Assessing the Implications of Financial/Real Interactions for Business Cycles in Macroeconometric Models

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Abstract

Many macroeconometric models are built to understand business cycles and (possibly) to predict recessions. But the methods applied to assess them are rarely of the form that one learns either whether they provide a good explanation of cycle characteristics or whether they can predict recessions. In this paper we review and apply techniques that do this. We apply the methods to two models which typify common approaches to including financial/real interactions - Gilchrist, Ortiz & Zakrajšek (2009) and Iacoviello & Neri (2010) - and find that the introduction of financial features into these models does not modify cycles to the extent that the empirical literature, such as IMF (2009), would suggest is needed.

1 Introduction

Many macroeconometric models are built to either produce an understanding of business cycles or to see if recessions can be predicted with them. Since the global financial crisis (GFC) there has been an explosion of models that aim to do one or both of these tasks in situations where it is felt that financial conditions are very important. Examples are Christensen & Dib (2008), Gilchrist et al. (2009), Liu, Wang & Zha (2009), Gertler & Kiyotaki (2010) and Iacoviello & Neri (2010). These papers deal with a number of issues such as credit availability, collateral, and the role of ‘animal spirits’ in initializing and propagating cycles. Generally the success of these modelling strategies is judged by: whether the model replicates some simple relationships; whether the models fit as well as a VAR; how the impulse responses change as a result of the financial constraints; and whether variance decompositions imply that financial factors are important. These are all useful tests to apply when considering model performance but, given that the models are often designed to explain business cycle outcomes, it would also seem useful to focus directly upon whether they provide a good match to these. In particular, we are interested in whether the models imply that the characteristics of the business cycle would be modified much by the presence of financial constraints.

Business cycles involve periods of recessions and expansions in the level of economic activity and therefore one needs some method of isolating when these occur in a given set of data

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representing economic activity i.e. one needs to find a set of turning points in the series. There is now a substantial literature on ways to do this. The first popular method was that set out in Bry & Boschan (1971), and it is widely used in industry to establish turning points in the series that are utilized to date recessions. A quarterly version of this, termed BBQ, that has had extensive use is set out in Harding & Pagan (2002).\(^1\) Variants of the BBQ algorithm have been used in much work that the IMF has done on the interactions between financial and real cycles - for example Claessens, Kose & Terrones (2011). As we are looking at models estimated on quarterly data it is proposed that the BBQ algorithm be applied to data simulated from the estimated macro-econometric models so as to capture the turning points in economic activity and hence to summarize the business cycle features of such models.

In many ways the best way to view the approach proposed in this paper is as providing another cut of the data, but one that is particularly informative when the focus is on the ability of models to generate realistic business cycles. The cut we perform basically combines together the moments of the growth rate in output - mean, variance and serial correlations- as shown in Harding & Pagan (2002). One could compute all these quantities separately, but it is the way that they are combined together which produces statistics which relate to the business cycle.

Many modern macroeconomic models, including those with financial/real interactions, do not provide the data generating process of the growth rates in economic activity. Instead, because the models are often set up so as to be stationary levels of output (around a deterministic trend path), it is felt that the data needs to be filtered before moments of the series are computed, and this is often done using filters such as Band Pass (e.g. Baxter & King (1999)) and Hodrick Prescott (Hodrick & Prescott 1997). However, the business cycle is about the level of output and the nature of cycles in it depends on the moments of the growth rate in output. Consequently, no filtering is required even when the level of output is non-stationary.\(^2\) Of course one could study turning points in the filtered series but then one is dealing with the growth cycle, not the business cycle.

In the next section we discuss some ways in which researchers have tried to investigate the causes of the business cycle and explain why some of the existing methods such as variance decompositions do not do this very effectively. We subsequently illustrate our methodology by selecting two models which typify common approaches to incorporating real/financial interactions. The first of these is presented in section 3 and is due to Gilchrist et al. (2009) (termed GOZ hereafter). It emphasises the financial accelerator. The second, described in Section 4, is due to Iacoviello & Neri (2010), and it revolves around the role of collateral.

Neither paper directly addresses whether the proposed models match the business cycle features seen in the data. Both papers report differences in impulse responses with and without financial factors, but these may or may not imply different cycles. In addition, GOZ present a decomposition of the level and volatility of the filtered component of GDP into contributions

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\(^1\)It is available at http://www.ncer.edu.au/data/ as a macro in Excel, as well as Matlab and Gauss programs. The Matlab version was used to generate the results in this paper.

\(^2\)As Zarnowitz & Ozyildirim (2006) say in relation to historical work: “...studies which defined business cycles as sequences of expansions and contractions in a large array of series representing the levels of total output, employment...".
from various identified shocks. One difficulty with the latter type of exercise is that, in this
decomposition, output must be fully explained by the model shocks, that is, there is no residual
term. Consequently, it is always unclear in such decompositions whether a shock accords with
its label or simply represents a residual.

Our approach therefore is to assess the models on other features, namely their ability to
capture the length, amplitude and other important characteristics of the business cycle with
and without financial effects. For this purpose we assemble some ‘stylised facts’ relating to
recessions and the role of credit. These are drawn from a number of sources, but principally
from the work of the International Monetary Fund (IMF) reported in the World Economic
Outlook, particularly the April 2009 issue. Because many of the measures the IMF use adopt
the method set out in Harding & Pagan (2002) for locating turning points we can utilize these
findings. We find that, while at least one of the models we study can replicate some of the
features set out in the IMF work, it is clear that neither model is able to fully capture financial
effects upon the business cycle.

In section 5 we ask whether the models are useful for predicting recessions. Because we
have a precise definition of a turning point it will become clear why prediction of recessions is
difficult. Nevertheless, we set out a framework that enables a researcher to perform some quick
checks on whether the model will have some predictive success. Finally, Section 6 concludes

2 Business Cycle Effects of Different Shocks

One way to assess the impact or relative importance of different shocks, including those which
are financial in nature, is to observe that, if there are as many shocks as variables in the model,
then one can exhaustively decompose the behaviour of any variable \(y_t\) into the contributions
from the shocks using

\[
y_t = y_0 + \sum_{i=1}^{K} y'_i = y_0 + \sum_{i=1}^{K} \sum_{j=1}^{t} C_{ij} \varepsilon_{ij},
\]

where \(\varepsilon_{it}\) are \(K\) uncorrelated (structural) shocks and the impulse responses of \(y_t\) to a unit rise
in \(\varepsilon_{it-j}\) are \(C_{ij}\). Here \(y'_i\) is the contribution to \(y_t\) of the \(i'th\) shock. Many researcher have used
this device to gain an impression of which shocks have been important in contractions and
expansions in economic activity. An alternative that is also widely used has been to look at the
forecast variance of \(y_{t+L} - y_t\). This can be written in terms of the sums of the products of \(C_{ij}^2\)
with the variance of the \(i'th\) shock. Consequently the fraction of the variance of \(y_{t+L} - y_t\) explained
by each of the shocks can be determined. The forecast variance decomposition technique is
widely used in VAR work. Often \(L\) is set to the “business cycle horizon” and the dominant
contributor to the variance of \(y_{t+L} - y_t\) is regarded as the “cause” of the business cycle. There
is no consensus on what \(L\) should be and choices have ranged between \(L = 12\) and \(L = 25\)
quarters. In this section we discuss a limitation of forecast error variance decompositions to
assess the importance of a particular shock which is overcome by using the business cycle dating
techniques presented in this paper.
One often finds that the information from a variance decomposition differs from that seen from looking at $y_i'$. An example is Galí (1992). By plotting $y_i'$ he concluded that demand shocks were important for recessions but, in contrast, his variance decomposition analysis showed that, when predicting variables 10 quarters out, 92% of the variance of the forecast error was explained by supply side shocks and only 3% was accounted for by demand shocks. This led him to comment (1992 p722) that, “The results here seem less akin to a traditional Keynesian view of economic fluctuations”. One reason for the discrepancy may be that the forecast error variance decompositions are averaging across the sample, whereas the analysis of $y_i'$ might have been influenced by just a few of the contractions (particularly if one is using visual evidence on $y_i'$ to assess which are the important shocks).

So we would want some evidence that the shocks identified as important by analysing $y_i'$ were also important "on average".

It is useful to work with the same model that Galí (1992) did in explaining our resolution of the conflict just noted. It was a SVECM Model. The variables in his system were the log level of GDP ($y_t$), the nominal interest rate ($i_t$), the log of the price level ($p_t$) and the log of the money supply ($m_t$). Galí then reduced the SVECM to an SVAR with variables $\Delta y_t$, $\Delta i_t$ and ECM terms $i_t - \Delta p_t$ and $\Delta m_t - \Delta p_t$. He proceeded with the assumptions that the nominal interest rate and inflation were both $I(1)$ processes, while the real interest rate and other growth variables were $I(0)$. The equations of the SVECM form a structural system with four structural shocks that were identified as aggregate supply $\varepsilon_{1t}$, money supply and demand ($\varepsilon_{2t}$ and $\varepsilon_{3t}$) and aggregate demand ($\varepsilon_{4t}$). Essentially it is an IS/LM model augmented by a Phillips curve. The system is recorded below with the contemporaneous part being highlighted.

\[
\begin{align*}
\Delta y_t &= \beta_{12}^0 \Delta i_t + \beta_{13}^0 (i_t - \Delta p_t) + \beta_{14}^0 (\Delta m_t - \Delta p_t) + \text{lags} + \varepsilon_{1t} \\
\Delta i_t &= \beta_{21}^0 \Delta y_t + \beta_{23}^0 (i_t - \Delta p_t) + \beta_{24}^0 (\Delta m_t - \Delta p_t) + \text{lags} + \varepsilon_{2t} \\
i_t - \Delta p_t &= \beta_{31}^0 \Delta y_t + \beta_{32}^0 \Delta i_t + \beta_{34}^0 (\Delta m_t - \Delta p_t) + \text{lags} + \varepsilon_{3t} \\
\Delta m_t - \Delta p_t &= \beta_{41}^0 \Delta y_t + \beta_{42}^0 \Delta i_t + \beta_{43}^0 (i_t - \Delta p_t) + \text{lags} + \varepsilon_{4t}.
\end{align*}
\]

The system was then identified by making a number of assumptions.

1. The shocks are uncorrelated among themselves.
2. Demand and monetary shocks have no long run effect on output.
3. Money shocks have no contemporaneous effect on output i.e. they only affect $y_{t+j}$ ($j \geq 1$) and not $y_t$.
4. In setting the money supply the monetary authority does not contemporaneously react to the inflation rate.

The impact of these restrictions upon the system and the estimation of it was discussed in Pagan & Robertson (1998). Here a longer data sample was used than Galí adopted but the parameter estimates of the system are not too far away from those reported in Pagan & Robertson (1998).
The estimated model is simulated to find the implied level of GDP (a deterministic trend that Galí subtracted was added back on to the cumulated values of $\Delta y_t$) and this simulated data was then used to investigate the nature of the business cycle implied by the model.\(^3\) Thereafter we omit some shocks in order to see what their role is in the business cycle.

Table 1: **US Business Cycle Characteristics, Galí’s SVAR Model**

<table>
<thead>
<tr>
<th>Durations (qtrs)</th>
<th>All shks $\varepsilon_{jt} = 0 \ (j = 2, 3, 4)$</th>
<th>$\varepsilon_{jt} = 0 \ (j = 2, 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contractions</td>
<td>4.4</td>
<td>2.9</td>
</tr>
<tr>
<td>Expansions</td>
<td>18.3</td>
<td>34.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amplitude (%)</th>
<th>Contractions</th>
<th>Expansions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contractions</td>
<td>-2.9</td>
<td>21.8</td>
</tr>
<tr>
<td>Expansions</td>
<td>-1.1</td>
<td>32.4</td>
</tr>
</tbody>
</table>

Table 1 shows that, if supply side shocks were the only factor present in the economy, the business cycle would be quite long, and that demand shocks are responsible for reducing the cycle to a length comparable to that actually observed. Money demand and supply shocks play almost no role since the cycle stays the same length even when they are deleted. Thus an implication of the analysis above would be that, if demand shock volatility can be reduced, then one might expect expansions to be of the order of ten years or so. So the implication from this Table is the same as Galí got from inspecting $y_t'$ and differs from his variance decomposition results. Notice that Table 1 is averaging across business cycles, so that the possible cause of divergence between the two measures mentioned earlier - that variance decompositions were looking at averages, while $y_t'$ were looking at particular cycles, is being controlled for.

The reason why forecast error variance decompositions at long horizons tell us little about business cycle determinants is simply that the turning points in the series $y_t$ (thought of as the log of the level of economic activity) are found from the DGP of $\Delta y_t$, as the latter is the indicator of whether $y_t$ is still rising (falling) or has started to fall (rise), and not that of a long difference $\Delta_L y_t$. To crystallize this, suppose we think of a recession being caused by the following event which partially defines a peak at time $t-1$: \{ $\Delta y_t < 0$, $\Delta_2 y_t < 0$ \} (this underlies the BBQ algorithm). Then when studying the emergence of a recession we are looking at the probability of such an event, and it will clearly depend upon the type of DGP that one has for $\Delta y_t$. We can be more specific than that, saying that the probability of a peak in $y_t$ will depend upon the $var(\Delta y_t)$ as well as the $corr(\Delta y_t, \Delta_2 y_t)$ and the latter will be determined by $var(\Delta y_t)$ and $cov(\Delta y_t, \Delta y_{t-1})$. Notice that it is the variance of $\Delta y_t$ which is important and not the variance of $\Delta_L y_t$, unless $L = 1$. If one chose $L = 2$ then that variance would depend on $var(\Delta y_t)$ and $corr(\Delta y_t, \Delta y_{t-1})$. Thus variance decompositions only relate to business cycle outcomes if $L$ is very small, and not if $L$ is large.

We now turn to an examination of the business cycle properties of some models which explicitly incorporate financial/real interactions.

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\(^3\)This simulation was also presented in Fry & Pagan (2005).

3.1 The Gilchrist et al Model

Gilchrist et al. (2009), (GOZ) allowed for an impact for financial conditions upon output using the financial accelerator mechanism described in Bernanke, Gertler & Gilchrist (1999). In brief, because asymmetric information exists between borrowers and lenders about the realized return on capital, the interest rate charged includes a premium above the risk free rate, known as the external finance premium, which depends on the leverage (or net worth) of the borrower. Consequently, shocks that, say, decrease the net worth of the borrower also increase the interest rate, and this constitutes the financial accelerator mechanism. Gilchrist et al. (2009) add this feature to a standard macroeconometric model set out by Smets & Wouters (2007) (SW). Specifically, the SW model is augmented by four equations:

\[
\begin{align*}
E_t \tilde{r}_{t+1}^K &= \frac{1 - \delta}{R_K + (1 - \delta)} E_t \tilde{q}_{t+1} + \frac{\bar{R}_K}{R_K + (1 - \delta)} E_t \tilde{m}_{t+1} - \tilde{q}_t \quad (1) \\
\tilde{s}_t &= E_t \tilde{r}_{t+1}^K - (\tilde{r}_t - E_t \tilde{r}_{t+1}) \quad (2) \\
\tilde{s}_t &= \chi(\tilde{q}_t + \tilde{k}_t - \tilde{n}_t) + \varepsilon_{t}^{d} \quad (3) \\
\tilde{n}_t &= \frac{\bar{K}}{\bar{N}} \tilde{r}_{t}^K - (\frac{\bar{K}}{\bar{N}} - 1)(\tilde{s}_{t-1} + \tilde{r}_{t-1} - \tilde{\pi}_t) + \theta \tilde{n}_{t-1} + \varepsilon_{t}^{NW} \quad (4)
\end{align*}
\]

where the over-bar indicates a steady state value, \( r_t^K \) is the rate of return on capital, \( q_t \) is the real price of capital, \( mpk_t \) is the marginal product of capital, \( s_t \) is the external finance premium, \( k_t \) is the capital stock, and \( n_t \) is entrepreneurs’ net worth. Tildes denote deviations from a steady state position. Of the coefficients, \( \delta \) is the depreciation rate of capital and \( \theta \) is the survival rate of entrepreneurs. Equation (2) defines the external finance premium as the difference between the expected rate of return on capital (which is determined by Equation (1)) and the real interest rate. Equation (3) shows how the external finance premium varies with the degree of leverage, and this is governed by the parameter \( \chi \). The shock \( \varepsilon_{t}^{d} \) captures fluctuations in the supply of credit unrelated to the leverage of the entrepreneurs. The evolution of the net worth of entrepreneurs in given in Equation (4), with the first term reflecting the leveraged return of capital and the second the cost of debt. A certain fraction of wealth disappears as entrepreneurs disappear \( ((1 - \theta)\tilde{n}_{t-1}) \). The shock \( \varepsilon_{t}^{NW} \) represents exogenous fluctuations to net worth.

3.2 Stylized Facts About Financial/Real Interactions

We might ask what features relating to business cycles and financial conditions are to be matched, since both the GOZ and IN models aim to measure the relationship between the two phenomena. Drawing on sources such as IMF (2009), the following four suggest themselves.
1. The length and amplitude of expansions and contractions with and without financial constraints. Empirical evidence would suggest that financial effects would tend to make expansions shorter and recessions longer.

2. Following IMF (2009) the probability of a recession would increase if there is a financial crisis. It was especially noticeable in their work that it was the probability of an investment recession that rose substantially in the presence of a crisis.

3. Recessions that are accompanied by a financial crisis are expected to last longer than those without. From the evidence presented a reasonable rule of thumb was that the duration of a recession would likely double in the presence of a crisis.

4. The Austrian economics view of business cycles was that they were mostly due to excessive credit expansion which stimulated unproductive investment. This eventually leads to a contraction in credit and a recession. *Prima facie* the GFC does seem to be an example of this, and there are also other booms and busts in history which generally accord with this description. Indeed there is a growing literature on credit cycles that emphasises pressure for banks to issue riskier loans during expansions in order to retain their market share. Therefore Aikman, Haldane & Nelson (2010) say “Credit lies at the heart of crises. Credit booms sow the seeds of subsequent credit crunches” (p.1) and note that

“On average, more than half of all financial crisis years across the 12 countries appear to have been preceded by a credit boom. Among Anglo-Saxon countries, such as the US, UK and Australia, closer to 75 per cent of crisis years occurred following a credit boom. This is relatively concrete evidence of the credit cycle having real and damaging effects on output.” (p.21)

The work utilizes the database constructed by Schularick & Taylor (2012) who conclude “credit aggregates contain valuable information about the likelihood of future financial crises.” (p. 1058-1059)

### 3.3 Matching of the GOZ Model and Business Cycle Features

We proceed to ask whether the GOZ model can replicate the four features listed above.

#### 3.3.1 Durations and Amplitudes with and Without Financial Constraints

To assess the business cycle properties of the GOZ model we simulate data (15,000 observations) from it and apply the BBQ cycle-dating procedure to the simulated data. Because the real variables in the GOZ model are log deviations from a constant growth path we need to add back a trend growth term to the simulated data to get a series on the level of GDP that can be used to determine the business cycle. We use the trend growth rate in GDP assumed by GOZ. Table 2 contains the cycle output for both the GOZ and SW models along with what we
would get when BBQ is applied to quarterly per capita US GDP data over the period 1973:Q1–2009:Q4. The Smets-Wouters results are found by using the parameters estimated by GOZ; in other words, they show the business cycle properties of the model when the financial accelerator mechanism is excluded.

Table 2: Business Cycle Characteristics – Data, SW and GOZ Models

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>SW model</th>
<th>GOZ model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Durations (qtrs)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>13.6</td>
<td>13.3</td>
<td>14.8</td>
</tr>
<tr>
<td>Contractions</td>
<td>4.8</td>
<td>4.5</td>
<td>4.2</td>
</tr>
<tr>
<td><strong>Amplitude (%)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>9.2</td>
<td>8.9</td>
<td>9.0</td>
</tr>
<tr>
<td>Contractions</td>
<td>-2.8</td>
<td>-1.9</td>
<td>-1.6</td>
</tr>
<tr>
<td><strong>Cumulative amplitude (%)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>132.4</td>
<td>99.2</td>
<td>107.9</td>
</tr>
<tr>
<td>Contractions</td>
<td>-8.1</td>
<td>-7.0</td>
<td>-5.2</td>
</tr>
</tbody>
</table>

The SW and GOZ models both show a good match to the business cycle characteristics, and overall the differences between them are not great. Thus the presence of a financial accelerator does not seem to have had a great effect upon business cycle outcomes, although it should be noted that these are averages, and credit availability may be important in particular cycles.

Some experiments can be conducted here. Doubling the standard deviation of the credit supply shocks in the GOZ model has a very small effect upon the cycle. It is necessary to make much bigger changes in order to have an impact, well outside the range of values of the external finance premium that have been observed. Thus, quadrupling the standard deviation reduces expansion length to 14.7 quarters and increases the amplitude of recessions, although only to –1.7 per cent. But it does this by producing premia of 1 000 basis points (and more). Doubling the coefficient \( \chi \) in Equation (3), which governs the sensitivity of the external finance premium to the entrepreneur’s net worth, has little impact on the business cycle characteristics.

### 3.3.2 Probability of Recession and Financial Crises

What is a financial crisis? As the external finance premium is the important financial cost in the model it would seem logical that a crisis would be evident if the premium went above some

4Alberto Ortiz kindly provided us with a Dynare program that simulated an updated version of the model they use, and some of their data for the period 1973:Q1–2009:Q4. The parameter values set in that code are the posterior mean and are different to those reported in their paper, in part due to a shorter estimation period being used, namely 1985:Q1–2009:Q4. As this is quite short for cycle dating we focus on the longer sample when comparing the models to the data. If instead the shorter sample was used then expansions would be longer and contractions less severe.

5Using the parameters from Smets & Wouters (2007), the results are not that different.

6It is interesting to note that, if money shocks are set to zero, the average duration of an expansion would increase by about two quarters and the amplitude of recessions would decline to -1.32%. This is consistent with the variance decomposition at an horizon of two quarters, which has money shocks accounting for about 11% of the volatility of GDP growth at that horizon.
threshold. As we don’t know what this threshold might be we look at how the probability of a recession changes as the external finance premium changes. One would expect a positive relationship, and perhaps even a non-linear response, as the external finance premium becomes larger. We can investigate the dependence of recessions upon the external finance premium by taking advantage of the binary nature of a recession indicator \( R_t \) (\( R_t = 1 \) if the economy is in recession but zero otherwise). Thus we are interested in \( \Pr(R_t = 1|s_{t-j}) \), where \( j \geq 0 \). One could compute this non-parametrically or by some parametric method. Given the binary nature of \( R_t \) it is attractive to use functions such as the Probit form i.e. \( \Phi(\alpha + \beta s_{t-j}) \), where \( \Phi(\cdot) \) is the cumulative standard normal distribution function. Of course there is nothing which guarantees that this is a good representation of the functional form for the probability, so it is always useful to check it against a non-parametric estimate. Because \( \Pr(R_t = 1|s_{t-j}) = E(R_t|s_{t-j}) \) the conditional probability can be found by applying standard non-parametric methods to estimate a conditional expectation. The non-parametric estimator’s limitations are that there may be few observations for extreme values of \( s_{t-j} \) and the estimated functional form may not be monotonic. Using the simulated data on \( R_t \) and \( s_t \) from the GOZ model Figure 1 fits \( \Pr(R_t = 1|s_t) \) using the Probit function.\(^7\) The Probit and non-parametric estimates agree quite well - the non-parametric estimates are slightly smaller but show the same pattern as \( s_t \) changes. Note that the unconditional probability of an NBER recession over 1953:Q2–2009:Q4 was approximately 0.16. Thus, while the rise in the external premium does increase the probability of a recession, it never gets to a standard critical value of \( .5 \) often used in predicting them.

Following Harding (2008) an alternative indicator of the relationship between recessions and the external finance premium comes from recognising that the recession states \( R_t \) generated by BBQ (and this is also true for NBER recession indicators) follow a recursive process of the form

\[
R_t = 1 - (1 - R_{t-1})R_{t-2} - (1 - R_{t-1})(1 - R_{t-2})(1 - \land_t) - R_{t-1}R_{t-2} \lor_t,
\]

where \( \land_t \) is a binary variable taking the value unity if a peak occurs at \( t \) and zero otherwise, while \( \lor_t \) indicates a trough. By definition \( \land_t = (1 - R_t)R_{t+1} \) and \( \lor_t = (1 - R_{t+1})R_t \). In BBQ,

\[
\land_t = 1(\{\Delta y_t > 0, \Delta_2 y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\})
\]

\[
\lor_t = 1(\{\Delta y_t < 0, \Delta_2 y_t < 0, \Delta y_{t+1} > 0, \Delta_2 y_{t+2} > 0\}),
\]

where \( \Delta_2 y_t = y_t - y_{t-2} \) will be six-monthly growth. Then one might condition upon the previous states as well as the external finance premium, to determine what the probability of going into a recession at time \( t \) is for a given finance premium and the knowledge that we were in expansion at \( t - 1 \) and \( t - 2 \). Using Equation (5) this probability will be

\[
\Pr(R_t = 1|R_{t-1} = 0, R_{t-2} = 0, s_t) = E(\land_{t-1}|s_t).
\]

Assuming a Probit form again, Figure 1 also gives a plot of this against \( s_t \). There is clearly

\(^7\)Setting \( j = 0 \) yields the best fit.
a big difference between it and Pr(R_t = 1|s_t). If it is known that one is in an expansion in
the preceding two periods the rise in the probability of a recession, even for large values of the
external finance premium, is very small. Indeed, the result suggests that the external finance
premium will not be useful for predicting recessions.

Figure 1: Probability of a Recession Given External Finance Premium

3.3.3 The Duration of a Recession and the Size of the External Finance Premium

The next question we seek to examine is whether the duration of recessions depends upon the
magnitude of the external finance premium. There are two ways this might be done. One is to
relate the durations of recessions to the external finance premium. To do this we regressed the
computed durations of recessions from the simulated data against the external finance premium
at the beginning of each recession. While this showed a positive relationship the connection
was very weak – even large changes in the premium only caused the duration to increase by a
fraction of a quarter.

Another method is to compute Pr(\land_{t+m} = 1|\land_t = 1, s_{t-j}), i.e. the probability that, condi-
tional on the external finance premium, in m periods time the economy will be at a trough,
given it was at a peak at time t. m is clearly the duration of the recession. Again there is an
issue of how one estimates this - either by estimating a Probit model or using a non-parametric
estimate. For large premia there are not that many observations so the non-parametric estimate
may not be that reliable over the complete range of values of s_{t-j}. Table 3 shows what this
probability is, using a Probit, for three levels of s_{t-2} and for m = 3, 4, 5 i.e. the probability
that a recession will last at least three, four or five quarters once it has begun. One might note
that the non-parametric estimates are smaller than the Probit estimates over the range from
24 to 481 basis points but the same type of increase is evident. So, if a financial crisis produces
long duration recessions we might expect to see the probabilities of getting longer recessions increase markedly with the level of the external finance premium. But this does not seem to be the case. Take $m = 5$. Whilst it is true that the probability of getting a five quarter recession increases with the level of the external finance premium, it is a very small increase. This weak effect agrees with the regression linking durations and the external finance premia noted above.

Table 3: **Probability of Recession for $m$ Periods as External Premium Varies in GOZ Model**

<table>
<thead>
<tr>
<th>External premium $\tilde{s}_t$ (basis points)</th>
<th>$m = 3$</th>
<th>$m = 4$</th>
<th>$m = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>0.30</td>
<td>0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>305</td>
<td>0.36</td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td>482</td>
<td>0.40</td>
<td>0.23</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Essentially, this computation addresses the often-quoted result that a recession associated with a financial crisis is around twice as long as one that does not have one. While we would think that a crisis would involve a high external finance premium, given that there would be little credit available, the GOZ model only delivers such a relationship between recessions and the external finance premium in a very weak way. One factor contributing to this might be the persistence in the growth in credit; the persistence in $\{\Delta \ln(1 - (1/l_t))\}$, where $l_t$ is leverage, which is used to form real credit growth, is considerably smaller in the model than in the data.\(^8\)

### 3.3.4 Credit Cycles

As mentioned earlier a recent emphasis has been placed on credit cycles where very fast credit growth is taken to lead to a crisis and recession. So it is of some interest to see if the GOZ model can produce such an outcome. The simplest way to assess this seemed to be to examine the probability of a recession as a function of credit growth before the emergence of a recession. Consequently, we looked at how the probability of a recession changed for various growth rates in credit over the past two years. The Probit estimates suggest that the probability is .23 when credit growth was just 11% but moved to .30 when it was 41%. However, because the unconditional probability of a recession is .22 this does not suggest a strong credit effect. It is worth noting that the result is in line with the results for the external finance premium. Specifically, if one first dates turning points in the levels of real credit and the level of GDP, then designates the binary numbers describing expansions and contractions in each of these by $S_c$ and $S_y$ respectively, it is found that the correlation between $S_c$ and $S_y$ is 0.3. Moreover, 65% of the time the credit and business cycles are in the same state. Real credit cycles are much shorter than cycles in GDP though, with a total cycle length of 12.6 quarters versus the 19 for GDP. The difference comes from the fact that, although the GOZ model implies the same

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\(^8\)See the appendix for the derivation of the real credit growth rate series.
growth rate in real credit as for GDP, there is much greater volatility in credit growth, and this shortens the cycle.

3.3.5 Relative performance of investment prediction

One of the observed cycle characteristics listed above was that there would be a stronger response by investment than output. We therefore studied the investment cycle present in the observed investment data used by GOZ, namely a chain-weighted index of per capita private fixed investment and durable goods. Here expansions were 8.8 quarters long on average and contractions were 6.6 quarters. So the contractions are longer and the expansions are shorter than for the cycle in aggregate activity. The model based investment expansion length was well below that in the data of 12 quarters. Figure 2 shows this, although it should be borne in mind that the unconditional probability of an investment recession is .43, much larger than for GDP.

Figure 2: Probability of an Investment Recession Given External Finance Premium in GOZ Model

A comparison with Figure 1 suggests that the probability of an investment recession tends to be much higher than that of output recessions for a given premium, reflecting in part the fact that investment is only one component of output, and therefore a very large negative growth rate in investment is needed to cause a decline in output. This is the case in the GOZ model; investment growth in the data is .504% per quarter whereas the model sets it to that of GDP growth of .39. The standard deviations of investment growth in model and data are much the same, being 2.39 and 2.46 respectively. Increasing the growth rate to .504 in line with the data only raises the duration of expansions marginally to 9 quarters. There is much more serial correlation in investment growth generated from the model - .64 - than in the data - .45. Simulations of an AR(1) model have shown that higher serial correlation has a tendency to reduce the duration of expansions.
despite using a relatively wide definition of investment in estimation, the parameters of the model (in particular $\alpha$, which governs its weight in the production function) imply a steady-state investment-to-output ratio of around 6.5 per cent, which is less than fixed non-residential investment’s share of output in the data.

In all, it appears that one needs to work with a broad set of investment expenditures, e.g. housing and consumer durables could be important to getting the quantitative financial effects right, and these probably should be integrated into the structure of the model. In turn this implies that collateral effects might be important and we turn to that in our investigation of the Iacoviello & Neri (2010) model in the next section.


4.1 The Iacoviello and Neri (2010) (IN) Model

One method for including financial factors into a macroeconomic model is to introduce a borrowing constraint. Notable examples of this approach are Kiyotaki & Moore (1997) and Iacoviello (2005). The borrowing constraint is motivated by the presence of asymmetric information between the lender and the borrower, namely that the lender is uncertain whether they shall be repaid, and therefore requires the borrower to post collateral. The collateral typically is a durable good, such as housing.

Iacoviello (2005) developed a model in which there were two groups of households, differing by their discount factors. Patient consumers invested in capital and housing as well as possibly lending to the impatient households. The impatient households invested only in housing and were subject to a borrowing constraint, namely that their lending was less than or equal to a constant fraction of the discounted value of their housing - the loan-to-value ratio. The borrowing constraint was assumed to always bind with equality so that the model may be log-linearized.

A limitation of the Iacoviello (2005) model is that the quantity of housing in the economy is fixed. The lack of any supply response potentially amplifies the response of house prices to shocks and may therefore increase the impact of the borrowing constraint. Davis & Heathcote (2005) introduced a second production sector, namely housing, into a real business cycle model. They demonstrated that their model was able to replicate several stylised facts with respect to residential investment, particularly that it was more than twice as volatile as business investment. Iacoviello & Neri (2010) introduce a similar second production sector into the Iacoviello (2005) model. The housing sector uses labour from both type of consumers, together with land and intermediate goods from the other production sector. Land is in fixed supply, but used to produce new housing, which may be purchased by either type of consumer. Technology in each production sector grows deterministically at different rates. The model also possesses sticky wages for each type of household in each sector, which considerably alters the dynamic
responses of the model to shocks. There are also sticky prices in the retail sector. The decisions made by households and firms are much the same as those in the GOZ model, except that credit constraints can limit expenditures, and so changes in the value of collateral can potentially have effects on cycles.

Iacoviello & Neri (2010) demonstrate that their model captures the procyclicality and sensitivity to interest rates of both house prices and residential investment. Housing sector specific shocks account for about half of the fluctuations in house prices and residential investment. These shocks also have spillovers to the rest of the economy.

4.2 Durations and Amplitudes

Using the parameter values provided in Iacoviello & Neri (2010) we simulate data (15,000 observations) from the IN model. Table 4 contains the results. Clearly there is a big gap between most of the business cycle characteristics from the model and the data. Therefore, we might ask whether the model and data summaries are significantly different. Simulating the model 1,000 times with a sample size equal to that of the data used in estimation, we find that it is virtually impossible to generate average expansion and contraction durations for GDP from the model that are consistent with the data. It is interesting that IN did not use GDP to estimate their model, only consumption, house prices and residential and aggregate investment, so there was no check on whether the model captured GDP.10 This suggests that one might want to look at whether the cycles in other series match what is in the data. For aggregate investment the average duration of expansions is 8.6 from the model, but 15.0 in the data, and simulations reveal that one cannot generate a number like the latter from the distribution of the average durations for investment expansions.11

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>IN</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Durations (qtrs)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>14.9</td>
<td>9.8</td>
</tr>
<tr>
<td>Contractions</td>
<td>6.0</td>
<td>4.0</td>
</tr>
<tr>
<td><strong>Amplitude (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>21.0</td>
<td>10.8</td>
</tr>
<tr>
<td>Contractions</td>
<td>−0.9</td>
<td>−0.4</td>
</tr>
<tr>
<td><strong>Cumulative amplitude (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>252.0</td>
<td>80.9</td>
</tr>
<tr>
<td>Contractions</td>
<td>−3.0</td>
<td>−1.1</td>
</tr>
</tbody>
</table>

10 Note that per capita GDP was defined as per capita consumption plus per capita investment so as to be consistent with the model i.e. the data in Table 4 is for this definition of “GDP”. Consequently, it is not what one would get if cycles in actual per capita GDP were dated.

11 It is interesting to note that the IN model gives a much greater role to monetary policy shocks than the GOZ and Galí models. A simple summary is that, for the two period horizon variance decomposition, monetary shocks account for 40% of the variance in the IN model. This remains much the same even if one takes a 20 quarter horizon.
Since a key parameter in the model is the loan to value ratio - set at .85 by IN - we investigate what happens as it changes. We consider two alternative values - .5 and .95. For the smaller value amplitudes of expansions are lessened and durations are increased, whereas the high value of .95 gives the opposite result. The differences come from different volatilities of GDP growth. The standard deviation of GDP growth is 1.36% when the loan to value ratio is .85 but 10% higher when it is set to .95. Thus, easier credit results in a more volatile economy. In a sense this is a story about imbalances. Keeping strong credit standards may the key to ensuring stability.

4.3 Probability of Recession and Financial Crises

There is no external finance premium in Iacoviello’s model, making it hard to define a crisis. But one might think that this would be related to the value of the collateral asset $\tilde{q}_t^h$. Thus we computed $\Pr(R_t|\tilde{q}_t^h)$ and $\Pr(R_t|\tilde{q}_{t-3}^h)$. Table 5 shows these probabilities (using the Probit form) as $\tilde{q}_t^h$ varies in the IN model. Estimates are given for the 10th, 50th and 90th percentiles of the values of $\tilde{q}_t^h$ generated in the simulations.

| Percentile | $\Pr(R_t|\tilde{q}_t^h)$ | $\Pr(R_t|\tilde{q}_{t-3}^h)$ |
|------------|-----------------|-----------------|
| .1         | .30             | .22             |
| .5         | .29             | .29             |
| .9         | .28             | .37             |

The unconditional probability of a recession is .29. It is interesting that the probability rises for higher values of the collateral asset price when it is lagged three quarters, but falls when it is contemporaneous. The latter result presumably reflects the fact that during a recession collateral asset values will be low, while the former agrees with the idea that a period of high collateral asset prices would lead to a strong investment response and subsequently a recession. One does see the probability of an investment recession rise with both $\tilde{q}_t^h$ and $\tilde{q}_{t-3}^h$, although much more strongly for the latter. In fact, while the probability of an investment recession is .26 at the 10th percentile, it has risen to .43 at the 90’th. Thus, just like the GOZ model, the effects of financial conditions on investment are stronger than the output effects, although one might think that they are not as strong as expected.

4.4 The Duration of a Recession and the Collateral Asset Price

Just as for the GOZ model we now look at whether a crisis leads to recessions of longer durations. Table 6 shows the same statistics as were computed for the GOZ model. The results

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12The model seems to explode if one tries a loan to value ratio just above .958.
13Inspection of turning points in $\tilde{q}_t^h$ and GDP show that the former generally precedes the latter.
are quite similar and, whilst crises do seem to imply somewhat longer recessions, the rise in the probability appears to be much smaller than one would expect from the evidence.

Table 6: Probability of Recession for m Periods as $q^h_t$ Changes, IN Model

<table>
<thead>
<tr>
<th>Percentile</th>
<th>m = 3</th>
<th>m = 4</th>
<th>m = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.29</td>
<td>.23</td>
<td>.13</td>
</tr>
<tr>
<td>.5</td>
<td>.29</td>
<td>.24</td>
<td>.15</td>
</tr>
<tr>
<td>.9</td>
<td>.29</td>
<td>.26</td>
<td>.17</td>
</tr>
</tbody>
</table>

5 Can a Model Predict Recessions?

To predict recessions with a model we need to predict $R_{t+1}$ using information available at $t$. We will assume here that $R_t = \{R_{t-1} = 0, R_t = 0\}$ is part of the information set, as well as some extra variables $z_t$. If it was believed that $R_t = 1$, then it must be the case that one would predict that $R_{t+1} = 1$ with probability one, since recessions have to be two periods long (by design). Consequently our focus is upon the case when $R_t = 0$. Of course in practice there is a now-casting issue involving the determination of $R_t$ before one can make a prediction of $R_{t+1}$.

Setting $R_{t-1} = 0, R_t = 0$ and using the recursive formula for $R_{t+1}$ in (5) shows that $R_{t+1}|R_t = \land_t$.\(^{14}\) Hence $E(R_{t+1}|R_t, z_t) = E(\land_t|z_t)$. Then, from the definition of $\land_t$(for BBQ) as

$$\land_t = 1(\{\Delta y_t > 0, \Delta_2 y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\})$$

$$= 1(\Delta y_t > 0)1(\Delta_2 y_t > 0)1(\Delta y_{t+1} < 0)1(\Delta_2 y_{t+2} < 0),$$

we have

$$\Pr(R_{t+1} = 1|R_t, z_t) = E\{1(\Delta y_t > 0)1(\Delta_2 y_t > 0)1(\Delta y_{t+1} < 0)1(\Delta_2 y_{t+2} < 0)|z_t\}$$

$$\leq E(1(\Delta y_{t+1} < 0)1(\Delta_2 y_{t+2} < 0)|z_t)$$

$$\leq E(1(\Delta y_{t+1} < 0)|z_t).$$

Equation (6) points to the fact that predicting a recession involves successfully predicting negative quarterly and six-monthly growth over the two quarters following on from the prediction point, (assuming of course that it is known that an expansion held at $t$ and $t-1$). Equations (7) and (8) are useful for providing some upper bounds to the probability of predicting a recession given any set of information. The last result is particularly useful as it is extremely simple to compute. Moreover, the ability to predict a negative growth rate in activity is common to

\(^{14}\)If $R_t$ is unknown using the recursive formula for $R_t$ and applying $\land_{t-1} \land_t = 0$ (since turning points must be two periods apart), we get $R_{t+1}|R_t = \land_{t-1} + \land_t$, and so one needs to allow for a possible turning point at $t - 1$ when considering the outcome for $R_{t+1}$. 

16
virtually all definitions of a recession. If one cannot predict negative growth then one won’t be able to predict recessions, as it represents an upper bound to \( \text{Pr}(R_{t+1} = 1 | R_t, z_t) \).

It might be asked why one didn’t just fit a discrete choice model such as Probit using \( R_{t+1} \) as the dependent variable and \( R_t, R_{t-1} \) as independent variables? As discussed in Harding & Pagan (2011) this cannot be done due to the restriction that phases must be at least two quarters in duration. It is not unusual to see Probit fits using \( R_{t+1} \) as the dependent variable but with (at most) \( R_t \) as an independent variable. However, the use of \( R_t \) as a regressor is illegitimate, as it is not a pre-determined variable, being constructed from future data - see Harding & Pagan (2011). One can compute a non-parametric solution by simply conditioning on the sample of observations that have \( R_t = 0, R_{t-1} = 0 \), but, for non-parametric methods to work, \( z_t \) must be of small dimension.\(^{15}\) It seems likely however that any model will imply a substantial number of variables in \( z_t \).

Turning to the bounds we can see that they are more straightforward to compute. If the model implies that \( \Delta y_{t+1} \) is normally distributed around \( E(\Delta y_{t+1} | z_t) = z'_t \beta_1 \) with variance \( \sigma^2 \), then \( E(1(\Delta y_{t+1} < 0) | z_t) = \Phi(-z'_t \beta_1 / \sigma) \), where \( \Phi \) is the cumulative standard normal density function. Similarly, if \( E(\Delta y_{t+1} | z_t) = z'_t \beta_1 \) and \( E(\Delta^2 y_{t+2} | z_t) = z'_t \beta_2 \), then we can find the requisite probability in Equation (7) from a bivariate normal with mean zero and covariance matrix given by the covariance of the errors for \( \Delta^2 y_{t+2} \) and \( \Delta y_{t+1} \). Of course these linear forms will be appropriate if \( \Delta y_t \) follows a VAR process. This is true of most macroeconomic models that are constructed. There is however a complication in that the macroeconomic models often contain latent variables, and it is the the joint vector of latent and observable variables which follow a VAR. Restricting attention to fitting a VAR to the observables alone may mean that it will be of higher order than that which includes the latent variables, although omission of variables need not always increase the order needed for the VAR. Now the GOZ model is a VAR(2) in all its variables but, since some of these are latent, it will be necessary to trial a higher order VAR to compute the relations between \( \Delta y_{t+1}, \Delta^2 y_{t+2} \) and the observable model variables.

Table 7 below performs an exercise with data on nine of the ten variables used by Gilchrist et al. (2009) in estimation. There are actually two missing series that they used for estimation and we were not able to get. One of these was the degree of leverage and the other the external finance premium. These came from some micro data sets. We substituted the BAA spread for the external finance premium but had to ignore the degree of leverage, and hence it effectively becomes a latent variable.\(^{16}\) If these two series had been available we could have computed \( E(\Delta y_{t+1} < 0 | z_t) \) and \( E(\Delta^2 y_{t+2} < 0 | z_t) \) with the Kalman predictor.

\(^{15}\)We computed \( \text{Pr}(R_t = 1|R_{t-1} = 0, R_{t-2} = 0, \tilde{q}_t) \) earlier using this method.

\(^{16}\)The Baa spread used is that available at the beginning of the quarter for which a prediction is to be made.
Table 7: First Period Probabilities of Recession Events and Predicted Growth Signs Using the GOZ Observables Fitted to a VAR(4)

<table>
<thead>
<tr>
<th>Peak</th>
<th>$1(\Delta y_{t+1} &lt; 0)1(\Delta_2 y_{t+2} &lt; 0)$</th>
<th>$1(\Delta y_{t+1} &lt; 0)$</th>
<th>$\Delta y_{t+1}$</th>
<th>$\Delta_2 y_{t+2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1977/3</td>
<td>.25</td>
<td>.36</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>1978/4</td>
<td>.52</td>
<td>.59</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>1981/1</td>
<td>.85</td>
<td>.87</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>1990/2</td>
<td>.20</td>
<td>.32</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>2000/4</td>
<td>.46</td>
<td>.56</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>2002/3</td>
<td>.35</td>
<td>.50</td>
<td>+</td>
</tr>
<tr>
<td>7</td>
<td>2007/4</td>
<td>.32</td>
<td>.47</td>
<td>+</td>
</tr>
</tbody>
</table>

We used the 15,000 observations from the simulations in section 3 to estimate the parameters $\beta_1$ and $\beta_2$ above. This was done by fitting VARs to the simulated data using just what would be the observed variables. VARs were fitted up to order five. By that order the probabilities of a recession seemed to have stabilized. Table 7 then shows these. This is done by computing the expectations in Equations (6) and (7) when $t$ corresponds to a peak in the data. We refer to these as “first period” probabilities as they are predictions of the first period of a recession. To be more specific, take the peak occurring in 1977/3. Since the recession started in 1977/4 we are therefore forming a prediction in 1977/3 about 1977/4 (of course this forecast is predicated on still being in an expansion in 1977/3). We need to emphasise that the cycle here is in per capita GDP and so there may be more recessions, as well as a difference in timing, to those presented by the NBER, which essentially relate to the level of economic activity.

Bearing in mind that the unconditional probability of a recession over this period is .27, that current data on items like consumption and investment would not have have been available when a prediction was made, and that the table gives upper bounds to the probability of a recession, at best it would seem that the GOZ model would have predicted two of the seven recessions. The biggest success would have been the second of the famous double dip recessions in the late 1970s and early 1980s, although it should be noted that, for the years leading up to 1981, the model predicts very high probabilities of a recession, reaching .90 (1980/1) and .99 (1980/2). Consequently, there were quite a few false predictions.\(^{17}\) The table also shows the signs predicted for the future quarterly and six-monthly growth rates that distinguish a recession.

Equation (7) has the implication that, if the information $z_t$ is to be useful in predicting recessions, it must be correlated with future shocks. A quick check on whether a model would be able to predict such a quantity is to ask how important the unpredictable part of future shocks are to these growth outcomes. Shocks, such as technology, often have an autoregressive structure, and it is the innovation (the unpredictable part) whose impact upon the business cycle is to be determined. We therefore simulate the GOZ model turning off the contemporaneous innovations. That is, the model is run with the current innovations set to zero, although they

\(^{17}\)Of course these are upper bounds and $\Pr(R_{t+1}|R_t, z_t)$ might be much smaller. But if so then it would seem that the model would fail to predict recessions at all.
are re-set to their actual values in later periods. To illustrate what is done, take an AR(1) 
\( \xi_t = \rho \xi_{t-1} + e_t \), where \( e_t \) is white noise. Defining \( \xi_t^- = \rho \xi_{t-1} \), we note that \( \xi_t \) and \( \xi_t^- \) differ only in that the current innovation is set to zero; in other words, \( \xi_t^- \) is the predicted value of \( \xi_t \) using information at \( t - 1 \).

Table 8 shows business cycle characteristics from the GOZ model with current innovations present (equivalent to basing the computation on \( \xi_t \)) and with them suppressed (equivalent to \( \xi_t^- \), and hence designated GOZ\(^-\)). It is clear that the innovations have a substantial effect upon the average cycle characteristics. Without these innovations, expansions become very long, and so there will be fewer recessions. Because these innovations are so important, and they are what makes \( \Delta y_{t+1} \) differ from \( E(\Delta y_{t+1}|z_t) \) in the analysis above, we can see why the GOZ model had limited success when it came to predicting recessions (on average). Repeating the exercise with the IN model one finds that, just as with the GOZ model, expansion durations double once current shocks are excluded. Thus this simple strategy of re-running the model with the innovations removed seems a good way to assess the likelihood of successful recession prediction in those cases where the assumptions such as normality needed to compute the probabilities in Table 7 do not hold.\(^{18}\)

Table 8: **Impact of Current Shocks on Business Cycles in GOZ Model**

<table>
<thead>
<tr>
<th>Durations (qtrs)</th>
<th>GOZ</th>
<th>GOZ(^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expansions</td>
<td>14.8</td>
<td>33.9</td>
</tr>
<tr>
<td>Contractions</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td><strong>Amplitude (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>9.0</td>
<td>15.9</td>
</tr>
<tr>
<td>Contractions</td>
<td>−1.6</td>
<td>−0.8</td>
</tr>
<tr>
<td><strong>Cumulative amplitude (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>107.9</td>
<td>461.8</td>
</tr>
<tr>
<td>Contractions</td>
<td>−5.2</td>
<td>−2.8</td>
</tr>
</tbody>
</table>

### 6 Conclusions

Models that incorporate financial features pertaining to credit and debt are increasingly appearing in the macroeconomic literature. To date much of the assessment of the success of this augmentation of traditional models has involved looking at two things - whether there is a difference between impulse responses with and without these financial shocks and whether variance decompositions suggest that financial shocks are important. Our paper provides a complementary approach, asking whether the augmented models provide a better explanation of the business cycle and whether they can replicate some stylised facts about the relationship between recessions and credit.

\(^{18}\)The IN model is an example of this as it was solved using a second order approximation, making it difficult to find the upper bounds to recession probabilities.
To demonstrate our approach we took two models representative of common ways of introducing financial factors into macroeconomic models – those of Gilchrist et al. (2009) and Iacoviello & Neri (2010). While financial factors can play a role in particular cycles, generally it seems that the average cycle characteristics of these models are not affected much by their introduction. Both models managed to replicate some of the stylised facts, but often these were not of a magnitude comparable to that evident in the data. This points to the need to either add extra features to the models or perhaps combine existing ones.

Finally, successful prediction of recessions ultimately involves an ability to predict the signs of future output growth rates. We demonstrate that a simple way to gauge whether the models examined here can predict recessions is to ask how important current shocks are for current output growth rates, as these are unpredictable using past inflation. In this paper we apply the test to models with financial factors and find that the current shocks are quite important. Consequently, these models imply that future growth rates in output are heavily dependent on future shocks. Since these are unpredictable using the information embodied in the models this severely limits their predictive ability.
A Derivation of Credit Growth Equations in the GOZ (2009) Model

It will be necessary to find the credit growth rates implied by the Gilchrist et al. (2009) model. Appendix A in Gilchrist et al. (2009) shows that the growth in real debt (credit), $\Delta \ln D_t$, equals

$$\Delta \ln D_t^* = \Delta q_t + \Delta k_t + \gamma + \Delta \ln(1 - l_t^{-1})$$

$$l_t = \exp((q_t^* + k_t^* - n_t^*)/100 + \ln(\bar{K}/\bar{N})),$$

It immediately follows that

$$\Delta \ln D_t = \Delta \ln(Q_t K_t) + \Delta \ln(1 - l_t^{-1}) = \Delta \tilde{q}_t + \Delta \tilde{k}_t + \gamma + \Delta \ln(1 - l_t^{-1}),$$

where $\tilde{q}_t = \ln(Q_t/\bar{Q})$ and $\tilde{k}_t = \ln(K_t/\bar{K} \gamma^t)$, since the steady state growth rate of capital will be the same as output ($\gamma$). Designating the ratio $K_t/N_t$ as $R_{KN,t}$, and using the fact that $\bar{Q} = 1$, an expression for $l_t$ is available from

$$\ln l_t = \ln Q_t + \ln R_{KN,t}.$$  

$$\ln l_t - \ln \bar{l} = \ln Q_t - \ln \bar{Q} + \ln R_{KN,t} - \ln \bar{R}_{KN}.$$  

$$\ln l_t = \tilde{q}_t + \tilde{k}_t - \tilde{n}_t + \ln \bar{R}_{KN}.$$  

$$\therefore \ l_t = \exp(\tilde{q}_t + \tilde{k}_t - \tilde{n}_t + \ln(\bar{K}/\bar{N})).$$

Now Gilchrist et al. (2009) measure variables in percentages so, designating these by a ‘*’, we get

$$l_t = \exp((q_t + k_t - n_t) + \ln(\bar{K}/\bar{N})).$$

Hence we have

$$\Delta \ln D_t^* = \Delta q_t + \Delta k_t + \gamma + \Delta \ln(1 - l_t^{-1})$$

$$l_t = \exp((q_t + k_t - n_t) + \ln(\bar{K}/\bar{N})).$$
References


