. In particular, PSY demonstrate via simulations that the GSADF test has significantly higher power than the SADF test in identifying the existence of explosive behavior. Furthermore, they show when there are multiple explosive episodes in the sample period, the GSADF test can estimate the origination and termination dates of those explosive episodes consistently, whereas the SADF can only consistently estimate dates associated with the first episode.[6]

Before outlining the GSADF test of PSY, we first introduce a backward sup ADF test. The backward sup ADF test implements a right-tailed unit root test (against an explosive alternative) repeatedly on a backward expanding sample sequence. Suppose r_1 is the (fractional) starting point of a regression sample and r_2 is the (fractional) ending points of the sample. The empirical regression model is

$$\Delta y_t = \alpha_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{i=1}^{k} \psi^i_{r_1,r_2} \Delta y_{t-i} + \varepsilon_t, \tag{1}$$

where k is the lag order and $\varepsilon_t \overset{iid}{\sim} N\left(0, \sigma^2_{r_1,r_2}\right)$. The number of observations in the regression is $T_w = \lfloor T r_w \rfloor$, where $\lfloor . \rfloor$ signifies the integer part of the argument, $r_w = r_2 - r_1$ is the (fractional) window size and T is the sample size. The ADF statistic (t-ratio) based on this regression is denoted by $ADF^{r_2}_{r_1}$.

For the backward sup ADF test, the ending point of the samples is fixed at r_2 and the starting point varies from 0 to $r_2 - r_0$ (it is equivalent to allowing the window size r_w to expand from r_0 to r_2).[7] The backward sup ADF statistic is defined as the sup value of the ADF statistic sequence, denoted by

$$BSADF_{r_2}\left(r_0\right) = \sup_{r_1 \in [0, r_2 - r_0]} \left\{ ADF^{r_2}_{r_1} \right\}.$$

The GSADF test can be viewed as a repeat implementation of a backward sup ADF test for each $r_2 \in [r_0, 1]$. The GSADF statistic is defined as the sup value of the backward sup ADF statistic

[6] It cannot consistently estimate the origination and termination dates associated with the subsequent episodes.

[7] The minimum window size r_0 is selected to ensure that there are sufficient observations to initiate the regressions.

sequence and denoted by[8]

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} \left\{ BSADF_{r_2}(r_0) \right\}.$$

The GSADF statistic is utilized to conduct inference of the existence of explosive behavior within the whole sample period. Suppose there is evidence of explosive behavior; one can then date stamp the occurrence periods using the backward sup ADF statistic. Specifically, we conclude that observation $\lfloor Tr_2 \rfloor$ belongs to an explosive phase in the trajectory given that

$$BSADF_{r_2}(r_0) > scv^{\alpha}(r_0),$$

where $scv^{\alpha}(r_0)$ is the $100(1-\alpha)\%$ (right-tail) critical value of the backward sup ADF statistic.

Notice that the backward sup ADF statistic $BSADF_{r_2}(r_0)$ is calculated using information up to period $\lfloor Tr_2 \rfloor$. It does not depend on future realisations and hence this strategy can serve as a warning mechanism for explosive behavior. A more detailed illustration of the GSADF test can be found in Phillips et al. (2011a).[9]

2.1 Data

The data series for this paper are taken from the Federal Reserve Bank of St. Louis's Federal Reserve Economic Data (FRED), and the U.S. Bureau of Economic Analysis (BEA) for the period 1982-2010. The PCE headline measure (PCEPI) is seasonally adjusted and at a monthly frequency, and comes from the GDP and components section of FRED. The PCE core measure (PCEPILFE) is also seasonally adjusted and obtained monthly from the same section of FRED, and excludes food and energy. The individual measures of food and energy are taken directly from the BEA, but at a quarterly frequency, and not seasonally adjusted. The food component is "food and beverages purchased for off-premises consumption" , and the energy component is "energy goods and services".

The headline PCE numbers, along with the food and energy indices are all deflated by core PCE, which we refer to as a relative measure and relative food and energy indices. This gives a measure of how each of the respective price indices are moving relative to core over time.[10] The base year is 2005. In the case of the food and energy indices a measure of core PCE at a quarterly frequency is used for deflation. The logarithm of each deflated series is then used when performing the tests.

2.2 Results

Table 1 shows the generalised SADF statistic, along with respective finite sample critical values.[11] As we can see from Table 1, we find evidence for explosive behavior in each of the data series tested. In

[8]Under the null hypothesis that y_t is a random walk with an asymptotically negligible drift (namely $y_t = dT^{-\eta} + y_{t-1} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0, \sigma^2)$, constant d and $\eta > 1/2$), the asymptotic distribution of the GSADF statistic is

$$\sup_{r_2 \in [r_0,1], r_1 \in [0, r_2 - r_0]} \left\{ \frac{\frac{1}{2} r_w \left[W(r_2)^2 - W(r_1)^2 - r_w \right] - \int_{r_1}^{r_2} W(r)\, dr \left[W(r_2) - W(r_1) \right]}{r_w^{1/2} \left\{ r_w \int_{r_1}^{r_2} W(r)^2\, dr - \left[\int_{r_1}^{r_2} W(r)\, dr \right]^2 \right\}^{1/2}} \right\},$$

where W is the standard Wiener process.

[9]The Gauss and Matlab programs for implementing this test are available for download from https://sites.google.com/site/shupingshi/PrgGSADF.zip?attredirects=0&d=1.

[10]This is the same as taking the ratio of each price index to core PCE each period and multiplying by a constant.

[11]The minimum window size is 36 for the (monthly) headline PCE index and 18 for the (quarterly) food and energy indexes. The lag order is determined by BIC with maximum lag length 12 for the headline PCE index and 6 for the food and energy indexes. The critical values are obtained from Monte Carlo simulations with 2,000 replications (parameters d and η in the null model are set to unity).

particular, for the relative measure between headline and core PCE, the generalised sup ADF statistic is 3.22, which is greater than the 99% critical value 3.12. This implies the existence of explosive behavior in the sample period. Similar results hold for the food and energy series. Each of generalised SADF statistics are above the 95% critical values. The exact periods of explosive behavior for each index (relative to core PCE) are depicted in Figures 2, 3, and 4.

Table 1: The generalized sup ADF test

	Relative PCE	Food & Beverage	Energy & Service
GSADF	3.22	3.72	3.45
90%	2.23	2.55	2.55
95%	2.52	3.07	3.07
99%	3.12	4.66	4.66

Note: The critical values are obtained from Monte Carlo simulations with 2,000 replications.

Figure 2: Explosive periods in headline PCE (deflated by core PCE)

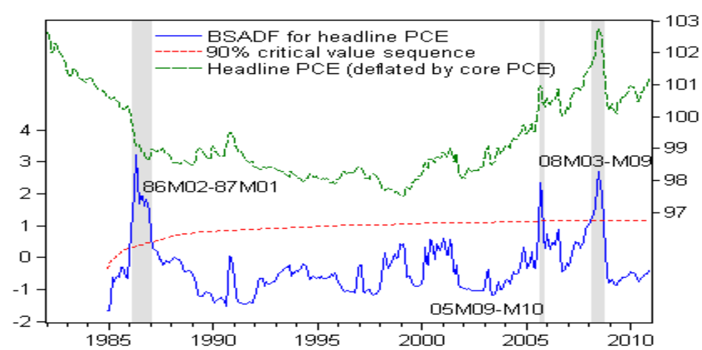


Figure 2 plots the backward SADF statistic sequence against its 90% critical value sequence for the relative PCE. We find explosive behavior whenever the BSADF statistic exceeds the critical value. The relative measure between headline and core PCE has three such events: FEB86-JAN87; SEP05-OCT05; and MAR08-SEP08. The first two periods of explosive behavior are likely to be driven by energy supply shocks. In particular, the first period may correspond to the OPEC collapse of 1986, which resulted in a surplus of oil on world markets. The second period may reflect the onset of Hurricane Katrina, which shut down many refineries in the Southern United States. The third period is the most dramatic as the explosive behavior in the relative measure lasts more than half a year.

To gain deeper insight we consider specific time series. Figures 3 and 4 show that both energy and food may seem to be causing this rise. Each of these series also show explosive behavior in 2008. Figure 3 confirms that periods of explosive behavior in the relative measure between headline and core PCE were due to energy shocks, as there are rises in energy in 1986 and 2004-2006. Interestingly, Figure 4 reveals that the explosive behavior in food in 2001-2002 does not translate into similar movements in headline PCE. This suggests that explosive energy price movements are more important than similar movements in food prices in generating explosive behavior for the aggregate measure of inflation.

Figure 3: Explosive periods in energy and service components (deflated by core PCE)

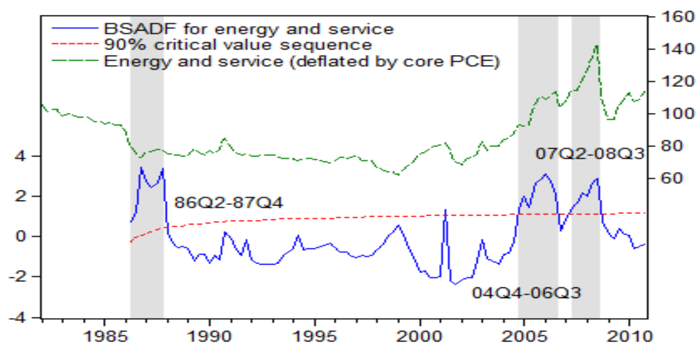
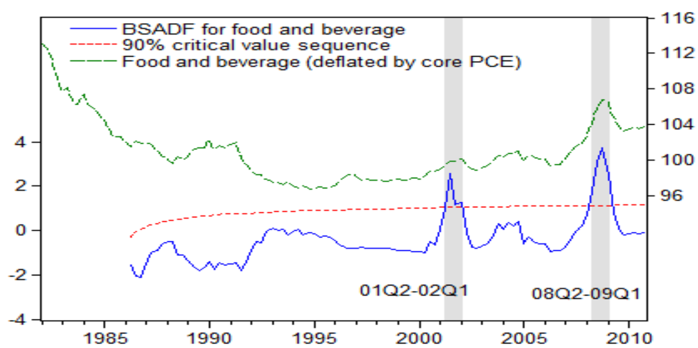


Figure 4: Explosive periods in food and beverage components (deflated by core PCE)



3 Explosiveness and Consumers' Inflation Expectations

A straight forward method to measure inflation expectations of consumers is to ask them to present quantitative estimates. For instance, each month, the University of Michigan's Survey Research Center assesses consumer sentiment by interviewing a random sample of approximately 500 U.S. households. As part of the survey, respondents are asked to forecast key macroeconomic variables, such as inflation, interest rates and unemployment. Alternatively the prices of index-linked financial securities, such as Treasury inflation protected securities (TIPS), could be used to provide market-based measures of inflation expectations and attitudes towards inflation risk.

Various inflation expectation measures are available.[12] In this section we focus on estimates from the University of Michigan's Survey of Consumer Attitudes and Behaviour (UM). As a robustness check and to allow for different time horizons in inflation expectations, we also consider the Federal Reserve Bank of Cleveland's monthly model-based inflation expectation measure (FRC).[13] Using these two measures we explore how these behave when we observe large deviations in the headline and core price measures.

Given the explosive deviations in the relative measure between headline and core PCE shown in the previous section, we next examine some implications for inflation expectations. Here, we argue that these explosive movements are in fact different than other movements in the relative measure

[12]Other estimates include the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF), inflation swap rates, and the difference between yields on nominal U.S. Treasury Notes and TIPS (Pasaogullari, 2011).

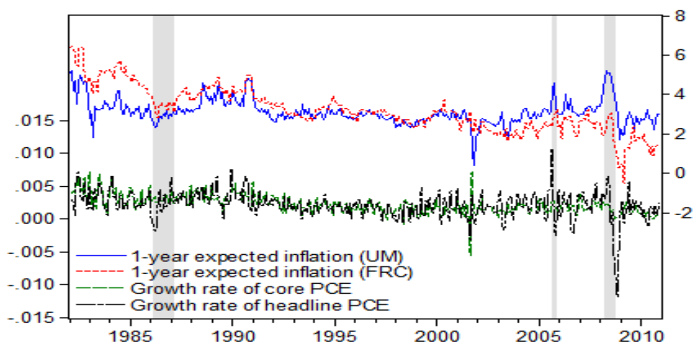
[13]The FRC's estimate of inflation expectations is based on a model that combines information from a number of sources, including the break-even rate derived from TIPS or survey-based estimates.

between headline and core PCE with regards to their impact on consumers' inflation expectations.

3.1 Descriptive Analysis

Figure 5 plots the one-year ahead inflation expectations from both the UM survey and the FRC estimation and the growth rates of headline PCE and core PCE over the period of January 1982 to December 2010. The shaded areas identify the explosive periods in the relative measure found in the previous section.

Figure 5: Inflation expectations and growth rates of headline PCE and core PCE



As we can see, there are sharp and pronounced rises in the UM measure of inflation expectations when the relative measure between headline and core PCE deviates explosively upward (2005 and 2008). In the sample period, the only other similar rises in inflation expectations occur around 1990, but the magnitude in this case is much smaller. It seems that explosive upward movements in prices may have a different impact on consumers' inflation expectations than in other periods. On the other hand, rises in the FRC estimate of inflation expectations are not as dramatic as those in the UM measure.

The largest fall in the UM inflation expectation measure follows the explosive rise in food and beverage prices in 2001-2002. This may be because food and beverages are commonly purchased items, and a rise in these prices have a larger impact on consumer expectations. Moreover, we find that the explosive downward deviation of headline from core PCE follows the OPEC collapse of 1985-1986 but does not result in a sharp and pronounced fall in expectations.

Table 2: Correlations between inflation expectations and inflation measures

	Δ Relative Inflation	Growth Rate of Headline PCE	Growth Rate of Core PCE
ΔInflation Expectations (UM)	0.432	0.308	-0.054
ΔInflation Expectations (FRC)	0.277	0.218	-0.005

Note: Δ is the first order difference operator.

Table 2 presents the correlation coefficients between the change of inflation expectations and different observed inflation measures. In particular, the correlations between the change of inflation expectations and the change of the relative inflation measure (i.e. 0.432 and 0.277) are larger than those between inflation expectations and the growth rates of headline PCE and core PCE. In addition, the growth rate of headline PCE is more related to inflation expectations than the growth rate of core

PCE as it has larger coefficients. This seems to suggest that inflation expectations are more closely related to the relative measure of inflation than core inflation itself. Furthermore, one can see that all three measures (i.e. relative, headline and core) are more strongly linked with consumers' inflation expectations (UM) than general inflation expectations (FRC).

3.2 Regression Analysis

While the previous analysis gives some interesting insights into inflation expectations and explosive behavior in headline PCE, a more formal analysis is needed to clarify this relationship. In an influential paper, Mankiw and Reis (2002) propose a model of sticky information where economic agents update their expectations only periodically because of the costs of collecting and processing information. Mankiw and Reis (2002) can generate disagreement in expectations that is endogenous to the model and correlated with aggregate variables. Based on this framework Mankiw et al. (2004), MRJ hereafter, document some important features of survey-based measures of inflation expectations. In particular, irrespective of the survey used, neither rationally generated nor adaptively generated expectations can fully account for the forecasts of inflation expectations. This latter claim is based on a series of regressions where inflation expectations from different surveys are regressed on possible explanatory variables such as inflation (π_t), unemployment (U_t) and interest rates (i_t). More specifically, MRJ propose the following specification

$$E_t\pi_{t+12} = \alpha + \beta\left(L\right)\pi_t + \gamma U_t + \kappa U_{t-3} + \delta i_t + \phi i_{t-3} + \varepsilon_t, \tag{2}$$

where $E_t\pi_{t+12}$ is the twelve-period ahead inflation expectation, L is lag operator and ε_t is the error term. The values of the regression coefficients indicate that survey respondents neither fully incorporate all of the past information ($\beta\left(1\right) < 1$), nor do they only use past information on inflation (i.e. MRJ reject the null hypothesis that $\gamma = \kappa = \delta = \phi = 0$).

In this paper, we use the MRJ basic setup to gauge how inflation expectations change during periods of explosive deviations in headline PCE. Specifically, we add a dummy variable (D_t) that captures periods of explosive deviation which we identified in the previous section. In particular, D_t has a value of 1 during periods of explosive deviation of headline PCE from core PCE, and 0 otherwise. This is generated based on the following criteria:

$$D_t = \begin{cases} 1 & \text{if } BSADF_t^h\left(r_0\right) \geq scv_t^{\alpha}\left(r_0\right) \\ 0 & \text{Otherwise} \end{cases},$$

where $BSADF_t^h\left(r_0\right)$ is the BSADF statistic for the relative measure between headline and core PCE and $scv_t^{\alpha}\left(r_0\right)$ is the finite sample critical value of the statistic.

The proposed regression model to study the relationship between inflation expectations and explosive inflation behavior is given by

$$\begin{aligned} E_t\pi_{t+j} &= \alpha + \beta\left(L\right)\pi_t + \gamma U_t + \kappa U_{t-1} + \delta i_t + \phi i_{t-1} \\ &\quad + \alpha' D_t + \beta\left(L\right)' D_t\pi_t + \gamma' D_t U_t + \kappa' D_t U_{t-1} + \delta' D_t i_t + \phi' D_t i_{t-1} + \varepsilon_t. \end{aligned} \tag{3}$$

where $E_t\pi_{t+j}$ is the j-period ahead inflation expectation. Our regression model includes U_{t-1} and i_{t-1} instead of U_{t-3} and i_{t-3}, as in MRJ. We adopt this method as the U.S. unemployment and interest rate data are released monthly, providing more data points for our analysis.[14] We include the year-on-year observed inflation for each of the previous three months as well.[15] If expectations change during explosive periods, the coefficients on the dummies should be jointly significant. Following MRJ, we also test for adaptive expectation for the normal periods ($\gamma = \kappa = \delta = \phi = 0$) and for the explosive periods ($\gamma = \kappa = \delta = \phi = \gamma' = \kappa' = \delta' = \phi' = 0$) respectively.

[14] It is also likely that consumers form their inflation expectations based on the newly released (monthly) data.

[15] This is restricted by the number of observations in the explosive regime.

Table 3: Test of adaptive expectations for one-year ahead inflation expectations

	UM Surveys				FRC Estimates			
	Baseline		Proposed		Baseline		Proposed	
$\beta(1)$	0.298***	(0.077)	0.213***	(0.056)	0.160***	(0.029)	0.148***	(0.038)
γ	-0.031	(0.057)	-0.039	(0.052)	0.017	(0.055)	-0.003	(0.057)
κ	0.056	(0.060)	0.092*	(0.053)	0.114**	(0.053)	0.139**	(0.054)
δ	0.259	(0.156)	0.224	(0.163)	0.300**	(0.123)	0.274*	(0.141)
ϕ	-0.258	(0.152)	-0.167	(0.154)	0.034	(0.126)	0.070	(0.143)
$\beta(1)'$			-0.110	(0.075)			-0.215***	(0.048)
γ'			0.048	(0.081)			0.324***	(0.077)
κ'			-0.057	(0.106)			0.095***	(0.085)
δ'			0.063	(0.117)			0.020	(0.165)
ϕ'			-0.328*	(0.127)			-0.328*	(0.169)
Adj. R^2	0.442		0.605		0.843		0.846	
Adaptive expectation?								
(i) Normal periods$^{(a)}$								
	$F_{4,305} = 0.892$		$F_{4,297} = 1.56$		$F_{4,305} = 88.03^{***}$		$F_{4,297} = 59.58^{***}$	
(ii) Explosive periods$^{(b)}$								
	-		$F_{8,297} = 31.12^{***}$		-		$F_{8,297} = 92.66^{***}$	
Joint sig. (dummy)								
	-		$F_{8,297} = 71.02^{***}$		-		$F_{8,297} = 37.94^{***}$	

Note: Parameters in parentheses are the Newey-West standard errors (lag truncation=5). ***, ** and * denote statistical significance at the 1%, 5% and 10% levels respectively. *(a)* $H_0 : \gamma = \kappa = \delta = \phi = 0$. *(b)* $H_0 : \gamma = \kappa = \delta = \phi = \gamma' = \kappa' = \delta' = \phi' = 0$.

Table 3 reports the estimation and hypothesis test results of a baseline model based on MRJ and our proposed model using the one-year ahead UM and FRC inflation expectation measures. The baseline model uses one month lags of the unemployment and interest rates as independent variables to make it consistent with our modification.[16]

As we can see from Table 3, when using the UM measure of inflation expectations, we cannot reject the null hypothesis of adaptive expectations over the sample period from the baseline model. Moreover, the dummy variables are jointly (and highly) significant and the adjusted R-square of the proposed model is much higher than that of the baseline model. This indicates that inflation expectations during periods when headline PCE deviates from core PCE in an explosive manner are quite relevant when forming expectations. Interestingly, the results indicate that consumers rely less on the past inflation during these explosive phases. In addition, consistent with the baseline model, we cannot reject adaptive expectations during normal periods. This is in sharp contrast to explosive periods where adaptive expectations are strongly rejected. This difference provides some evidence that expectations change during such periods. Moreover, past interest rates also help predict consumers' inflation expectations during these periods.

Finally, regressions based on the FRC estimate of inflation expectations provide a different story. The adjusted R-square of the proposed model is quite close to that of the baseline model. Moreover, both models reject the null hypothesis of adaptive expectations for the whole sample period. The distinct results provided by the UM measure and the FRC estimates indicate that general inflation expectations behave quite differently from those of consumers with explosive deviations in the relative measure of headline PCE and core PCE. This provides evidence that not all agents in the economy

[16] The regression starts from December 1984 since the minimum window size in the explosive test is 36. The estimation and test results of the baseline model are not sensitive to the lag selection of inflation.

form inflation expectations in the same manner. This fact increases the difficulty of anchoring inflation expectations.[17]

3.3 The Explosive Upward and Downward Deviations

We further investigate the individual impact of the explosive upward and downward deviations on inflation expectations. Specifically, we replace dummy variable D_t in equation (3) with D_{1t} for explosive upward deviations and D_{2t} for explosive downward deviations, where D_{1t} and D_{2t} are defined as follows

$$
\begin{aligned}
D_{1t} &= \begin{cases} 1 & \text{if } BSADF_t^h(r_0) \geq scv_t^\alpha(r_0) \text{ and } H_t \geq H_{t-1} \\ 0 & \text{Otherwise} \end{cases}, \\
D_{2t} &= \begin{cases} 1 & \text{if } BSADF_t^h(r_0) \geq scv_t^\alpha(r_0) \text{ and } H_t < H_{t-1} \\ 0 & \text{Otherwise} \end{cases},
\end{aligned}
$$

and H_t is the current period relative measure between headline and core PCE. Structurally, the new regression model is

$$
\begin{aligned}
E_t\pi_{t+j} &= \alpha + \beta(L)\pi_t + \gamma U_t + \kappa U_{t-1} + \delta i_t + \phi i_{t-1} \\
&\quad + \alpha' D_{1t} + \beta(L)' D_{1t}\pi_t + \gamma' D_{1t}U_t + \kappa' D_{1t}U_{t-1} + \delta' D_{1t}i_t + \phi' D_{1t}i_{t-1} \\
&\quad + \alpha'' D_{2t} + \beta(L)'' D_{2t}\pi_t + \gamma'' D_{2t}U_t + \kappa'' D_{2t}U_{t-1} + \delta'' D_{2t}i_t + \phi'' D_{2t}i_{t-1} + \varepsilon_t. \qquad (4)
\end{aligned}
$$

Table 4: Test of adaptive expectations for Michigan median inflation expectations: Model (4)

$\beta(1)$	0.209***	(0.057)	$\beta(1)'$	0.414***	(0.022)	$\beta(1)''$	-0.078	(0.061)
γ	-0.037	(0.053)	γ'	0.416***	(0.087)	γ''	-0.119	(0.105)
κ	0.091*	(0.053)	κ'	-0.495***	(0.113)	κ''	-0.036	(0.128)
δ	0.224	(0.167)	δ'	0.348***	(0.079)	δ''	-0.048	(0.189)
ϕ	-0.164	(0.158)	ϕ'	-0.194**	(0.088)	ϕ''	-0.264	(0.185)
Adj. R^2		0.600						
Adaptive expectation?								
(i) Normal periods			$F_{4,289} = 1.60^{(a)}$					
(ii) Explosive periods			$F_{8,289} = 1181.43^{***(b)}$					
Joint sig. (dummy)			$F_{16,289} = 5321.99^{***}$					

Note: Parameters in parentheses are the Newey-West standard errors (lag truncation=5). ***, ** and * denote statistical significance at the 1%, 5% and 10% levels respectively. *(a)* $H_0: \gamma = \kappa = \delta = \phi = 0$. *(b)* $H_0: \gamma' = \kappa' = \delta' = \phi' = \gamma'' = \kappa'' = \delta'' = \phi'' = 0$.

Table 4 presents the model estimates and hypothesis testing results using the Michigan median inflation expectations. Table 4 shows the hypothesis test results of the regression with upward and downward explosive dummies, (4), are the same as those of the regression with a combined explosive dummy, (3). Specifically, we fail to reject the hypothesis of adaptive expectations in normal periods and we strongly reject it in explosive periods.

Furthermore, one can see that parameters related to both current and past unemployment rates are individually *significant* during the upward explosive periods and *insignificant* during the downward explosive periods. This suggests that the rejection of adaptive expectation hypothesis arises

[17] We have also explored the impact of the explosive deviations on inflation expectations with different time horizons. See Appendix A for details.

from periods when headline PCE deviates explosively upward from core PCE. Consumers also utilise both current and past unemployment and interest rates in forming inflation expectation during those periods. In other words, they are more forward-looking as they rely less on past inflation.

4 Markov-Switching and Consumers' Inflation Expectations

The strategy of PSY used in the previous section implicitly classifies the relative measure between headline and core PCE into an explosive regime and a non-explosive regime. We demonstrate above that when there is explosive behavior in the relative measure between headline and core, consumers depart from adaptive inflation expectation and utilise information other than past inflation to forecast inflation.

In this section, we consider an alternative to the previous regime classification. The new framework assumes that the relative measure has two regimes and these two regimes switch from one to the other in a Markov pattern. More structurally, the Markov-switching model is specified as follows:

$$\triangle y_t = \alpha + \beta y_{t-1} + \sum_{j=1}^{2} \psi_j \triangle y_{t-j} + \varepsilon_t, \varepsilon_t \sim N\left(0, \sigma_{s_t}^2\right). \tag{5}$$

where y_t is the relative measure between headline and core PCE and $\sigma_{s_t} = \sigma_0 + S_t(\sigma_1 - \sigma_0)$. The state variable S_t takes value 0 or 1 and is governed by a first order Markov-chain, namely

$$P\{S_t = 0|S_{t-1} = 0\} = p \text{ and } P\{S_t = 1|S_{t-1} = 1\} = q.$$

The corresponding log likelihood function[18] is

$$l(y_1, \cdots, y_T; \Psi) = \sum\nolimits_{t=1}^{T} \log \sum\nolimits_{s_t \in \{0,1\}} f(y_t|\Upsilon_{t-1}, S_t = s_t; \Psi) \times \Pr\{S_t = s_t|\Upsilon_{t-1}; \Psi\},$$

where Ψ contains all of the unknown parameters and Υ_{t-1} is the information set available at period $t-1$. The model is estimated using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm with 100 sets of randomly generated start-up values; we choose the one associated with the largest likelihood value. The smoothed probabilities are calculated according to Kim (1994).

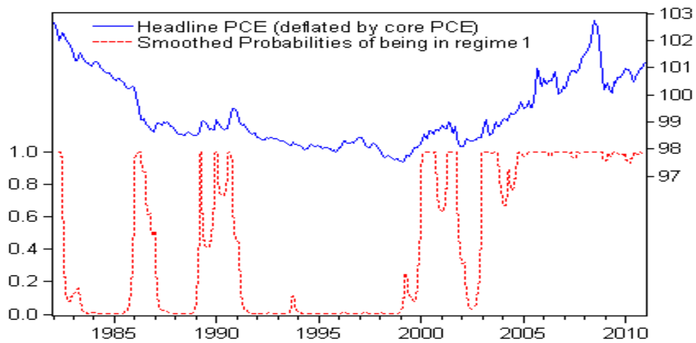
Table 5: Estimates of the Markov-switching model

α	1.253	(3.30)	β	-0.013	(-3.33)
ψ_1	0.456	(8.46)	ψ_2	-0.202	(-4.07)
σ_0	0.063	(15.38)	σ_1	0.202	(14.29)
p	0.960	(49.04)	q	0.955	(34.01)

Note: Figures in parentheses are t-statistics.

Table 5 presents estimates associated with the Markov-switching model. Notice that the estimate of β is smaller than zero. Regime 1 has larger conditional standard deviation (σ_1) than regime 0. In particular, σ_1 is 3.21 times larger than σ_0. That is, based on the Markov-switching model, the sample period can be separated into a normal regime (regime 0) and a volatile regime (regime 1). The smoothed probabilities of being in the volatile regime are displayed in Figure 6. As evident by this figure, there is a high probability of being in the volatile regime (greater than 0.5) during the periods

Figure 6: The smoothed probabilities of being in regime 1



of 1986 OPEC collapse, 1990, 2000-2001, and 2003 onwards.[19]

In order to explore how consumers' inflation expectations change over these two regimes, we replace the dummy variable D_t in equation (3) by the smoothed probabilities of being in regime 1 (p_t^s). The new model for inflation expectations is

$$\begin{aligned} E_t\pi_{t+12} &= \alpha + \beta(L)\pi_t + \gamma U_t + \kappa U_{t-1} + \delta i_t + \phi i_{t-1} \\ &\quad + \alpha' p_t^s + \beta(L)' p_t^s \pi_t + \gamma' p_t^s U_t + \kappa' p_t^s U_{t-1} + \delta' p_t^s i_t + \phi' p_t^s i_{t-1} + \varepsilon_t. \end{aligned} \tag{6}$$

If parameters related to the auxiliary variable p_t^s are jointly significant, it suggests that inflation expectations in regime 0 and regime 1 are different. This could potentially be due to the fact that consumers rely on different information in these two regimes (i.e. the use of unemployment and interest rates to help predict). Alternatively, consumers use the same information set but the dependence level of their forecast on the information varies across regimes.

Table 6: Test of adaptive expectations for the Michigan median inflation expectations: Model (6)

$\beta(1)$	0.287***	(0.067)	$\beta(1)'$	0.015	(0.140)
γ	-0.070	(0.047)	γ'	0.029	(0.137)
κ	0.058	(0.049)	κ'	0.036	(0.141)
δ	0.031	(0.143)	δ'	0.167	(0.297)
ϕ	0.025	(0.144)	ϕ'	-0.216	(0.311)
Adj. R^2				0.496	
Adaptive expectation in explosive periods?$^{(a)}$				$F_{8,297} = 1.20$	
Joint significance test of parameters related to auxiliary variable				$F_{8,329} = 2.14^{**}$	

Note: Parameters in parentheses are the Newey-West standard errors (lag truncation=5). *** and ** denotes statistical significance at the 1% and 5% level respectively. *(a)* $H_0: \gamma = \kappa = \delta = \phi = \gamma' = \kappa' = \delta' = \phi' = 0$.

The estimation results of model (6) using the UM inflation expectations, along with related hypothesis tests, are presented in Table 6. We fail to reject the joint insignificance of $\gamma, \kappa, \delta, \phi, \gamma', \kappa', \delta', \phi\prime$. This suggests that consumers do not rely on unemployment and interest rate information to predict

[18] We resort to the Quasi-Bayesian approach (Hamilton, 1991) to address of the problem of unbounded likelihood (Day, 1969). However, this adjustment does not change the estimation results of the relative PCE.

[19] We have considered an alternative specification of the Markov-switching model which allows all model parameters to switch across regimes. The smoothed probabilities of being in the volatile regime obtained from the alternative specification are almost identical to those calculated from model (5). See Appendix B for more details.

inflation in both regimes. Therefore, we cannot reject adaptive expectations in both regimes.[20] This finding is consistent with the baseline model where we cannot reject adaptive expectations for the whole sample period. Nevertheless, it is in sharp contrast to the finding of model (3). According to model (3), adaptive expectations do not hold in periods of explosive divergence of headline PCE from core PCE. In other words, when consumers observe explosive deviations of food and energy prices from other prices, they change their way of forming inflation expectation; whereas they still rely on adaptive expectation when observe volatility changes in the relative prices.

Finally, we note that the adjusted R-square of model (3) is higher than that of model (6), i.e. $0.605 > 0.496$. This suggests that the explosiveness indicator (i.e. the dummy variable D_t obtained from the PSY strategy) performs better than the volatility indicator (i.e. the smoothed probability p_t^s obtained from the Markov-switching model) in capturing the dynamics of consumers' inflation expectations. These findings suggest that inflation expectations can be better forecasted when exploiting the explosive behavior of the relative measure between headline and core PCE highlighting the potential benefits for monetary policy.

5 Conclusion

We apply the recently proposed test of Phillips et al. (2011a) to identify episodes where headline PCE deviates from core PCE in an explosive manner.[21] We also isolate the components of the headline measure which are responsible for this behavior. We find there has been explosive behavior in headline PCE relative to core PCE on three occasions since 1982. It also seems that explosive behavior in the headline PCE series on these occasions is driven by similar behavior in energy prices. Two of these periods correspond to energy supply shocks (the OPEC collapse of 1986 and Hurricane Katrina). The third period of explosive behavior was from March 2008 to September 2008.

Identifying these periods is important because we find evidence suggesting that consumer inflation expectations behave differently under normal and explosive periods. During non-explosive periods we are unable to reject the hypothesis that inflation expectations are formed by adaptive expectations. But the hypothesis of adaptive expectations is decisively rejected during explosive periods. Interestingly, we find that this rejection is due to consumers' behaviour during upward explosive periods. Consumers rely less on past inflation and also utilise both current and past unemployment and interest rates in forming inflation expectation when headline PCE deviates explosively upward from core PCE.

We also consider a Markov-switching process for the relative measure between headline and core PCE when analyzing inflation expectations. We find that explosive deviations of the relative measure are more important than the relative volatility implied by the Markov switching model when studying inflation expectations. The findings of this paper suggest that sharp differences between food and energy prices relative to other prices in the economy (especially upward deviations) can impact the way in which consumer inflation expectations evolve. This possibility may complicate the conduct of monetary policy and suggests the need to consider both headline and core PCE when designing monetary policy.

References

Bernanke, Ben S., "Inflation Expectations and Inflation Forecasting,"

[20]Notice that all parameters related to the auxiliary variable are individually insignificant. The joint significance of parameters related to the auxiliary variable indicates that the extent to which consumers relied on past inflation changed slightly across these two regimes.

[21]Other test focusing on specific commodity prices have been done by Shi and Arora (2012), Phillips and Yu (2011), and Sanders and Irwin (2010).

http://www.federalreserve.gov/newsevents/speech/bernanke20070710a.htm 2007. At the Monetary Economics Workshop of the National Bureau of Economic Research Summer Institute, Cambridge, Massachusetts.

Bodenstein, Martin, Christopher J. Erceg, and Luca Guerrieri, "Optimal Monetary Policy with Distinct Core and Headline Inflation Rates," *Journal of Monetary Economics*, October 2008, *55* (Supplemen), S18–S33.

Bullard, James, "Measuring Inflation: The Core is Rotten," *Review*, 2011, (July), 223–234.

Day, N.E., "Estimating the Components of a Mixture of Normal Distribution," *Biometrika*, 1969, *56*, 463–474.

Hamilton, James D., "A Quasi-Bayesian Approach to Estimating Parameters for Mixture of Normal Distribution," *Journal of Business and Economic Statistics*, 1991, *9*, 27–39.

Kim, Chang Jin, "Dynamic Linear Models with Markov-switching," *Journal of Econometrics*, 1994, *60*, l–22.

Mankiw, N. Gregory and Ricardo Reis, "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics*, November 2002, *117* (4), 1295–1328.

Mankiw, N.Gregory, Ricardo Reis, and Justin Wolfers, "Disagreement About Inflation Expectations," *NBER Macroeconomics Annual 2003*, 2004, pp. 209–248.

Mehra, Yash P. and Bansai Sawhney, "Inflation Measure, Taylor Rules, and the Greenspan-Bernanke Years," *Economic Quarterly*, 2010, (2Q), 123–151.

Motley, Brian, "Should Monetary Policy Focus on "Core" Inflation?," *FRBSF Economic Letter*, 1997, (Apr 18).

Pasaogullari, Mehmet, "Recent Developments in Inflation Expectations," *Economic Trends*, 2011.

Phillips, Peter C. B. and Jun Yu, "Dating the Timeline of Financial Bubbles During the Subprime Crisis," *Quantitative Economics*, 2011, *2* (3), 455–491.

Phillips, Peter C.B., Shuping Shi, and Jun Yu, "Testing for Multiple Bubbles," *Singapore Management University, Center for Financial Econometrics Working Paper 2010-03*, 2011.

_ , Y. Wu, and Jun Yu, "Explosive Behavior in the 1990s Nasdaq: When did Exuberance Escalate Asset Values?," *International Economic Review*, 2011, *52*, 201–226.

Sanders, Dwight R. and Scott H. Irwin, "A Speculative Bubble in Commodity Futures Prices? Cross-sectional Evidence," *Agricultural Economics*, 01 2010, *41* (1), 25–32.

Shi, Shuping and Vipin Arora, "An Application Of Models Of Speculative Behaviour To Oil Prices," *Economics Letters*, 2012, *115*, 469–472.

Thornton, Daniel L., "Measure for Measure: Headline versus Core Inflation," *Monetary Trends*, 2007, (Sep).

_ , "Core versus Headline Inflation: An Opportunity for Greater Transparency," *Economic Synopses*, 2011.

Trehan, Bharat, "Household Inflation Expectations and the Price of Oil: It's Deja Vu All Over Again," *FRBSF Economic Letter*, 2011, *2011-16*.

6 Appendix A: Robustness Check

We have also explored the impact of the explosive deviations on inflation expectations with different time horizons. Table 7 presents the model estimates and hypothesis testing results using the one-year and five-year ahead FRC estimates of inflation expectations (we do not have the five-year ahead inflation expectation data of consumers). For the five-year ahead inflation expectation, the proposed model has an almost identical adjusted R-square with the baseline model, which suggests similar fits of these two models. This can also be seen from the similar estimates of the proposed model and the baseline model. Like the one-year ahead expectation measure, we reject adaptive expectations for the whole sample period. Therefore, we conclude that the explosive deviation does not have obvious impact on both the one-year ahead and the five-year ahead inflation expectation measures of FRC.

Table 7: Test of adaptive expectations for the FRC estimates of inflation expectations

	One-year ahead				Five-year ahead			
	Baseline		Proposed		Baseline		Proposed	
$\beta(1)$	0.160***	(0.029)	0.148***	(0.038)	0.075***	(0.025)	0.079**	(0.032)
γ	0.017	(0.055)	-0.003	(0.057)	0.068*	(0.039)	0.059	(0.041)
κ	0.114**	(0.053)	0.139**	(0.054)	0.111**	(0.044)	0.122***	(0.047)
δ	0.300**	(0.123)	0.274*	(0.141)	0.413***	(0.123)	0.398***	(0.145)
ϕ	0.034	(0.126)	0.070	(0.143)	-0.065	(0.129)	-0.050	(0.153)
$\beta(1)'$			-0.215***	(0.048)			-0.155***	(0.036)
γ'			0.324***	(0.077)			0.148**	(0.064)
κ'			0.095***	(0.085)			0.060	(0.085)
δ'			0.020	(0.165)			-0.014	(0.154)
ϕ'			-0.328*	(0.695)			-0.134	(0.165)
Adj. R^2	0.843		0.846		0.848		0.846	
Adaptive expectation?								
(i) Normal periods$^{(a)}$								
	$F_{4,305} = 88.03^{***}$		$F_{4,297} = 59.58^{***}$		$F_{4,305} = 83.16^{***}$		$F_{4,297} = 52.86^{***}$	
(ii) Explosive periods$^{(b)}$								
	-		$F_{8,297} = 90.66^{***}$		-		$F_{8,297} = 286.44^{***}$	
Joint significance test of parameters related to dummy								
	-		$F_{8,297} = 37.94^{***}$		-		$F_{8,297} = 11.45^{***}$	

Note: Parameters in parentheses are the Newey-West standard errors (lag truncation=5). ***, ** and * denote statistical significance at the 1%, 5% and 10% levels respectively. *(a)* $H_0 : \gamma = \kappa = \delta = \phi = 0$. *(b)* $H_0 : \gamma = \kappa = \delta = \phi = \gamma' = \kappa' = \delta' = \phi' = 0$.

7 Appendix B: An Alternative Markov-Switching Model

We consider an alternative specification of the Markov-switching model, namely

$$\triangle y_t = \alpha_{s_t} + \beta_{s_t} y_{t-1} + \sum_{j=1}^{2} \psi_{s_t,j} \triangle y_{t-j} + \varepsilon_t, \varepsilon_t \sim N\left(0, \sigma_{s_t}^2\right), \tag{7}$$

where y_t is the relative measure between headline and core PCE, $\alpha_{s_t} = \alpha_0 + S_t(\alpha_1 - \alpha_0), \beta_{s_t} = \beta_0 + S_t(\beta_1 - \beta_0), \psi_{s_t,j} = \psi_{0,j} + S_t(\psi_{1,j} - \psi_{0,j})$ and $\sigma_{s_t} = \sigma_0 + S_t(\sigma_1 - \sigma_0)$. Table 8 presents estimates associated with the Markov-switching model. Notice that estimates β_0 and β_1 are both smaller than zero. Regime 1 has larger conditional mean (α_1) and conditional standard deviation (σ_1) than those of regime 0. In particular, σ_1 is 3.23 times larger than σ_0. Furthermore, the likelihood ratio test indicates that σ_0 is significantly different from σ_1.

Table 8: Estimates of the Markov-switching model: relative measure between headline and core PCE

α_0	1.450	(3.21)	α_1	2.526	(1.79)
β_0	-0.015	(-3.24)	β_1	-0.025	(-1.78)
ψ_{01}	0.368	(3.37)	ψ_{11}	0.511	(6.36)
ψ_{02}	-0.147	(-1.85)	ψ_{11}	-0.262	(-3.25)
σ_0	0.061	(10.47)	σ_1	0.197	(14.04)
p	0.952	(36.77)	q	0.946	(28.42)
Likelihood ratio stat. ($\sigma_0 = \sigma_1$)			55.408 [0.000]		

Note: Figures in parentheses are t-statistics. Figures in the square bracket is p-value.

The smoothed probabilities of being in regime 1 are displayed in Figure 7. We can see that the smoothed probabilities of being in the volatile regime obtained from model (5) and (7) are almost identical. Therefore, it is reasonable to expect that regime classifications based on model (7) would have a similar impact on consumers' inflation expectation as those based on model (5).

Figure 7: The smoothed probabilities of being in regime 1: Model (7)