Liquidity, Credit Frictions, and Optimal Monetary Policy

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Abstract

We study optimal monetary policy in a New Monetarist framework with banking, private liquidity, and credit frictions. We show that whenever part of the decentralized transactions are allowed to use deposit claims backed by interest-bearing assets, the optimal policy is a non-Friedman-rule liquidity trap. In contrast, the Friedman rule is optimal if there are no credit frictions, or if the economy is monetarily underdeveloped.

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1 Introduction

This paper studies optimal monetary policy within a New Monetarist Framework which highlights three essential features of modern monetary and financial systems. First, banking and inside money play important roles in facilitating decentralized transactions and in providing insurance against liquidity needs. Second, public and private liquidity coexist, and interest-bearing assets potentially command a liquidity premium as they back deposit claims that can be traded in part of the decentralized exchanges. And third, the creation of private liquidity is subject to credit frictions. These features introduce constraints and tradeoffs that policy makers are confronted with when choosing the welfare-maximizing inflation and interest rates.

With quasilinear preferences, the constraints are summarized in an arbitrage condition which says that the real interest rate is bounded from below by the rate of deflation and bounded from above by the time discount rate. It does not have to equal the discount rate unless the deflation rate is set so. The possibility of a positive liquidity premium on interest-bearing assets breaks the tight link between a Friedman-rule equilibrium, where both the deflation rate and the real interest rate equal the discount rate, and a liquidity-trap equilibrium, where the deflation rate equals the real interest rate, but the latter is strictly lower than the discount rate.

The tradeoffs arise from the tension between decentralized transactions and entrepreneurial activities which provide productive intertemporal transformation of consumption opportunities. The desire to raise the surplus from decentralized transactions leads to a large demand for interest-bearing assets and hence high real interest rates. The high real interest rates, however, might reduce the supply of private interest-bearing assets.
The government can remedy this reduction by increasing the supply of public liquidity. But with credit frictions, high real interest rates also produce large deadweight losses due to entrepreneurial bankruptcy. The increase in public liquidity cannot make up for the harm done to entrepreneurial activities. We show that, in a benchmark New Monetarist framework, these constraints and tradeoffs imply the suboptimality of Friedman rule and the optimality of a liquidity trap. The Friedman rule is suboptimal because it forces the equilibrium real interest rate to take the maximum value, which is too high from the welfare perspective.

Our analysis is built on Williamson (2012), who shows that the Friedman rule is optimal when all the interest-bearing assets are publicly supplied. He also contends that the optimality of Friedman rule is not plausible, and resorts to the costs of operating a currency system to generate the suboptimality of the Friedman rule. Although the second part of his paper does consider private liquidity and credit frictions, the analysis of optimal monetary policy is not extended accordingly. We believe that a serious investigation of this issue is important for the following reasons. First, credit frictions changes the analysis of optimal monetary policy in a fundamental way, and the conclusions reached are radically different in settings with and without credit frictions. Second, the current generation of search-theoretic models of money deems it a central task to integrate mainstream macroeconomic analysis into monetary theory with explicit microfoundations, as in Lagos and Wright (2005). Our paper emphasizes the importance of financial intermediation and entrepreneurial activities in centralized markets for the design of monetary policy in the New Monetarist framework. The frictions associated with these activities have been in the center stage of the large macroeconomic literature with financial fac-

We view our results as complementary to those in Williamson (2012). In fact, the two different reasons for the suboptimality of the Friedman rule correspond to two polar cases. The costs of operating a currency system prescribe that the optimal type of equilibrium is one with plentiful interest-bearing assets where interest-bearing assets command zero liquidity premium and dominate currency in rate of return. In contrast, credit frictions imply that the optimum involves a liquidity-trap equilibrium, where the liquidity premium is positive and the rates of return on currency and interest-bearing assets are equalized, i.e., the nominal interest rate is zero. Both types of optimal equilibrium represent large deviations from reality: the nominal interest rate is usually positive, and the liquidity premium is rarely zero. Although it is obvious that the observed behavior of interest rates does not necessarily coincide with the prescription of theory, we conjecture that a complete theory of optimal monetary policy would need to take into account both reasons and would predict something in between the two extremes. Hence the purpose of this paper is not to literally advocate the optimality of liquidity traps, but to examine thoroughly the implications of credit frictions.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents the main results. The last section concludes.
2 The Model

2.1 Liquidity Demand

There are two types of agents, buyers and sellers, and in each period $t$ there is a centralized market (CM) in the first subperiod and a decentralized market (DM) in the second subperiod. In the DM only sellers can produce, and only buyers can consume. Buyers can also produce in the CM. Preferences are additively separable over time, with the common discount factor $\beta \in (0, 1)$, and are linear in CM consumption and leisure. For buyers, DM consumption $x$ yields utility $u(x)$, which is strictly increasing, strictly concave, and twice continuously differentiable, with $u(0) = 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $-xu''(x)/u'(x) < 1$ for all $x \geq 0$. Assume that there exists some $\hat{x} > 0$ such that $u(\hat{x}) = \hat{x}$.

There are a fraction $\rho \in (0, 1]$ of DM meetings that are “nonmonitored,” where buyers can only use currency to acquire goods from sellers. The remaining fraction $1 - \rho$ of DM meetings are “monitored,” where buyers are allowed to use deposit claims on banks, in addition to currency. In each period a set of one-period lived competitive banks form in the CM, before the depositors, i.e., buyers, know whether they will be in a nonmonitored or monitored meeting in the subsequent DM. A bank acquires $m$ units of currency and $a$ units of interest-bearing assets, both in real terms, and offers a deposit contract to each of its ex ante identical depositors. The contract provides insurance against the need for liquidity in different types of transactions. It specifies that each depositor who will be in a nonmonitored meeting withdraws $m'/\rho$ units of currency, and who will be in a monitored meeting receives the right to trade away deposit claims on $m - m' + a - a'$ units of the bank’s assets, and receives the returns on $a'/1 - \rho$ units of interest-bearing assets in the next CM.
Let $\phi_t$ denote the price of currency in units of goods in the period-$t$ CM. And let $q_{t+1}$ and $r_{t+1} = \frac{\phi_{t+1}}{\phi_t} q_{t+1}$ denote, respectively, the gross nominal and real interest rates on government bonds that are issued in period-$t$ CM and pay off in the next CM. As both government bonds and privately created interest-bearing assets play the same role in backing deposit claims traded in monitored DM meetings, in equilibrium the real return on private assets is also given by $r_{t+1}$. Consider stationary equilibria where real quantities are constant over time, implying that that the gross real return on currency equals the deflation rate, i.e., $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ for all $t$. Assuming that buyers make take-it-or-leave-it offers to sellers, an optimal deposit contract entails

$$\frac{\beta}{\mu} u'(x_n) = \beta r u'(x_m) = 1,$$

where $x_n = \frac{\beta m'}{\mu}$ is the nonmonitored buyer’s consumption, and $x_m = \beta r \frac{a - a'}{1 - \rho} + \frac{\beta m - m'}{1 - \rho}$ is the monitored buyer’s consumption in the DMs. In addition, the following arbitrage condition must hold:

$$\frac{\phi_{t+1}}{\phi_t} \leq r_{t+1} \leq \frac{1}{\beta}.$$  \hspace{1cm} (2)

The first inequality in (2) reflects the zero bound on the nominal interest rate, and the second inequality is a boundedness condition imposed by quasilinear preferences. The potential for $r$ to fall below $\beta$ comes from a liquidity premium on interest-bearing assets, derived from their role in facilitating monitored DM exchanges.

The arbitrage condition (2), together with the first-order conditions (1), implies $x_n \leq x_m \leq x^*$, where $x^*$ is the efficient level of DM consumption defined by $u'(x^*) = 1$. The magnitudes of DM consumptions and their gaps from the efficient level depend on asset returns. In particular, $x_n$ is increasing in the return on currency $\frac{1}{\mu}$ and $x_m$ is increasing in the return on interest-bearing assets $r$, as long as they do not exceed the upper limit.
2.2 Liquidity Supply

Both government bonds and private interest-bearing assets can be used to back deposit claims traded in monitored DM exchanges. In this subsection we focus on the supply of private liquidity. During the CM of each period, a continuum of one-period lived entrepreneurs with unit mass is born. An entrepreneur born in the CM of period \( t \) consumes only in the CM of period \( t + 1 \), is risk neutral, and has no endowment during her lifetime. Each entrepreneur has access to an indivisible investment project, which requires one unit of consumption good in the CM of period \( t \) to operate, and yields a gross return of \( \omega \) in the next CM, where \( \omega \) is distributed according to the c.d.f. \( F(\omega) \) on \([0, \infty)\), with associated p.d.f. \( f(\omega) \). We make the common assumption in the incentive-contract literature that the hazard function, \( \lambda(\omega) \equiv \frac{f(\omega)}{1-F(\omega)} \) is increasing in \( \omega \). Investment project returns are independent across entrepreneurs. An entrepreneur can observe costlessly the realization of \( \omega \) on her own project, but any other individual needs to expend a fixed cost, denoted by \( \gamma \), in order to observe the same object. The verification cost can in principle be entrepreneur-specific, and we will consider two economies which are differentiated by the distribution of verification costs across entrepreneurs. Before that we present some general results applicable to both economies.

With costly state verification and deterministic monitoring, the efficient lending arrangement is for individuals to act as perfectly-diversified intermediaries, i.e., banks, and for the banks to sign “standard debt contracts,” in Gale and Hellwig (1985)’s term, with the entrepreneurs.\(^1\) A contract between a bank and an entrepreneur with verification cost \( \gamma \)

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\(^1\) For models that deliver deterministic monitoring endogenously, see Krasa and Villamil (2000) and Khalil, Martimort, and Parigi (2007). And for models that feature imperfectly diversified banks, see
specifies a noncontingent payment $R$ such that if $\omega \geq R$ then the entrepreneur pays $R$ to the bank, otherwise the bank incurs the verification cost, observes the entrepreneur’s output $\omega$, and seizes all of it. The expected payoffs to the bank and the entrepreneur from the loan contract are, respectively,

$$\pi (R, \gamma) = [1 - F (R)] R + \int_0^R (\omega - \gamma) dF (\omega)$$

and

$$\psi (R) = \int_R^\infty (\omega - R) dF (\omega).$$

The sum of $\pi (R, \gamma)$ and $\psi (R)$ is the expected net output (net of monitoring costs) from the project:

$$y (R, \gamma) \equiv \pi (R, \gamma) + \psi (R) = \bar{\omega} - \gamma F (R),$$

where $\bar{\omega}$ denotes the mean value of $\omega$. Here $\gamma F (R)$ reflects the deadweight loss due to costly monitoring. In equilibrium the bank receives a certain one-period return $r$ per unit lent to entrepreneurs. That is, $\pi (R, \gamma) = r$ for every funded project. Furthermore, participation by the entrepreneur requires $\psi (R) \geq 0$ or $\bar{\omega} \geq \gamma F (R) + r$.

A bank in the model accepts deposits from buyers in the CM, and acquires a portfolio of currency, government bonds, and loans to entrepreneurs. Both government bonds and bank loans are interest-bearing assets that back deposit claims traded in monitored DM exchanges. As an important object in our analysis, the equilibrium quantity of bank loans depends crucially on the distribution of verification costs across entrepreneurs.

**Economy E.** The entrepreneur-specific verification cost is a continuous random variable. Let $G (\gamma)$ denote the distribution of verification costs across entrepreneurs, with Krasa and Villamil (1992) and Zeng (forthcoming) for example.
continuous p.d.f. \( g(\gamma) \), and with support \([0, \infty)\). An entrepreneur, when born, receives a draw \( \gamma \) from the distribution \( G(\cdot) \). We assume that the project return \( \omega \) is independent of \( \gamma \). As will be clear momentarily, this specification gives rise to an aggregate loan supply schedule that is *elastic* with respect to the real interest rate.

Under the increasing hazard assumption for the project return distribution \( F(\cdot) \), for any given \( \gamma \) there exists a value of \( R \), denoted by \( \hat{R}(\gamma) \), such that \( \pi(R, \gamma) \) is maximized at \( R = \hat{R}(\gamma) \), where \( \hat{R}(\gamma) \) satisfies \( \lambda\left(\hat{R}(\gamma)\right) = \frac{1}{\gamma} \). There is a cutoff type \( \gamma^* \) such that a project is funded if and only if \( \gamma \leq \gamma^* \). The total loan quantity is thus given by \( L = G(\gamma^*) \).

Let \( R^* \equiv \hat{R}(\gamma^*) \). Then \( \gamma^* \) and \( R^* \) are determined jointly by

\[
\lambda(R^*) = \frac{1}{\gamma^*},
\]

\[
\pi(R^*, \gamma^*) = r.
\]

For an entrepreneur with \( \gamma < \gamma^* \), the non-contingent payment on the loan, denoted by \( R(\gamma) \), is the smaller one of the two values of \( R \) that satisfy \( \pi(R, \gamma) = r \). It is not difficult to show that a decrease in the required rate of return \( r \) raises \( \gamma^* \), implying that the total loan quantity \( L \) is larger when \( r \) is lower. The decrease in \( r \) lowers the noncontingent payment \( R^* \) and the expected monitoring cost \( \gamma^* F(R^*) \) on the cutoff project type. It also reduces the noncontingent payment \( R(\gamma) \) and the expected monitoring cost \( \gamma F(R(\gamma)) \) for all projects with \( \gamma < \gamma^* \). Denote \( \gamma^* \) and \( R^* \) as functions of \( r \) by \( \gamma^*(r) \) and \( R^*(r) \), respectively. For analytic convenience, we make the following assumption.

*Assumption E.* \( \bar{\omega} - \gamma^* \left( \frac{1}{r} \right) \cdot F \left( R^*(\frac{1}{r}) \right) \geq \frac{1}{\beta} \).

This assumption has two implications. First, all funded projects generate nonnegative surplus in any possible type of equilibria. The surplus from a project with monitoring
cost $\gamma$ and noncontingent payment $R$, from the perspective of time-$t$ CM, is

$$s(R, \gamma) = \beta y(R, \gamma) - 1 = \beta [\tilde{\omega} - \gamma F(R)] - 1.$$  

In any equilibrium, we have $r \leq \frac{1}{\beta}$ and $\gamma^*(r) F(R^*(r)) \leq \gamma^*(\frac{1}{\beta}) F\left(R^*\left(\frac{1}{\beta}\right)\right)$. Meanwhile, $\gamma F(R(\gamma)) \leq \gamma^*(r) F(R^*(r))$ for all $\gamma \leq \gamma^*(r)$. Hence $s(R(\gamma), \gamma) \geq 0$ for all funded projects in any equilibrium under Assumption E. Second, the entrepreneur’s participation constraint, $\tilde{\omega} \geq \gamma F(R) + r$, is satisfied whenever her project is funded. In fact, the participation constraint can be rewritten as $\frac{1}{\beta} s(R, \gamma) + \left(\frac{1}{\beta} - r\right) \geq 0$. It clearly holds for all funded projects in any equilibrium, since both terms in the preceding inequality are nonnegative. Essentially, Assumption E says that there is no tension between the bank, the entrepreneur, and the society whenever the bank finds a project fundable in equilibrium.\(^2\)

**Economy I.** A fraction $\eta$ of entrepreneurs have the same monitoring cost $\gamma^l$, while the others have monitoring cost $\gamma^h > \gamma^l$. We assume the following.

*Assumption I.* $\gamma^l \leq \gamma^*\left(\frac{1}{\beta}\right)$ and $\gamma^h > \tilde{\omega}$.

This assumption implies that the fraction $\eta$ of entrepreneurial projects with monitoring cost $\gamma^l$ are funded in all possible types of equilibria, and that those remaining projects with monitoring cost $\gamma^h$ are never funded. The total loan quantity in this economy is thus given by $L = \eta$, which is *inelastic* with respect to the real interest rate $r$. For the funded projects, the noncontingent payment is given by $R(\gamma^l)$, which is the smallest value of $R$.

\(^2\)In general, the nonnegative surplus condition is harder to satisfy than the entrepreneur’s participation condition. If Assumption E fails, then it is possible that in an equilibrium with positive liquidity premium ($r < \frac{1}{\beta}$), a project satisfies the entrepreneur’s participation condition, i.e., $\frac{1}{\beta} s(R, \gamma) + \left(\frac{1}{\beta} - r\right) \geq 0$, but fails the nonnegative surplus condition, i.e., $s(R, \gamma) < 0$ for some funded project. This possibility arises when the liquidity premium on interest-bearing assets, $\frac{1}{\beta} - r$, is sufficiently large. A project that generates negative surplus can still be funded as it generates loans backing deposit claims that are useful in facilitating some DM transactions.
satisfying $\pi(R, \gamma^l) = r$.

2.3 Welfare

If we weight utilities of all agents equally, then the surplus from DM activities is proportional to

$$W^D = \rho [u(x_n) - x_n] + (1 - \rho) [u(x_m) - x_m],$$

where $x_n$ and $x_m$ satisfy $\frac{\beta}{\mu} u'(x_n) = \beta r u'(x_m) = 1$. Denote the DM consumptions as functions of asset returns by $x_n\left(\frac{1}{\mu}\right)$ and $x_m(r)$, respectively. It is straightforward to show that $W^D$ is increasing in $\frac{1}{\mu}$ for $\frac{1}{\mu} \leq \frac{1}{\beta}$, and increasing in $r$ for $r \leq \frac{1}{\beta}$ if $\rho < 1$. On the other hand, the surplus from CM activities is proportional to

$$W^C = \int_{0}^{\gamma^*(r)} s(R(\gamma), \gamma) dG(\gamma).$$

The aggregate welfare is thus proportional to $W = W^D + W^C$. It is important to note that CM activities do not drop out when calculating the total surplus, despite that utility is linear in CM consumption and leisure with equal weights. This is because the presence of entrepreneurial projects offers opportunities to productively transform consumption from one period to another.

The effect of a change in the real interest rate $r$ on $W^C$ differs from its effect on $W^D$. If $\rho < 1$ then $W^D$ is strictly increasing in $r$ for $r < \frac{1}{\beta}$, and is maximized at $r = \frac{1}{\beta}$. In contrast, an increase in $r$ has a negative effect on $W^C$. In general, this negative effect occurs on two margins. The first is the intensive margin: the increase in $r$ lowers $s(R(\gamma), \gamma)$ for all $\gamma \leq \gamma^*$. The second is the extensive margin: the increase in $r$ lowers $\gamma^*$ so that less projects are funded. By differentiation,

$$\frac{dW^C}{dr} = \int_{0}^{\gamma^*(r)} \frac{ds(R(\gamma), \gamma)}{dr} dG(\gamma) + \left(\frac{d\gamma^*}{dr}\right) [s(R(\gamma), \gamma) g(\gamma)]|_{\gamma=\gamma^*(r)},$$
where
\[ \frac{d\sigma}{dr} (R(\gamma), \gamma) = \beta \frac{-\gamma \lambda (R(\gamma))}{1 - \gamma \lambda (R(\gamma))} < 0 \]
for \( \gamma \leq \gamma^* (r) \) and
\[ \frac{d\gamma^*}{dr} = \frac{-1}{F (R^*(r))} < 0. \]

For Economy I, where the total loan quantity is inelastic, only the intensive-margin effect is present, while for Economy E, both effects obtain.

The negative dependence of \( W_C \) on \( r \) implies that among the possible equilibrium values of the real interest rate, i.e., those on the interval \( \left[ \frac{1}{\beta}, \frac{1}{\beta} \right] \), \( r = \frac{1}{\beta} \) minimizes \( W_C \). But the same value of \( r \) maximizes \( W_D \). Hence there is a tension between DM and CM surpluses. This is one of the tradeoffs that we will explore in the following analysis of optimal monetary policy.

### 3 Optimal Monetary Policy

#### 3.1 Monetary Policy Making: Constraints and Tradeoffs

To help shed light on the general nature of monetary policy making in the New Monetarist framework, let us first briefly review the case where all DM meetings are nonmonitored \((\rho = 1)\). Since banking and inside money do not play a role and only currency can be used in decentralized tradings, we refer to such an economy as “monetarily undeveloped.” As it turns out, the analysis of monetary policy in this world is similar to the standard neoclassical theory. In a monetarily undeveloped world, the liquidity premium on interest-bearing assets is always zero, and with quasi-linear preferences, the real interest rate \( r \) always equals the time discount rate \( \frac{1}{\beta} \) in equilibrium, regardless of the government’s choice of the growth and composition of its liabilities. The broader idea is that the equilibrium
real interest rate is determined purely by “real” factors in such an environment. Since
the Fisher relation $r = \frac{\mu}{\beta}$ must hold in equilibrium, there is a one-to-one correspondence
between inflation $\mu$ and the nominal interest rate $q$, so that the control of inflation can
be equated to the control of the nominal rate. Moreover, there is a tight link between a
liquidity trap equilibrium ($q = 1$) and a Friedman-rule equilibrium ($\mu = \beta$). The mone-
tary authority can change the nominal rate, but is powerless in influencing the real rate.
Taking $r = \frac{1}{\beta}$ as given, the welfare-maximizing policy is to set $\mu = \beta$ or equivalently
$q = 1$, implementing the Friedman-rule/liquidity-trap equilibrium. By focusing on mon-
etarily undeveloped economies, the usefulness of standard theories for monetary policy
analysis is actually quite limited, as they ignore banking and the liquidity associated with
interest-bearing assets.

The New Monetarist framework with banking provides a substantial departure from
the standard analysis of interest rate determination and monetary policy making. As long
as some of DM transactions use deposit claims backed by interest-bearing assets ($\rho < 1$),
and as long as interest-bearing assets are sufficiently scarce, there is a positive liquidity
premium on these assets ($r < \beta$). In general, the equilibrium value of the real interest
rate is restricted by the arbitrage condition, $\frac{1}{\mu} \leq r \leq \frac{1}{\beta}$.\(^3\) The government has some
room in choosing the desired real interest rate, and the choice is not independent of its
choice of inflation. Whenever $\mu > \beta$, reducing inflation compresses the range of possible
equilibrium interest rates. If $\mu$ is set equal to $\beta$, then the government effectively restricts
itself to the only possible equilibrium real interest rate $r = \frac{1}{\beta}$. This value, although equals
the one in the standard theories, is in fact selected by the government’s inflation policy. In

\(^3\)The condition can equivalently be seen as an equilibrium restriction on the nominal interest rate and
inflation, taking the form $1 \leq q \leq \frac{\mu}{\beta}$. 

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this framework, the control of inflation cannot be equated to the setting of interest rates. Although setting $\mu = \beta$ automatically pins down $q = 1$, setting the nominal interest rate to zero does not necessarily lead to $\mu = \beta$: it only eliminates the gap between the rates of return on currency and interest-bearing assets ($\frac{1}{\mu} = r$). Banking and the liquidity associated with interest-bearing assets breaks the tight link between liquidity-trap and Friedman-rule equilibria.

The preceding discussion pertains to the constraints faced by the monetary authority when choosing inflation and interest rates. Since the choice of inflation determines the rate of return on currency, the same arbitrage condition restricts the choice of the rates of return on currency and interest-bearing assets. Respecting this constraint, the monetary authority must take into account the following tradeoffs when deciding on the optimal configuration of asset returns:

1. the tradeoff between the surplus from monitored DM transactions $(1 - \rho) [u(x_m(r)) - x_m(r)]$ and the surplus from CM activities $W^C(r)$ faced by the choice of $r$, and
   
2. the tradeoff between the surplus from nonmonitored DM transactions $\rho [u(x_n \left( \frac{1}{\mu} \right)) - x_n \left( \frac{1}{\mu} \right)]$ and the set $\{W^C(r) : r \geq \frac{1}{\mu}\}$ faced by the choice of $\mu$.

In the first tradeoff, an increase in $r$ raises the surplus from monitored DM transactions but reduces the surplus from CM financial intermediation and entrepreneurial activities. In the second tradeoff, a decrease in $\mu$ raises the rate of return on currency and hence the surplus from nonmonitored DM transactions. Meanwhile, the increase in the rate of return on currency eliminates the lower end of possible equilibrium rates of return on interest-bearing assets, since $r$ is bounded from below by $\frac{1}{\mu}$. The remaining possible equilibrium values of the surplus from CM activities are thus smaller. Below we analyze...
the implications of these tradeoffs for optimal monetary policy.

3.2 The Suboptimality of Friedman Rule and the Optimality of 
Liquidity Traps

The tradeoffs described in the previous subsection hinge critically on the presence of credit frictions. Absent credit frictions, the surplus from CM financial intermediation and entrepreneurial activities is independent of the real interest rate \( r \), as there is no agency cost arising from costly monitoring. Since the surplus from DM transactions are strictly increasing in asset returns as long as they fall below \( \frac{1}{\beta} \), the Friedman rule, whereby \( \mu = \beta \), is optimal as it maximizes the equilibrium return on currency, and at the same time forces the equilibrium return on interest-bearing assets to take the highest possible value \( (r = \frac{1}{\beta}) \).

If we introduce credit frictions as in Economy E or Economy I, then the surplus from CM activities depends negatively on the real interest rate \( r \). However, if the agents are living in a monetarily undeveloped world where interest-bearing assets are not useful in any DM meetings \( (\rho = 1) \), then the monetary authority is not faced with exploitable tradeoffs between DM and CM surpluses. This is because it has no way to influence the liquidity premium on interest-bearing assets, which is identically zero, and hence no way to influence the equilibrium real interest rate. In this case the Friedman rule is still optimal since it maximizes the surplus from (nonmonitored) DM transactions.

**Proposition 1.** The Friedman rule is optimal if there are no credit frictions, or if the economy is monetarily undeveloped \((\rho = 1)\).

Now consider monetarily developed economies with credit frictions, where interest-bearing assets back deposit claims that can be used in monitored DM meetings \( (\rho < 1) \).
In this case the monetary authority is faced with exploitable tradeoffs as described in the previous subsection. And these tradeoffs require us to re-examine the issue of optimal monetary policy. It turns out that both tradeoffs call for deviations from the Friedman rule. In the first tradeoff, reducing the real interest rate \( r \) from its upper limit \( \frac{1}{\beta} \) has a positive first-order effect on the surplus from CM activities \( W^C(r) \). That is, \( \frac{dW^C}{dr} \bigg|_{r = \frac{1}{\beta}} < 0 \).

Meanwhile, this policy change has no first-order effect on the surplus from monitored DM transactions, \((1 - \rho) [u(x_m(r)) - x_m(r)]\). This is because \( \frac{\partial W^D}{\partial x_m} = (1 - \rho) [u'(x_m) - 1] \) equals zero when \( r = \frac{1}{\beta} \). Likewise, reducing the return on currency \( \frac{1}{\mu} \) from \( \frac{1}{\beta} \) has no first-order effect on the surplus from nonmonitored DM transactions in the second tradeoff, since \( \frac{\partial W^D}{\partial x_n} = \rho [u'(x_n) - 1] \) equals zero when \( \frac{1}{\mu} = \frac{1}{\beta} \). However, the decrease in \( \frac{1}{\mu} \) allows the real interest rate \( r \) to be lowered, thanks to the arbitrage condition. And this has a positive effect on the surplus from CM activities \( W^C(r) \). These considerations lead to the conclusion that the Friedman rule is suboptimal. By setting \( \mu = \beta \), the Friedman rule forces the equilibrium real interest rate \( r \) to take the maximum possible value \( \frac{1}{\beta} \), which is too high from the welfare perspective. Now, for any given \( r < \frac{1}{\beta} \), it is optimal to set \( \frac{1}{\mu} = r \) so as to maximize the surplus from nonmonitored DM transactions, taking as given the surplus from monitored DM transactions and the surplus from CM activities. Hence the optimal monetary policy involves setting the nominal interest rate to zero and thus equating the rates of return on currency and interest-bearing assets.

**Proposition 2.** In both Economy E and Economy I, whenever part of the DM transactions are allowed to use deposit claims backed by interest-bearing assets \((\rho < 1)\), the optimal policy is a non-Friedman-rule liquidity trap, where \( \mu > \beta \) and \( r = \frac{1}{\mu} \).

In the literature, other reasons for the suboptimality of the Friedman rule have been
proposed. For example, Williamson (2012) argues that the costs associated with operating a currency system can make it optimal to tax currency-using transactions by lowering the return on currency from the time discount rate $\frac{1}{\beta}$. In this paper we show that the suboptimality of the Friedman rule is an inherent property of the New Monetarist Framework with banking and credit frictions, without resorting to the costs of operating a currency system. This finding complements the other reasons for the suboptimality of the Friedman rule, such as the one proposed in Williamson (2012).

For the sake of comparing the optimal policy in Economy I to that in Economy E, we consider a normalization of Economy I. Denote the optimal real interest rate of Economy E by $r^*$, and let $\bar{\eta} = G(\gamma^*(r^*))$. Further, let $\bar{\gamma}^l$ satisfy$^4$

$$\bar{\eta} \frac{\bar{\gamma}^l \lambda(R(\bar{\gamma}^l))}{1 - \bar{\gamma}^l \lambda(R(\bar{\gamma}^l))} = \int_{0}^{\gamma^*(r^*)} \frac{\gamma \lambda(R(\gamma))}{1 - \gamma \lambda(R(\gamma))} dG(\gamma).$$

The normalization guarantees that the total loan quantity is the same in Economy I with $(\eta, \gamma^l) = (\bar{\eta}, \bar{\gamma}^l)$ and in Economy E with $r = r^*$, and further that the intensive-margin effect of a change in the real interest rate on the surplus from CM activities is the same in both economies when $r = r^*$. For Economy E where the total loan quantity is elastic with respect to the real interest rate, there is also a positive extensive-margin effect of a decrease in $r$ on $WC$ under Assumption E. This latter effect is absent in Economy I where the total loan quantity is interest-inelastic. Hence the overall effect on $WC$ of a change in the real interest rate is stronger in Economy E. The following proposition obtains.

**Proposition 3.** Let Economy I be parameterized by $(\eta, \gamma^l)$ and let $\rho < 1$. There exists a neighborhood for $(\eta, \gamma^l)$ around $(\bar{\eta}, \bar{\gamma}^l)$ such that the optimal real interest rate

$^4$Such a $\bar{\gamma}^l$ exists and is unique under the increasing hazard assumption for the distribution of $\omega$.
(resp. inflation rate) is lower (resp. higher) in Economy E than in any Economy I with \((\eta, \gamma')\) in that neighborhood.

3.3 Policy Implementation

Suppose the monetary authority has direct control over \((\mu, \zeta)\), where \(\mu\) is the gross growth rate of total government liabilities and \(\zeta\) is the ratio of the supply of government bonds to currency.\(^5\) We treat fiscal policy as passive and assume that the path of lump-sum taxes changes passively to support given paths for nominal government liabilities. The monetary authority may choose to implement particular types of equilibria with specific configurations of asset returns, subject to the equilibrium restriction \(\frac{1}{\mu} \leq r \leq \frac{1}{\beta}\). Let \(\Theta_F, \Theta_P, \Theta_S, \) and \(\Theta_L\) denote the sets of \((\mu, \zeta)\) that implement Friedman-rule equilibria \((\frac{1}{\mu} = r = \frac{1}{\beta})\), equilibria with plentiful interest-bearing assets \((\frac{1}{\mu} < r = \frac{1}{\beta})\), equilibria with scarce interest-bearing assets \((\frac{1}{\mu} < r < \frac{1}{\beta})\), and liquidity-trap equilibria \((\frac{1}{\mu} = r < \frac{1}{\beta})\), respectively. Then,

\[
\Theta_F = \{ (\mu, \zeta) : \mu = \beta \},
\]

\[
\Theta_P = \left\{ (\mu, \zeta) : \mu > \beta, \frac{1 - \rho}{\rho} - \frac{L(\frac{1}{\mu})}{m(\mu)} < \zeta < \frac{(1 - \rho)x^*}{m(\mu)} - \frac{L(\frac{1}{\beta})}{m(\mu)} \right\},
\]

\[
\Theta_S = \left\{ (\mu, \zeta) : \mu > \beta, \frac{1 - \rho}{\rho} - \frac{L(\frac{1}{\mu})}{m(\mu)} < \zeta < \frac{(1 - \rho)x^*}{m(\mu)} - \frac{L(\frac{1}{\beta})}{m(\mu)} \right\},
\]

\[
\Theta_L = \left\{ (\mu, \zeta) : \mu > \beta, -1 - \frac{L(\frac{1}{\mu})}{m(\mu)} < \zeta \leq \frac{1 - \rho}{\rho} - \frac{L(\frac{1}{\beta})}{m(\mu)} \right\},
\]

where \(m(\mu)\) solves

\[
\frac{\beta}{\mu} u' \left( \frac{\beta m}{\mu \rho} \right) = 1.
\]

\(^5\)Williamson (2012) specifies the fraction of currency in total government liabilities, denoted by \(\delta\). Obviously, \(\zeta \equiv \frac{1}{\beta} - 1\). For expositional purpose, we find it more convenient to work with \(\zeta\). But the two approaches are entirely equivalent.
In Economy I, $L \left( \frac{1}{\beta} \right) = L \left( \frac{1}{\mu} \right) = \eta$, while in Economy E, $L \left( \frac{1}{\beta} \right) < L \left( \frac{1}{\mu} \right)$ whenever $\mu > \beta$.

Equilibria with plentiful interest-bearing assets are important cases to look at because they are generally optimal in settings with no credit frictions, and the optimality of Friedman rule obtains as limit cases of such settings. Consider the costs of operating a currency system as in Williamson (2012). With these costs the Friedman rule is suboptimal, and the optimal type of equilibria feature plentiful interest-bearing assets. This is because it is optimal to tax those DM transactions that use currency, so that the welfare-maximizing return on currency is less than $\frac{1}{\beta}$, and nonmonitored depositors consume less than the first-best amount $x^{*}$ in the DMs. But for monitored depositors it is still optimal to set $r = \frac{1}{\beta}$ and let them consume $x^{*}$. As the costs of operating a currency system go to zero, the Friedman rule becomes optimal in the limit.

In order to implement an equilibrium with plentiful interest-bearing assets, the monetary authority needs to set $\mu > \beta$ and $\zeta \geq \frac{(1-\rho)x^{*}}{m(\mu)} - \frac{L \left( \frac{1}{\beta} \right)}{m(\mu)}$, where $m(\mu)$ solves (5). The intuition is as follows. With $\frac{1}{\mu} < r = \frac{1}{\beta}$, monitored depositors consume the efficient amount $x^{*}$ and only use deposit claims backed by interest-bearing assets in DM meetings, while nonmonitored depositors demand currency of the amount $m(\mu)$. Absent private liquidity, the supply of government bonds relative to currency must be at least $\frac{(1-\rho)x^{*}}{m(\mu)}$. But there are privately supplied interest-bearing assets in the amount $L \left( \frac{1}{\beta} \right)$. This reduces the needed supply of government bonds relative to currency by $\frac{L \left( \frac{1}{\beta} \right)}{m(\mu)}$.

As we have shown earlier, the introduction of credit frictions in a monetarily developed economic system breaks the optimality of the Friedman rule without resorting to the costs of operating a currency system. Moreover, for any given deviation from the Fried-
man rule \((\mu > \beta)\), credit frictions also render equilibria with plentiful interest-bearing assets suboptimal. The optimal type of equilibria are instead liquidity-trap equilibria. To implement a liquidity-trap equilibrium, the monetary authority needs to set \(\mu > \beta\) and \(\zeta \in \left(-1 - \frac{L(\frac{1}{\mu})}{m(\mu)}, \frac{1-e}{\rho} - \frac{L(\frac{1}{\mu})}{m(\mu)}\right]\). In this equilibrium currency and interest-bearing assets are perfect substitutes in DM meetings, and the amount of currency must be at least \(\rho\) times the total amount of assets so as to satisfy the needs arising from nonmonitored transactions. Hence without private liquidity, the ratio of government bonds to currency must be less than or equal to \(\frac{1-e}{\rho}\), and must be greater than \(-1\) in order to support positive consumption in the DMs. This ratio is reduced by \(\frac{L(\frac{1}{\mu})}{m(\mu)}\) with privately supplied interest-bearing assets in the amount \(L(\frac{1}{\mu})\).

The critical difference between an equilibrium with plentiful interest-bearing assets and a liquidity-trap equilibrium is that the liquidity premium on interest-bearing assets is zero in the former but is positive in the latter. The intuition for why it is not optimal to eliminate the liquidity premium when there are credit frictions is as follows. Although public and private interest-bearing assets are perfect substitutes from the perspective of monitored DM transactions, they are not if CM activities are taken into account. The creation of private interest-bearing assets facilitates the productive intertemporal transformation of goods by entrepreneurs. This function is not shared by public interest-bearing assets, and will be hurt by high real interest rates. Whether the total loan quantity is interest-elastic or not, high real interest rates lead to large deadweight losses from frequent monitoring. When the loan quantity is interest-elastic, as in Economy E, setting the real interest rate high reduces the supply of private liquidity. Although the government can offset this reduction by increasing the supply of public liquidity in order
to maintain the desired level of DM consumption, it cannot remedy the harm done to CM entrepreneurial activities. In other words, privately supplied interest-bearing assets are not truly plentiful in an equilibrium with “plentiful” interest-bearing assets.

In general, moving from an equilibrium with plentiful interest-bearing assets to a liquidity trap equilibrium requires monetary expansion in order to lower the real interest rate. The monetary expansion takes the form of reducing the supply of government bonds relative to currency $\zeta$, achievable by a one-time open market purchase. Comparing Economy E with Economy I, the elasticity of private liquidity makes the required monetary expansion greater in Economy E.

**Proposition 4.** The deviation from an equilibrium with plentiful interest-bearing assets to a liquidity-trap equilibrium with the same inflation rate requires greater monetary expansion in Economy E than in Economy I. That is, $\inf_{\Theta_P} \zeta - \sup_{\Theta_L} \zeta$ is larger in Economy E for any given $\mu > \beta$.

4 Conclusions

In this paper we have examined optimal monetary policy in a New Monetarist Framework with banking, private liquidity, and credit frictions. The presence of credit frictions and the existence of decentralized exchanges that are allowed to use deposit claims backed by interest-bearing assets are necessary conditions for the optimality of a positive liquidity premium on interest-bearing assets. With buyers making take-it-or-leave-it offers in the DMs, it is shown that the Friedman rule is suboptimal and that the optimal policy implements a liquidity-trap equilibrium. If buyers do not have all the bargaining powers in DM meetings, then deviations from the Friedman rule do have first-order effects on the
surplus from DM transactions, as in Lagos and Wright (2005). This considertation may or may not lead to modifications of the results in the paper, depending on whether these effects are strong enough. But the main insight of our analysis, namely, that banking, liquidity, and credit frictions introduce constraints and tradeoffs that monetary policy makers are confronted with when choosing the optimal configuration of asset returns, remains intact and will prove useful in future research in the area.

References


