Prudential Capital Controls: The Impact of Different Collateral Constraint Assumptions*

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This version: March, 2013

Abstract

A fast-growing literature on small open economy models with pecuniary externalities has provided the theoretical ground for the policy analysis of macro-prudential regulations and bailouts. Benigno, Chen, Otrok, Rebucci, and Young (2012a) recently showed that the macro-prudential regulations are desirable if the policy instrument is restricted to capital controls. Using the framework of Jeanne and Korinek (2010), we show that this result depends on the functional form of the collateral constraint which households are faced with. If households collateralize their assets that they purchase at the same time with their borrowing, capital controls in the form of bailout subsidy during crises can be optimal because they can achieve the first best allocation. If, on the other hand, the maximum borrowing is constrained by their assets that they have purchased before they borrow, a stronger case can be made for ex ante macro-prudential regulations.

JEL Classification: E32, G01, G18

Keywords: Financial crises, Credit externalities, Bailouts, Macroprudential policies

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*We would like to thank seminar participants at Hitotsubashi University, Kobe University, the 14th Annual Macro Conference and the Banking Theory Study Group for helpful discussions. Takayuki Tsuruga gratefully acknowledges the financial support of a Grant-in-Aid for Scientific Research. Views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

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1 Introduction

Recent studies on financial crises have highlighted the importance of prudential controls on capital inflow and macro-prudential regulations to prevent inefficient boom and bust cycles. Among others, Jeanne and Korinek (2010, hereafter JK), Bianchi (2011), and Bianchi and Mendoza (2012) focus on prudential capital controls and emphasize that market-determined asset prices or real exchange rates can give rise to pecuniary externalities that distort financing decisions of economic agents (e.g., over-borrowing). A key ingredient in their theory is a collateral constraint that depends on the market value of collateral. Based on the framework, they suggest that the government impose a Pigouvian tax on debt during normal times to internalize the externalities.

While their result provides the theoretical ground for macro-prudential regulations, Benigno, Chen, Otrok, Rebucci, and Young (2012a, BCORY) show that a “price support policy” during a crisis, which can be interpreted as ex post bailouts, can outperform the macro-prudential policy measures in terms of welfare. More specifically, using the model by Bianchi (2011), they show that, if the government has alternative distortionary policy instruments other than capital controls (e.g., tax/subsidy on consumption or collateral assets), the collateral constraint can be always removed as well as the inefficiency arising from the externalities and thus the first best allocation can be achieved. Their finding implies that the prudential capital controls are not optimal in general. They argue that the macro-prudential policies would be desirable when capital controls are the only policy instrument that is available to the government.

This paper shows that the bailout subsidy can achieve the first best allocation even in the case where capital controls are the only policy instrument. Unlike BCORY, we use a variant of the JK model. We show that capital controls, or capital stimulus in fact, during crises can outperform the prudential capital controls if the households collateralize their assets that they purchase at the same time of their borrowing. We call such a collateral constraint end-of-period collateral constraint, in comparison with the JK’s original beginning-of-period collateral constraint where the households collateralize their assets that they have purchased before their borrowing decisions are made.

While our result is closely related to that of BCORY, it has important differences from theirs in terms of implications for crisis management. In their model, the government has an instrument to intervene the markets of collateral. The government can directly support the collateral price such that the support completely removes the collateral constraint at
the time of crises, by appropriately choosing a distortionary subsidy on collateral. In our model, the government can achieve the first best allocation, even when the government has no access or intention for outright asset purchase program and continues to rely on capital controls. Furthermore, the mechanism behind the intervention is different from BCORY. Under the end-of-period collateral constraint, the greater holdings of collateral can expand the household’s borrowing capacity. When an adverse state of the economy precipitates a crisis, the government can encourage debt holdings via the subsidy on debt and prevent fire sales of collateral by shifting the (net) supply curve for collateral leftward. This decreased (net) supply for collateral can inflate the collateral price until the inflated collateral price expands the borrowing capacity to the first best level of borrowing. Unlike BCORY, the bailout subsidy does not remove the collateral constraint and the constraint is binding in equilibrium. What happens here is that the collateral constraint binds but the maximum borrowing is “constrained” to the first best level of borrowing. This is feasible when the price of collateral is sufficiently inflated.

The fact that the collateral constraint is not removed by the subsidy gives rise to another important difference between BCORY’s result and ours. The difference lies in the activation of each policy. In the price support policy suggested by BCORY, the commitment to intervene the collateral market makes households form the expectations that the collateral constraint is always slack. Consequently, no states of the economy precipitate crises and no policy activation is in fact needed. By contrast, under the subsidy on debt in our model, the policy must be activated, whenever it is needed. This is because the households know that the collateral constraint is binding under the optimal bailout subsidy, whenever the state of the economy is about to precipitate the crisis.

To conclude, our paper’s findings implies that the policy prescriptions toward financial stability may be sensitive to the assumptions regarding the collateral constraint. The decision between which assumption to use mainly depends on which is more empirically plausible for the collateral constraint. Our finding is that the decision between the two is not innocuous.

2 Two Collateral Constraints

The baseline model is a small open economy following JK. Suppose that the utility of identical atomistic households is given by \( u(c_0) + u(c_1) + c_2 \), where \( u(\cdot) \) is assumed to take a CRRA form. The riskless world interest rate is set to zero, for simplicity. The households’ income consists of an consumption endowment \( e \) and the return on collateral \( y \). The endowment
is received in period 1 and not pledgeable to foreign lenders. The return on collateral is obtained in period 2 and is used for repayment of the households’ debt. Following JK, we assume that the return on collateral is acquired by only domestic agents. The price of the collateral for period $t$ is denoted by $p_t$. In JK, the households are assumed to hold one unit of collateral in the beginning of period 0.

The domestic households’ budget constraints for each period are given by:

\begin{align*}
c_0 + p_0 \theta_1 &= d_1 + p_0 \quad (1) \\
c_1 + d_1 + p_1 \theta_2 &= e + d_2 + p_1 \theta_1 \quad (2) \\
c_2 + d_2 &= \theta_2 y, \quad (3)
\end{align*}

where $d_t$ is the debt at the beginning of the period $t$. The domestic households in period $t$ trade $\theta_{t+1}$ in a competitive market to carry their wealth into period $t + 1$, but the foreign agents do not participate in the market for the collateral. In this model, the linear utility in period 2 implies that the first best level of consumption in period 0 and 1 is unity.

In what follows, we introduce a collateral constraint for $d_2$ into the model. As JK discuss, a low value of $e$ may result in the binding collateral constraint and precipitate a crisis (e.g., credit crunch or sudden stop in capital inflow). With the binding collateral constraint, the desired borrowing in perfect financial markets is generally impossible and the households must accept a large reduction in $c_1$. As such, the period 1 corresponds to the period of a sudden stop or credit crunch.

### 2.1 The Beginning-of-period Collateral Constraint

Each household is faced with the collateral constraint of the form:

$$d_2 \leq \phi \theta_1 p_1, \quad (4)$$

where $\theta_1$ is the quantity of collateral goods at the beginning of period 1. The parameter $\phi \in (0, 1]$ represents the ceiling of the leverage in the collateral constraint. In a symmetric equilibrium, we must have $\theta_1$ of unity because the supply of collateral asset is inelastic and is normalized to one. Not surprisingly, the collateral constraint generates the asset-debt loop that involves pecuniary externalities. When a sufficiently low $e$ takes place, the collateral constraint binds. The households try to increase net supply of their collateral to prevent consumption reduction. The households’ deleverage results in declines in asset price and the
decline in $p_1$ further tightens their collateral constraints. Note that each atomistic household takes $p_1$ as given. In the equilibrium, however, the households’ price-taking decision as a whole has the general equilibrium effect on $p_1$. Because of the pecuniary externalities, each competitive domestic borrower fails to internalize the general equilibrium effect. In JK’s model, liquid net worth held by the domestic households, $m_1 = e - d_1$, has a nonnegligible impact on the asset price. A well-known result in JK and others is that the laissez-faire equilibrium is not generally Pareto-efficient.  

JK show that, in their stochastic model where $e$ is random, macro-prudential Pigouvian tax $\tau$ can replicate the second best allocation solved by the constrained social planner. Replace period-1 budget constraint (2) by

$$ c_1 + (1 + \tau) d_1 - T + p_1 \theta_2 = e + d_2 + p_1 \theta_1, $$

where $T = \tau d_1$. In JK, the following macro-prudential tax is proposed:

$$ \tau = \frac{\mathbb{E}_0 [\lambda_{sp} p'(m_1)]}{\mathbb{E}_0 [u'(c_1)]}, $$

where $\mathbb{E}_0$ denotes the expectations operator conditional on the information at $t = 0$.  
In (6), $\lambda_{sp}$ is the Lagrange multiplier for the collateral constraint that the social planner faces and $p'(m_1)$ is the derivative of $p_1$ with respect to the level of the liquid net worth $m_1$. The shadow price of holding debt $\lambda_{sp}$ and the asset price function $p(m_1)$ are obtained from the constrained social planner’s problem where she internalizes the effect of the liquid net worth on $p_1$.  
Based on (6), JK argue $p'(m_1)$ to be positive and call for a Pigouvian taxation.

### 2.2 The End-of-period Collateral Constraint

We depart from JK by considering a variant of (4):

$$ d_2 \leq \phi \theta_2 p_1, $$

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<sup>1</sup> For example, Bianchi (2010), Benigno, Chen, Otrok, Rebulli, and Young (2013) and Mendoza (2010).

<sup>2</sup> More specifically, the information set does not include the realization of $e$.

<sup>3</sup> See JK for the detail of the constrained social planner’s problem who maximizes the households’ utility subject to the budget constraints and the collateral constraint given by $d_2 \leq \phi \theta_1 p(m_1)$. The price function $p(m_1)$ here is obtained from the first order conditions of the constrained social planner’s problem. It is expressed as $p(m_1) = y/u'(c_1) = y/u'(d_2 + m_1)$. 

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where the value of collateral is evaluated by the end-of-period holding of collateral $\theta_2$, rather than $\theta_1$. As discussed in JK, their key results remain the same across the two collateral constraints: $^4$ (i) the same collateral constraint in equilibrium (i.e., $d_2 \leq \phi p_1$); (ii) the same asset-debt loop involving pecuniary externalities; and (iii) the same form of the Pigouvian tax. These results remain unaltered even under the infinite horizon setting. $^5$

3 Results

3.1 The optimal bailout subsidy

We consider the impact of the two different assumptions regarding collateral constraint on bailout policies. For expositional purpose, we simplify the JK’s model by assuming deterministic $e$, rather than stochastic $e$, to obtain the explicit solution for the optimal bailout subsidy.

To introduce bailouts, we replace (2) by

$$c_1 + d_1 + p_1 \theta_2 = e + (1 + s) d_2 - S + p_1 \theta_1,$$

where $s > 0$ is the subsidy on debt. Following the literature, we assume that the government runs a balanced budget: $S = sd_2$, which ensures that the households’ resource is kept unchanged intra-temporally. Note that the bailouts here are not intertemporal transfers of resources from the government to the constrained household. Rather, we keep focusing on the same instrument of the government that intervenes the same credit market. The only distinction between JK and ours is, while using the same policy instrument, whether to raise the cost of debt before a crisis or to reduce it during a crisis. Our scope contrasts sharply with BCORY who suggest intervention in some other markets, including market for collateral assets, to support the collateral prices.

The households maximize their utility $u(c_0) + u(c_1) + c_2$, subject to the budget constraints

$^4$See the footnote 4 in JK.

$^5$Jeanne and Korinek (2011) numerically reconfirm the robustness to the assumptions on collateral constraints in Appendix A.2.
(1), (8), (3) and the end-of-period collateral constraint (7). The first order conditions are

\[ u'(c_0) = u'(c_1) \]  
\[ (1 + s) u'(c_1) = 1 + \lambda_m \]  
\[ p_1 = \frac{y}{u'(c_1) - \lambda_m \phi}, \]  

and \( p_0 = p_1 \). Here \( \lambda_m \) represents the Lagrange multiplier for (7). In (10), the household chooses \( d_2 \) by comparing the marginal cost \( 1 + \lambda_m \) on the right hand side with the marginal benefit \( (1 + s) u'(c_1) \) on the left hand side. Clearly, a higher subsidy on debt encourages the household to hold more debt during the crisis (i.e., \( t = 1 \)). Equation (11) points to the asset pricing equation, which is affected by the return \( y \) and the discount factor \( 1/|u'(c_1) - \lambda_m \phi| \).

We are ready to state the main result. The optimal \( s \) can fully replicate the unconstrained first best allocation even with the Lagrange multiplier of the collateral constraint being strictly positive. Bearing that \( \lambda_m \) is a function of \( s \), Proposition 1 summarizes the main result.

**Proposition 1** Suppose that the household maximizes the utility of \( u(c_0) + u(c_1) + c_2 \) subject to (1), (8), (3) and the end-of-period collateral constraint (7). Then, there exists \( s^* \) with which decentralized equilibrium achieves the unconstrained first best allocation, where \( s^* \) is equal to the equilibrium shadow value of increasing debt in the decentralized economy:

\[ s^* = \lambda_m (s^*) \geq 0. \]  

**Corollary 1** If \( 0 < \phi < (2 - e)/y \), \( s^* \) is given by

\[ s^* = \frac{1}{\phi} - \frac{y}{2 - e} > 0. \]  

Otherwise, \( s^* = 0. \)

**Proof.** It is straightforward to obtain the unconstrained first best allocation: \( c_{FB,0} = c_{FB,1} = 1, c_{FB,2} = y - 2 + e, \) and \( d_{FB,2} = 2 - e. \) For \( (2 - e)/y \leq \phi \leq 1, \) the optimal bailout subsidy is trivial: \( s^* = \lambda_m (0) = 0 \) and the collateral constraint does not bind under \( s^*. \) For \( 0 < \phi < (2 - e)/y, \) \( d_{FB,2} > \phi y. \) If \( s^* \) exists, it must be the case that \( \lambda_m (s^*) > 0 \) and \( d_{FB,2} = \phi p_1. \) Substituting the unconstrained first best allocation into (10) and (11) yields \( s = \lambda_m (s) \) and \( p_1 = y/(1 - s \phi), \) respectively. We can solve for \( s^* \) in (13) by eliminating \( p_1 \) from \( p_1 = y/(1 - s \phi) \) and \( d_{FB,2} = \phi p_1. \) The solution \( s^* \) confirms \( s^* = \lambda_m (s^*). \)
3.2 Interpretation

Our result indicates that, if the collateral constraint (4) in JK is replaced by (7), the bailout policy $s^*$ can fully avoid fire sales of collateral and achieve the allocation as if the collateral constraint were not binding. Without the subsidy, the binding collateral constraint prompts fire sales of collateral because the households try to increase the net supply of collateral in an attempt to prevent reductions in their consumption. Fire sale externalities further tighten the collateral constraint, creating the debt-asset loop. With the bailout subsidy, however, the government can avoid the fire sales of collateral and can achieve the first best allocation. The debt-asset loop does not play out in this case. This result also means that the bailout policy $s^*$ is strictly preferred to the prudential policy $\tau$, because the prudential policy can only replicate the constrained social planner’s allocation.

To understand the result, consider the case where $0 < \phi < (2 - \epsilon) / y$. In this case, the government intervenes into the credit market with a positive subsidy. With the bailout subsidy, the households increase borrowing and shifts the net supply curve (i.e., $\theta_1 - \theta_2$) for collateral leftward. As the result of the leftward shift of the net supply curve, the price of collateral is inflated. The asset price inflation enhances the households’ borrowing capacity $\phi p_1$ until the first best level of the period-2 debt is attained: $d_{FB,2} = \phi p_1$. Put differently, $d_{FB,2}$ uniquely determines the target level of the asset price for the government:

$$p_1 = \frac{d_{FB,2}}{\phi} = \frac{2 - \epsilon}{\phi} > y,$$

where the inequality is ensured by the assumption of $\phi < (2 - \epsilon) / y$.

With the end-of-period collateral constraint (7), the households take into account the fact that holding more collateral increases their borrowing capacity. Thus, the first order condition (11) shows that $p_1$ is affected by $\lambda_m \phi$, the shadow value of holding – or not fire-selling – the collateral. Using (10), the asset pricing equation (11) reduces to

$$p_1 = \frac{y}{[1 - (1 + s) \phi] u'(c_1) + \phi}.$$ 

This equation suggests that the government can fully control the price of collateral by $s$ at any level of $p_1 > 0$. If the government sets $s = s^* = \lambda_m (s^*)$,

$$p_1 = \frac{y}{1 - s^* \phi}. \tag{15}$$
That is, \( s^* \) can be consistent with the target price given by (14), while satisfying all first order conditions and resource constraints. Equating (14) with (15) yields the solution for \( s^* \).

With the beginning-of-period collateral constraint (4), however, the households cannot increase the borrowing capacity because \( \theta_1 \) is pre-determined at \( t = 1 \) when they determine \( d_2 \). The asset pricing equation is \( p_1 = y/u(c_1) \) with (4). At the first best allocation, it must be the case that

\[
p_1 = y, \tag{16}
\]

instead of (15). Consequently, any subsidy, \( s \), cannot achieve the first best allocation because the target asset price requires \( p_1 > y \). This implies that full bailouts are impossible with (4).

Using a two sector model of Bianchi (2011), BCORY argue that, when the government has policy instruments other than capital controls, it can achieve the unconstrained first best allocation. In their model, the income from tradable and non-tradable endowments can be pledged as collateral. Further, the government makes a commitment that, during crises, it supports the real exchange rate (the relative price of non-tradables to tradables) by a subsidy on consumption of these goods. BCORY show that the commitment of the government can fully eliminate the collateral constraint in achieving the unconstrained first best allocation. In addition, owing to the commitment to eliminating the collateral constraint, households behaves as if they were in economy without the collateral constraint. In other words, the collateral constraint never binds and the price support policy is never activated in equilibrium.

In our model, by contrast, the government has only the single policy instrument of capital controls. But the government can achieve the first best allocation, \( c_{FB,t} \) for \( t = 0, 1, 2 \) and \( d_{FB,t} \) for \( t = 1, 2 \). The government makes a commitment to bailing out the credit-constrained households during crises and supporting the price of collateral as in BCORY. However, the government’s intervention does not remove the collateral constraint. Surprisingly, the government which subsidizes debt holdings increases the shadow price of holding debt as indicated by (12). While increasing the shadow price of holding debt, the subsidy also increases the shadow value of holding collateral. Because the asset price under the end-of-period collateral constraint is positively correlated with the shadow value of holding collateral, the asset price can be inflated with the subsidy on debt. Therefore, the Lagrange multiplier under the optimal bailout subsidy remains strictly positive in equilibrium whenever the government needs to intervene the credit market. Because the strictly positive Lagrange multiplier will be actually observed in equilibrium by the households, the bailout subsidy in our model must be activated whenever needed.
As a final remark, while we have assumed that $e$ is deterministic in this illustration, the result can be easily extended into stochastic environment. In the next section, we briefly show that our result applies to a more general, infinite horizon setting with an occasionally binding collateral constraint.

4 The infinite horizon case

We consider the stochastic infinite horizon model, similar to Jeanne and Korinek (2011) and Bianchi and Mendoza (2012). The households choose $d_{t+1}$ and $\theta_{t+1}$ to maximize

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where $\beta$ is the discount factor satisfying $\beta \in (0, 1)$. Each household is faced with the period-by-period budget constraint:

$$c_t + d_t + p_t \theta_{t+1} = \theta_t e_t + (1 + s_t) \frac{d_{t+1}}{R} - S_t + p_t \bar{\theta},$$

and the end-of-period occasionally binding collateral constraint:

$$\frac{d_{t+1}}{R} \leq \phi p_t \theta_{t+1}.$$ 

In this maximization problem, $d_{t+1}$ is non-state-contingent one-period debt. The real interest rate on the non-state-contingent debt is $R > 1$ rather than unity. Every period, the households receive the exogenous endowment of collateral assets $\tilde{\theta}$, which is normalized to one. He receives a stochastic income $e_t$ based on the predetermined share of collateral assets $\theta_t$. The budget constraint here is basically the same as (8), but also similar to (3) in terms of the returns on collateral. Finally, the collateral constraint is the same as (7) except for $R$.

The first order conditions are standard:

$$\left(1 + s_t\right) u'(c_t) = \beta R \mathbb{E}_t u'(c_{t+1}) + \lambda_{mt}$$

$$p_t = \beta \frac{\mathbb{E}_t [u'(c_{t+1}) e_{t+1}]}{u'(c_t) - \lambda_{mt} \phi}$$

$$0 = \left[ \phi p_t \theta_{t+1} - \frac{d_{t+1}}{R} \right] \lambda_{mt}, \lambda_{mt} \geq 0, \text{ and } \phi p_t \theta_{t+1} - \frac{d_{t+1}}{R} \geq 0.$$
In equilibrium, the markets for collateral and consumption goods clear: \( \theta_{t+1} = \bar{\theta} \) and \( c_t + d_t = e_t + d_{t+1}/R \) for all \( t \), respectively. As before, we assume a balanced budget of the government: \( S_t = s_t d_{t+1}/R \).

The first best allocation must satisfy the following first order conditions:

\[
\begin{align*}
    u' (c_{FB,t}) &= \beta \mathbb{E}_t [u' (c_{FB,t+1})] \\
    p_{FB,t} &= \beta \mathbb{E}_t [u' (c_{FB,t+1}) e_{t+1} / u' (c_{FB,t})]
\end{align*}
\]

which yield the policy functions \( c_{FB,t} = c_{FB} (d_t, e_t) \) and \( d_{FB,t+1} = d_{FB} (d_t, e_t) \) along with the asset pricing function obtained from the model without the collateral constraint \( p_{FB,t} = p_{FB} (d_t, e_t) \), all of which are functions of the state variables \( (d_t, e_t) \).\(^6\) The following proposition indicates that there exists the state-contingent optimal subsidy \( s_t = s_t (d_t, e_t) \) consistent with the first best allocation in the stochastic infinite horizon model.

**Proposition 2** Suppose that the household maximizes (17) subject to (18) and the end-of-period collateral constraint (19). Then, there exist a price function \( p_s (d_t, e_t) \) and the subsidy \( s^* (d_t, e_t) \), with which the decentralized equilibrium characterized by (18) - (22) achieves the unconstrained first best allocation \( \{ d_{FB,t+1}, c_{FB,t} \}^{\infty}_{t=0} \). Furthermore, the subsidy \( s^*_t = s^* (d_t, e_t) \) is proportional to the Lagrange multiplier for (19):

\[
s^* (d_t, e_t) = \frac{\lambda_m (d_t, e_t; s^*_t)}{u' [c_{FB} (d_t, e_t)]} \geq 0,
\]

where \( \lambda_m (d_t, e_t; s_t) \) represents the Lagrange multiplier for the collateral constraint, given \( s_t \).

**Corollary 2** If \( d_{FB} (d_t, e_t) / R > \phi p_{FB} (d_t, e_t) \), \( s^* (d_t, e_t) \) is given by

\[
s^* (d_t, e_t) = \frac{1}{\phi} - \frac{p_{FB} (d_t, e_t)}{d_{FB} (d_t, e_t) / R} > 0.
\]

Otherwise, \( s^* (d_t, e_t) = 0 \).

**Proof.** We consider (i) \( d_{FB} (d_t, e_t) / R \leq \phi p_{FB} (d_t, e_t) \) and (ii) \( d_{FB} (d_t, e_t) / R > \phi p_{FB} (d_t, e_t) \) for the states of the economy \( (d_t, e_t) \). Here, the conditions distinguish whether or not the\(^6\)In this formulation, we abstract frictions in debt adjustment (e.g., a risk premium, or debt adjustment cost) from the model, for expositional purpose. It is well-known that the first best allocation in small open economy models without the collateral constraint results in nonstationary debt holdings. (Schmitt-Grohe and Uribe, 2003). Nevertheless, even when we include such frictions into our model (e.g., the risk-premium in \( R \) in (18)), our result in this section remains unaltered.
first best level of debt is feasible for a particular state of the economy. For each case, we will confirm that the first order conditions (20) – (22) are satisfied under \( s^* (d_t, e_t) \) when they are evaluated at the first best allocation \( c_{FB} (d_t, e_t) \) and \( d_{FB} (d_t, e_t) \). Consider the states satisfying (i). If \( s^* (d_t, e_t) = 0 \), \( \lambda_m [d_t, e_t, s^* (d_t, e_t)] = 0 \) because the first best allocation is trivially achieved. Then, the efficiency conditions of (20) and (21) are the same as (23) and (24). Therefore, \( p_s (d_t, e_t) = p_{FB} (d_t, e_t) \) and \( s^* (d_t, e_t) \) is confirmed to be optimal for the states satisfying (i).

Next, for the states satisfying (ii), consider the price of collateral that achieve \( d_{FB} (d_t, e_t) \) with the binding collateral constraint. If \( p_s (d_t, e_t) = d_{FB} (d_t, e_t) \), then (22) is satisfied together with \( \lambda_m [d_t, e_t; s^* (d_t, e_t)] \geq 0 \). Combining (25), (21), and (27) yields
\[
\frac{d_{FB} (d_t, e_t)}{\phi R} = \beta \mathbb{E}_t \left( u' \left\{ c_{FB} \left[ d_{FB} (d_t, e_t) \right], e_{t+1} \right\} e_{t+1} \right) \left( 1 - s^*_t \phi \right),
\]
for all states \((d_t, e_t)\) satisfying (ii). Using the above equation and (24), we can solve for \( s^*_t = s^* (d_t, e_t) \) and the solution turns out to be (26). Because \( s^*_t \) is chosen to satisfy (28), (22) and (21) are obviously satisfied at the first best allocation. Finally, we show that (20) can be derived from (23) and (25). In particular,
\[
\left[ 1 + s^* (d_t, e_t) \right] u' [c_{FB} (d_t, e_t)] = \beta \mathbb{R} \mathbb{E}_t \left( u' \left\{ c_{FB} \left[ d_{FB} (d_t, e_t) \right], e_{t+1} \right\} \right) + \lambda_m [d_t, e_t; s^* (d_t, e_t)].
\]
Hence, (20) is satisfied under \( s^*_t = s^* (d_t, e_t) \).

The proposition confirms that, even in the stochastic infinite horizon model, the optimal bailout function \( s^* (d_t, e_t) \) can achieve the first best allocation as in the three period model. The mechanism behind the result is the same as that in the three period model. With the end-of-period collateral constraint, the price of collateral is affected by the shadow value of holding collateral \( \lambda_m (d_t, e_t, s^*_t) \phi \) and the optimal bailout subsidy is proportional to \( \lambda_m (d_t, e_t, s^*_t) \).

Equation (28) corresponds to (15) in the three period model. In other words, the government can fully control the asset price to avoid fire sales by changing \( s_t \).

Again, the optimal bailout subsidy achieves the first best allocation under the end-of-
period collateral constraint, because the shadow value of holding collateral affects the price of collateral. With the beginning-of-period collateral constraint, the asset pricing function is

\[ p_t = \beta E_t [\frac{u'(c_{t+1}) e_{t+1}}{u'(c_t)}], \]

which is independent of \( \lambda_{m,t} \) and no \( s_t \) can inflate the asset price to the target level of \( p_t \) indicated by (27).

A few remarks are in order. First, the state-contingent optimal bailout subsidy \( s^*_t \) is easy to calculate because (26) indicates that the optimal bailout subsidy is a function of \( d_{FB,t+1} \) and \( p_{FB,t} \). If one is interested only in the optimal size of intervention and frequencies of the activations of the optimal ex-post intervention, the corollary suggests that there is no need to simulate in the non-linear model with an occasionally binding collateral constraint. He could calibrate the policy functions for \( d_{FB,t+1} \) and \( p_{FB,t} \) from the model without the collateral constraint and plug the solutions into (26). Without the collateral constraint, the policy functions are well approximated by the linearized model, which further simplifies computation. The information regarding on the collateral constraint is the ceiling of the leverage \( \phi \). Hence, when the tightness of financial frictions can be calibrated, he could easily assess the feasibility of the optimal bailout subsidy by the size of \( s^*_t \) and the frequency of policy activations.

Second, although the computational simplicity of \( s^* (d_t, e_t) \) is appealing, the proposition has some limitations regarding the characteristics of the collateral asset in the model. In the proposition, we assumed, for simplicity, that the returns on the collateral asset are confined to the return in period \( t+1 \). In this sense, the collateral asset in our model is not the one like the capital as in Mendoza (2010) or the land as in Bianchi and Mendoza (2012) where the returns on the collateral assets are received over infinitely many future periods. On the other hand, they employ the end-of-period collateral constraint where the shadow value of holding collateral affects the price of collateral. Thus, there might be a possibility that the first best allocation can be achieved by the bailout subsidy on debt, though introducing returns in future periods generally makes it difficult to prove the possibility. Nevertheless, even when the first best allocation cannot be achieved, there is another possibility that an allocation that the bailout policy achieve is close to the first best allocation and thus is better than the allocation that the prudential policies in their models can implement. To evaluate whether the bailout policy outperforms the prudential policy in the model where the capital or the land can be pledged as collateral, the quantitative analysis would be called for.
Finally, it is generally impossible for the subsidy on debt to achieve the first best allocation in the class of the models by Bianchi (2011) and Benigno, Chen, Otrok, Rebucci and Young (2013). In these models, the amount of borrowing is constrained by a fraction of households’ current income. In Bianchi’s (2011) model, the market value of tradable and non-tradable endowments can be pledged as collateral. Because the households cannot control the quantity of endowments, the shadow value of holding collateral does not emerge in the asset pricing equation. This means that the government cannot use the subsidy on debt to control the price of collateral and thus it fails to achieve the first best allocation. In the model by Benigno, Chen, Otrok, Rebucci and Young (2012b, 2013), they assume that tradable and non-tradable goods are produceable in the economy. In our context, this change in the assumption is similar to the change from the beginning-of-period to the end-of-period collateral constraint. The reason is that this production-economy assumption allows the household to choose the labor supply endogenously and the price of collateral is affected by the shadow value of producing more collateral. In particular, the endogenously chosen labor supply gives rise to the substitutions across labor and consumption goods. The substitution between labor and consumption goods then may affect the price of collateral and create the price of collateral that depends on the shadow value of producing collateral. In this case, the subsidy on debt during crises may increase the welfare such that it exceeds the level that the prudential capital control can achieve. As Benigno, Chen, Otrok, Rebucci and Young (2012b) recently discussed, however, only the subsidy on debt during crises cannot achieve the first best allocation in this class of the production-economy models. This is because the substitutions across labor and consumptions would be distorted by the intervention and the distortions cannot be removed without additional policy instruments of the government.

5 Concluding remarks

This paper analyzed the scope for the ex-post bailout policy in managing financial crises. Using the simple framework employed by JK, we showed that the subsidy on debt during crises could overperform the prudential capital controls in terms of welfare. Our result is closely related to that of BCORY who recently found that a price support policy during crises can be strictly preferred to the prudential capital controls if such policy instruments are available to the government. But our result has notable differences from them. We showed that, even without alternative policy instruments other than capital controls, the government could achieve the first best allocation via capital controls during crises. The
key element for this result is the timing of the purchase of collateral. If households can collateralize their asset that they purchase at the same time with their borrowing, the asset price would be influenced by the shadow value of holding collateral and the government can control the shadow value via the bailout subsidy. If we employ the JK’s original formulation of the collateral constraint, however, the same policy fails to achieve the first best allocation.

We presented the results analytically. Another analytical result in our analysis would be that the optimal bailout subsidy function can be computed without solving the model with an occasionally binding collateral constraint. Using the calibrated optimal bailout subsidy function, we could investigate the policy in the light of empirical perspective. On the other hand, for analytical simplicity, we considerably simplified many aspects of the model especially in the stochastic infinite horizon model. To extend the analysis into more general framework comparable with other previous studies, the quantitative assessment would be called for. All of these would be important steps for future research.

References


